Inclusive quarkonium production phenomenology and tools overview

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Aussois

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RadCor4HEF (“Radiative Corrections for Heavy Quark(-onium) Production in High-Energy Factorisation”)

[1]IJClab, Orsay

Outline

1. A short theory/pheno. overview on inclusive quarkonium production in pp/ep collisions
2. Introduction to the discussion of tools for NLO QCD computations with quarkonia

Motivations (I): understanding hadronisation

Description of production of any high-$p_T (\gg \Lambda_{\rm QCD})$ hadrons in QCD = (perturbative) production of quarks/gluons + *hadronisation.*

1. For light and heavy-light hadrons, hadronisation is studied phenomenologically:
   - Fragmantation Functions: based on factorisation theorems, fitted to describe data (first attempts to compute on the lattice)
   - Monte-Carlo models: hard to derive from QCD Lagrangian (string-based in Pythia, cluster hadronisation in Herwig,...)
2. Quarkonia – "Hydrogen atoms of QCD" ⇒ corrections to the "naive" quark model should be suppressed by powers of relative velocity (v) of heavy quarks in the bound state:

$$
\begin{aligned}
|J/\psi\rangle &= O(1)\left|c\bar{c}\left[{}^3S_1^{(1)}\right]\right\rangle + O(v)\left|c\bar{c}\left[{}^3P_J^{(8)}\right]+g\right\rangle \\
&+ O(v^{3/2})\left|c\bar{c}\left[{}^1S_0^{(8)}\right]+g\right\rangle + O(v^2)\left|c\bar{c}\left[{}^3S_1^{(8)}\right]+gg\right\rangle + \ldots,
\end{aligned}
$$

3. ⇒ let's try to use understand production of quarkonia. **This understanding will be a small-v limit for any future theory of hadronisation!**

Motivations (II): quarkonia as tools

If hadronisation mechanism **was well understood,** *then quarkonium production would be:*

1. An excellent tool to study gluon content of a proton/nucleus:
   - Small (or negligible) “valence” c and b content – production predominantly through coupling to gluons at high energies
   - Clean experimental signatures for J/ψ, $\Upsilon(nS)$, ...
   - relatively small $M_{J/\psi} \simeq 3GeV$ – *access to very small* $x \sim Me^{-y}/\sqrt{s} \sim 10^{-4} - 10^{-6}$ at the LHC.
2. A tool to study double/multiple parton scattering: due to significant cross sections of multiple/associated production and lower p_T/scales in comparison to vector bosons/jets
3. A probe for QGP: melting/recombination/parton energy loss could be studied
4. A tool to study of c-Higgs and b-Higgs couplings through associated production and Higgs decays
5. ...

Quarkonium production models

Unfortunately no existing model can describe all data on inclusive quarkonium hadro/photo/electro/e^+e^- production and polarisation observables.
Three classes of models:

1. (Improved) Colour Evaporation Model assumes "democracy" of colour/orbital momentum/spin states of the $Q\bar{Q}$-pair
2. NRQCD factorisation: based on the hierarchy of different colour/orbital momentum/spin states of the $Q\bar{Q}$-pair in the v-expansion for the quarkonium state
3. Colour Singlet Model: only **colour-singlet** $Q\bar{Q}$ pairs with the same orbital momentum/spin as corresponding potential-model state hadronise to the quarkonium.

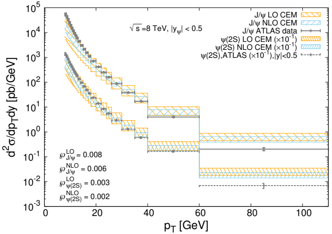
Colour Evaporation Model

The $c\bar{c}$ pairs with $M_{c\bar{c}} < 2m_D$ can not hadronise to the pair of D-mesons. Where do they go?

CEM assumes that all of them hadronise to quarkonia with the same probability $F_{J/\psi}$, $F_{\psi(2S)}$, ... :

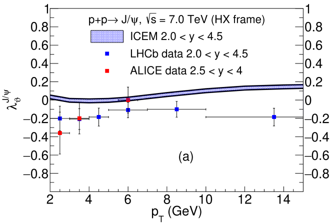
$$\sigma_{J/\psi} = F_{J/\psi} \times \int\limits_{m_{J/\psi}}^{2m_D} dM_{c\bar{c}} \frac{d\sigma_{c\bar{c}}}{dM_{c\bar{c}}}.$$

CEM@NLO [Lansberg et al. '20]



Unpolarised production at high-p_T

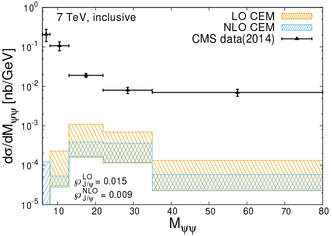
[Vogt, Chung '21]

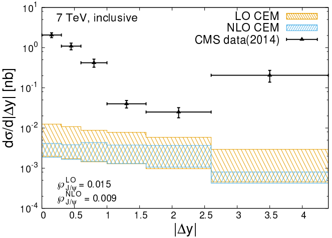


Problems of CEM

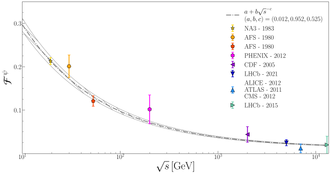
Pair production ($F_{2J/\psi} = (F_{J/\psi})^2$)

[Lansberg et al. '20] :

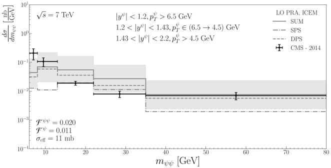




Energy-dependence (=non-universality) of $F_{J/\psi}$ [Saleev, Chernyshev '22]



Pair production ($F_{2J/\psi} \neq (F_{J/\psi})^2$ + DPS) [Saleev, Chernyshev '22] :



Quarkonium in the potential model

Cornell potential:

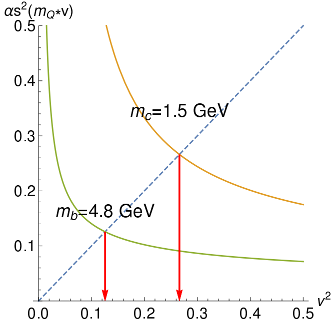
$$V(r) = -C_F \frac{\alpha_s(1/r)}{r} + \sigma r,$$

neglect linear part, because quarkonium is "small" (~ 0.3 fm) $\rightarrow$ Coulomb wavefunction (for effective mass $\frac{m_1 m_2}{m_1+m_2} = \frac{m_Q}{2}$):

$$R(r) = \frac{\sqrt{m_Q^3 \alpha_s^3 C_F^3}}{2} e^{-\frac{\alpha_s C_F}{2} m_Q r}$$

$$\langle v^2 \rangle = \frac{C_F^2 \alpha_s^2}{2}, \langle r \rangle = \frac{3}{2C_F} \frac{1}{m_Q v}$$

$$\Rightarrow \boxed{\alpha_s^2(m_Q v) \simeq v^2}$$



Non-relativistic QCD

The velocity-expansion for quarkonium eigenstate is a copy of corresponding arguments from atomic physics:

$$\begin{aligned} |J/\psi\rangle &= O(1)\left|c\bar{c}\left[{}^3S_1^{(1)}\right]\right\rangle + O(v)\left|c\bar{c}\left[{}^3P_J^{(8)}\right]+g\right\rangle \\ &+ O(v^{3/2})\left|c\bar{c}\left[{}^1S_0^{(8)}\right]+g\right\rangle + O(v^2)\left|c\bar{c}\left[{}^3S_1^{(8)}\right]+gg\right\rangle + \ldots, \end{aligned}$$

for validity of this arguments, we should work in *non-relativistic EFT*, dynamics of which conserves number of heavy quarks. In such EFT, $Q\bar{Q}$-pair is produced in a point, by local operator:

$$\mathcal{A}_{\rm NRQCD} = \langle J/\psi + X|\, \chi^\dagger(0)\kappa_n\psi(0)\, |0\rangle\,,$$

Different operators "couple" to different Fock states:

$$\chi^\dagger(0)\psi(0) \leftrightarrow \left|c\bar{c}\left[{}^1S_0^{(1)}\right]\right\rangle,\ \chi^\dagger(0)\sigma_i\psi(0) \leftrightarrow \left|c\bar{c}\left[{}^3S_1^{(1)}\right]\right\rangle,$$
$$\chi^\dagger(0)\sigma_i T^a\psi(0) \leftrightarrow \left|c\bar{c}\left[{}^3S_1^{(8)}\right]\right\rangle,\ \chi^\dagger(0)D_i\psi(0) \leftrightarrow \left|c\bar{c}\left[{}^1P_1^{(8)}\right]\right\rangle,\ldots$$

squared NRQCD amplitude (=LDME):

$$\sum_X |\mathcal{A}|^2 = \langle 0|\, \underbrace{\psi^\dagger\kappa_n^\dagger\chi a^\dagger_{J/\psi}a_{J/\psi}\chi^\dagger\kappa_n\psi}_{\mathcal{O}_n^{J/\psi}}\, |0\rangle = \left\langle \mathcal{O}_n^{J/\psi}\right\rangle,$$

Non-relativistic QCD

Velocity-scaling of LDMEs follows from velocity-scaling of corresponding Fock states and of operators $\chi^\dagger \kappa_n \psi$:

	${}^1S_0^{(1)}$	${}^3S_1^{(1)}$	${}^1S_0^{(8)}$	${}^3S_1^{(8)}$	${}^1P_1^{(1)}$	${}^3P_0^{(1)}$	${}^3P_1^{(1)}$	${}^3P_2^{(1)}$	${}^1P_1^{(8)}$	${}^3P_0^{(8)}$	${}^3P_1^{(8)}$	${}^3P_2^{(8)}$
η_c	1		v^4	v^3					v^4			
J/ψ		1	v^3	v^4						v^4	v^4	v^4
h_c			v^2		v^2							
χ_{c0}				v^2		v^2						
χ_{c1}				v^2			v^2					
χ_{c2}				v^2				v^2				

Note that:

- Colour-singlet LDMEs are LO in v for S-wave states $\Rightarrow$ *Colour-Singlet Model*
- For P-wave states the CS and CO LDMEs are of the same order $\Rightarrow$ *mixing*
- Connection between LDMEs for η_c and J/ψ through *Heavy-Quark Spin Symmetry*

Matching procedure between QCD and NRQCD:

$$v \ll 1 : \mathcal{A}_{\rm QCD}(gg \to Y_{Q\bar{Q}(v)}) = \sum_n f_n \left\langle Y_{Q\bar{Q}(v)} \right| \chi^\dagger(0)\kappa_n\psi(0) \left|0\right\rangle + O(v^{\#}),$$

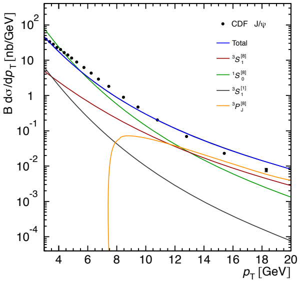
$\Rightarrow$ NRQCD factorization formula (“theorem”) [Bodwin, Braaten, Lepage 95’] :

$$\sigma(gg \to \mathcal{H} + X) = \sum_n \sigma(gg \to Q\bar{Q}[n] + X) \left\langle \mathcal{O}_n^{\mathcal{H}} \right\rangle .$$

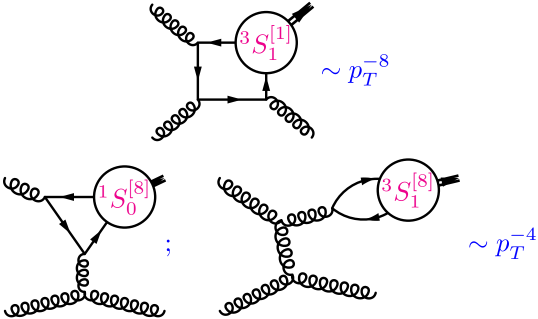
NRQCD factorisation: p_T-behaviour in pp

$$\frac{d\sigma}{dp_T^2}(pp \to \mathcal{H} + X) = \sum_n \frac{d\sigma}{dp_T^2}(pp \to Q\bar{Q}[n] + X)\,\langle \mathcal{O}_n^{\mathcal{H}} \rangle\,.$$

NLO, plot from hep-ph/1403.3970 :



At **LO**:



Potential NRQCD

The NRQCD logic can be pushed even further by assuming that $mv^2 \ll mv$ and dynamics at the scale mv^2 is strongly-coupled [Brambilla et.al., '22] (Talk by Xiangpeng Wang on Tuesday) . **At LO in v:**

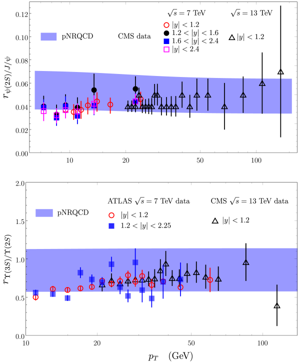
Prompt cross section ratios:

$$\boxed{\langle \mathcal{O}^{\mathcal{H}}({}^3S_1^{[1]})\rangle = \frac{3N_c}{2\pi}|R_{\mathcal{H}}(0)|^2,}$$

$$\langle \mathcal{O}^{\mathcal{H}}({}^3P_J^{[8]})\rangle = \frac{2J+1}{18N_c}\frac{3|R_{\mathcal{H}}(0)|^2}{4\pi}\mathcal{E}_{00},$$

$$\langle \mathcal{O}^{\mathcal{H}}({}^1S_0^{[8]})\rangle = \frac{1}{6N_c m^2}\frac{3|R_{\mathcal{H}}(0)|^2}{4\pi}c_F^2\mathcal{B}_{00},$$

$$\langle \mathcal{O}^{\mathcal{H}}({}^3S_1^{[8]})\rangle = \frac{1}{2N_c m^2}\frac{3|R_{\mathcal{H}}(0)|^2}{4\pi}\mathcal{E}_{10;10},$$

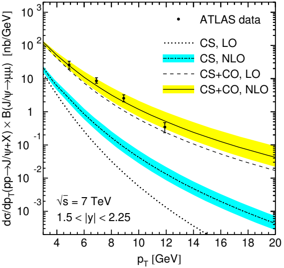


where $|R_{\mathcal{H}}(0)|^2$ – radial wave function at the origin from **potential model** for the quarkonium $\mathcal{H}$, and $\mathcal{E}_{00}$, $\mathcal{B}_{00}$, $\mathcal{E}_{10;10}$ – chromo electric/magnetic field correlators over **QCD vacuum**.

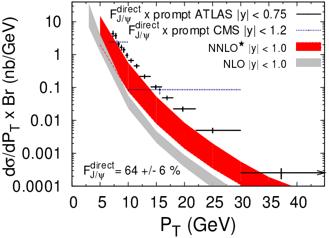
NRQCD factorisation: what does work?

- *Un-polarized* p_T distributions of J/ψ, χ_{cJ} in hadro- and photoproduction, as well as e^+e^- data can be described. The same is true for $\Upsilon(nS)$, $\chi_{bJ}(nS)$.
- Solves the problem of non-cancelling IR divergence at NLO in CSM for P-wave states production and decay through mixing with ${}^3S_1^{(8)}$ or ${}^1S_0^{(8)}$ states at $O(v^2)$.
- Covers the gap between CSM (@LO and NLO) and data at high-p_T in hadroproduction, due to contribution of CO states. If NNLO corrections in CS are as large as needed to close this gap, then perturbative expansion is just useless and we should stop doing quarkonia.

NLO NRQCD, [Butenschön, Kniehl, '11]



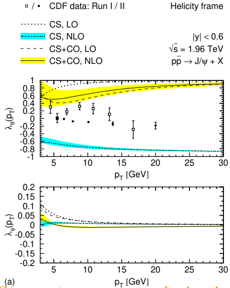
NLO and NNLO* CSM [Lansberg '11]



Problems: Polarisation

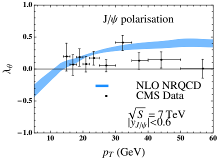
LDME fit	J/ψ hadropr.	J/ψ photopr.	J/ψ polar.	η_c hadropr.
Butenschön et al.	✓($p_T > 3$ GeV)	✓	✗	✗
Chao et al. + η_c	✓($p_T > 6.5$ GeV)	✗	✓	✓
Zhang et al.	✓($p_T > 6.5$ GeV)	✗	✓	✓
Gong et al.	✓($p_T > 7$ GeV)	✗	✓	✗
Chao et al.	✓($p_T > 7$ GeV)	✗	✓	✗
Bodwin et al.	✓($p_T > 10$ GeV)	✗	✓	✗

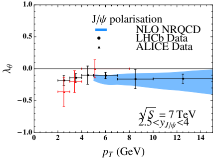
Global fit [Butenschön, Kniehl, '12]



Strong transverse polarisation due to $^3S_1^{[8]}$ and $^3P_J^{[8]}$ states at high p_T

Example hadroproduction dominated fit [Chao et.al., '14]

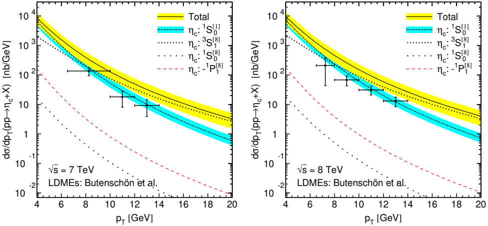




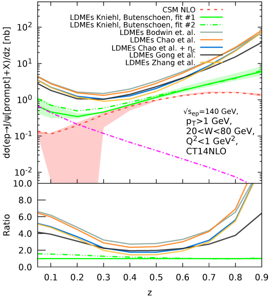
Problems: HQSS and photoproduction

LDME fit	J/ψ hadropr.	J/ψ photopr.	J/ψ polar.	η_c hadropr.
Butenschön et al.	✓($p_T > 3$ GeV)	✓	✗	✗
Chao et al. + η_c	✓($p_T > 6.5$ GeV)	✗	✓	✓
Zhang et al.	✓($p_T > 6.5$ GeV)	✗	✓	✓
Gong et al.	✓($p_T > 7$ GeV)	✗	✓	✗
Chao et al.	✓($p_T > 7$ GeV)	✗	✓	✗
Bodwin et al.	✓($p_T > 10$ GeV)	✗	✓	✗

Global fit [Butenschön, Kniehl, '12]



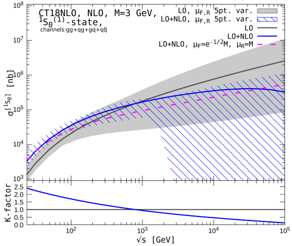
J/ψ-photoproduction at the EIC vs $z = (p_{J/\psi}P)/(qP)$, using NLO calculation data by M. Butenschön



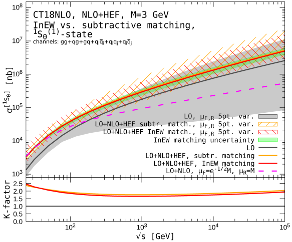
p_T-integrated cross sections, another "puzzle"?

Inclusive η_c-hadroproduction (CSM): [Lansberg, Ozcelik '20; Lansberg, M.N., Ozcelik '22]

NLO:

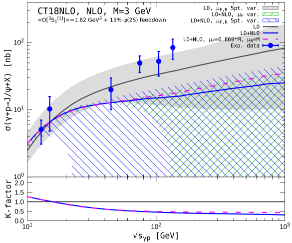


NLO+HEF matching:

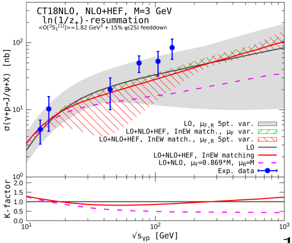


Inclusive J/ψ-photoproduction (CSM): [Lansberg et al. '21; Lansberg, M.N., Ozcelik, '23 (*in prep.*)]

NLO:



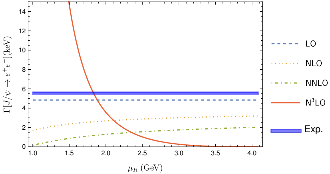
NLO+HEF matching:

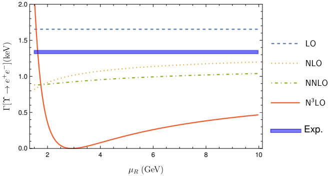


Prospects for NNLO

Tere is no NNLO calculations for heavy qarkonium **production** yet, but the calculations for **decay rates** are rather advanced:

1. For $J/\psi \to \mu^+\mu^-$ the NNLO[Beneke, Smirnov] and (very recent!) N^{3}LO results are known [Feng et.al., 2207.14259] :





2. For $\eta_c \to \gamma\gamma$ the NNLO result was obtained in [Abreu et.al., 2211.08838] and similar behaviour of radiative corrections was found.

- Should we expect strong perturbative instability for production cross section at $p_T < M$ due to bound-state effects? Maybe separation of corrections between LDMEs and hard part is not optimal in NRQCD?
- The NLO corrections to parton$\to Q\bar{Q}[n]$ fragmentation functions tend to be moderate $\Rightarrow$ NNLO will stabilize at $p_T > M$ and $p_T \gg M$.

The LO calculation in NRQCD

Projection of the usual QCD amplitude for $Q\bar{Q}$-production on the certain $Q\bar{Q}[^{2S+1}L_J^{[1,8]}]$ NRQCD state:

- Introduce relative momentum $k_Q = \frac{p}{2} + q$, $k_{\bar{Q}} = \frac{p}{2} - q$, p-quarkonium momentum: $p^2 = M^2 = 4m_Q^2$.
- Spin projection:

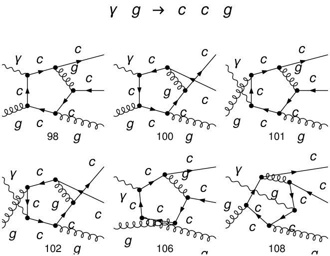
$$\bar{u}_\alpha(k_Q)\mathcal{M}_{\alpha\beta}v_\beta(k_{\bar{Q}}) = \mathcal{M}_{\alpha\beta}\ [v_\beta(k_{\bar{Q}})\bar{u}_\alpha(k_Q)] = \mathrm{tr}\left\{\mathcal{M}\ [v(k_{\bar{Q}}) \otimes \bar{u}(k_Q)]\right\}$$

$$S = 1 : v(k_{\bar{Q}}) \otimes \bar{u}(k_Q) \to \frac{1}{\sqrt{M^3}}\left(\frac{\not{p}}{2} - \not{q} - \frac{M}{2}\right)\gamma_\rho\left(\frac{\not{p}}{2} + \not{q} + \frac{M}{2}\right)$$

$$S = 0 : v(k_{\bar{Q}}) \otimes \bar{u}(k_Q) \to \frac{1}{\sqrt{M^3}}\left(\frac{\not{p}}{2} - \not{q} - \frac{M}{2}\right)\gamma_5\left(\frac{\not{p}}{2} + \not{q} + \frac{M}{2}\right)$$

- $L = 0 \Rightarrow$ set $q = 0$ (i.e. $M = 2m_c$).
- $L = 1 \Rightarrow$ take a derivative $\partial/\partial q^\mu$ and then set $q = 0$.
- The L and S can be added using Clebsch-Gordan coefficients
- Colour-singlet state: $\delta_{ij}/\sqrt{N_c}$, Colour-octet state $\sqrt{2}T^a_{ij}$
- Standard convention for LDMEs: divide squared amplitude by $N_{\text{col.}}N_{\text{pol.}}$ with $N_{\text{pol.}} = 2J + 1$ and $N_{\text{col.}} = 2N_c$ for CS and $N_c^2 - 1$ for CO.

Peculiarities of the loop correction in quarkonium case



If $q^\mu = (0, m_Q \mathbf{v})^\mu \neq 0$ these diagrams contain **Coulomb divergence,** related with the behaviour of the continuum-spectrum Coulomb wave-function at origin [Sommerfeld '39]

$$|R(0)|^2 = 1 + \frac{\alpha_s}{2\pi} \frac{C_F}{v^2} + \ldots$$

If we set $q = 0$ from the beginning we get **linearly-dependent denominators**:

$$D_1 = l^2, \quad D_2 = \left(\frac{p}{2} - l\right)^2 - m_Q^2, \quad D_3 = \left(\frac{p}{2} + l\right)^2 - m_Q^2,$$
$$D_2 + D_3 - 2D_1 = 0,$$

but the Coulomb divergence is automatically put to zero by dimensional regularisation.

Infrared divergences in real-emission corrections

The integral over Phase-Space of additional parton is **logarithmically-divergent** if

- **Soft:** The **energy** (k^0) of a **gluon** attached to *initial/final state gluon or quark* lines is small: $\int_0 dk^0 \times (1/k^0) \times (k^0)^{-2\epsilon}$.
- **FS-collinear:** The **angle** (θ) between two $g+g$ or $q+\bar{q}$ is small: $\int_0 d\theta \times (1/\theta) \times (\theta)^{-\epsilon}$
- **IS-collinear:** The **transverse momentum** ($\mathbf{k}_T$) of g or q is small: $\int_0 d\mathbf{k}_T^2 \times (1/\mathbf{k}_T^2) \times (\mathbf{k}_T^2)^{-\epsilon}$.

All these divergences can be regularised in **dimensional regularisation** and lead to $1/\epsilon$ and $1/\epsilon^2$ poles in $\epsilon = (4-D)/2$. In case with **no identified hadrons**, the $1/\epsilon^2$ and $1/\epsilon$ soft and FS-collinear poles **cancel** against corresponding poles in the **loop correction**. The IS-collinear divergences are absorbed by the *redefinition* of collinear PDFs at NLO, which is closely related to the origins of the DGLAP evolution.

New IR divergences in case of P-wave

Production of S**-wave CS/CO** follows the pattern described in the previous slide at NLO. For the P**-wave** new divergences appear.

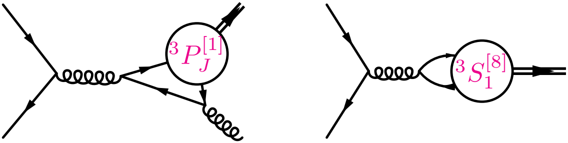
Simple example: The process

$$q + \bar{q} \to Q\bar{Q}[{}^3P_J^{[1]}] + g,$$

naively should not contain IR-divergences, because $g \to Q\bar{Q}[{}^3P_J^{[1]}]$ transition is forbidden. However:

$$s \to M^2 : |\mathcal{M}(q + \bar{q} \to Q\bar{Q}[{}^3P_J^{[1]}] + g)|^2 \quad \propto \quad \alpha_s(2J+1)\frac{(M^4 - \hat{t}^2)\hat{t}^2}{M^4(\hat{s} - M^2)^4} \times |\mathcal{M}(q + \bar{q} \to Q\bar{Q}[{}^3S_1^{[8]}])|^2.$$

This new IR divergence can be absorbed through mixing between ${}^3P_J^{[1]}$ and ${}^3S_1^{[8]}$ LDMEs.



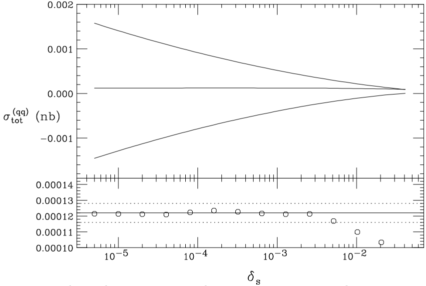
How to manage IR divergences?

Several options exist:

- Do the PS-integral in $D = 4 - 2\epsilon$ dimensions **analytically**: *easy* if the NLO real-emission is $2 \to 2$, *hard* in all other cases
- **Phase-space slicing** separate the computation with small parameters $\delta_s \ll 1$ and $\delta_c \ll 1$. For soft divergences:

$$\underbrace{\int d^{D}\Phi_k |\mathcal{M}|^2 \theta(\sqrt{s}\delta_s - k^0)}_{\text{divergent, computed analytically}} + \underbrace{\int d^{4}\Phi_k |\mathcal{M}|^2 \theta(k^0 - \sqrt{s}\delta_s)}_{\text{finite, computed numerically}}$$

[Harris, Owens, '01]



State of the art method in quarkonium production computations at NLO until recently.

How to manage IR divergences?

- Subtractive methods:

$$\underbrace{\int d^{D}\Phi_k \mathrm{ST}}_{\text{divergent, computed analytically}} + \underbrace{\int d^{4}\Phi_k (|\mathcal{M}|^2 - \mathrm{ST})}_{\text{finite, computed numerically}}$$

  - The subtraction term ST should reproduce all soft and collinear singularities of $|\mathcal{M}|^2$.
  - The subtraction term should be sufficiently simple to be *integrable analytically in D-dimensions*
  - The algorithm for constructing ST should be **general** and easy to implement
- **Catani-Seymour dipole subtraction** has recently been extended to cases with quarkonia [Butenschön, Kniehl, '21, '22]
- **Frixione-Kunszt-Signer subtraction** method is more popular in the NLO community nowadays and can be also extended to the quarkonium case

Conclusions and outlook

- Uncertainty in the quarkonium production mechanism limits our ablilty to use quarkonia to study something else. On the other hand it stays one of a few open problems in SM physics.
- Despite its simplicity (I)CEM has several interesting phenomenological features.
- NRQCD continues to get more constrained theoretically ($\rightarrow$ pNRQCD) but “closes” only with *hadroproduction* data at relatively large p_T. *Contradiction with photoproduction OR polarisation and η_c-production in pp?*
- Colour-Singlet contribution is clearly large in several observables (η_c-production, pair production, photoproduction, ...) but requires very large higher-order corrections to describe high-p_T J/ψ and $\Upsilon(nS)$ production
- The technology of NLO calculations in NRQCD is now mature and should be pushed towards automation

LDME fits

LDME fit	J/ψ hadropr.	J/ψ photopr.	J/ψ polar.	η_c hadropr.	$J/\psi + Z$
Butenschön et al.	✓($p_T > 3$ GeV)	✓	✗	✗	✗
Chao et al. + η_c	✓($p_T > 6.5$ GeV)	✗	✓	✓	✗
Zhang et al.	✓($p_T > 6.5$ GeV)	✗	✓	✓	✗
Gong et al.	✓($p_T > 7$ GeV)	✗	✓	✗	✗
Chao et al.	✓($p_T > 7$ GeV)	✗	✓	✗	✗
Bodwin et al.	✓($p_T > 10$ GeV)	✗	✓	✗	✗
Brambilla et al.	✓($p_T > 9$ GeV)	✗	✓	(✗✓)	✓