

Dreams of a Physics Without Lagrangians: The Long S-Matrix Programme

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Differential time-evolution equations have long been the paradigm examples of laws of nature. Yet, the 20th century saw the rise to prominence of atemporal conditions and principles in many areas of physics. This paper looks at a particularly radical strand of this tendency: the attempt to do away with time-evolution equations entirely in relativistic quantum theory. I make a case for recognising a continuous tradition of these attempts—which I call the long S-matrix programme—stretching from the 1940s to the 1970s.

The story starts with Heisenberg's introduction of the S-matrix in the 40s. He believed that the divergence difficulties facing quantum field theory (QFT) stemmed from the use of a differential time evolution equation. Not unreasonably, since the dynamical equations governing locally interacting fields contained products of field operators at coincident space-time points, and this even today, is seen as the root of many of the foundational difficulties with the theory. Heisenberg's idea was that one could, instead, set up a relativistic scattering theory by imposing conditions directly on the S-matrix, with unitarity and Lorentz invariance being the two he came up with. Rather than dismissing Heisenberg's approach as an oddity of pre-war physics which was entirely swept aside by the success of renormalized quantum electrodynamics, I argue that a number of influential programmes in the 50s and 60s were direct continuations of Heisenberg's dream of a Lagrangian free physics.

The consensus that emerged amongst these followers was that a causality condition of some kind needed to be added to Heisenberg's list of principles. Different causality conditions were developed, leading to a family of sibling research programmes:

- i) Causal perturbation theory: Stueckelberg and later Bogoluibov developed a causality condition designed to set up a series expansion for the S-matrix without starting from a Lagrangian. This approach remained peripheral in the 50s and 60s but would later be adopted by mathematical physicists attempting to reconstruct the conventional perturbative formalism in a more rigorous way.
- ii) Axiomatic quantum field theory: Somewhat surprisingly, the early work of Haag and Wightman can be seen as a continuation of the S-matrix concept, with the microcausality condition now being imposed on the basic structures of the theory. Early axiomatic QFT was also driven by a desire to avoid writing down particular Lagrangians and to progress as far as possible using only general principles.
- iii) The 60s S-matrix programme: Chew's S-matrix approach to strong interactions in the 60s did not come out of nowhere. It was a continuation of work on dispersion relations and n-point functions in the 50s which had often been allied to early axiomatic QFT. The analyticity conditions which were now imposed on the S-matrix were interpreted as expressing a new causality condition and a new concerted effort was made to construct a Lagrangian free phenomenological description of strong interactions.

While the dream of completely eliminating Lagrangians from high energy theory never came to pass, many of these sub-programmes were ultimately rebranded and remain influential today, especially in mathematical physics.

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