Beam Loading

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CERN

RF for Accelerators

26 June 2023

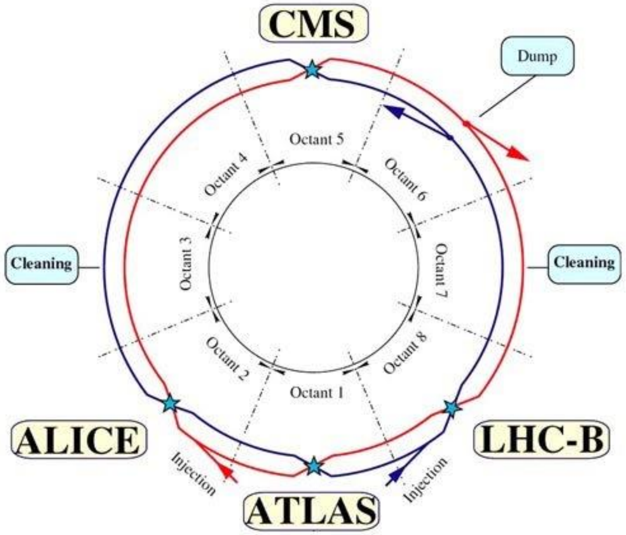
Outline

- Introduction
- RF cavity parameters
  - Shunt impedance, beam loading, power coupling
- Fundamental theorem of beam loading
- Passage of a bunches through a cavity
  - Single passage or bunches with large spacing
  - Multiple bunch passages
- Steady state beam loading and partial filling
  - Few bunches with large spacing
- Summary

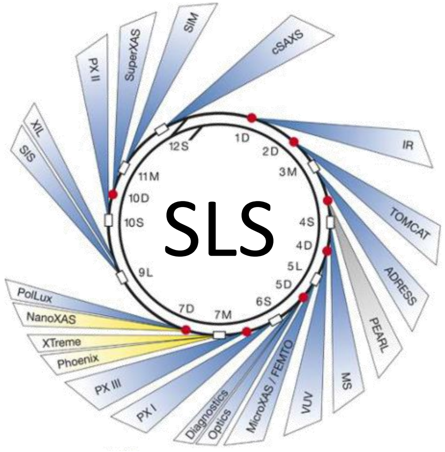
Introduction

What do these devices in common?

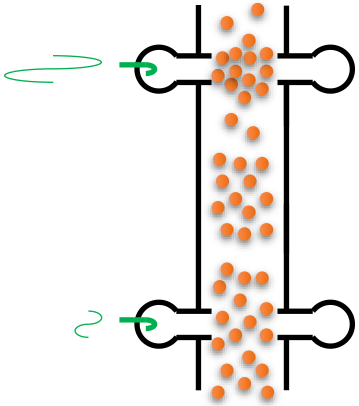
Hadron collider



Electron storage ring



Klystron amplifier

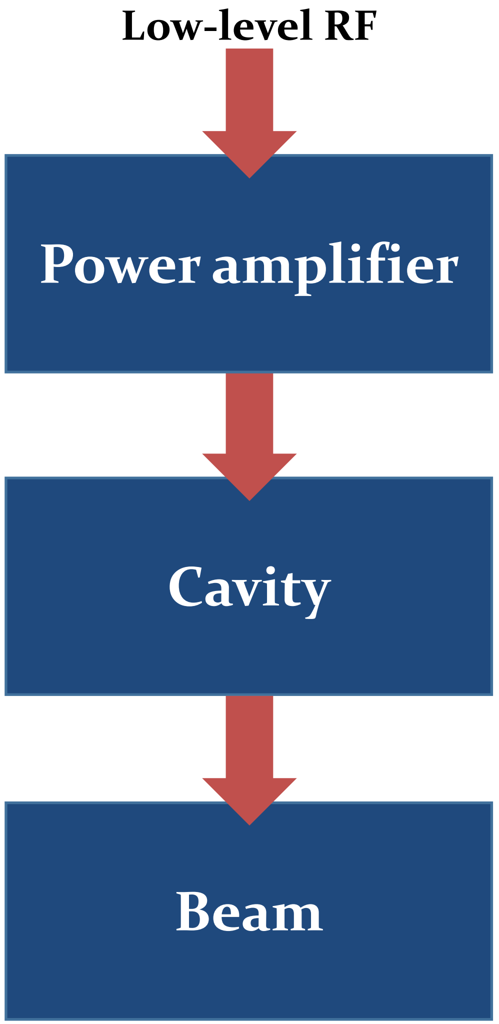


Microwave oven

→ **They all suffer from or make use of beam loading**

Introduction

- The radiofrequency (RF) system should provide
  - → Energy to the beam
  - → Longitudinal focusing
- Intended energy flow usually from cavity to beam
- But beam also likes to influence the field in the cavity



Low-level RF

Power amplifier

Cavity

Beam

Introduction

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  - → Energy to the beam
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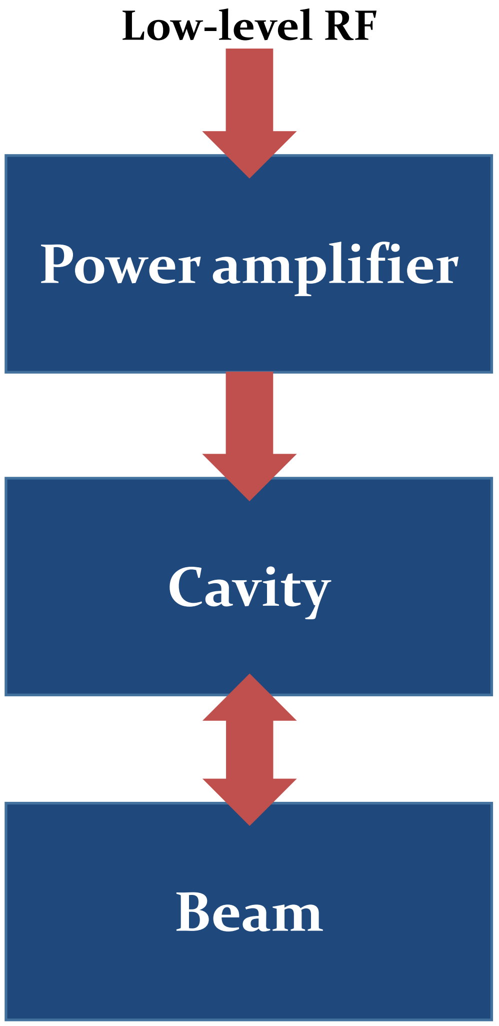
→Beam loading


Low-level RF

Power amplifier

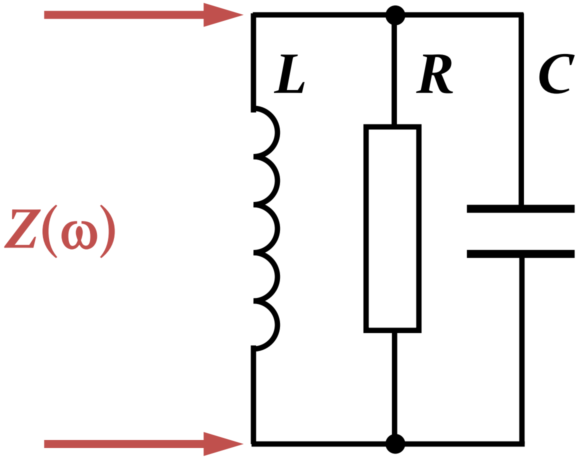
Cavity

Beam

RF cavity

Cavity parameters

- **The resonance of a cavity can be understood as simple parallel resonant circuit described by *R*, *L*, *C***



$$\frac{1}{Z(\omega)} = \frac{1}{R} + \frac{1}{i\omega L} + i\omega C$$

with $$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$E = \frac{1}{2}CV^2 = \frac{1}{2}LI^2$$

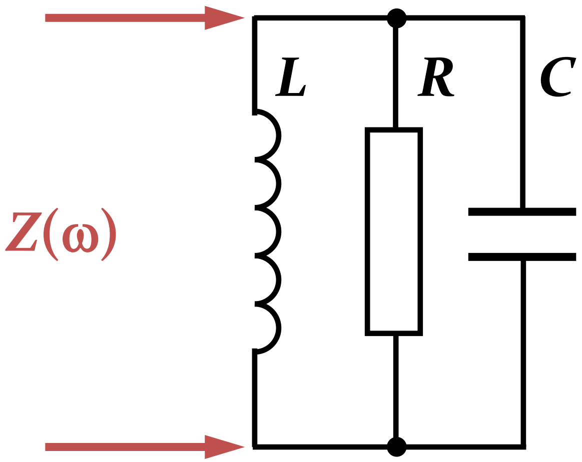
$$Q = \omega_0 \frac{\text{Stored energy}}{\text{Average power loss}} = \frac{\omega_0 E}{P}$$

$$P = \frac{1}{2}\frac{U^2}{R} = \frac{1}{2}I^2 R$$

$$Q = \omega_0 RC = \frac{R}{\omega_0 L}$$

Cavity parameters

- **The resonance of a cavity can be understood as simple parallel resonant circuit described by *R, L, C***



$$\frac{1}{Z(\omega)} = \frac{1}{R} + \frac{1}{i\omega L} + i\omega C$$

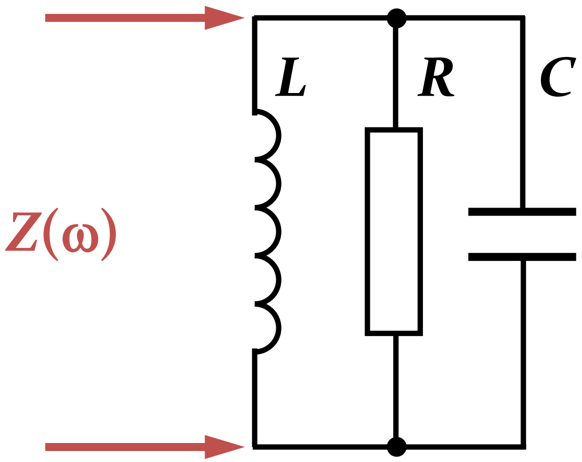
with $$\omega_0 = \frac{1}{\sqrt{LC}}$$

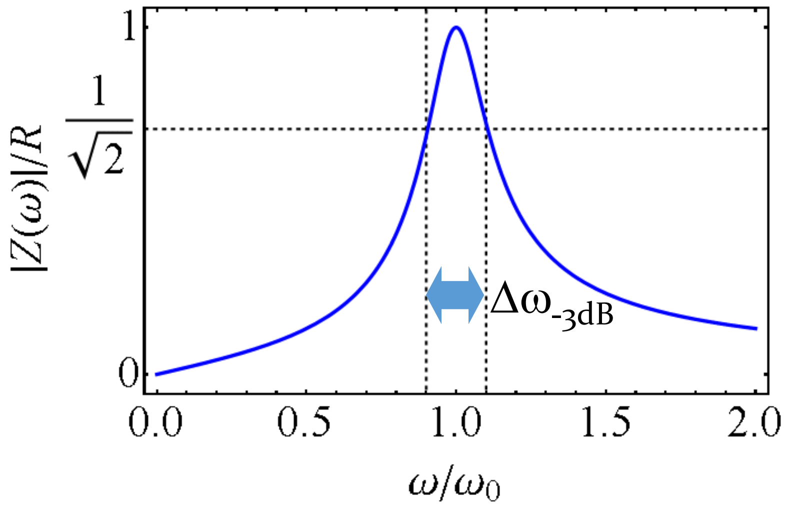
$$Q = \omega_0 RC = \frac{R}{\omega_0 L} \qquad Z(\omega) = \frac{R}{1 + iQ\left(\frac{\omega^2 - \omega_0^2}{\omega\omega_0}\right)} \simeq \frac{R}{1 + 2iQ\frac{\Delta\omega}{\omega_0}}$$

→ **Resonant circuit can also be described by *R, R/Q,* ω_o or any other set of three parameters**

Cavity parameters

- The resonance of a cavity can be understood as simple parallel resonant circuit described by *R, L, C*





$$Q = \omega_0 RC = \frac{R}{\omega_0 L} \qquad Z(\omega) = \frac{R}{1 + iQ\left(\frac{\omega^2 - \omega_0^2}{\omega\omega_0}\right)} \simeq \frac{R}{1 + 2iQ\frac{\Delta\omega}{\omega_0}}$$

→ Resonant circuit can also be described by *R*, *R/Q*, ω_o or any other set of three parameters

Cavity parameters

- Most common choice by cavity designers ω_o, R, R/Q – why?

- Resonance frequency, ω_o
  → Exactly defined for given application, e.g. $h\omega_{\mathrm{rev}}$

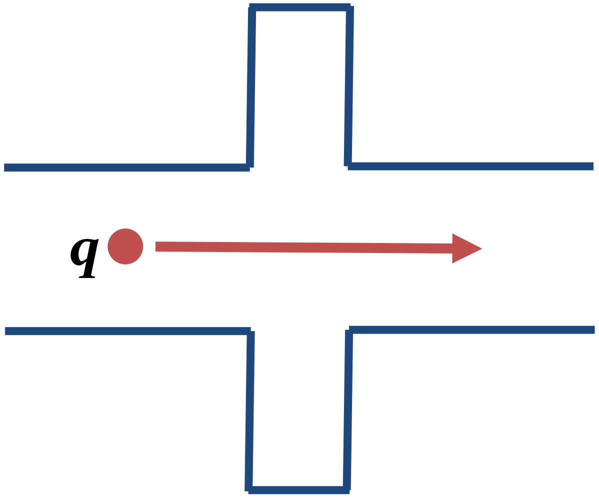
- Shunt impedance, R
  → Power required to produce a given voltage without beam

- “R-upon-Q”, R/Q
  → Defined only by the cavity geometry
  → Criterion to optimize a geometry
  → Detuning with beam proportional to R/Q

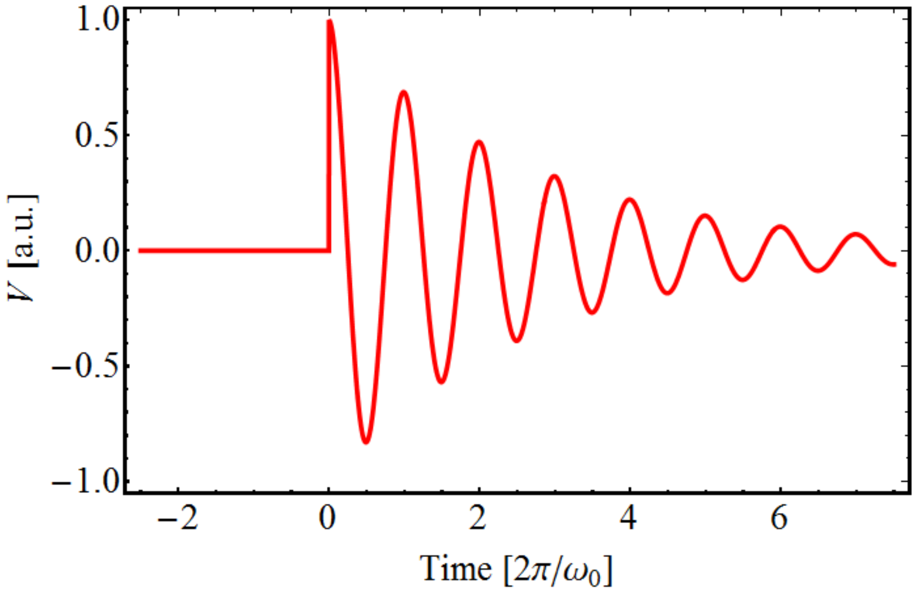
Why *R*/*Q*?

→ **Charged particle experiences cavity gap as capacitor**

Cavity



Beam induced voltage



$$q = V_{b0}C$$

$$Q = \omega_0 RC \quad \rightarrow \quad \frac{1}{C} = \left(\frac{R}{Q}\right)\omega_0$$

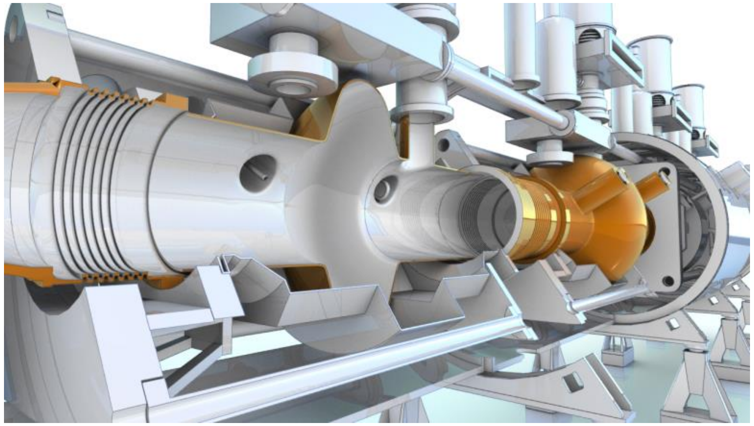
$$V_{b0} = \frac{q}{C} \propto \frac{R}{Q}$$

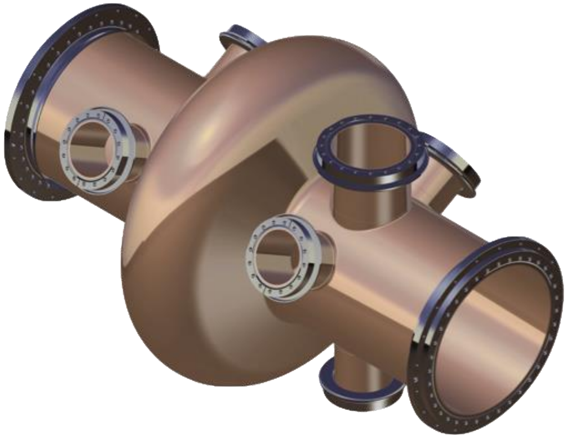
→ **Cavity geometry with small *R*/*Q* to reduce beam loading**

Example: 400 MHz cavities in LHC

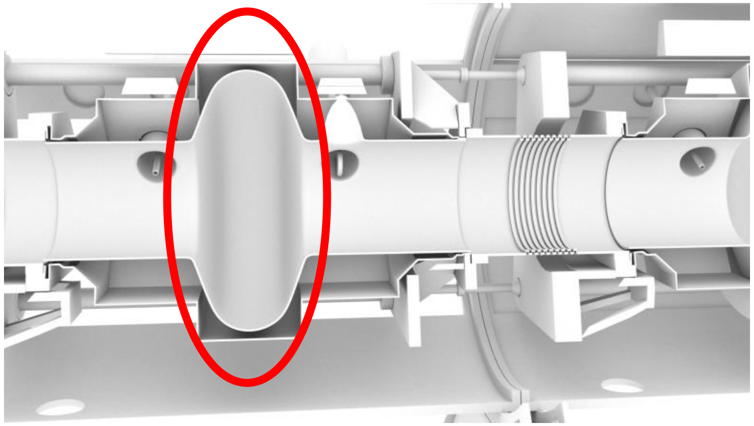
→ Reduce beam loading in RF cavities

→ **Shunt impedance, *R*, low for small *R*/*Q* with normal conducting cavities → superconducting cavities in LHC**





Bell shape: *R*/*Q* ~ 44 Ω, 400 MHz

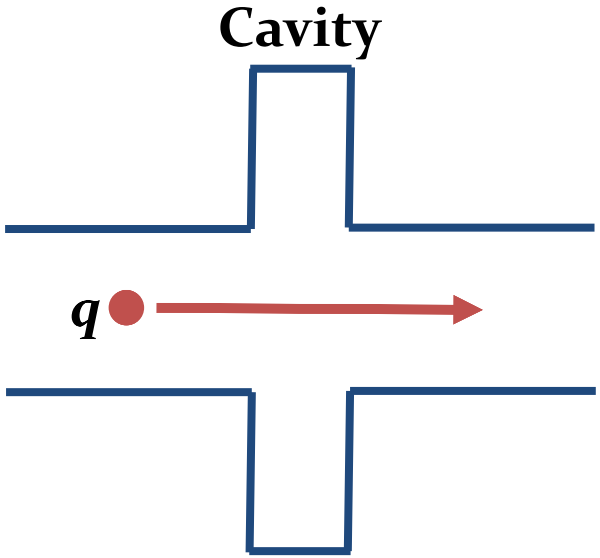


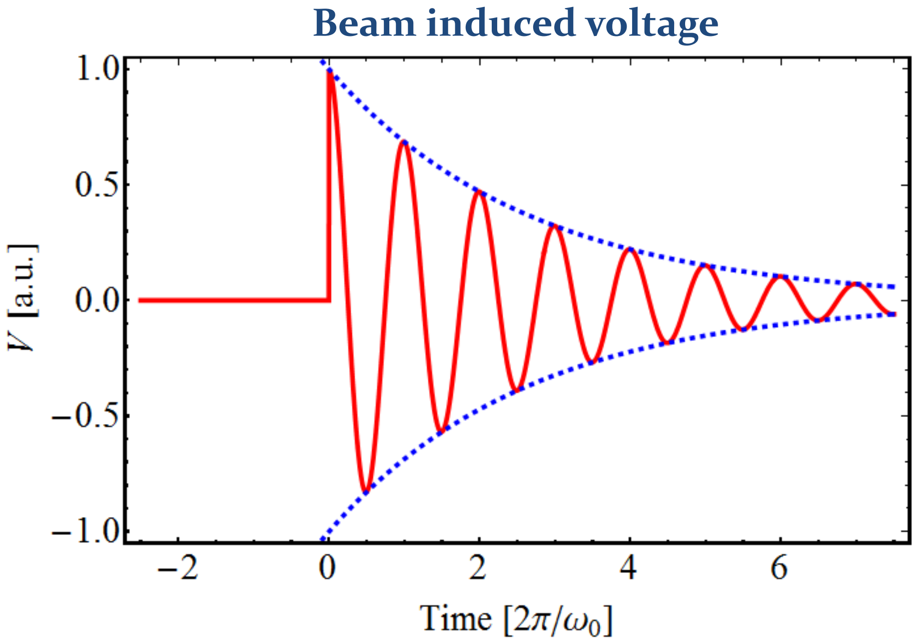
→ **2×8 cavities, 5.3 MV/m**

$$\frac{1}{Q_\mathrm{L}} = \cancelto{\sim 0}{\frac{1}{Q_0}} + \frac{1}{Q_\mathrm{ext}}$$

Field decay in cavity

→ **After passage of charge: energy and fields decay exponentially**





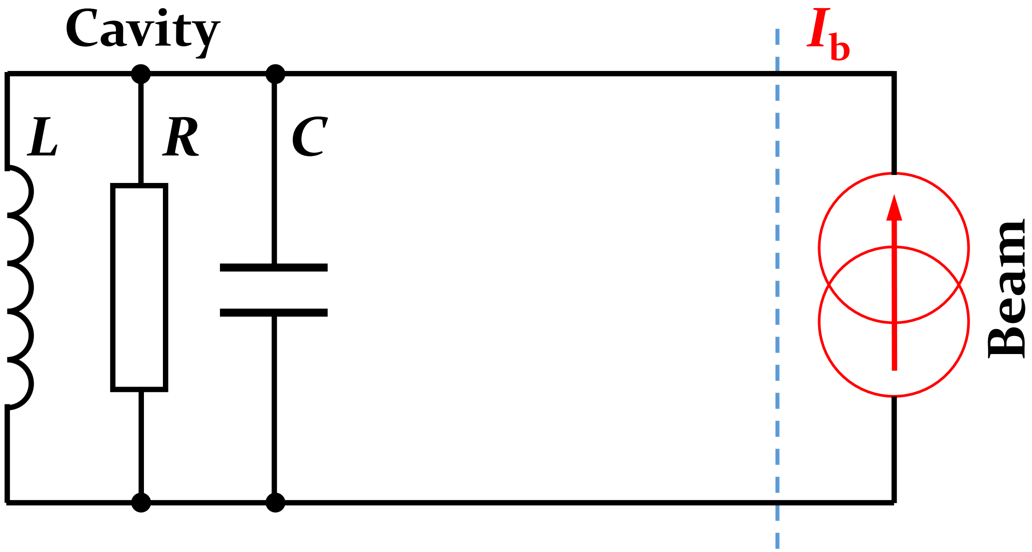
→ **Energy:** $W(t) = W_0 e^{-\frac{\omega_0}{Q}t}$

→ **Voltage:** $V(t) = V_{\mathrm{b}0} e^{-\frac{\omega_0}{2Q}t} = V_{\mathrm{b}0} e^{-t/T_{\mathrm{f}}}$ **and**

→ **Filling time:** $T_{\mathrm{f}} = \dfrac{2Q}{\omega_0}$

Coupling power into a cavity

- **Connection of cavity to power amplifier**
  - → **Capacitive:** Capacitor coupling electrically to the gap
  - → **Inductive:** Coupling loop in region of large magnetic field



Coupling power into a cavity

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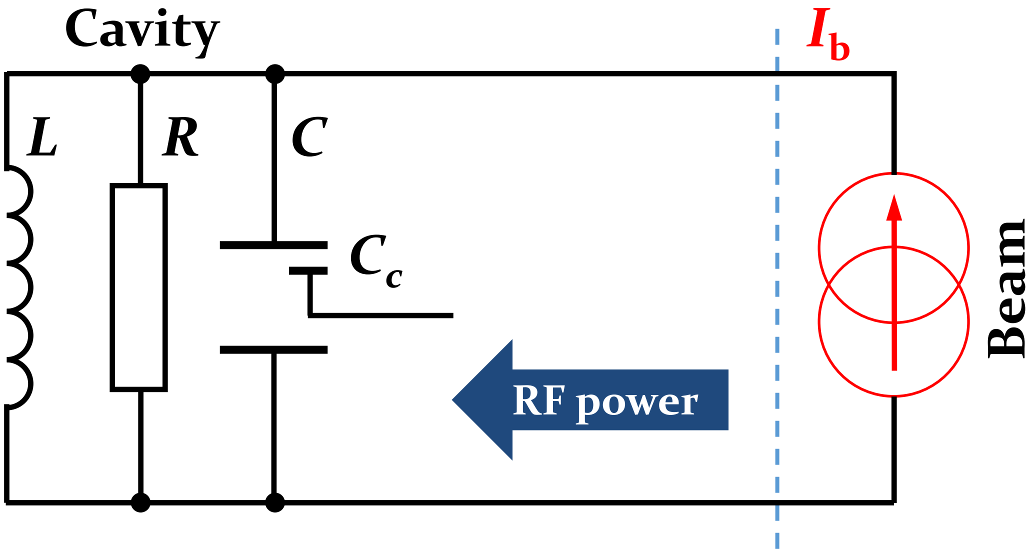

Capacitive coupler of CERN PS 40 MHz

Coupling power into a cavity

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  - → **Capacitive:** Capacitor coupling electrically to the gap
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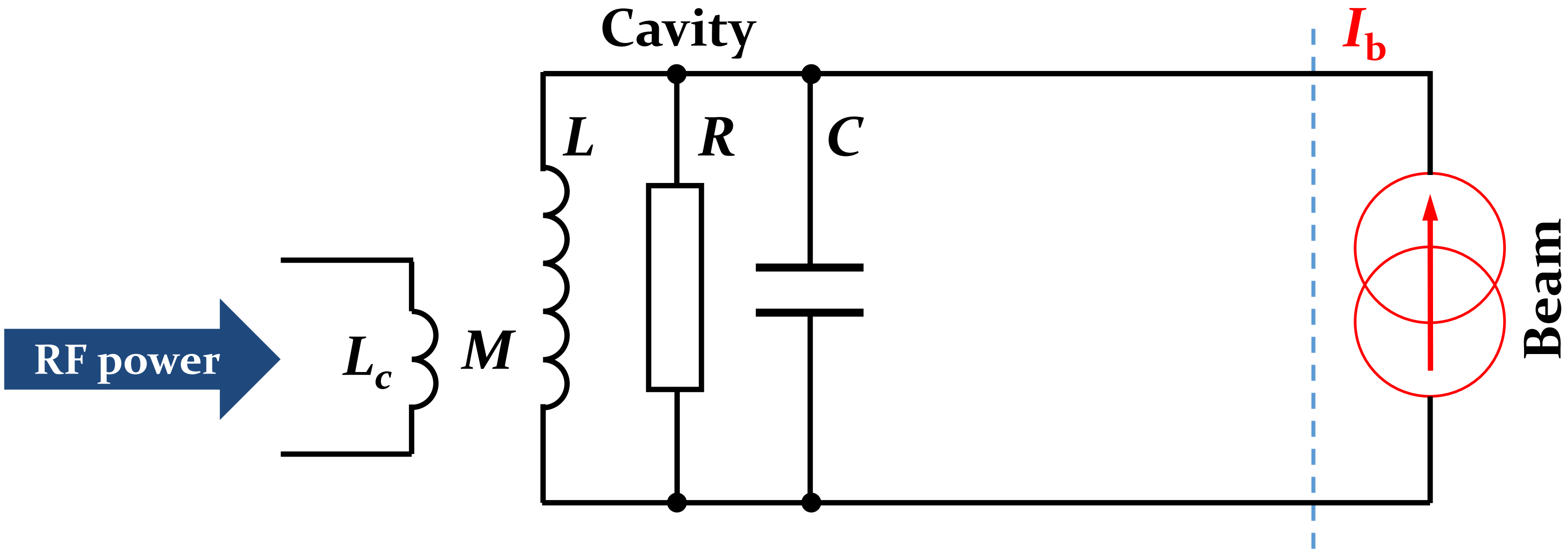
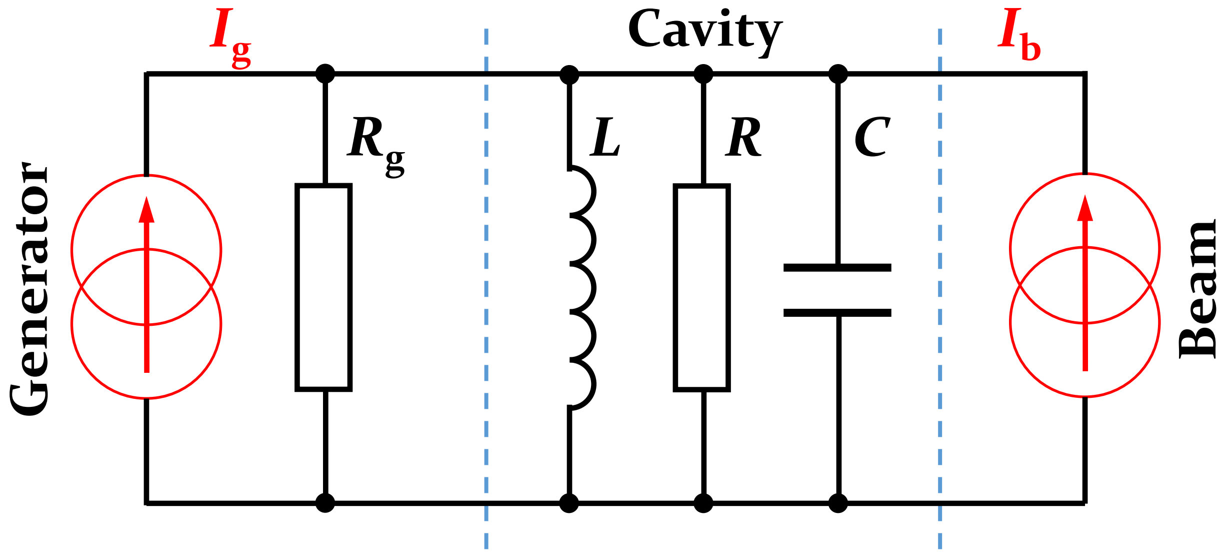


Main coupler PSI cyclotron

L. Stingelin

Coupling power into a cavity

- → **Output impedance loads the resonant circuit:** $R_g \,||\, R$
- → **Reduction of quality factor:** $Q_o \rightarrow Q_L$
- → **Coupling coefficient, β, defines coupling ratio**



$$\boxed{\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{ext}}} = \frac{1}{Q_0} + \frac{\beta}{Q_0}$$

$$\frac{1}{R_L} = \frac{1}{R} + \frac{1}{R_g} = \frac{1}{R} + \frac{\beta}{R}$$

$$Q_L = Q_0 \frac{1}{1+\beta} \qquad Q_{ext} = \frac{Q_0}{\beta}$$

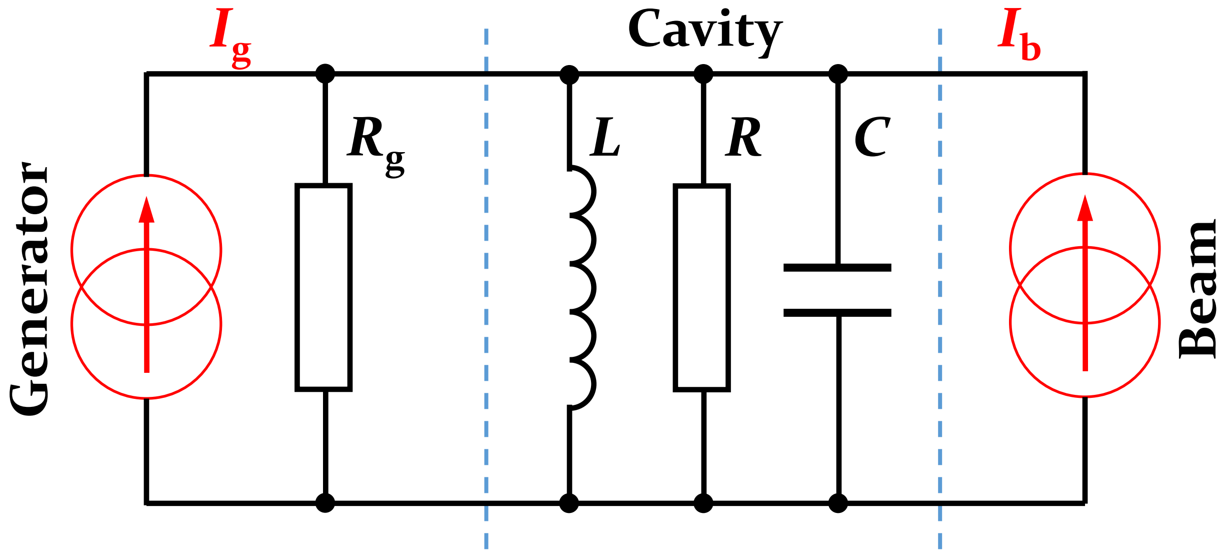
$$R_L = R \frac{1}{1+\beta} \qquad R_g = \frac{R}{\beta}$$

Coupling power into a cavity

→ Output impedance loads the resonant circuit: $R_g \,||\, R$

→ Reduction of quality factor: $Q_o \rightarrow Q_L$

→ Coupling coefficient, β, defines coupling ratio



1. Generator output impedance is not a physical resistor
   → Generator does not experience own output impedance
2. Beam experiences output impedance of generator as resistor
   → $R_g \,||\, R$ relevant for beam loading

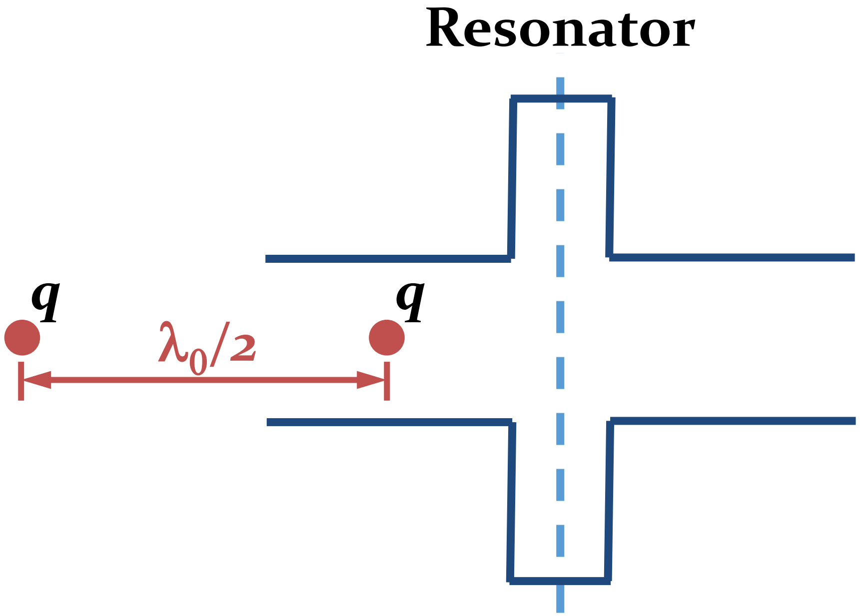
Fundamental theorem of beam loading

Initially empty cavity

Which fraction does a charge experience of its induced voltage?

- **Equal charges passing through cavity at distance** $\lambda_o/2 = \pi c_o/\omega_o$

→ **Principles:** energy conservation **and** superposition

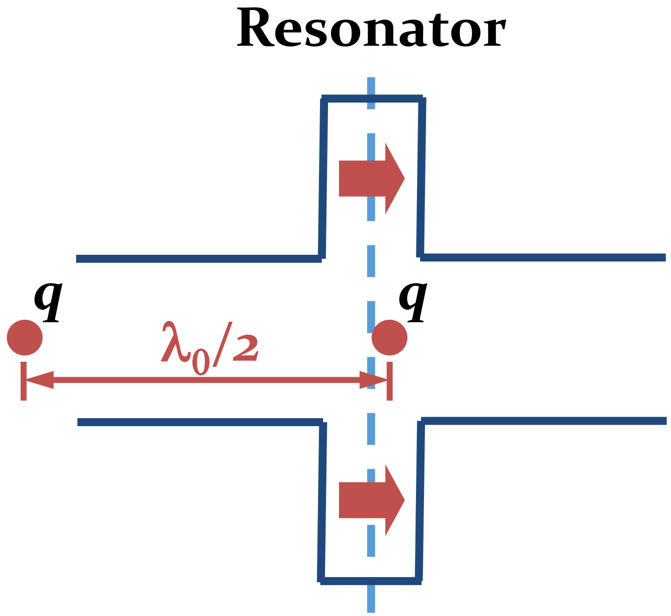


→ $W = 0$ **and** $V_c = 0$

After passage of first charge

- 1^{st} charge passes through the cavity and induces voltage
- Fraction, *r* describes part of induced voltage affecting itself:

$$\Delta U_1 = r \cdot qV_{\mathrm{b1}}$$

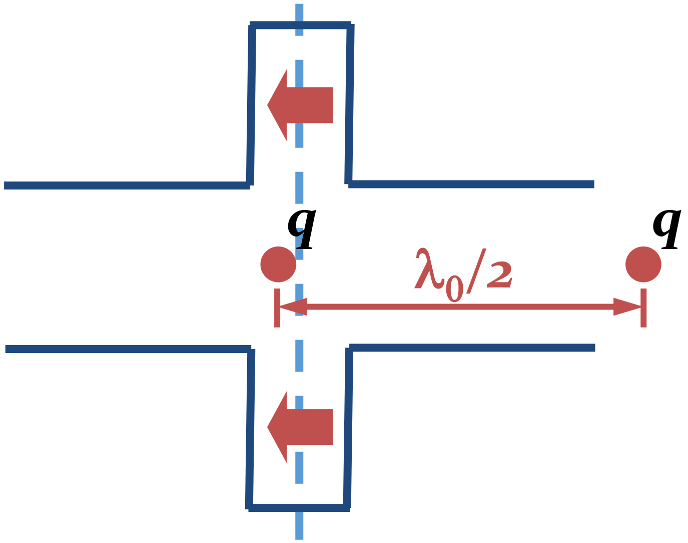


$$\rightarrow \quad W = \Delta U_1 \quad \text{and} \quad V_c = V_{\mathrm{b1}}$$

Before passage of 2nd charge

- 2nd charge passes through the cavity
- Affected by induced field of 1st charge

Resonator

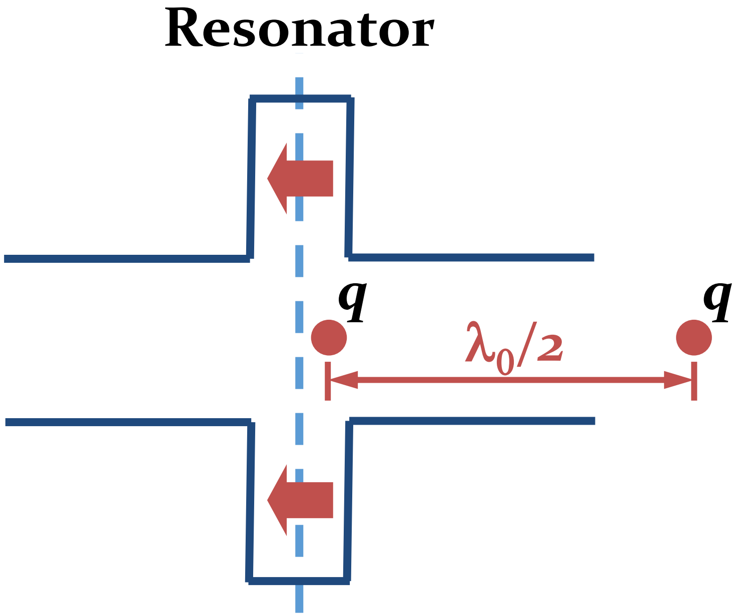


→ $W = \Delta U_1$ **and** $V_c = -V_{\mathrm{b}1}$

Passage of 2nd charge

- 2nd charge passes through the cavity
- Affected by induced field of 1st charge and its own induced

$$\Delta U_2 = -qV_{\mathrm{b1}} + r \cdot qV_{\mathrm{b2}}$$

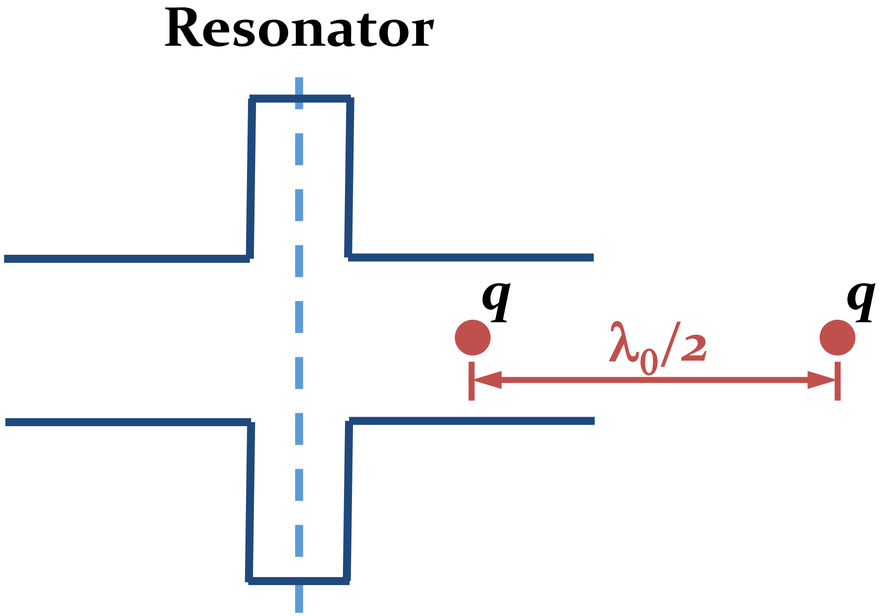


→ $W = \Delta U_1$ **and** $V_c = -V_{\mathrm{b1}}$

After passage of first bunch

- After passage of 2nd charge through the cavity

→ Takes the same energy as brought into cavity by 1st charge



→ $W = 0$ **and** $V_c = -V_{b1} + V_{b2} = 0$

Ratio of induced field

→ **Total energy brought in and taken out of cavity must be zero**

$$\begin{aligned}\Delta U_1 + \Delta U_2 &= 0\\ r \cdot qV_{\mathrm{b1}} - qV_{\mathrm{b1}} + r \cdot qV_{\mathrm{b2}} &= 0\\ r\left(V_{\mathrm{b1}} + V_{\mathrm{b2}}\right) &= V_{\mathrm{b1}}\\ 2r \cdot V_{\mathrm{b0}} &= V_{\mathrm{b0}}\\ \rightarrow r &= \frac{1}{2}\end{aligned}$$

→ **Fundamental theorem of beam loading:**

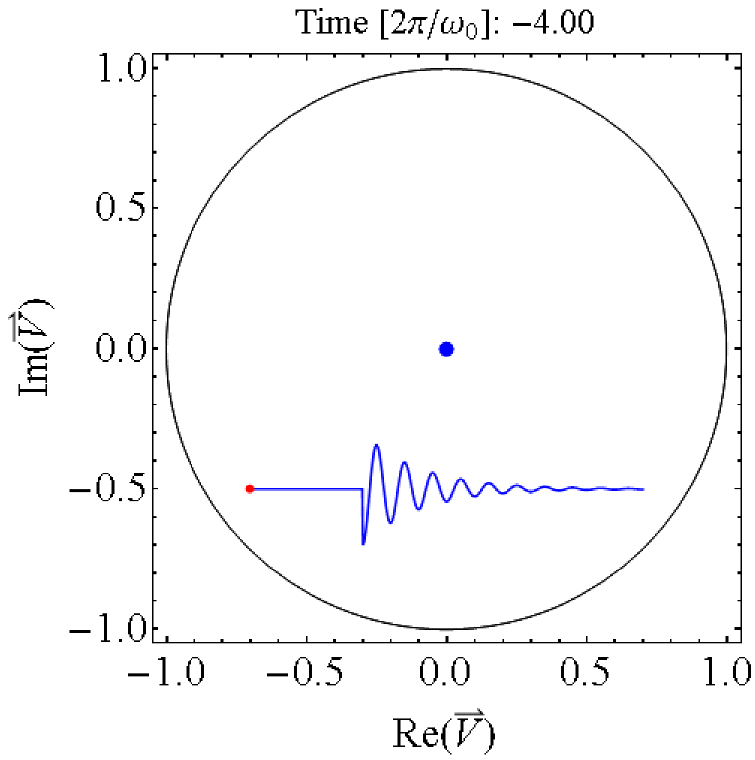
Charge passing through a resonator sees ½ of its induced voltage: $V_b = ½\, V_{bo}$

Single passage through a cavity

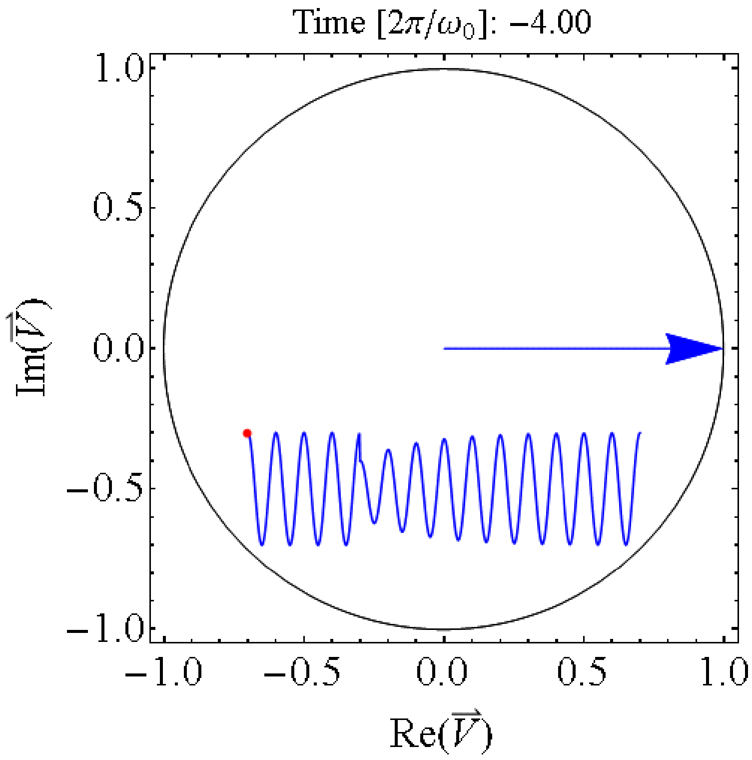
Vector representation

- **Passing charge induces voltage**
- **Voltage vector rotates with resonance frequency of cavity**

Cavity without external drive



Cavity driven by external source

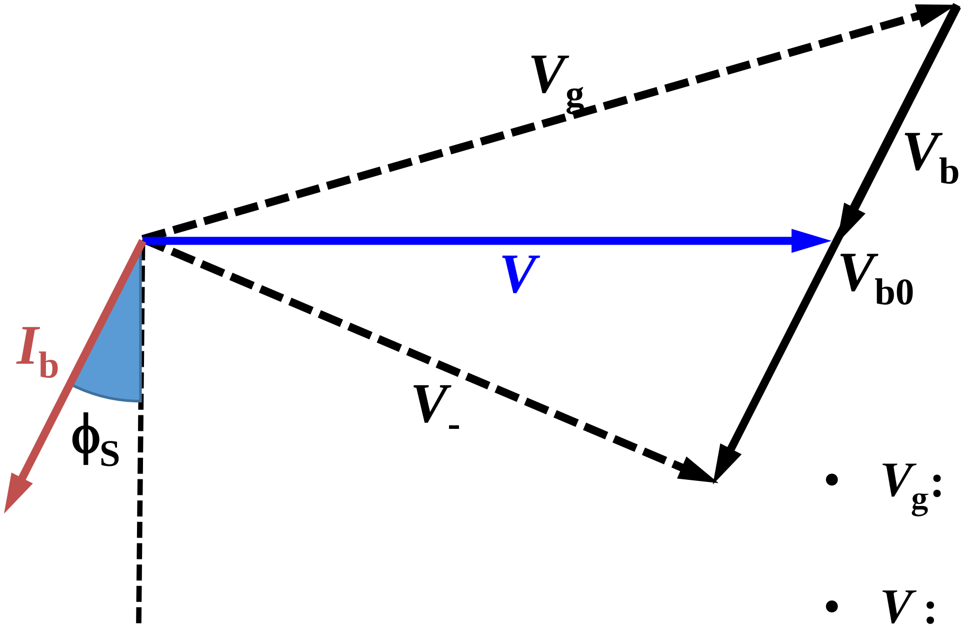


→ **Vector rotation with ω_o not relevant**

→ **Need cavity voltage at arrival of next charge**

Single passage

- Vector diagram at the instant of the bunch passage:



- V_g: Generator driven voltage before bunch passage
- V_-: Voltage after bunch passage
- V: Net voltage seen by beam

→ **Vector sum:** $$\vec{V} = \vec{V}_g + \vec{V}_\mathrm{b} = \vec{V}_\mathrm{g} + \frac{1}{2}\vec{V}_\mathrm{b0}$$

→ **Induced voltage changes cavity phase: detuning**

→ **De-phase generator to obtain expected net voltage**

Multiple passages through a cavity

Multiple passage of bunches

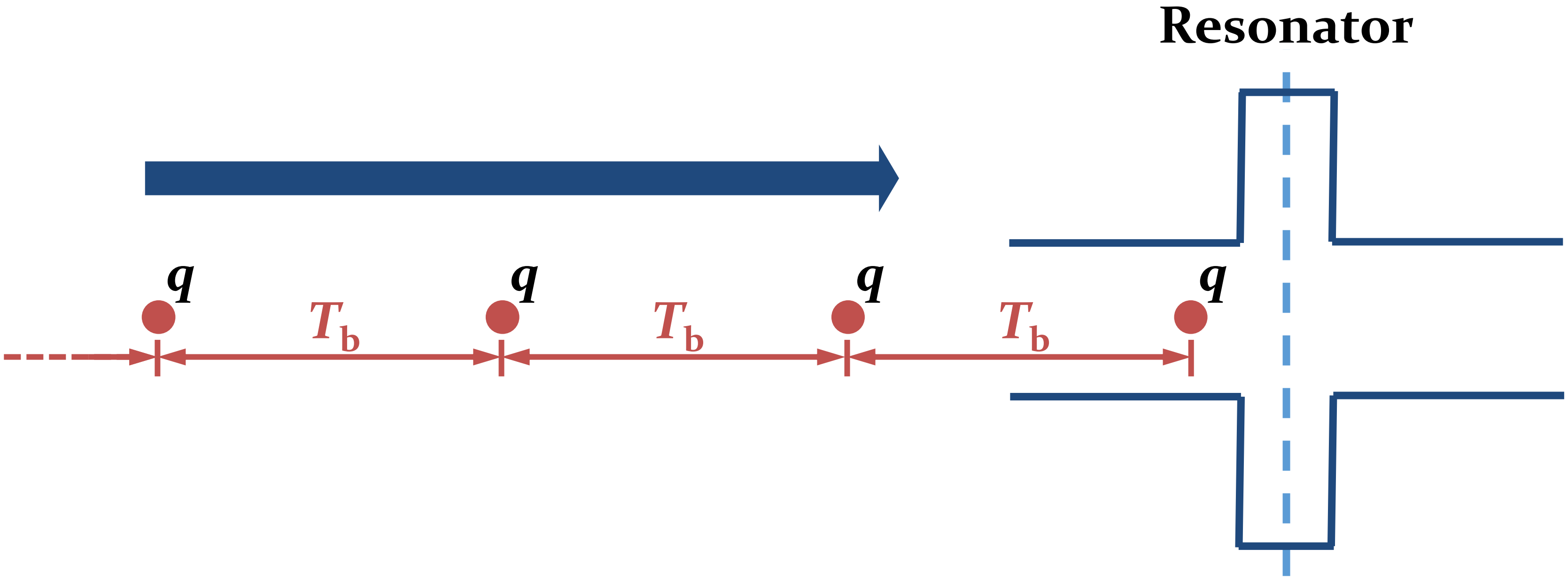
- Resonator excited by chain of charges or particle bunches

1. Fields in resonator decay from one charge to the next

   → Single passage case

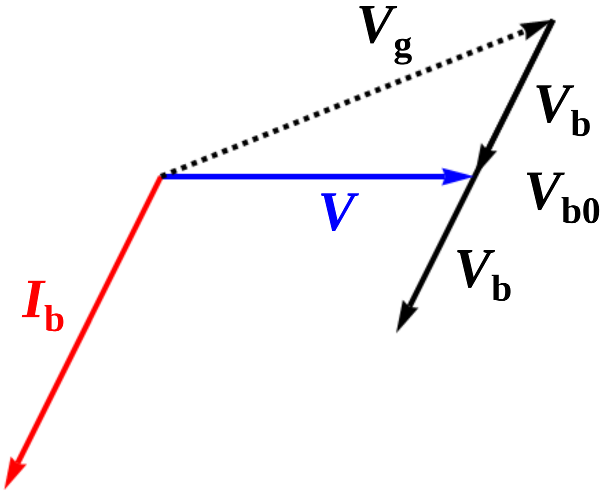
2. Field from previous still present

   → Accumulation of induced voltages



Multiple passages

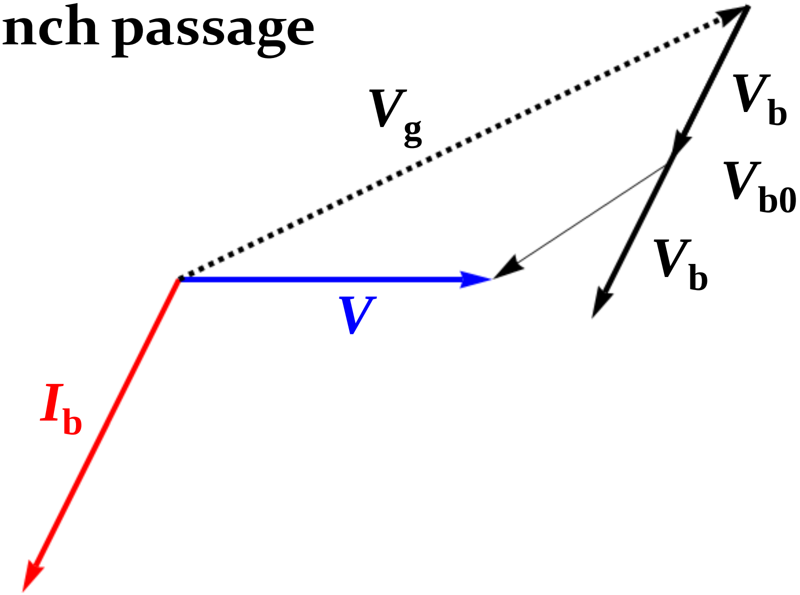
- Arrange generator phase and voltage for real net voltage
- After 1st bunch passage



$$\rightarrow \quad \vec{V} = \vec{V}_g + \frac{1}{2}\vec{V}_{b0}$$

Multiple passages

- **Arrange generator phase and voltage for real net voltage**
- **After 2nd bunch passage**



→ $$\vec{V} = \vec{V}_g + \frac{1}{2}\vec{V}_{b0} + \vec{V}_{b0} e^{-\delta} e^{i\Psi}$$

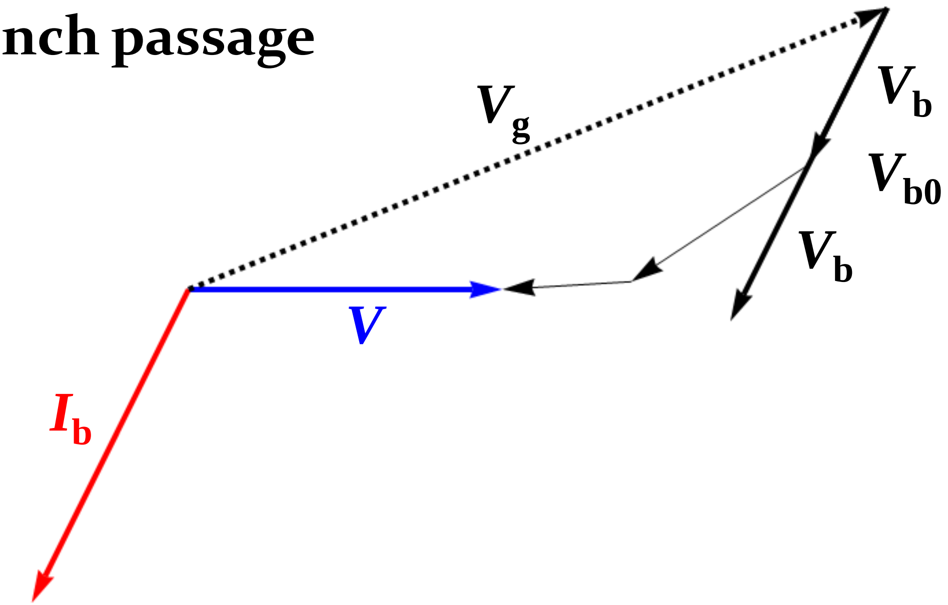
→ **Induced voltage of 1st passage decayed:** $e^{-\delta}$ **with** $\delta = \frac{T_b}{T_f}$

→ **Phase advance between two bunches:** $e^{-i\Psi}$ **with**

$$\Psi = \omega_0 T_b - 2\pi h_b$$

Multiple passages

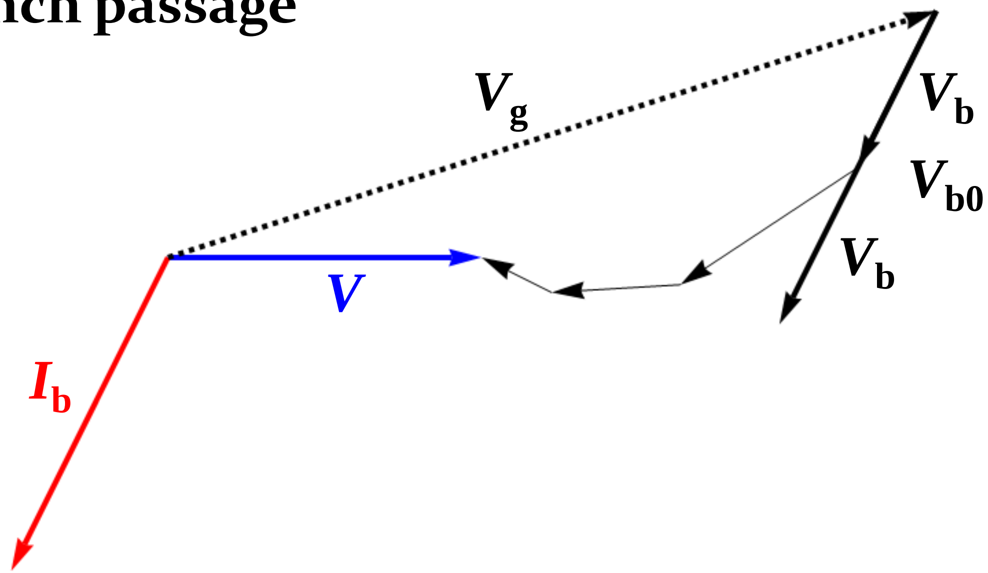
- **Arrange generator phase and voltage for real net voltage**
- **After 3$^{\text{rd}}$ bunch passage**



$$\rightarrow \quad \vec{V} = \vec{V}_{\rm g} + \frac{1}{2}\vec{V}_{\rm b0} + \vec{V}_{\rm b0}e^{-\delta}e^{i\Psi} + \vec{V}_{\rm b0}e^{-2\delta}e^{2i\Psi}$$

Multiple passages

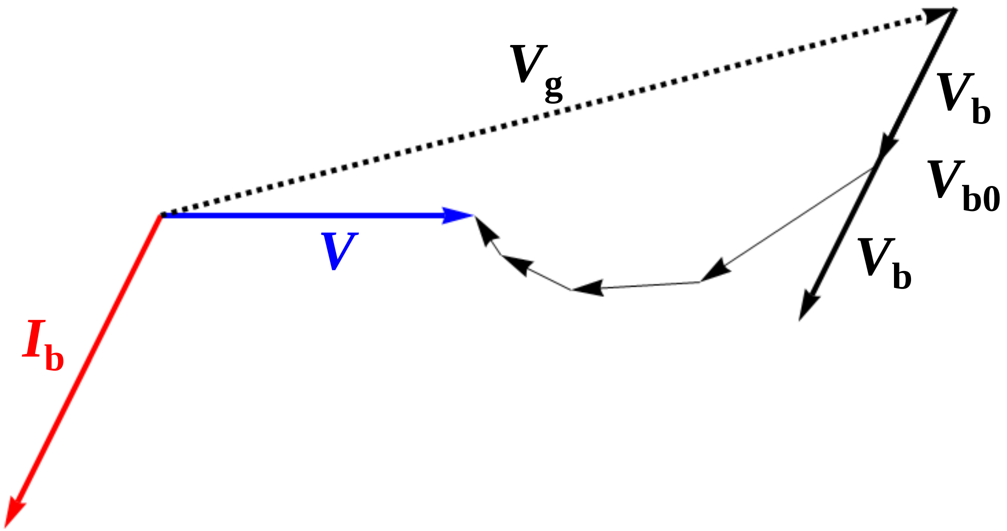
- **Arrange generator phase and voltage for real net voltage**
- **After 4th bunch passage**



$$\rightarrow \quad \vec{V} = \vec{V}_g + \frac{1}{2}\vec{V}_{b0} + \vec{V}_{b0}\left(e^{-\delta}e^{i\Psi} + e^{-2\delta}e^{2i\Psi} + \ldots\right)$$

Multiple passages

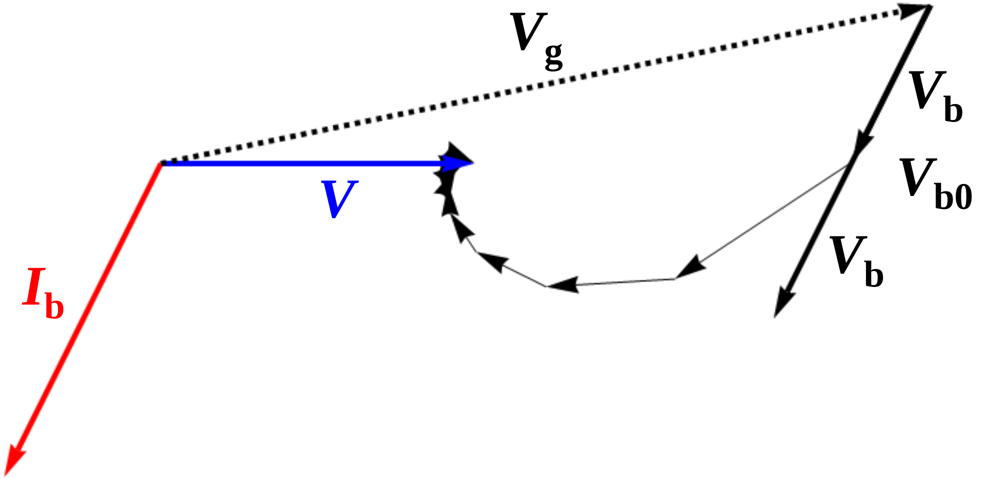
- **Arrange generator phase and voltage for real net voltage**
- **After 5th bunch passage**



$$\rightarrow \quad \vec{V} = \vec{V}_{\mathrm{g}} + \frac{1}{2}\vec{V}_{\mathrm{b0}} + \vec{V}_{\mathrm{b0}}\left(e^{-\delta}e^{i\Psi} + e^{-2\delta}e^{2i\Psi} + \ldots\right)$$

Multiple passages

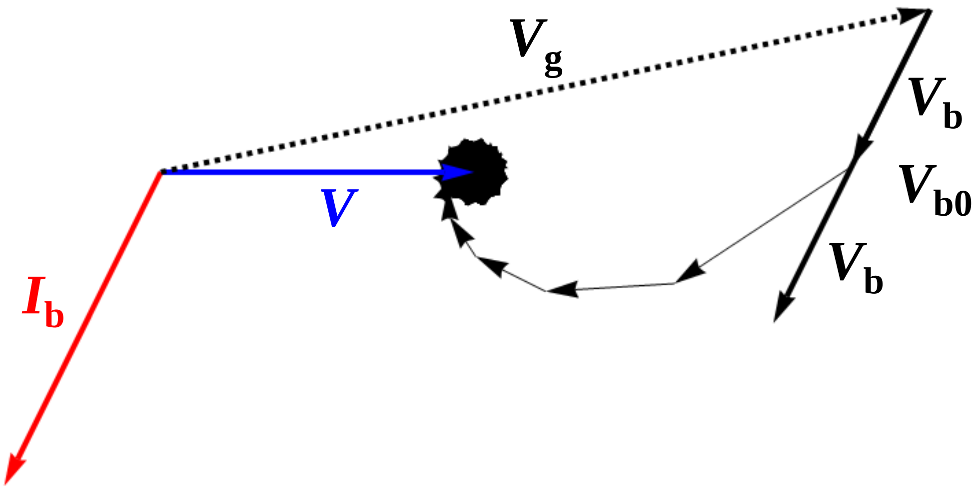
- Arrange generator phase and voltage for real net voltage
- After 10th bunch passage



$$\rightarrow \quad \vec{V} = \vec{V}_g + \frac{1}{2}\vec{V}_{b0} + \vec{V}_{b0}\left(e^{-\delta}e^{i\Psi} + e^{-2\delta}e^{2i\Psi} + \ldots\right)$$

Multiple passages

- Arrange generator phase and voltage for real net voltage
- After 100th bunch passage



$$\rightarrow \quad \vec{V} = \vec{V}_g + \frac{1}{2}\vec{V}_{b0} + \vec{V}_{b0}\left(e^{-\delta}e^{i\Psi} + e^{-2\delta}e^{2i\Psi} + \ldots\right)$$

→ **Infinite passages:** $1 + e^{-\delta}e^{i\Psi} + e^{-2\delta}e^{2i\Psi} + \ldots = \frac{1}{1 - e^{-\delta}e^{i\Psi}}$

General beam induced voltage

$$\vec{V}_{\mathrm{b}} = \vec{V}_{\mathrm{b0}} \left(\frac{1}{1 - e^{-\delta} e^{i\Psi}} - \frac{1}{2} \right)$$

Separate real and imaginary part:

$$V_{\mathrm{b}} = V_{\mathrm{b0}} \left[F_1(\delta, \Psi) + i F_2(\delta, \Psi) \right]$$

$$F_1(\delta, \Psi) = \frac{1 - e^{-2\delta}}{2(1 - 2e^{-\delta} \cos \Psi + e^{-2\delta})}$$

$$F_2(\delta, \Psi) = \frac{e^{-\delta} \sin \Psi}{1 - 2e^{-\delta} \cos \Psi + e^{-2\delta}}$$

Change of variables

- **Variables for damping, δ, and bunch-by-bunch phase advance, Ψ, not very practical**

→ **New variables with RF system parameters:**

1. Coupling coefficient, β

$$1+\beta = \frac{Q_0}{Q_\mathrm{L}}$$

2. Cavity tuning angle, ϕ_c

$$\tan\phi_c = 2Q_\mathrm{L}\frac{\omega_0-\omega}{\omega_0}$$

$$Z_\mathrm{L}(\omega) = \frac{R}{1+2iQ_\mathrm{L}\frac{\Delta\omega}{\omega_0}}$$

$$\Big\downarrow$$

$$Z_\mathrm{L}(\phi_\mathrm{c}) = \frac{R}{1-i\tan\phi_\mathrm{c}}$$

$$\boxed{\begin{aligned}\delta &= \delta_0(1+\beta)\\ \Psi &= \delta_0(1+\beta)\tan\phi_c\end{aligned}}$$

$$\delta_0 = \frac{T_\mathrm{b}}{T_\mathrm{f0}} \approx \frac{2Q_0}{\omega_0}$$

Beam induced voltage in new variables

$$V_\mathrm{b} = I_\mathrm{b} R \delta_0 \left[F_1(\beta, \phi_c) + i F_2(\beta, \phi_c) \right]$$

$$F_1(\beta, \phi_c) = \frac{1 - e^{-2\delta_0 (1+\beta)}}{2D}$$

$$F_2(\beta, \phi_c) = \frac{e^{-\delta_0 (1+\beta)} \sin \left[\delta_0 (1+\beta) \tan \phi_\mathrm{c} \right]}{D}$$

with denominator

$$D = 1 - 2e^{-\delta_0 (1+\beta)} \cos \left[\delta_0 (1+\beta) \tan \phi_\mathrm{c} \right] + e^{-2\delta_0 (1+\beta)}$$

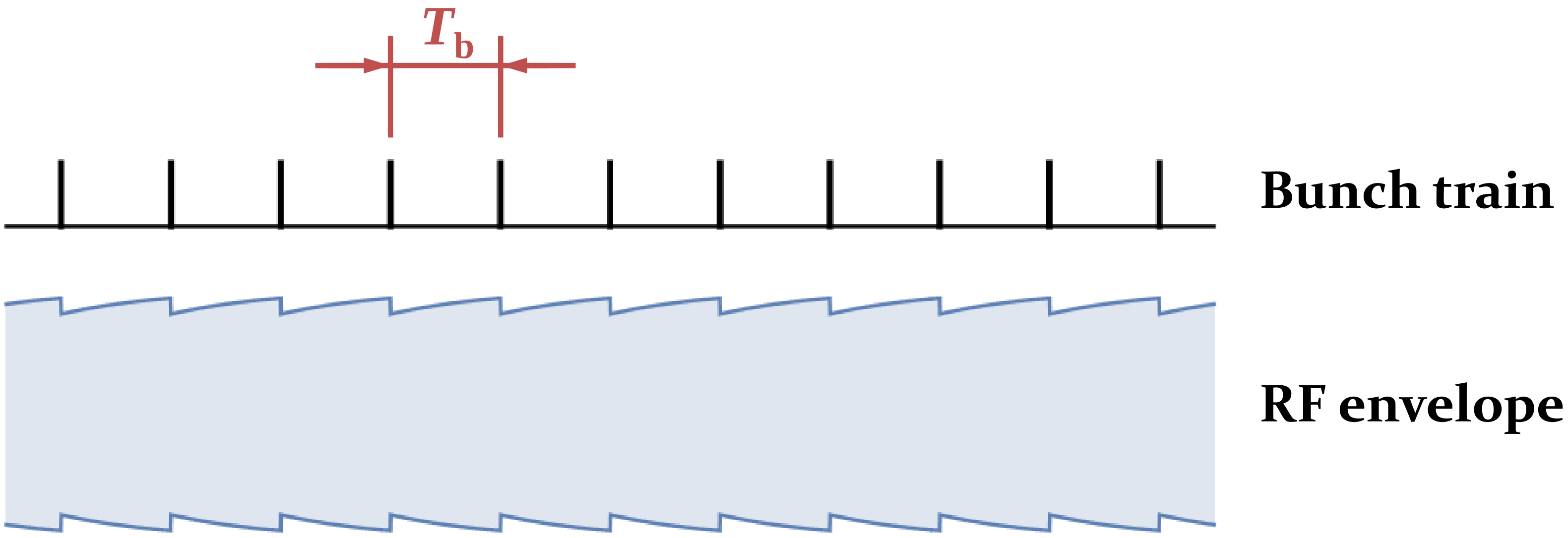
→ **Numerical computations required for analysis**

→ **Let us look at a particularly relevant approximation:** $\delta_0 \simeq 0$

Approximation

→ **Bunch distance short compared filling time:** $\delta_0 \simeq 0$

→ **Approximate terms including** $\mathcal{O}(\delta_0^2)$



Approximation

→ **Bunch distance short compared filling time:** $\delta_0 \simeq 0$

→ **Approximate terms including** $\mathcal{O}(\delta_0^2)$

$$V_\mathrm{b} = I_\mathrm{b} R \delta_0 \left[F_1(\beta, \phi_c) + i F_2(\beta, \phi_c) \right]$$

$$F_1(\beta, \phi_\mathrm{c}) \simeq \frac{1}{\delta_0 (1+\beta)(\tan^2 \phi_\mathrm{c} + 1)}$$

$$F_2(\beta, \phi_\mathrm{c}) \simeq \frac{\tan \phi_\mathrm{c}}{\delta_0 (1+\beta)(\tan^2 \phi_\mathrm{c} + 1)}$$

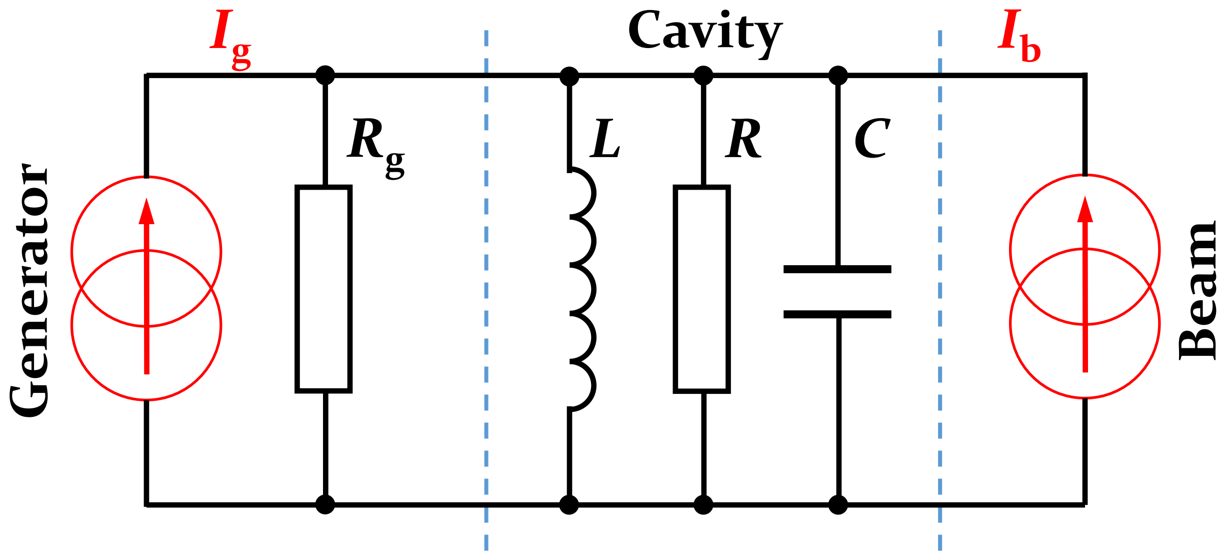
$$V_\mathrm{b} \simeq \frac{I_\mathrm{b}}{(1+\beta)} \frac{R}{1 - i \tan \phi_\mathrm{c}} = I_\mathrm{b} \frac{Q_\mathrm{L}}{Q_0} Z_\mathrm{L}(\phi_\mathrm{c})$$

→ **Ohm’s law for the loaded cavity impedance: steady state case**

Steady state beam loading

Equivalent circuit model

- **Lumped element circuit model for steady state case**

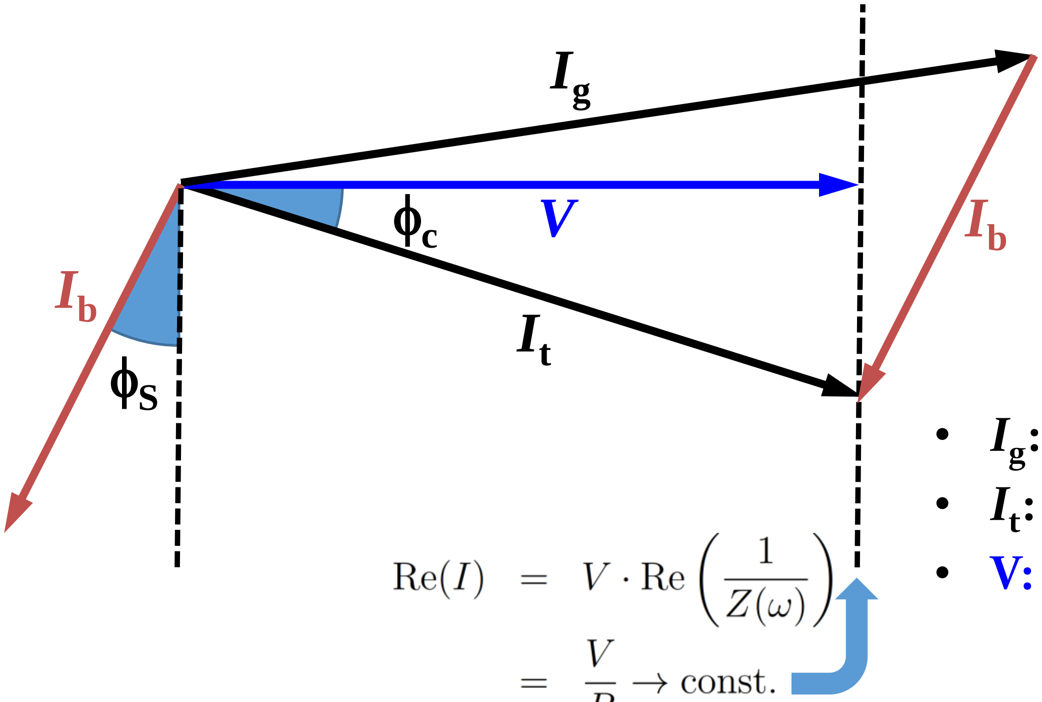


→ **Total current:** $\vec{I}_\mathrm{t} = \vec{I}_\mathrm{g} + \vec{I}_\mathrm{b}$

→ **Power required from generator:** $P_\mathrm{g} = \frac{1}{2} R_\mathrm{g} I_\mathrm{g}^2$

Steady state

- Vector diagram for passage of continuous bunch train



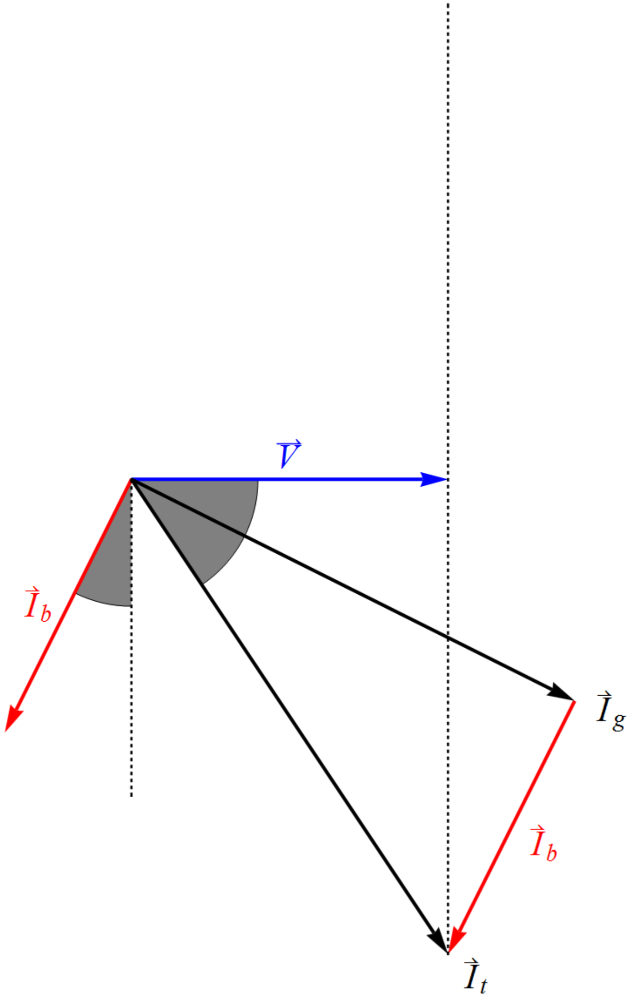
$$\begin{aligned}\mathrm{Re}(I) &= V \cdot \mathrm{Re}\left(\frac{1}{Z(\omega)}\right) \\ &= \frac{V}{R} \rightarrow \text{const.}\end{aligned}$$

- I_g: Generator current
- I_t: Total current
- V: Net voltage seen by beam

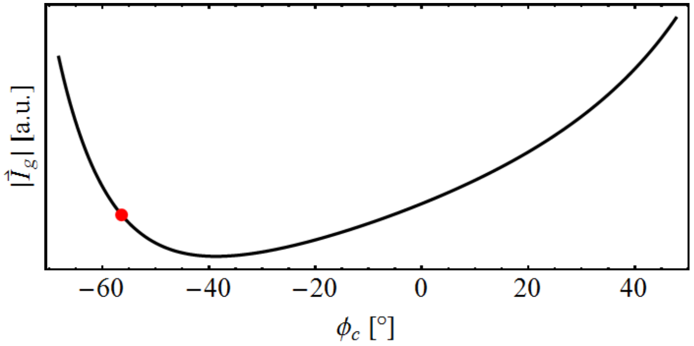
→ Parameters to achieve minimum generator current?

Steady state: minimum generator current

- Vector diagram for passage of continuous bunch train



Generator current, I_g



- I_g: Generator current
- I_t: Total current
- V: Net voltage seen by beam

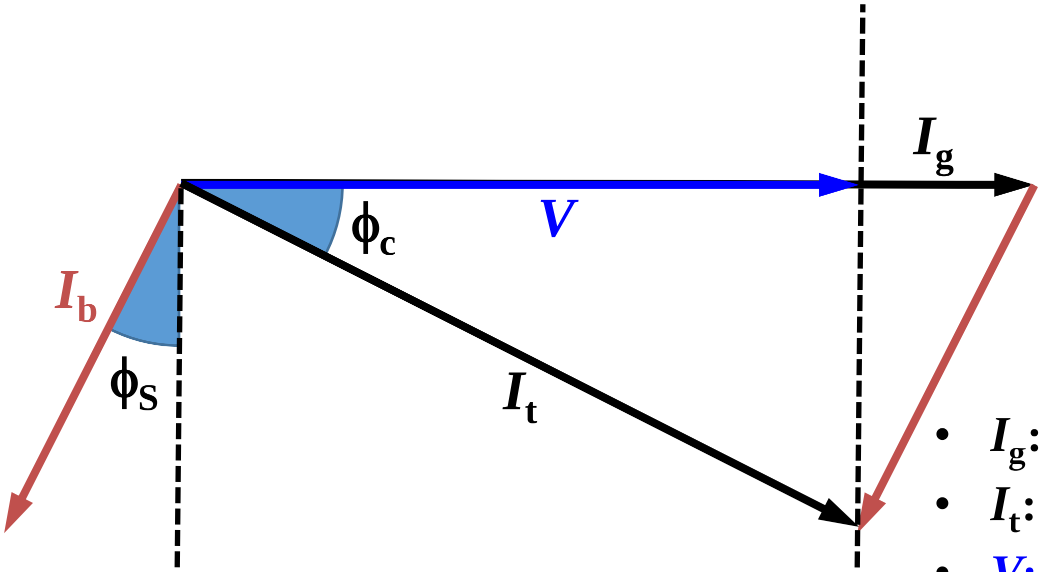
Lowest power (current I_g)

→ Generator current, I_g in phase with voltage

→ Resistive load with beam

Steady state: minimum generator current

- Vector diagram for passage of continuous bunch train



- I_g: **Generator current**
- I_t: **Total current**
- V: **Net voltage seen by beam**

$$\rightarrow \qquad \tan\phi_c = -\frac{I_b}{V}\frac{R}{1+\beta}\cos\phi_S$$

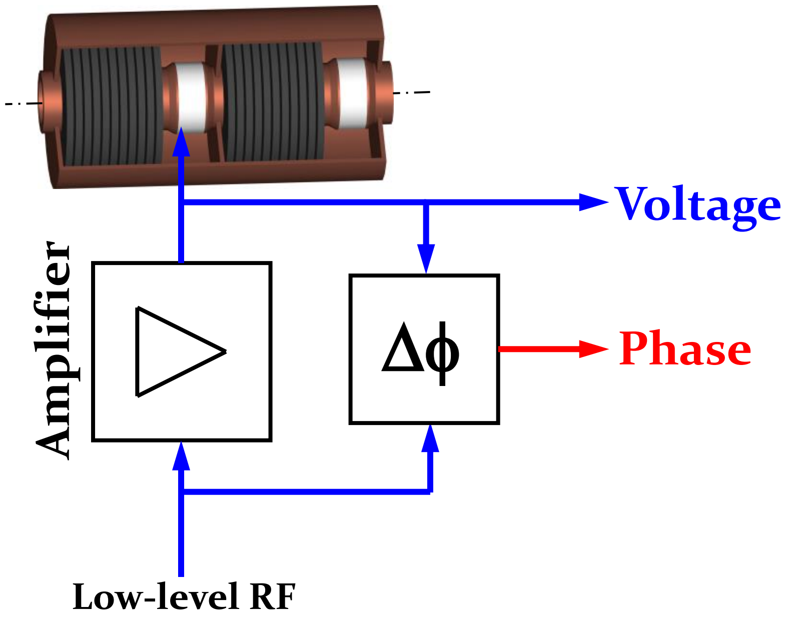
$$= -\frac{I_b}{V}Q_L\left(\frac{R}{Q_0}\right)\cos\phi_S$$

$$\beta = 1 + I_b\frac{R}{V}\sin\phi_S$$

Example: Cavity dephasing in PS

- **Voltage descent with beam:**

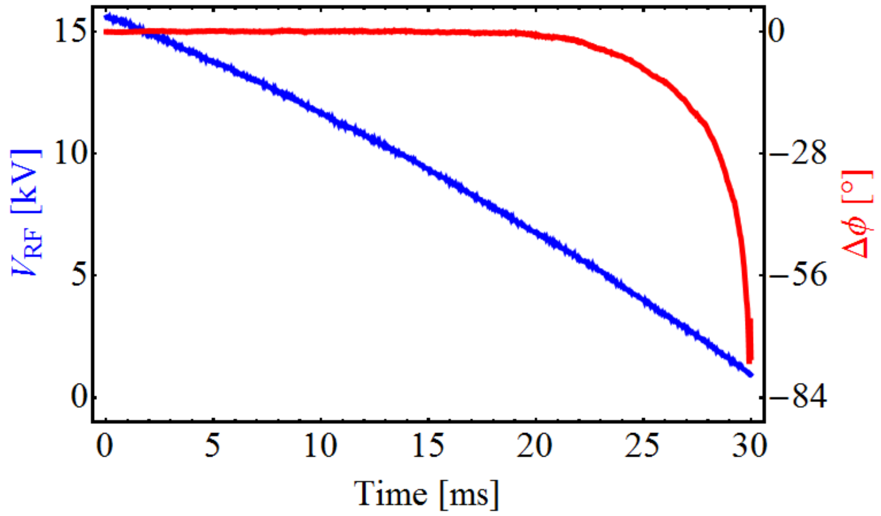
$$\frac{\Delta\omega}{\omega_0} \propto I_\mathrm{b} \cdot \frac{1}{V}$$



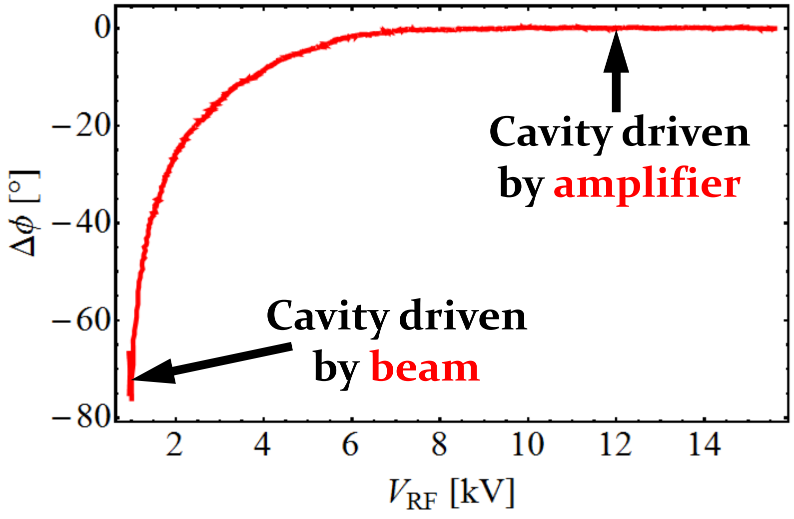
→ **Tuning loop recovers cavity resonance frequency**

→ **Dephasing at low RF voltage**

Voltage and phase vs. time



Phase versus voltage



Steady state: minimum generator current

- **Minimum power:** $$P_\mathrm{g} = \frac{V^2}{2R} + V I_\mathrm{b,DC} \sin\phi_\mathrm{S}$$

  $\left(I_\mathrm{b} \simeq 2I_\mathrm{b,DC} \right.$ **short bunches** $\left. \right)$

Resistive losses in cavity

Power delivered to beam

1. **Optimum detuning:** $$\frac{\omega - \omega_0}{\omega_0} = \frac{\Delta\omega}{\omega_0} = \frac{1}{2}\frac{I_\mathrm{b}}{V}\left(\frac{R}{Q_0}\right)\cos\phi_\mathrm{S}$$

   → **Cavity and beam appear as resistive load to generator**

   → **Automatically adjusted by cavity tuning loop**

2. **Optimum coupling:** $$\beta = 1 + I_\mathrm{b}\frac{R}{V}\sin\phi_\mathrm{S}$$

   → **Usually mechanically fixed by construction**

Example: LHC power coupler

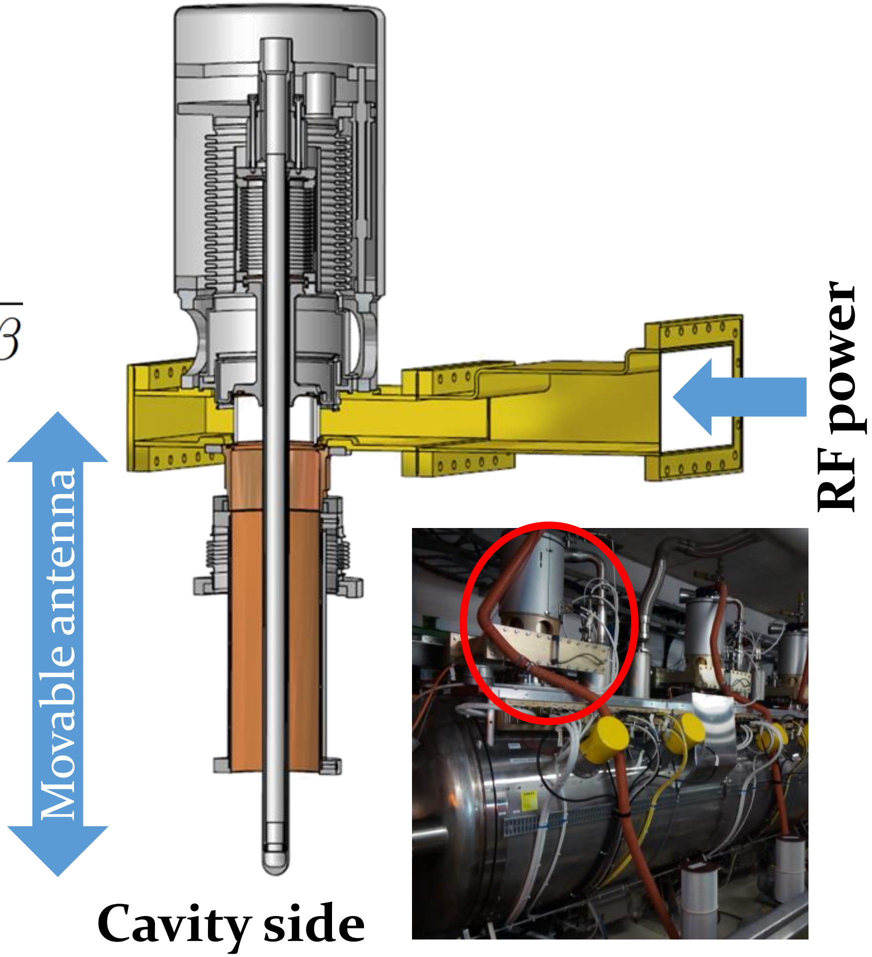
- **Control of both cavity resonance frequency and coupling**
- **Optimize quality through Q_{ext} for injection and storage**

$$\frac{1}{Q_{\mathrm{L}}} = \cancelto{\sim 0}{\frac{1}{Q_0}} + \frac{1}{Q_{\mathrm{ext}}}$$

$$Q_{\mathrm{L}} = Q_0 \frac{1}{1+\beta}$$

Loaded quality factor:

Mode	Q_{L}, Q_{ext}	Comment
Injection	$\sim 2 \cdot 10^4$	Suppress transients
Collision	$\sim 6 \cdot 10^4$	Maximum voltage



Filling pattern with gaps

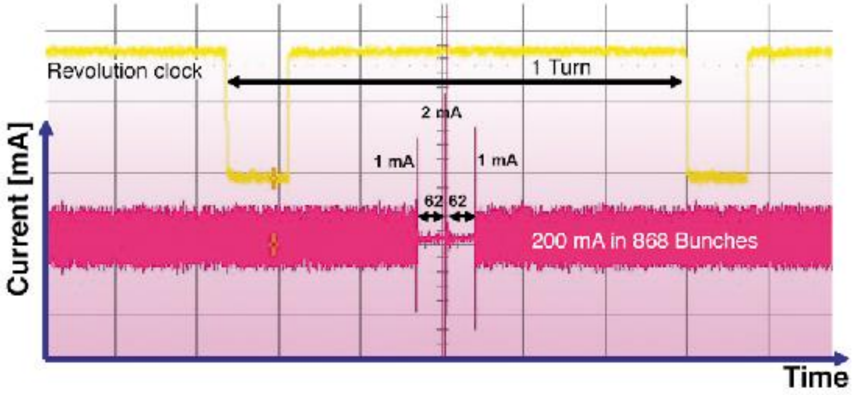
Why leaving a gap and not filling full ring?

→ **Electron storage rings:** **Clear ions attracted by electron beam**

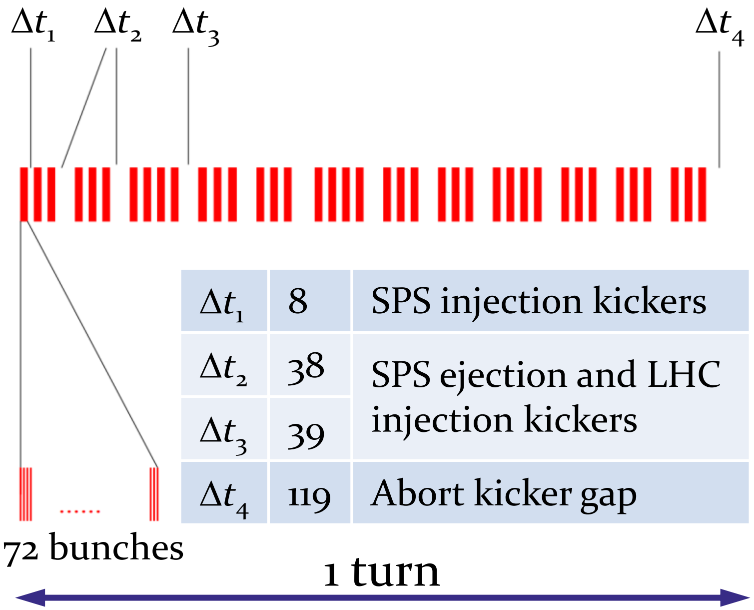
→ **Hadron accelerator:** **Leave gap for kicker magnets at injection/ejection**

ESRF: 7/8 + 1 filling mode

868	23 uA/b	200 mA in 7/8 train
2	1 mA/b	Marker bunches
1	2 mA	Single bunch
2×62	<2 pA/b	Gap



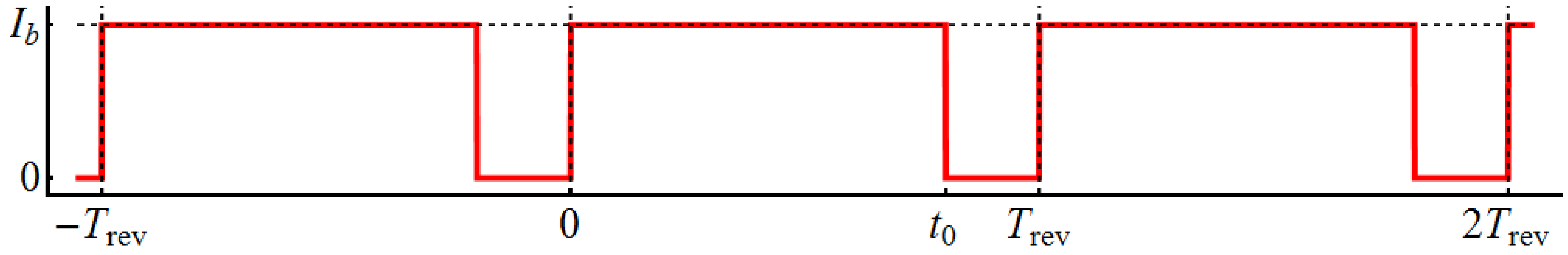
LHC: original nominal



Δt_1	8	SPS injection kickers
Δt_2	38	SPS ejection and LHC injection kickers
Δt_3	39	
Δt_4	119	Abort kicker gap

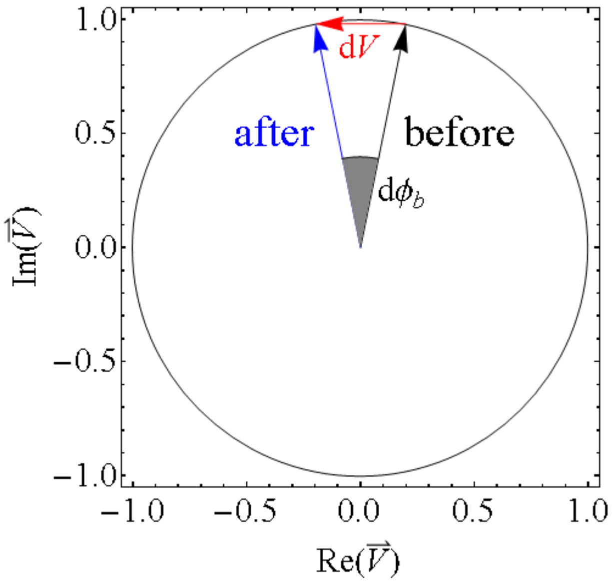
Beam loading with gaps

- **Limitations:** $\delta_0 \simeq 0$, **no acceleration, lossless cavity**



- **Phase change due to cavity detuning:** $d\phi_\mathrm{a} = \boxed{\Delta\omega\, dt}$

- **Phase change due to induced voltage:** $d\phi_\mathrm{b} = \boxed{\frac{1}{V}\, dV}$ **with**



$$dV = \frac{1}{2}\left(\frac{R}{Q_0}\right)\omega_0\, I_\mathrm{b}(t)\, dt$$

→ **Total phase advance:**

$$d\phi = \left[\Delta\omega - \frac{1}{2}\left(\frac{R}{Q_0}\right)\frac{\omega_0}{V}\, I_\mathrm{b}(t)\right]\, dt$$

Beam loading with gaps

→ **Periodicity condition** $\int_{1\mathrm{turn}} d\phi = 0$ **to get average detuning**

$$\Delta\omega_0 = \frac{1}{2}\left(\frac{R}{Q_0}\right)\frac{\omega_0}{V}\frac{1}{T_{\mathrm{rev}}}\int_0^{T_{\mathrm{rev}}} I_{\mathrm{b}}(t)\,dt = \frac{1}{2}\left(\frac{R}{Q_0}\right)\frac{\omega_0}{V}\bar{I}_{\mathrm{b}}$$

→ **and phase along the circumference**

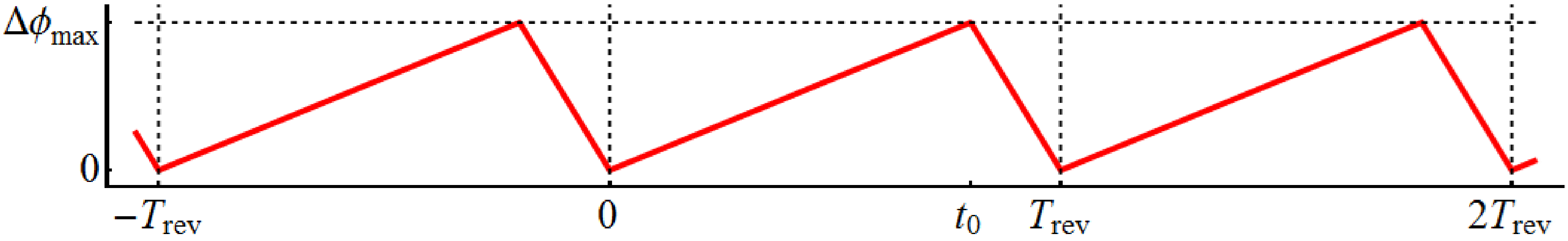
$$\phi(t) = \int_0^t d\phi = \frac{1}{2}\left(\frac{R}{Q_0}\right)\frac{\omega_0}{V}\int_0^t \left[\bar{I}_{\mathrm{b}} - I_{\mathrm{b}}(t)\right]\,dt$$

→ **Phase changes linearly for $I_b(t)$ = const. during beam region**

Maximum phase excursion

- **Maximum phase excursion**

$$\Delta\phi_{\mathrm{max}} = \frac{1}{2}\left(\frac{R}{Q_0}\right)\frac{\omega_0}{V}\bar{I}_{\mathrm{b}}\left(T_{\mathrm{rev}} - t_0\right) = \Delta\omega_0\left(T_{\mathrm{rev}} - t_0\right)$$



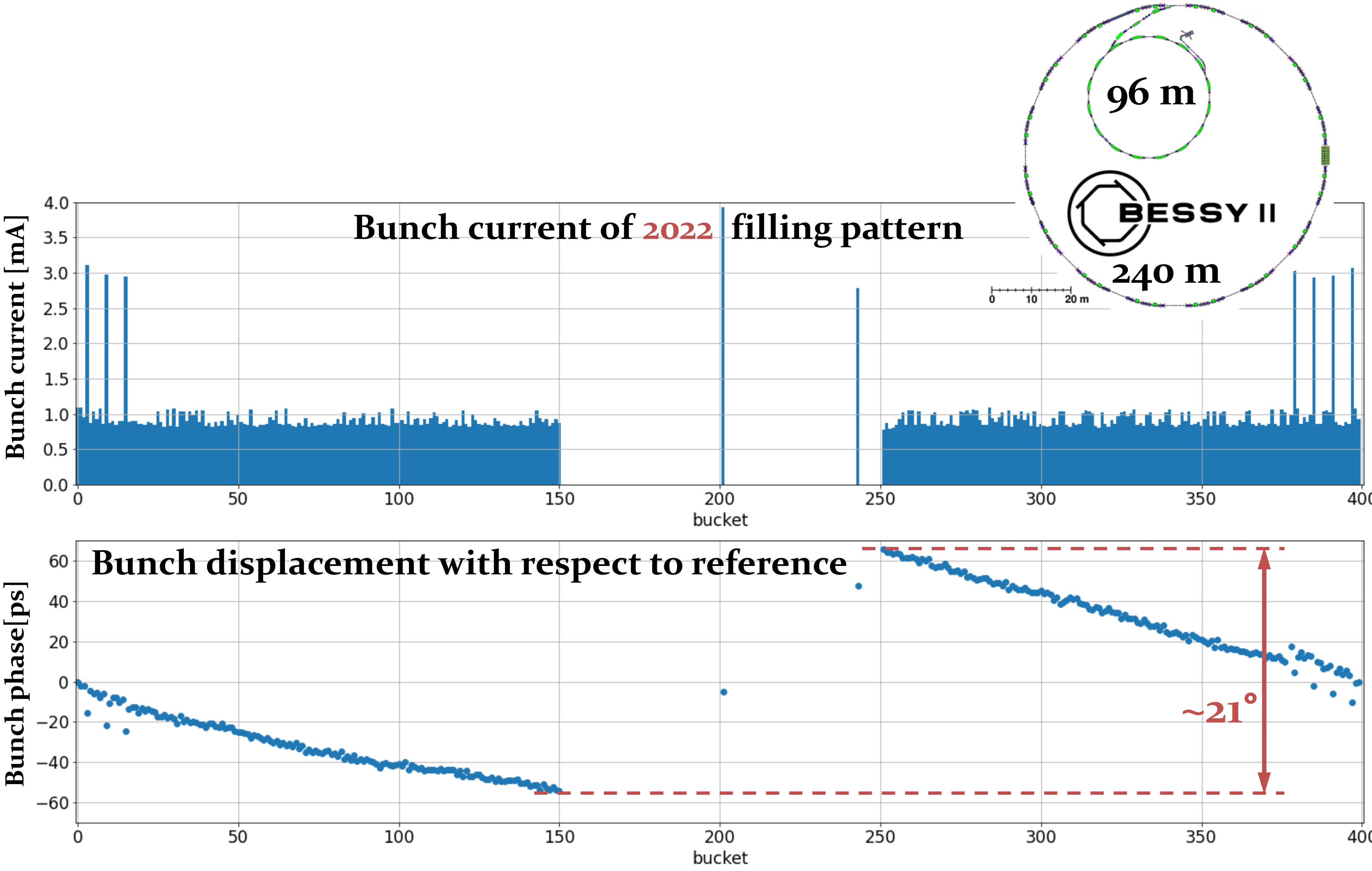
→ **Displaces timing of synchrotron radiation pulses**

→ **Longitudinally moves collision point in collider**

→ **Compromise between RF power and collision point**

Example: Electron storage ring

- Transient beam loading in electron storage ring BESSY II



Bunch current of 2022 filling pattern

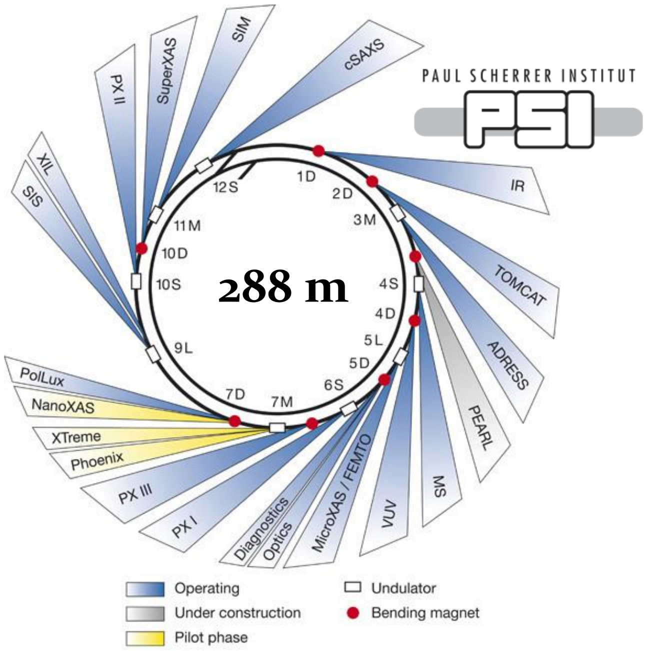
Bunch displacement with respect to reference

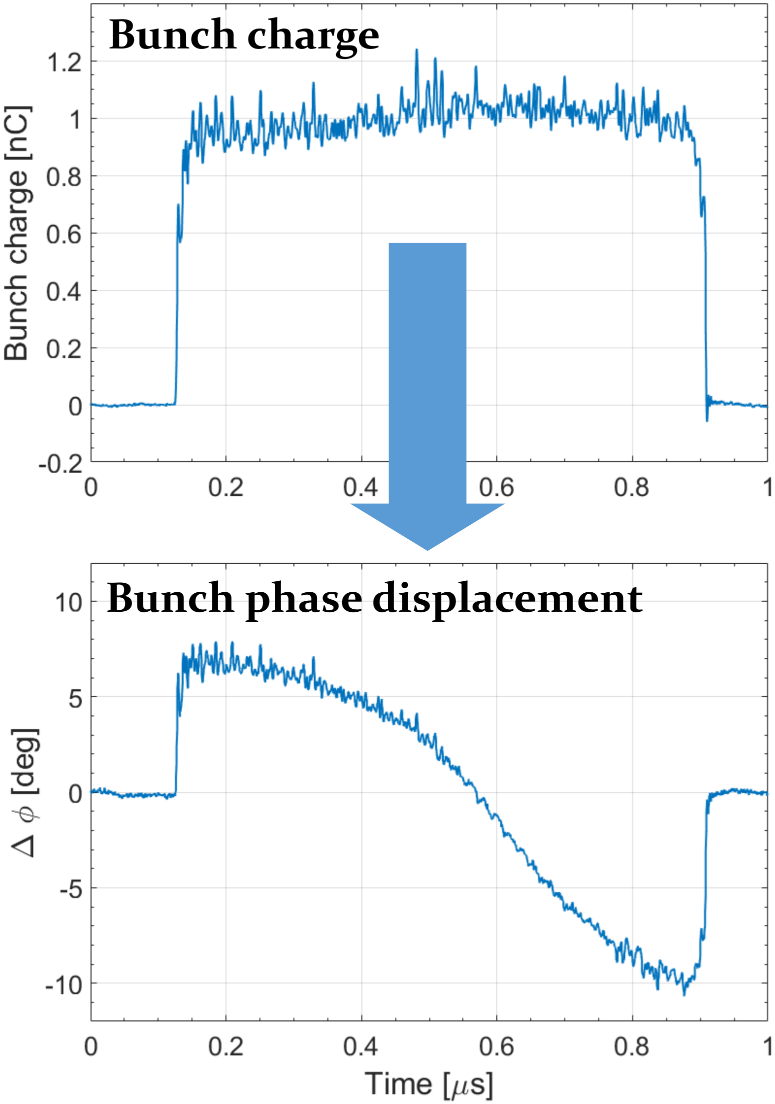
M. Ries, G. Rehm, A. Matveenko, G. Schiwietz et. al.

→ Synchrotron radiation light pulses slightly shifted in time

Example: Electron storage ring

- **Transient beam loading in** **electron storage ring SLS**

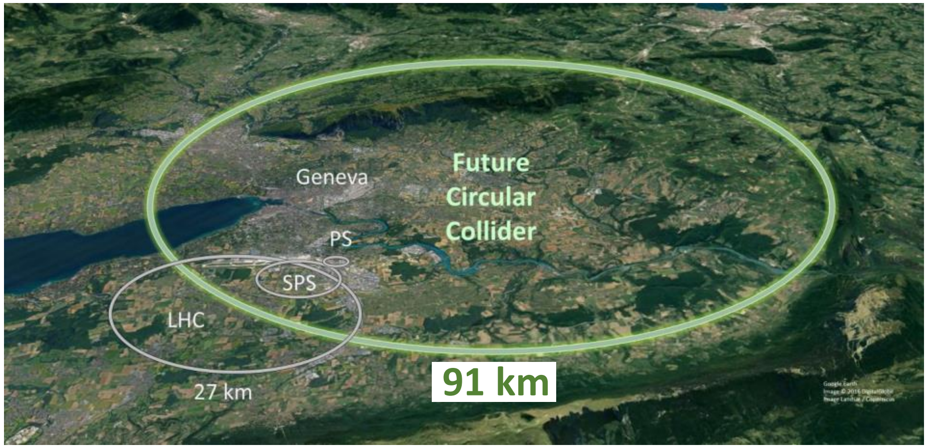




J. Kallestrup et. al.

Example: FCC-hh (hadron-hadron)

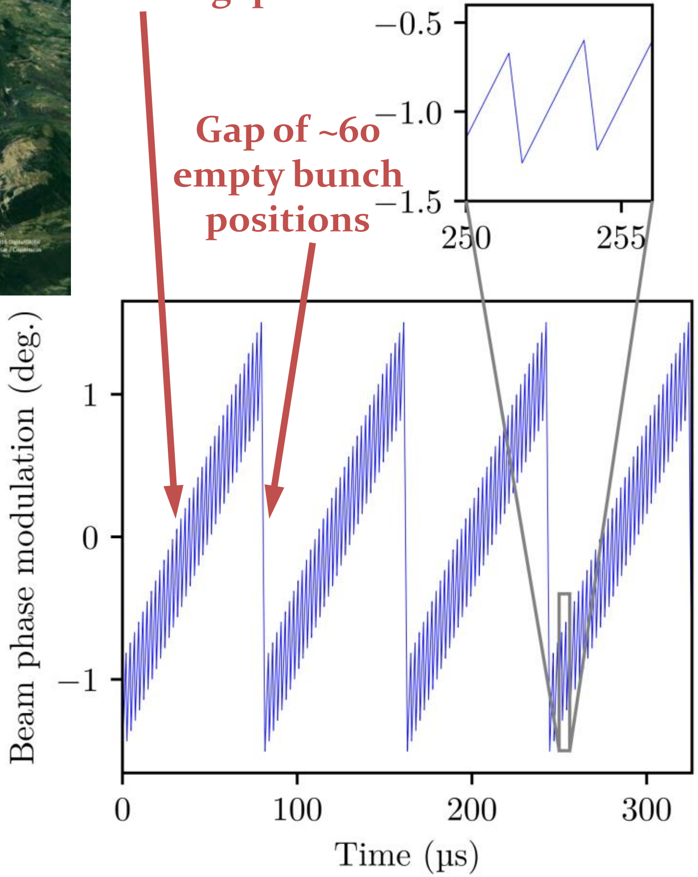
- Proposed future circular collider



- Machine protection requires
  - → Four batches per turn
  - → Gaps of ~1.5 µs

→ Full-detuning causes a bunch phase modulation of ~2°

→ Position of collision point modulated



Batch of 2600 bunches with short gaps

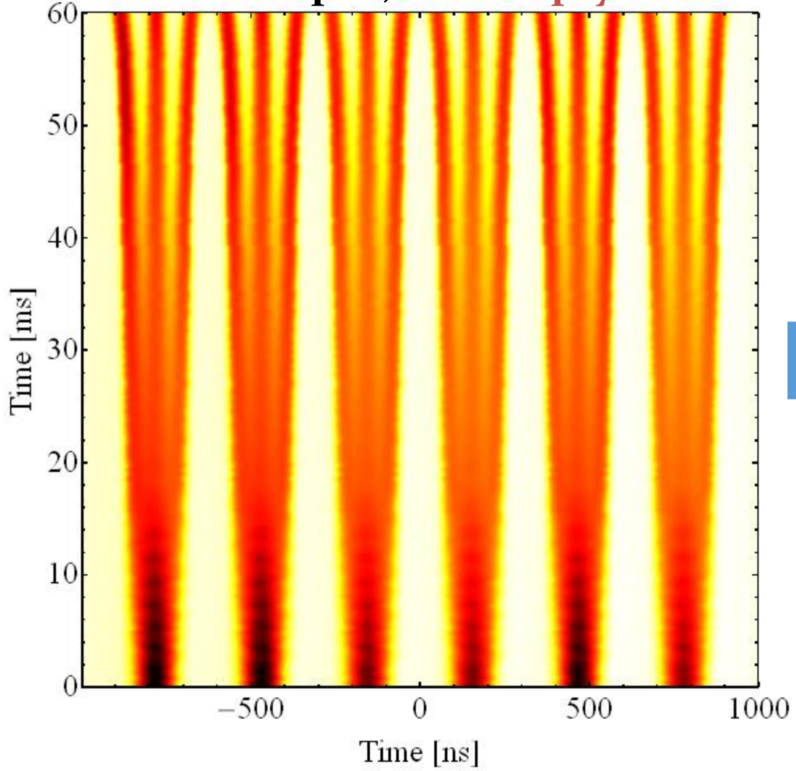
Gap of ~60 empty bunch positions

I. Karpov, P. Baudrenghien, IPAC19, MOPGW084

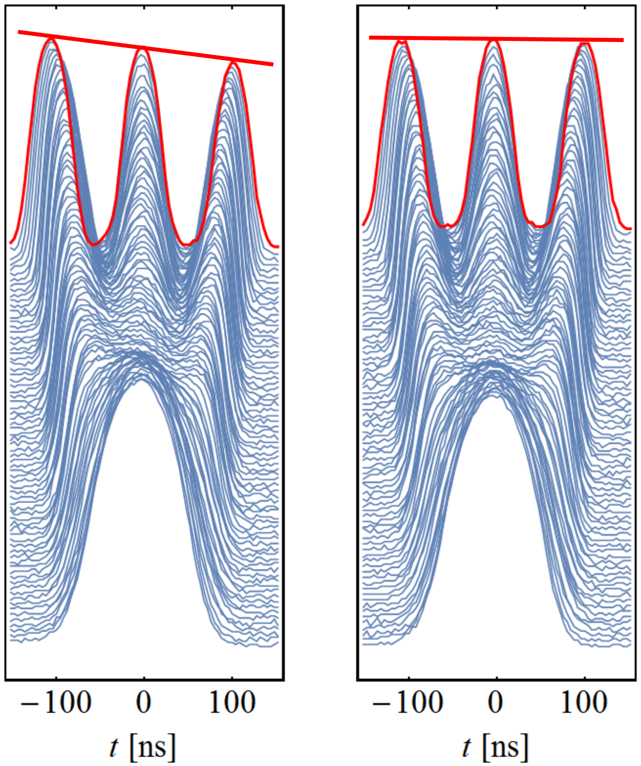
Transient beam loading between RF systems

- **Triple splitting of LHC-type beams in CERN PS requires three RF systems (h = 7, 14 and 21) in phase at degree level**

6 bunches split, one empty bucket



First bunch **Last bunch**



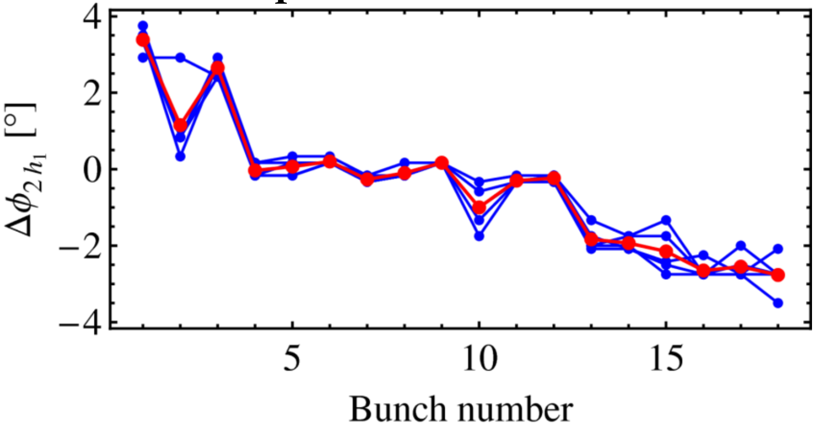
→ **Transient beam loading: relative phases different for 1st bunch**

→ **Bunch-by-bunch intensity variations in LHC**

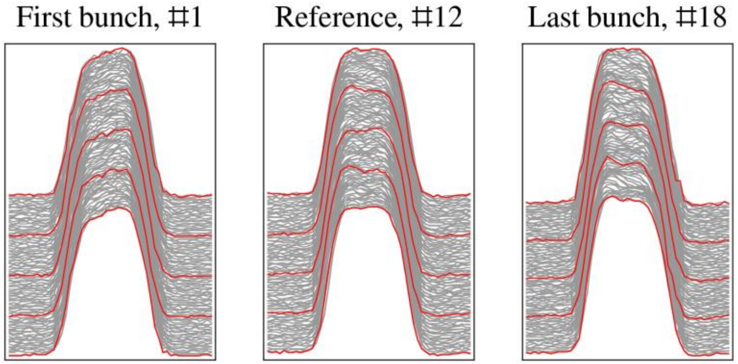
Transient beam loading between RF systems

→ **Fast phase measurement to directly observe relative changes**

Relative phase of 10 MHz and 20 MHz



Effect on bunch shape



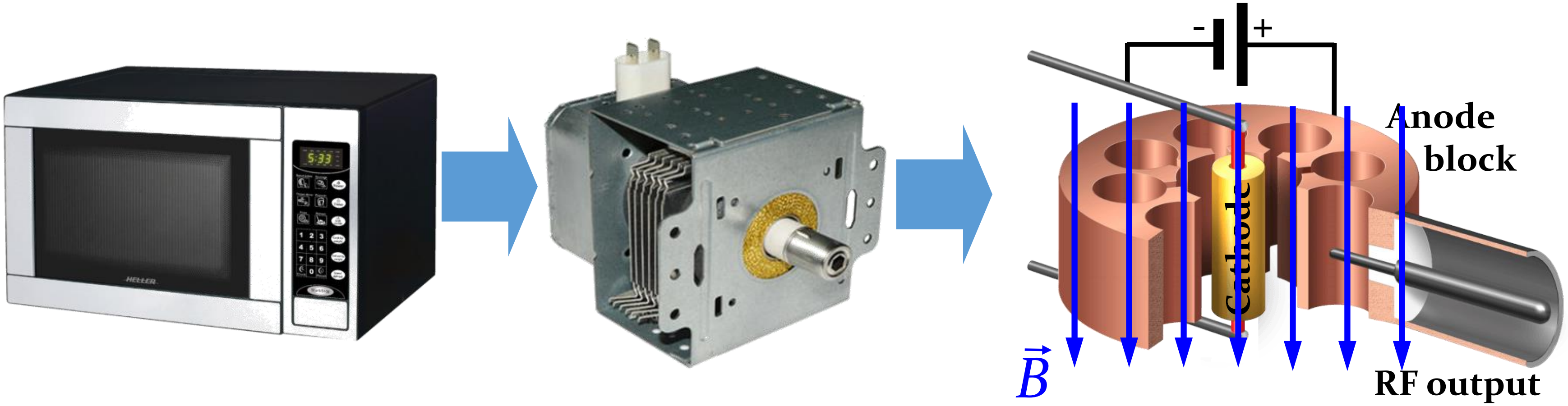
- **Cavity detuning not an option**
  → **Would even enhance phase modulation along batch**

- **Feedback systems**
  → **Counteract beam loading with additional RF power**
  → **Stabilize phase**

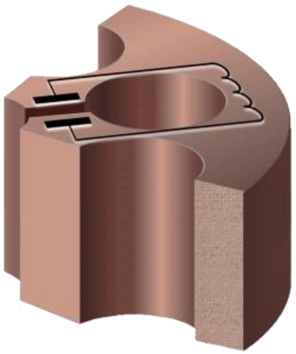
Beam loading in microwave oven?

Beam loading in microwave oven?

- **Microwave ovens use magnetrons as RF power source**



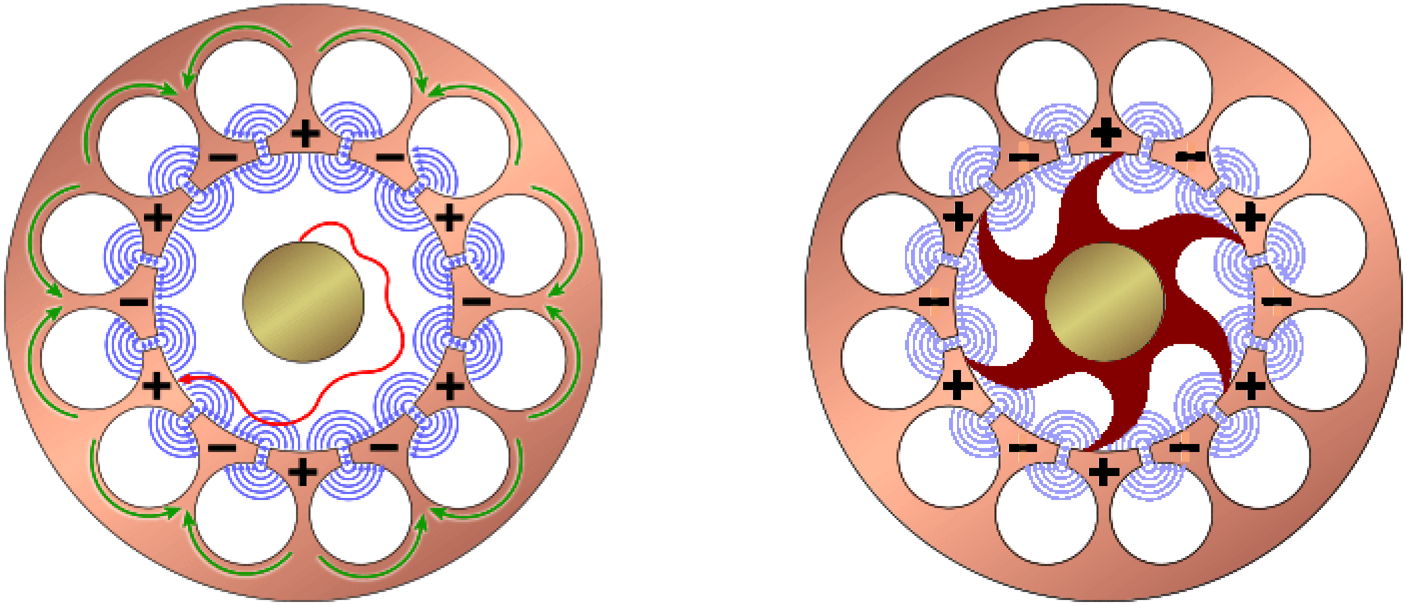
- **Anode block consists of ring of cavity resonators**
- **Electrons from the cathode accelerated towards anode (cavities)**
- **Perpendicular magnetic field causes cyclotron motion**



Ch. Wolff, http://www.radartutorial.eu/08.transmitters/Magnetron.en.html

Beam loading in microwave oven?

- **Magnetron as RF power source**



Ch. Wolff, http://www.radartutorial.eu/08.transmitters/Magnetron.en.html

→ Electron flow from cathode to anode self-bunched under influence of oscillating fields in anode resonators

→ Bunched electrons excite RF fields → beam loading!

→ Food gets heated

Summary

- **RF cavity parameters**
  - → System of cavity, coupling and amplifier
- **Single and multi-passage of bunches through a cavity**
  - → Fundamental theorem of beam loading
  - → Multiple passages limiting case of steady state
- **Steady state beam loading**
  - → Minimize RF power by detuning and coupling
- **Partial filling**
  - → Modulation of bunch phase and RF voltage
- **Magnetron principle**
  - → Heating food with beam loading

A big Thank You

to all colleagues providing support, material and feedback

Philippe Baudrenghien, Thomas Bohl, Anny Gora, Wolfgang Höfle, Andreas Jankowiak, Erk Jensen, Ivan Karpov, AlexandreLasheen, Eric Montesinos, Elena Shaposhnikova, Lukas Stingelin, Frank Tecker, Christian Wolff and many more…

Thank you very much for your attention!

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Approximations

→ **2nd order Taylor expansion for** $\delta_0 \simeq 0$

$$\begin{aligned}
e^{-\delta_0(1+\beta)} &\simeq 1 - \delta_0(1+\beta) + \frac{1}{2}\delta_0^2(1+\beta)^2 \\
e^{\delta_0(1+\beta)} &\simeq 1 + \delta_0(1+\beta) + \frac{1}{2}\delta_0^2(1+\beta)^2 \\
\cos\left[\delta_0(1+\beta)\tan\phi_\mathrm{c}\right] &\simeq 1 - \frac{1}{2}\delta_0^2(1+\beta)^2\tan^2\phi_\mathrm{c} \\
\sin\left[\delta_0(1+\beta)\tan\phi_\mathrm{c}\right] &\simeq \delta_0(1+\beta)\tan\phi_\mathrm{c}
\end{aligned}$$

Approximations: F_1

→ **Simplification of real part $F_1(\beta,\phi_c)$ for** $\delta_0 \simeq 0$

$$\begin{aligned}
F_1 &= \frac{1-e^{-2\delta_0(1+\beta)}}{2\{1-2e^{-\delta_0(1+\beta)}\cos\left[\delta_0(1+\beta)\tan\phi_c\right]+e^{-2\delta_0(1+\beta)}\}} \\
&= \frac{e^{\delta_0(1+\beta)}-e^{-\delta_0(1+\beta)}}{2\{e^{\delta_0(1+\beta)}-2\cos\left[\delta_0(1+\beta)\tan\phi_c\right]+e^{-\delta_0(1+\beta)}\}} \\
&\simeq \frac{1+\delta_0(1+\beta)+\frac{1}{2}\delta_0^2(1+\beta)^2-1+\delta_0(1+\beta)-\frac{1}{2}\delta_0^2(1+\beta)^2}{2\{1+\delta_0(1+\beta)+\frac{1}{2}\delta_0^2(1+\beta)^2-2\left[1-\frac{1}{2}\delta_0^2(1+\beta)^2\tan^2\phi_c\right]+1-\delta_0(1+\beta)+\frac{1}{2}\delta_0^2(1+\beta)^2\}} \\
&= \frac{2\delta_0(1+\beta)}{2\{2+\delta_0^2(1+\beta)^2-2+\delta_0^2(1+\beta)^2\tan^2\phi_c\}} \\
&= \frac{\delta_0(1+\beta)}{\delta_0^2(1+\beta)^2+\delta_0^2(1+\beta)^2\tan^2\phi_c} \\
&= \frac{1}{\delta_0(1+\beta)(\tan^2\phi_c+1)}
\end{aligned}$$

Approximations: F_2

→ **Simplification of real part $F_2(\beta,\phi_c)$ for** $\delta_0 \simeq 0$

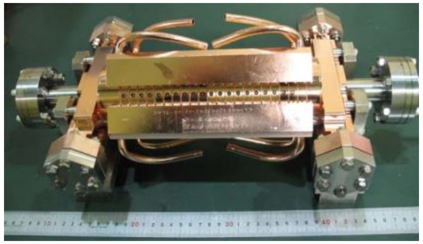
$$
\begin{aligned}
F_2 &= \frac{e^{-\delta_0(1+\beta)}\sin\left[\delta_0(1+\beta)\tan\phi_\mathrm{c}\right]}{1-2e^{-\delta_0(1+\beta)}\cos\left[\delta_0(1+\beta)\tan\phi_\mathrm{c}\right]+e^{-2\delta_0(1+\beta)}}\\
&= \frac{\sin\left[\delta_0(1+\beta)\tan\phi_\mathrm{c}\right]}{e^{\delta_0(1+\beta)}-2\cos\left[\delta_0(1+\beta)\tan\phi_\mathrm{c}\right]+e^{-\delta_0(1+\beta)}}\\
&\simeq \frac{\delta_0(1+\beta)\tan\phi_\mathrm{c}}{1+\delta_0(1+\beta)+\frac{1}{2}\delta_0^2(1+\beta)^2-2\left[1-\frac{1}{2}\delta_0^2(1+\beta)^2\tan^2\phi_\mathrm{c}\right]+1-\delta_0(1+\beta)+\frac{1}{2}\delta_0^2(1+\beta)^2}\\
&= \frac{\delta_0(1+\beta)\tan\phi_\mathrm{c}}{2+\delta_0^2(1+\beta)^2-2+\delta_0^2(1+\beta)^2\tan^2\phi_\mathrm{c}}\\
&= \frac{\delta_0(1+\beta)\tan\phi_\mathrm{c}}{\delta_0^2(1+\beta)^2+\delta_0^2(1+\beta)^2\tan^2\phi_\mathrm{c}}\\
&= \frac{\tan\phi_\mathrm{c}}{\delta_0(1+\beta)(\tan^2\phi_\mathrm{c}+1)}
\end{aligned}
$$

Frequency and wavelength ranges

PS longitudinal damper

PS main RF system

SPS 200 MHz



CLIC 12 GHz

100 kHz
3 km

1 MHz
300 m

10 MHz
30 m

100 MHz
3 m

1 GHz
30 cm

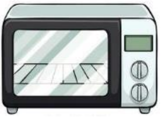
10 GHz
3 cm

100 GHz
3 mm

Long wave

Medium/ short wave

VHF



Microwave links