High Beta Cavities I and II
Breakdown and Multipactor

Overall introduction

What lies behind these titles?

High Beta Cavities I and II
Breakdown and Multipactor

High beta cavities – Refers to those cavities used when a particle beam is ‘fully’ relativistic, that is when we can put *v*=c in equations with negligible error. So not in an injector! Examples can be found in a linac or storage ring, or in an electron linac.

Breakdown and Multipactor – Multi-physics process that emerge, beyond Maxwell’s equations, when power and field levels are increased beyond a certain value. They can be annoying, and may limit the performance of a system. Breakdown especially is very, very complicated.

How are we going to approach these topics?

High beta cavities – For clarity, we will develop the basic concepts of acceleration of relativistic particles with a focus on traveling wave acceleration, as used in for example electron linacs like in XFELs. Understanding can be relatively easily transferred to standing wave cavities and to use in rings.

Breakdown – Using more precise language, vacuum breakdown, or vacuum arcing. This is an effect seen in dc and rf systems where the insulation provided by vacuum gives way to a conductive plasma. It can limit the maximum achievable accelerating gradient. I will introduce the subject from the perspective of high-gradient linear collider development.

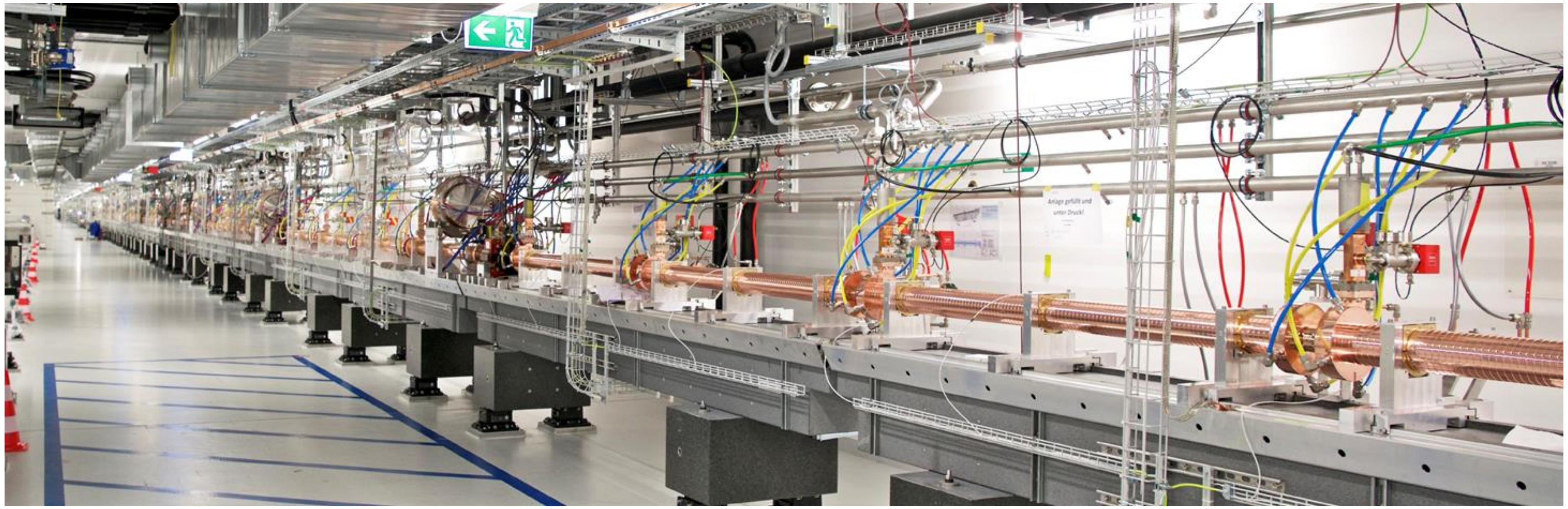
Multipactor - A lower-field phenomenon that is a result of a resonant excitation of electron trajectories in the vacuum, with a current that grows exponentially due to electron-surface interactions.

High Beta Acceleration: Introduction

$$\beta = \frac{v}{\mathrm{c}} \cong 1$$

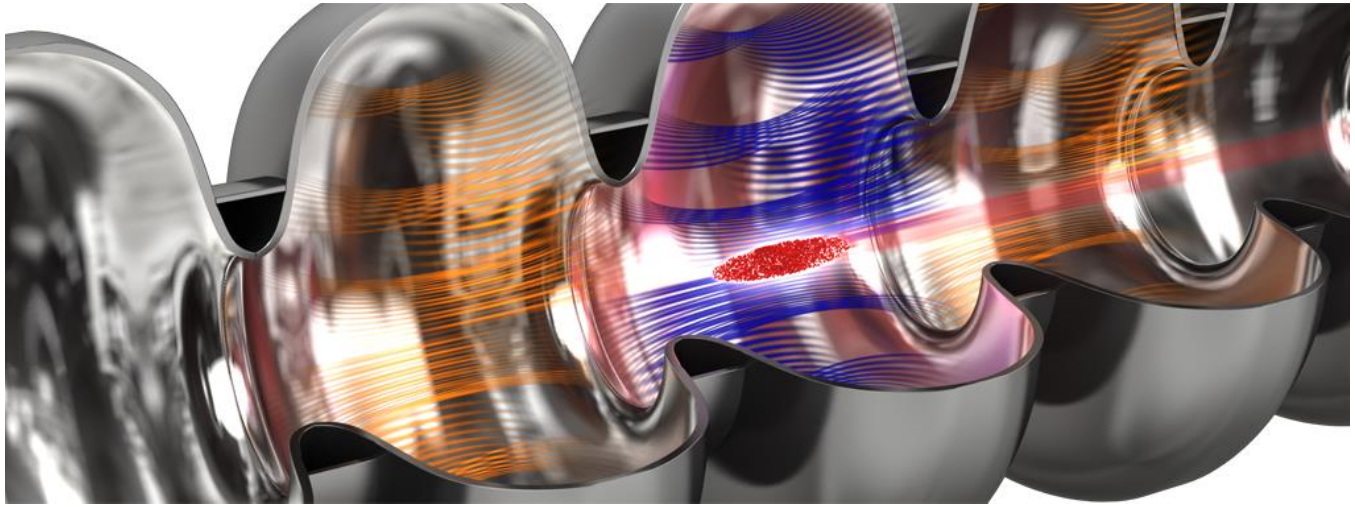
Examples of β=1 rf systems

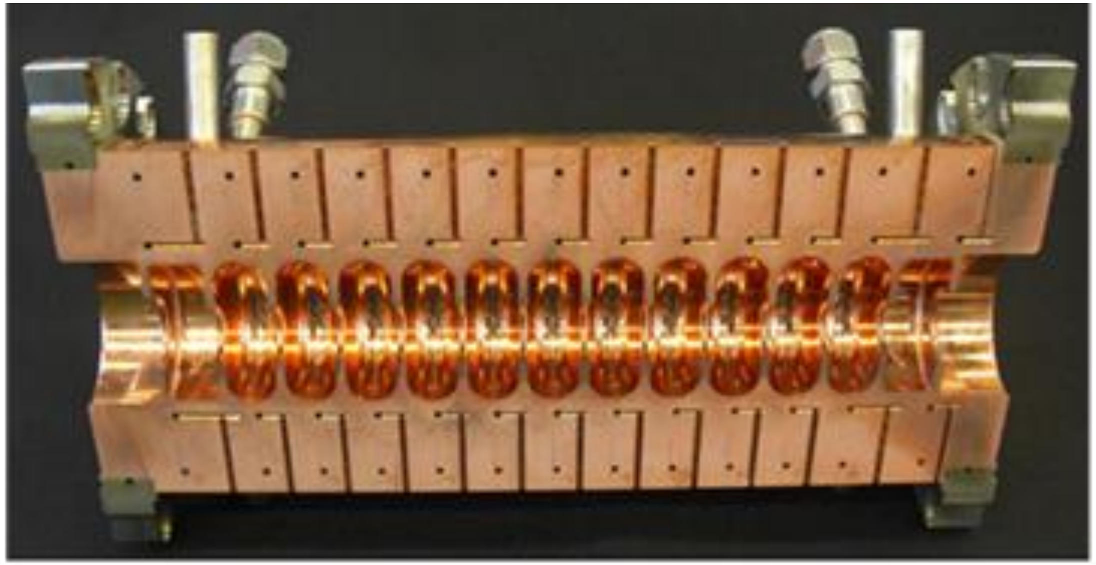
European XFEL, DESY
Superconducting 1.3 GHz, standing wave



SwissFEL, PSI
Normal conducting 5.7 GHz, travelling wave

Examples of β=1 rf systems - inside





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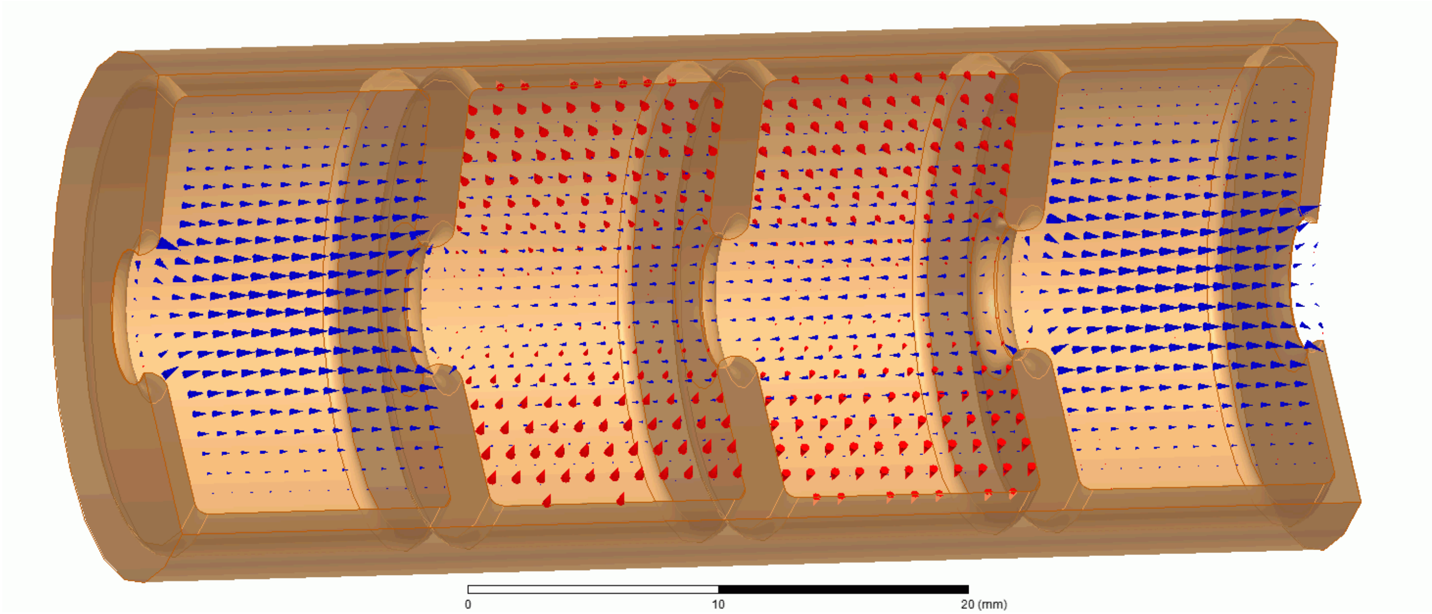
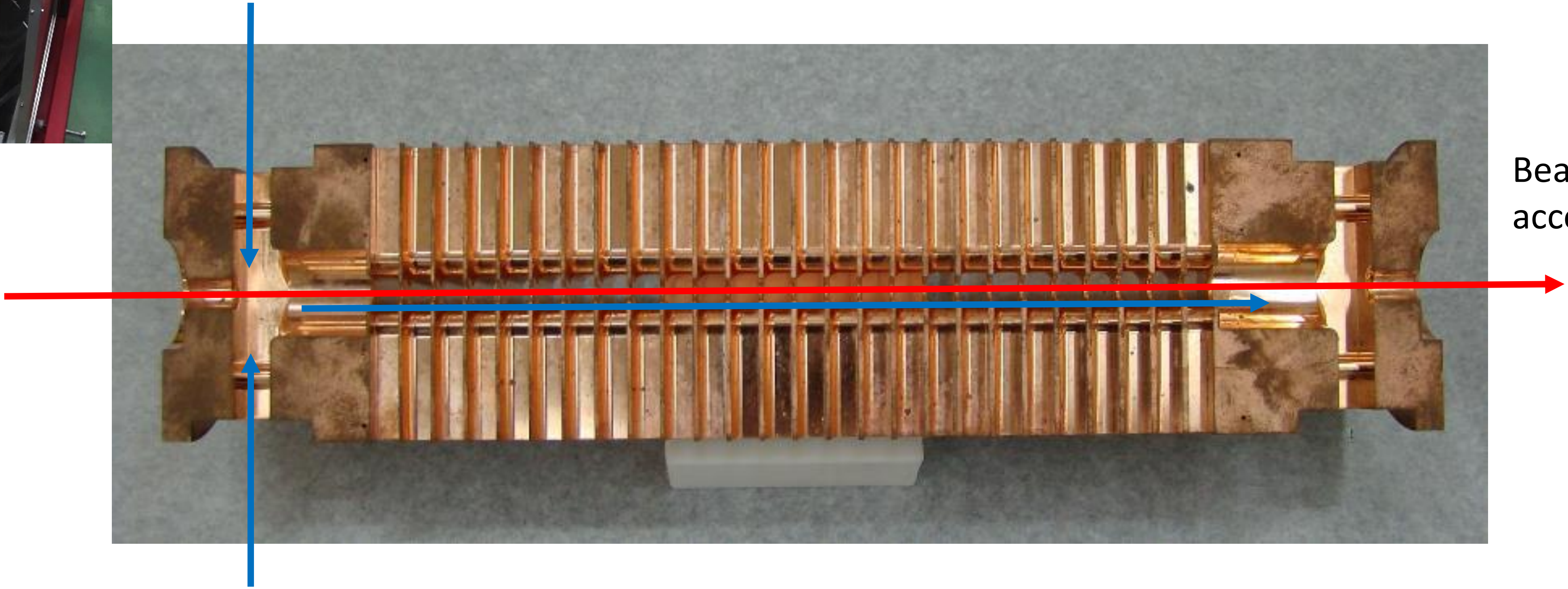
Traveling wave acceleration


0
10
20 (mm)

Klystron

rf power in

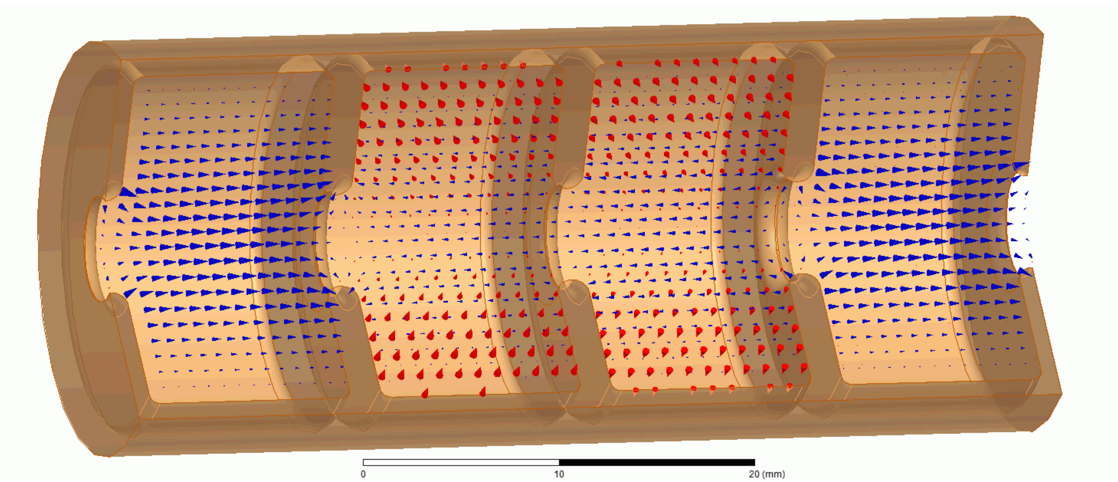


Beam accelerated

rf power flows along structure

A number of important concepts underlie traveling wave accelerating structures:

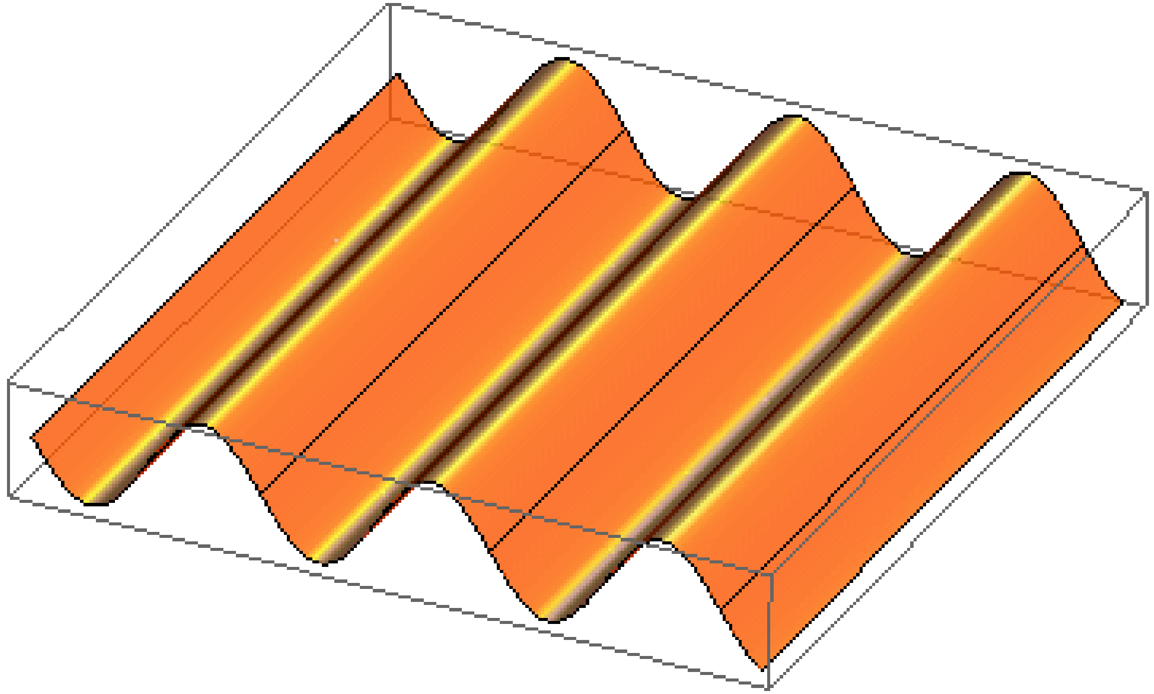
1. Phase velocity
2. Periodic boundary conditions
3. Group velocity
4. (Orientation of the electric field relative to power flow)



You may have seen these concepts already this school, but they are so important that I will go through them again.

Phase velocity

The field of a wave travelling in free space

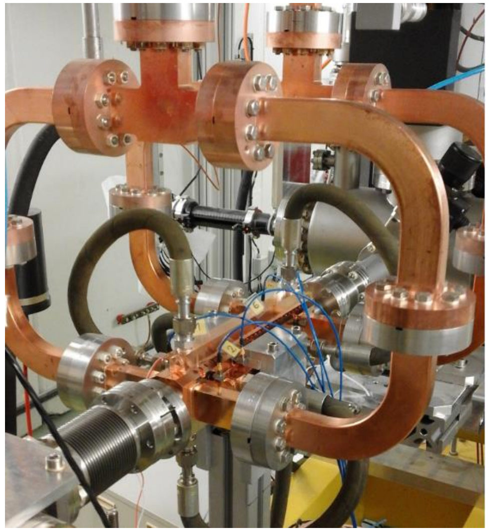


Free-space electromagnetic waves travel at the speed of light – both phase and group velocity are equal to c. First we will look at phase velocity:

$$\vec{E}(z,t) = E_0 \hat{x} e^{i(kz-\omega t)}$$

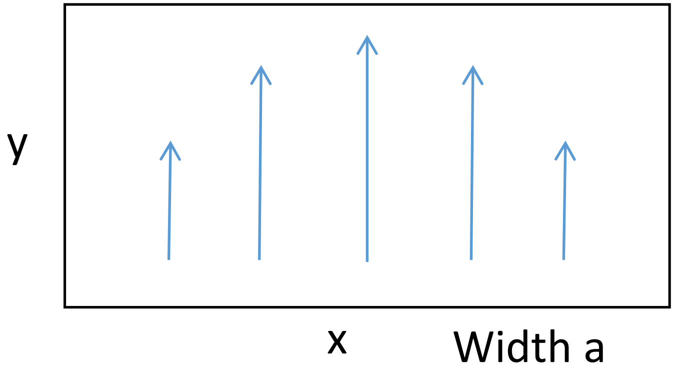
$$k = \frac{\omega}{c} \qquad \lambda = \frac{2\pi}{k}$$

Uniform (rectangular) waveguide



WR90 waveguides feeding a prototype CLIC structure.

The lines are electric field



Field solution

$$E_y = E_0 \sin\left(\frac{\pi}{a} x\right) e^{i(\omega t - k_z z)}$$

$$k_z = \sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{\pi}{a}\right)^2} \qquad \lambda = \frac{2\pi}{k}$$

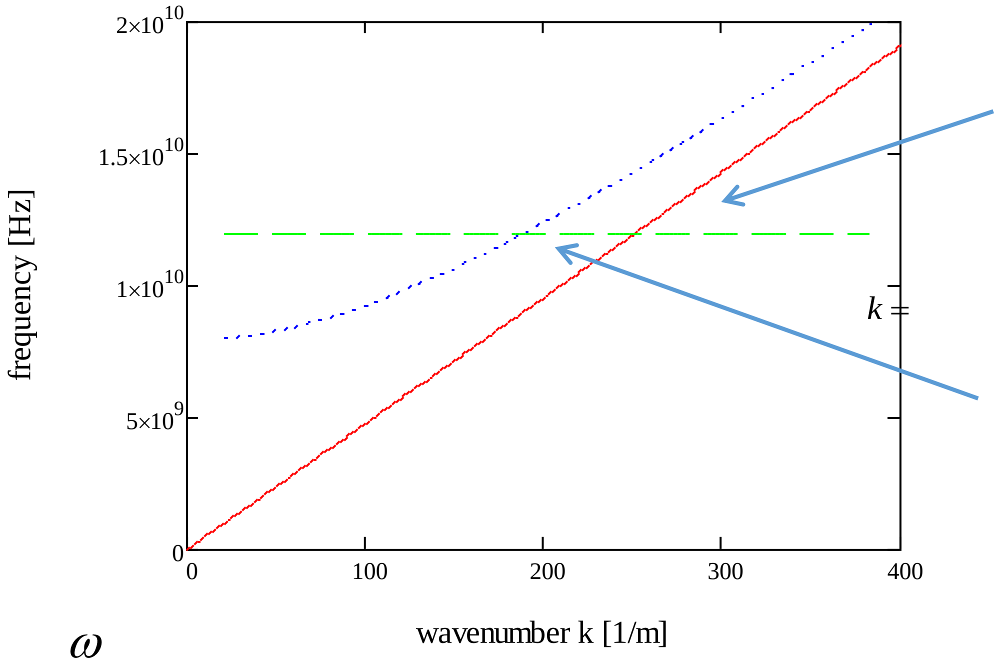
Cutoff frequency

In a waveguide k is smaller than in free space, so wavelength longer!
Same equation holds true for all uniform waveguides.

Wavelength in another picture – the dispersion curve.

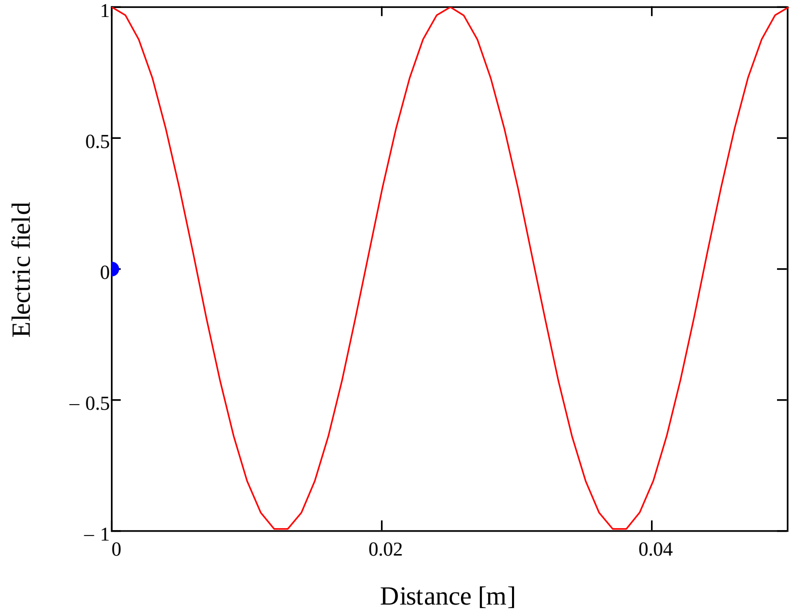
$$v_{phase} = \frac{\omega}{k}$$

$$k = \frac{\omega}{c}\sqrt{1-\left(\frac{\omega_0}{\omega}\right)^2}$$

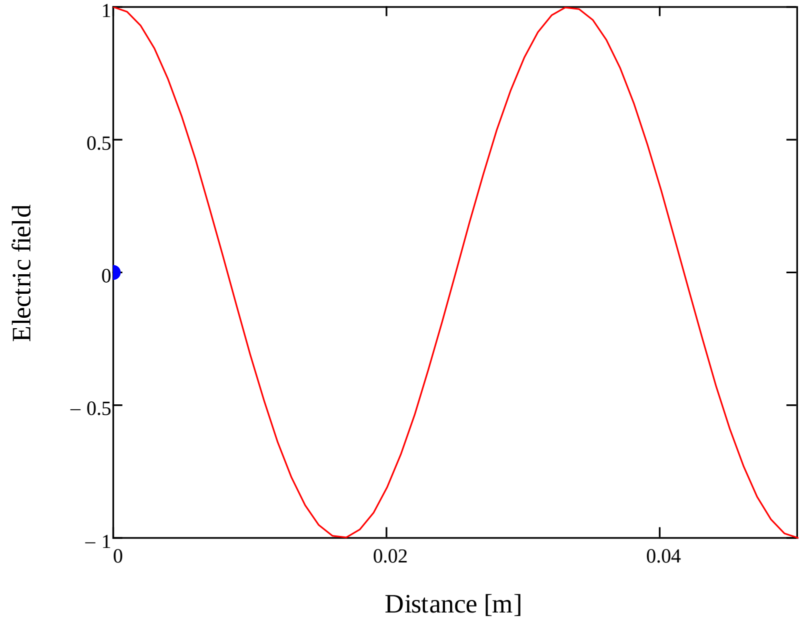


$$k = \frac{\omega}{c}$$

Horizontal green line: waveguide k is 0.75 of free space k at 11.994 GHz

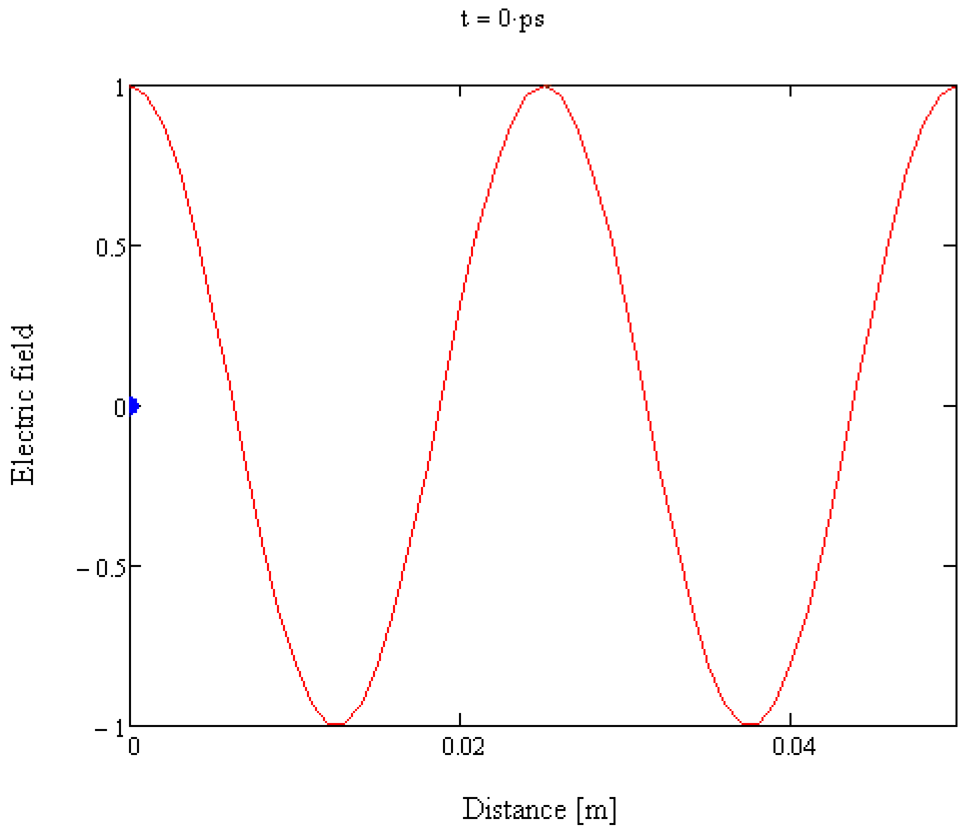


case 1



case 2

A first view of travelling wave acceleration

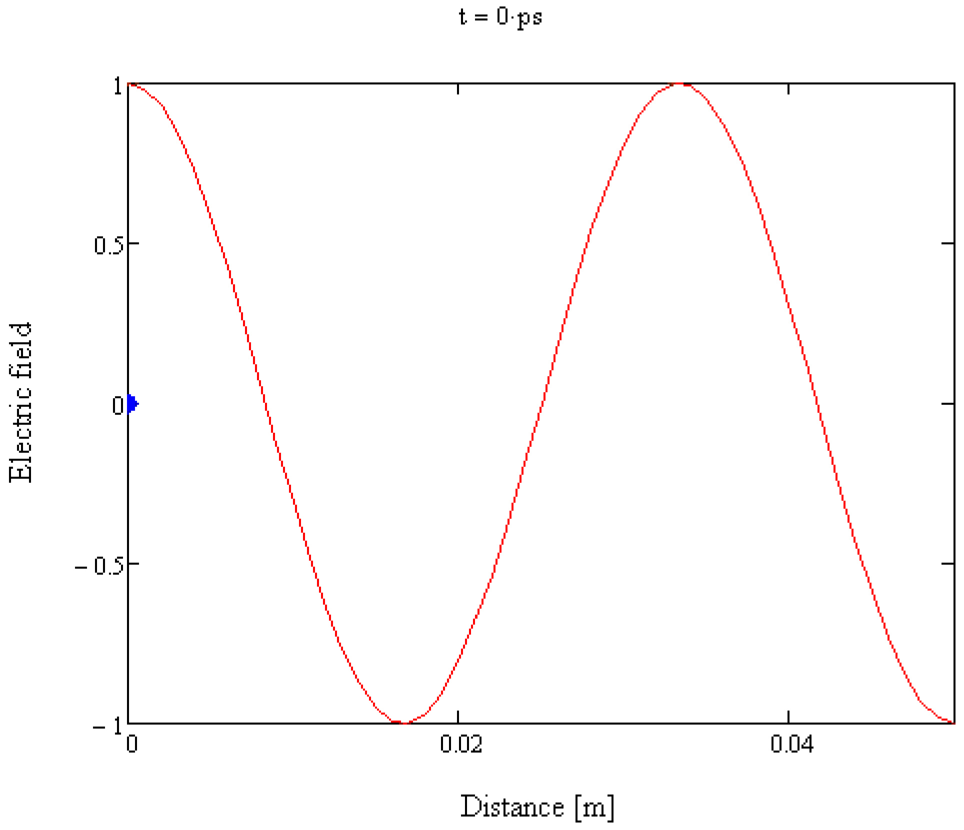


Beam (blue dot) travels with the speed of light.
$z(t)=ct$

$$E(z,t) = \mathrm{Re}(e^{\mathrm{i}(kz-\omega t)})$$

Case 1: Wavelength is equal to free space wavelength, phase velocity equal to c.

But in a uniform waveguide v_p > c:

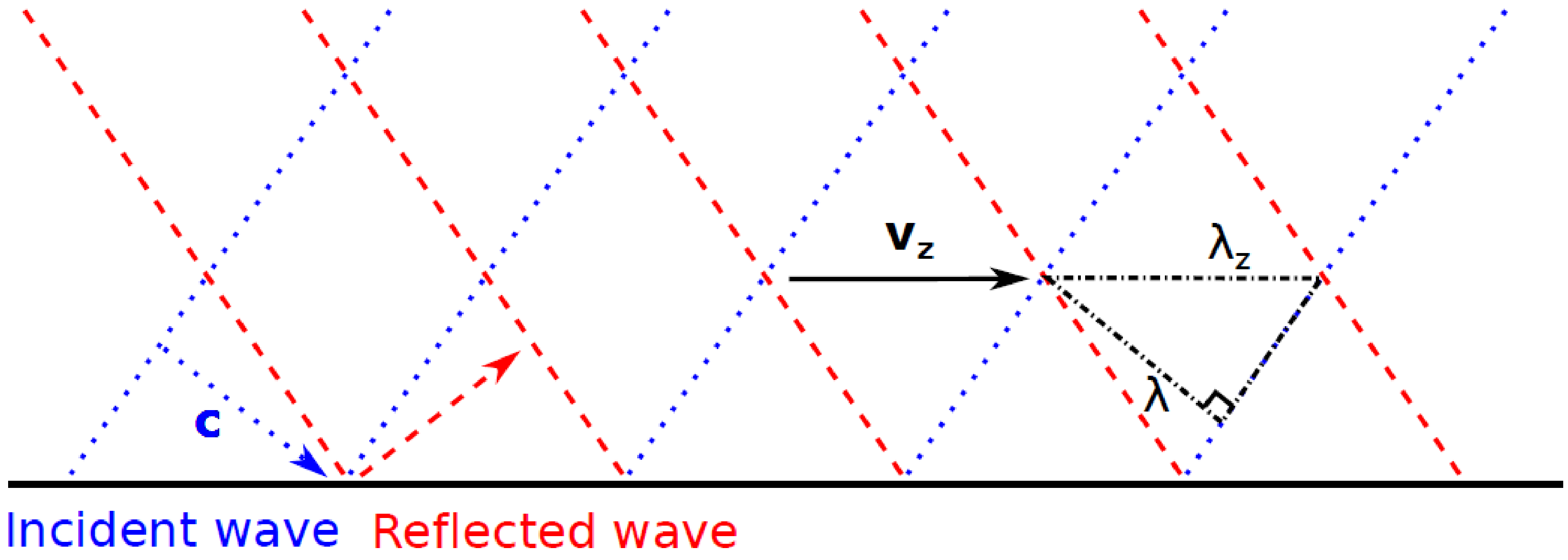


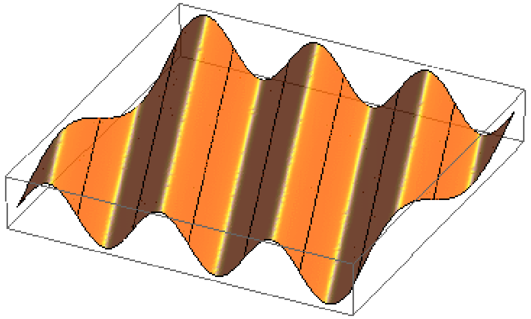
Beam (blue dot) travels with the speed of light.
$x(t)=ct$

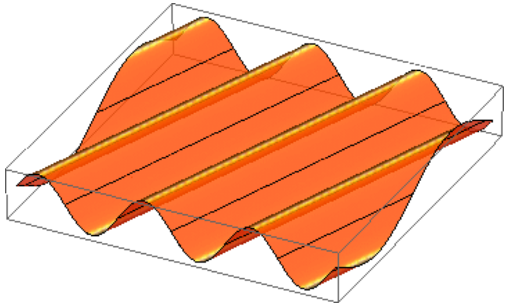
$$E(z,t) = \mathrm{Re}(e^{\mathrm{i}(kz-\omega t)})$$

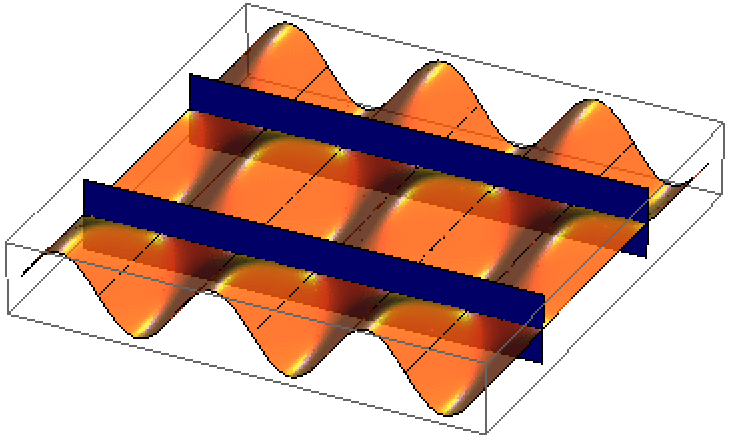
Case 2: Wavelength is equal to free space wavelengthx4/3, phase velocity equal to 4/3xc.

A physical picture of why v_p is always greater than c in a uniform waveguide.





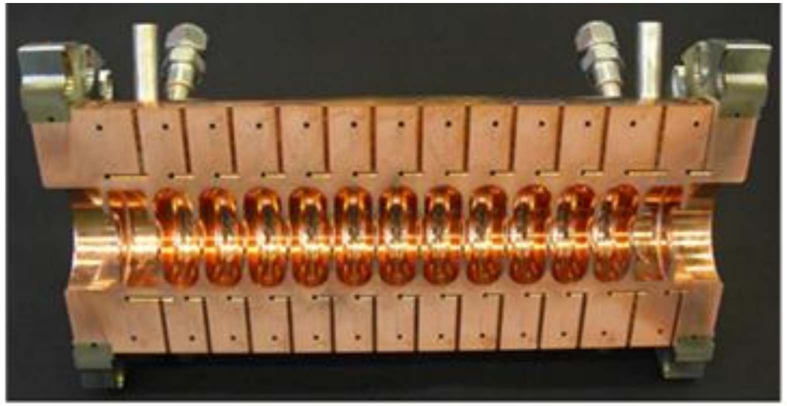




Periodic boundary conditions

So how can we slow phase velocity to down to c?

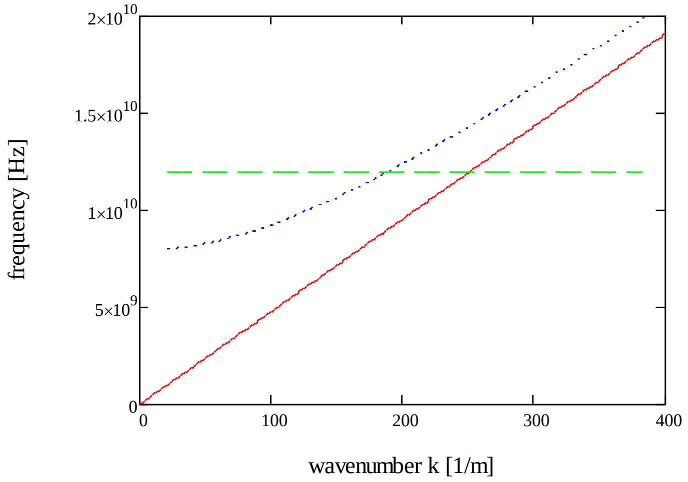
By introducing periodic structures!

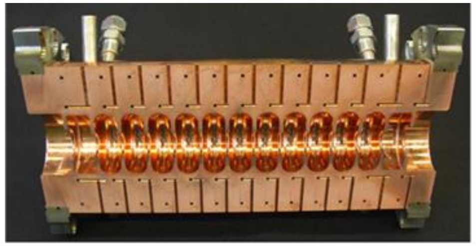


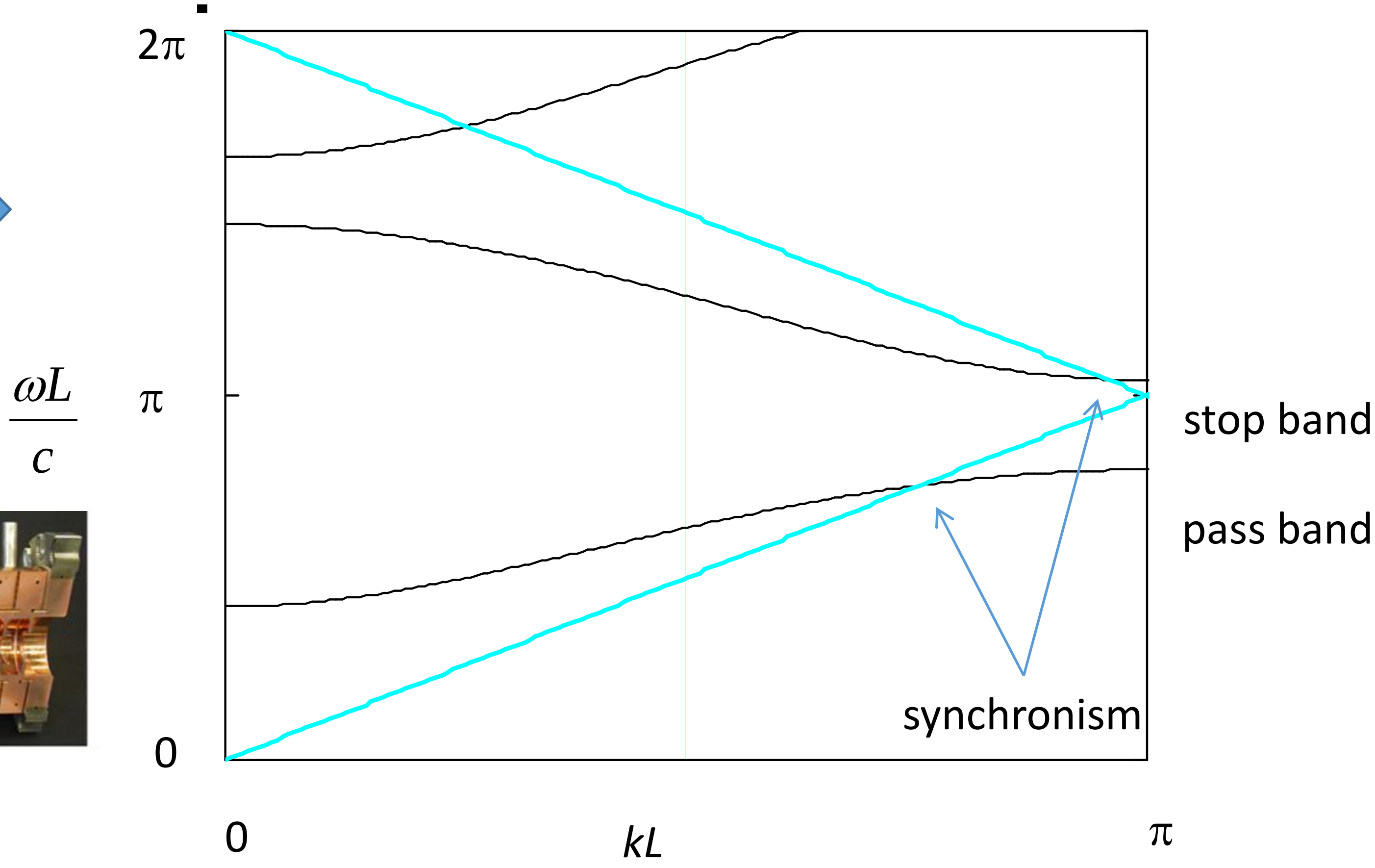
Floquet's theorem states, translated to rf language, that periodic boundary conditions give solutions with same field in every cell, just differing by a complex phase advance.
It turns out that this allows us to set up fields with arbitrary phase advance!
See also Bloch's theorem.

The Brillouin diagram

Band structure can be determined with equivalent circuit model, matrix formalism and periodic boundary conditions in EM simulation codes.



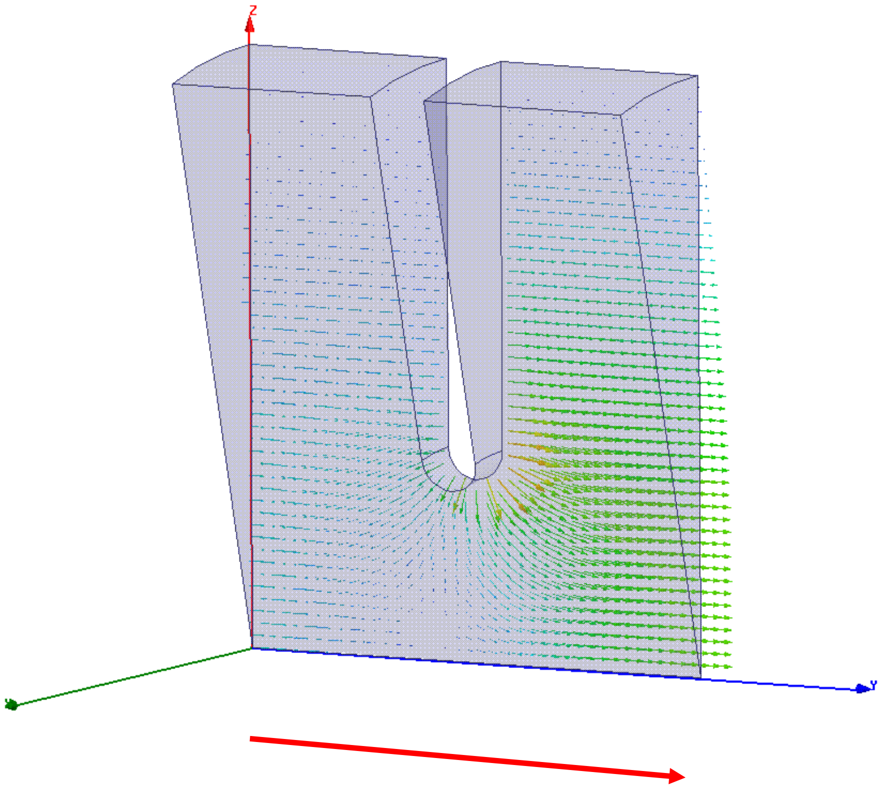


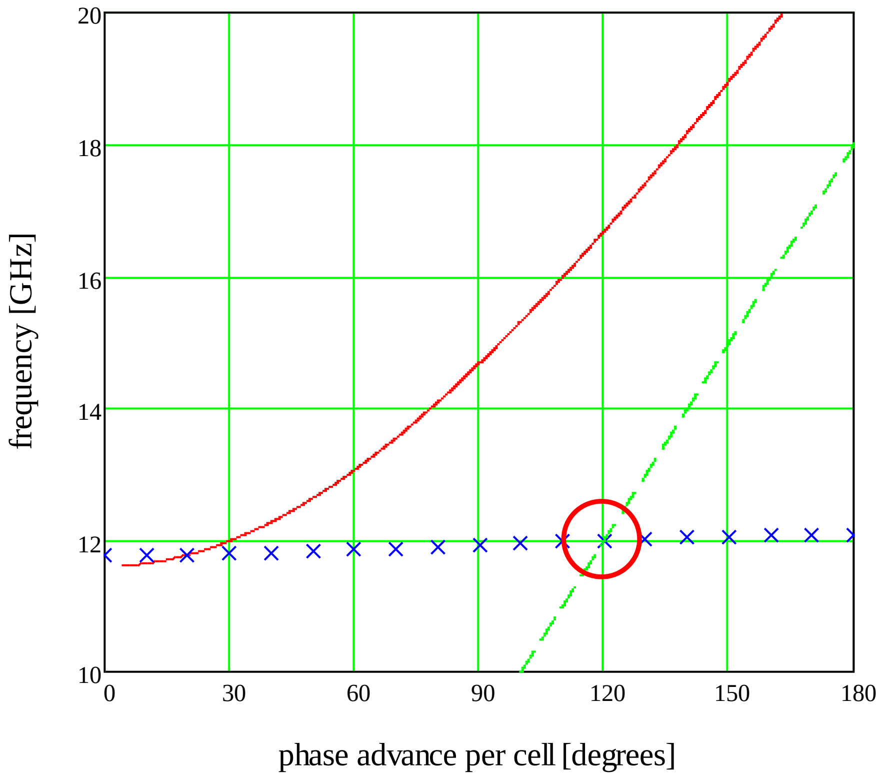


A periodic geometry modifies the dispersion curve of the uniform waveguide.

Frequency vs phase advance per period, which is *kL*.

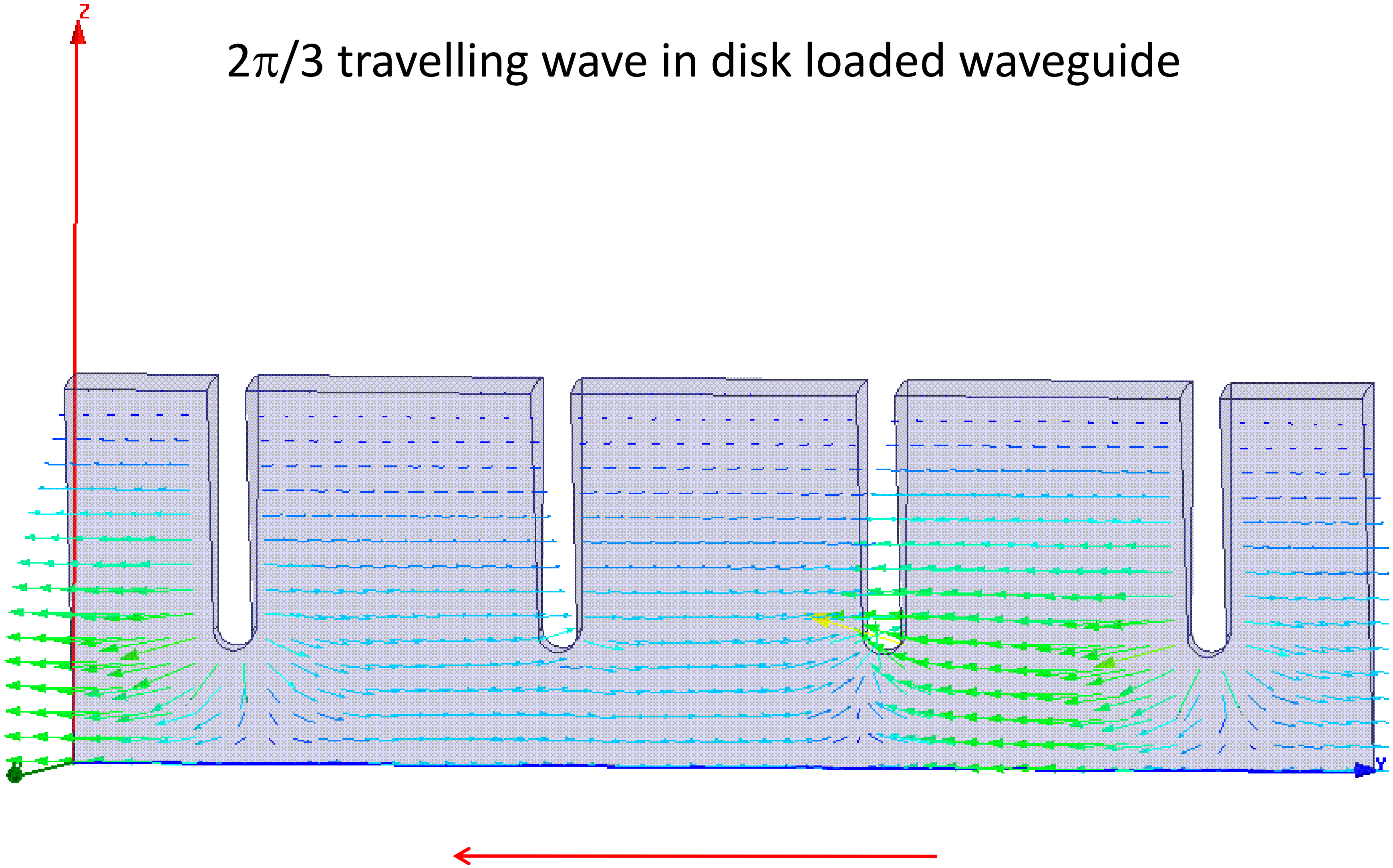
Single cell electric field pattern 2π/3 phase advance





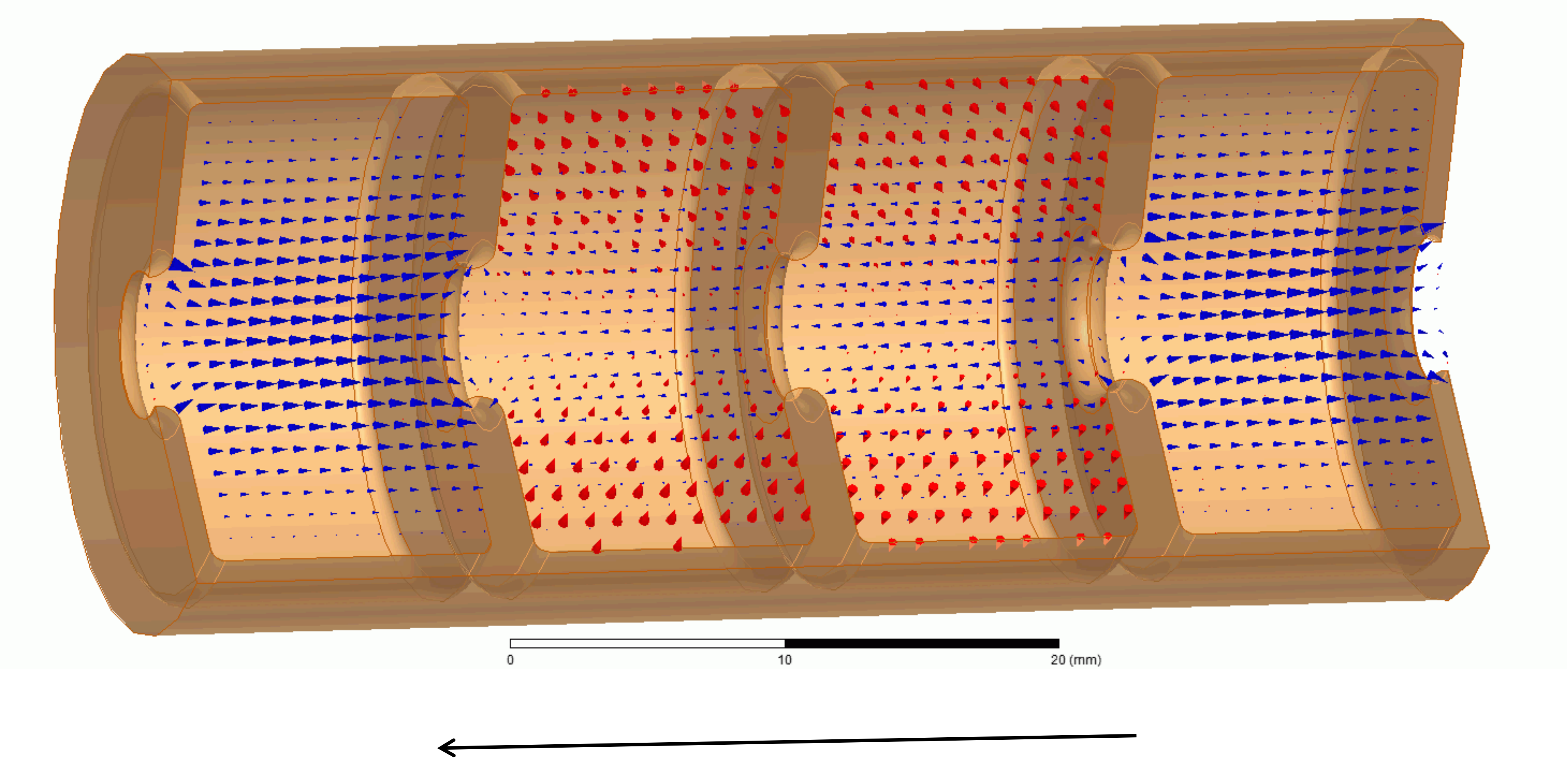
Phase synchronism means time for beam to get across cell is the same as accelerating phase to get across cell.

2π/3 travelling wave in disk loaded waveguide



Phase propagation direction

Another view



beam propagation direction

Group velocity

In review - phase velocity is the speed of propagation of a field pattern.

It needs to be matched to the speed of the particle we want to accelerate, which is usually c, so that we can stay matched up with a maximum electric field.

There is another velocity in electromagnetism which is group velocity.

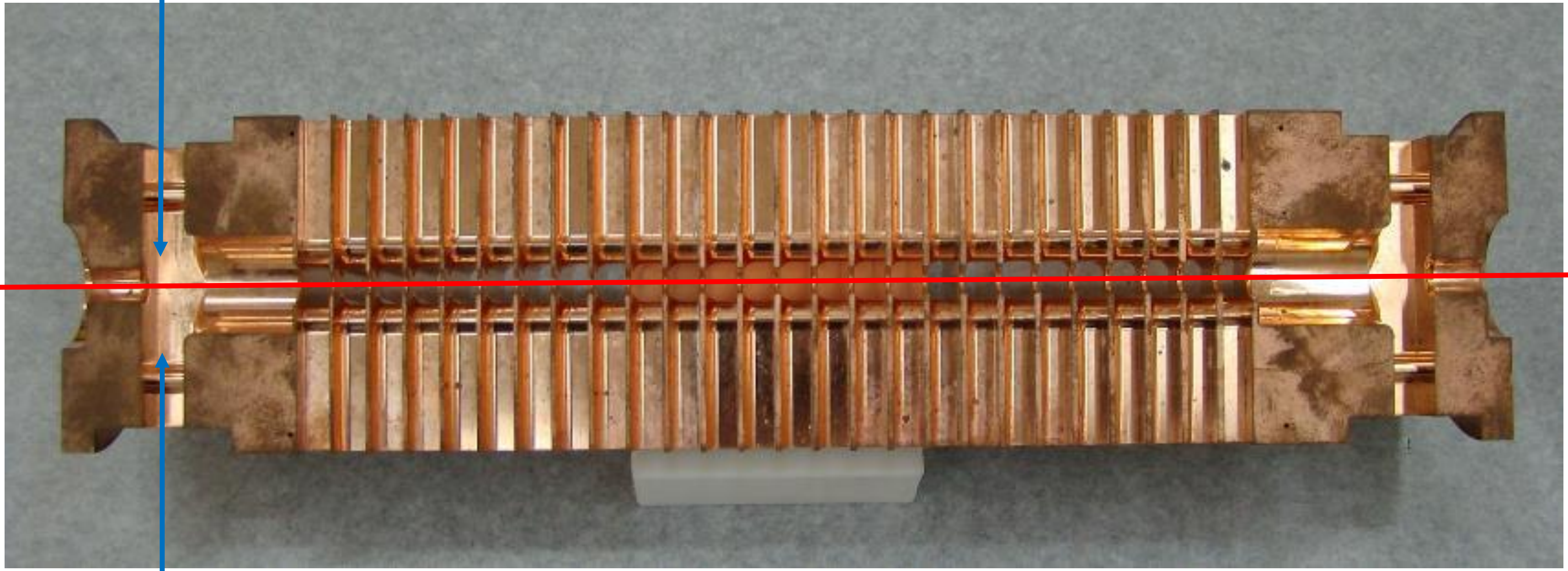
This is the speed at which energy propagates through a system.

Phase velocity

Group velocity

Group velocity, along with length, gives the fill time of an accelerating structure. Fill time is a crucial parameter for example for the pulse length needed from a klystron.

rf power in



Beam accelerated

Power flows along structure with group velocity

$$P = v_g W'$$

To introduce group velocity we need to introduce two concepts stored energy and power flow:

The stored energy of an electromagnetic field is given by:

$$\mathrm{U} = \int \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2\right) dV$$

Local power flow is given by the real part of the Poynting vector ***S***

$$\boldsymbol{S} = \frac{1}{\mu_0} \boldsymbol{E} \times \boldsymbol{B}$$

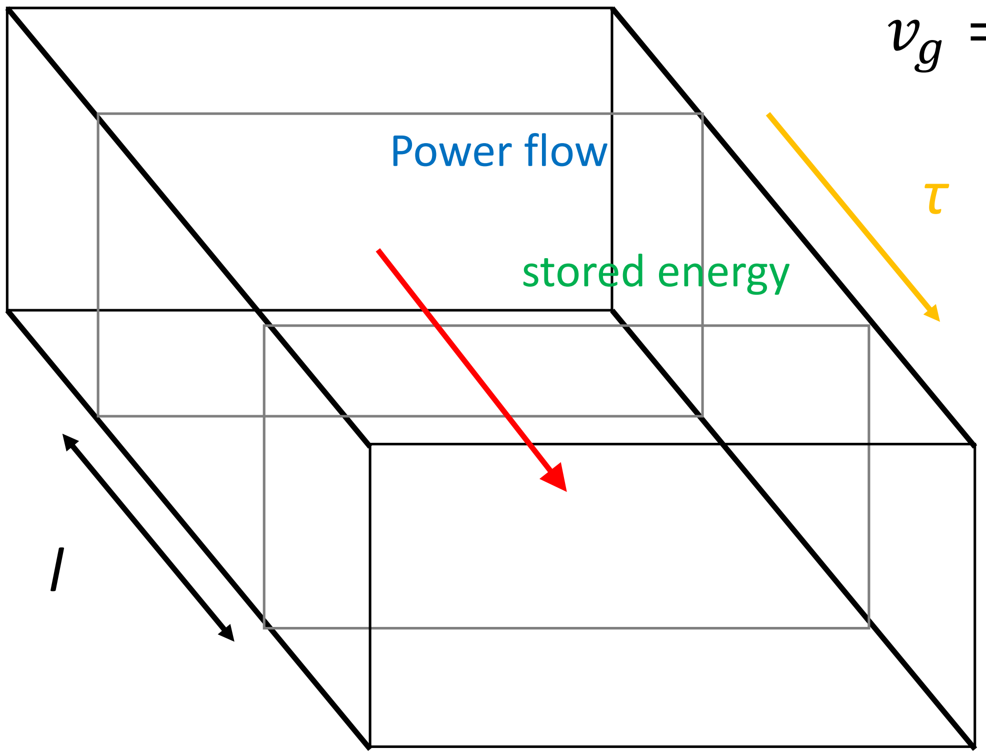
Total power flow is given by the integral of the Poynting vector over a surface, the cross section of a waveguide for example.

$$P = \int n \cdot S \, dA$$

Group velocity

We will get group velocity by first calculating the group delay, that is time it takes power to flow through a given section of waveguide. We'll get velocity by dividing by the length.

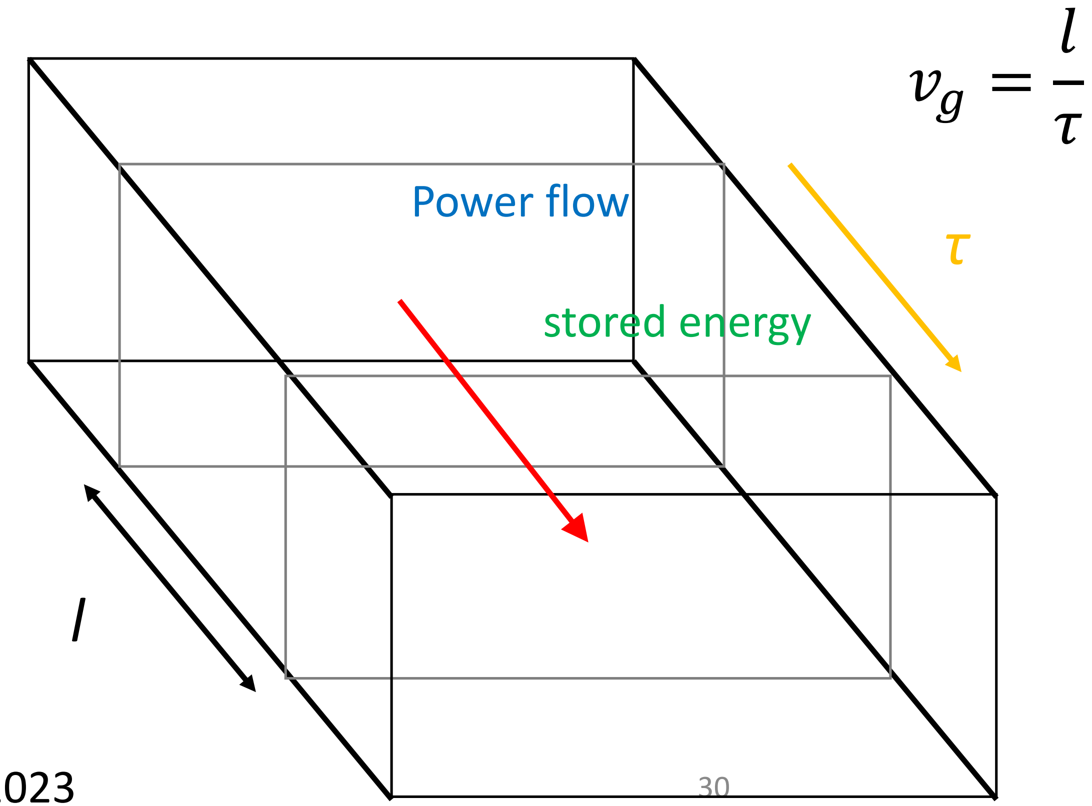
$$v_g = \frac{l}{\tau}$$



There are two ways of looking at this.

Energy-based: $$\tau_{delay} = \frac{Energy\ stored}{power\ through} = \frac{\int \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2\right) dV}{\oint \boldsymbol{n} \cdot \boldsymbol{S} dA}$$

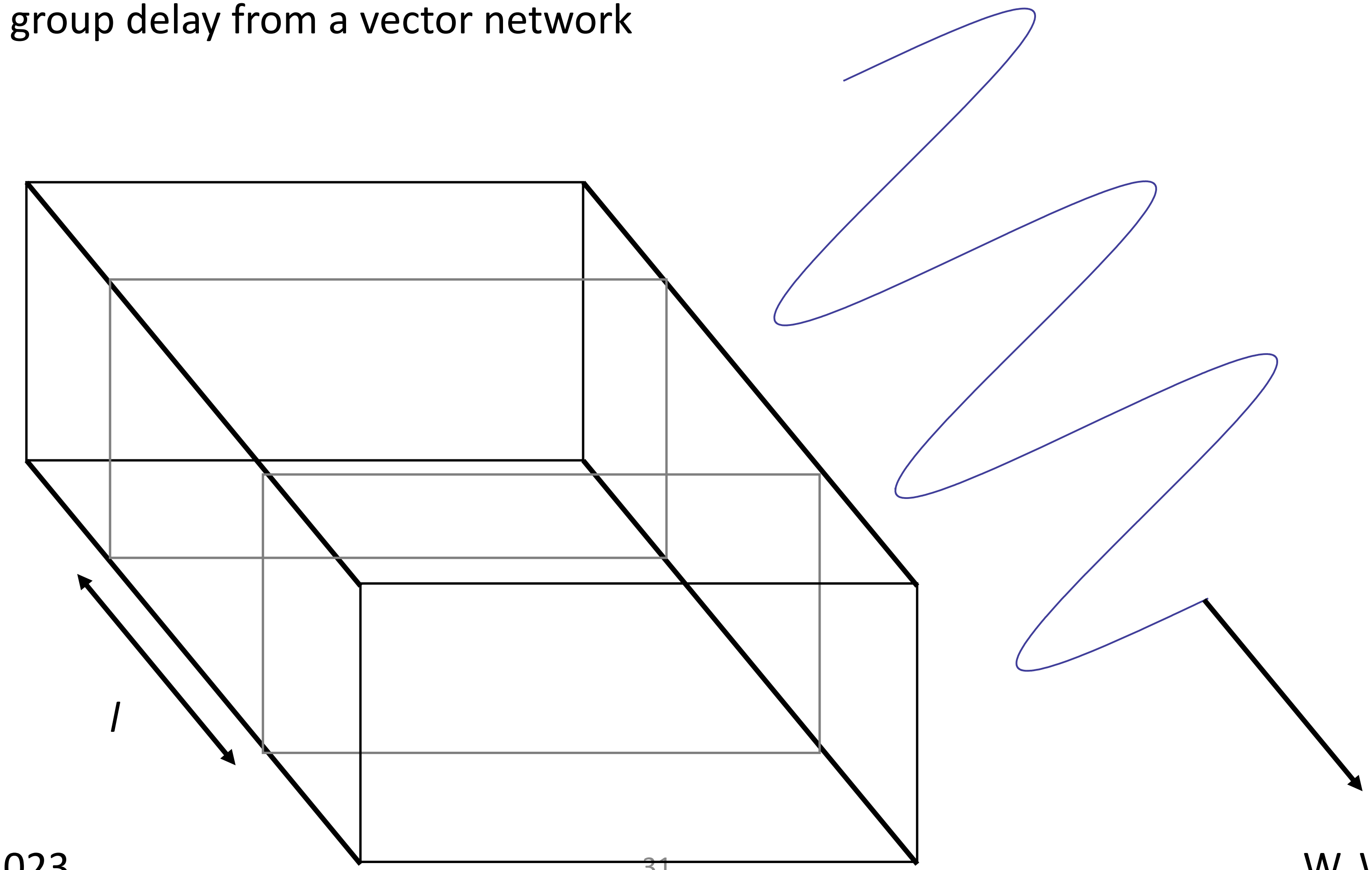
This can be very efficient on a computer because you only need fields at a single frequency and the quantities are integrals (and not derivatives).



And phase-based: $\tau_{delay} = \frac{d\varphi}{d\omega}$ $v_{group} = \frac{l}{\tau} = \frac{1}{\frac{dk}{d\omega}} = \frac{d\omega}{dk}$

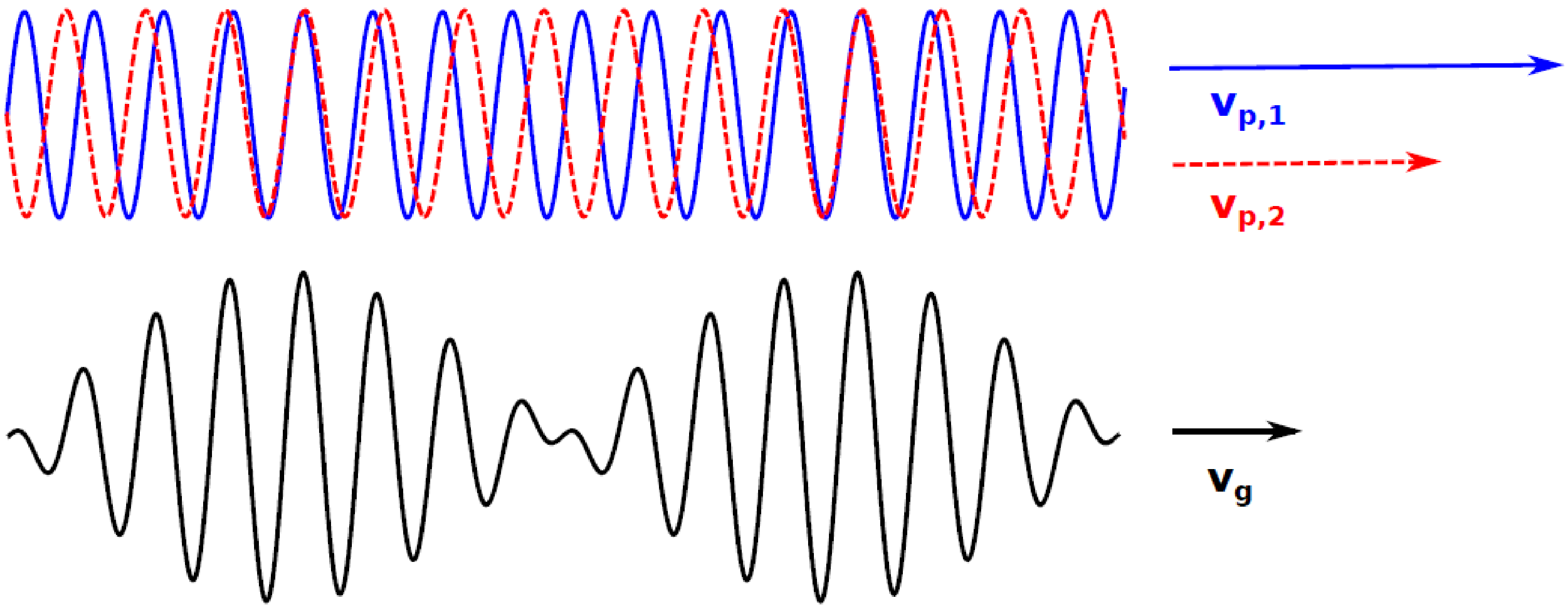
Intuitively - you know how many cycles are sitting inside the system . Many cycles and you get a big change in phase across the system for a given shift in frequency.

This is particularly effective for reading dispersion curves. It also how you get group delay from a vector network analyser.



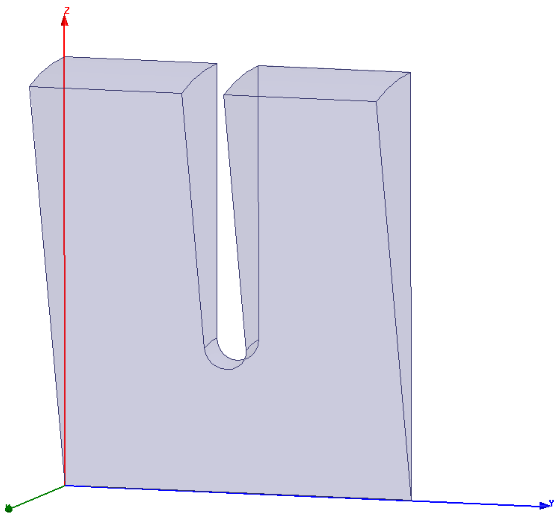
A graphical explanation for group velocity

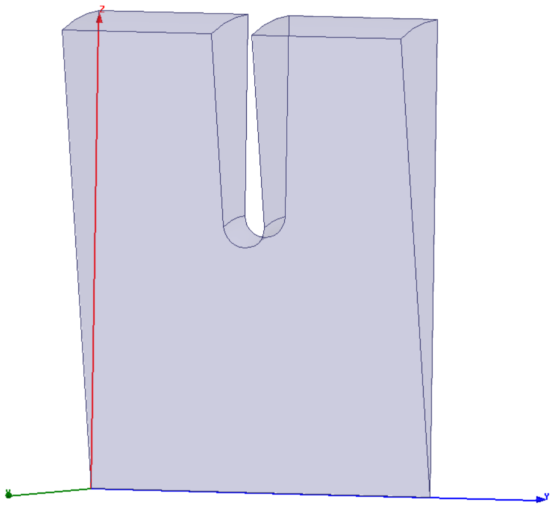
$$v_{group} = \frac{d\omega}{dk}$$



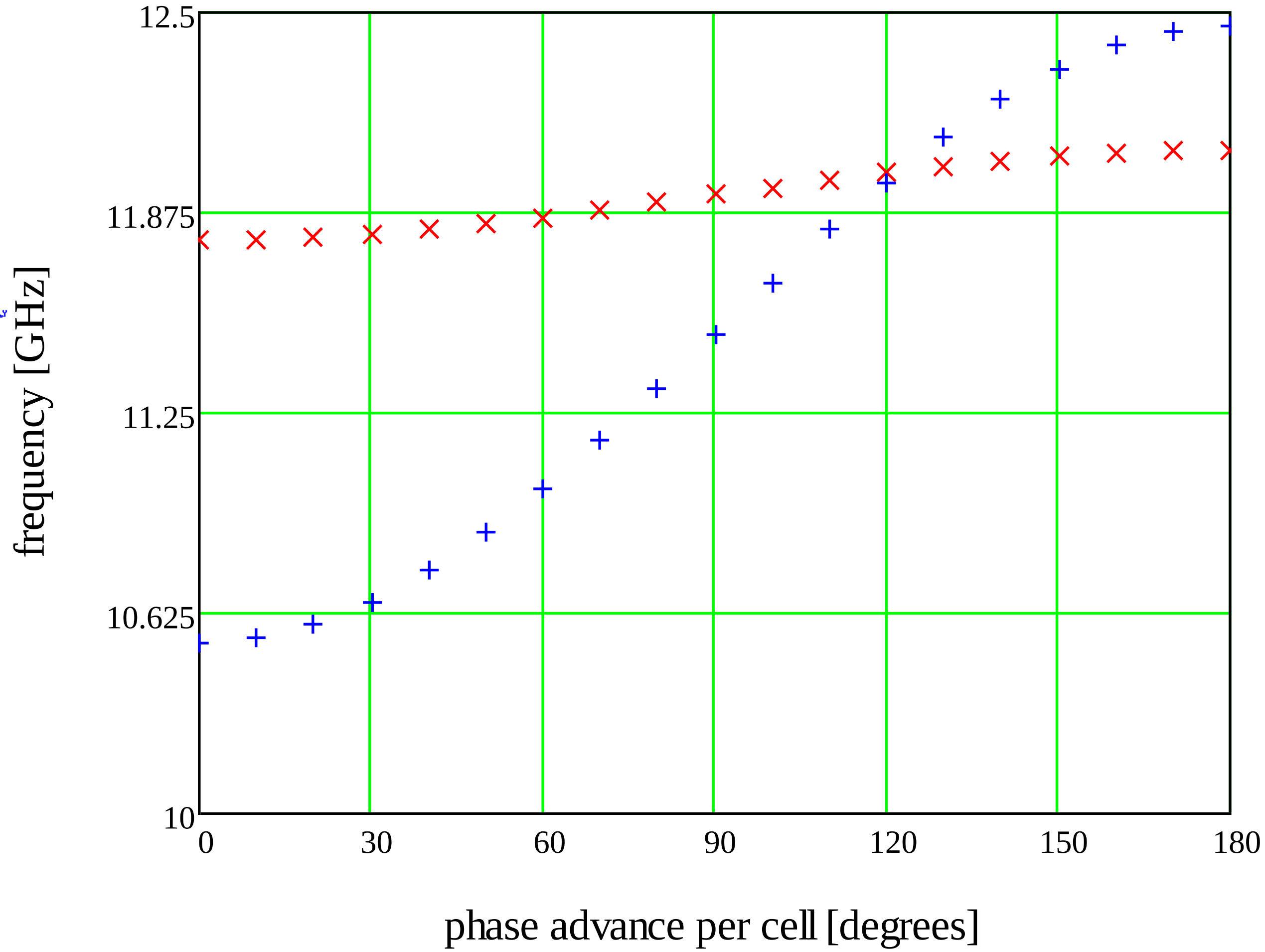
Phase and group velocity in a dispersive medium

Two different aperture geometries. The same phase velocity for the 2π/3 mode but different group velocity. This is given by the slope of the dispersion curve.





$$v_{phase} = \frac{\omega}{k} \qquad v_g = \frac{\partial \omega}{\partial k}$$



Now for UNIFORM CROSS SECTION WAVEGUIDES ONLY

$$v_{phase} = \frac{\omega}{k}$$

$$v_{group} = \frac{d\omega}{dk}$$

In a uniform waveguide

$$k^2 = \frac{\omega^2}{c^2} - \frac{\omega_0^2}{c^2}$$

cutoff frequency

$$\frac{dk}{d\omega} = \frac{1}{c^2}\frac{\omega}{k}$$

$$v_{phase} v_{group} = c^2$$