das globale Relativitätsprinzip

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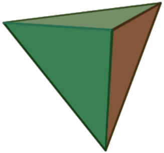
Three steps Philosophiae naturalis

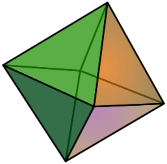
- There are natural laws
- we have to determine, what they are
- we have to understand why the laws are the way they are.

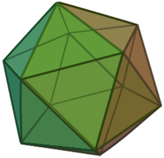
As Einstein said: I am interested to know whether God had a choice when He created the world.

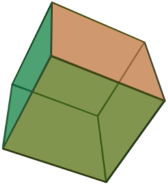
Leukippos, Demokritos
Emptiness → space
Fullness → atoms

Plato, Empedokles, Aristoteles
Elements


Tetrahedron
(fire)


Octahedron
(air)


Icosahedron
(water)


Cube
(earth)

Dodekahedron → fifth element (quintessence)
prediction of the theory
mathematical basis

Space $\rightarrow$ Spacetime

with structure: $R_{\mu\nu\alpha\beta}$

and dynamic: $R_{\mu\nu} = \kappa T_{\mu\nu}$ (Einstein)

Matter: gauge theories (symmetry)

Groups and representations
Many possibilities
Constraint: anomaly cancellation

Symmetry breaking (Higgs): not considered here.
Naturalness was always somewhat suspect as a principle: why should one expect to explain features of the symmetry-breaking when the symmetry itself is not understood?

Ellis, Gaillard, Zumino (1980)

$N = 8$ Supergravity
$SO(8)$ symmetry too small
Hidden $SU(8)$

assumption 1: $SU(8)$ becomes dynamical
assumption 2: also superpartners dynamical
assumption 3: anomaly free subset of $SU(5)$
assumption 4: non-chiral part mass to Planck mass
Leaves $3(5 + \bar{5}) + 9(1)$
and supersymmetry, which has to be broken

Nowadays $N = 8$ supergravity is not anymore considered fundamental (non-renormalizable).

Gross, Harvey, Martinec, Rohm (1985)

Heterotic string
Anomaly free superstring 10 dimensions
Gauge group $E(8) \times E(8)$

assumption 1: compactification to 4-D
assumption 2: one $E(8)$ disappears
assumption 3: Calabi-Yau manifold to break $E(8)$
assumption 4: topology to get 3 generations
assumption 5: some form of supersymmetry breaking

Nowadays string theory has many vacua and no unique gauge theory is expected.

What do we learn?

- Assuming one knows the fundamental laws of physics and only has to construct the standard model out of these is not very promising. Therefore use experiment.
- anomalies are important
- topology is important

Paradigm shift

Dear Fabiola,

I want to congratulate CERN for the great running of the machine and the brilliant work of the detectors. I think the results are great, I have rarely seen such a convincing null-experiment. I am sure this will lead to the long overdue paradigm-shift away from the view ” the standard model is wrong and we will have to see what is beyond” towards the view ” we know the standard model is true and we have to understand why”.

In the attachment I give you my answer.

good luck, Jochum

The question we therefore want to discuss is whether in some sense the laws of nature are the unique possible ones!

Is there a reason for the choice of gauge group and representations?

Phys. Rev. D76, 121702 (R) (2007);

General Relativity and Gravitation 43, 2467 (2011).

I. Rabi: Who ordered that?
on the discovery of the muon

A. Einstein: Did God have a choice when He created the world?

Anomalies !

L. Alvarez-Gaume, E. Witten, gravitational anomalies;
Nucl. Phys. B234 (1986), 309.
We do not know under what conditions such phenomena occur in general relativity.

E. Witten, global gravitational anomalies;
Commun. Math Phys. 100 (1985), 227.
The choice of S4 corresponds to treating four dimensional space time as Minkowski space. In the long run, a more delicate choice will be necessary to accomodate cosmological considerations. It may be that eventually global anomalies will have cosmological applications, restricting the large scale topology of space-time.

J.J. van der Bij:
Apparently it is the opposite, all topology is allowed, but the particle types are restricted. The topology I use is the one with few observational limits.

Principle of global relativity

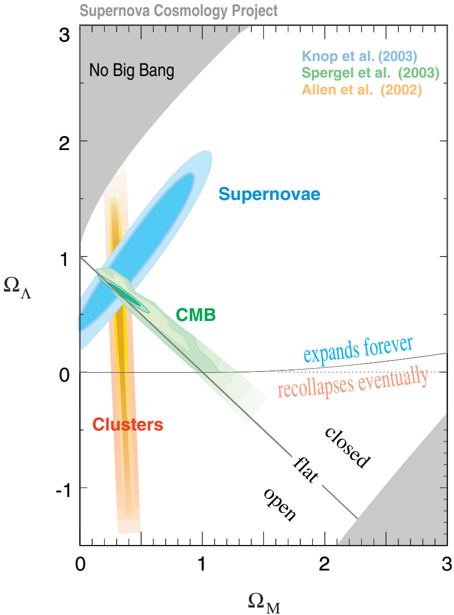
Gravity is a geometrical theory. The Einstein equations allow for different topologies. The matter fields live in these geometrical backgrounds. The matter equations should be consistent with any form of compactification (or more general "every" topology) of spacetime consistent with the Einstein equations.

This allows for topological anomalies that can constrain the matter content!

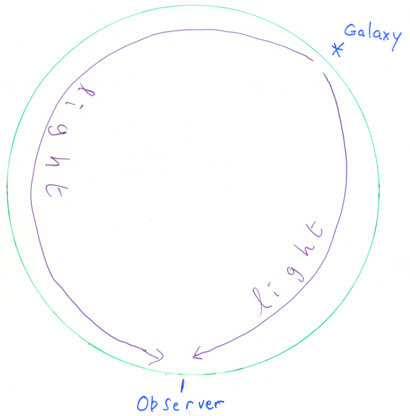
Example of topology in the universe

Higher dimensional Kaluza-Klein universe
Example: $M_4 \times U(1)^n$ with radius of U(1) going to zero
Difficult in practice

Therefore we try the opposite and assume that the universe was three dimensional

Supernova Cosmology Project
Knop et al. (2003)
Spergel et al. (2003)
Allen et al. (2002)
No Big Bang
Supernovae
CMB
Clusters
expands forever
recollapses eventually
closed
flat
open
Ω_Λ
Ω_M
3
2
1
0
-1
0
1
2
3

Cosmic topology



- multiple images: difficult, evolution
- cosmic microwave background: circles in the sky

Theory: In flat space, topology can always be at too large a scale to be seen directly

Bianchi-I universes

Flat, homogeneous, non-isotropic
Pancake picture

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2) + b^2(t)dz^2$$

dx,dy topology R^2; dz topology S_1

Late in time $\frac{\dot{a}}{a} \sim \frac{\dot{b}}{b}$, therefore isotropy

This is generically true when there is a positive cosmological constant (Wald's theorem 1983)
So present day isotropy says little about the (very) early universe.

At small t approximate Kasner solution (1925)

$$ds^2 \cong -dt^2 + dx^2 + dy^2 + t^2 dz^2$$

therefore the third dimension gets compactified to zero at early times

For instance exact dust universe ($\lambda = 0$)

$$g_{zz} = M^{1/3} t / (t + \Sigma)^{1/3}$$

$$g_{xx} = g_{yy} = M^{2/3} (t + \Sigma)^{2/3}$$

M and Σ are integration constants

Suggested topology of the universe $M_3 \times S_1$

S_1 The radius may be too large to see the topology at the present time
However a preferred direction may be visible

There appears to be an allignment of low multipoles along a preferred axis in the data

This could be explained in an inflationary Bianchi-I model

3 dimensional Yang-Mills theory

$$\mathcal{L} = -\frac{1}{2} Tr\, F_{\mu\nu} F^{\mu\nu} - i\, m\, \epsilon^{\mu\nu\rho}\, Tr(A_\mu \partial_\nu A_\rho + A_\mu A \nu A \rho)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g[A_\mu, A_\nu]$$

There is a gauge invariance under $A_\mu \to U^{-1}(\partial_\mu / g + A_\mu) U$
Under large gauge transformations the action shifts by
$8\pi^2 im/g^2$

So full invariance leads to a quantization condition

$g_{YM} = \frac{4\pi m}{g^2}$ must be integer

Renormalization

$$q_{YM}^{ren} = q_{YM}^{0} + C(G) + sign(m_f) N_f C(R)$$

$SU(N)$: $C(G) = N$
fundamental fermion: $C(R) = 1/2$
So one needs an even number of fermions. This stays true even when $m_f = 0$.

In four dimensions there is a similar effect when one starts with Weyl-fermions in a $M_3 \times S_1$ spacetime. There a Chern-Simons like term is generated

$$\mathcal{L}_{CSlike} = m_{ph} n^{\alpha} \epsilon_{\alpha\mu\nu\rho} A^{\mu} F^{\nu\rho}$$

Three dimensional gravity

$$\mathcal{L} = -(1/\kappa^2)\sqrt{g}R - \frac{i}{4\kappa^2\mu}\epsilon^{\mu\nu\lambda}(R_{\mu\nu ab}\omega_{\lambda}^{ab} + \frac{2}{3}\omega_{\mu a}^{b}\omega_{\nu b}^{c}\omega_{\lambda c}^{a}).$$

$q_{gr} = \frac{6\pi}{\mu\kappa^2}$ must be integer

Renormalization

R.D. Pisarski, S. Rao, J.J. van der Bij; Phys. Lett. B179, 87 (1986).

$$q_{gr}^{ren} = q_{gr}^{0} + \frac{1}{8}N_g\ sign(m_g) - \frac{1}{16}\,N_f\,sign(m_f)$$

N_g is the number of vector bosons
N_f is the number of fermions

assume $q_{gr} = 0$ (Einstein equations)
consistency: $N_f \mp 2N_g = 0 \mod(16)$

Stronger conditions

isotropization: $q_{gr}^{ren} = 0$

vectors and fermions separately consistent:

$$N_g = 0 \mod(8)$$

$$N_f = 0 \mod(16)$$

In combination vectors $SU(5)$: 24
fermions $SO(10)$: 16

$$2 \times 24 - 3 \times 16 = 0$$

Basically unique if also:
1) fermions automatically anomaly free, i.e. no $SU(n)$:
2) fermions in fundamental representation

Speculations

- Symmetry breaking:
  $SU(5)$ decomposition: $16 = 10 + \bar{5} + 1$.

  $$SU(3) \to +, \quad SU(2) \to -, \quad U(1) \to +$$

  $$10 \to +, \qquad \bar{5} \to -, \qquad 1 \to -$$

  $2 \times (8 - 3 + 1) - 3 \times (10 - 5 - 1) = 0$
  possible: $SU(5) \to SU(3) \times SU(2) \times U(1)$
  impossible: $SU(5) \to SU(4) \times U(1)$
- more conditions
- other compactifications
- underlying structure
- quantum gravity → spacetime 3-D at the Planck scale ?

Predictions from 2007 (present status) and remarks

- No new fermions at the LHC (confirmed)
- No new vector bosons at the LHC (confirmed)
- Presence of a preferred direction in the universe (sort of there in 2007, confirmed by Planck).
- Many applications in solid state physics: graphene, toplogical insulators ...

More positive prediction from 2012

European Physics Letters (EPL), 100, 29003 (2012).

$SU(5)$ Unification

Extra fermion fields are needed, but multiple of 16.
Solution: a Dirac **24**

Symmetry breaking by **24** Higgs field.
Unification is easy, both **F** and **D** term of $SU(5)$ possible.

$$\mathbf{F}_{abc} = \mathbf{Tr}([T_a, T_b]T_c)$$

$$\mathbf{D}_{abc} = \mathbf{Tr}(\{T_a, T_b\}T_c)$$

$$\mathbf{24} = (\mathbf{8}, \mathbf{1}, \mathbf{0}) \oplus (\mathbf{1}, \mathbf{3}, \mathbf{0}) \oplus (\mathbf{3}, \mathbf{2}, -\mathbf{5}/\mathbf{6}) \oplus (\mathbf{3}, \mathbf{2}, \mathbf{5}/\mathbf{6}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{0})$$

Dark matter candidate: A Dirac triplet with mass 1.9 TeV.

Phenomenology

$$\textbf{Triplet} = (\mathbf{T}^{+}, \mathbf{T^0}, \mathbf{T}^{-})$$

The neutral field is the dark matter.
The charged field is heavier by 166 MeV, due to the Coulomb energy. Decay in missing energy plus soft pion.

Can this be seen?

HL-LHC : NO !

Direct search : NO ! (No = at the very least very unlikely)

FERMI: strong constraints , but very large uncertainties due to halo models.
Cerenkov Telescope Array (CTA): should see this without much problems !! (maybe)

Rabi's question: who ordered that?

Answer: the early universe.

Einstein's question: did God have a choice?

Answer: no, because He has to use perfect symmetry.
However the devil may have had something to do with the Higgs sector.