# **RF** Manipulations I



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CERN



The CERN Accelerator School

#### **RF for Accelerators**

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## Outline

#### Introduction

Longitudinal beam dynamics

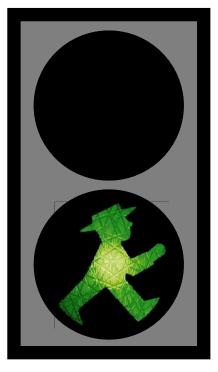
#### • Single-harmonic RF

- Bunching and de-bunching
- Bunch rotation
- Controlled longitudinal blow-up

#### • Double-harmonic RF

- Rebucketing
- Bunch merging
- Multiple bunch splitting
- Batch compression
- Double RF system
- Non-sinusoidal RF voltages
- Sequences, design and implementation
- Summary

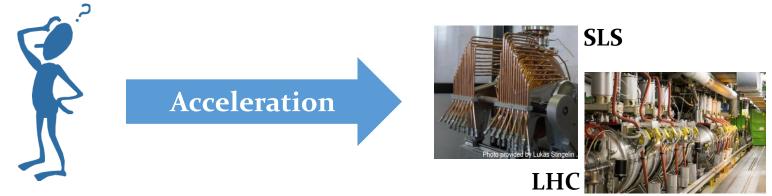




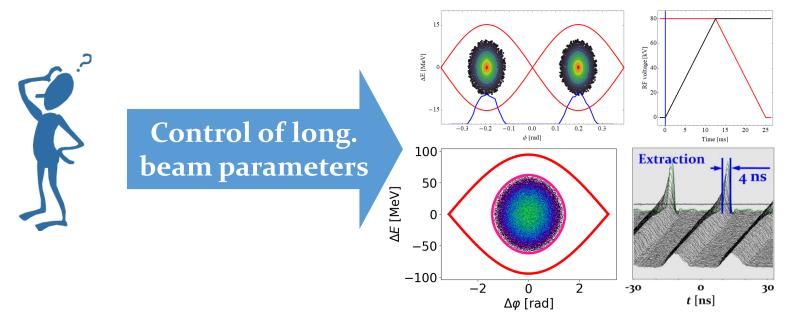
## Let's go!

#### Introduction

#### What can you do with RF in an accelerator?



#### Only acceleration? → RF can do much more!



## Impact and application of RF

RF parameter	Beam parameter
<ul> <li>Frequency, <i>f</i><sub>RF</sub></li> <li>Harmonic number, <i>h</i></li> </ul>	→ Bunch spacing: $1/f_{RF}$ and multiples → Bunch length and pattern
$-\pi -\pi/2  0  \pi/2  \pi$ $\phi$	<ul> <li>→ Orbit length: 2πR is multiple of RF wavelength</li> <li>→ Radial offset</li> <li>→ Beam energy: approximately proportional to orbit length</li> </ul>
• Amplitude, <i>V</i> <sub>RF</sub>	→ Bunch length
• Phase, ø	$\rightarrow$ Position of beam in time or phase
Beyond acceleration	→ Control over longitudinal beam parameters

→ Impacts some transverse properties

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#### **Motivation**

• Energy gain per turn in a hadron synchrotron

 $\Delta E_{\rm turn} = 2\pi q \rho R \dot{B}$ 

 $\rightarrow$  Grows with size of accelerator,  $\rho \cdot R$ 

Low and medium energy < ~50 GeV	High energy synchrotrons and colliders > ~50 GeV
<ul> <li>Moderate RF voltage</li></ul>	<ul> <li>Large RF voltages for fast</li></ul>
requirements: < ~ 1 MV <li>Non-relativistic beam</li>	acceleration: several MV <li>Ultra-relativistic beam</li>
→ Revolution frequency	→ Tiny revolution
sweeps	frequency increase
→ <b>Tuneable</b> (ferrite) <b>or</b>	<ul> <li>→ Fixed-frequency RF</li></ul>
<b>wideband</b> RF systems	systems <li>• Short bunches in collision</li>
→ RF frequencies	→ RF frequencies
below 50 MHz	above 50 MHz

### Motivation

Low and mediu	ım energy	High energy syn colliders	nchrotrons and
→ RF frequenc below 50 MF		→ RF frequenc above 50 MF	
CERN PS	2.810 MHz	<b>CERN SPS</b>	200 MHz
<b>BNL AGS</b>	1.64.5 MHz	<b>BNL RHIC</b>	198 MHz
FNAL Booster	3853 MHz	LHC	<b>400 MHz</b>

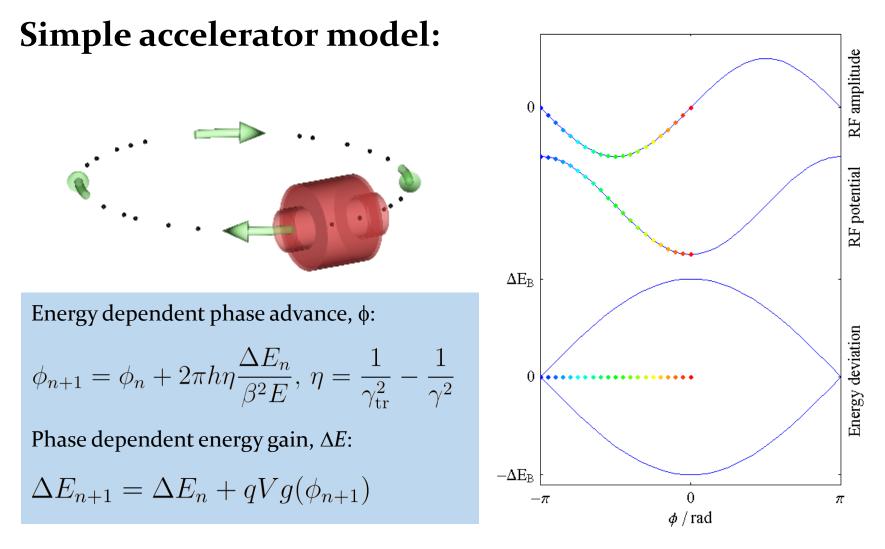
- → Need RF to increase RF frequency in chain of synchrotrons
- → **Beam parameters** for an experiment, e.g., short bunches
- → Longitudinal stacking and accumulation of beam





Longitudinal beam dynamics

#### **RF voltage, potential and bucket**



Works for arbitrary shape of acceleration amplitude  $g(\phi)$ 

## Longitudinal beam dynamics – single RF

• Construct Hamiltonian from equations of motion

$$\frac{d}{dt}\phi = \frac{h\eta\omega_{\text{rev}}}{pR} \left(\frac{\Delta E}{\omega_{\text{rev}}}\right)$$

$$\frac{d}{dt} \left(\frac{\Delta E}{\omega_{\text{rev}}}\right) = \frac{qV}{2\pi} \left(\sin\phi - \sin\phi_{\text{S}}\right)$$
same  $\frac{dq}{dt} = \frac{\partial H}{\partial p}$ 

$$\frac{dp}{dt} = -\frac{\partial H}{\partial q}$$

$$q = \phi \qquad p = \frac{\Delta E}{\omega_{\text{rev}}}$$

$$H(p,q) = T(p) + W(q)$$

- Hamiltonian constant on trajectory
- $\rightarrow$  'Energy conservation'

 $H(p,q) = H_{\text{trajectory}}$ 

#### **Continuous approximation – single RF**

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#### Single-harmonic RF system

$$\frac{d}{dt}\phi = \frac{h\eta\omega_{\rm rev}}{pR} \left(\frac{\Delta E}{\omega_{\rm rev}}\right)$$
$$\frac{d}{dt} \left(\frac{\Delta E}{\omega_{\rm rev}}\right) = \frac{qV}{2\pi} (\sin\phi - \sin\phi_{\rm S})$$
$$H\left(\phi, \frac{\Delta E}{\omega_{\rm rev}}\right) = \frac{1}{2} \frac{h\eta\omega_{\rm rev}}{pR} \left(\frac{\Delta E}{\omega_{\rm rev}}\right)^2 + \frac{qV}{2\pi} [\cos\phi - \cos\phi_{\rm S} + (\phi - \phi_{\rm S})\sin\phi_{\rm S}]$$
with  $\phi = \phi_{\rm S} + \Delta\phi$  this becomes

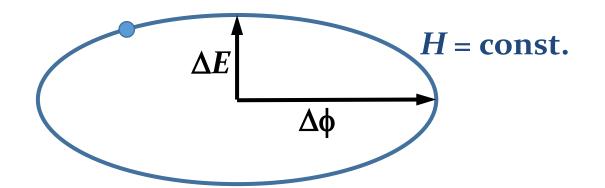
$$H\left(\Delta\phi,\frac{\Delta E}{\omega_{\rm rev}}\right) = \frac{1}{2}\frac{h\eta\omega_{\rm rev}}{pR}\left(\frac{\Delta E}{\omega_{\rm rev}}\right)^2 + \frac{qV}{2\pi}\left[\cos(\phi_{\rm S}+\Delta\phi) - \cos\phi_{\rm S} + \Delta\phi\sin\phi_{\rm S}\right]$$

 $\rightarrow$  Conventional longitudinal beam dynamics  $\rightarrow$  E. Shaposhnikova

#### Linear part of non-linear bucket

$$H\left(\Delta\phi,\frac{\Delta E}{\omega_{\rm rev}}\right) \simeq \frac{1}{2} \frac{h\eta\omega_{\rm rev}}{pR} \left(\frac{\Delta E}{\omega_{\rm rev}}\right)^2 - \frac{1}{2} \frac{qV}{2\pi} \cos\phi_{\rm S} \Delta\phi^2$$

- In the centre of the bucket,  $\Delta \phi \ll 1 \rightarrow \text{particles}$  move on elliptical trajectories in  $\Delta \phi \Delta E$  phase space
- Hamiltonian is constant on these trajectories

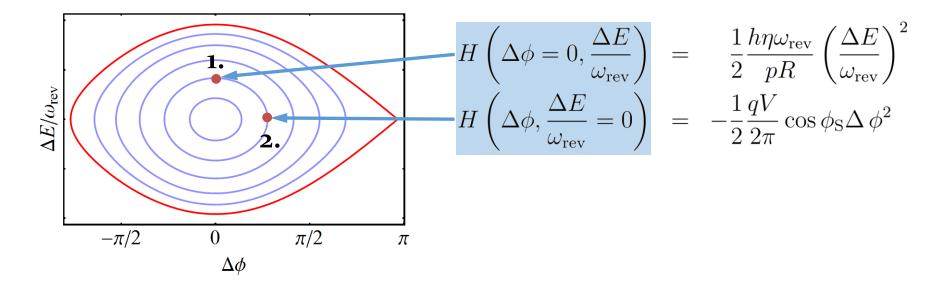


• In the bucket centre, particles oscillate with the synchrotron frequency,  $\omega_s = 2\pi f_s$ 

$$\omega_{\rm S}^2 = -\frac{h\eta\omega_{\rm rev}qV\cos\phi_{\rm S}}{2\pi pR} \qquad \eta = \frac{1}{\gamma_{\rm tr}^2} - \frac{1}{\gamma^2}$$

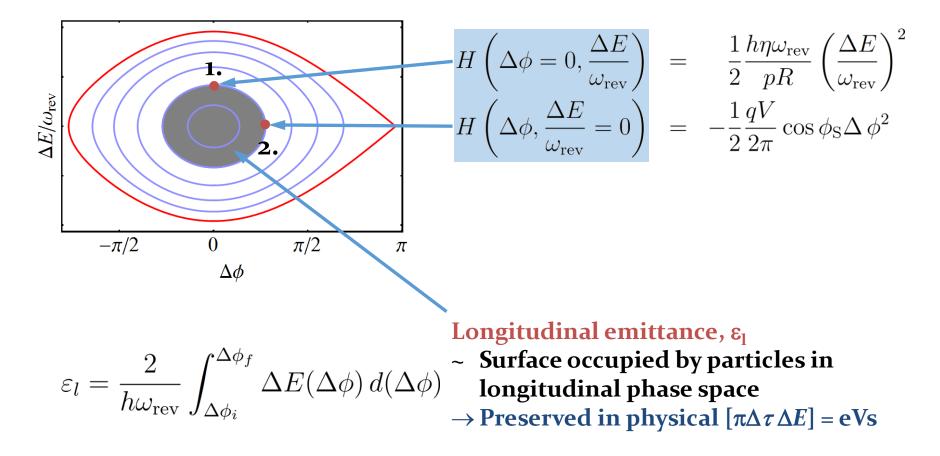
#### Longitudinal emittance

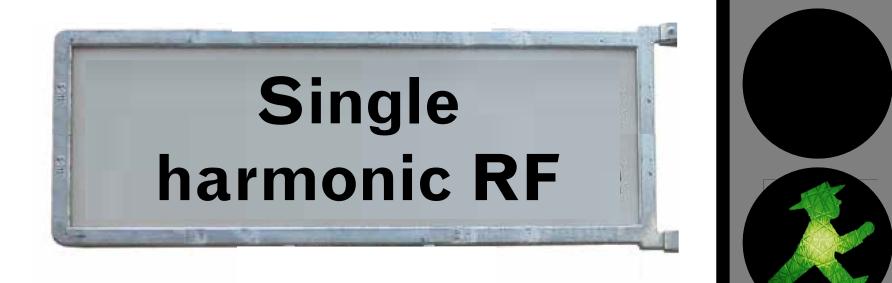
- Compare two particles on the same trajectory
  - 1. No phase deviation 2. No energy deviation



#### Longitudinal emittance

- Compare two particles on the same trajectory
  - 1. No phase deviation 2. No energy deviation

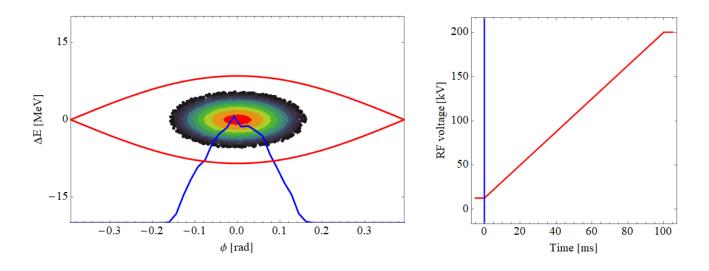




## **RF voltage variations**

#### Most simple RF manipulation

• Change bunch length by simply changing RF voltage



- Bunches get shorter, but not much
- Which voltage function is optimal to get from V<sub>i</sub> to V<sub>f</sub>?

#### **Optimum voltage changes**

- Which voltage function is optimal to get from  $V_i$  to  $V_f$ ?
- → **Definition** adiabaticity parameter

Relative change of synchrotron frequency  $\frac{d\omega_{\rm S}/dt}{\omega_{\rm S}}$ during one period  $T_{\rm S} = \frac{2\pi}{\omega_{\rm S}}$  $\alpha = 2\pi \frac{1}{\omega_{\rm S}^2} \frac{d\omega_{\rm S}}{dt}$ 

 $\rightarrow$  Derive voltage functions with constant  $\alpha$ 

#### **Optimum voltage changes**

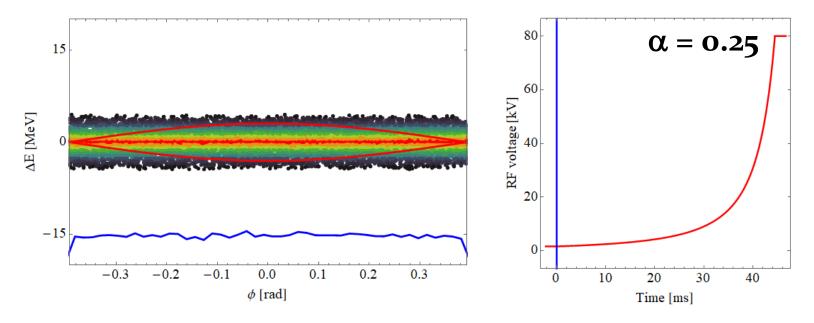
- Which voltage function is optimal to get from V<sub>i</sub> to V<sub>f</sub>, within the time, τ ?
- $\rightarrow$  Synchrotron frequency depends on RF voltage

$$\alpha = 2\pi \frac{1}{\omega_{\rm S}^2} \frac{d\omega_{\rm S}}{dt} \longrightarrow \alpha = \frac{2\pi}{\omega_{\rm rev}} \sqrt{\frac{2\pi E\beta^2}{h|\eta|q}} \cdot \frac{1}{2V_{\rm RF}(t)^{3/2}} \frac{d}{dt} V_{\rm RF}(t)$$

 $\rightarrow$  Voltage function with constant adiabaticity,  $\alpha$ 

$$V_{\rm RF}(t) = \frac{V_{\rm i}}{\left(1 - \frac{t}{\tau} \frac{\sqrt{V_{\rm f}} - \sqrt{V_{\rm i}}}{\sqrt{V_{\rm f}}}\right)^2}$$

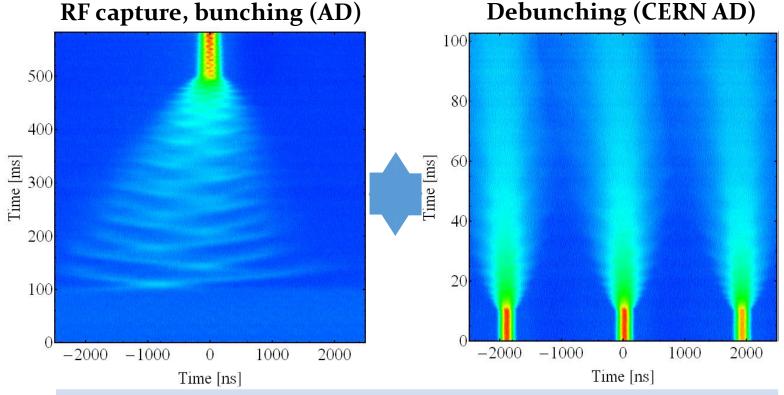
- 1. Start from coasting beam, for example injected Linac
- 2. Switch RF on at low voltage,  $V_i$
- 3. Raise voltage to the desired level,  $V_{\rm f}$



→ Clean capture

### RF capture, debunching and rebunching

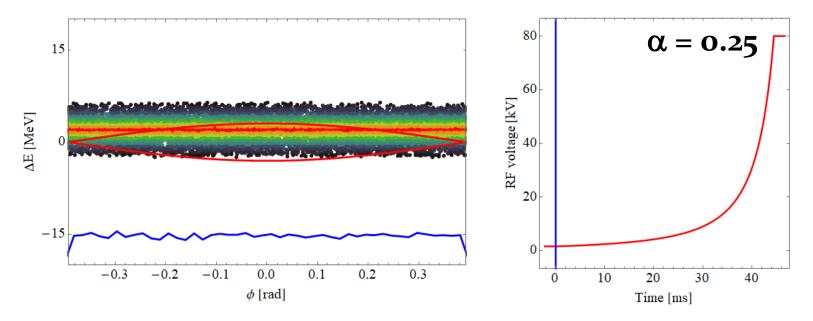
- Bunch a beam without RF structure, e.g. after injection
- De-bunching → Rebunching: Change harmonic number



#### Strengths and weaknesses

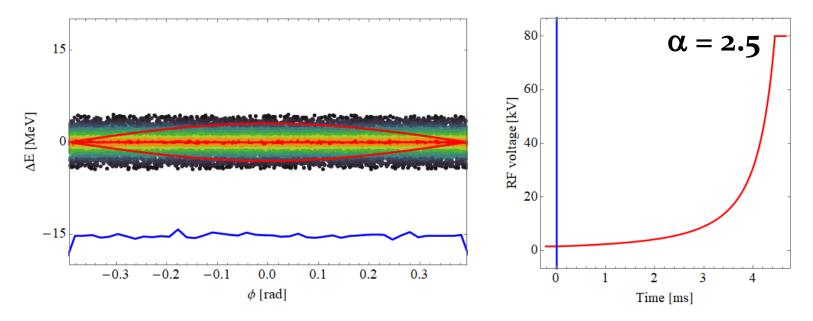
- + Simple manipulation, e.g. to change harmonic number
- Need the right RF frequency (exactly  $n \cdot f_{rev}$ ) for capture
- No RF control while beam is debunched

- 1. Start from coasting beam, for example injected Linac
- 2. Switch RF on at low voltage,  $V_i$
- 3. Raise voltage to the desired level,  $V_{\rm f}$



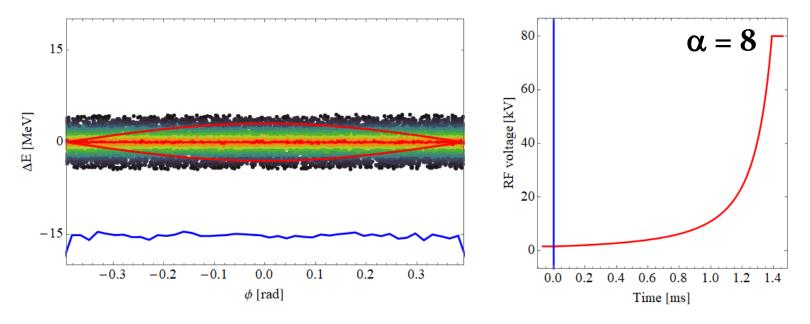
→ Some filamentation

- 1. Start from coasting beam, for example injected Linac
- 2. Switch RF on at low voltage,  $V_i$
- 3. Raise voltage to the desired level,  $V_{\rm f}$



→ Some filamentation

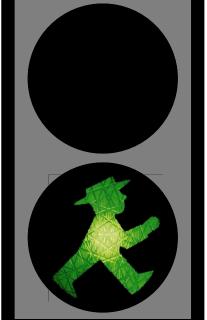
- 1. Start from coasting beam, for example injected Linac
- 2. Switch RF on at low voltage,  $V_i$
- 3. Raise voltage to the desired level,  $V_{\rm f}$



- → Beam more rotating then being captured
- $\rightarrow$  Slow and fast RF manipulations, depending on  $\alpha$ 
  - → Period of synchrotron frequency is the reference



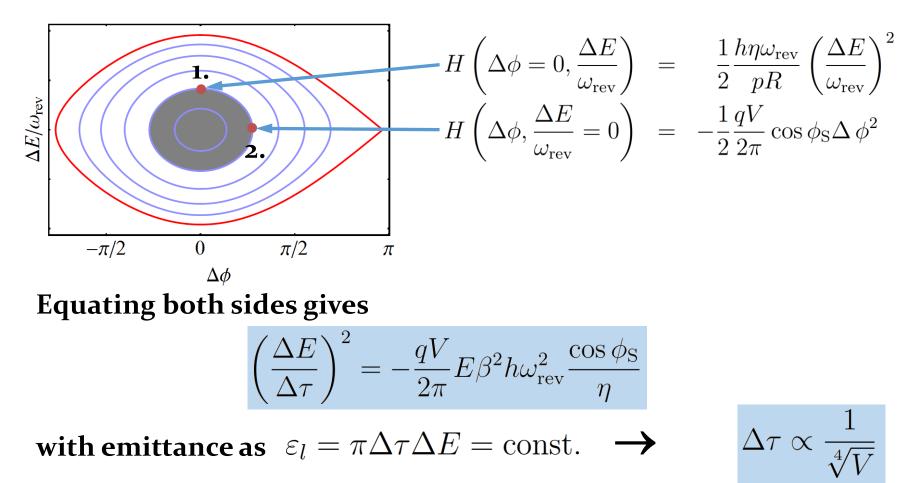
## fast variations



## **Bunch rotation**

### **Change RF voltage to change bunch length?**<sup>25</sup>

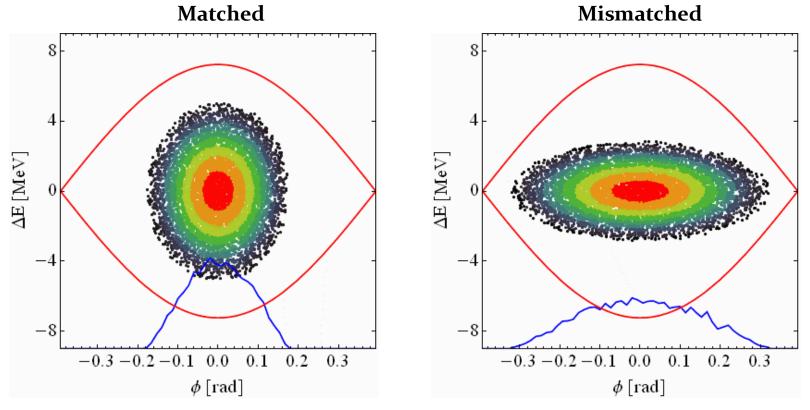
#### → Calculate aspect ratio of bucket trajectories



- $\rightarrow$  Not efficient at all
- $\rightarrow$  16 times more RF voltage needed to cut bunch length in half

#### Abrupt change of RF voltage

- → Individual particles in matched bunch oscillate but no macroscopic motion
- → Abruptly changing the RF voltage flips particles to new trajectories

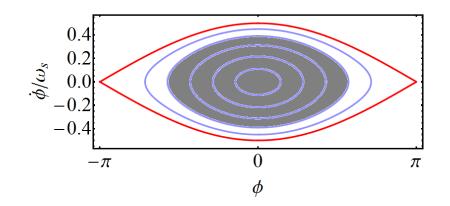


→ The bunch distribution seems to rotate
→ Exchange of bunch length and momentum spread

#### Introduce sudden change: bunch rotation

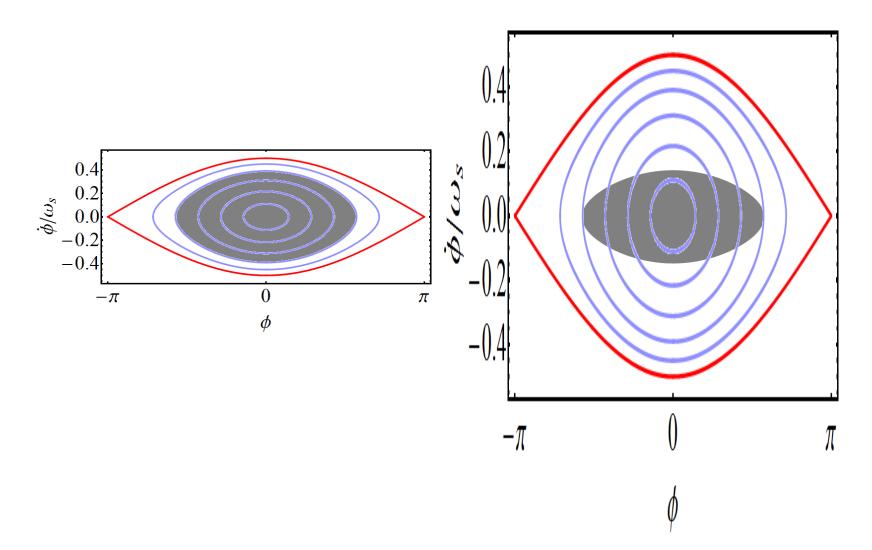
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- $\rightarrow$  Quickly exchange longitudinal phase space behind bunch
- $\rightarrow$  Increase RF voltage much faster than period of  $f_{\rm S}$



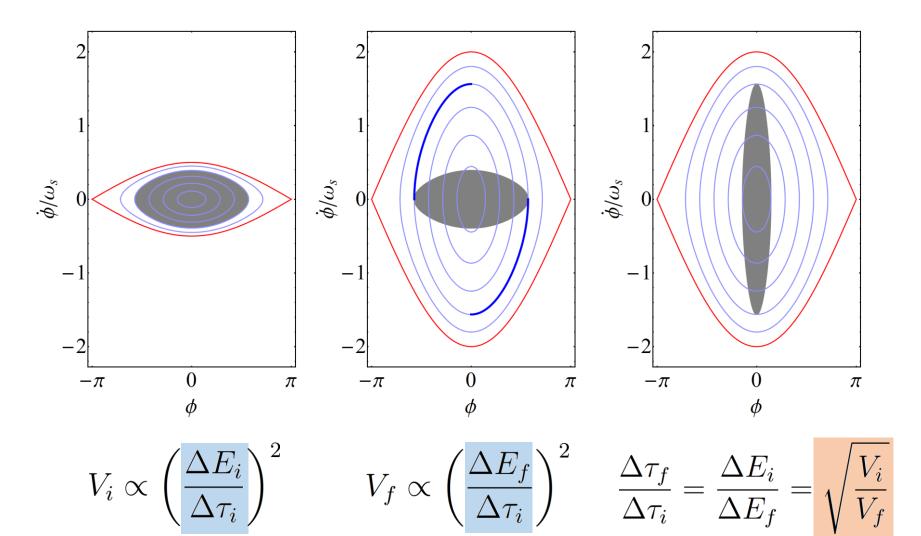
#### Introduce sudden change: bunch rotation

- $\rightarrow$  Quickly exchange longitudinal phase space behind bunch
- $\rightarrow$  Increase RF voltage much faster than period of  $f_{\rm S}$



#### Introduce sudden change: bunch rotation

 $\rightarrow$  Switch RF voltage much faster than period of  $f_{\rm S}$ 

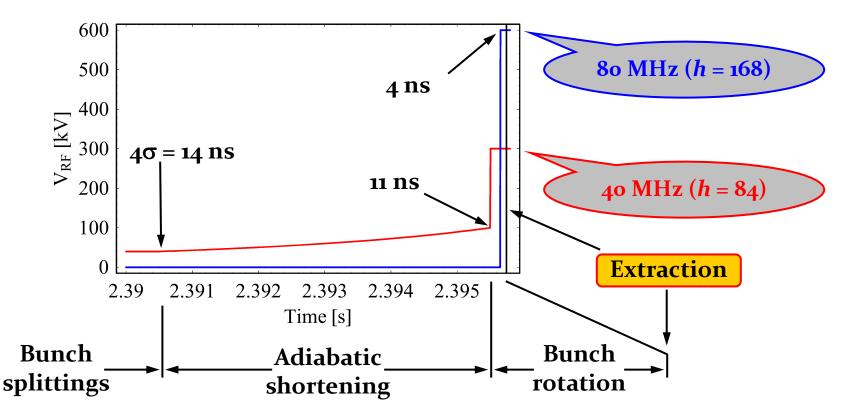


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#### **Example: PS to SPS transfer at CERN**

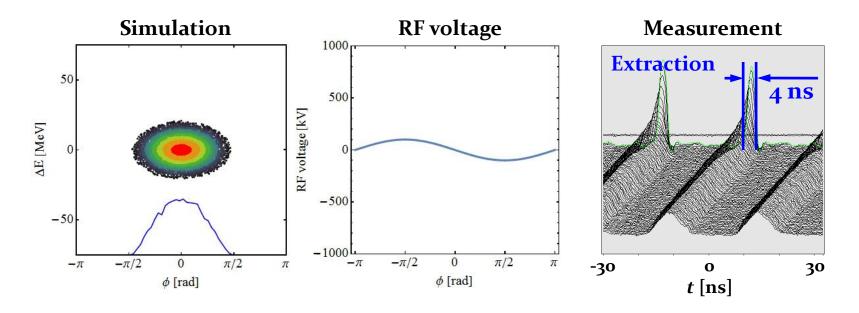
• Fit 14 ns long bunches into 5 ns long buckets in the SPS

 $\rightarrow$  Double-step bunch rotation



#### **Example: rotation at PS-SPS transfer**

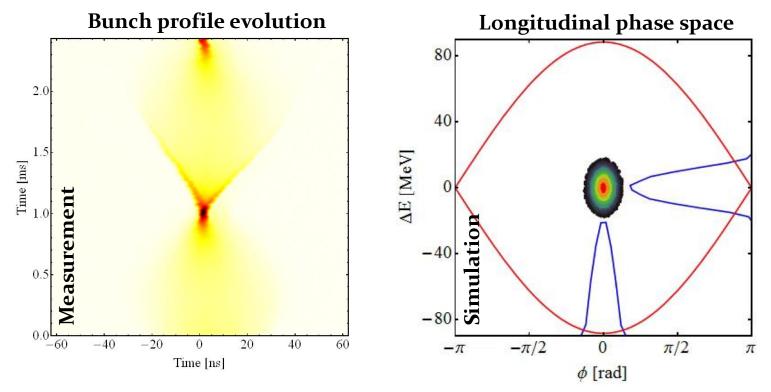
- $\rightarrow$  Bunch length now proportional to  $\sqrt{V}$  and not  $\sqrt[4]{V}$
- $\rightarrow$  Can save enormous RF voltage
- $\rightarrow$  Bunch shortening from 14 ns to 4 ns (ratio ~3.5)
- $\rightarrow$  Starting from 100 kV at 40 MHz
- $\rightarrow$  Slow shortening would require 100 kV  $\cdot$  3.  $5^4 \sim 15$  MV
- $\rightarrow$  Installed RF voltage is only about 1.2 MV



#### Bunch stretching at unstable fixed point

Need large momentum spread for slow extraction

- 1. Jump RF phase such that bunch at unstable fixed point
- 2. Jump back
- 3. Let bunch rotate, switch RF off at large momentum spread



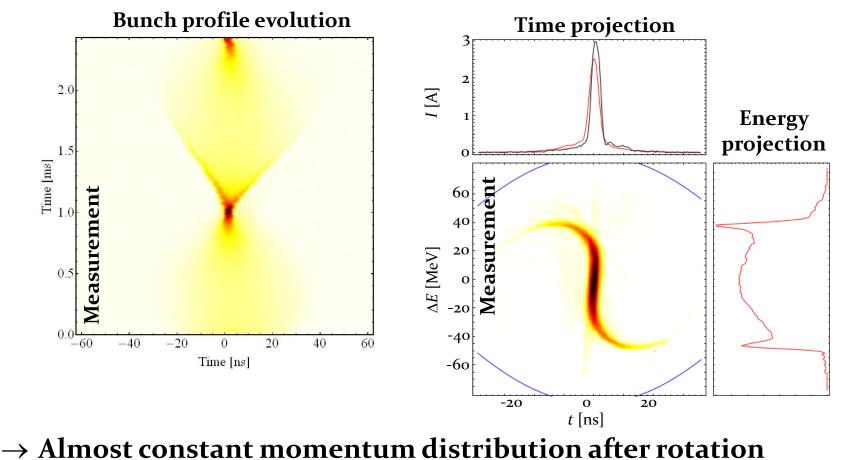
 $\rightarrow$  Non-linearity of bunch rotation helps

### **Example: using the non-linearity**

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Need large momentum spread for slow extraction

- 1. Jump RF phase such that bunch at unstable fixed point
- 2. Jump back
- 3. Let bunch rotate, switch RF off at large momentum spread



### **Comparison of bunch rotation techniques**

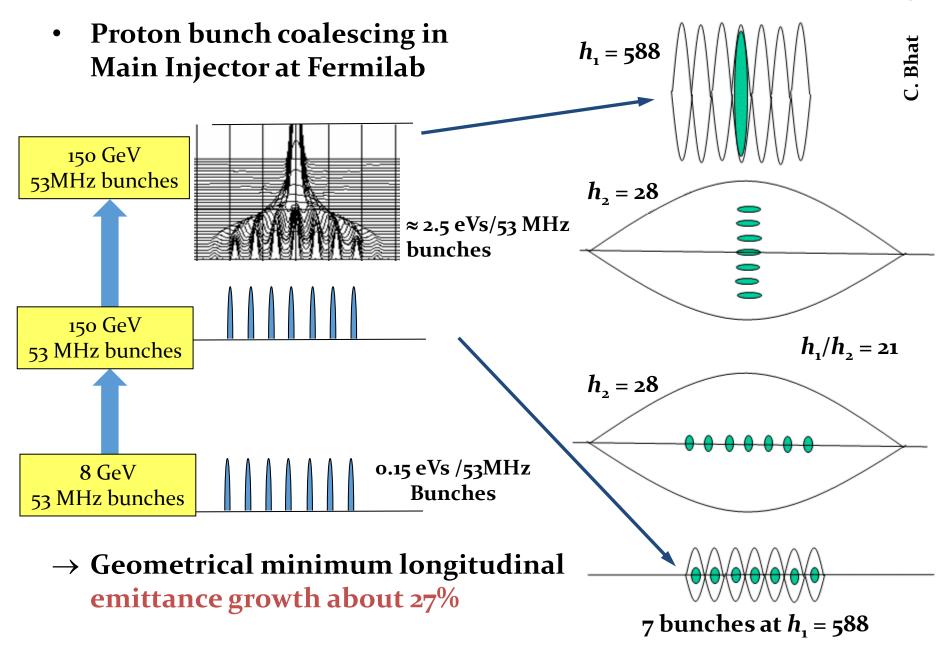
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 $\rightarrow$  Bunch length proportional to  $\sqrt{V}$ 

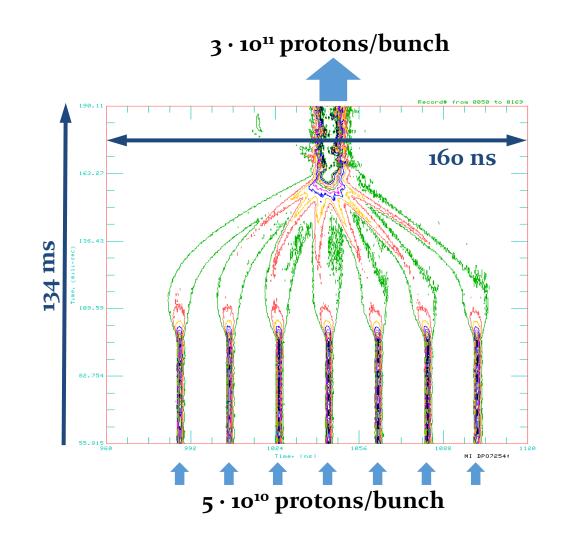
Which one to choose?

RF voltage jump	Jump to unstable fixed-point and back
+ Easy implementation: just raise voltage quickly	<ul> <li>+ Bunch always kept in bucket with large RF voltage</li> <li>+ Controlled RF phase jumps straightforward with digital RF sources</li> </ul>
<ul> <li>Power demand on RF systems during voltage jump</li> <li>Bunch kept at low RF voltage before</li> </ul>	<ul> <li>Non-linearity → rotation of more than π/2 in longitudinal phase space</li> </ul>

#### **Multi-bunch rotation at Fermilab: coalescing**<sup>35</sup>



### **Example: Main ring bunch coalescing**



→ Clever combination of bunch rotation and subsequent harmonic handover (rebucketing)

# Controlled longitudinal emittance blow-up

### **Controlled emittance blow-up**

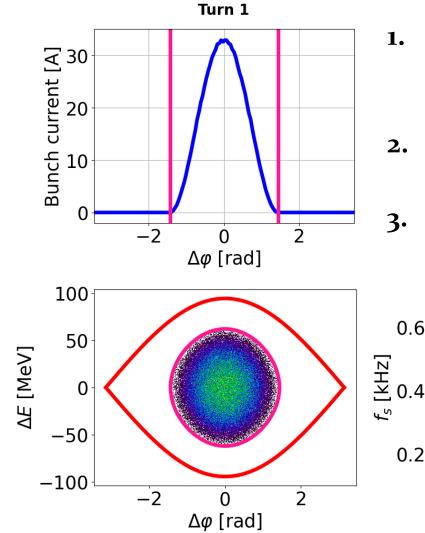
- How to reduce longitudinal density and increase bunch length in well-defined way?
- $\rightarrow$  Shake the RF bucket
- → Introduce some diffusion



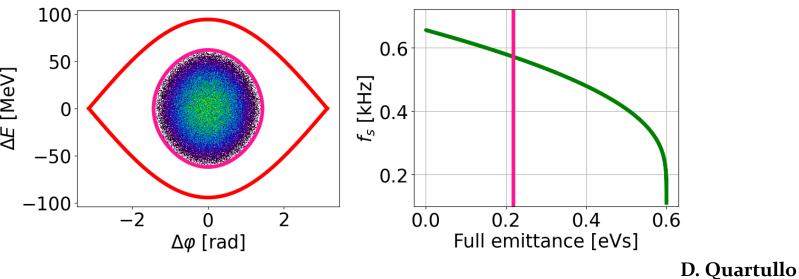
	Main RF system only		Higher-harmonic RF
1.	Phase noise modulation of main RF in fixed frequency band • Variant: fixed-frequency excitation during acceleration → Bunch Spreader	2.	Separate higher-harmonic RF system with periodic phase modulation

### **Emittance control with phase noise**

2. Phase noise modulation of main RF in fixed frequency band

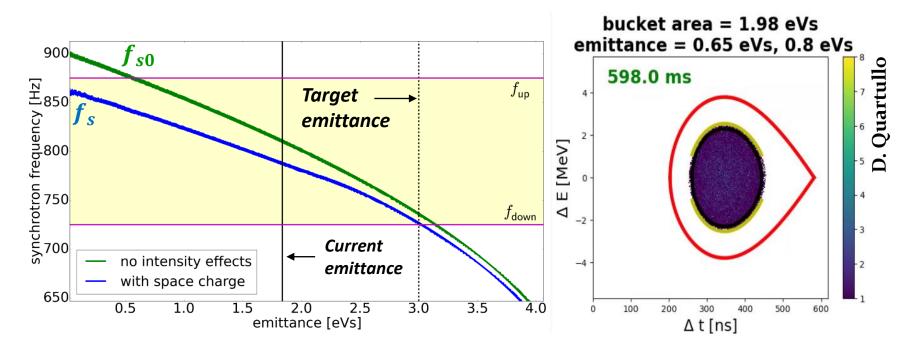


- 1. Choose upper frequency to cover synchrotron frequency at bunch centre
- 2. Choose lower frequency to match target emittance
- 3. Excite



### **Example: Phase noise blow-up in PS Booster**

- Noise excitation of bunch by band-width limited noise
- → Controlled longitudinal blow-up in the PSB



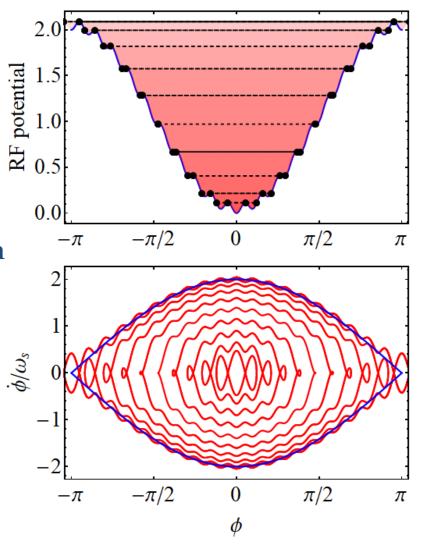
#### Strengths and weaknesses

- + Single harmonic, no need for 2<sup>nd</sup> RF system
- + Few parameters: band of excitation, amplitude
- Difficult to achieve smooth blow-up

### **Controlled emittance blow-up**

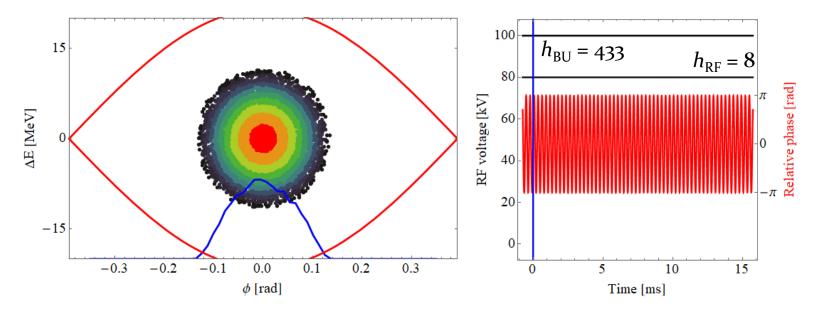
- How to reduce longitudinal density and increase bunch length in well-defined way?
- $\rightarrow$  Shake the RF bucket
- $\rightarrow$  Introduce some diffusion

1. Higher-harmonic RF with periodic phase modulation



### Higher-harmonic RF system

- Prefer high harmonic ratios, ideally > ~20
- Voltage ratios of the order of ~1

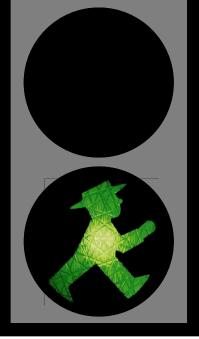


#### Strengths and weaknesses

- + Blow-up easily controllable: RF voltage and duration  $(h_{BU}/h_{RF}>20)$
- + Equalizes bunch-by-bunch parameters differences
- + Control of longitudinal distribution
- Requires additional RF system
- Difficult to adjust for smaller ratios  $h_{\rm BU}/h_{\rm RF}$



h and  $n \cdot h$ 



### **Arbitrary RF waveform**

**Replace**  $V \sin \phi \rightarrow V g(\phi) \rightarrow \text{arbitrary amplitude}$ 

**Equations of motion** 

$$\frac{d}{dt}\phi = \frac{h\eta\omega_{\rm rev}}{pR} \left(\frac{\Delta E}{\omega_{\rm rev}}\right)$$

$$\frac{d}{dt} \left(\frac{\Delta E}{\omega_{\rm rev}}\right) = \frac{qV}{2\pi} \left[g(\phi) - g(\phi_{\rm S})\right]$$
same structure
$$\frac{dq}{dt} = \frac{\partial H}{\partial p}$$

$$\frac{dp}{dt} = -\frac{\partial H}{\partial q}$$

#### The Hamiltonian describing the system becomes

$$H\left(\phi, \frac{\Delta E}{\omega_{\rm rev}}\right) = \frac{1}{2} \frac{h\eta \omega_{\rm rev}}{pR} \left(\frac{\Delta E}{\omega_{\rm rev}}\right)^2 - \frac{qV}{2\pi} \left[\int g(\phi) d\phi - g(\phi_{\rm S})\phi\right]$$

$$\eta = \frac{1}{\gamma_{\rm tr}^2} - \frac{1}{\gamma^2}$$

### **Arbitrary RF waveform**

$$H\left(\phi, \frac{\Delta E}{\omega_{\rm rev}}\right) = \frac{1}{2} \frac{h\eta \omega_{\rm rev}}{pR} \left(\frac{\Delta E}{\omega_{\rm rev}}\right)^2 - \frac{qV}{2\pi} \left[\int g(\phi) d\phi - g(\phi_{\rm S})\phi\right]$$
$$\mathbf{x} W(\phi)$$

→ RF potential  $W(\phi) = \frac{1}{\cos \phi_{\rm S}} \left[ \int g(\phi) \, d\phi - g(\phi_{\rm S}) \phi \right]$ → and without acceleration  $W(\phi, \phi_{\rm S} = 0) = \int g(\phi) \, d\phi$ 

#### Integral of RF voltage $\rightarrow$ RF potential

Two sinusoidal RF voltages

### Sinusoidal double-harmonic RF

• Double-harmonic sum voltage

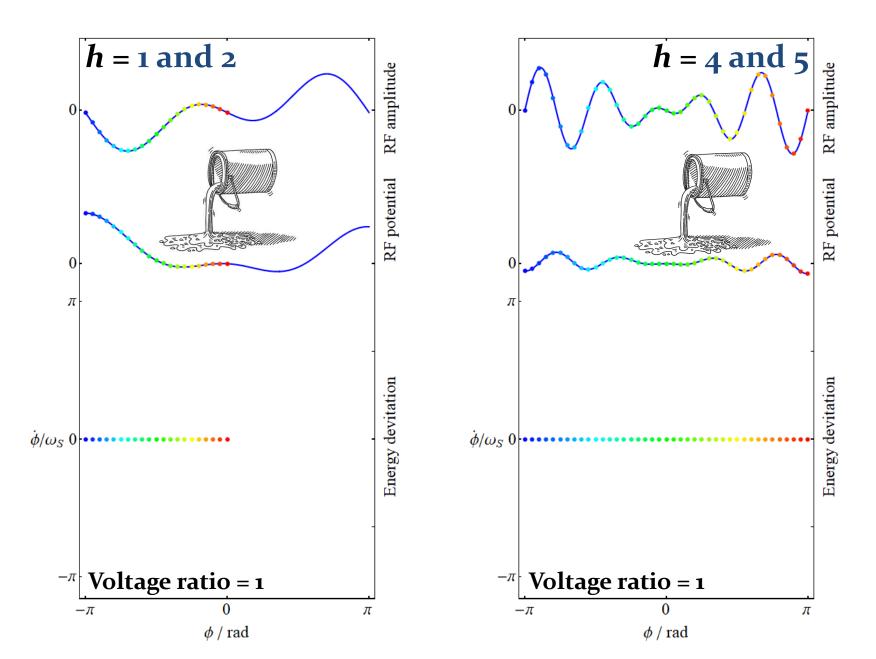
 $V_{\rm RF}(\phi) = V_1 \sin(h_1\phi + \phi_1) + V_2 \sin(h_2\phi + \phi_2)$ 

• Corresponding RF potential

$$W(\phi) = \frac{V_1}{h_1} \left[ \cos \phi_1 - \cos(h_1 \phi + \phi_1) \right] + \frac{V_2}{h_2} \left[ \cos \phi_2 - \cos(h_2 \phi + \phi_2) \right]$$

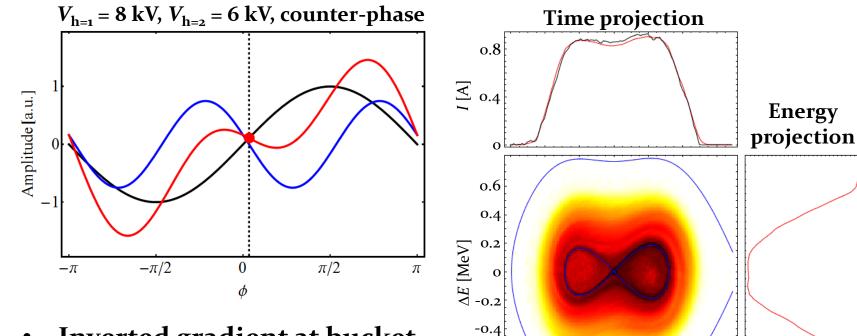
→ Actually one relative phase between both RF systems

### **Examples of double-harmonic RF voltage**



**Example application: space charge in PSB** <sup>49</sup> **RF amplitude**  $V_{\rm RF}(\phi) = V_1 \sin(h_1\phi + \phi_1) + V_2 \sin(h_2\phi + \phi_2)$ 

→ Space charge ∝ instantaneous current



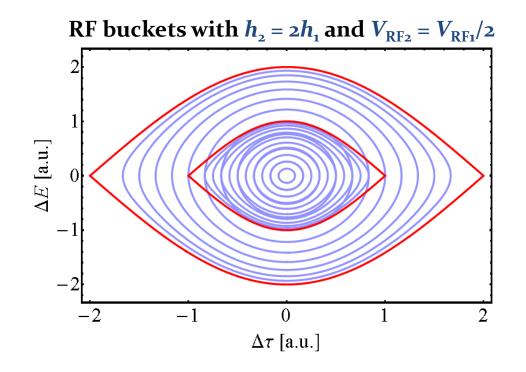
- Inverted gradient at bucket centre
- Flattened bunch with  $t = \frac{-400}{t \, [ns]} e^{-400}$ reduced peak current  $\rightarrow$  Space charge reduction at low energy

-0.6

# Rebucketing

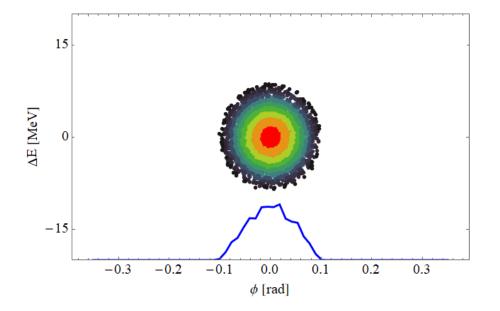
- Change of harmonic number from bucket centre to centre
- → Phase space aspect ratio:

$$\Delta E = \beta \omega_{\rm rev} \sqrt{\frac{q \mathcal{V}}{2\pi} E \hbar} \left| \frac{\cos \phi_0}{\eta} \right| \cdot \Delta \tau$$



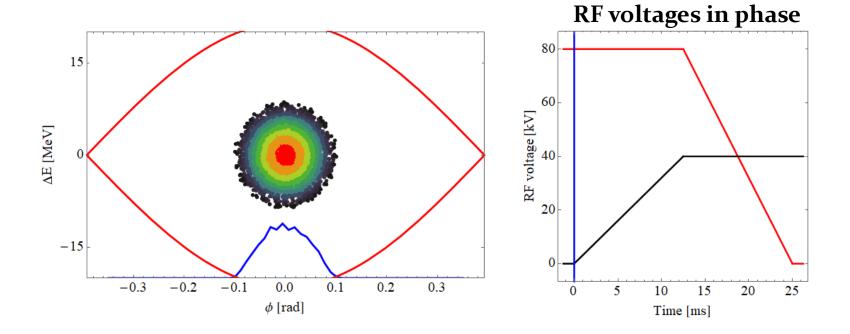
- Change of harmonic number from bucket centre to centre
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- Change of harmonic number from bucket centre to centre
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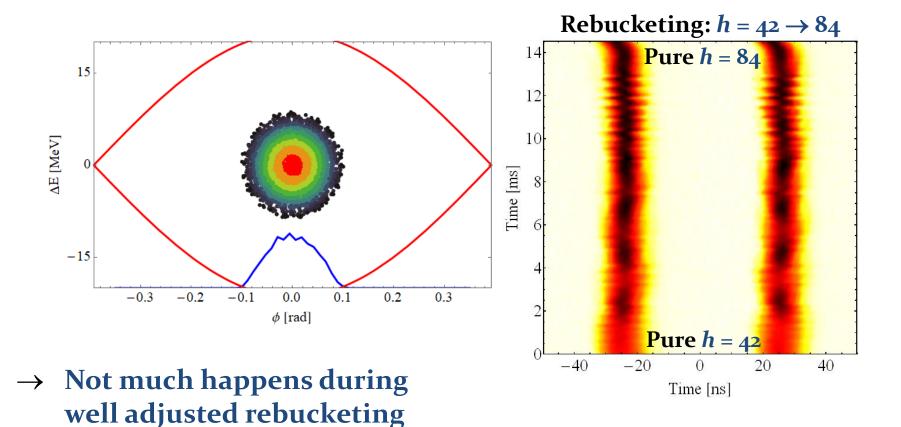
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- Change of harmonic number from bucket centre to centre
- → Phase space aspect ratio:

$$\Delta E = \beta \omega_{\rm rev} \sqrt{\frac{q \mathcal{V}}{2\pi} E \hbar} \left| \frac{\cos \phi_0}{\eta} \right| \cdot \Delta \tau$$

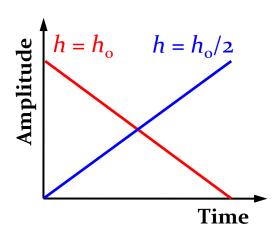
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Bunch merging and splitting

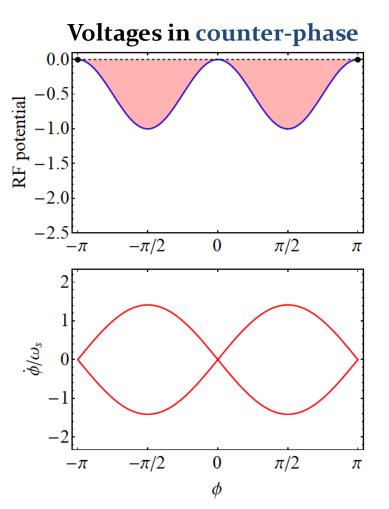
### Merging two bunches into one

- Double intensity per bunch
- No change of transverse beam parameters → more brightness
- $\rightarrow$  Increase RF voltage on  $h = h_o/2$  while decrease on  $h_o$

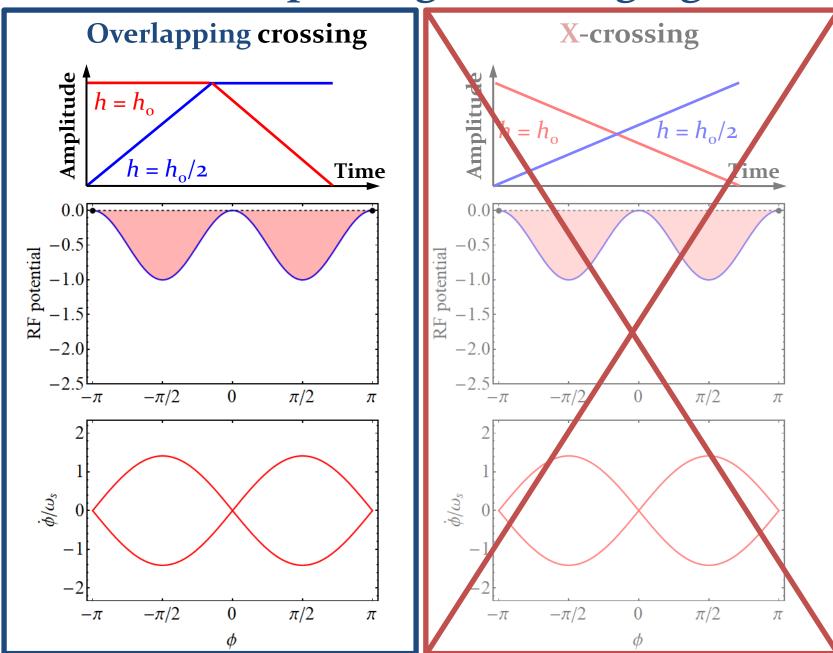




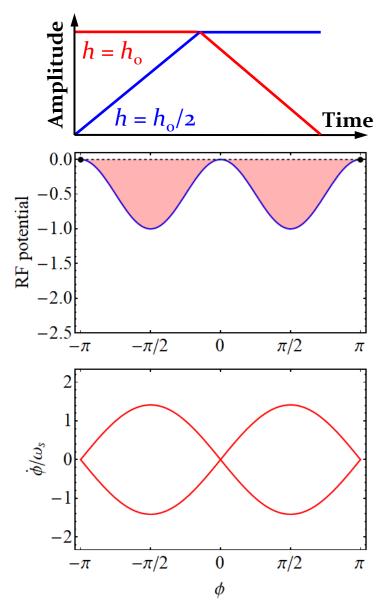
→ Not fully profiting from available RF voltage at both harmonics

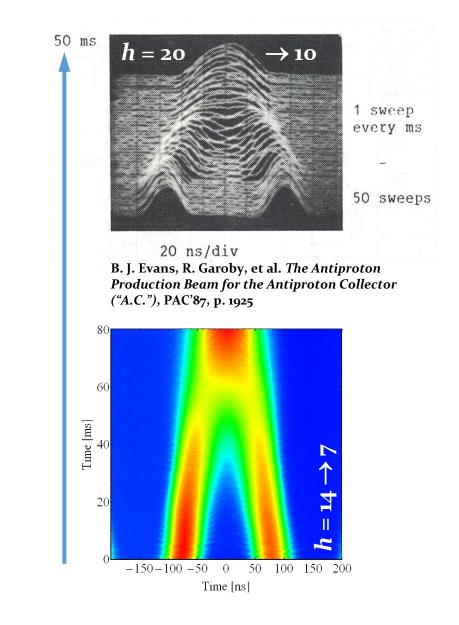


### **Bunch splitting and merging**

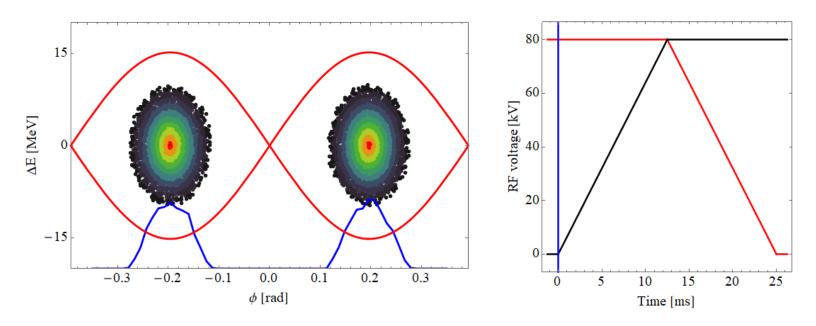


#### **Overlapping crossing**





#### → Simulation of an adiabatic manipulation

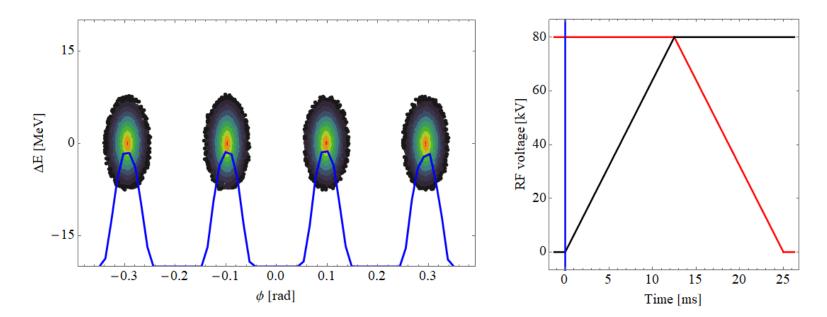


- Longitudinal distribution ideally unchanged
  - $\rightarrow$  Central part of the bunch remains in the centre

#### Strengths and weaknesses

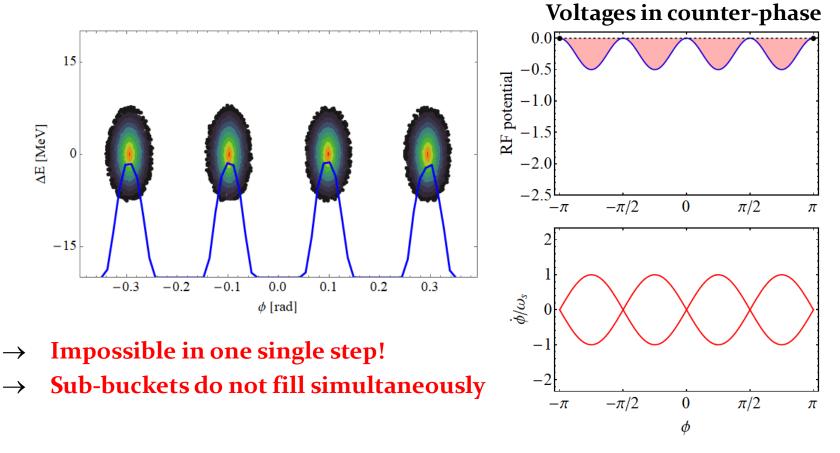
- + Simple manipulation to increase intensity per bunch
- Initial bunches must have identical longitudinal emittance
- Relative RF phase critical to avoid blow-up

- Generalization of bunch merging for multiple bunches?
- $\rightarrow \text{ Direct hand-over from } h \rightarrow h/n$



→ Impossible in one single step!

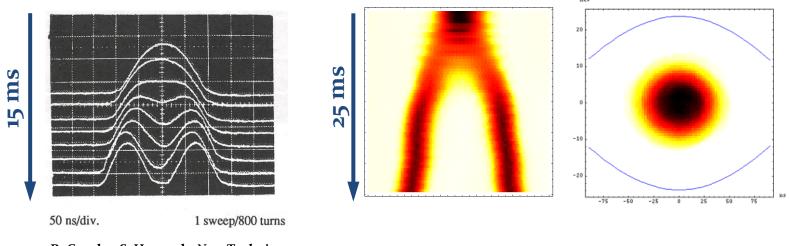
- Generalization of bunch merging for multiple bunches?
- $\rightarrow \text{ Direct hand-over from } h \rightarrow h/n$



→ Sequential manipulation of two mergings instead:  $h \rightarrow h/2 \rightarrow h/4$ 

### **Bunch splitting**

• Inverse bunch merging → Bunch Splitting

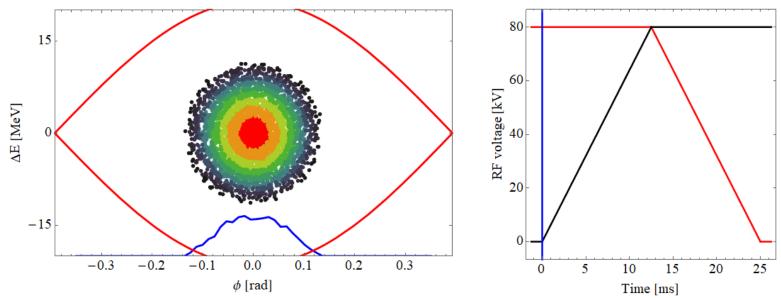


R. Garoby, S. Hancock, New Techniques for Tailoring Longitudinal Density in a Proton Synchrotron, EPAC'94, p. 282

- → Increase number of bunches and RF frequency
- → **Reduce bunch spacing**
- → Distribute intensity more equally around circumference

### **Bunch splitting**

• Inverse bunch merging → Bunch Splitting



- → Increase number of bunches and RF frequency
- → **Reduce bunch spacing**

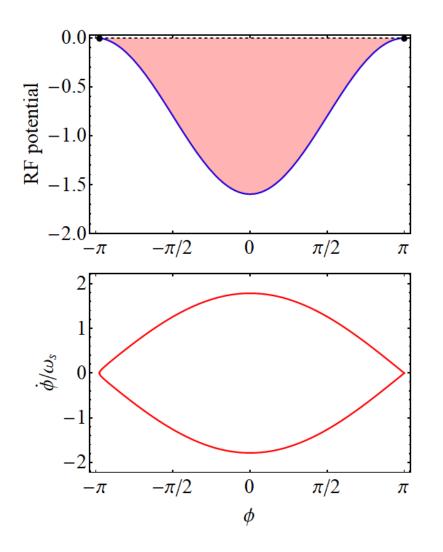


#### Strengths and weaknesses

- + Simple manipulation, to increase intensity per bunch
- Relative RF phase critical to avoid blow-up, especially for small bucket filling factors

### Extension of bunch splitting $\rightarrow$ Triple split

- $\rightarrow$  Apply RF voltage at 3 harmonics *h*, 2*h* and 3*h* at the same time
- → Form sub-buckets of equal size during process



1. RF voltages at *h* and 3*h* in phase

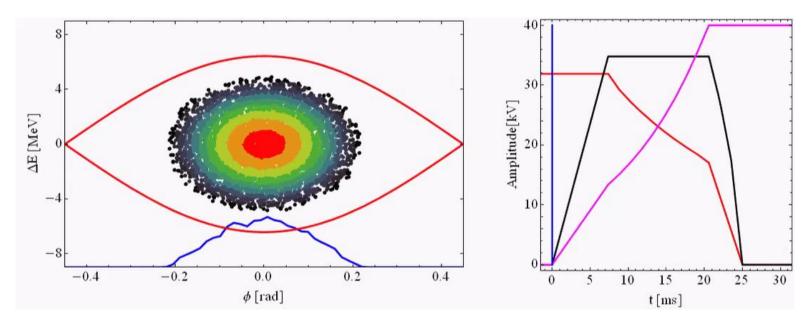
64

2. Voltage at 2*h* in counterphase to bucket particles outside

→ Defines voltage programs during the process

### **Bunch splitting and merging**

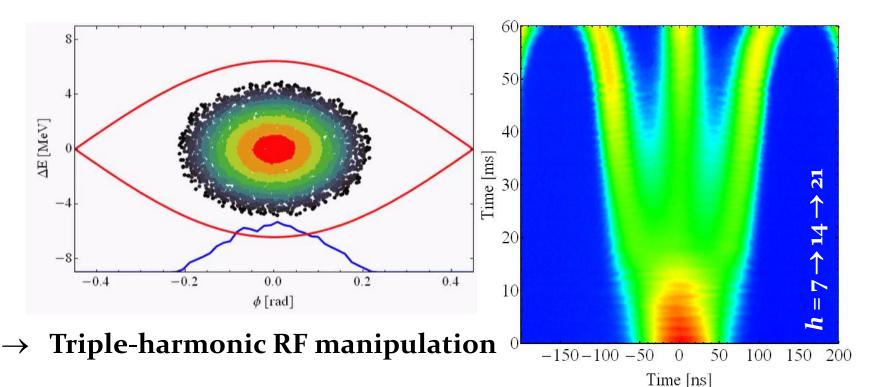
- $\rightarrow$  Apply RF voltage at 3 harmonics *h*, 2*h* and 3*h* at the same time
- → Form sub-buckets of equal size during process



- → Triple-harmonic RF manipulation
- ✓ Works on paper

### **Bunch splitting and merging**

- $\rightarrow$  Apply RF voltage at 3 harmonics *h*, 2*h* and 3*h* at the same time
- → Form sub-buckets of equal size during process



... and with beam as well!

### Summary

- RF can do so much more than just acceleration
- →Adiabaticity and synchrotron frequency decide whether a manipulation is slow or fast
- The RF potential is the integral of voltage
   → Fill occupied phase space into the bucket



# Thank you very much for your attention!

# **RF** Manipulations II



H. Damerau

CERN



The CERN Accelerator School

#### **RF for Accelerators**

30 June 2023



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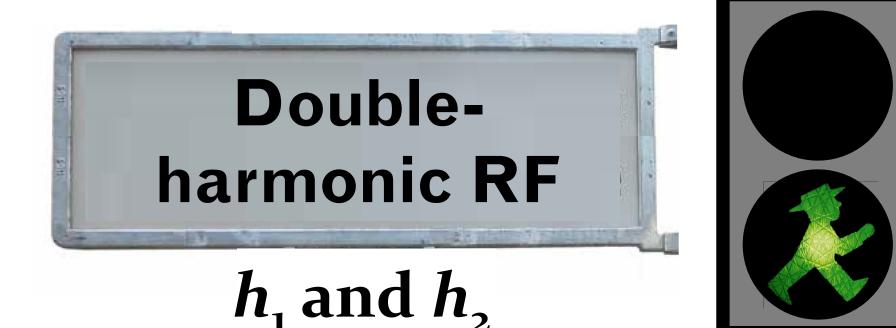
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### Outline

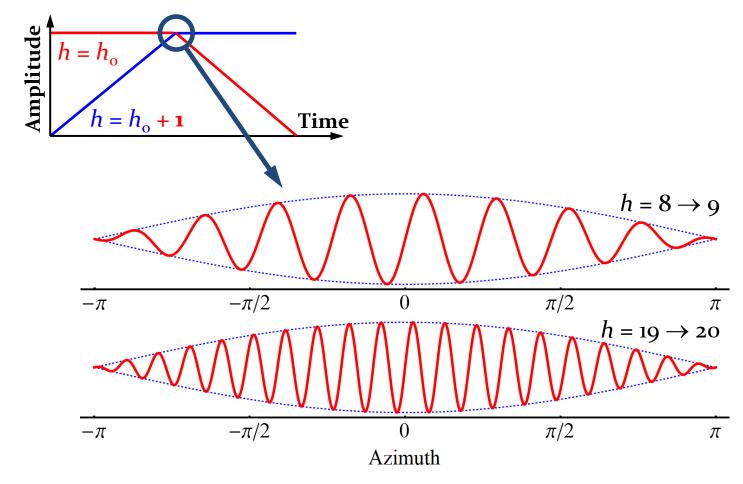
- Introduction
- Single-harmonic RF
- Double-harmonic RF
  - Rebucketing, bunch merging and splitting
  - Batch compression
- Double RF system
  - Slip stacking
- Non-sinusoidal RF voltages
  - Barrier bucket manipulations
- Sequences, design and implementation
  - Batch compression, merging and splitting
  - A real world example
- Summary



# **Batch compression**

#### **Change harmonic in small steps**

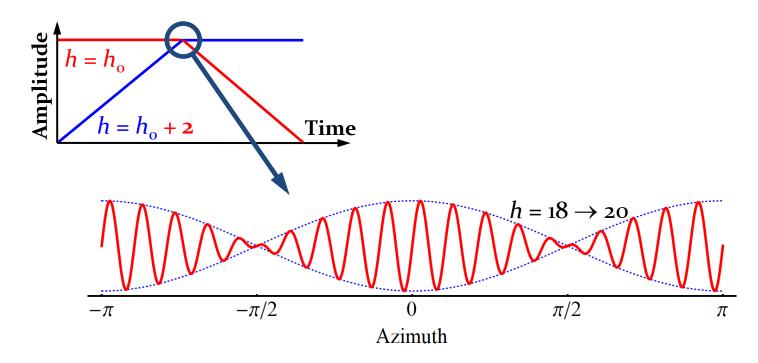
- $\rightarrow$  Control spacing between bunches  $\rightarrow$  proportional to  $T_{rev}/h$
- $\rightarrow$  Slowly change harmonic:  $h \rightarrow h + n$  with n = 1, 2, 3, ...



→ Amplitude modulation for RF voltage envelope

#### **Change harmonic in small steps**

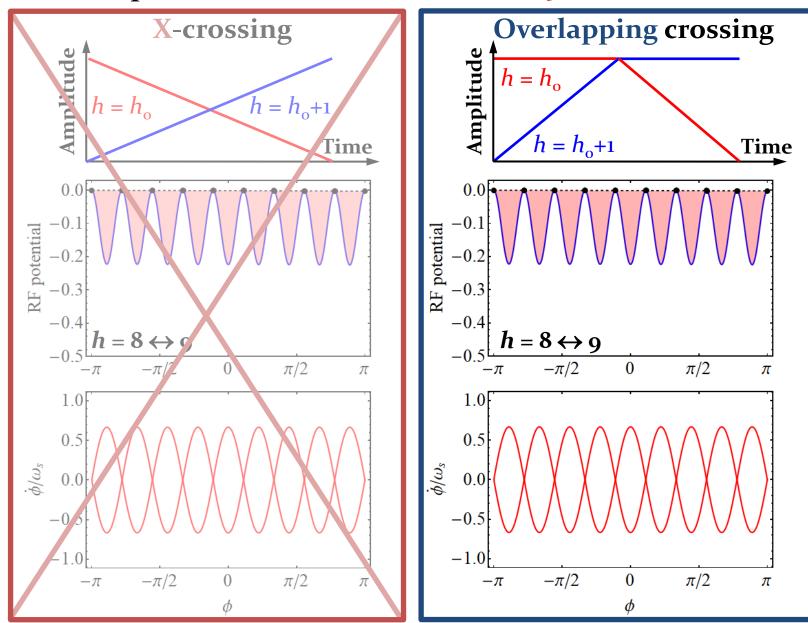
- $\rightarrow$  Control spacing between bunches  $\rightarrow$  proportional to  $T_{\rm rev}/h$
- $\rightarrow$  Slowly change harmonic:  $h \rightarrow h + n$  with n = 1, 2, 3, ...



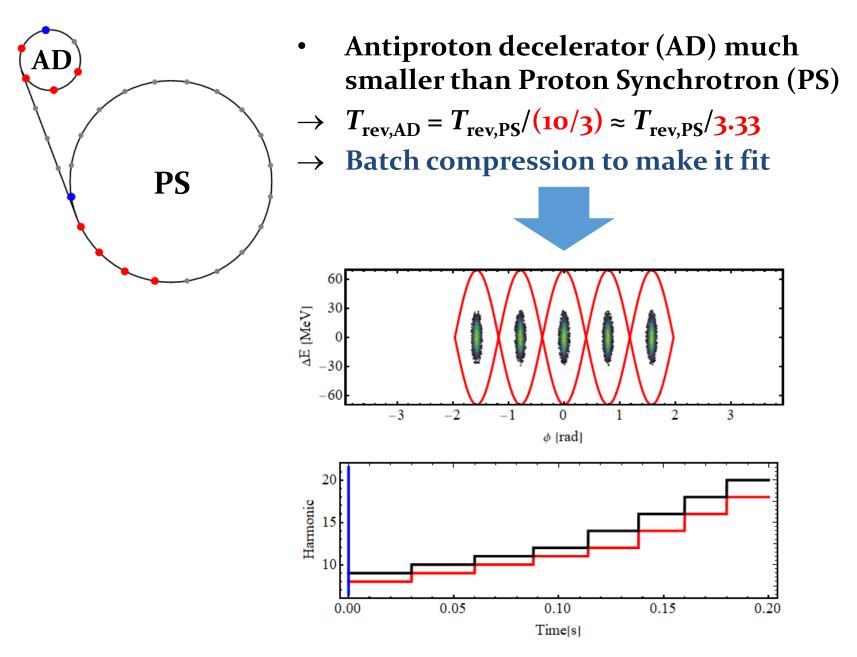
→ Maximum length of bunch train limited to  $2\pi/\Delta h$  to avoid azimuth regions with no RF

#### **RF potential and buckets during handover**

• Example: hand-over from h = 8 to h = 9 (and back)

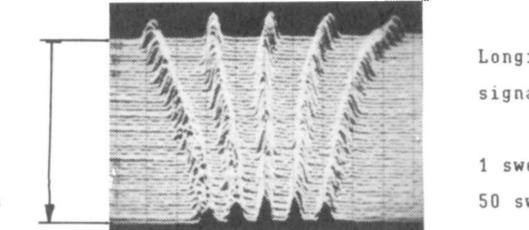


#### **Example batch compression for AD**



#### **Example batch compression for AD**

 $h = 10 \rightarrow 12 \rightarrow 14 \rightarrow 16 \rightarrow 18 \rightarrow 20$ 



Longitudinal PU signal 1 sweep/2.8 ms 50 sweeps - 20 ns/div.

Time

140 ms

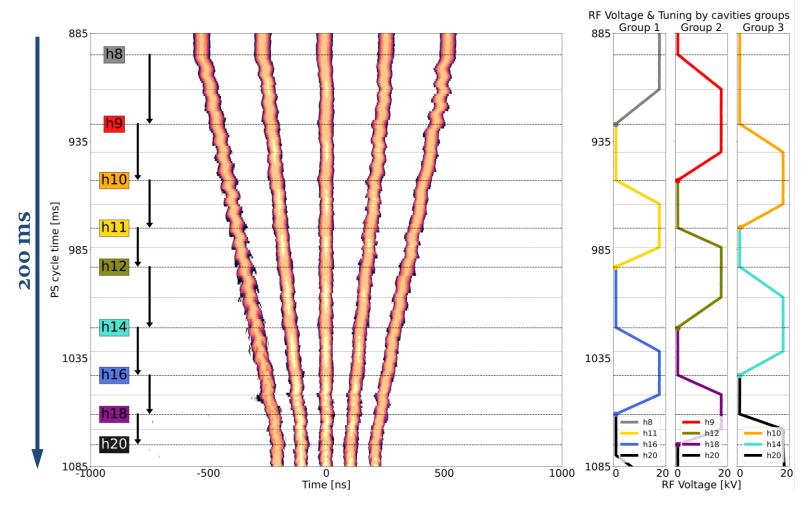
R. Cappi, B. J. Evans, R. Garoby, *Status of the anti-proton production beam in the CERN PS*, Part. Accel. 26 (1990), p. 217

#### Strengths and weaknesses

- + Hand-over from bucket centre to bucket centre
   → Robust RF manipulation
- + No particles at unstable fixed point
- Requires two groups of active RF cavities, ideally with a 3<sup>rd</sup> preparing for subsequent harmonic
- Complex RF voltage programmes

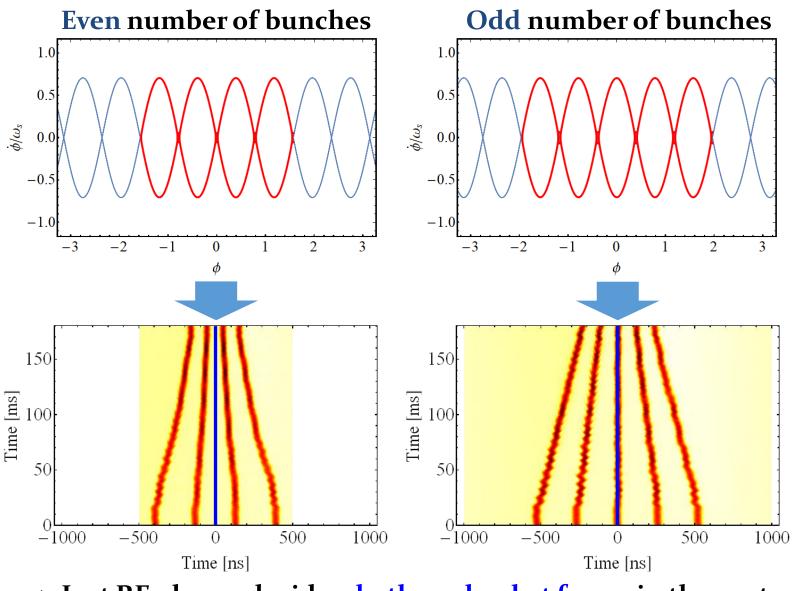
#### **Example batch compression for AD**

 $h = 8 \rightarrow 9 \rightarrow 10 \rightarrow 11 \rightarrow 12 \rightarrow 14 \rightarrow 16 \rightarrow 18 \rightarrow 20$ 



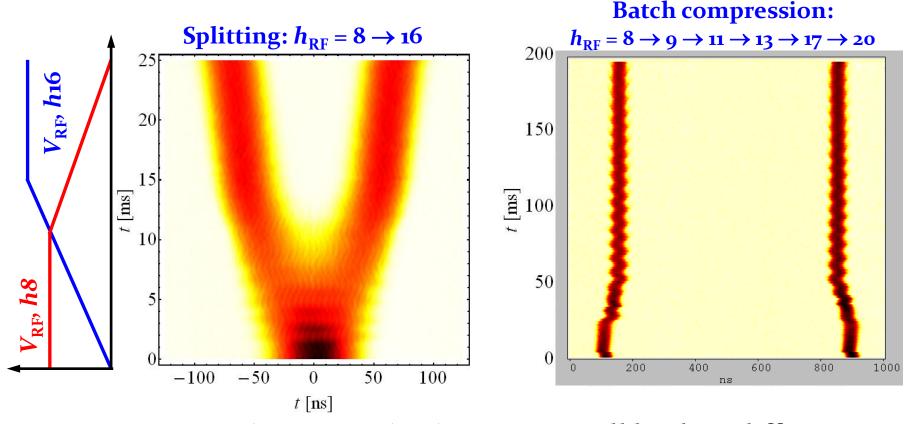
C. Lombard et al., Improved antiproton production beam at CERN, PAC2023

#### Symmetry of batch compression



→ Just RF phases decide whether a bucket forms in the center

#### **Symmetry of RF manipulations**

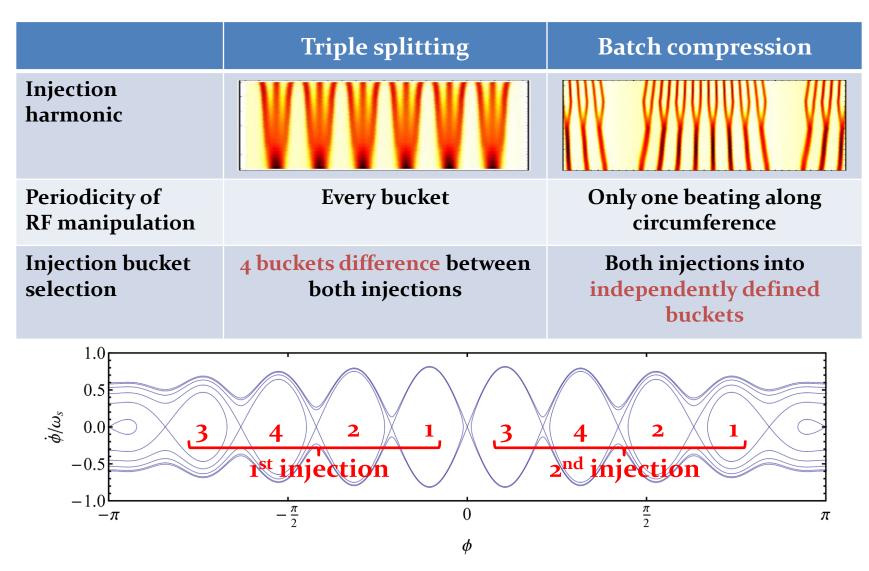


- Works in every bucket
- $\rightarrow$  Periodicity: h = 8

- All buckets different (even and odd harm.)
- $\rightarrow$  Periodicity: h = 1

 $\rightarrow$  All RF sources must be synchronous with respect to  $f_{\rm rev}$  at any harmonic

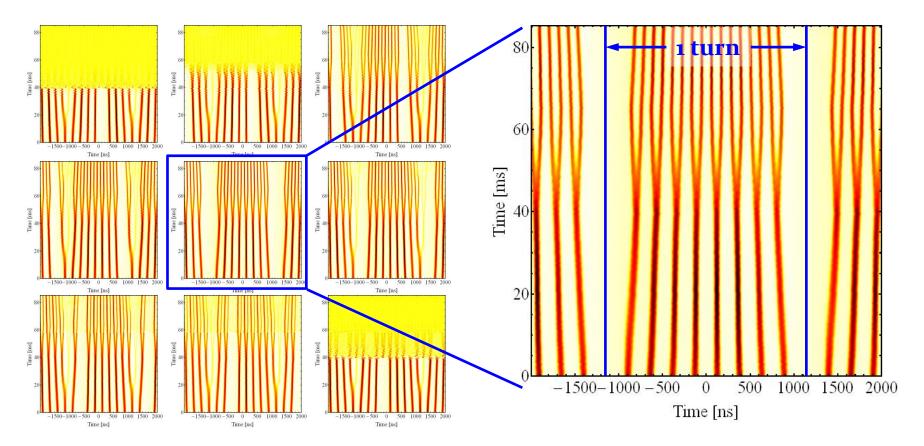
### **Bucket numbering for RF manipulations**



→Must inject into the correct bucket numbers

#### **Bucket numbering**

- Bunches must be placed into the correct buckets
- $\rightarrow$  Must respect bucket number  $\rightarrow$  azimuth relative to phase of h = 1





## Two beams

## **Slip stacking**

#### Momentum, radial position and frequency

- Ideal beam circulates with the expected revolution frequency  $(\Delta f = \mathbf{o})$  on the central orbit  $(\Delta R = \mathbf{o}) \rightarrow \Delta p = \mathbf{o}$
- **Real beam** behaviour is calculated using

Variables	Equations
B, p, R	$\frac{dp}{p} = \gamma_{\rm tr}^2 \frac{dR}{R} + \frac{dB}{B}$
f, p, R	$\frac{dp}{p} = \gamma^2 \frac{df}{f} + \gamma^2 \frac{dR}{R}$
B,f,p	$\frac{dB}{B} = \gamma_{\rm tr}^2 \frac{df}{f} + \frac{\gamma^2 - \gamma_{\rm tr}^2}{\gamma^2} \frac{dp}{p}$
B,f,R	$\frac{dB}{B} = \gamma^2 \frac{df}{f} + (\gamma^2 - \gamma_{\rm tr}^2) \frac{dR}{R}$

C. Bovet, R. Gouiran, I. Gumowski, K. H. Reich, A selection of formulae and data useful for the design of A.G. synchrotrons, CERN-MPS-SI-Int-DL-70-4

#### Momentum, radial position and frequency

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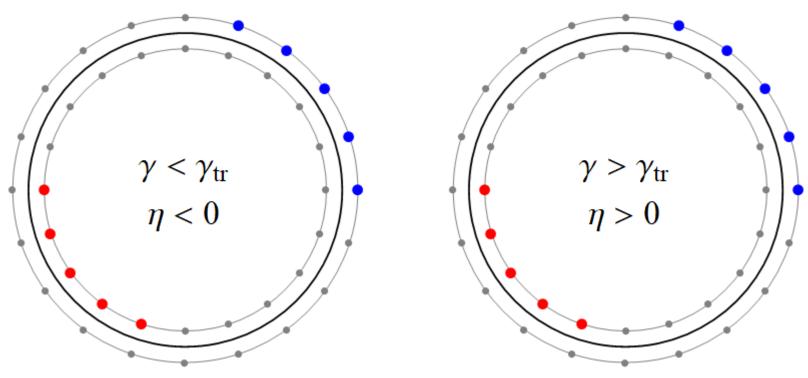
- Ideal beam circulates with the expected revolution frequency  $(\Delta f = \mathbf{o})$  on the central orbit  $(\Delta R = \mathbf{o}) \rightarrow \Delta p = \mathbf{o}$
- **Real beam behaviour is calculated using**

Variables	Equations	C. Bovet, R. Gouiran, I. Gumowski, K. H. Reich, A selection of formulae and data useful for the design of A.G. synchrotrons, CERN-MPS-SI-Int-DL-70-4
B, p, R	$\frac{dp}{p} = \gamma_{\rm tr}^2 \frac{dR}{R} + \frac{dB}{B}$	
f,  p,  R	$\frac{dp}{p} = \gamma^2 \frac{df}{f} + \gamma^2 \frac{dR}{R}$	
B,f,p	$\frac{dP}{B} = \gamma_{\rm tr}^2 \frac{df}{f} + \frac{\gamma^2 - \gamma_{\rm tr}^2}{\gamma^2} \frac{dp}{p}$	At constant bending field
B,f,R	$\frac{dP}{B} = \gamma^2 \frac{df}{f} + (\gamma^2 - \gamma_{\rm tr}^2) \frac{dR}{R}$	<i>f</i> , <i>p</i> and <i>R</i> are equivalent

→ RF frequency controls beam momentum and radial position ...when the beampipe is wide enough

#### **Two co-rotating beams simultaneously?**

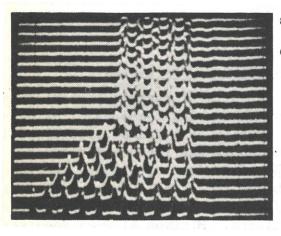
• Difference in revolution frequencies causes phase slip



- 1. Increase intensity by slip-stacking of bunches
- 2. Reduce bunch spacing  $\rightarrow$  Interleaved slip-stacking

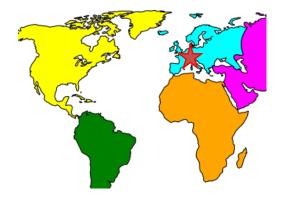
#### Two co-rotating beams simultaneously

• First slip-stacking ('azimuthal combination') in CERN PS



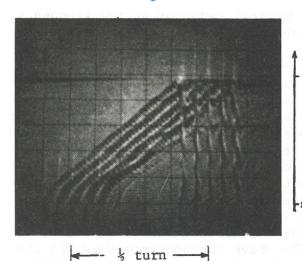
scope trigger on f<sub>2</sub> 1 trace/50 turns

bunch combination



injection

 $E_{\rm kin}$  = 800 MeV



**p** = 26 GeV

fast ejection

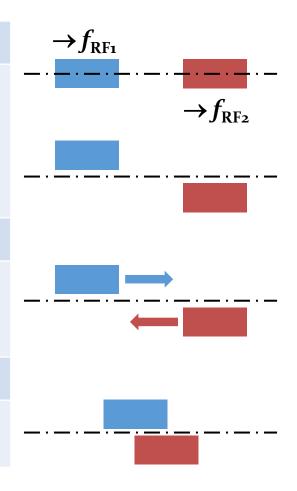
start separation of beams with

D. Boussard,Y. Mizumachi, Production of beams with high line-density by azimuthal combination of bunches in a synchrotron, PAC'79, p. 3623

## Slip stacking procedure

#### 1. Separate RF for two beams to split RF

- Inject with momentum offset
- RF frequency modulation for bunches separated in time
  - Sufficient RF system bandwidth for modulation at  $f_{rev}$
- 2. Drift/slippage with  $\Delta f_{\rm RF} = f_{\rm RF1} f_{\rm RF2}$
- Both RF frequencies active simultaneously
- Only minor perturbations of buckets with sufficient  $\Delta f_{RF}$
- 3. Approach of  $f_{\rm RF1}$  and  $f_{\rm RF2}$
- Carefully optimized compromise between adiabaticity and bucket perturbations



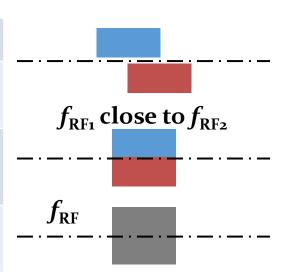
87



## Slip stacking procedure



- 3. Approach of  $f_{\rm RF1}$  and  $f_{\rm RF2}$ 
  - Carefully optimized compromise between adiabaticity and bucket perturbations
- 4. Recapture to common RF frequency:  $f_{RF1} = f_{RF1} + f_{RF2}/2$
- Carefully optimized compromise between adiabaticity and bucket perturbations



Restaurant RF

Menu à 15€ Entrée + plat/plat + dessert

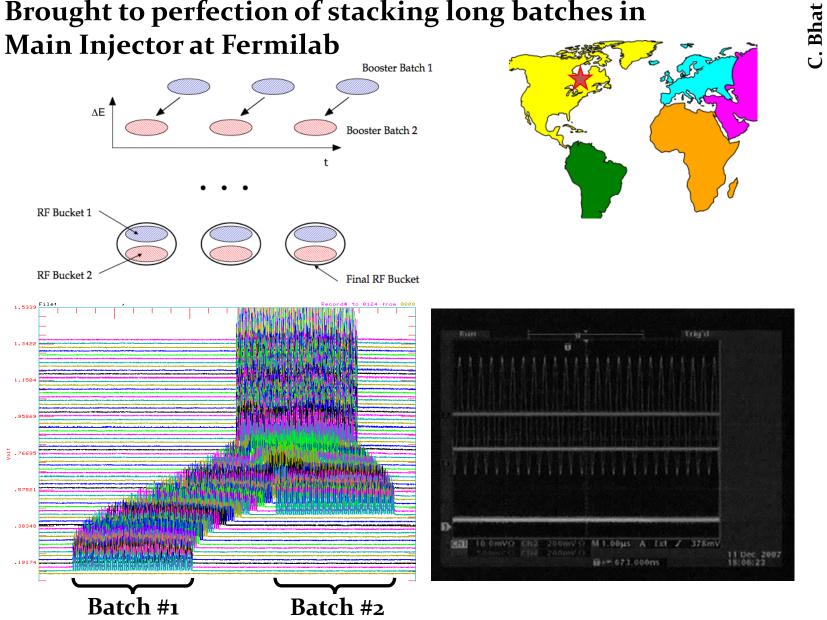
> Menu à 20€ Entrée + plat + dessert

Fromage: supplement 5  $\in$ 

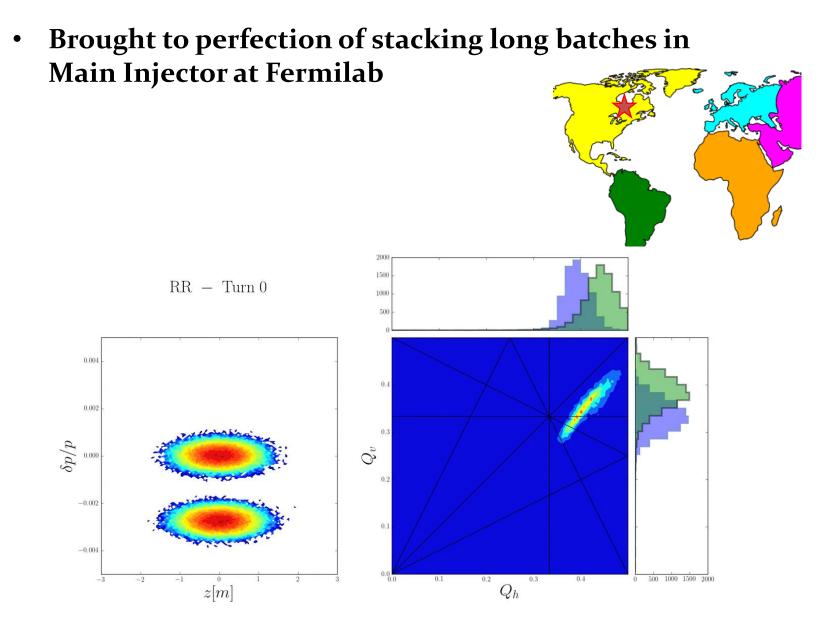


### **Slip stacking**

Brought to perfection of stacking long batches in • Main Injector at Fermilab

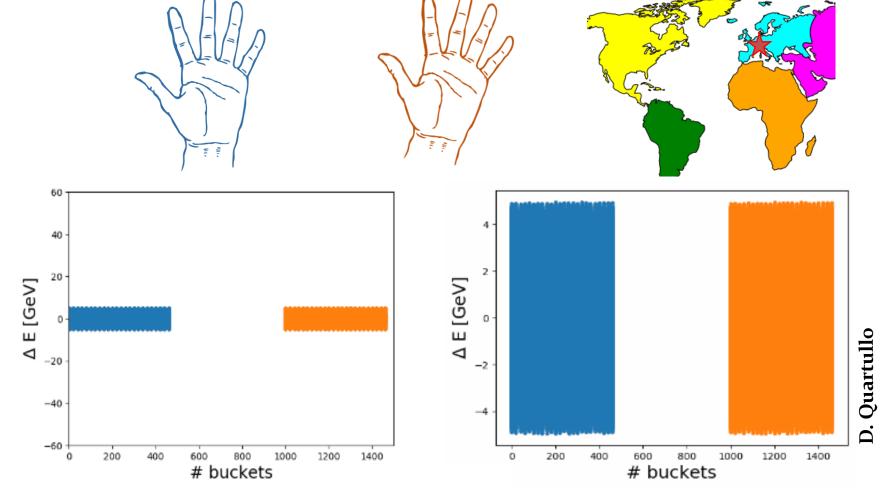


### **Slip stacking**



#### **Example: interleaved slip-stacking**

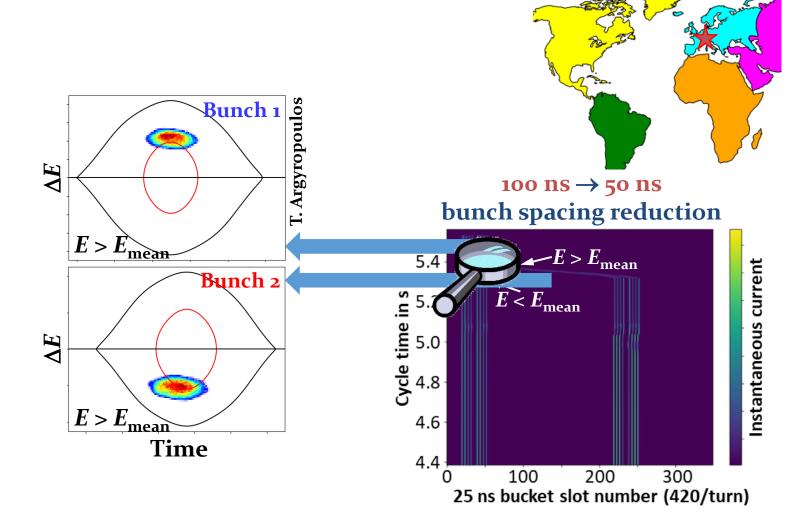
Reduce bunch spacing by interleaved slip-stacking



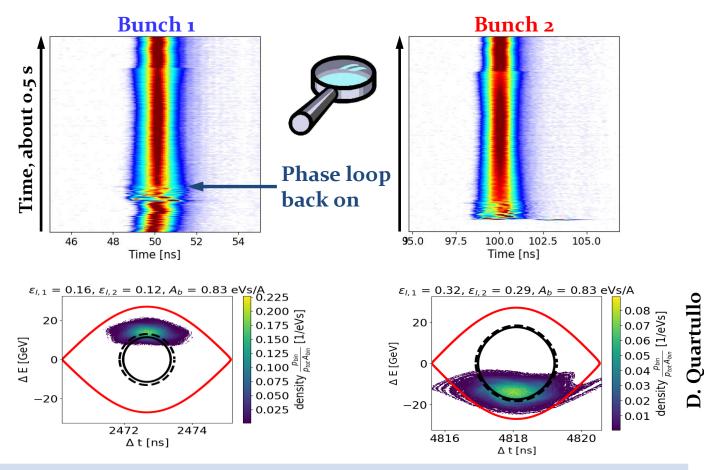
- $\rightarrow$  Reduce bunch spacing from 100 ns to 50 ns
- $\rightarrow$  Ion beams for LHC in the SPS at CERN

#### **Example: interleaved slip-stacking**

• Reduce bunch spacing by interleaved slip-stacking: SPS



#### **Example: interleaved slip-stacking**

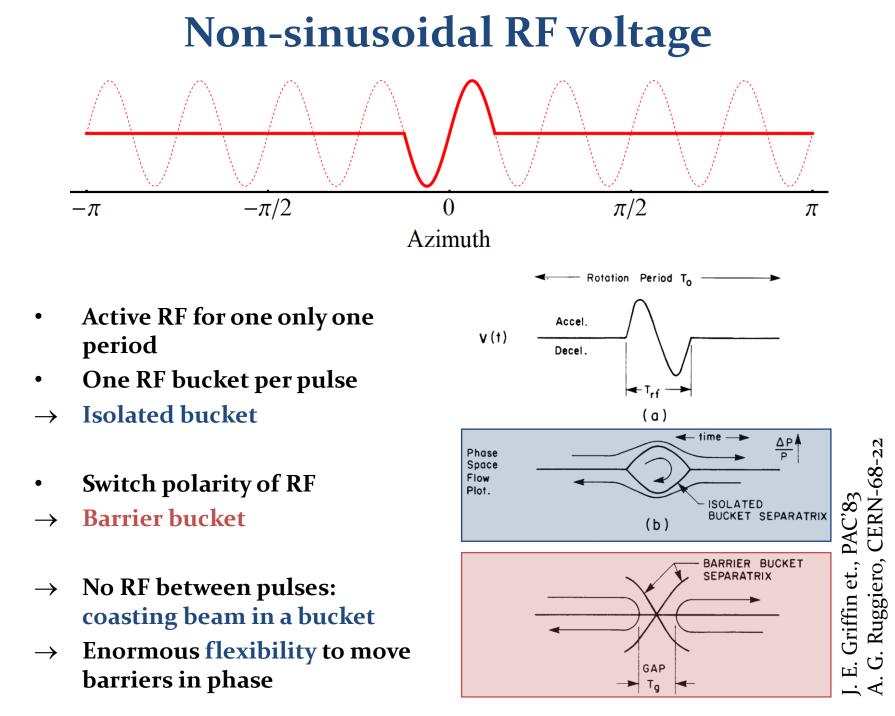


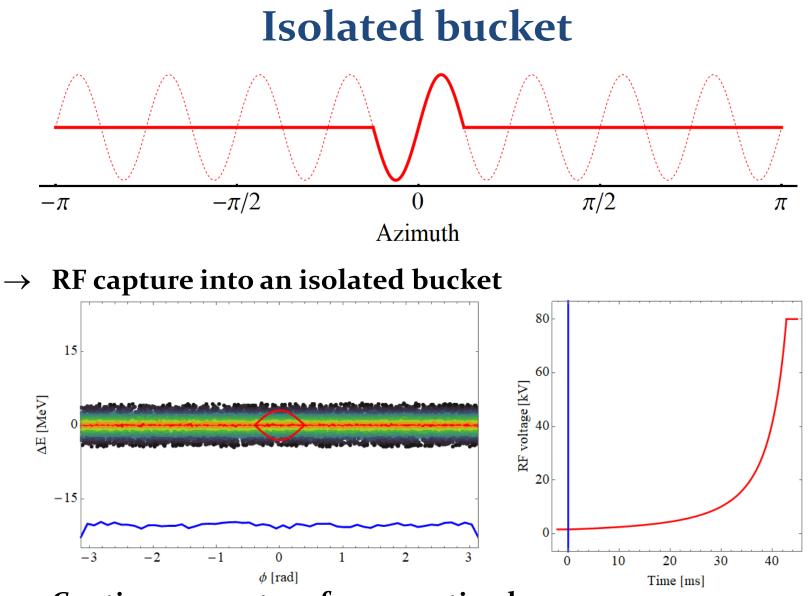
#### Strengths and weaknesses

- + Powerful stacking scheme
- + Partial scheme combined with injection and extraction
- Large emittance growth at recapture
- **Complex implementation:** modulation, two RF frequencies



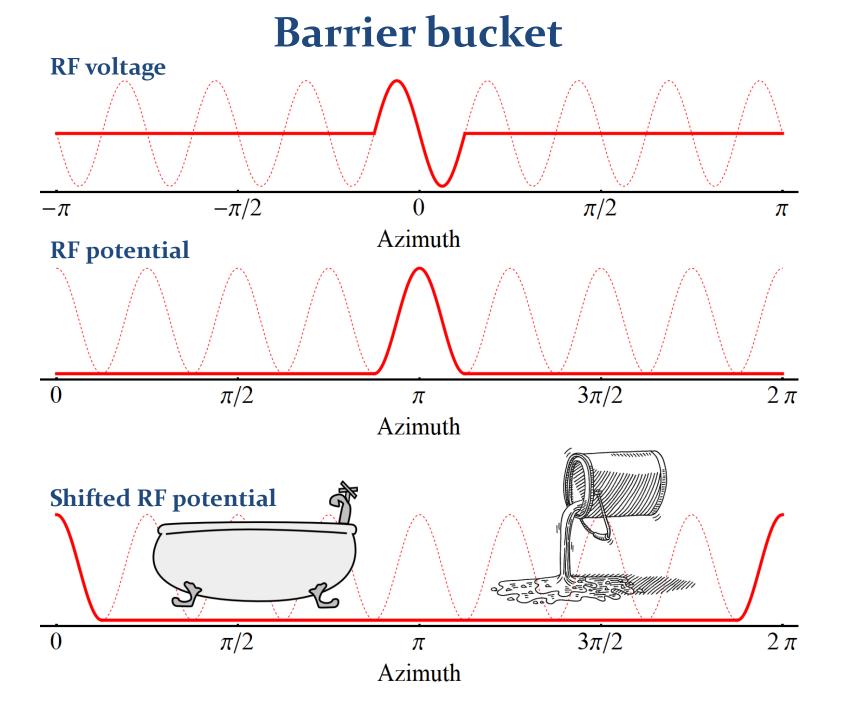
## **Isolated and barrier bucket**

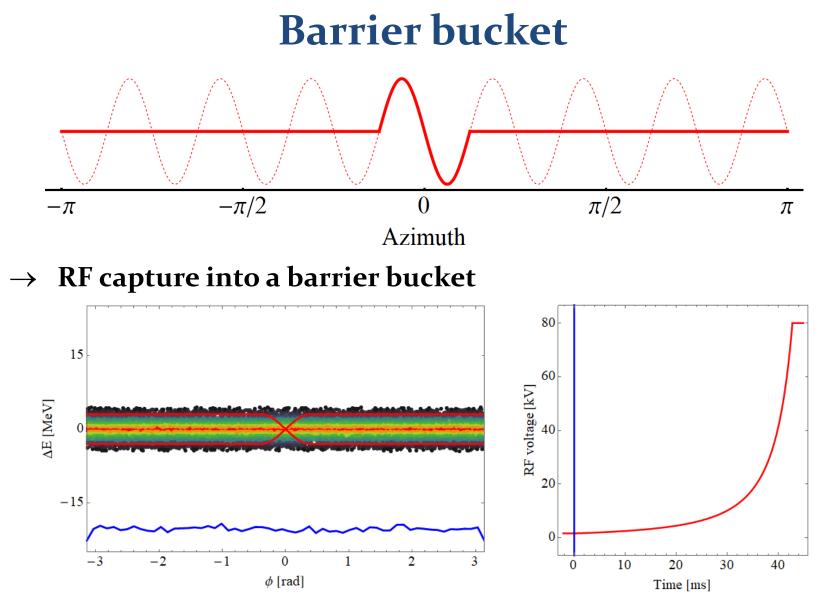




→ Continuous capture from coasting beam

→ Little practical application until today



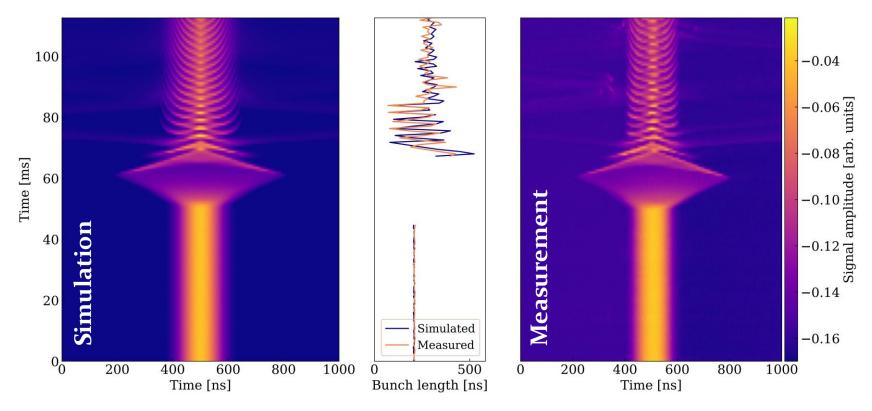


→ Combine advantages of continuous and bunched beam

→ Most simple application: Generate particle free gap

### **Barrier-bucket application in CERN PS**

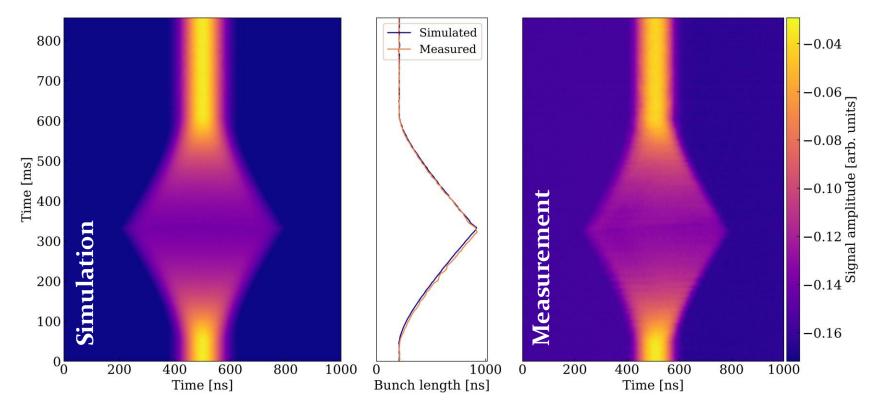
• Compress or stretch bunch by moving phase of RF barriers



- → Perfect bunch length manipulation, but slow
- → Bunch oscillations very well predicted by simulations

### **Barrier-bucket application in CERN PS**

• Compress or stretch bunch by moving phase of RF barriers

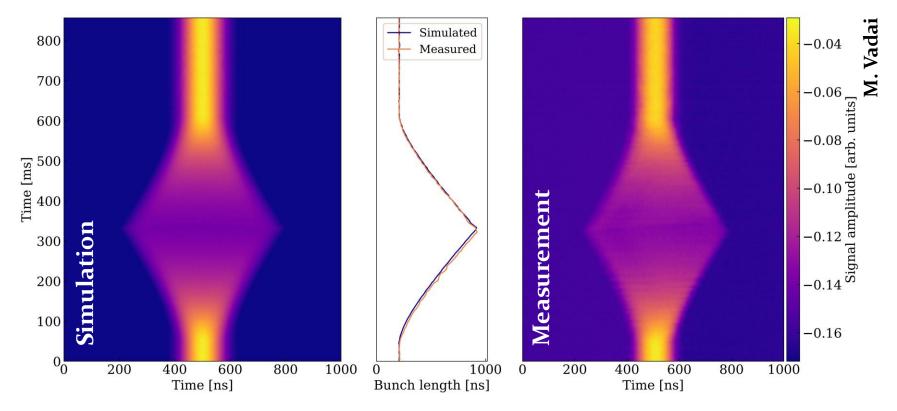


→ Perfect bunch length manipulation

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### **Barrier-bucket application in CERN PS**

• Compress or stretch bunch by moving phase of RF barriers

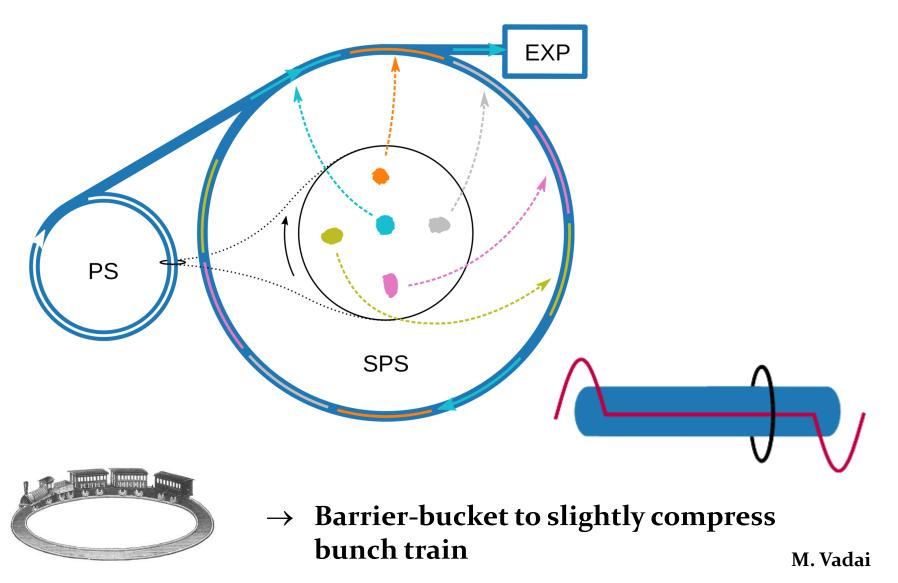


#### → Strengths and weaknesses

- + Very flexible, control of bunch length and peak current
- Difficult to generate large RF voltage with wide-band RF system
- Compensation of beam loading at high intensity

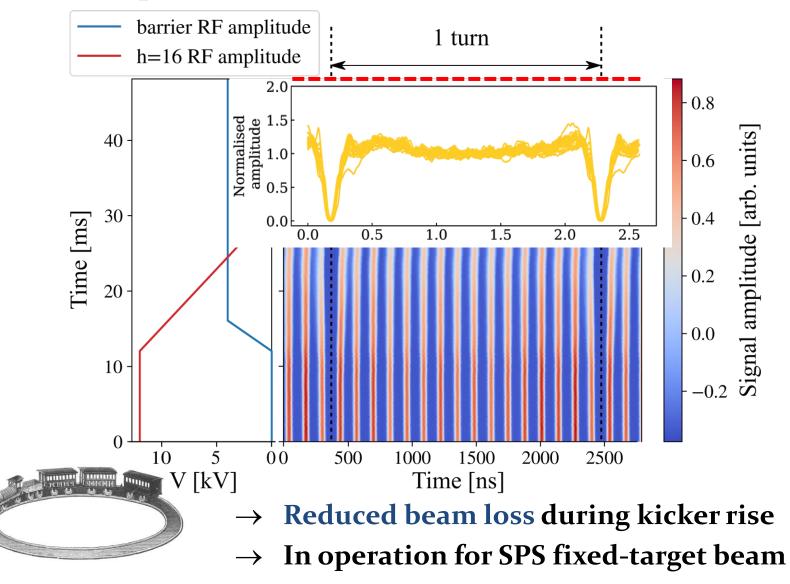
#### Loss reduction with barrier-bucket transfer

• RF manipulation allows generating a gap for extraction kicker



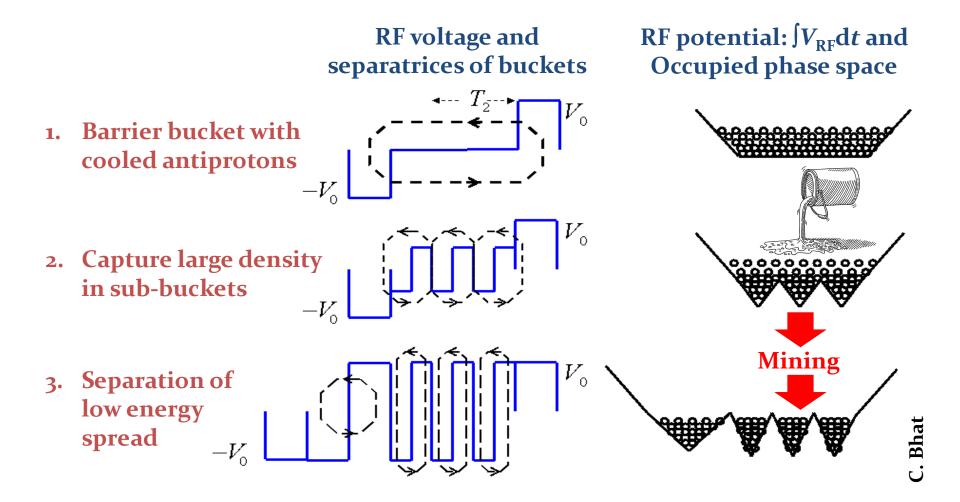
#### Loss reduction with barrier-bucket transfer

• RF manipulation to extract (almost) un-bunched beam



#### Momentum mining at Fermilab

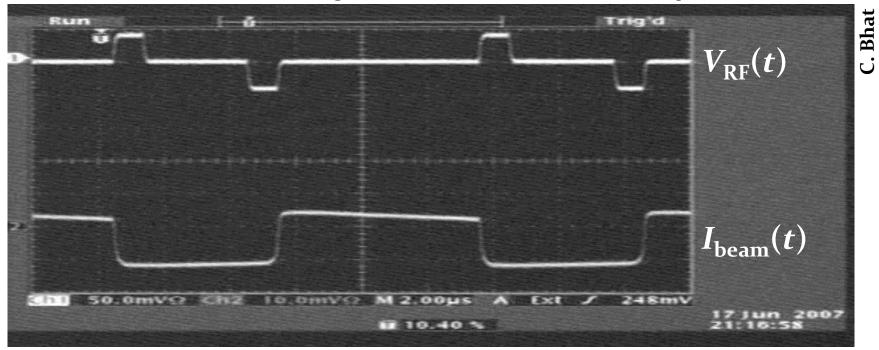
- Mining of antiprotons at large longitudinal phase space density
- $\rightarrow$  Developed at Fermilab for proton-antiproton collider Tevatron



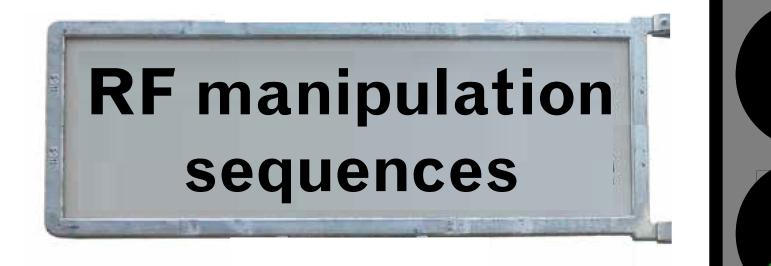
#### **Momentum mining Fermilab**

- Mining of antiprotons at large longitudinal phase space density
- $\rightarrow$  Developed at Fermilab for proton-antiproton collider Tevatron

Momentum mining in action in the Fermilab Recycler



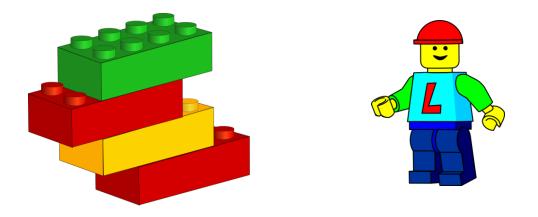
#### → One of the most evolved RF manipulation ever performed!



# **Example: BCMS beam at CERN**

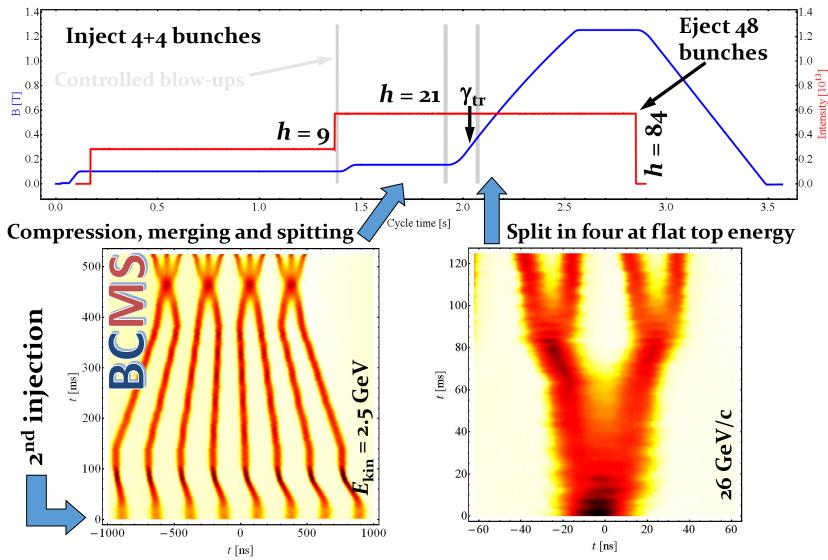
#### **RF** manipulation sequences

• Single RF manipulation often not sufficient to make a beam



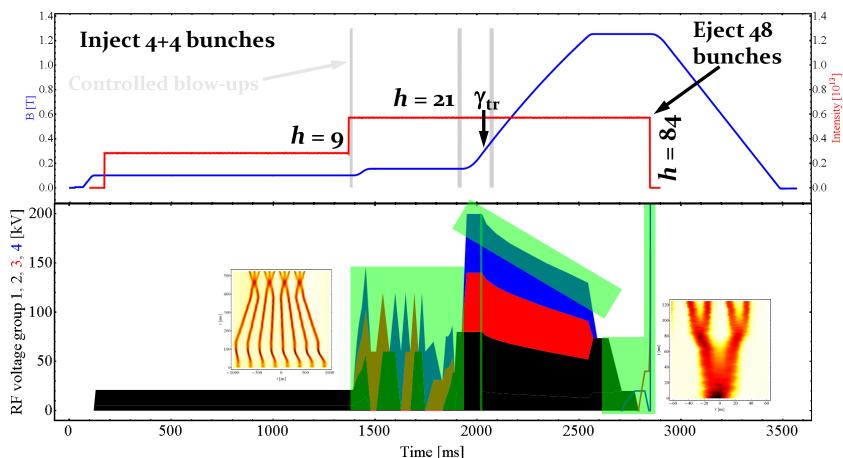
→ Real-life RF manipulations sequence of basic building blocks

#### **Example: LHC-type beam in the CERN PS**



• RF manipulations control all longitudinal parameters

# **Example: LHC-type beam in the CERN PS**



- $\rightarrow$  RF manipulation from 8 bunches in h = 9 to 12 in h = 21
- → Transition crossing
- $\rightarrow$  RF voltage reduction during acceleration
- $\rightarrow$  Splitting at the flat-top
- → Bunch shortening (rotation) before extraction



# **Design and implementation**

#### Example: Ions with 75 ns bunch spacing

Question:

Can the Proton Synchrotron produce ion bunches with a spacing of 75 ns for LHC?



111

→ Needs an RF manipulation!

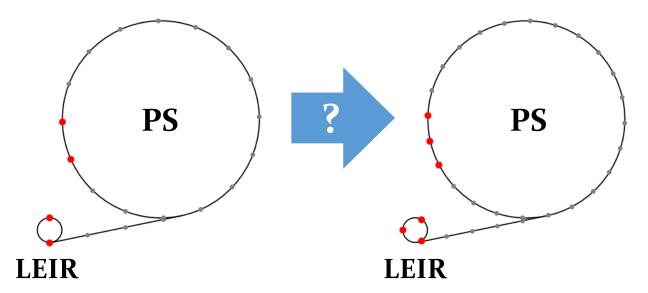
**Step 1:** Check frequency ranges of existing RF systems

- 75 ns corresponds to h = 28 (13.25 MHz)
  - ✓ Within range of existing 13.3/20 MHz cavities
- Ions for PS injected from Low Energy Ion Ring (LEIR)  $\rightarrow 2\pi R_{\text{LEIR}} = 2\pi R_{\text{PS}}/8 \rightarrow 8 \times h_{\text{LEIR}} = 2\pi R_{\text{PS}}$ 
  - $\rightarrow$  Standard transfer of two bunches:  $h_{\text{LEIR}} = 2 \rightarrow h_{\text{PS}} = 16$

#### **Step 1: Check of frequencies for 3 bunches**

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- Ions for PS injected from Low Energy Ion Ring (LEIR)
  - $\rightarrow 2\pi R_{\rm LEIR} = 2\pi R_{\rm PS}/8 \rightarrow 8 \times h_{\rm LEIR} = 2\pi R_{\rm PS}$
  - $\rightarrow$  Standard transfer of two bunches:  $h_{\text{LEIR}} = 2 \rightarrow h_{\text{PS}} = 16$

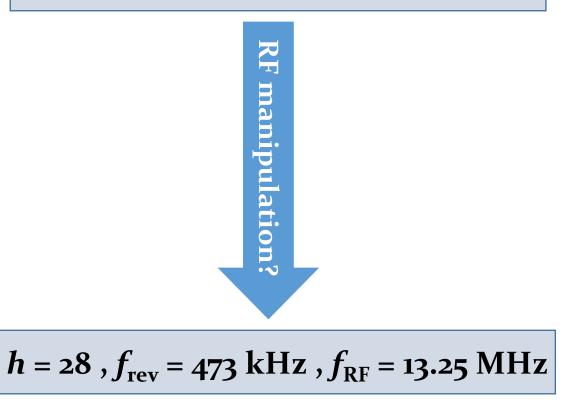


- LEIR:  $h_{\text{LEIR}} = 2 \rightarrow 3$ • PS:  $h_{\text{PS}} = 16 \rightarrow 24$   $\begin{cases} f_{\text{RF}} = 3.2 \text{ MHz} \rightarrow 4.8 \text{ MHz} \end{cases}$ 
  - ✓ Within frequency range of LEIR and PS main RF systems

#### Step 1: Harmonic number sequence

• Injection:

$$h = 24, f_{rev} = 202 \text{ kHz}, f_{RF} = 4.8 \text{ MHz}$$



• Flat-top:

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#### Step 1: Harmonic number sequence

• Injection:

$$h = 24, f_{rev} = 202 \text{ kHz}, f_{RF} = 4.8 \text{ MHz}$$





- Flat-top: h = 28,  $f_{rev} = 473$  kHz,  $f_{RF} = 13.25$  MHz
- $\rightarrow$  Upper frequency too high: 24  $\cdot$  473 kHz = 11.4 MHz > 10 MHz
- $\rightarrow$  Maximum harmonic up to flat-top: h = 21
- $\rightarrow$  Introduce batch expansion h = 24  $\rightarrow$  21 at intermediate energy

#### Step 1: Harmonic number sequence

• Injection:



$$h = 24, f_{rev} = 202 \text{ kHz}, f_{RF} = 4.8 \text{ MHz}$$

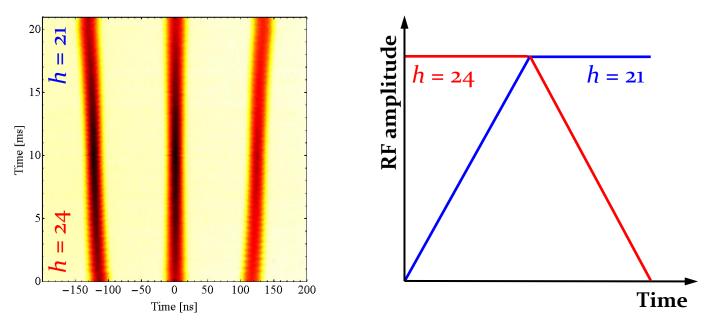
- 1. Acceleration to intermediate plateau energy at h = 24
- 2. Batch expansion  $h = 24 \rightarrow 21$
- 3. Acceleration to flat-top at h = 21
- 4. Batch compression  $h = 21 \rightarrow 28$

• Flat-top:

$$h = 28$$
,  $f_{rev} = 473$  kHz,  $f_{RF} = 13.25$  MHz

#### ✓ Sequence respects all frequency limitations of RF systems

- 2. Batch expansion  $h = 24 \rightarrow 21$ 
  - ✓ Large voltage of main RF system:  $\Sigma V_{\rm RF}$  = 200 kV

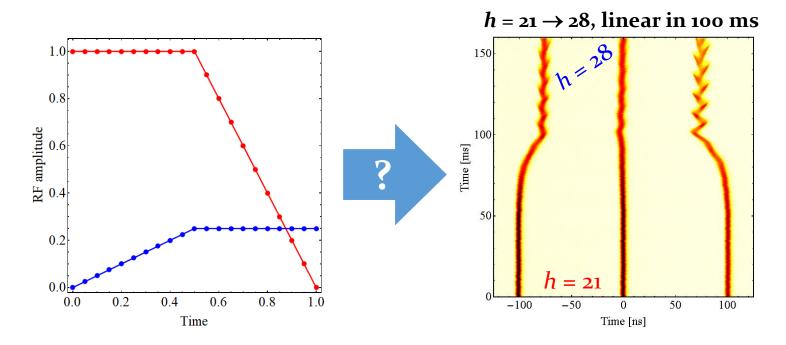


- $\rightarrow$  Huge bucket areas
- $\rightarrow$  Simple linear voltage functions are sufficient

- 4. Batch compression  $h = 21 \rightarrow 28$ 
  - Asymmetric voltage capabilities:

```
h = 21 up to 200 kV
h = 28 only 20 kV
```

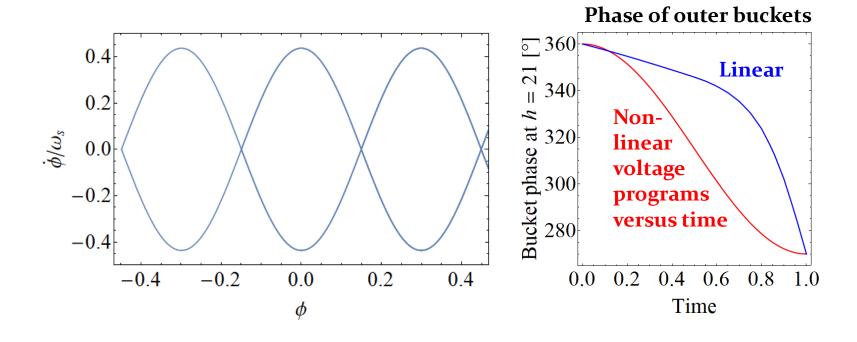
 $\rightarrow$  Compromise: handover 80 kV  $\rightarrow$  20 kV



- 4. Batch compression  $h = 21 \rightarrow 28$ 
  - Asymmetric voltage capabilities:

```
h = 21 up to 200 kV
h = 28 only 20 kV
```

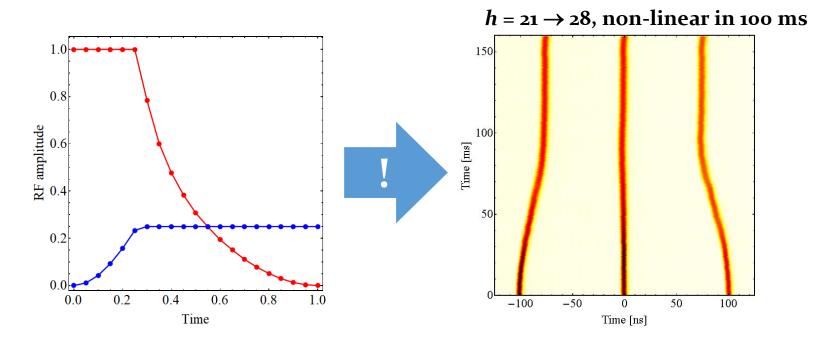
 $\rightarrow$  Compromise: handover 80 kV  $\rightarrow$  20 kV



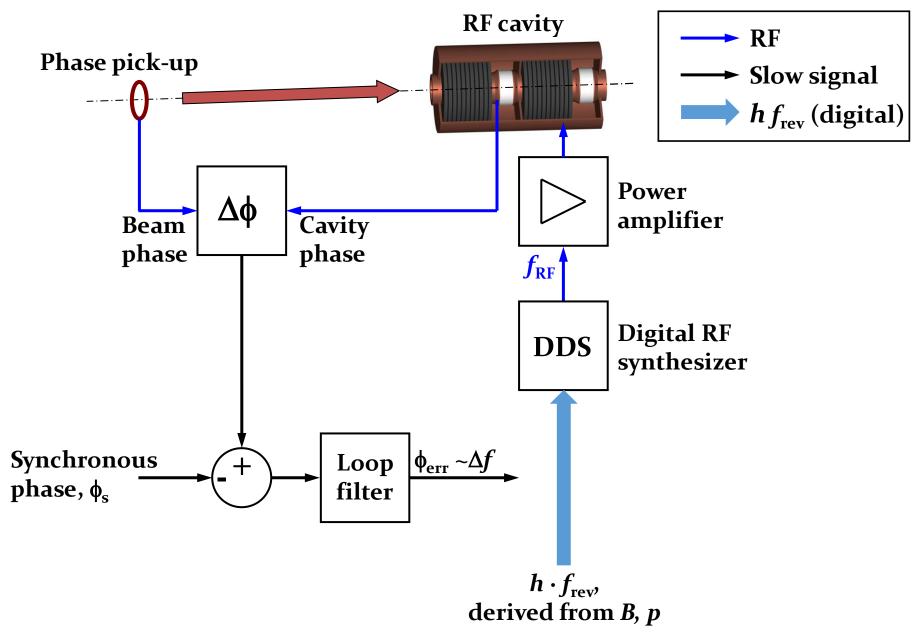
- 4. Batch compression  $h = 21 \rightarrow 28$ 
  - Different voltage capabilities:

```
h = 21 up to 200 kV
h = 28 only 20 kV
```

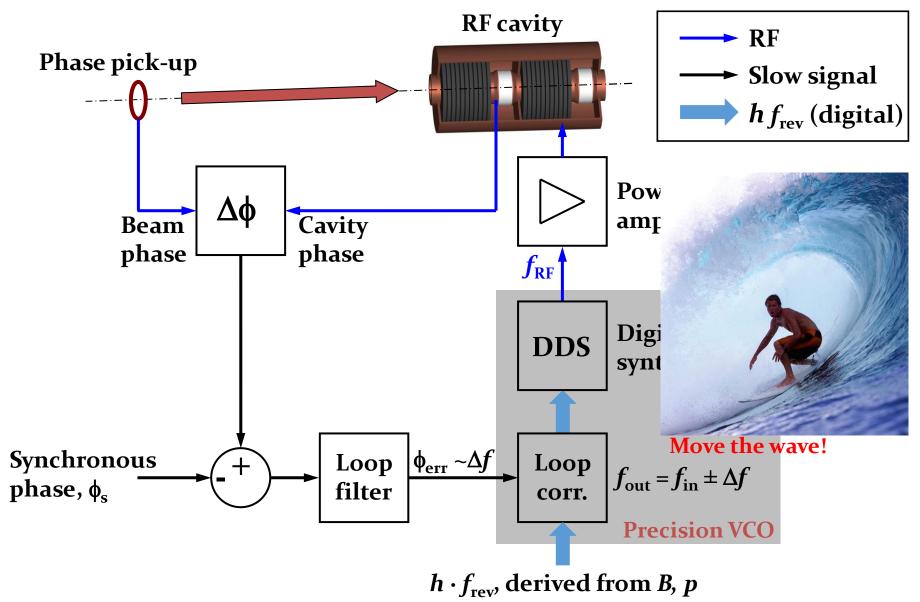
 $\rightarrow$  Compromise: handover 80 kV  $\rightarrow$  20 kV



#### Step 3: Implementation with beam phase loop<sup>120</sup>



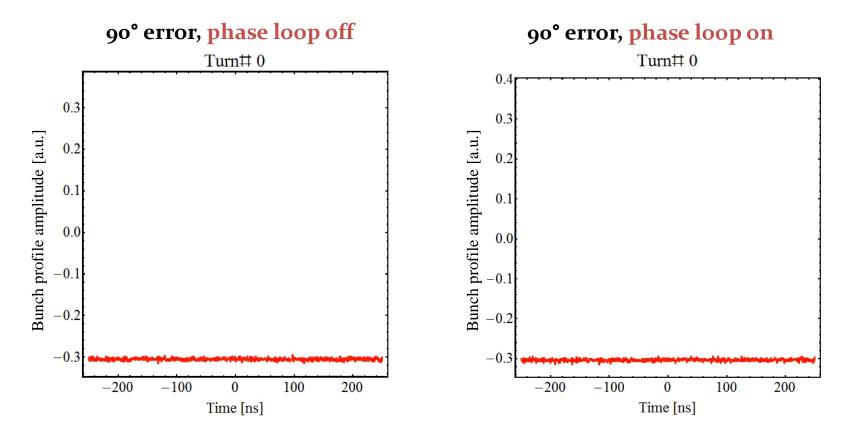
# Step 3: Implementation with beam phase loop<sup>121</sup>



→ Phase-locked loop with beam phase as reference for RF system

# **Step 3: Effect of beam phase loop at injection**<sup>122</sup>

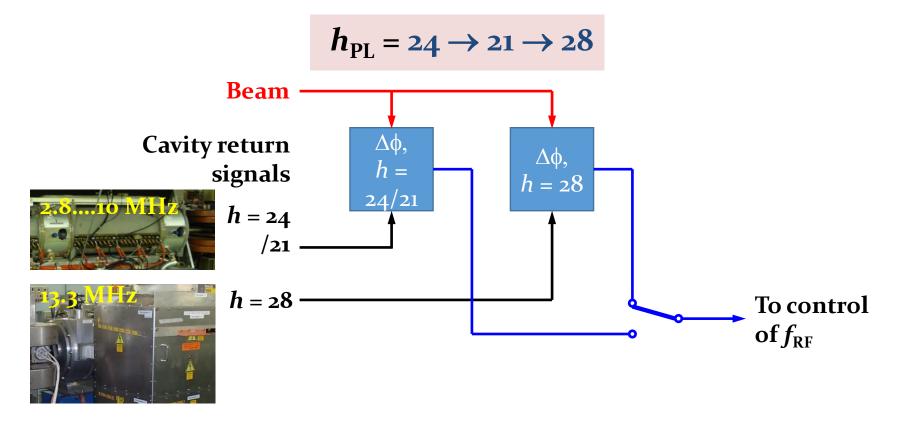
• Example: Injection of a bunch from PS Booster into PS



→ Essential in hadron accelerators to keep RF locked to beam
→ Mitigates common-mode dipole oscillations

# Step 3: Implementation with beam phase loop<sup>23</sup>

- Need beam phase loop closed during RF manipulations
   → Prevent excitation of dipole oscillations during process
- $\rightarrow$  Harmonic number sequence of beam phase loop

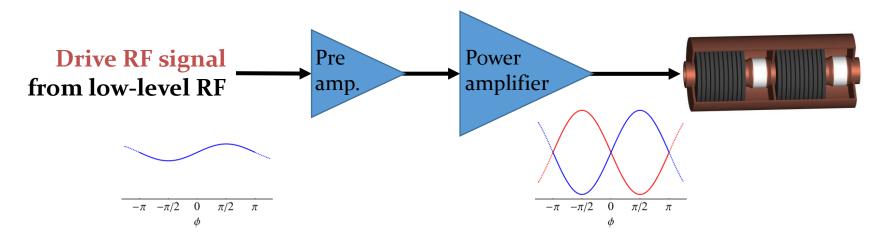


# **Recipe for RF manipulation design**

Item		Remark
1.	Define sequence of harmonic numbers	<ul> <li>→ Constrained by RF frequencies of existing systems</li> <li>→ Propose minimal extension</li> </ul>
2.	<ul> <li>(a) Assume simple (or time- normalized) voltage programs and check evolution of bucket position and area</li> <li>(b) Optimize voltage functions versus time</li> </ul>	<ul> <li>→ Avoid abrupt changes of bunch phases</li> <li>→ Respect adiabaticity</li> <li>↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓</li></ul>
3.	Design <mark>phase loop</mark> harmonic sequence	→ As few harmonic number changes as possible



# **Polarity of RF systems**



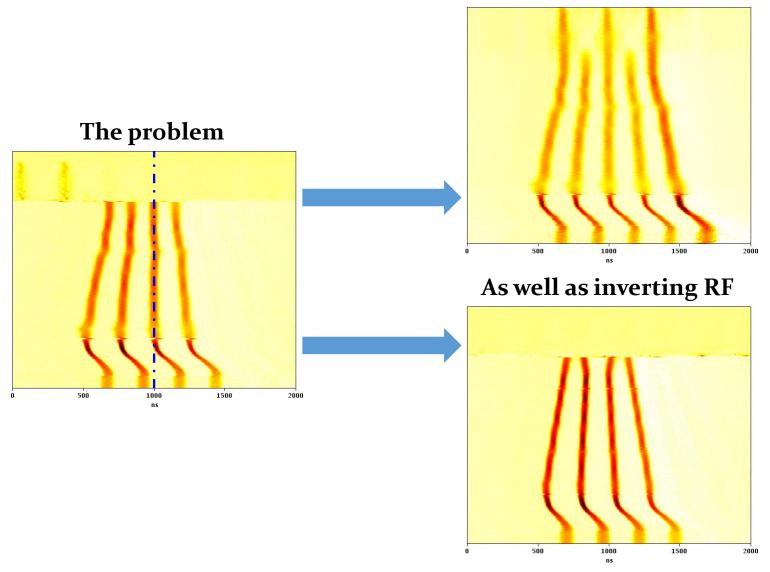
- What is the polarity of your RF system?
- Is your amplifier straight-through or inverting?

• Why bother? → Irrelevant for most accelerators

#### 'First' beam in Proton Synchrotron (2018)

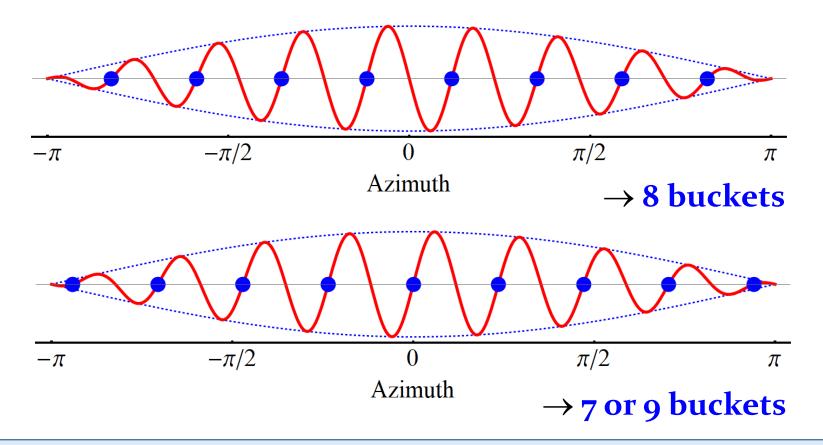
• Impossible to establish symmetric batch compression of four bunches

#### One more bunch would do



#### 'First' beam in Proton Synchrotron (2018)

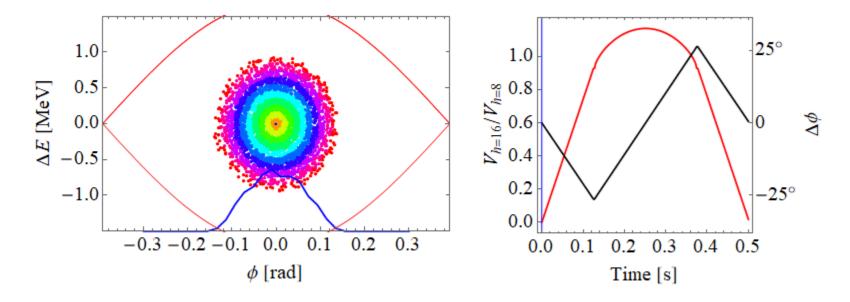
→ A small preamplifier upgrade had unintentionally and unexpectedly changed the polarity of the gain!



→ Gain polarity of amplifiers becomes relevant with RF voltage at multiple harmonics

#### Turn a bunch inside out

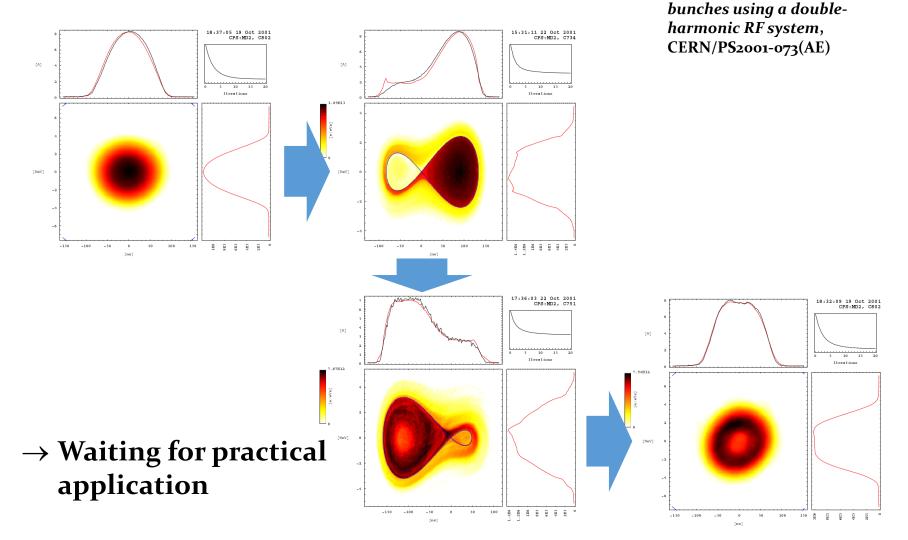
- Bunch core denser that tails
- → Can core and tails be exchanged to flatten a bunch?
- → Voltage and phase programs calculates to suck bunch into emerging bucket next to it



→ Asymmetric merging with an empty bucket



#### Turn a bunch inside out: hollow bunch



#### • Beam test in the CERN PS Booster

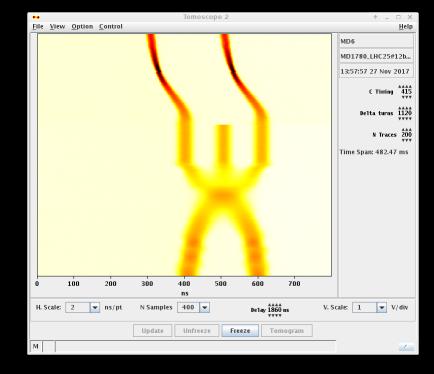
130

C. Carli, Creation of hollow

# **Summary**

- RF can do so much more than just acceleration
- The RF potential is the integral of voltage
   → Fill occupied phase space into the bucket
- Adiabaticity and synchrotron frequency decide whether a manipulation is slow or fast
- → You will define the next generation of RF manipulations to come
- → Looking forward to seeing your new, unimageable RF manipulations





# A big Thank You

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