CAS RF School
Transverse deflecting cavities

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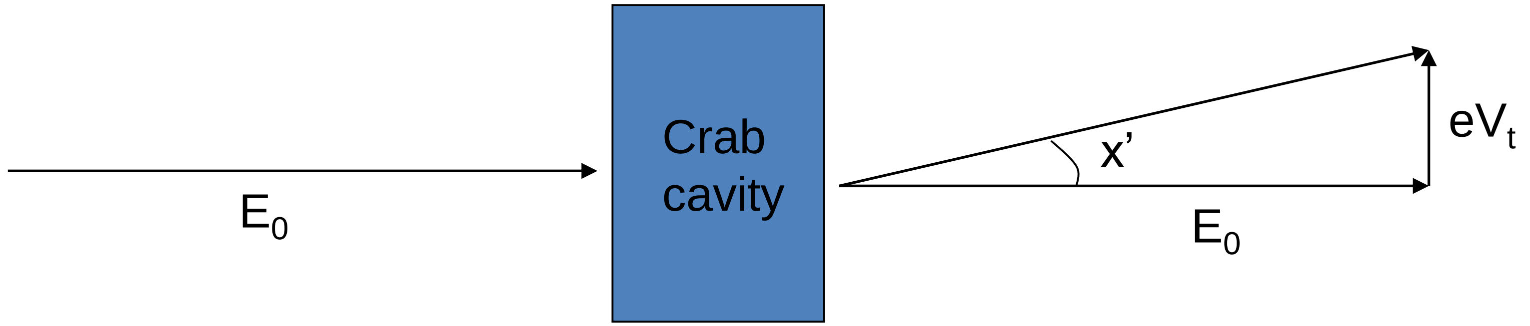
Transverse Kicks

- If we apply a **kick** of voltage V_t to a charged particle it gain a transverse energy qV_t
- If the electron has a longitudinal energy E_0 then the electron will have a trajectory with angle,
  - $x' = arctan(qV_t/E_0) \sim qV_t/E_0$

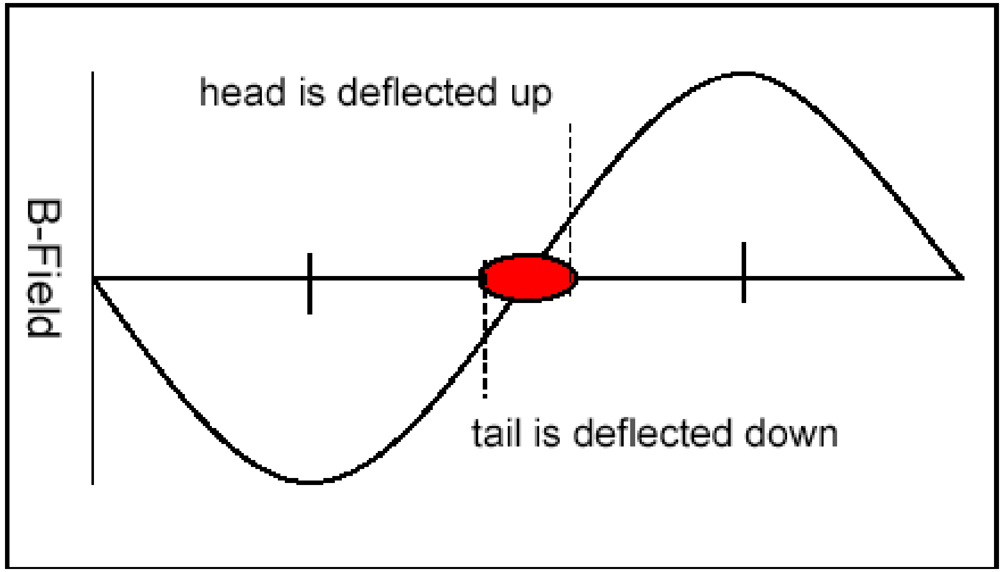


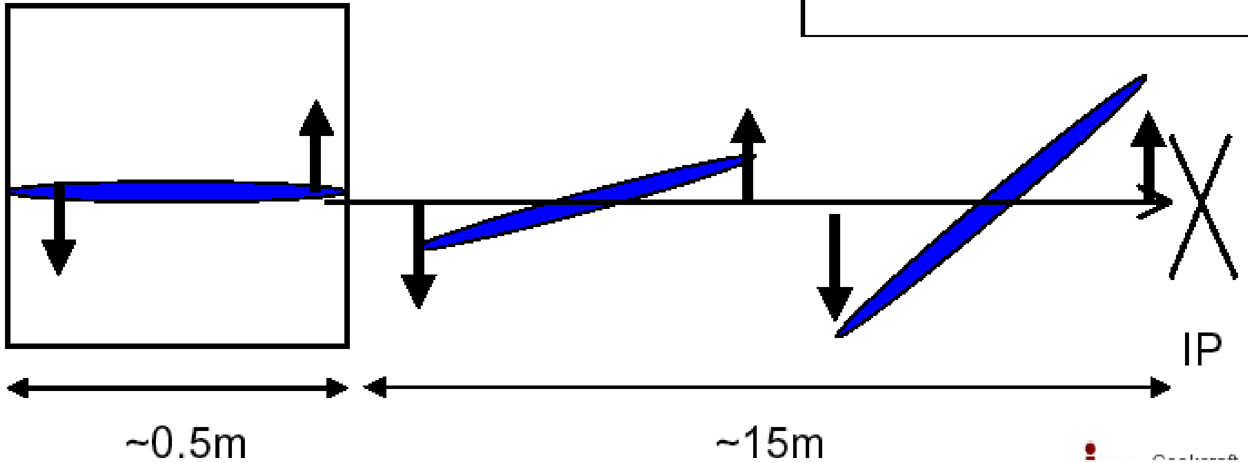
- The transfer matrix element R_{12} relates the final offset x_{IP} to the initial angle x' hence
  - $x_{IP} = R_{12}\, qV_t/E_0$

Kick or rotation

As the transverse kick varies **sinusoidally** in time, the finite bunch width means that each part of the bunch receives a different kick.

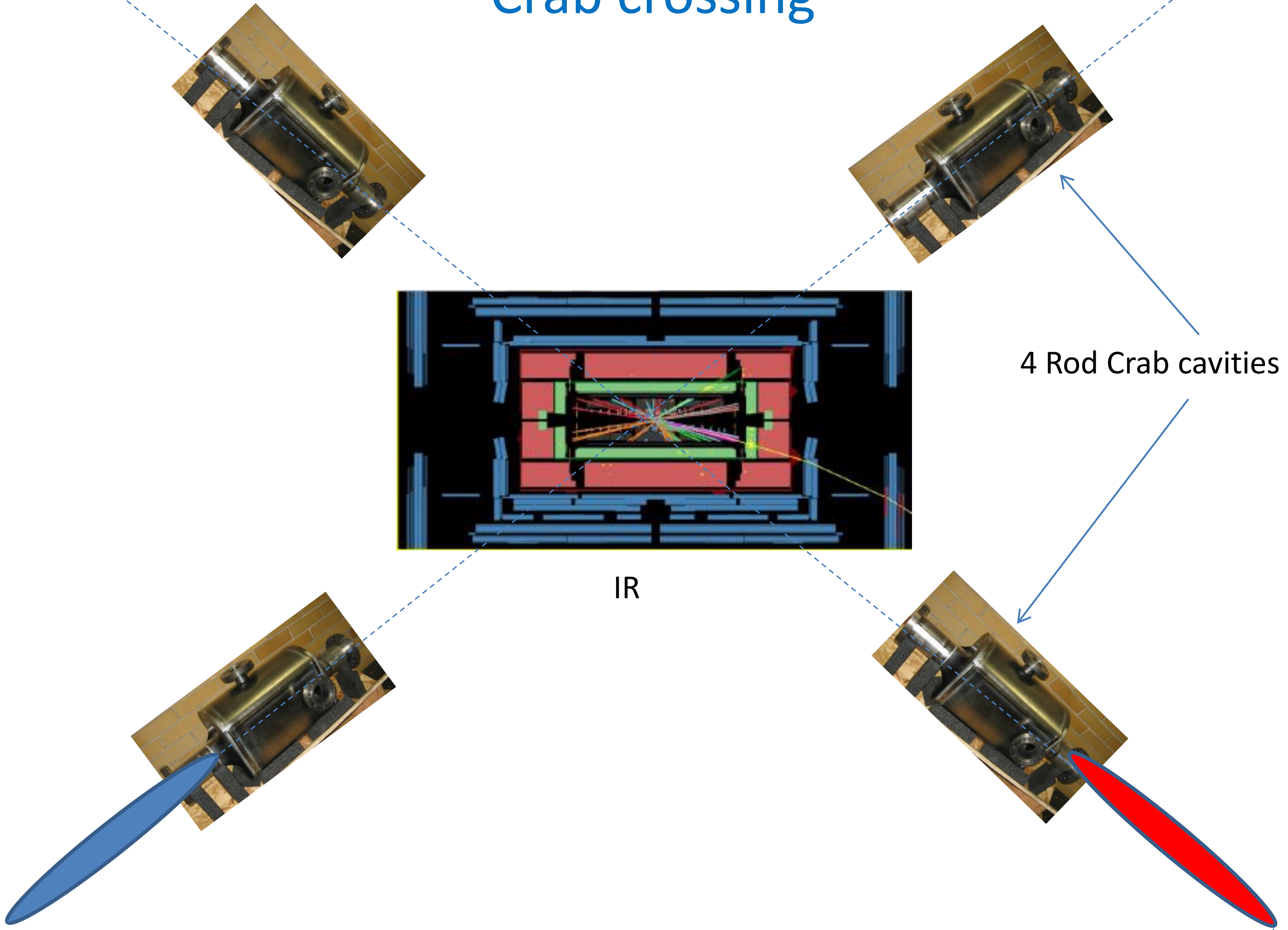
If the centre of the bunch is synchronised to pass through the cavity at the **zero crossing**, the **head and tail of the bunch will receive equal and opposite kicks**. Causing the bunch to appear to **rotate** as it travels towards its destination.





If a cavity is in rotating phase we call it a crab cavity

Crab crossing

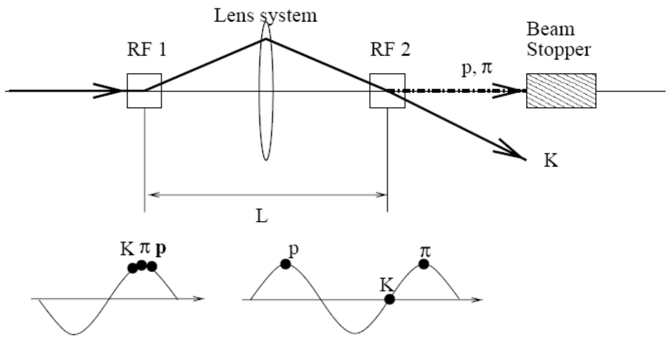


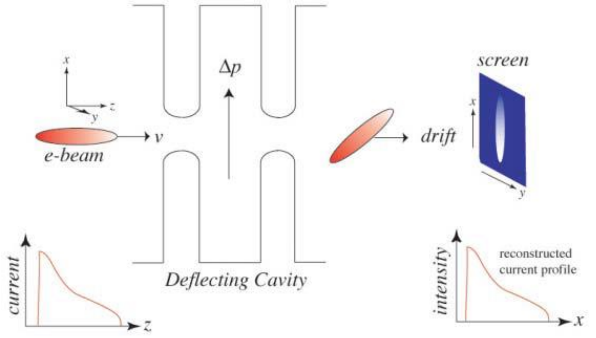
Applications of Deflecting Cavities

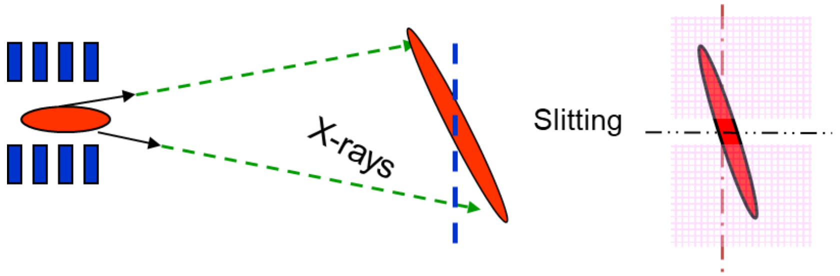
- Particle separation (Kaon separated from pions)
- Temporal beam diagnostics
- Crab-crossing in colliders
- X-ray pulse compression
- Emittance exchange
- Beam splitters (to different user stations)

- **<u>Basically any application where the kick changes too fast for a magnet or kicker</u>**

- Choppers are very similar but will not be discussed here!







Transverse Kicks

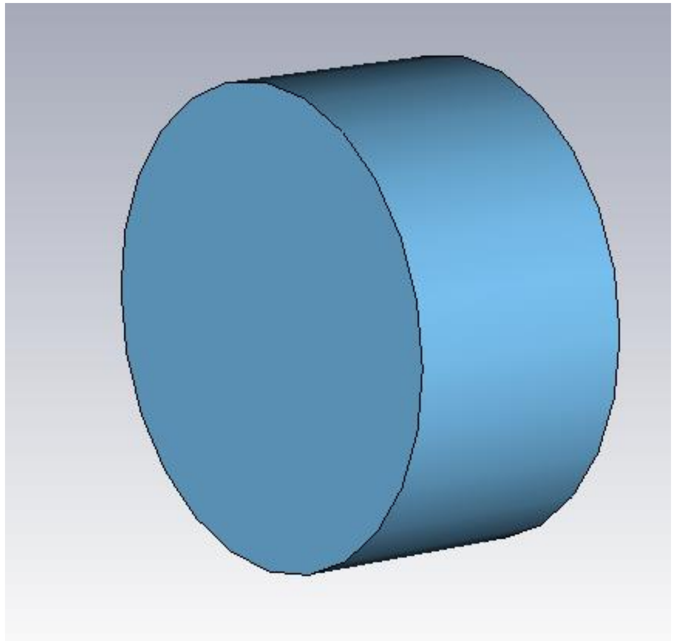
- The force on an electron is given by

$$F = e\left(E + v \times B\right)$$

- If an electron is travelling in the z direction and we want to kick it in the x direction we can do so with either
  - An electric field directed in x
  - A magnetic field directed in y
- The electric and magnetic field may be **in or out of phase** either **increasing the kick or cancelling it**
- At a high enough beam velocity E and cB look identical (as change in beam direction is negligible) and hence its common to discuss an equivalent **transverse voltage, V_t**

$$V_t = \int (E + v \times B)\, dz$$

Pillbox Cavity Fields



Wave equation in cylindrical co-ordinates

$$\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial\varphi^2}+\mu\varepsilon\omega^2-k_z{}^2\right]\psi=0$$

Solution to the wave equation

$$\psi = A_1 J_m(k_t r)e^{\pm im\varphi}$$

- Transverse Electric (TE) modes

$$H_z(r,\varphi)=A_1 J_m\left(\frac{\varsigma'_{m,n}\, r}{a}\right)e^{\pm im\varphi} \qquad H_t=\frac{ik_z a^2}{\varsigma'^2_{m,n}}\nabla_t H_z \qquad E_t=-\frac{i\mu\omega a^2}{\varsigma'^2_{m,n}}\left(\hat{z}\times\nabla_t H_z\right)$$

- Transverse Magnetic (TM) modes

$$E_z(r,\varphi)=A_1 J_m\left(\frac{\varsigma_{m,n}\, r}{a}\right)e^{\pm im\varphi} \qquad E_t=\frac{ik_z a^2}{\varsigma^2_{m,n}}\nabla_t E_z \qquad H_t=\frac{i\varepsilon\omega a^2}{\varsigma^2_{m,n}}\left(\hat{z}\times\nabla_t E_z\right)$$

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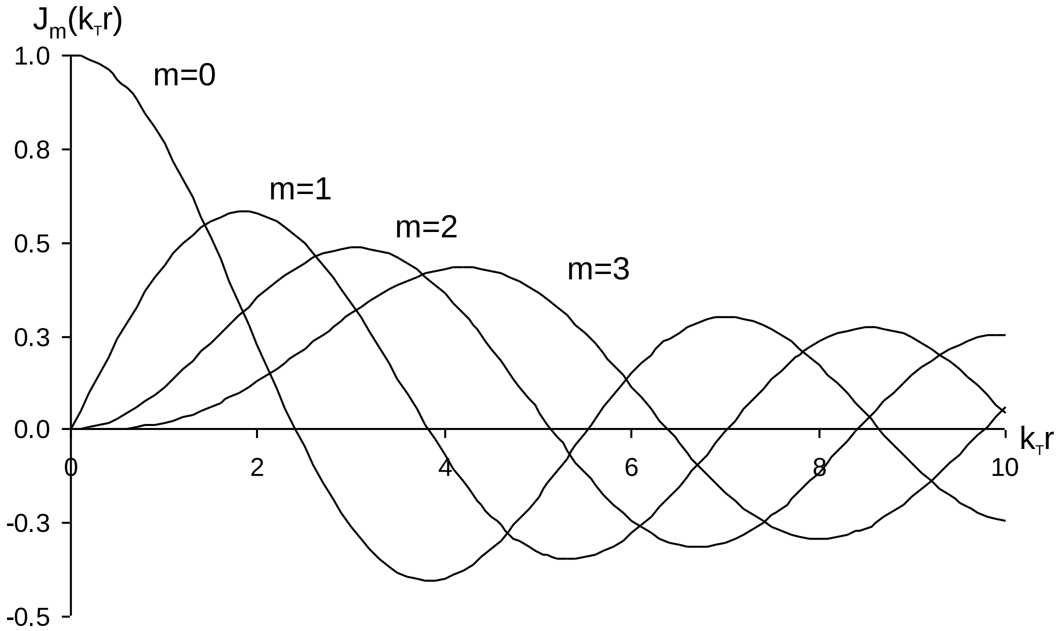
Bessel Function



First four Bessel functions.

- E_z (TM) and H_z (TE) **vary radially** as **Bessel functions** in pill box cavities.
- All functions have zero at the centre except the **0th order Bessel functions**.

- One of the transverse fields varies with the **differential of the Bessel function J'**
- All J' are zero in the centre except the **1st order Bessel functions**
- As we can only get transverse fields on axis with fields that vary with Differential Bessel functions of the 1st kind only modes of type $\mathbf{TM_{1np}}$ or $\mathbf{TE_{1np}}$ can kick electrons on axis.
- We call these modes **dipole** modes

TM_{110} Dipole Mode

$$E_z = E_0 J_1(k_t r)\cos(\varphi)$$

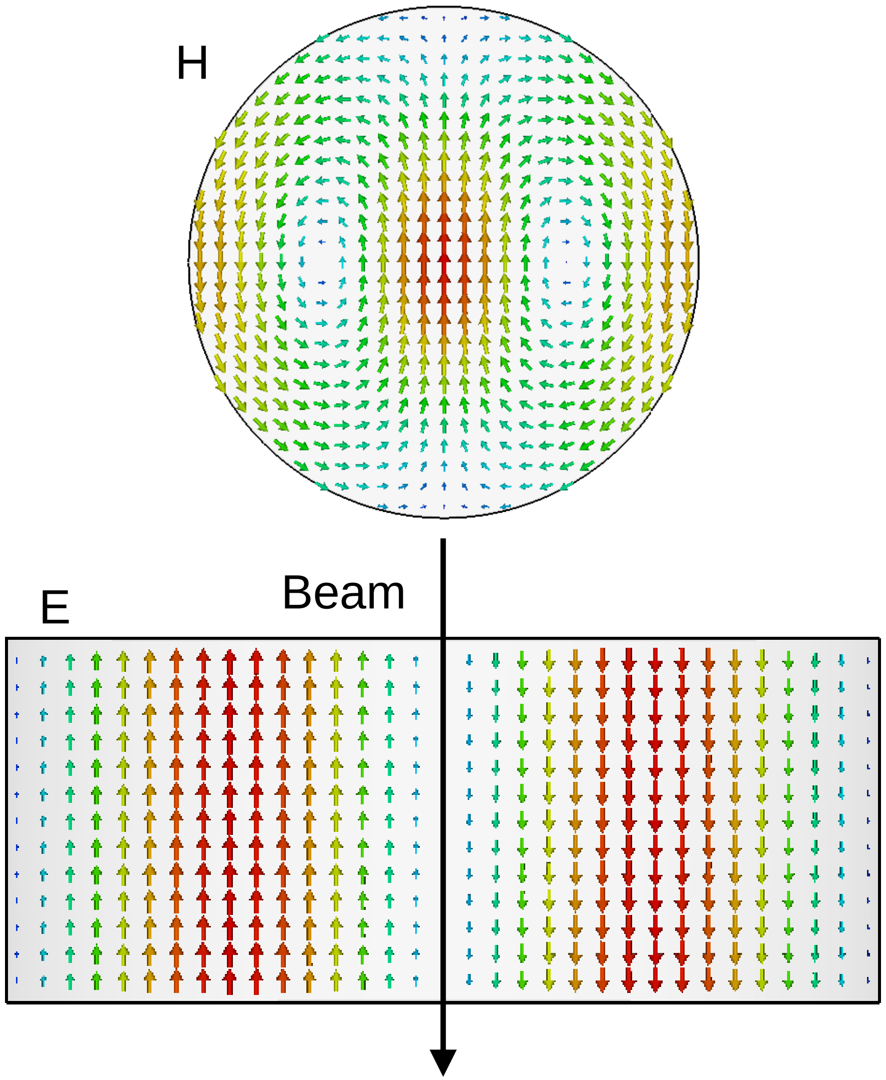
$$H_z = 0$$

$$H_r = \frac{i\omega\varepsilon}{k_t{}^2 r} E_0 J_1(k_t r)\sin(\varphi)$$

$$H_\varphi = \frac{-i\omega\varepsilon}{k_t} E_0 J_1'(k_t r)\cos(\varphi)$$

$$E_\varphi = \frac{-ik_z}{k_t{}^2 r} E_0 J_1(k_t r)\sin(\varphi)$$

$$E_r = \frac{-ik_z}{k_t} E_0 J_1'(k_t r)\cos(\varphi)$$



TE_{111} Dipole Mode

$$H_z = H_0 J_1(k_t r)\sin(\varphi)$$

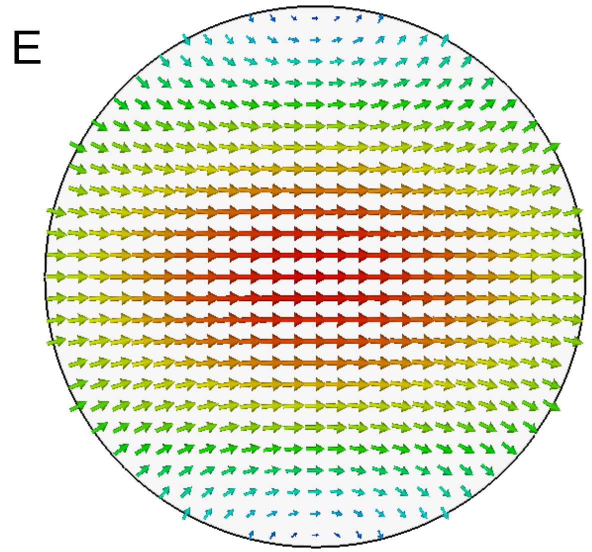
$$E_z = 0$$

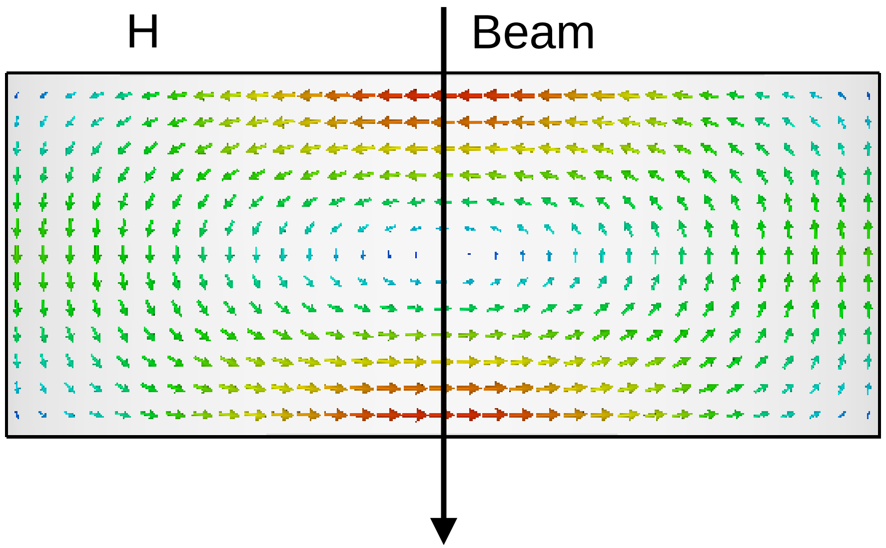
$$H_r = \frac{-ik_z}{k_t} H_0 J_1'(k_t r)\sin(\varphi)$$

$$H_\varphi = \frac{-ik_z}{k_t^2 r} H_0 J_1(k_t r)\cos(\varphi)$$

$$E_\varphi = \frac{i\omega\mu}{k_t} H_0 J_1'(k_t r)\sin(\varphi)$$

$$E_r = \frac{-i\omega\mu}{k_t^2 r} H_0 J_1(k_t r)\cos(\varphi)$$





Panofsky-Wenzel Theorem

If we rearrange Farday's Law ($\nabla \times E = -\frac{dB}{dt}$)and integrating along z we can show

$$c\int_0^L dzB\left(z,\tau = z/c\right) = c\int_0^L dz\int_{t_0}^{z/c} dt\left(\frac{\partial E_\perp\left(z,t\right)}{\partial z} - \nabla_\perp E_z\left(z,t\right)\right)$$

Inserting this into the Lorentz transverse force equation gives us

$$\int_0^L dz\left(E_\perp\left(z,z/c\right) + cB\left(z,z/c\right)\right) = c\int_0^L dz\int_{t_0}^{z/c} dt\left(\frac{dE_\perp\left(z,t\right)}{dz} - \nabla_\perp E_z\left(z,t\right)\right)$$

for a closed cavity where the 1st term on the RHS is zero at the limits of the integration due to the boundary conditions this can be shown to give

$$\int_0^L dz\left(E_\perp\left(z,z/c\right) + cB\left(z,z/c\right)\right) = -c\int_0^L dz\int_{t_0}^{z/c} dt\left(\nabla_\perp E_z\left(z,t\right)\right)$$

Panofsky-Wenzel Theorem

$$\int_0^L dz\left(E_\perp\left(z,{}^z\!/_c\right)+cB\left(z,{}^z\!/_c\right)\right)=-c\int_0^L dz\int_{t_0}^{z/c} dt\left(\nabla_\perp E_z\left(z,t\right)\right)$$

As the electrons have a large longitudinal energy we can approximate the kick from the magnetic field as equivalent to an electric field of magnitude E=cB. Hence we can define a transverse voltage

$$V_\perp=\int_0^L dz\left(E_\perp\left(z,{}^z\!/_c\right)+cB\left(z,{}^z\!/_c\right)\right)$$

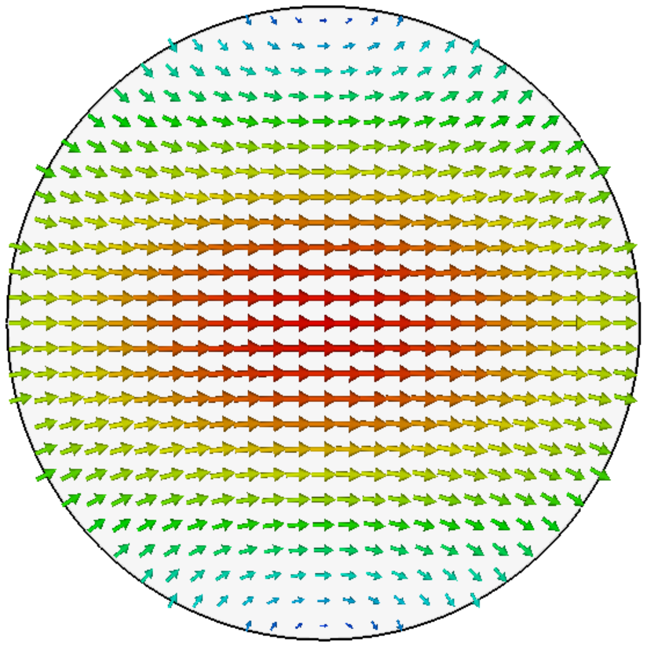
$$V_\perp=-c\int_0^L dz\int_{t_0}^{z/c} dt\left(\nabla_\perp E_z\left(z,t\right)\right)$$

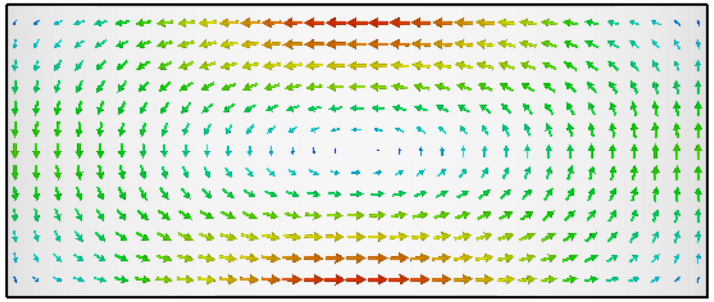
$$V_\perp=-\frac{ic}{\omega}\int_0^L dz\nabla_\perp E_z\left(z,{}^z\!/_c\right)\sim-\frac{ic}{\omega}\frac{mV_\parallel}{r^m}$$

This means the transverse voltage is given by the rate of transverse change of the longitudinal voltage (for particles travelling close to c).

TE modes

- The transverse kick is proportional to the rate of **radial change in the E_z field**.
- TE modes **do not have longitudinal electric fields** so they **cannot kick** an electron beam.
- But the TE110 mode has transverse E and B fields what happens to their kick?
- The transverse **kick due to the electric fields and the magnetic fields completely cancel** each other out if they have the same magnitide
- As the magnetic kick is proportional to particle velocity for **low beta cavities TE modes can be used** for deflecting





Transverse Shunt Impedance

For dipole modes, m=1, so the transverse voltage is given by

$$V_{\perp} == -\frac{ic}{\omega}\frac{V_{\parallel}}{r}$$

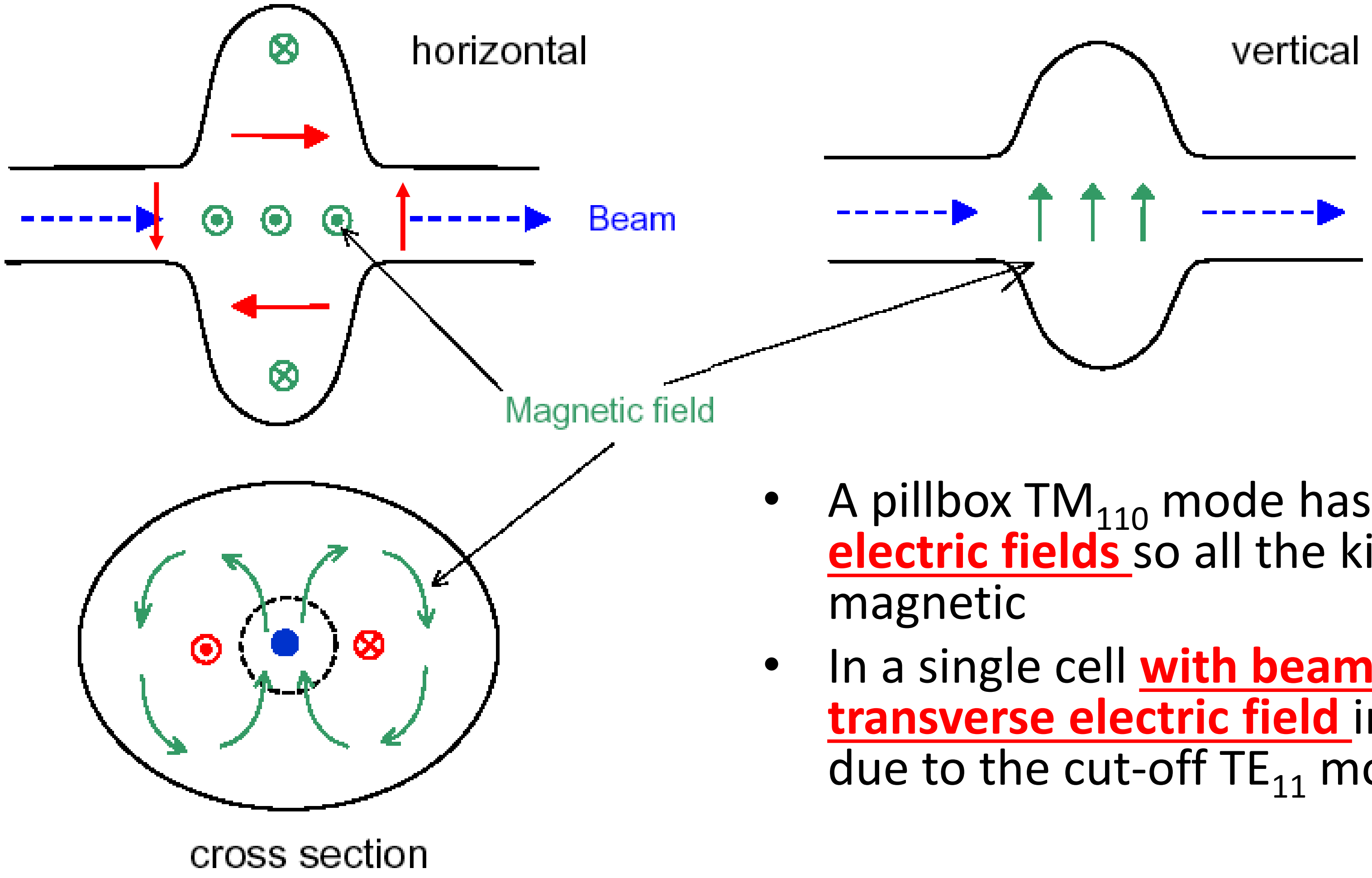
For calculating required power we use a modified transverse shunt impedance definition

$$R_{\perp} = \frac{1}{2}\frac{\left|V_{\perp}\right|^{2}}{P_{c}}$$

We also use a modified transverse R/Q definition (like when calculating dipole wakefields) except this definition is in the units of Ohms and is in a more convenient form for calculating power and energy requirements for deflecting cavities.

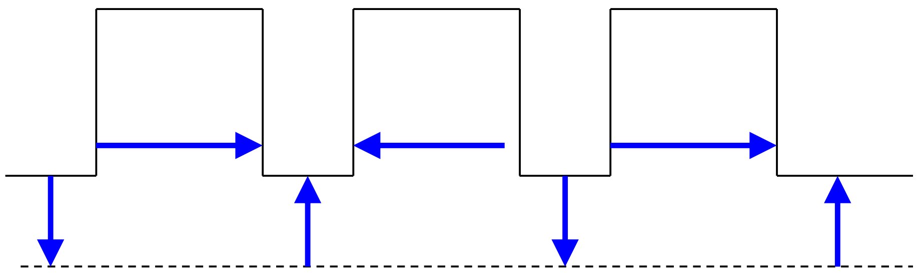
$$\frac{R}{Q} = \frac{\left|V_{\perp}\right|^{2}}{2\omega U} = \frac{\left|V_{\parallel}(r)\right|^{2}}{2\omega U}\left(\frac{c}{\omega r}\right)^{2}$$

Single-cell crab cavity



- A pillbox TM_{110} mode has **no transverse electric fields** so all the kick would be magnetic
- In a single cell **with beampipes** cause a **transverse electric field** in the beampipes due to the cut-off TE_{11} mode.

Iris loaded deflectors

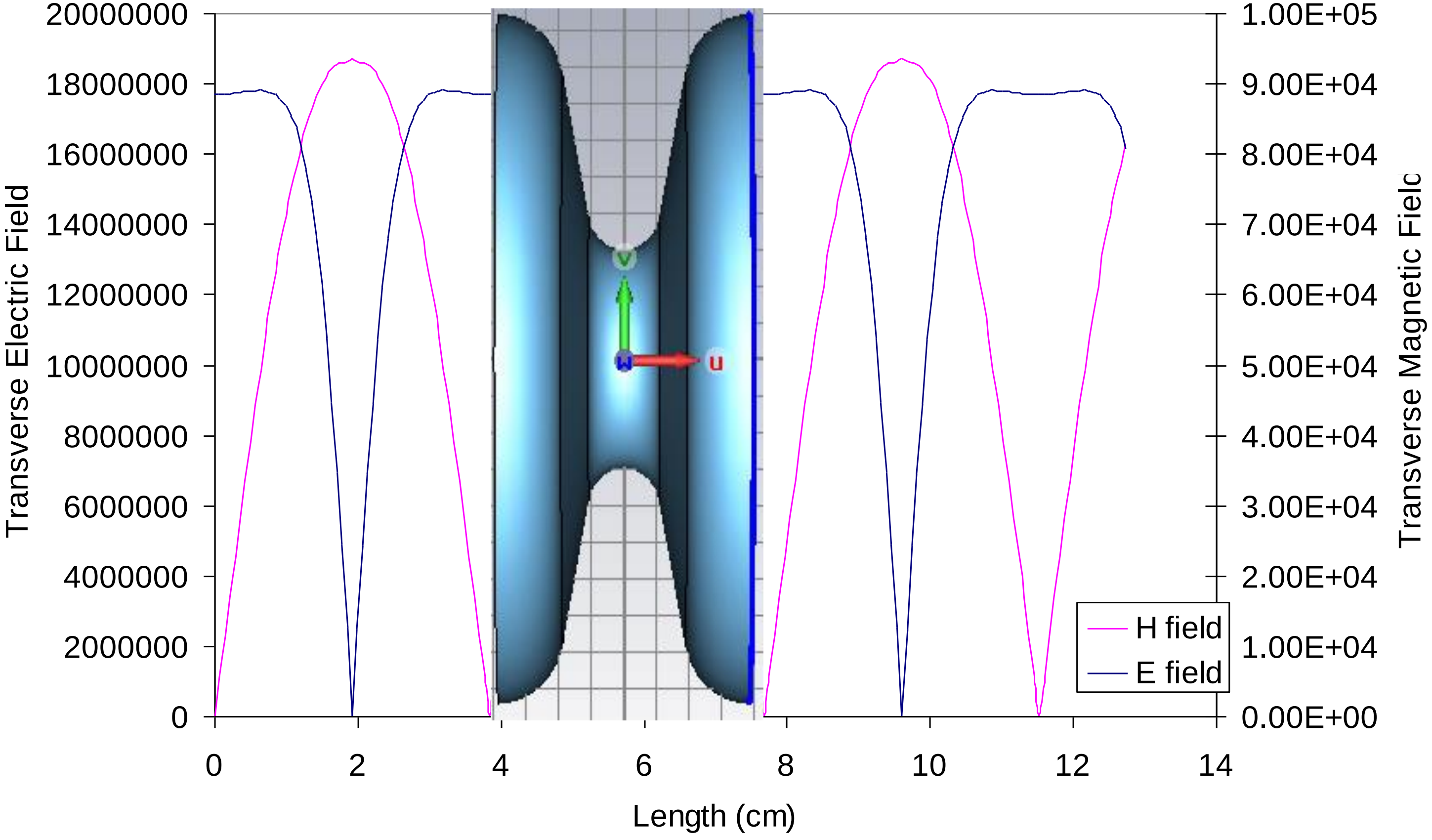


When we add the **iris** the **TM_{110} mode** in the cavity **couples** to the **TE_{11} mode** of the iris.

- The fields near the centre of the cavity becomes

$E_x = \mathcal{E}\frac{k}{4}\left(a^2 + x^2 - y^2\right)\sin z \cos\omega t,$	$cB_x = \mathcal{E}\frac{k}{2}xy\cos kz\sin\omega t,$
$E_y = \mathcal{E}\frac{k}{2}\,xy\,\sin kz\cos\omega t,$	$cB_y = -\mathcal{E}\frac{1}{k}\left(\frac{(ka)^2}{4} - 1 + \frac{k^2(x^2-y^2)}{4}\right)\cos kz\sin\omega t,$
$E_z = \mathcal{E}\,x\cos kz\cos\omega t,$	$cB_z = -\mathcal{E}\,y\sin kz\sin\omega t.$

Fields seen on-axis

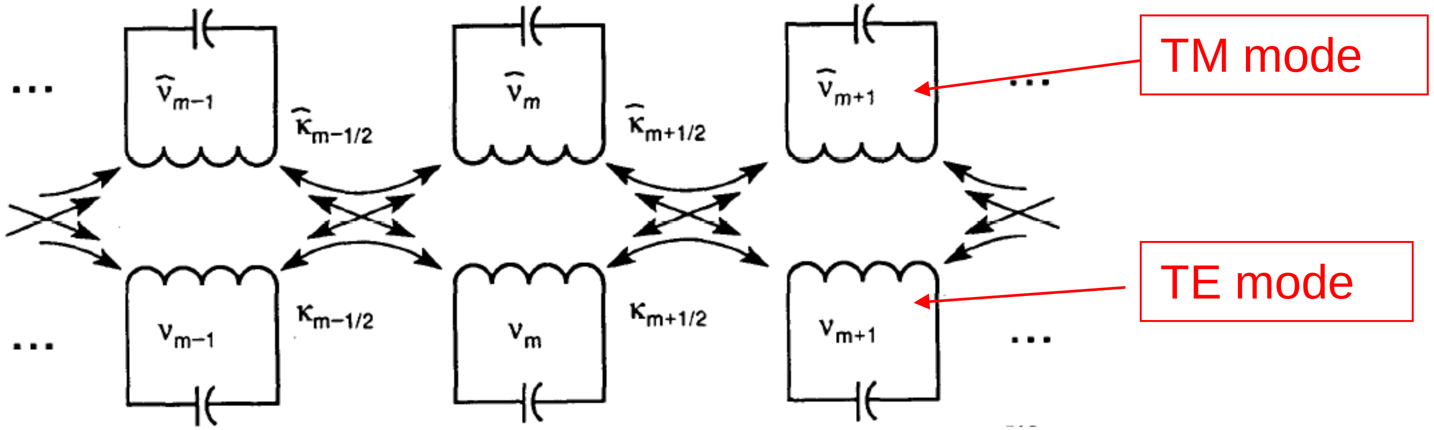


The electric and magnetic fields are 90 degrees out of phase in **both space and time** so that their kicks coherently add.

The electric field is in the iris and the magnetic field is in the cavity

Equivalent Circuit

- To find the dispersion of the deflecting cavity an equivalent circuit can be constructed.
- In order to obtain accurate results we need to include the **TE mode** as well as the TM mode in the cavity. This leads to a **two-chain model**
- **The iris coupling is magnetic at low frequency and electric at high frequency so is this a forwards or backwards travelling wave?**

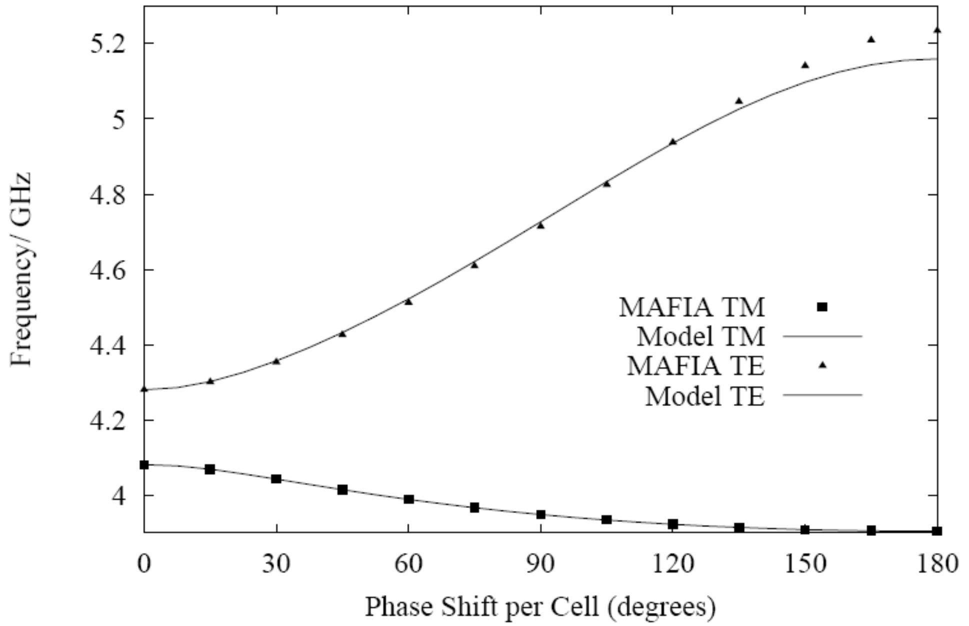


Each mode couples to its nearest neighbour of both modes

$$\left(\frac{1}{\nu^2}-\lambda\right) f_m - \frac{\kappa}{2} f_{m+1} - \frac{\kappa}{2} f_{m-1} = -\frac{\sqrt{\kappa\bar{\kappa}}}{2} f_{m+1} + \frac{\sqrt{\kappa\bar{\kappa}}}{2} f_{m-1}$$

$$\left(\frac{1}{\bar{\nu}^2}-\lambda\right) \bar{f}_m - \frac{\bar{\kappa}}{2} \bar{f}_{m+1} - \frac{\bar{\kappa}}{2} \bar{f}_{m-1} = \frac{\sqrt{\kappa\bar{\kappa}}}{2} \bar{f}_{m+1} - \frac{\sqrt{\kappa\bar{\kappa}}}{2} \bar{f}_{m-1}$$

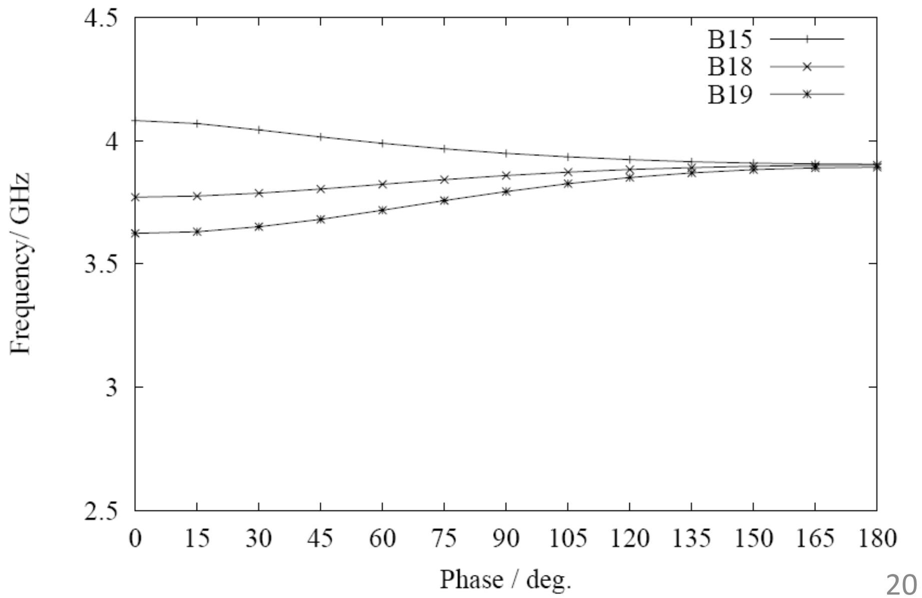
Dispersion Diagram



The two-chain model creates **two eigenmode passbands**, a TM-like hybrid and a TE-like hybrid. Neither has an exact sinusoidal dependence due to the TM-TE mixing. Very shallow near the pi mode

The **lower mode is magnetically coupled** and is a backwards travelling wave, **the upper mode is electric coupled** and is a forwards travelling wave

As the cell to cell coupling of the eigenmode can occur via the TE mode, the cell-to-cell coupling parameter can be **capacitive or inductive** depending on the exact dimensions. A **large iris means the lower mode becomes capacitive** and we get a forwards travelling wave too



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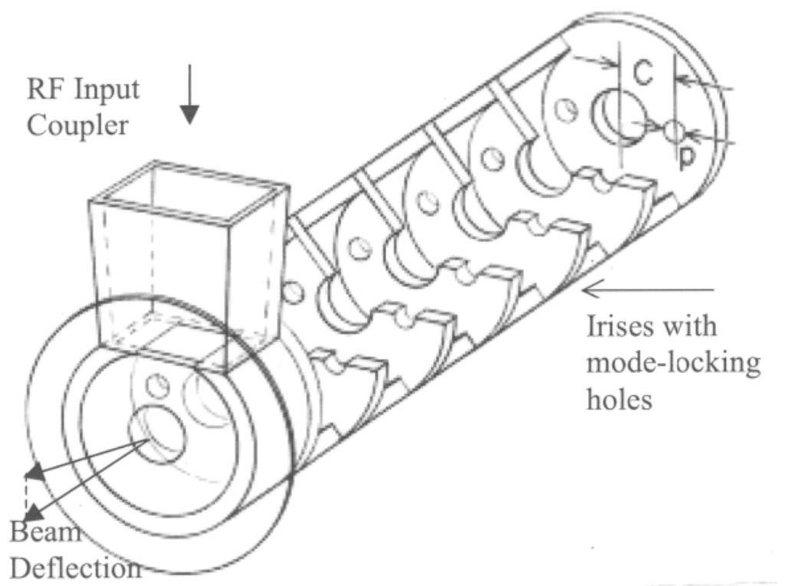
Travelling wave Cavities

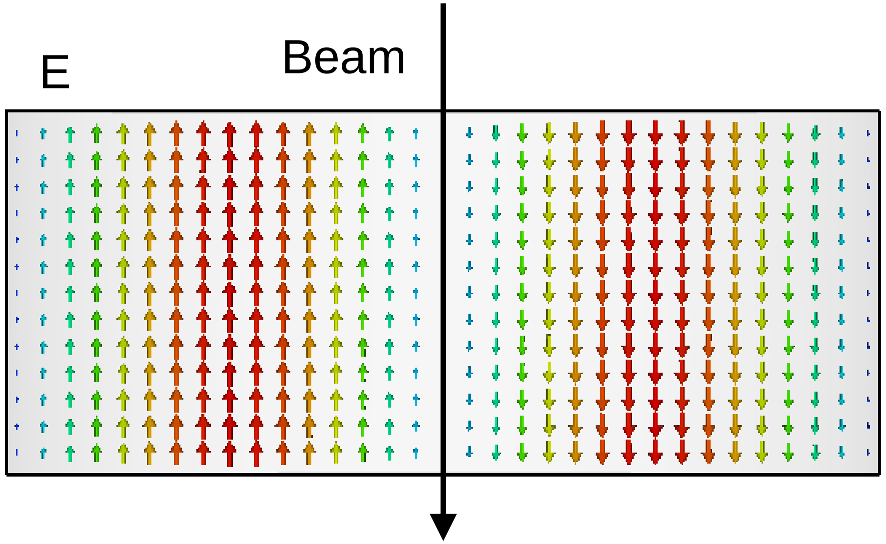
- Like accelerating cavities we can also use travelling wave deflecting cavities.
- These cavities are less sensitive to temperature, can have more cells per cavity and fill faster.
- The down side is they require more RF power.
- Most diagnostic cavities and fast separators are travelling wave to take advantage of fast filling times.



Beam-loading

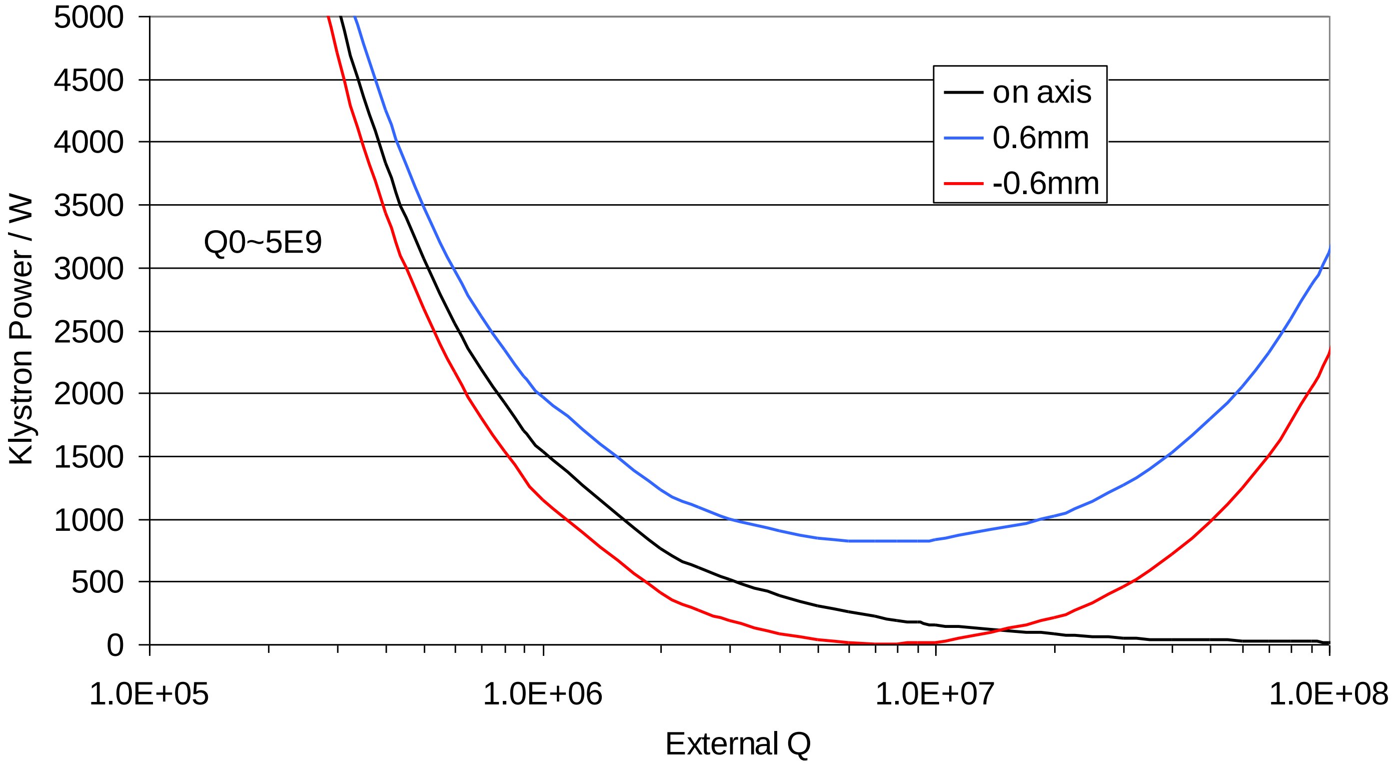
As pointed out by Panofsky and Wenzel in 1956, deflection from *E* and *B* in a TM mode add - but this means **large E_z near but not at cavity center** axis.

As the Ez field is zero on axis the **beam-loading is zero on axis** but like the Ez field it **varies linearly with offset** as the beam goes off-axis radially. The **beam-loading** can be either **positive or negative** depending on the beam position.



The **decelerating** field is **90 degrees out of phase** with the **deflecting field**. Hence the beam-loading in deflecting phase is zero, but is maximum when in crabbing phase.

Dipole Beam-loading

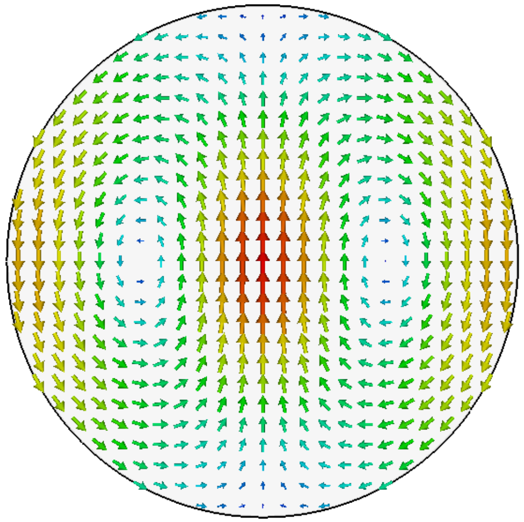


As the beam-loading can be positive or negative, **the beam can either give or take power from the cavity**. This makes control hard as the beam position jitters.

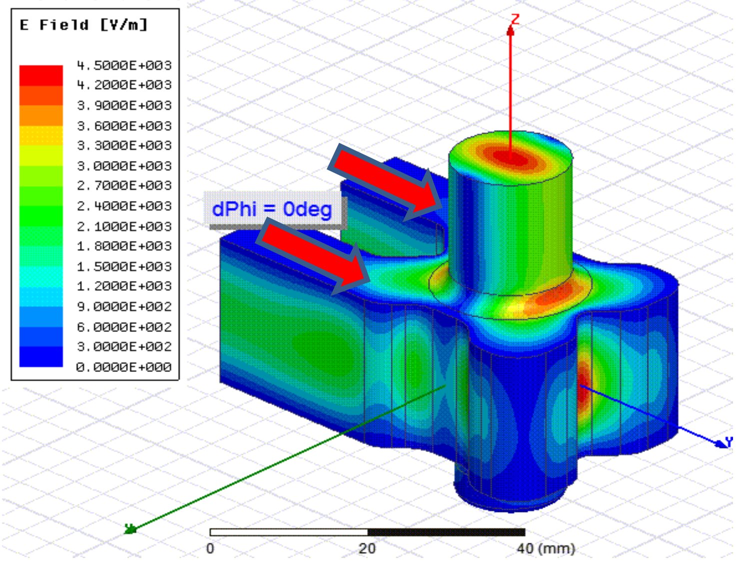
It could even be possible (but not advisable) to run the cavity without an RF amplifier using an offset beam.

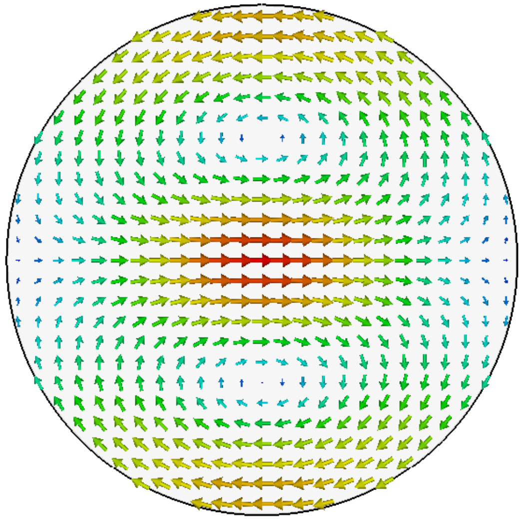
Mode Polarisation

- Dipole modes have a distinct **polarisation** ie the field points in a given direction and the **kick is in one plane,** horizontal or vertical.
- In a cylindrically symmetric cavity this polarisation could take any angle.
- In order to set the polarisation we make the cavity **slightly asymmetric** (racetrack cross section, squeezing, coupling slots or dimples).
- This will set up two dipole modes in the cavity each at 90 degrees to each other and **we can select the correct polarisation by exciting the correct frequency.**



- One mode will be the operating mode, the other is referred to as the same order mode (SOM) and is unwanted.
- In PolariX both modes are at the same frequency. Variable polarization is made possible via the circular TE_{11} Mode Launcher

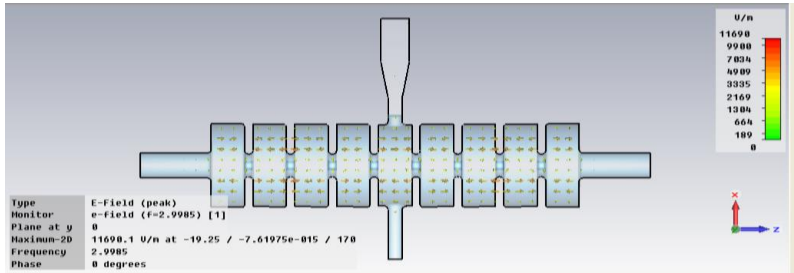


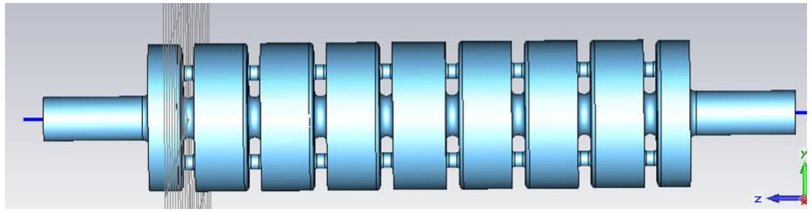


Standing wave Deflector

In a standing wave deflector, in crabbing mode, **<u>the center of the bunch is deflected first up then down</u>** in a single cell so that the net deflection is zero

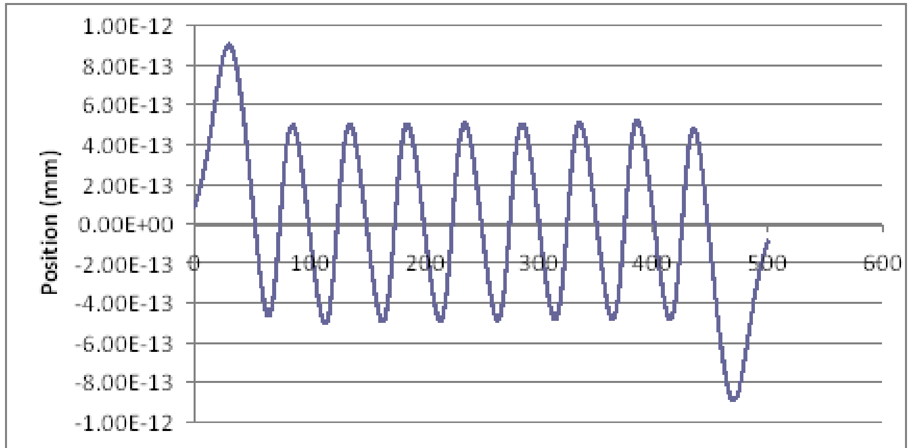
In the end cell the electric field **<u>decays into the beampipe</u>** so the centre of the bunch sees **<u>more deflection in the first half</u>** of the cell so the net deflection is **<u>only cancelled at the end cell on the other side</u>**



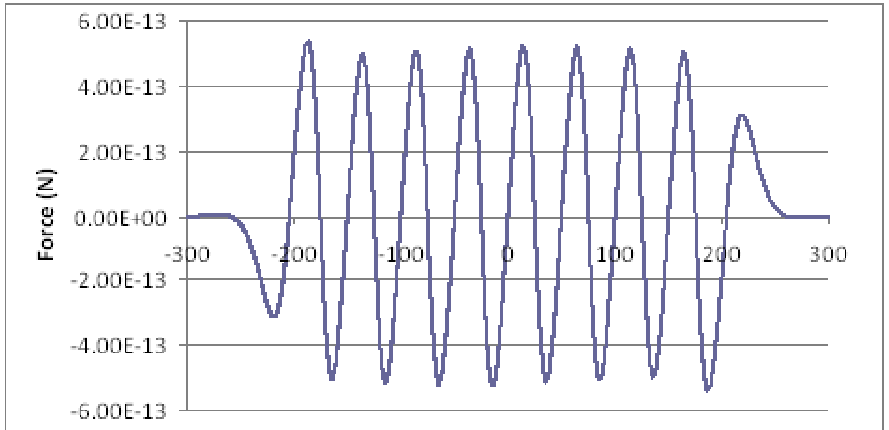


At low beam energy this can result in a large beam offset

For low energy deflectors used in injectors the end cells need to be shorter to balance the kicks locally

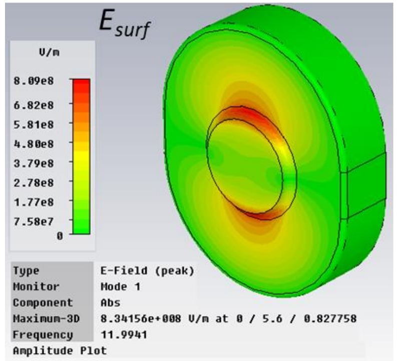


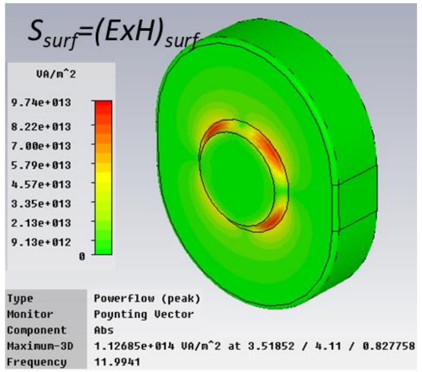
Original design

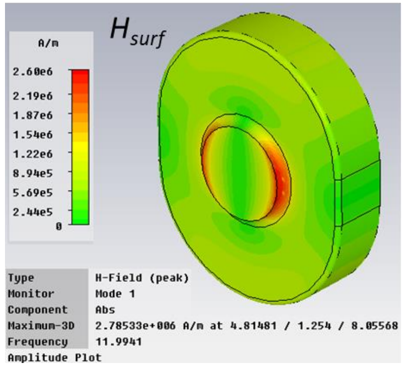


New design

Peak Fields





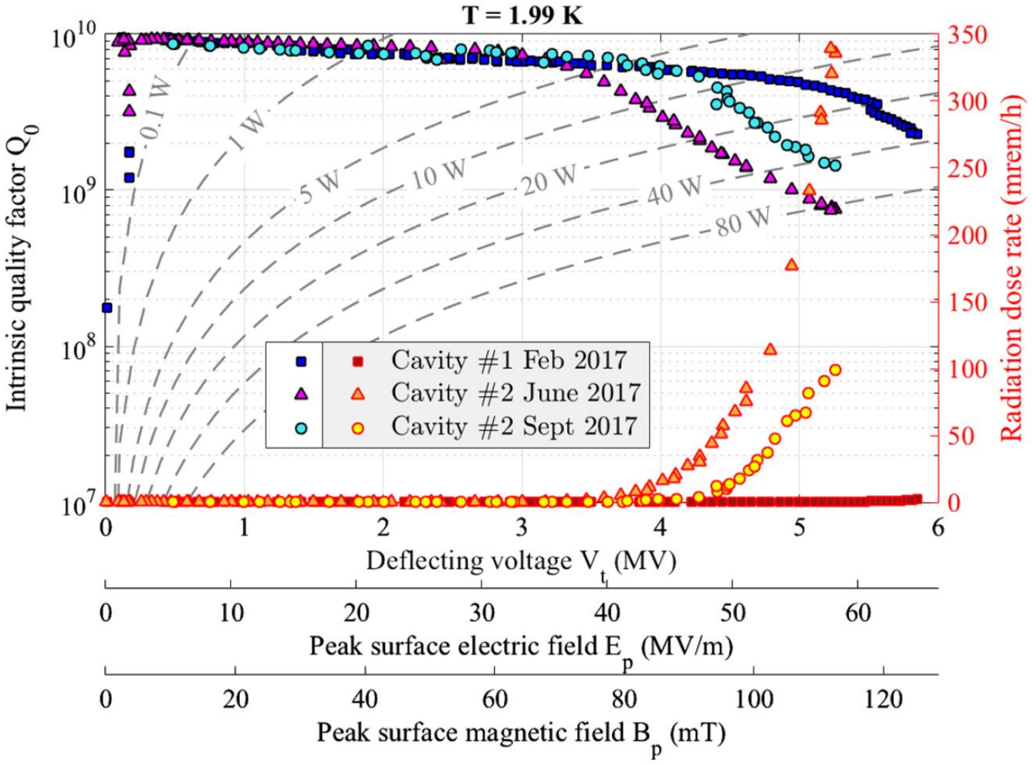


Dipole cavities **have much larger peak surface magnetic fields** than surface electric fields.

This ***limits the gradient*** in SRF cavities compared to acc. Cavities

For normal conducting cavities its not clear what the limits are as E is smaller, B is larger and S_c is about the same (see Walters lecture)

NCRF TWS	CLIC T24 (unloaded)	LCLS deflector
Input Power	37.2 MW	20 MW
Gradient	100 MV/m	22 MV/m
Peak surf. E-field	219 MV/m	115 MV/m
Peak surf. H-field	410 kA/m	405 kA/m
Peak Sc [3]	3.4 MW/mm^2	3.5 MW/mm^2



SRF Cavity type	mode	Frequency GHz	B_{max} mT	E_{max} MV/m
TESLA	TM010	1.3	105	50
CKM	TM110	3.9	80	18.5

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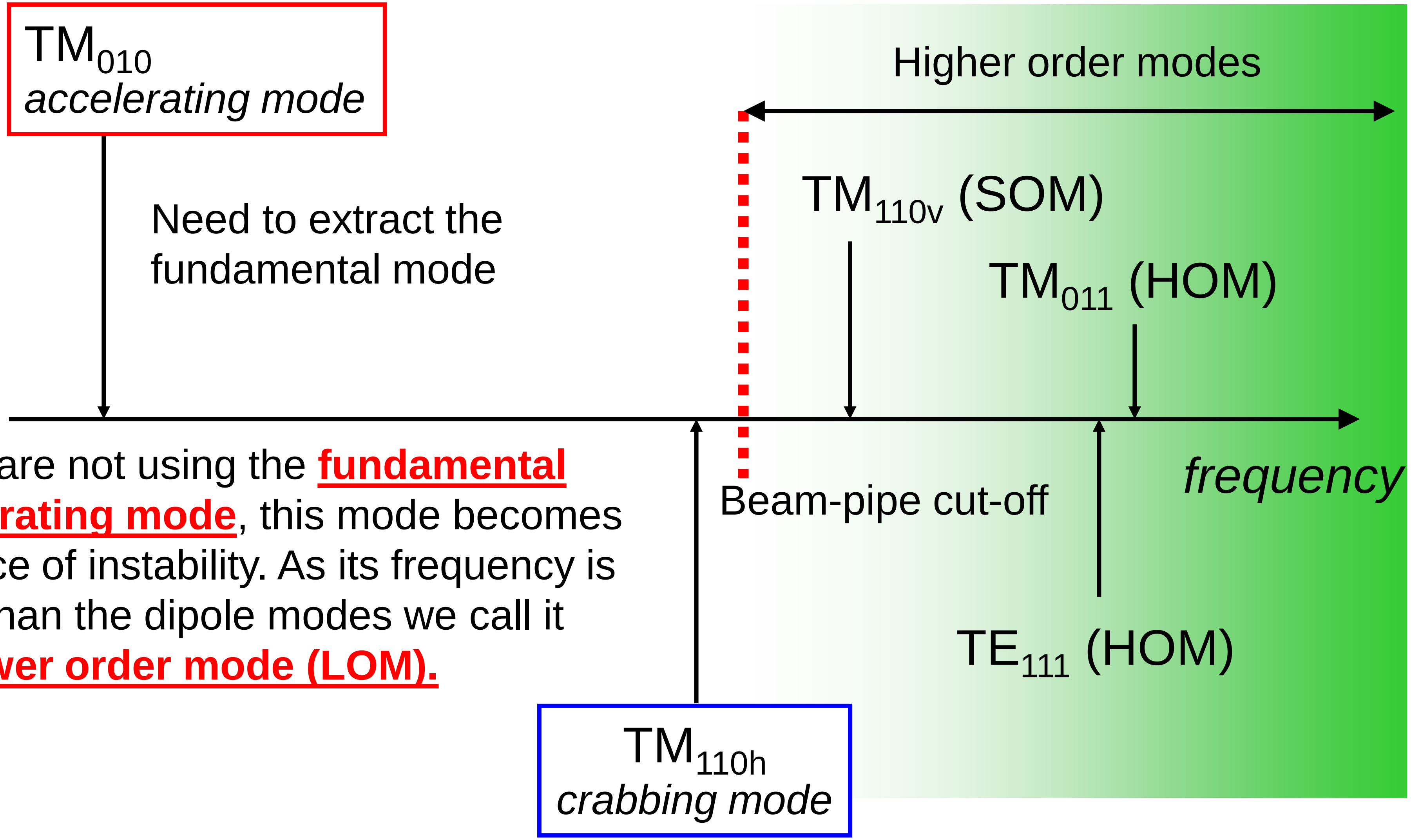
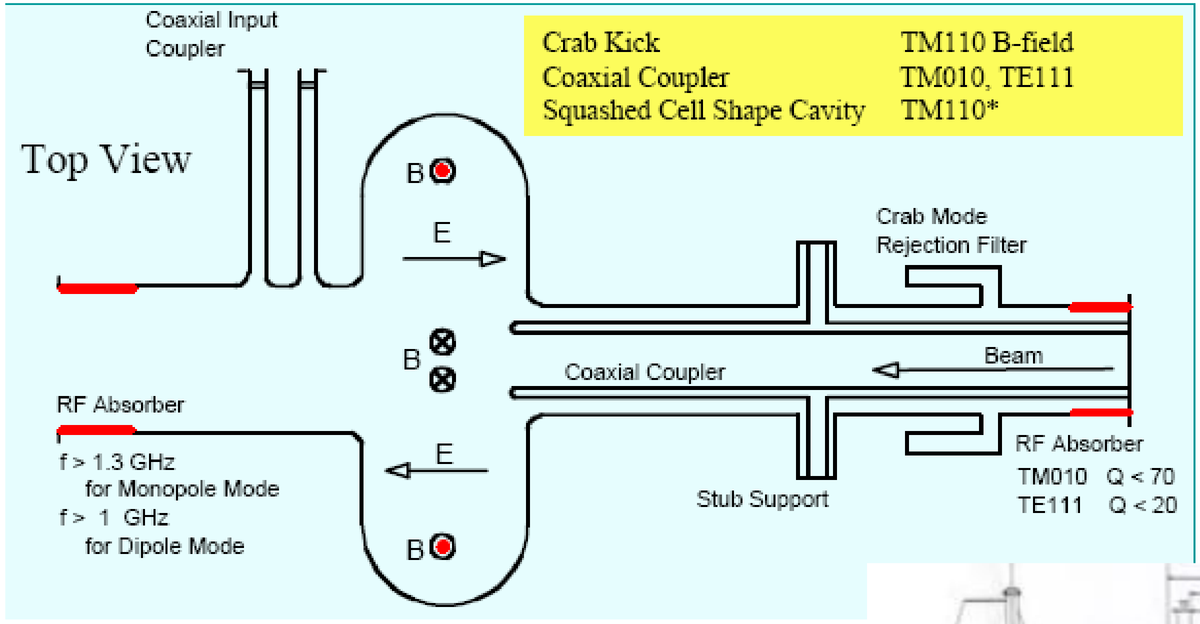
Lower and Higher Order Modes



As we are not using the **fundamental accelerating mode**, this mode becomes a source of instability. As its frequency is lower than the dipole modes we call it **the lower order mode (LOM).**

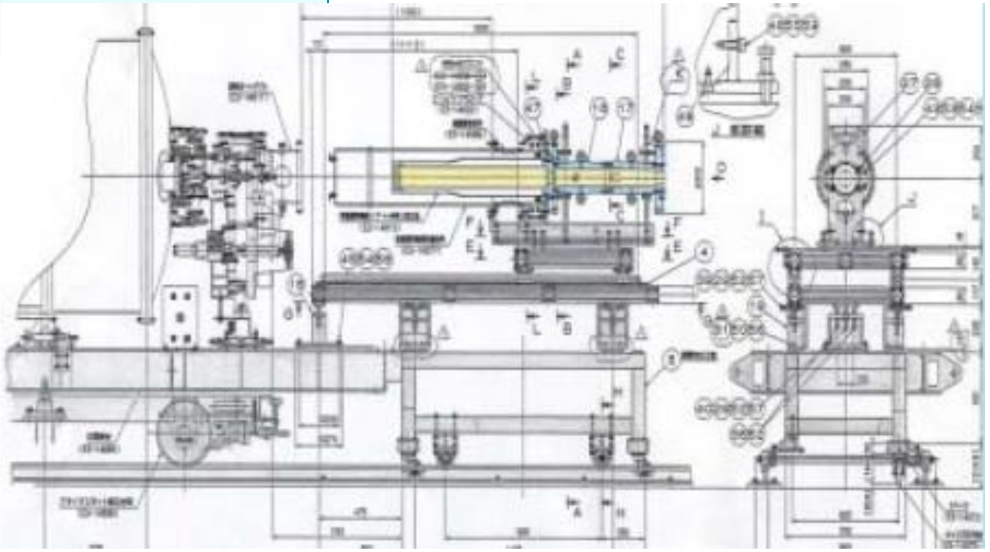
KEKB Coaxial Damper



- The cavity has special **hollow coaxial dampers** to deal with the monopole mode (LOM) of the cavity.
- If the **coax is centred it will not couple** to the dipole mode as the dipole modes are **cut-off in the beam-pipe**. Only the TEM mode exists.

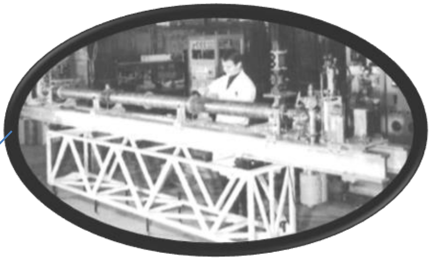
If the coax is off centre the crab mode can couple to the **TEM coax mode**, hence a rejection filter is used.

Alignment is not easy with such a long coupler.

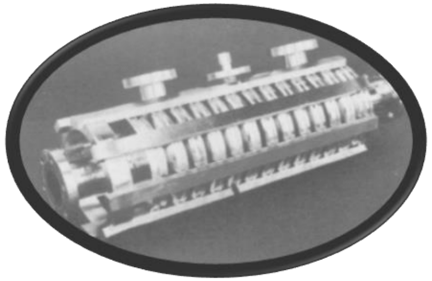


Early History of deflectors

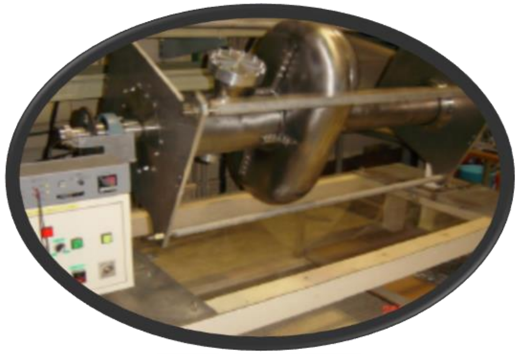
CERN, Montague Jan 1965 ,Bernard and Lengler 1969, 2π/3 2855 MHz 100 Cells



CERN-Karlsruhe, 1970, 104 cell S-band pi/2 SRF



KEK, 1991-2009, single cell elliptical SRF, 500 MHz

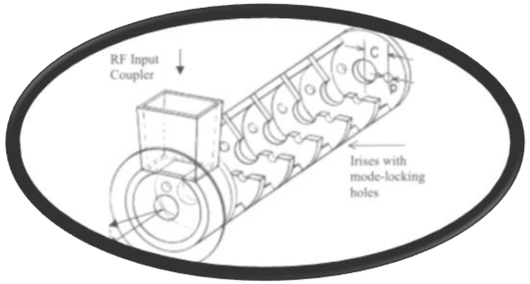


1965

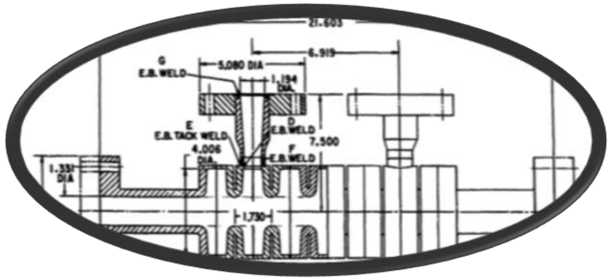
1985

2005

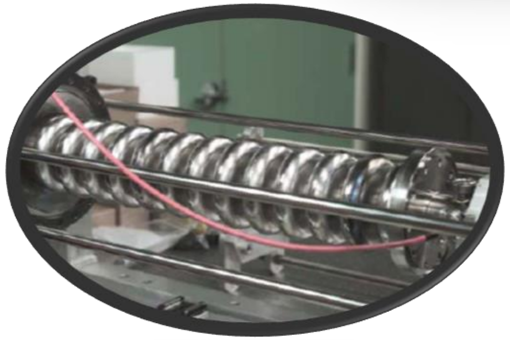
SLAC, Loew 1965, 2π/3 mode traveling wave, Frequency=2856 MHz



NAL, 1973, pi mode SRF, X-band

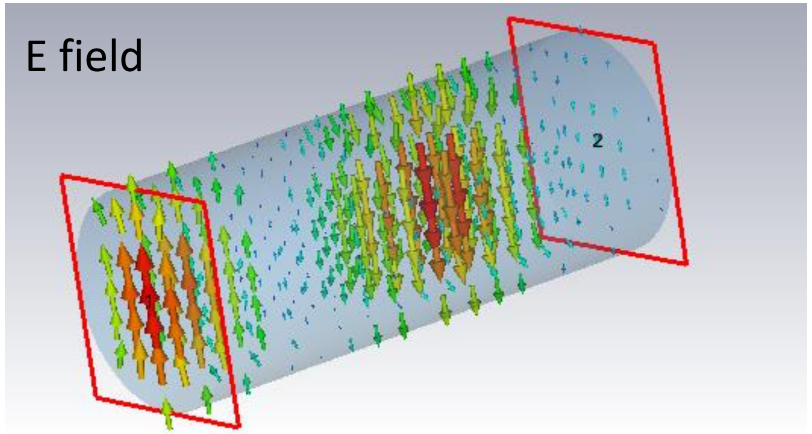


FNAL, CKM, Prototpe only, 3.9 GHz 13 cell SRF

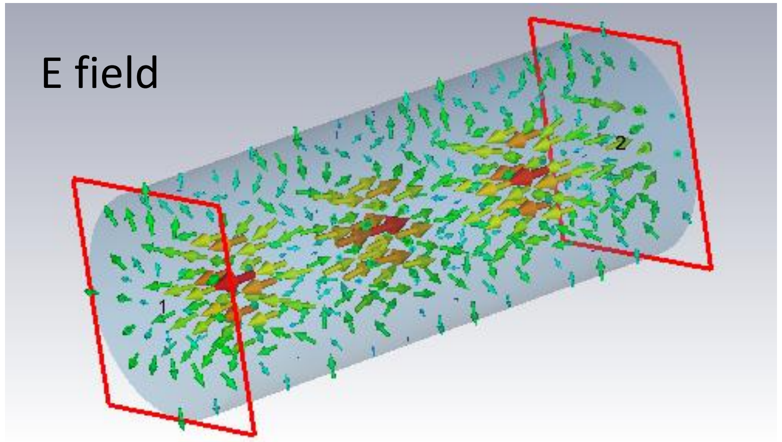


TM, TE and TEM modes

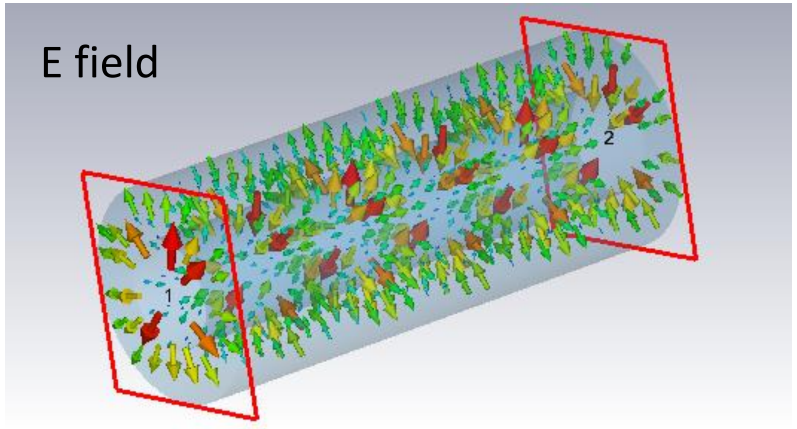
Transverse magnetic modes (TM) only have transverse magnetic fields but always have longitudinal electric fields (good for defecting)



Transverse electromagnetic modes (TEM) have no longitudinal fields. They need two isolated conductors. Frequency is not dependant on transverse size so can work at very low frequencies.

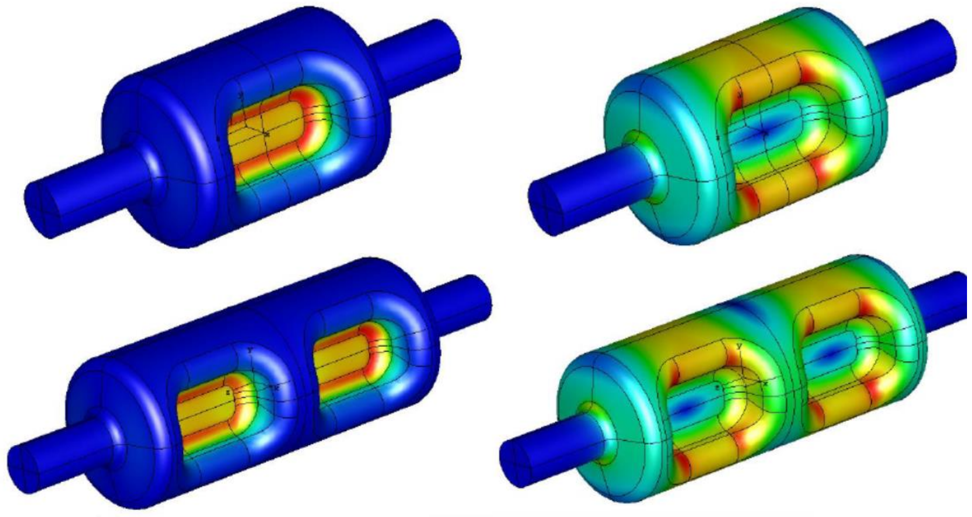


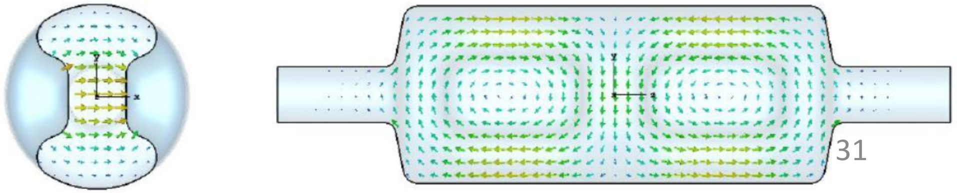
Transverse electric (TE) modes only have transverse electric fields (Ez=0). They are smaller than TM mode cavities.

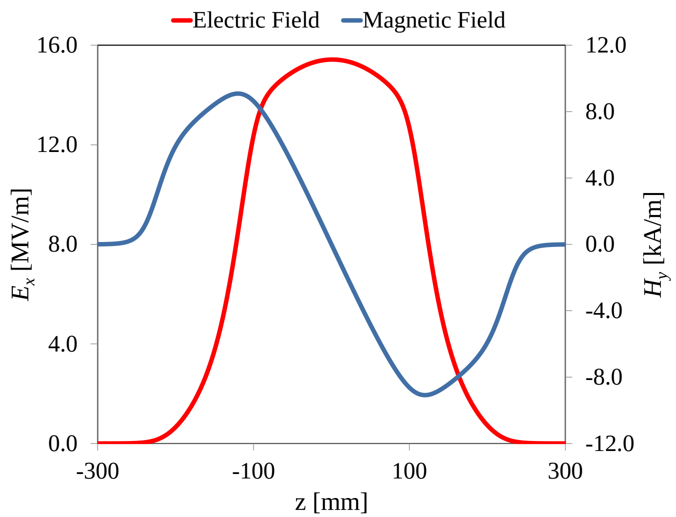


TE type deflectors (RF dipoles)

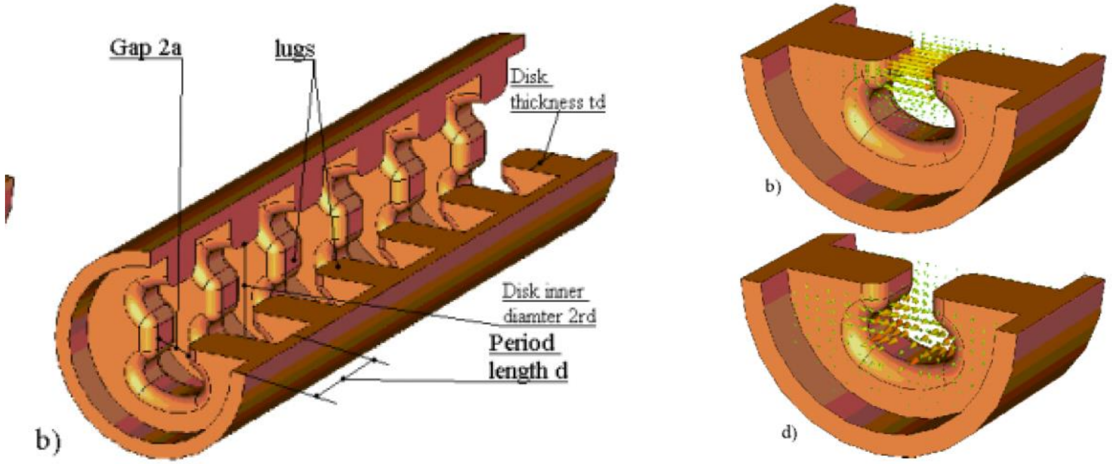
- In RADAR engineering it is normal to use ridged waveguide to reduce the waveguide size by adding capacitance. This makes them ideal for low frequency or compact deflectors
- TE modes in constant cross section waveguides have **no Ez field** but if we interface the ridged waveguide with a standard waveguide we get a **hybrid mode coupling to TM modes** in the ends which **provide an Ez field**.
- The RFD is baseline for the HL-LHC upgrade for the horizontal crossing and for the ILC.

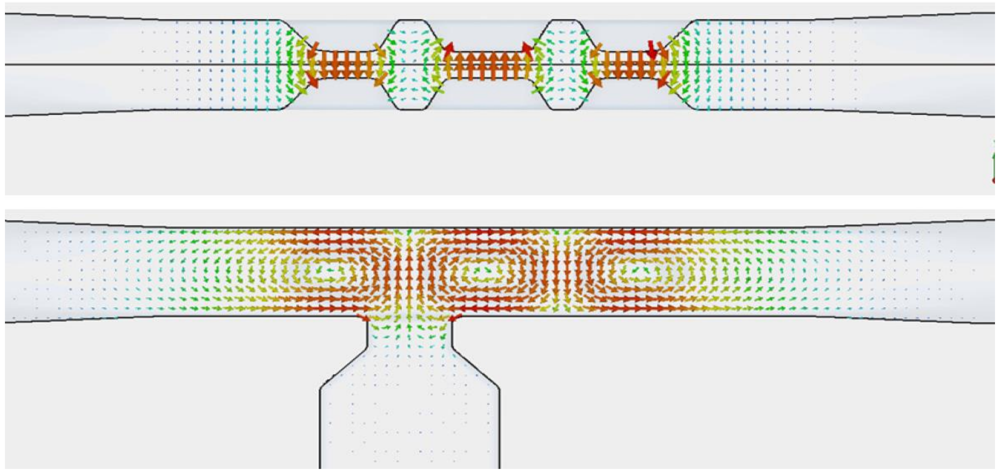






Other TE type cavities





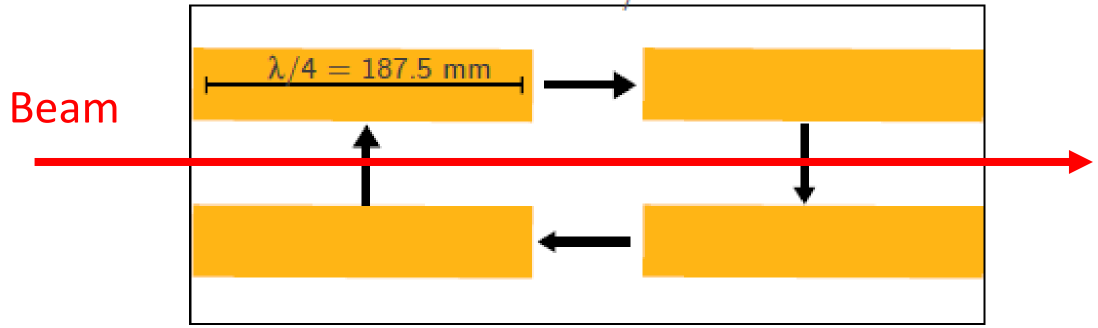
- A recent cavity proposal by Paramonov utilises a periodic ridged waveguide loaded cavity to reduce the cavity diameter by a factor of two.
- This structure is designed to be a π mode standing wave cavity.

- As the wave is only above cut-off in the ridged section the **beampipe doesn't need tapered down**
- The beampipe has a lower cutoff due to the size and provides excellent damping of HOMs which just propagate out the ends
- Also easier to coat with thin film SRF coatings
- Examples are the CERN-WOW or the FNAL-QMiR cavities

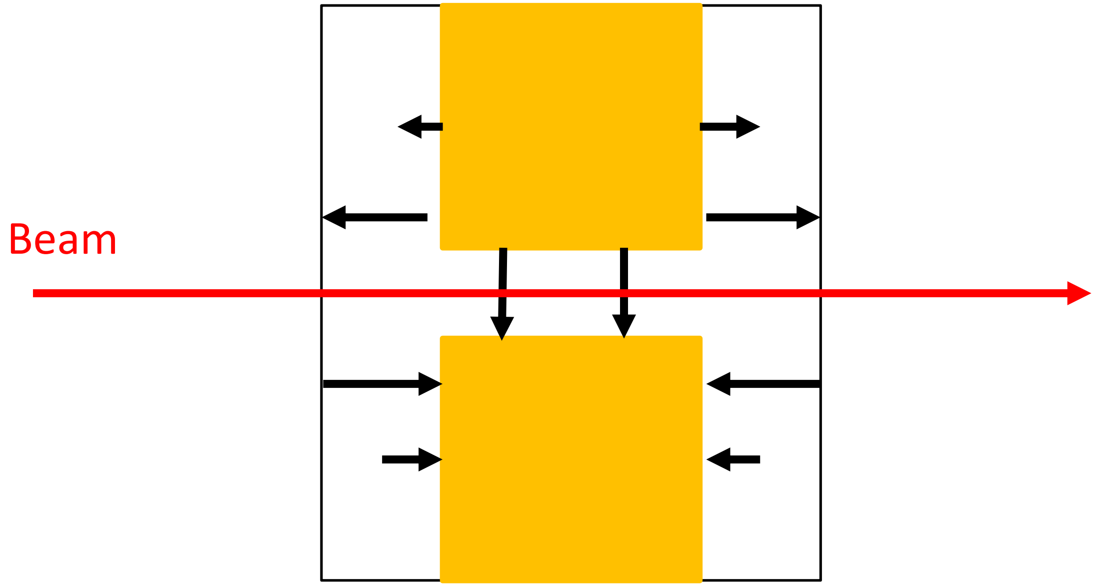
TEM cavities

- **TEM modes** have only transverse fields but the **resonance is not dependent on transverse size** only on longitudinal size (but coordinate system doesn't need to conform to the beam)
- We have two options
  - A **4-rod cavity** sets up an Ez field in a gap between a **longitudinal line**. Two lines are needed to provide the transverse fields
  - A **double quarter wave (DQW)** uses a gap in a **transverse orientated halfwave cavity** to set up a transverse field. Ez comes from the orientation

Four co-linear λ/4 resonators

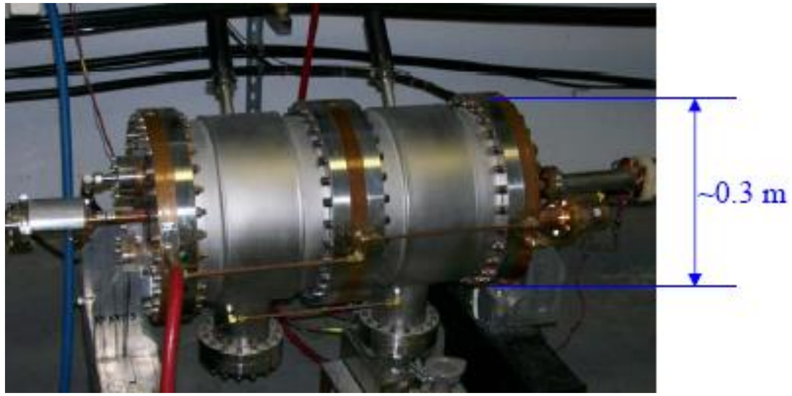


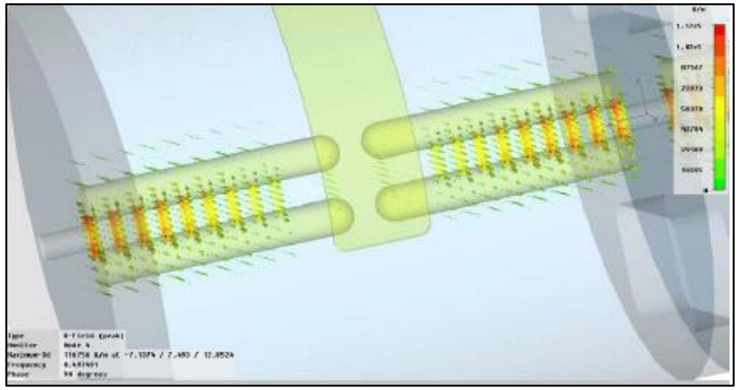
4 eigenmodes, mode 2 is our crab mode

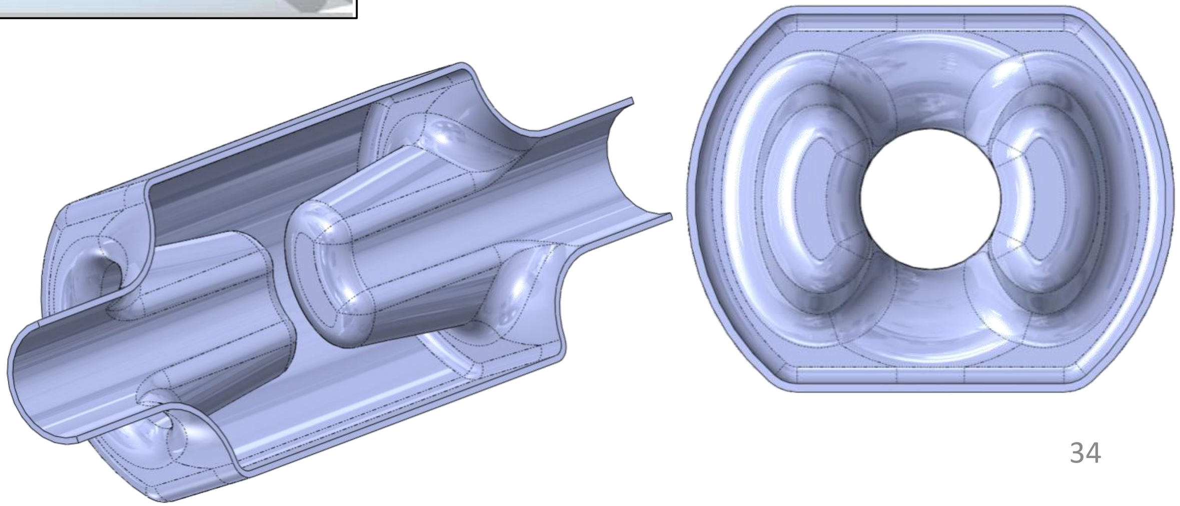


4 rod deflectors

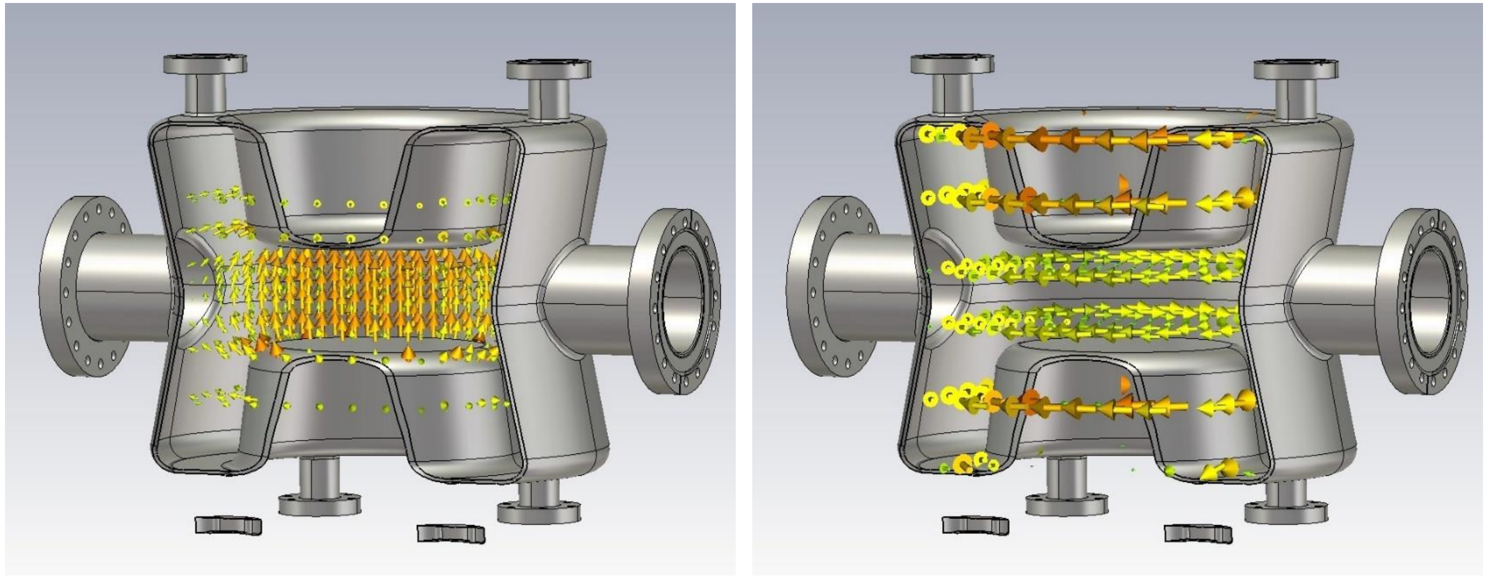
- CEBAF currently uses a compact normal conducting separator. 30 cm diameter at 500 MHz
- Also an initial suggestion for LHC in SRF version
- These cavities are **the most transversely compact** so ideal at low frequency
- The have low fields in the crabbing mode on the outer walls **so simple access for HOM damping**
- But has a **lower order mode**



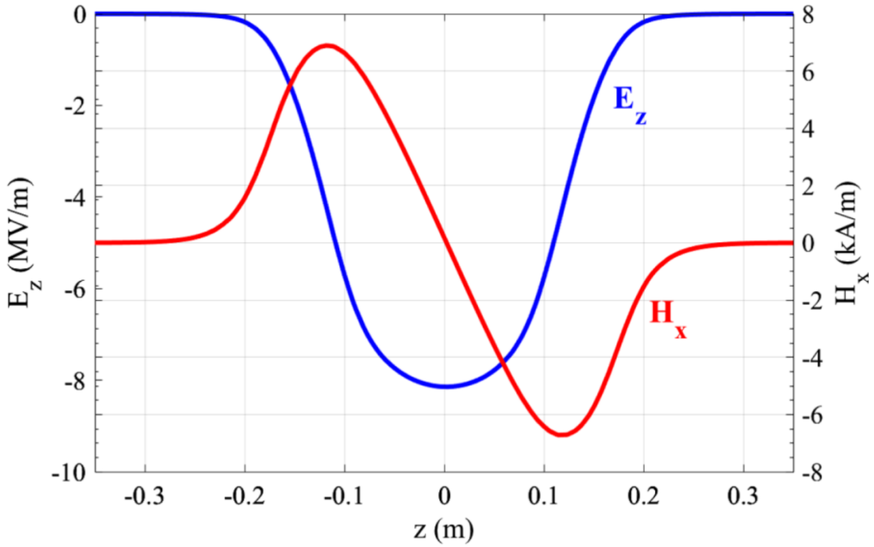




DQW cavities



- DQW cavities are compact in one axis. They are just under a half wavelength in the vertical and longitudinal planes (long. For synchronization of E and H fields with beam)
- They **do not have a lower order mode**



- DQW cavities are currently baseline for HL-LHC (vertical crossing) and EIC
- Conceptually they are very similar to RF dipoles as neither are true TE or TEM modes, both are hybrid. The main difference is a DQW has cylindrical symmetry and the RFD has rectangular symmetry which effects HOM couplers, and mechanical aspects.

The Cockcroft Institute of Accelerator Science and Technology

Lancaster University

The end

The CERN Accelerator School