Theory progress in quarkonium production in nucleus-nucleus collisions

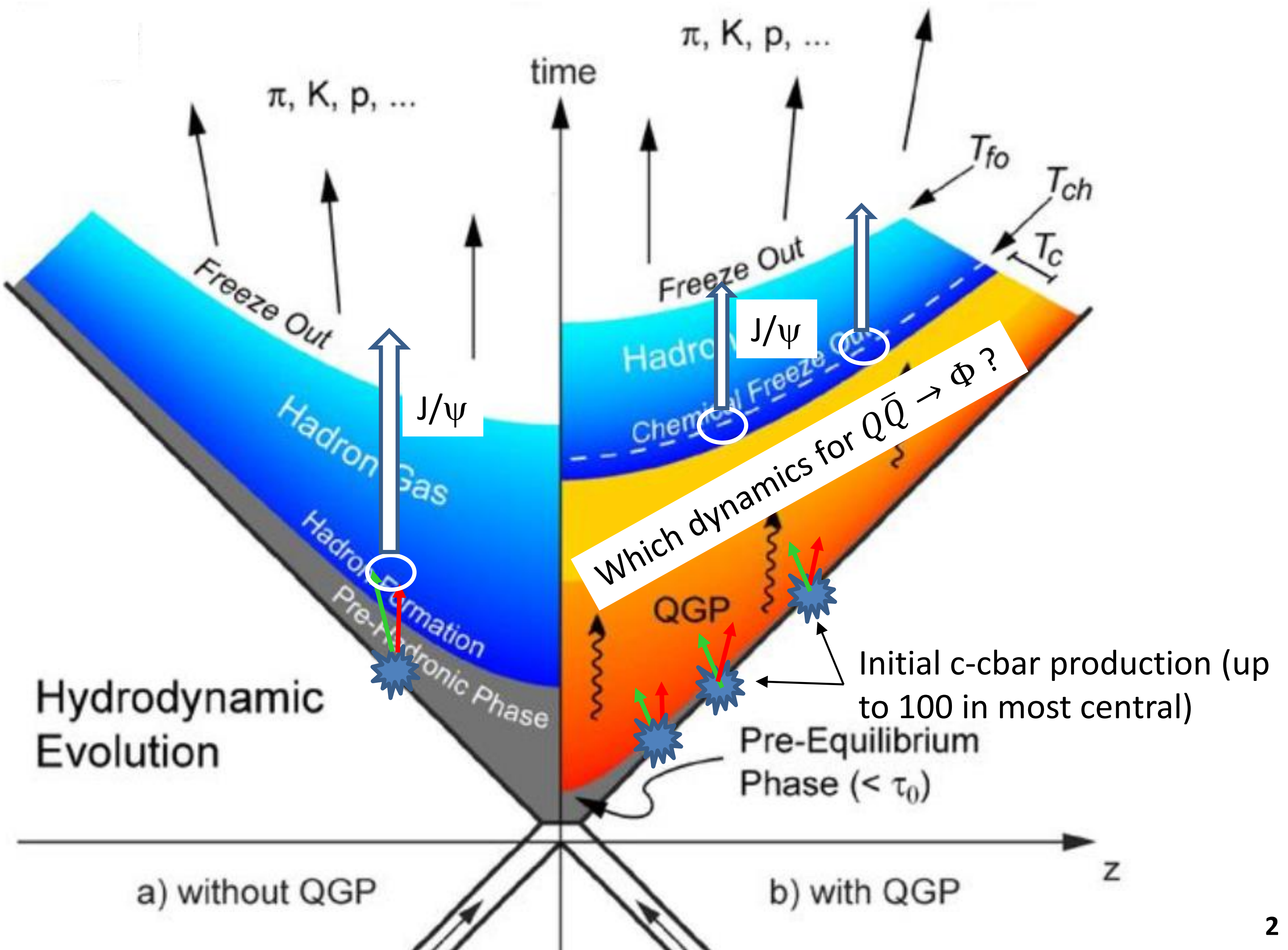
Pol B Gossiaux, SUBATECH (NANTES)

Quarkonia as Tools 2023

Aussois, 9-13 Janvier 2023

and Pays de la Loire

The fragmentation functions for HIC ?



The 3 classes of Models

- The "color evaporation model" ⇔ redistribution of the energy levels

Statistical Hadronization Model :

All possible correlations between initial heavy quarks is suppressed due to the strong fields in the QGP => at the chemical freeze out, various quarkonia are produced according to **statistical weights**… (We are thus not probing much of the QGP)

- The "color singlet model" ⇔ deals with the problem using a truncated basis

Transport models :

Time differential equations on the **phase space distribution** of quarkonia, relying on both dissociation and recombination cross sections… neglects quantum/microscopic features

- The "NRQCD approach" ⇔ in principle the best approach (but hard to solve)

Open quantum approaches :

Preserves at most the quantum feature of the $Q\overline{Q}$ system and coherence while dealing with the energy-momentum exchanges with the QGP environment.

The 3 classes of Models

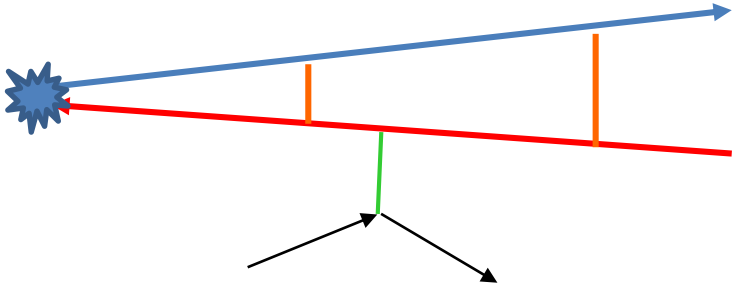
- **Statistical Hadronization Model**

- **Transport models**
- **Open quantum approaches**

Share the same central concepts (even with different realizations)

Central ingredients : what do we test ? Both the medium … and the theory

- Temperature field of the medium (space-time dependent)
- Strength between Q and $\bar{Q}$ (in medium screened-potential)
- Coupling of the QGP with the $Q\bar{Q}$ as well as QGP DoF. For instance, in HTL theory, thermal gluon propagator



Some recent results & reviews

Selection of a couple of topics due to the lack of (preparation) time… If you are not glad with it:

- Open quantum approaches : M. Strickland @ SQM 2021, St. Delorme @ QAT 2022, PB Gossiaux @ SQM 2022
- Revival of EMMI Rapid Reaction Task Force (GSI dec 2022) : https://indico.gsi.de/event/15946 (day 1: review of models)
- Reviews by Akamatsu (2009.10559) and Yao (2102.01736)
- He, Rapp and Van Hees (2204.09299)

Some news on Statistical Hadronization

Historical reminder :

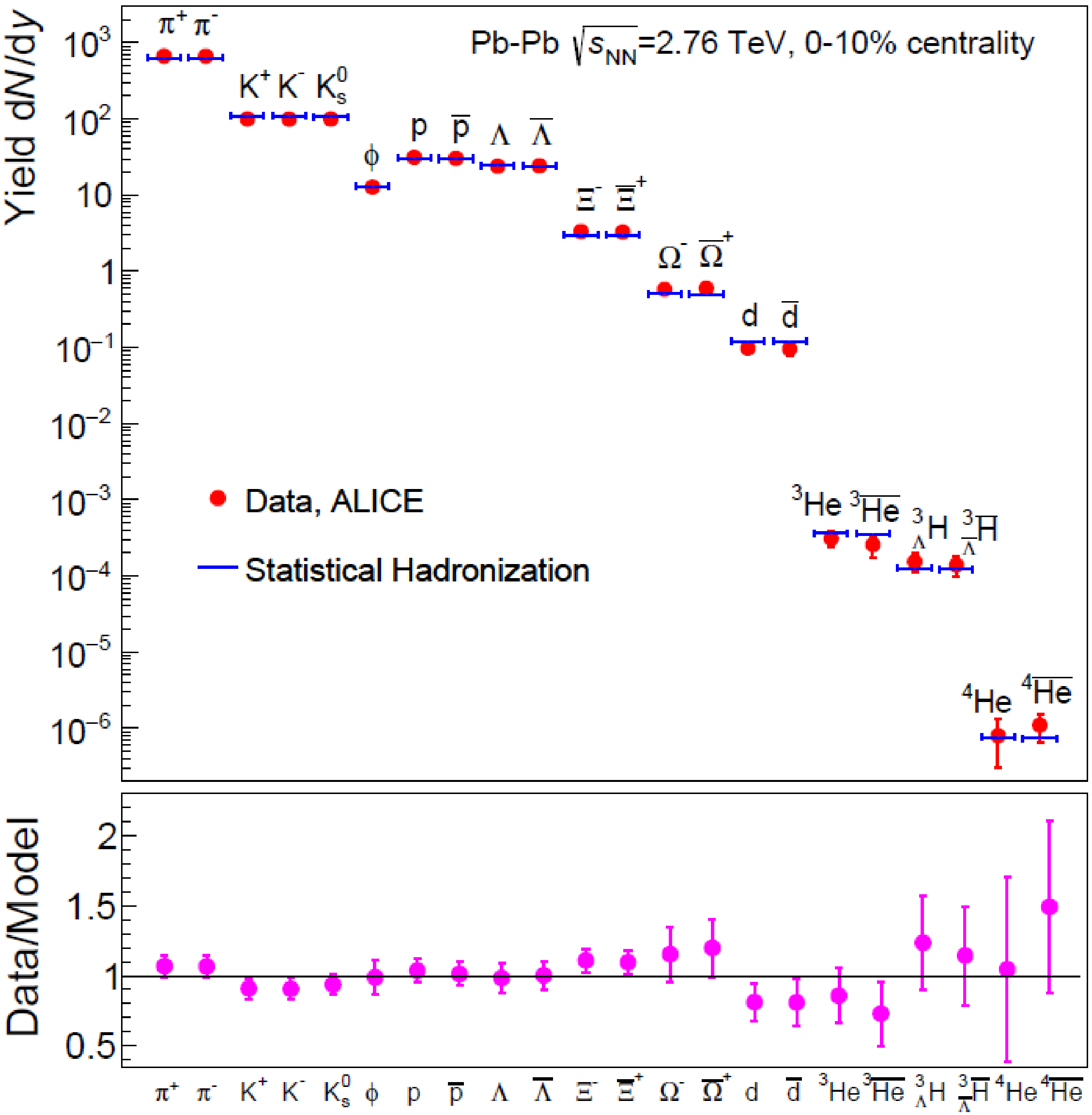
In the light sector, the total multiplicities are beautifully described by the SH hypothesis

Common parameters :

$$T_{CF} = 156.6 \pm 1.7 \text{ MeV}$$
$$\mu_B = 0.7 \pm 3.8 \text{ MeV}$$
$$V_{\Delta y=1} = 4175 \pm 380 \text{ fm}^3$$

$$\chi^2/N_{df} = 16.7/19$$

Also works for loosely bound states (snowball in the fire challenge)…



A. Andronic (HH-WINC 2022)

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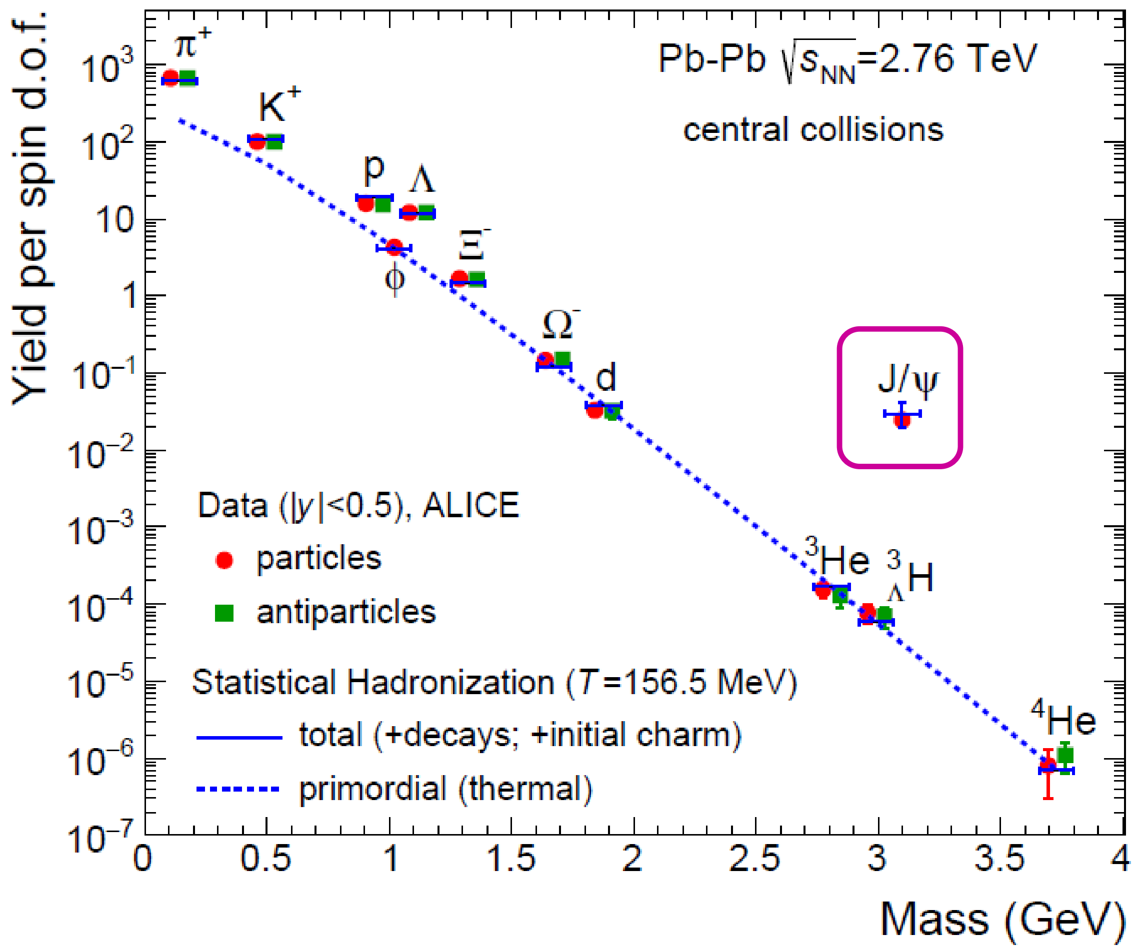
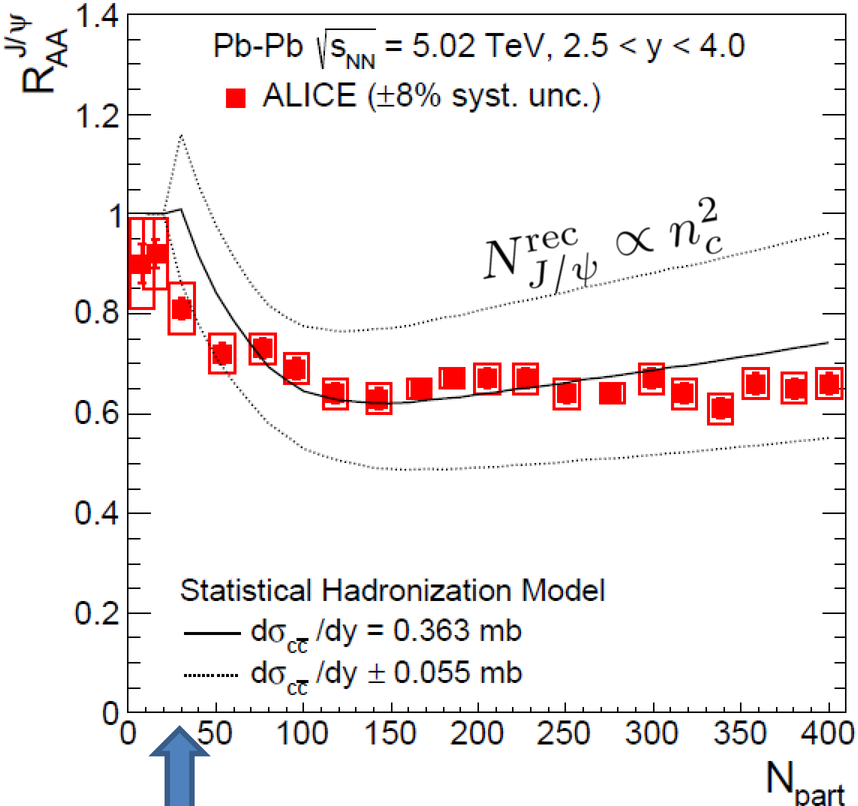
… as well as for J/ψ (provided one assumes that c and cbar quarks are conserved from Initial State)



A. Andronic et al. PLB 797 (2019) 134836

Some news on Statistical Hadronization

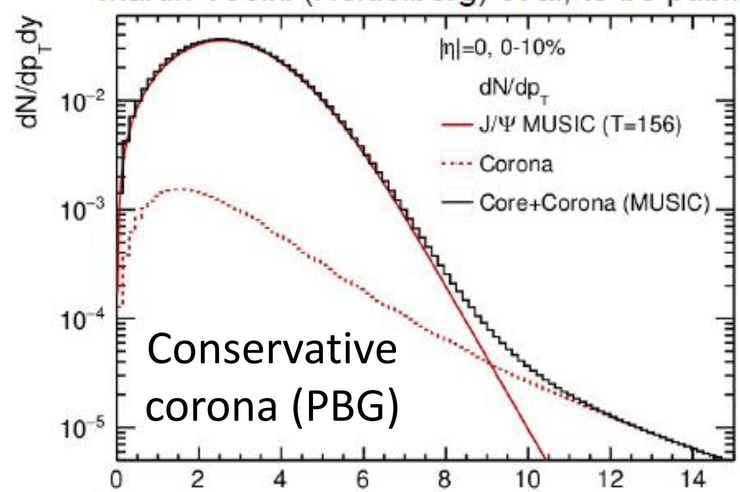
Most recent J/ψ predictions at LHC :

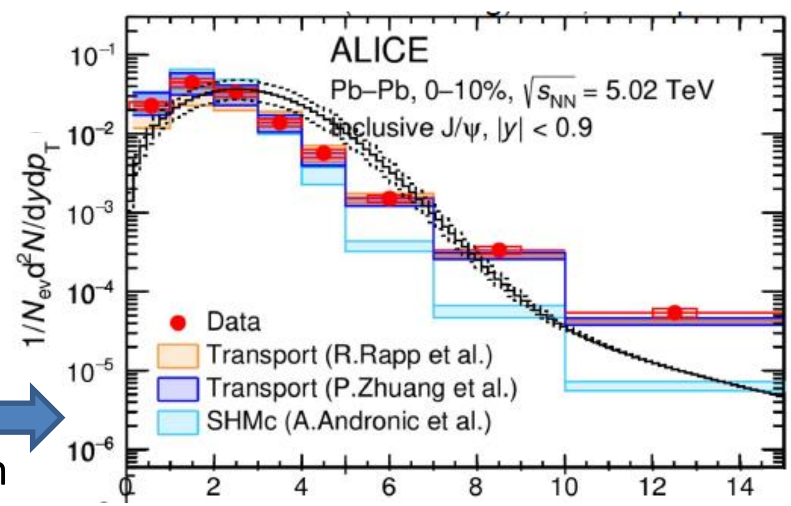


Saturation and even small rising trend as a function of the # of participants : sign for (re)combination

p_T spectra: New approach: Cooper-Frye freeze-out of MUSIC at 156.5 MeV instead of blast wave.

Martin Voelkl (Heidelberg) et al, to be publ.



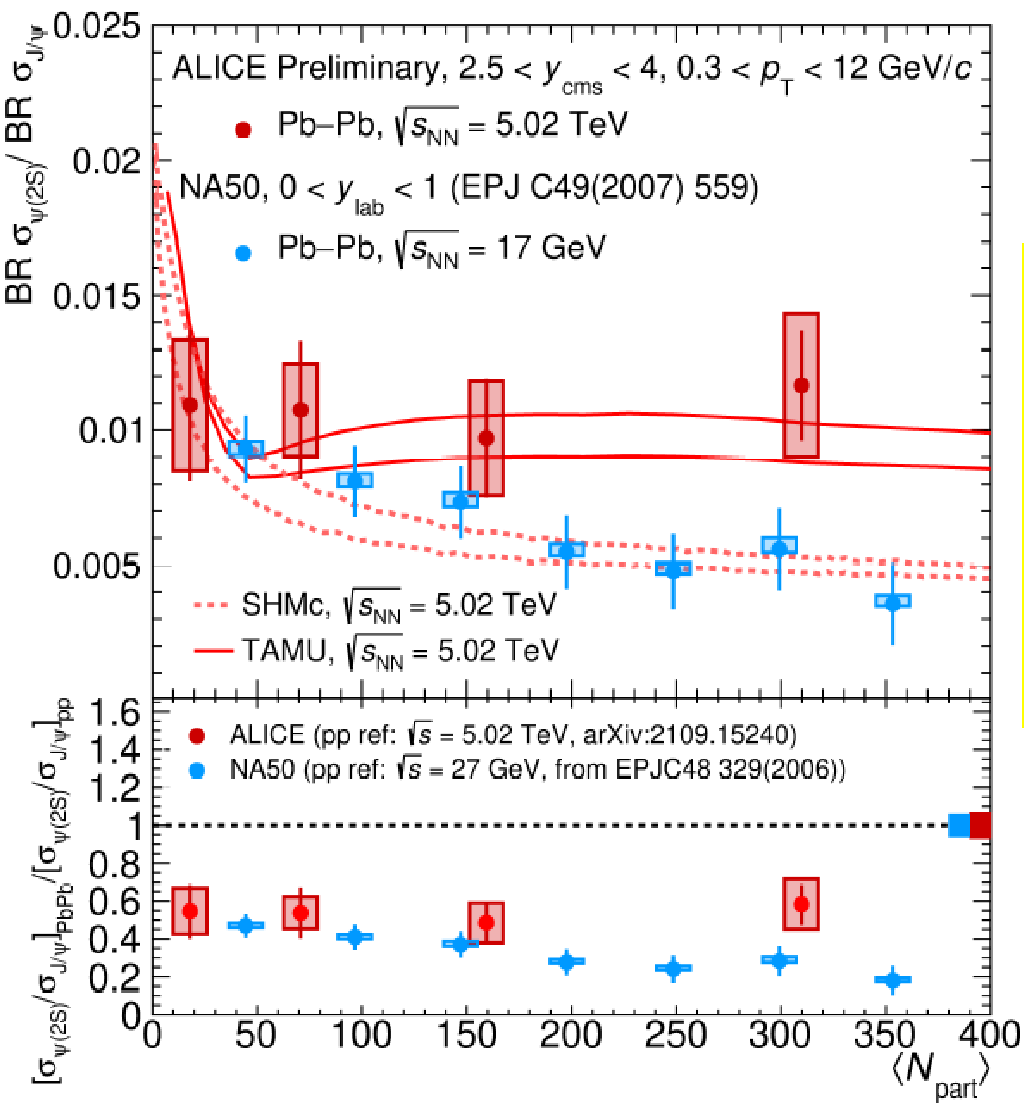


Clear sign of non thermal production for p_T>5 GeV/c

J. Stachel, EMMI RRTF 2023

Some news on Statistical Hadronization

Ψ(2S) at LHC :



Failure of SHM @ LHC. Was working fine at SPS… and by construction, the ratio should be beam-energy invariant (only the freeze out T matters)

J. Stachel, EMMI RRTF 2023:

within stat. hadronization approach, an unexpected result
→ little room to accommodate in a likely physical scenario

- larger common freeze-out temperature ☹
- larger freeze-out temperature for ψ(2S) vs J/ψ ☹

Future opportunities :

- Better precision
- χ_c state => T_{FO} from quarkonium spectra

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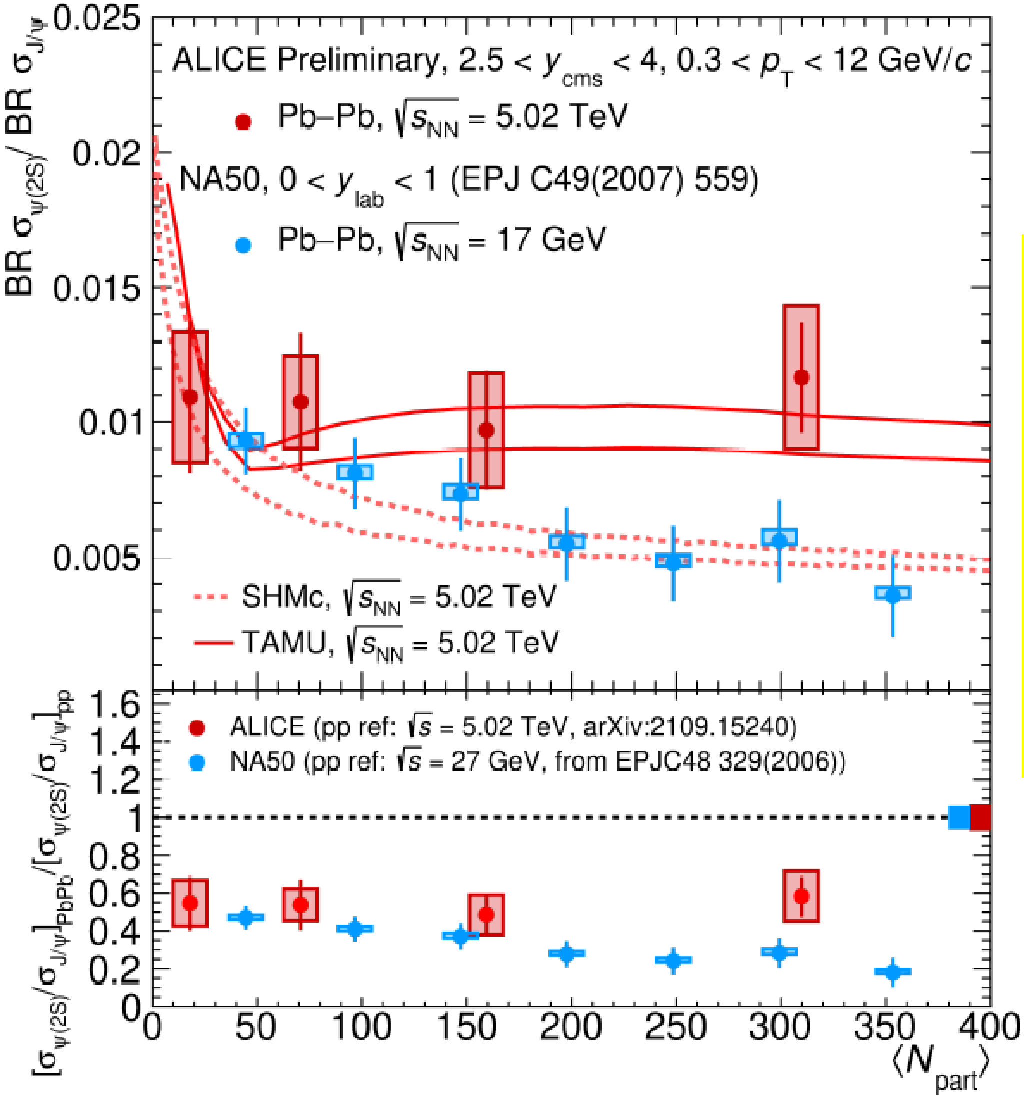

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→ little room to accommodate in a likely physical scenario

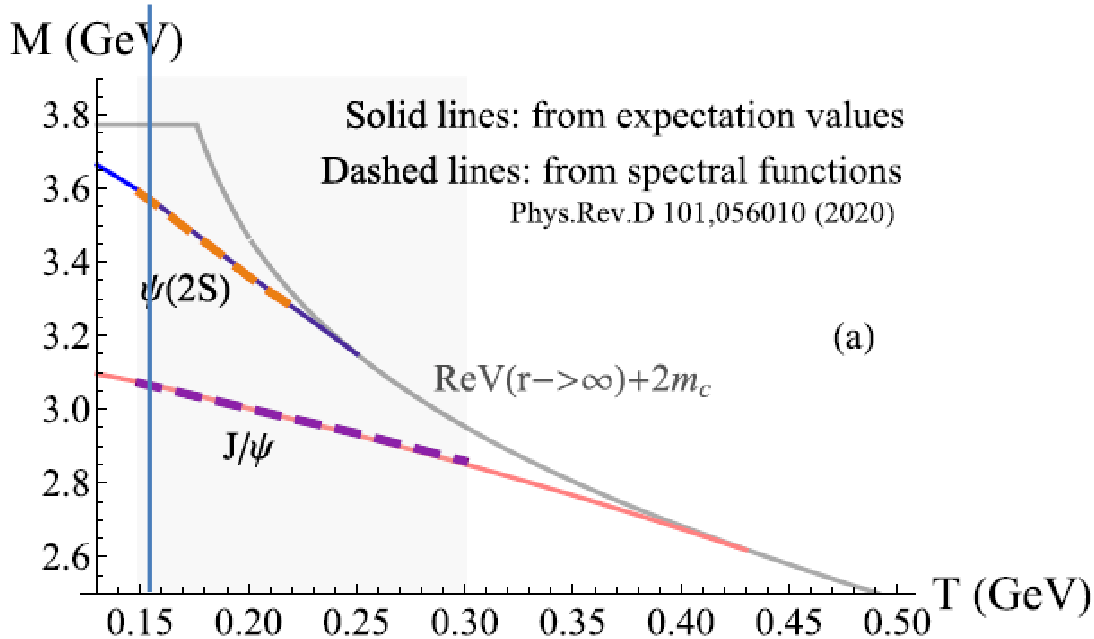
- larger common freeze-out temperature ☹
- larger freeze-out temperature for ψ(2S) vs J/ψ ☹
- PB: ψ(2S) @ FO ≠ vacuum ψ(2S)

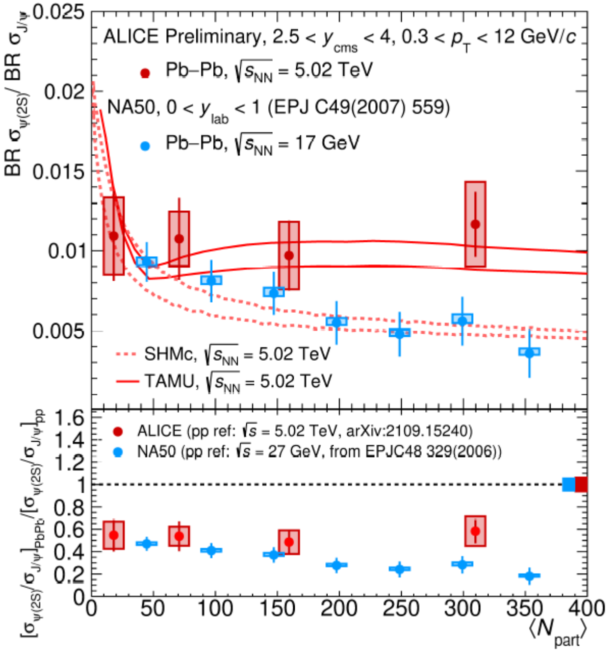
Future opportunities :

- Better precision
- χ_c state => T_{FO} from quarkonium spectra

Some news on Statistical Hadronization

Ψ(2S) at LHC : role of the mass difference





Statistical yield : $\frac{N_{\psi(2S)}}{N_{\psi(1S)}} \propto e^{-\frac{\Delta M}{T_{\rm FO}}}$ with $\Delta M(T_{\rm FO}) \approx \Delta M(T_{\rm vacuum}) - 75 \pm 25 {\rm MeV}$

Factor x (1.4 – 2) on the ψ(2S)/ψ(1S) ratio

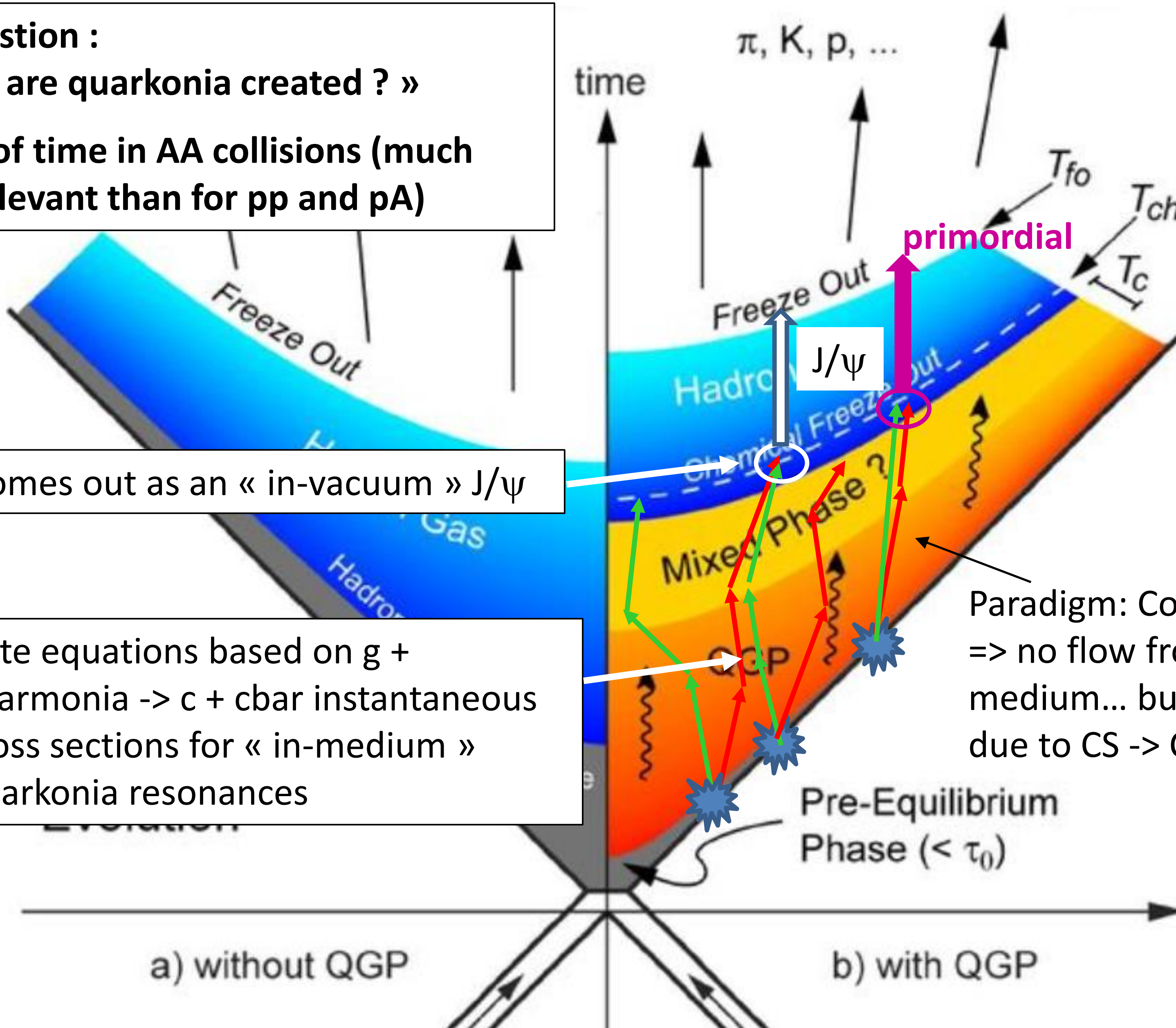
⇒ need for a fine understanding of in-medium quarkonia states

⇒ Expected gain of considering several quarkonium states… Important question of “quarkonium spectrum in a single $Q\bar{Q}$ state and associated thermalization”

Charmonia in transport models

Key question :
« When are quarkonia created ? »

Notion of time in AA collisions (much more relevant than for pp and pA)



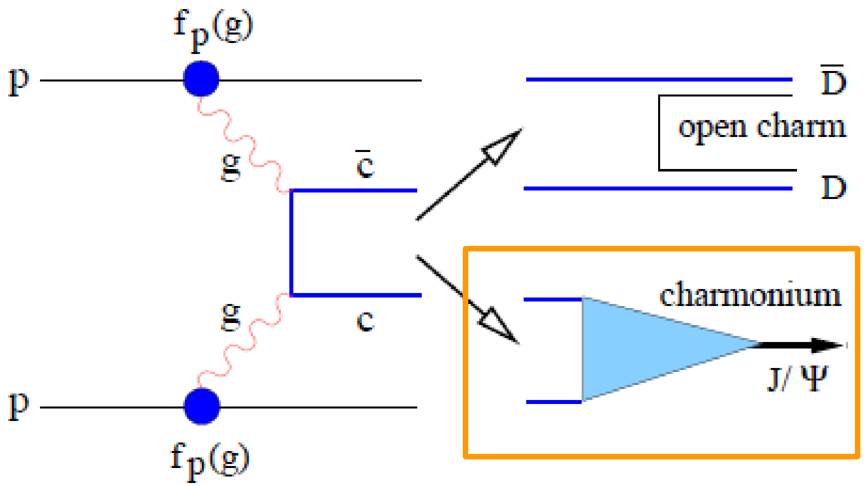
Finally comes out as an « in-vacuum » J/ψ

Rate equations based on g + charmonia -> c + cbar instantaneous cross sections for « in-medium » quarkonia resonances

Paradigm: Color singlet => no flow from the medium… but could be due to CS -> CO* -> CS

Reminder : main ingredients of a transport model (and a bit of critical considerations)

Picture behind transport theory :



Open heavy flavor and quarkonia assumed to be uncorrelated

Formed after some "formation time" τ_f (typically the time to grow -> fully bloomed states), usually assumed to be independent of the surrounding medium

Decoupled production of various HF mesons and quarkonia (neglect of quantum coherence)

In fact the decoupling from quantum states -> classical probabilities happens after the so-called decoherence time τ_{dec} which depends on the surrounding medium

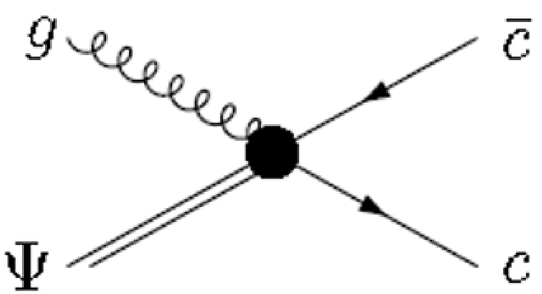
Rate equations

$$\frac{dN(t)}{dt} = -\Gamma(T(t))\,(N(t) - N^{\mathrm{eq}}(T(t)))$$

Loss Gain

For instance, Bhanot-Peskin gluo-dissociation

$$\sigma_{J/\psi}(\omega) = A_0 \frac{(\omega/\epsilon_{J/\psi} - 1)^{3/2}}{(\omega/\epsilon_{J/\psi})^5}$$



ω: gluon energy in the quarkonium rest frame

$$\Gamma_\Psi(T) = \int \frac{d^3p}{(2\pi)^3} v_{i\Psi}\sigma_{i\Psi\to X} f_i(m_i, T)$$

Dissociation rate

Parton Density

$$N_{\mathrm{final}} = N_0 \times \underbrace{e^{-\int_{t_0}^{+\infty}\Gamma_\Psi(T(t))\mathrm{d}t}}_{R_{AA}}$$

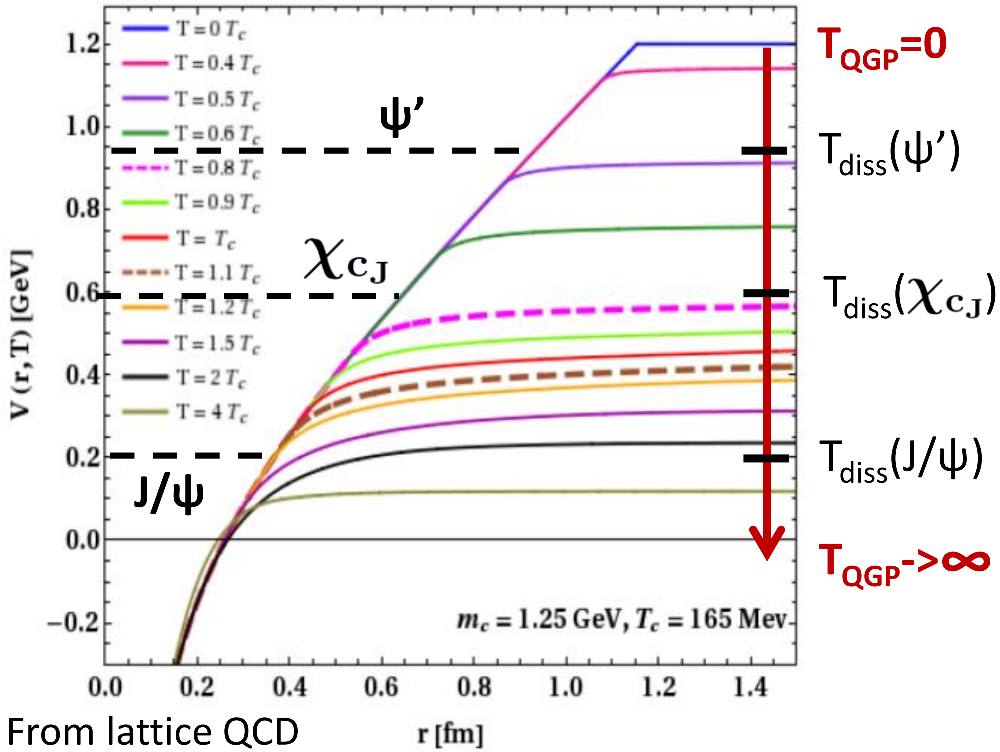
If just suppression

Various states are still decoupled in their evolution… while in principle, one could have some gluon-induced “conversion” (not implemented in any model to my knowledge)

Historical models

Sequential suppression (Matsui and Satz)

$Q\bar{Q}$ color potential



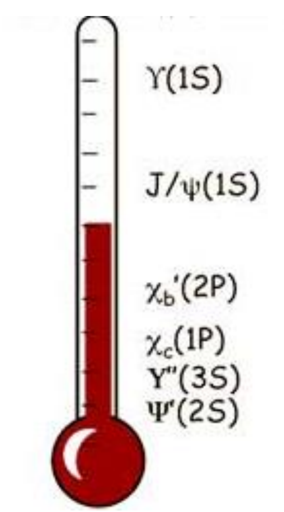
From lattice QCD

T ↗ => screening ↗ => progressive states melting

« all or nothing »:

- If $T_{early\ QGP} > T_{diss}$ => the state is not produced
- If $T_{early\ QGP} < T_{diss}$ => the state is produced like in pp

=> *Quarkonia as early QGP thermometer*



T. Matsui and H. Satz, Phys. Lett. B 178 (1986) 416

Possible Combined suppression + Rate equations

$$\frac{dN(t)}{dt} = -\Gamma(T(t))\,(N(t) - N^{\mathrm{eq}}(T(t)))$$

Loss Gain

- With Γ = Infinity it T > T_{diss} (instantaneous dissociation)
- Dissociation rates evaluated with "in-medium" quarkonia wave functions (François Arleo, early 2000)

Do we really believe that a quarkonia wanna-be that feels a temperature $T_{diss} + \varepsilon$ at the formation time as 0 probability to emerge as a quarkonia ?

$\Rightarrow$ Uncertainties that prevents to make genuine precision physics based on solid underlying QCD quantities

$\Rightarrow$ Need for a better formalism (that would allow to keep track of the microscopic degrees of freedom during the whole QGP evolution)

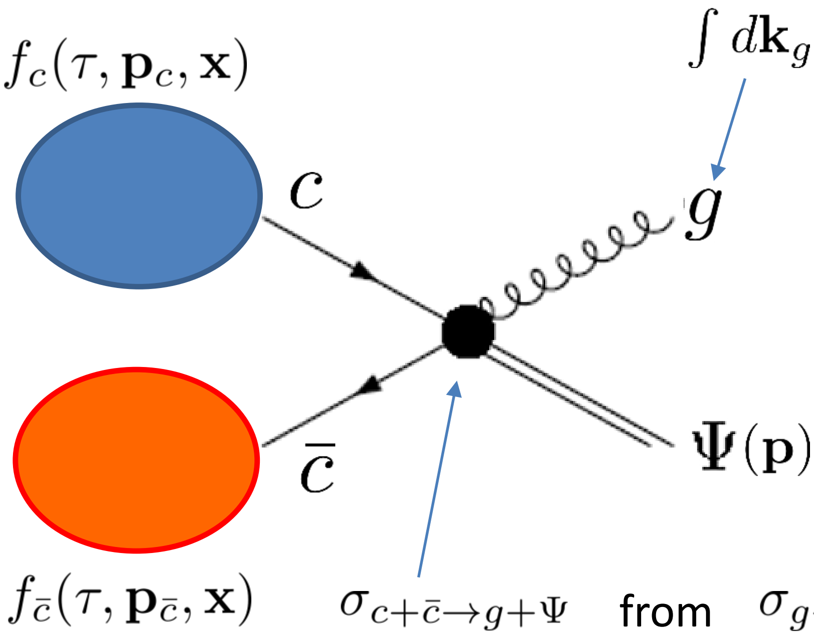
Rate equations

$$\frac{dN(t)}{dt} = -\Gamma(T(t))\,(N(t) - N^{eq}(T(t)))$$

Loss Gain

Recombination known to play a major role at LHC

$\beta_\Psi(\tau, \mathbf{x}, \mathbf{p}) \equiv$

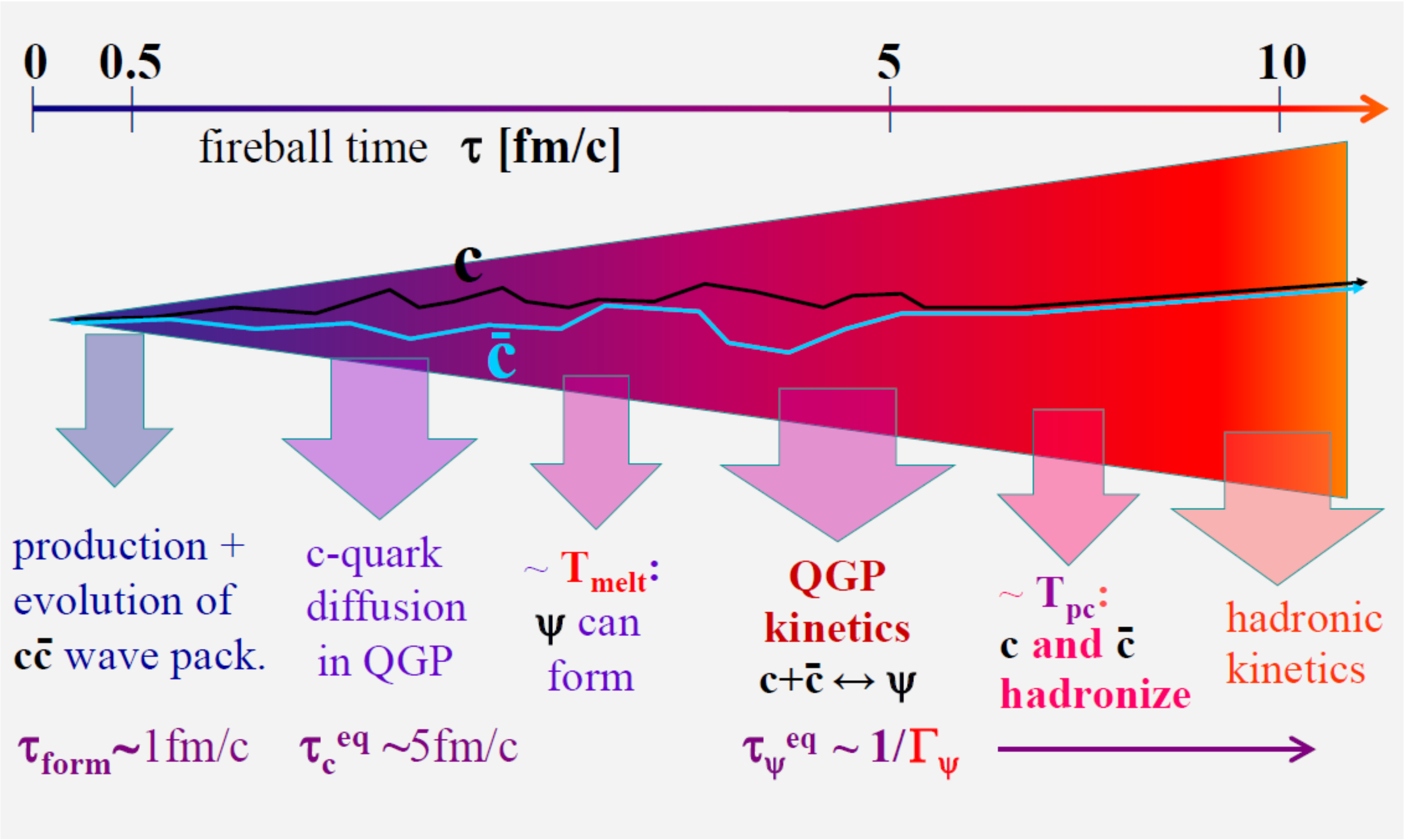


$\sigma_{c+\bar{c}\to g+\Psi}$ from $\sigma_{g+\Psi\to c+\bar{c}}$ by detailled balance

- Question about the underlying c and cbar distributions (are they "statistically equilibrated")
- The "better formalism" must be able to describe the equilibration of a (quantum) system with an external "reservoir" .

Charmonia in transport models

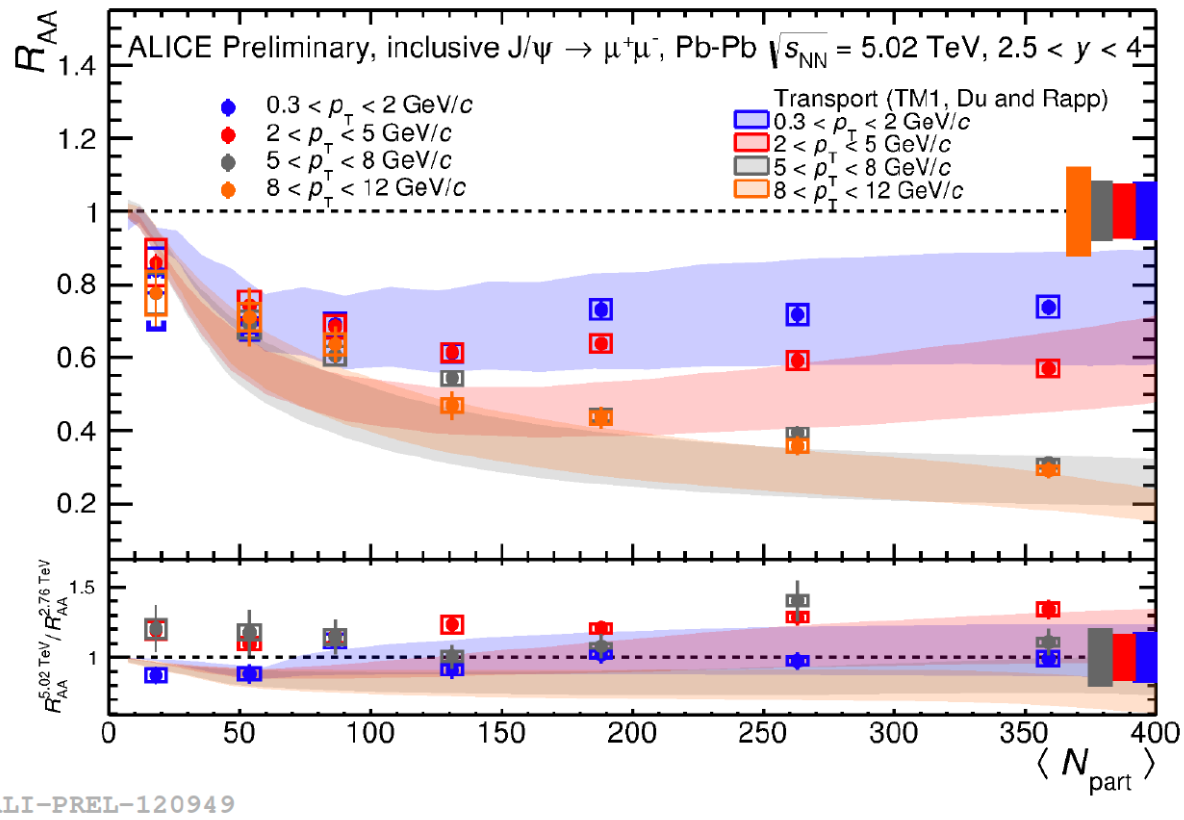
Rapp and Du *Nucl.Phys.A* 967 (2017) 216-224



Charmonia does not exist

Charmonia do exist and dissociate/recombine following semi-classical laws

TAMU et al: some illustrative results

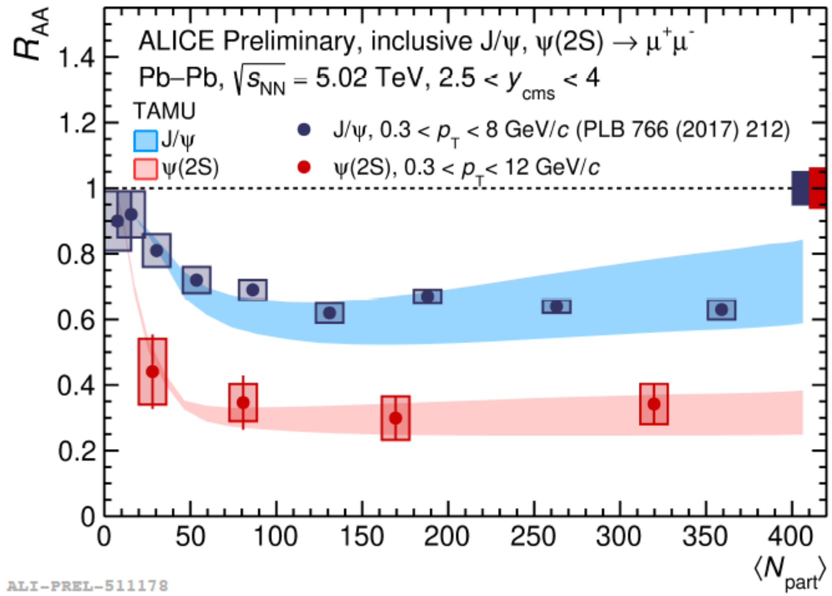


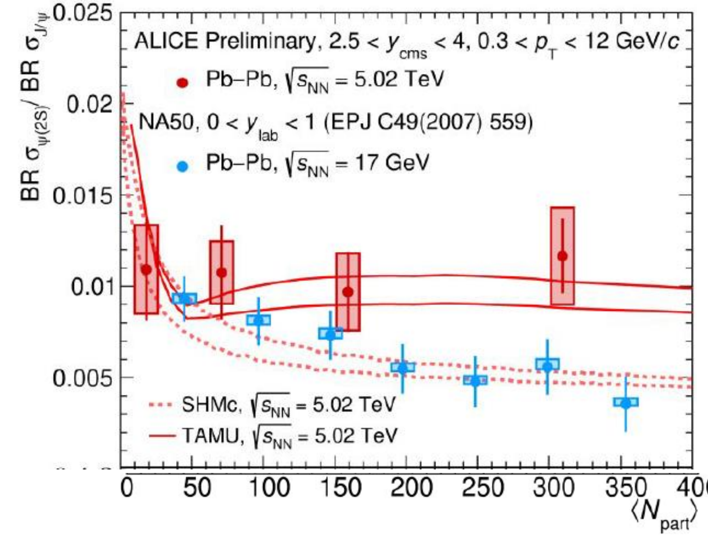
- In transport theory, primordial component is mandatory to reproduce the absolute production as a function of centrality & p_T class

Not simple statistical hadronization at the end of the QGP

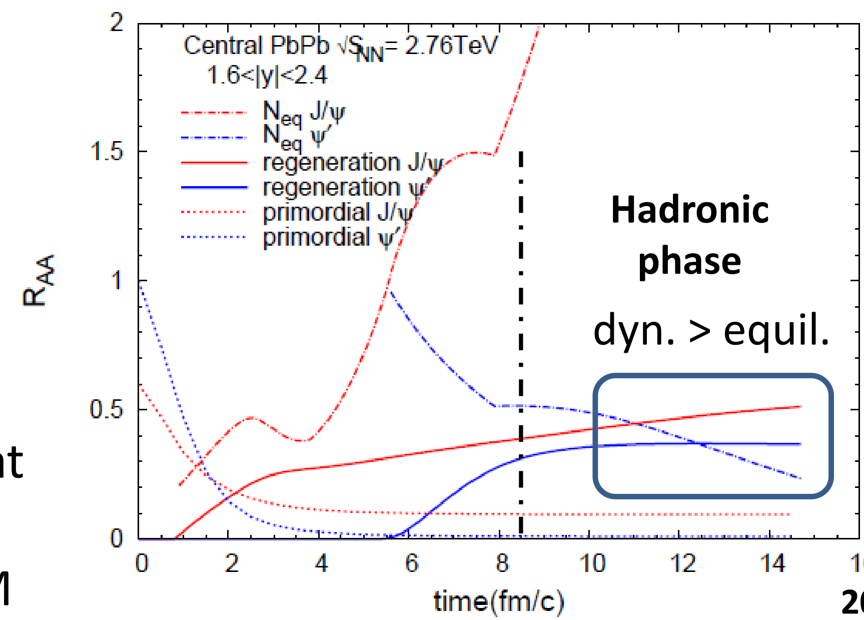
TAMU et al: some illustrative results

Recent precise measurement of ψ(2S) by ALICE





- Found in good agreement with TAMU (Du and Rapp 2015) : late formation due to hadronic channels
- Rather early production of J/y bound state, gets reinforced over time
- Freeze out of ψ(2S) in the hadronic phase at a value > equilibrium value… based on equilibrium rates => no counterpart in SHM



Reaction rate approach by TAMU

Differential observables (augmented Reaction Rate as Boltzmann eqs are not solved)

- Primordial $f_\Psi^{\rm prim}(p_t, x_t, \tau) = f_\Psi(p_t, x_t - v_t(\tau - \tau_0), \tau_0) e^{-\int_{\tau_0}^{\tau} d\tau' \alpha_\Psi(p_t, x_t - v_t(\tau-\tau'), \tau')}$

  (same method as in Tsinghua)

  +leakage effect

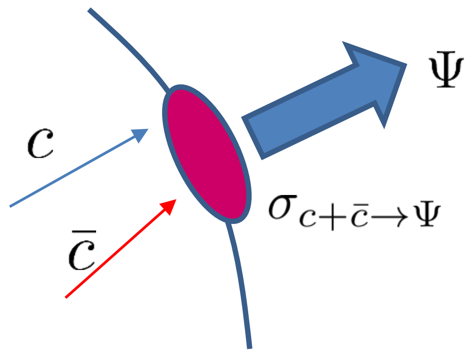
  + formation time at high p_T

- Regeneration ; p_T spectrum : from blast wave formula (until recently)

$$\frac{\mathrm{d}N_\Psi^{\rm reg}}{p_t \mathrm{d}p_t} \propto m_t \int_0^R r dr K_1\left(\frac{m_t \cosh\rho}{T}\right) I_0\left(\frac{p_t \sinh\rho}{T}\right)$$

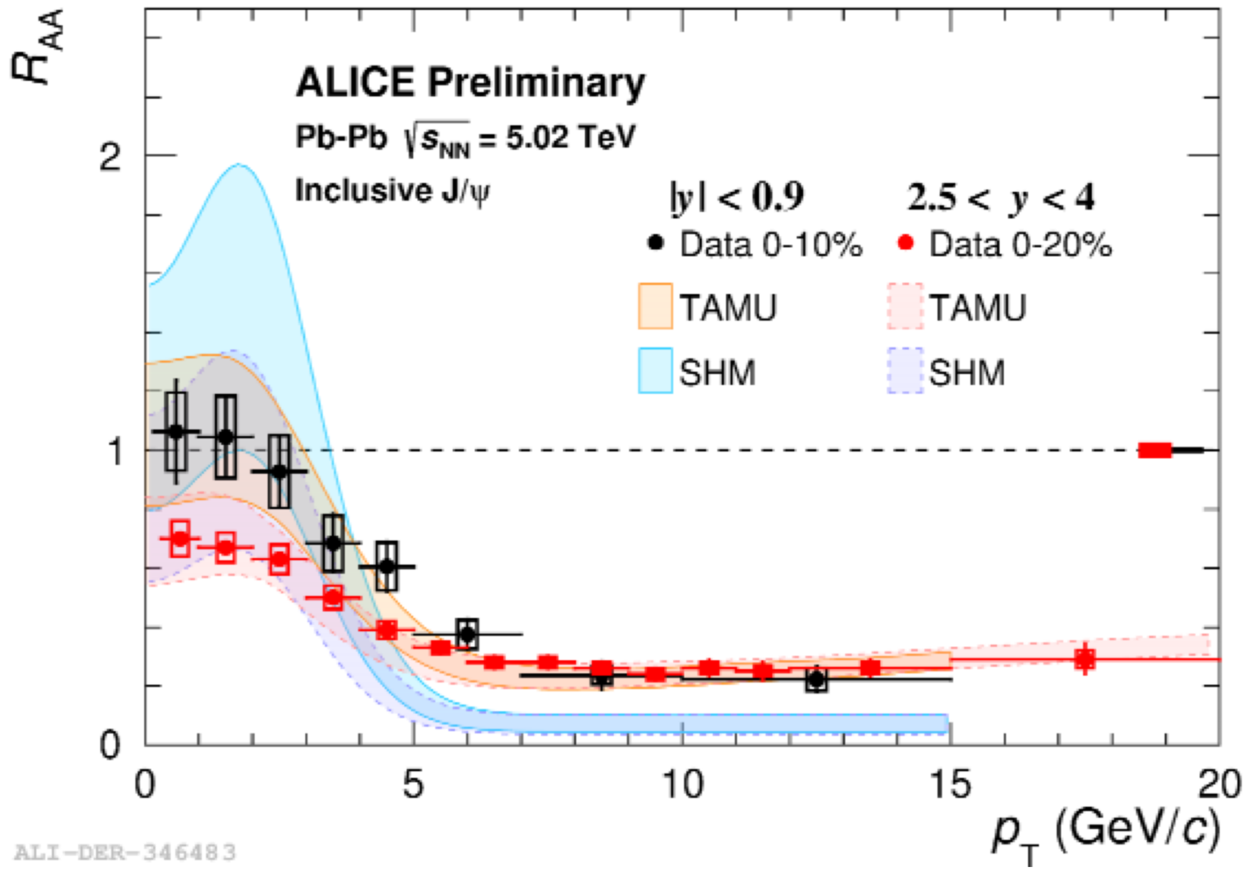
- More recently: Regeneration ; azimuthal flow : « Resonance Reaction Model »

L. Ravagli and R. Rapp, Phys. Lett. B 655, 126 (2007)



Taken as Breit-Wigner shaped

TAMU et al: some illustrative results



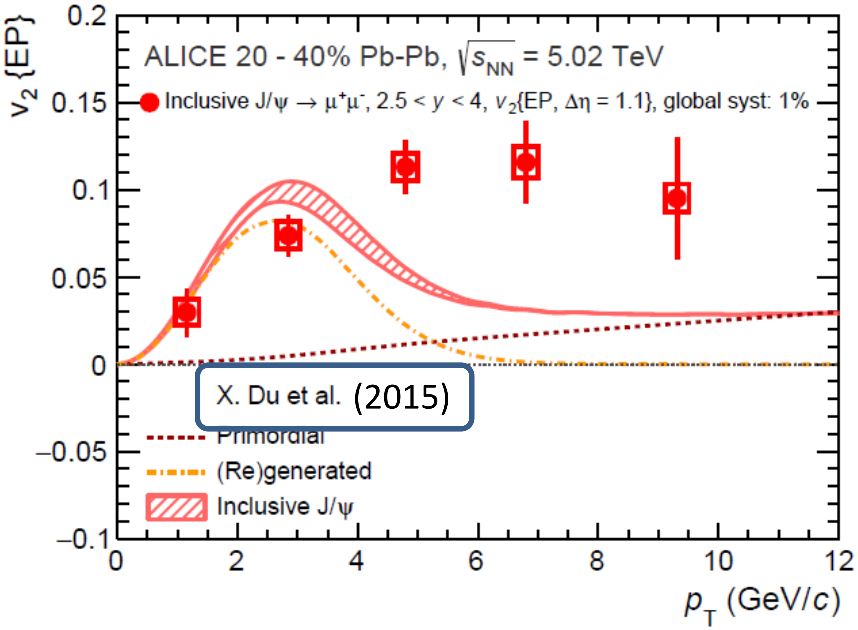
- In transport theory, primordial component is mandatory to reproduce the absolute production at intermediate and high p_T

Not simple statistical hadronization at the end of the QGP

TAMU et al: some illustrative results

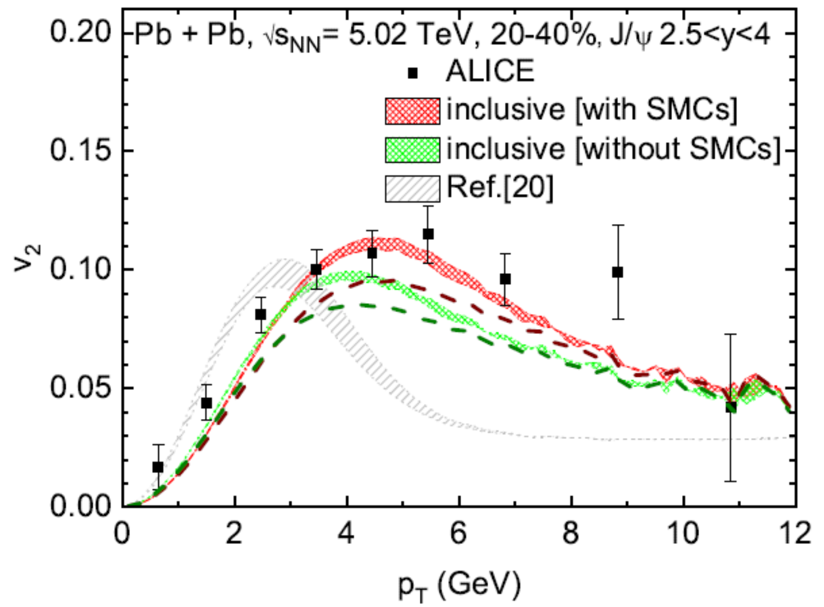
Elliptic Flow

Ph D Audrey Francisco 2018 (Subatech)



Quite similar to Tsinghua (no B feeddown)

He et al, Phys.Rev.Lett.128, 162301 (2022))



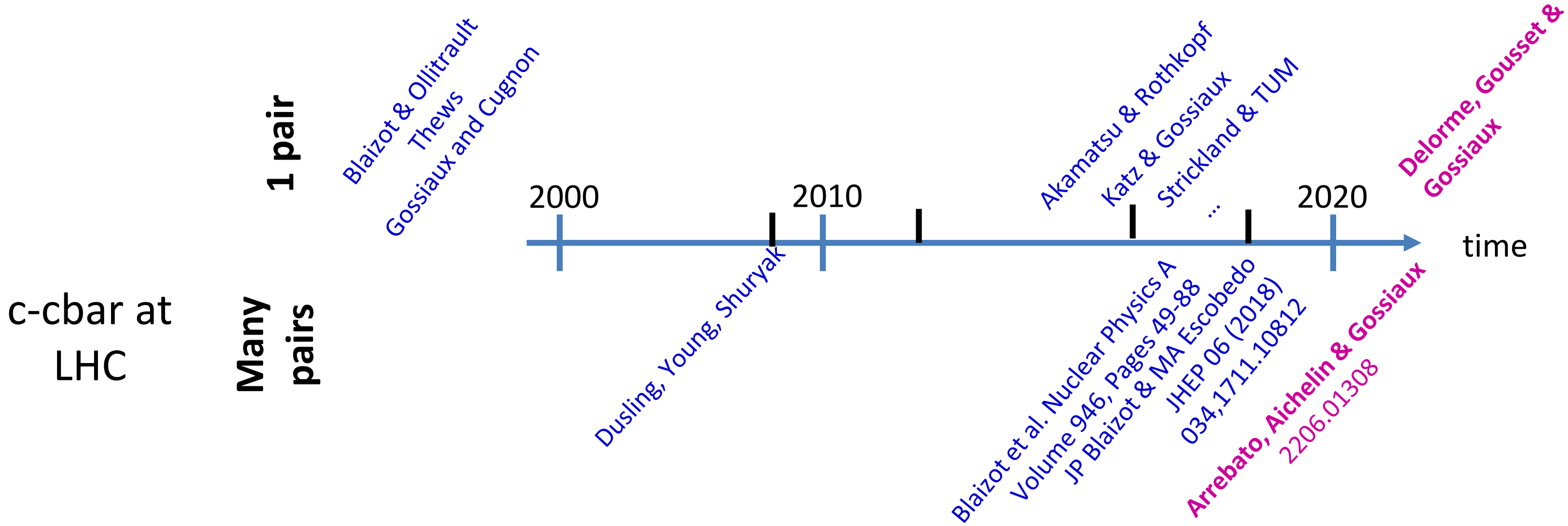
New calculation by TAMU collaboration

- Good agreement for low p_T, where J/ψ formation proceeds through recombination at FO
- ~~Disagreement from intermediate p_T on, where primordial production start having a large weight (crucial for the $R_{AA}(p_T)$)~~

Transport models are nice for phenomenology but may ignore important quantum features

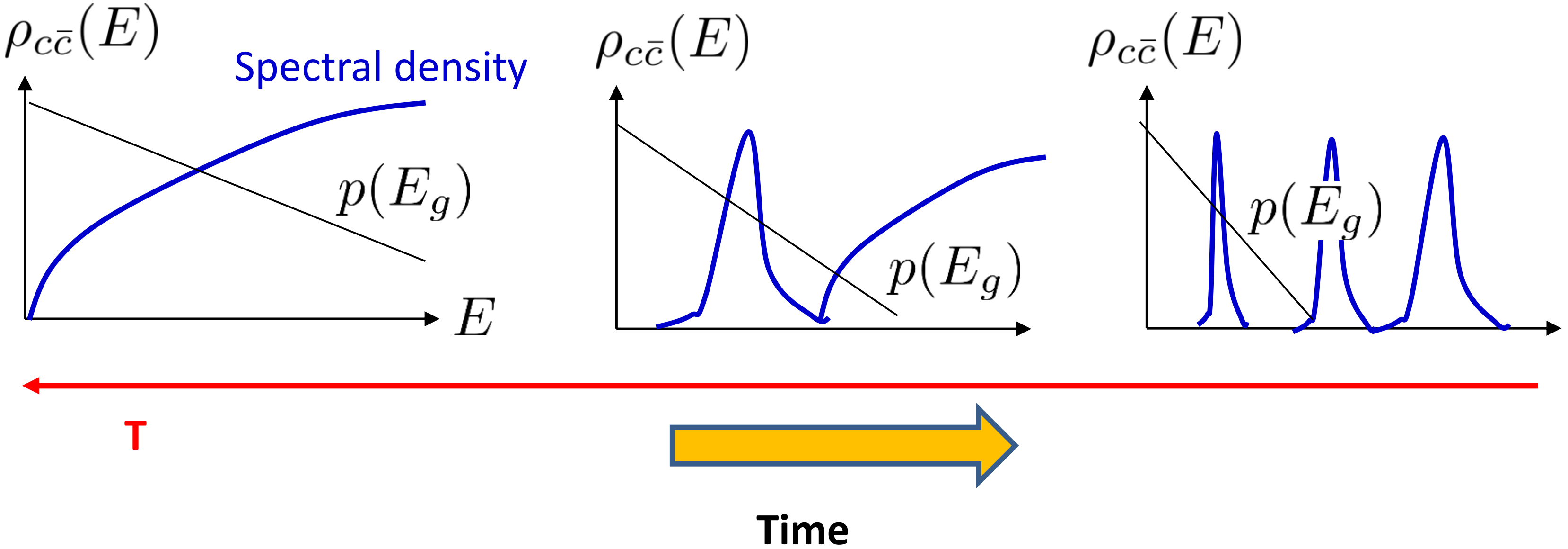
Several motivations to go microscopic & quantum

- The in-medium quarkonia are not born as such. One needs to develop an **initial compact (quantum) coherent state** to fully bloomed quarkonia
- The dissociation-recombination reactions affecting quarkonia are **not instantaneous**… In dense medium, the notion of cross section should be replaced by the more rigorous open-quantum system approach (**continuous transitions including interference**)
- Better take into account the complicated **structure of in-medium quarkonia spectral functions**
- Better suited for **« from small to large »**



Pioneering work of **Blaizot and Escobedo** for many c-cbar pairs (NRQCD) => mixed Fokker-Planck + gain/loss rates for color transitions; awaits for implementation in realistic conditions

Charmonia in a microscopic theory
Several regimes / effects



Multiple scattering on quasi free states

Gluo-dissociation of well identified levels by scarce “high-energy” gluons (dilute medium => cross section ok)

Well identified formalisms (Quantum Master Equation, Boltzmann transport, Stochastic equations,…) in well identified regimes, but continuous evolution and no unique framework continuously applicable (to my knowledge)

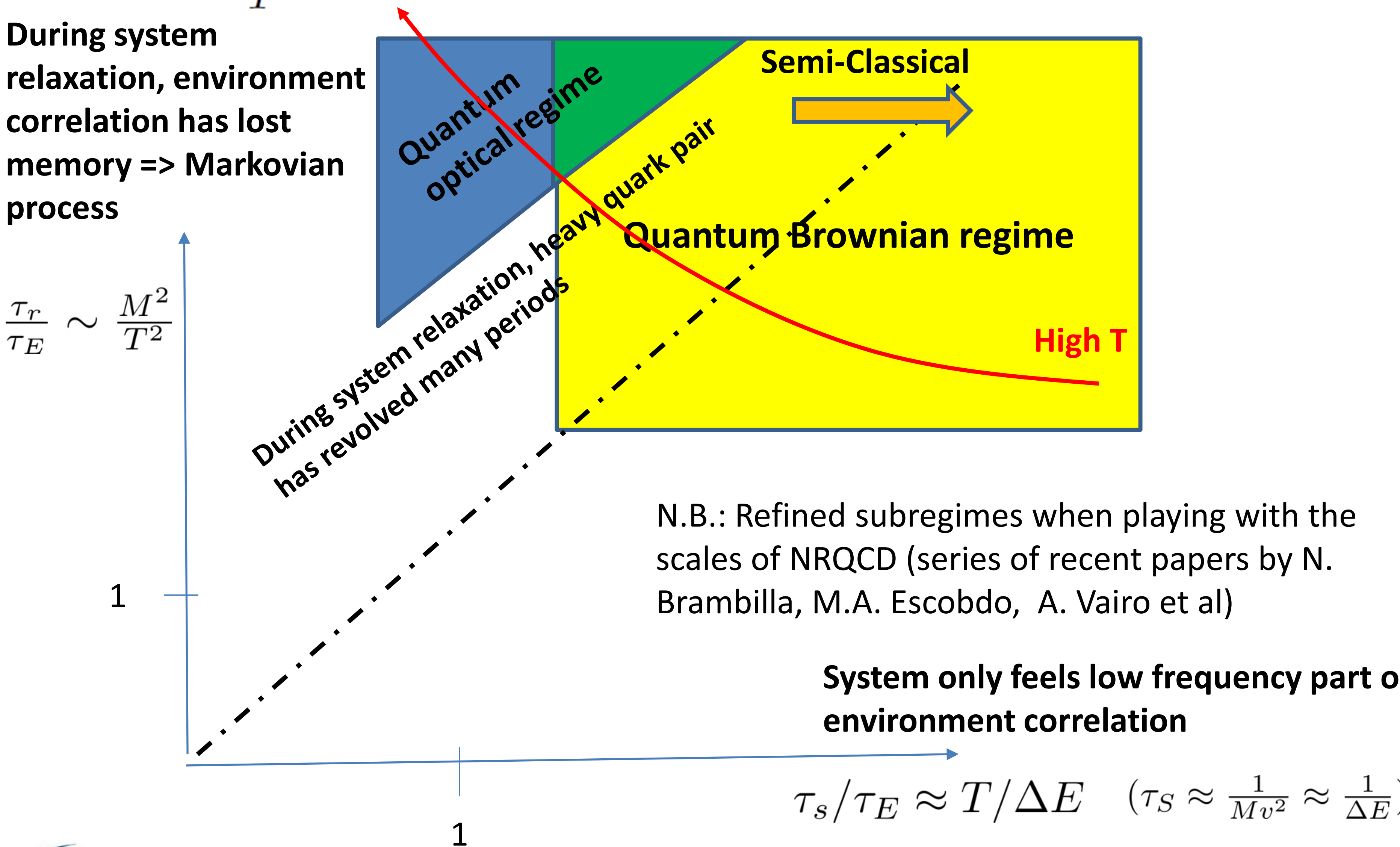
Yet, still a need to define the equivalent of a formation – dissociation rate

τ_E: environment correlation time τ_S: system intrinsic time scale τ_R: system relax time

$$\tau_E \sim \frac{1}{T}$$

$$\tau_r \sim \frac{1}{\Gamma} \approx \frac{1}{\alpha T (m_D^2 \langle r^2 \rangle)} \sim \frac{M^2}{T^3} \gg \tau_E$$

During system relaxation, environment correlation has lost memory => Markovian process



N.B.: Refined subregimes when playing with the scales of NRQCD (series of recent papers by N. Brambilla, M.A. Escobdo, A. Vairo et al)

System only feels low frequency part of environment correlation

Not clear all states goes from one regime to the other at the same T

How can we restore the quantumness of Quarkonia treatment (in interaction with some environment) ?

- Statistical mechanics is about averages… averaging wave functions makes no sense.

  Deals with density matrix for the full state (quarkonia + environment)

$$\hat{\rho} = |\psi\rangle\langle\psi| \text{ with } |\psi\rangle = |\psi_{\mathrm{QGP}}\rangle \otimes |\psi_{Q\bar{Q}}\rangle$$

Possible to make statistical averages on the environment (« tracing out ») while still preserving all coherence at the level of Q-Qbar

$$\mathcal{D}_{Q\bar{Q}} = \mathrm{tr}_{\mathrm{QGP}}(\rho)$$ + Evolution equation

Quantum Master Equation

Not the same basis !!!

Quantum Optical Regime	Quantum Brownian Motion
Vacuum bound states	Usual space variables
$\mathcal{D}_{Q\bar{Q}}(N, N')$	$\mathcal{D}_{Q\bar{Q}}(x_Q, x_{\bar{Q}}, x'_Q, x'_{\bar{Q}})$
Low T	high T

Like : rigorously derived from fundamental principles

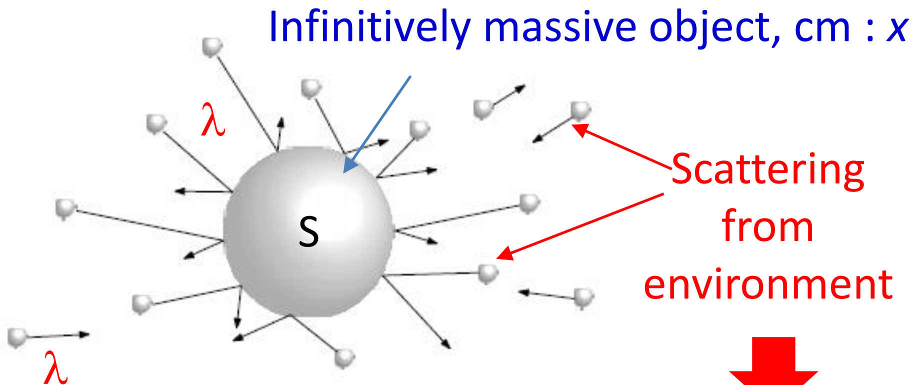
Dislike : computer demanding… currently impossible for several pairs

Decoherence from system-env. interaction

Quantitative model :

$$\hat{H} = \hat{H}_S + \hat{H}_E + \hat{H}_{\rm int}$$

Reduced density matrix $\mathcal{D}_S(\mathbf{x},\mathbf{x}')$



$$\frac{\partial \mathcal{D}_S(\mathbf{x},\mathbf{x}',t)}{\partial t} = -F(\mathbf{x}-\mathbf{x}')\mathcal{D}_S(\mathbf{x},\mathbf{x}',t)$$

Fluctuations !

Decoherence factor:

$$F(\mathbf{x}-\mathbf{x}') = \int dq\rho(q)v(q)\int \frac{d\hat{n}d\hat{n}'}{4\pi}\left(1-e^{iq(\hat{n}-\hat{n}')\cdot(\mathbf{x}-\mathbf{x}')/\hbar}\right)\underbrace{|f(q\hat{n},q\hat{n}')|^2}_{\frac{d\sigma}{d\Omega(\hat{n},\hat{n}')}}$$

Short wave length (λ << Δx)

$$F(\mathbf{x}-\mathbf{x}') = \Gamma_{\rm tot}$$

Total collision rate

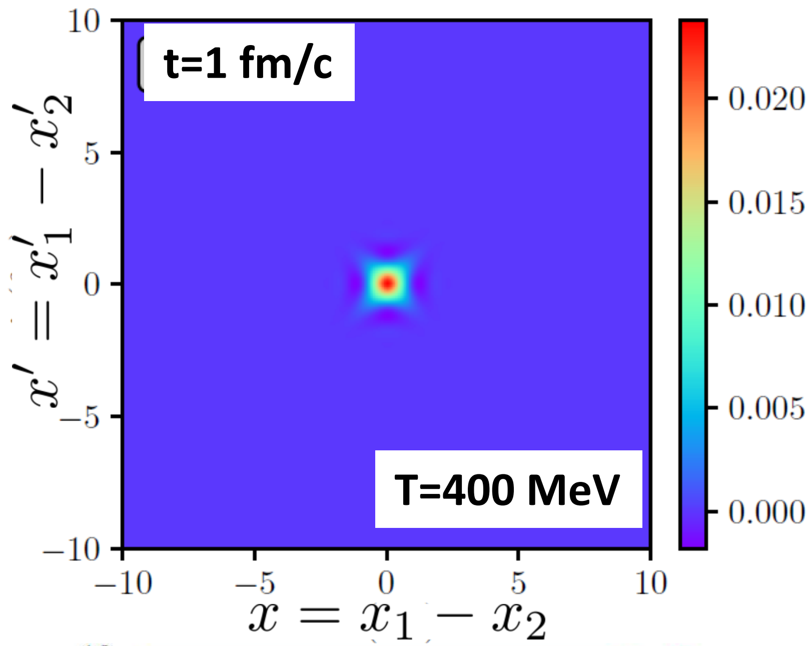
Long wave length (λ >> Δx)

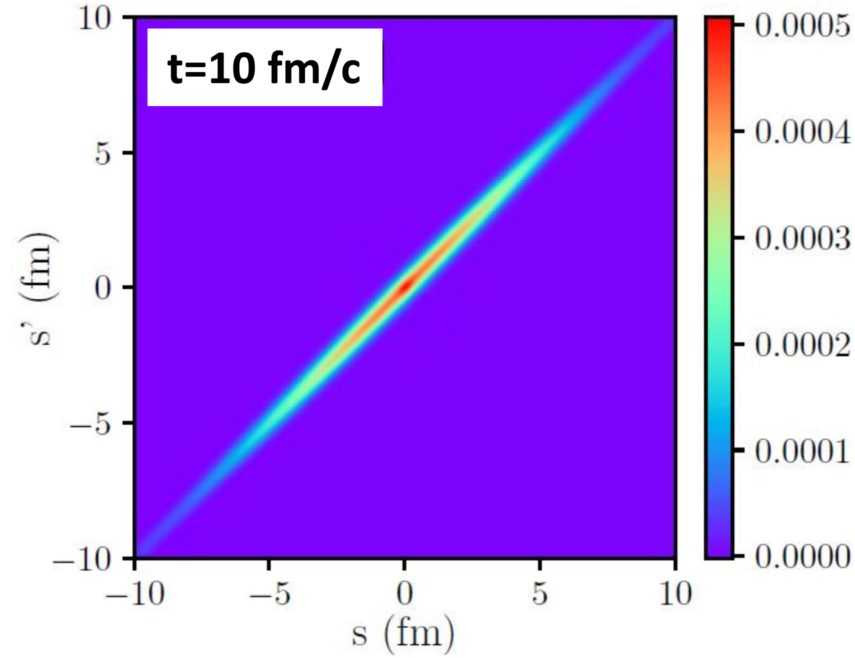
$$F \approx \int dq\rho(q)v(q)q^2\sigma_{\rm transp}(q)\times(\mathbf{x}-\mathbf{x}')^2 \approx \kappa(\mathbf{x}-\mathbf{x}')^2$$

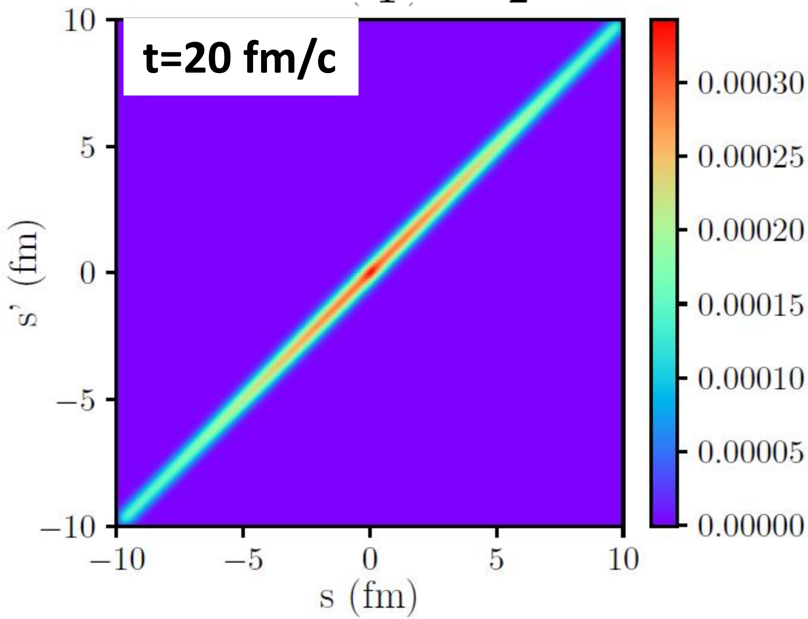
Suppresses coherence at large x-x' : classicalization

For small objects, coherence can be preserved over long times (several “cycles”)

Decoherence from system-env. interaction







$$\mathcal{D}_S(x,x',t) \sim \mathcal{D}_S(x,x',0)e^{-\Lambda(x-x')^2 t}$$

- Compactification along the short diagonal = « classicalization »
- $t_{\rm dec} \sim \frac{1}{\kappa(\Delta x)^2} \sim \frac{1}{TM\eta_D(\Delta x)^2}$

Einstein relation Single part. relaxation rate

$$t_{\rm dec} \sim \frac{\tau_R^{\rm single}}{\frac{1}{\lambda_{th}^2}(\Delta x)^2} \sim \tau_R^{\rm single} \times \left(\frac{\lambda_{\rm th}}{\Delta x}\right)^2$$

Blaizot-Escobedo Quantum Master Equation

Series expansion in τ_E/τ_S

Compact form: $\frac{d\mathcal{D}_Q}{dt} = \mathcal{L}\mathcal{D}_Q$ with $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \cdots$

$$\mathcal{L}_0\,\mathcal{D}_Q \equiv -i[H_Q, \mathcal{D}_Q],$$

$$\mathcal{L}_1\,\mathcal{D}_Q \equiv -\frac{i}{2}\int_{\boldsymbol{x}\boldsymbol{x}'} V(\boldsymbol{x}-\boldsymbol{x}')[n^a_{\boldsymbol{x}} n^a_{\boldsymbol{x}'}, \mathcal{D}_Q],$$

Mean field hamiltonian

$$\mathcal{L}_2\,\mathcal{D}_Q \equiv \frac{1}{2}\int_{\boldsymbol{x}\boldsymbol{x}'} W(\boldsymbol{x}-\boldsymbol{x}')\left(\{n^a_{\boldsymbol{x}} n^a_{\boldsymbol{x}'}, \mathcal{D}_Q\} - 2 n^a_{\boldsymbol{x}} \mathcal{D}_Q n^a_{\boldsymbol{x}'}\right),$$

Im. Potential… contains all transition physics

Fluctuations => decohence, Linblad form

$$\mathcal{L}_3\,\mathcal{D}_Q \equiv \frac{i}{4T}\int_{\boldsymbol{x}\boldsymbol{x}'} W(\boldsymbol{x}-\boldsymbol{x}')\left([n^a_{\boldsymbol{x}}, \dot{n}^a_{\boldsymbol{x}'}\mathcal{D}_Q] + [n^a_{\boldsymbol{x}}, \mathcal{D}_Q \dot{n}^a_{\boldsymbol{x}'}]\right)$$

Dissipation

N.B. : Dissipation is NOT of the Linbladian form => the evolution breaks positivity.

Positivity and Linblad form can be restored at the price of extra subleading terms* :

$$\left\{\left(n^a_{\mathbf{x}} - \frac{i}{4T}\dot{n}^a_{\mathbf{x}}\right)\left(n^a_{\mathbf{x}'} + \frac{i}{4T}\dot{n}^a_{\mathbf{x}'}\right), \mathcal{D}_{Q\bar{Q}}\right\} - 2\left(n^a_{\mathbf{x}} + \frac{i}{4T}\dot{n}^a_{\mathbf{x}}\right)\mathcal{D}_{Q\bar{Q}}\left(n^a_{\mathbf{x}'} - \frac{i}{4T}\dot{n}^a_{\mathbf{x}'}\right)$$

$\mathcal{L}_2$

Subatech

* As well as another time discretization

B-E Quantum Master Equation: QCD case

singlet density matrix

$$\frac{d}{dt}\begin{pmatrix}\mathcal{D}_s\\ \mathcal{D}_o\end{pmatrix} = \mathcal{L}\begin{pmatrix}\mathcal{D}_s(\mathbf{r}_{rel},\mathbf{r}'_{rel},t)\\ \mathcal{D}_o(\mathbf{r}_{rel},\mathbf{r}'_{rel},t)\end{pmatrix}$$

octet density matrix

$$\mathcal{L} = \begin{pmatrix}\mathcal{L}_{ss} & \mathcal{L}_{so}\\ \mathcal{L}_{os} & \mathcal{L}_{oo}\end{pmatrix}$$

singlet-octet transitions

2 coupled color representations (singlet octet)

Alternate choice : $\begin{pmatrix}\mathcal{D}_0\\ \mathcal{D}_8\end{pmatrix}$ Off color-equilibrium component

With (infinite mass limit)

$$\mathcal{D}_8(r,t) \sim \mathcal{D}_8(r,0)e^{-N_c\Gamma(r)t} \to 0$$

Color equilibration

Still semi-classical approximation (power series in *y*).

Example of the $\mathcal{D}_s$ evolution (after SC expansion)

$$\begin{aligned}
(D_s|\mathcal{L}|\mathcal{D}) = &\left(2i\frac{\nabla_r\cdot\nabla_y}{M} + i\frac{\nabla_R\cdot\nabla_Y}{2M} + iC_F\boldsymbol{y}\cdot\boldsymbol{\nabla}V(\boldsymbol{r})\right)D_s\\
&-2C_F\Gamma(\boldsymbol{r})(D_s - D_o)\\
&-\frac{C_F}{4}\left(\boldsymbol{y}\cdot\mathcal{H}(\boldsymbol{r})\cdot\boldsymbol{y}\,D_s + \boldsymbol{y}\cdot\mathcal{H}(0)\cdot\boldsymbol{y}\,D_o\right)\\
&-C_F\boldsymbol{Y}\cdot[\mathcal{H}(0)-\mathcal{H}(\boldsymbol{r})]\cdot\boldsymbol{Y}\,D_o\\
&+\frac{C_F}{2MT}\left[\nabla^2W(0) - \nabla^2W(\boldsymbol{r}) - \boldsymbol{\nabla}W(\boldsymbol{r})\cdot\boldsymbol{\nabla}_r\right](D_s - D_o)\\
&-\frac{C_F}{2MT}\left(\boldsymbol{y}\cdot\mathcal{H}(\boldsymbol{r})\cdot\boldsymbol{\nabla}_y D_s + \boldsymbol{y}\cdot\mathcal{H}(0)\cdot\boldsymbol{\nabla}_y D_o\right)\\
&-\frac{C_F}{2MT}\boldsymbol{Y}\cdot[\mathcal{H}(0)-\mathcal{H}(\boldsymbol{r})]\cdot\boldsymbol{\nabla}_Y D_o.
\end{aligned}$$

Our current projects (St. Delorme, A. Daddi, Th. Gousset, JP Blaizot)

<u>Our Goal:</u>

- Gain insight on the quarkonium dynamics inside the QGP by **solving exactly the B-E equations** for a single $c\bar{c}$ pair without performing the Semi-Classical approximation:
  - Evolution of the density matrix
  - Evolution of states probabilities over time
  - Singlet-octet transitions
  - Color relaxation time
  - …
- Comparison with the semi-classical approach for a various range of QGP temperatures (should be fine at large temperature… but down to ?)

- Possibly design improved algorithm for intermediate temperatures

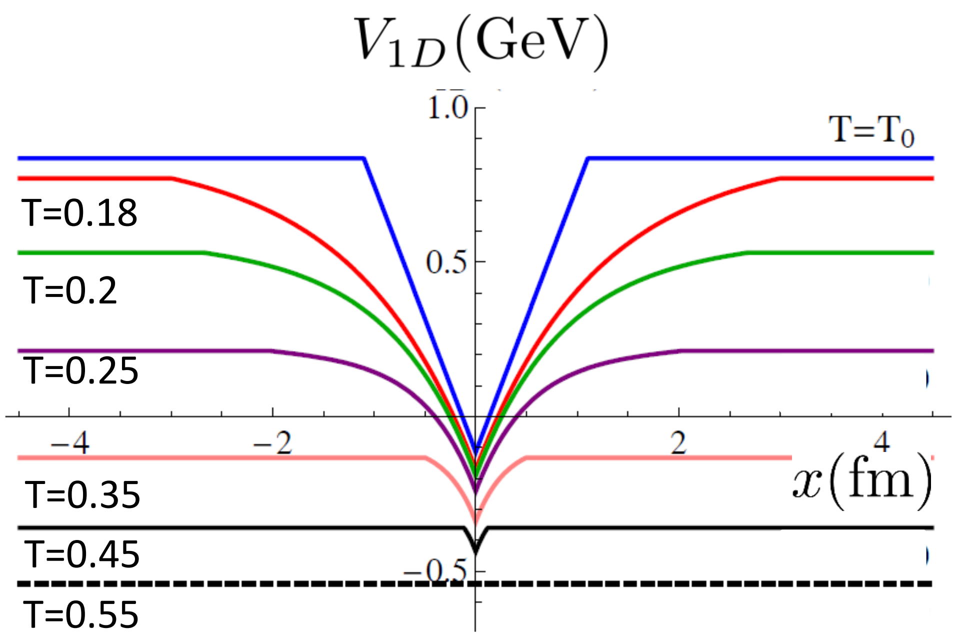
Further implementation features

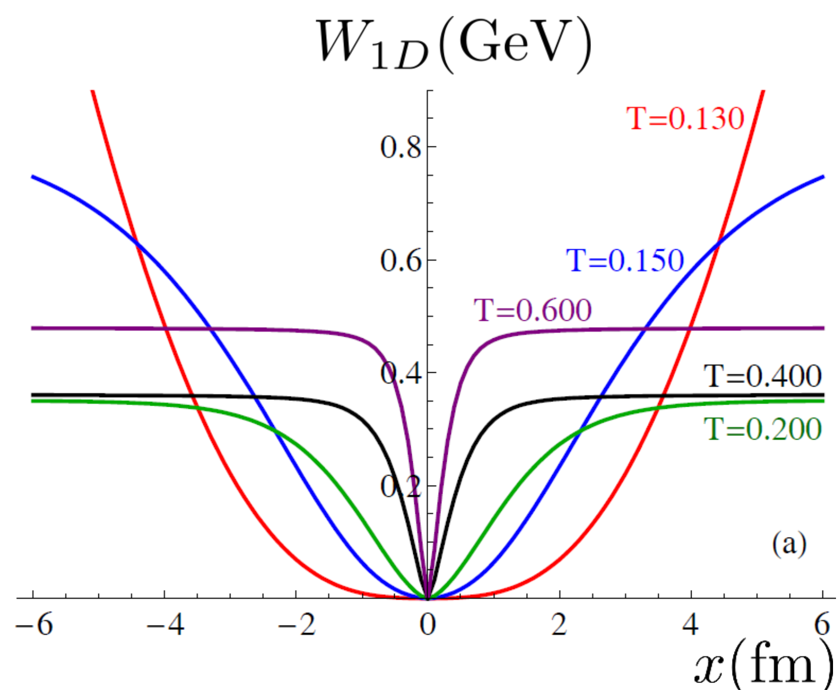
- 1D grid for both $s \in [-s_{\max}, +s_{\max}]$ and $s' \in [-s_{\max}, +s_{\max}]$

!!! Not the radial decomposition of $\mathcal{D}_{c\bar{c}}(\vec{s}, \vec{s}\,')$ which is more cumbersome

Even states will be considered as « S like » while odd states will be considered as « P like » states

Need to design a realistic 1D bona fide potential V + i W (based on 3D lQCD results, tuned to reproduce 3D mass spectra and decay widths)



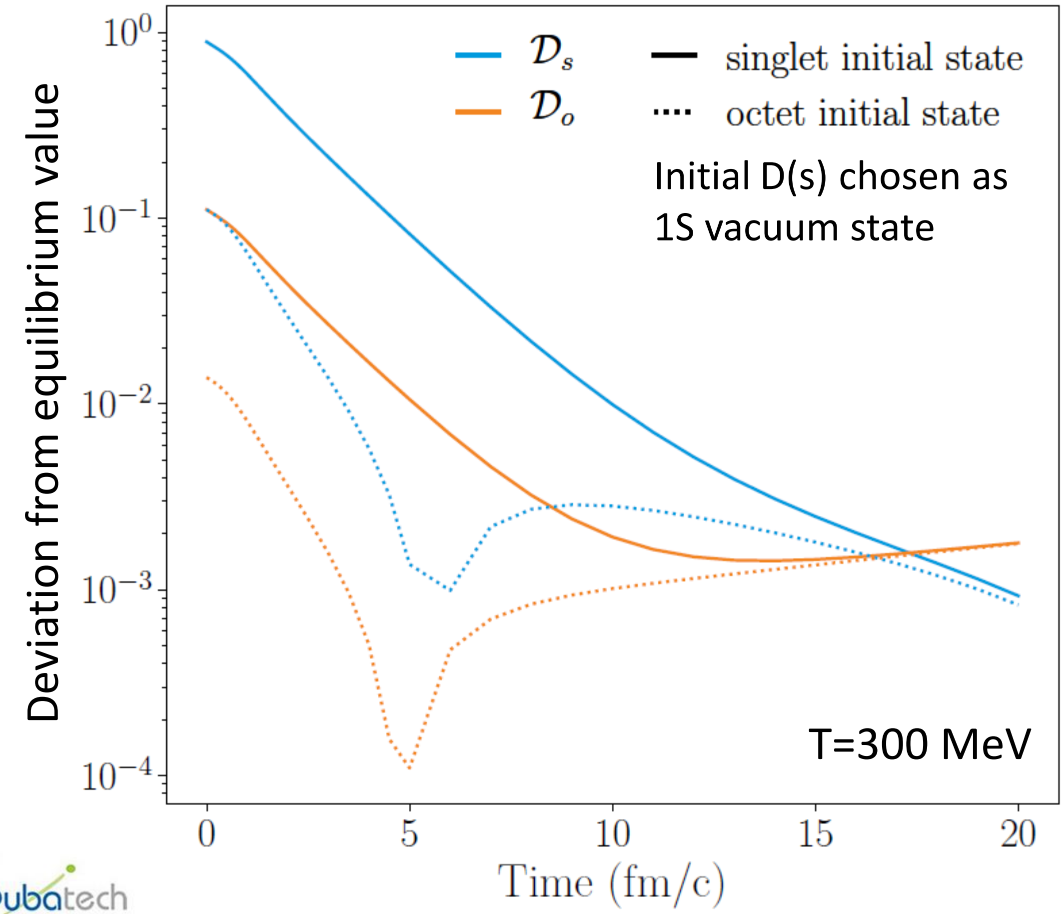


R. Katz et al, Eur. Phys. J. A (2022) 58:198

Results for the c-cbar system

Color Dynamics : Singlet – octet probabilities:

- Starting from singlet (———) or octets (- - - - -) states, one expects some equilibration / thermalisation -> asymptotic values : $D_s^{\rm eq} = D_o^{\rm eq} = \frac{1}{9} \quad (1+8) \times \frac{1}{9}$
- Study the deviations $|D_s - D_s^{\rm eq}|$ and $|D_o - D_o^{\rm eq}|$

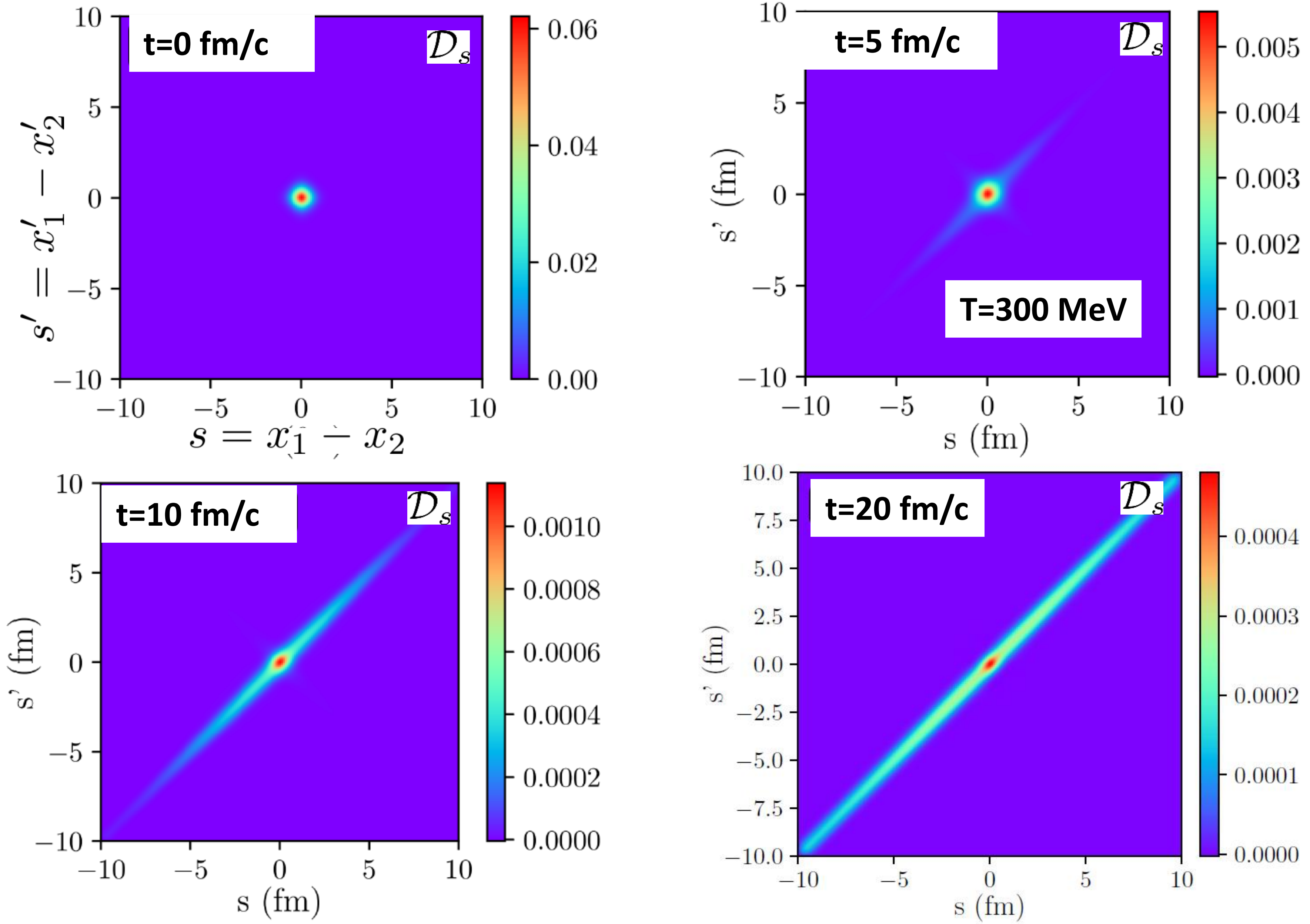


- At early times : Quasi exponential behaviour exp(-t/τ), with thermalisation time $\tau_o < \tau_s \approx 2$ fm/c
- At later time : Saturation possibly due to the grid size.
- Color appears to thermalize on time scales < QGP life time, but not instantaneoulsy.

Subatech

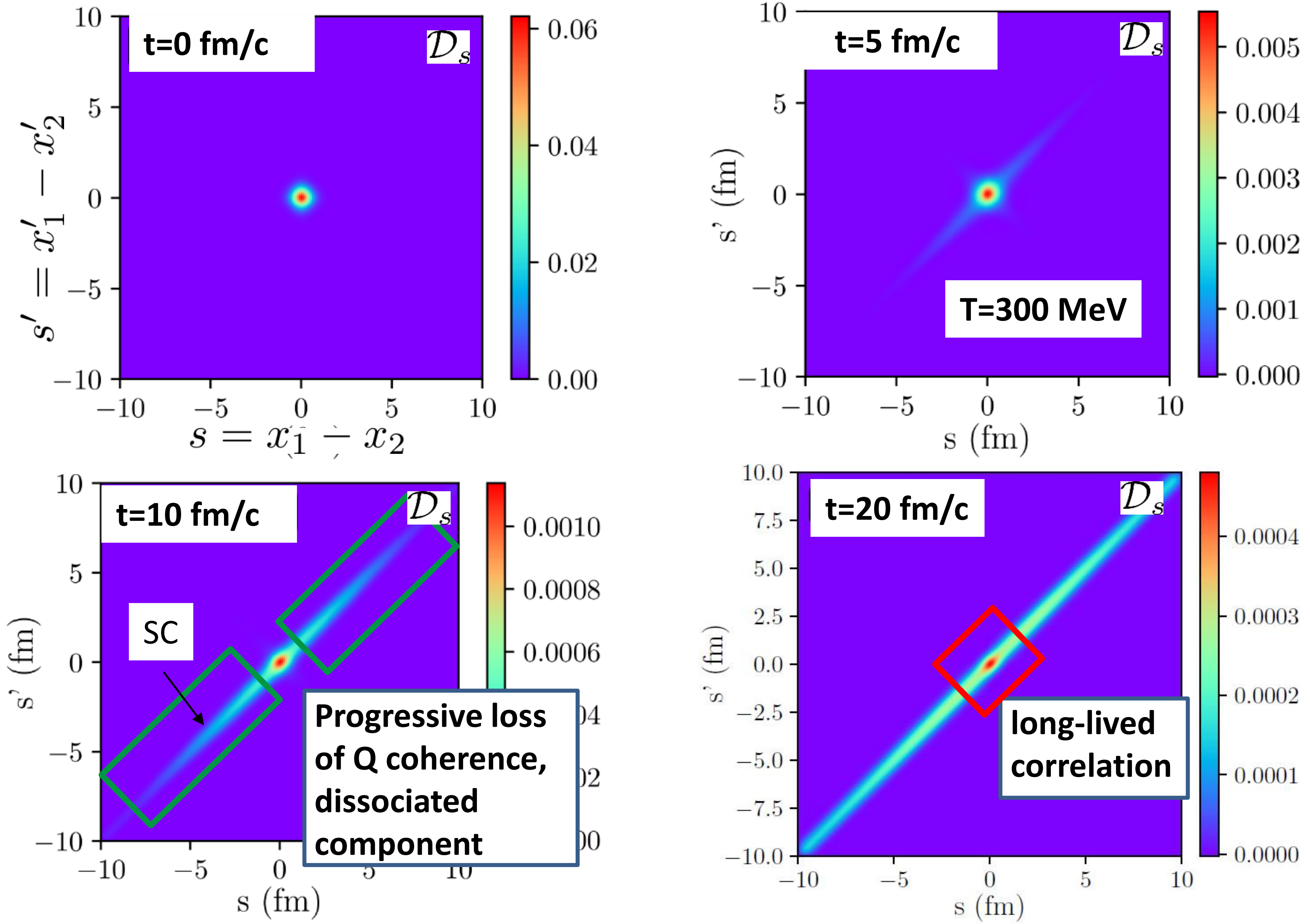
Evolution of the Density matrix

1S singlet initial state:



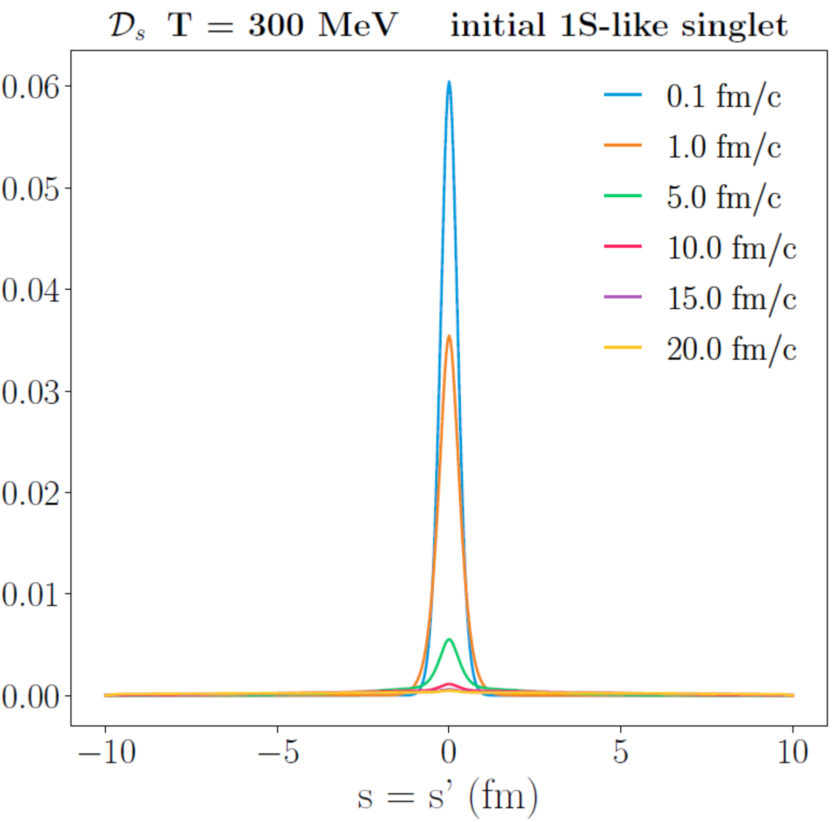
Evolution of the Density matrix

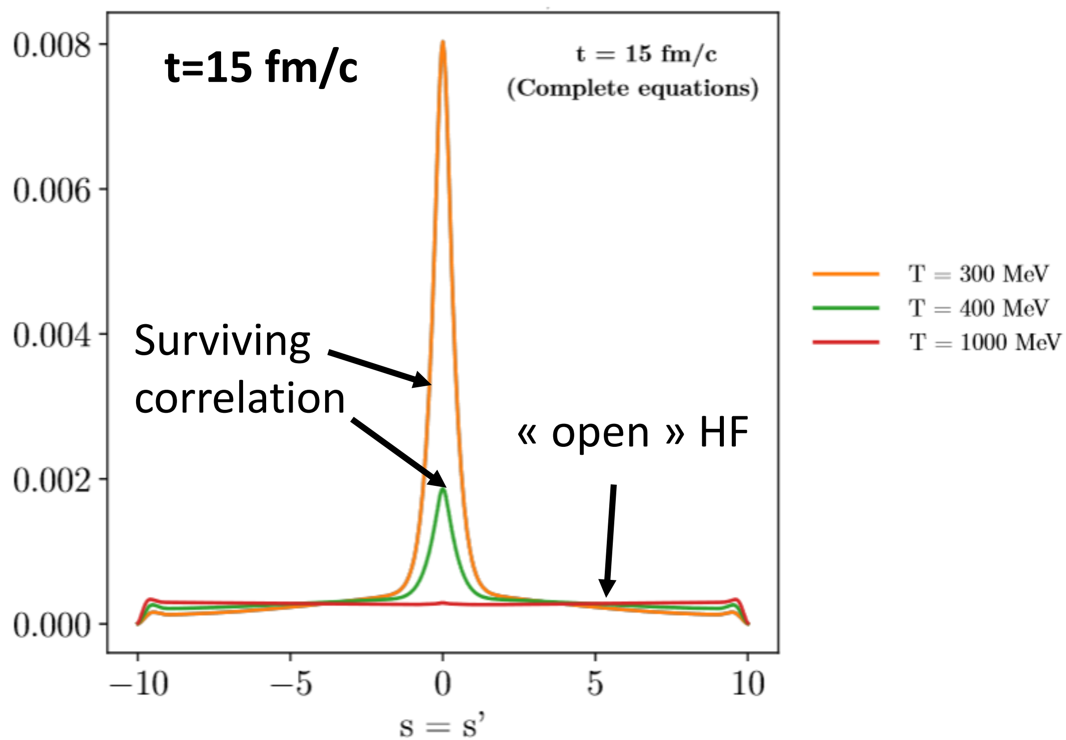
1S singlet initial state:



Evolution of the Density

$$\rho_s(s) = D_s(s, s' = s)$$



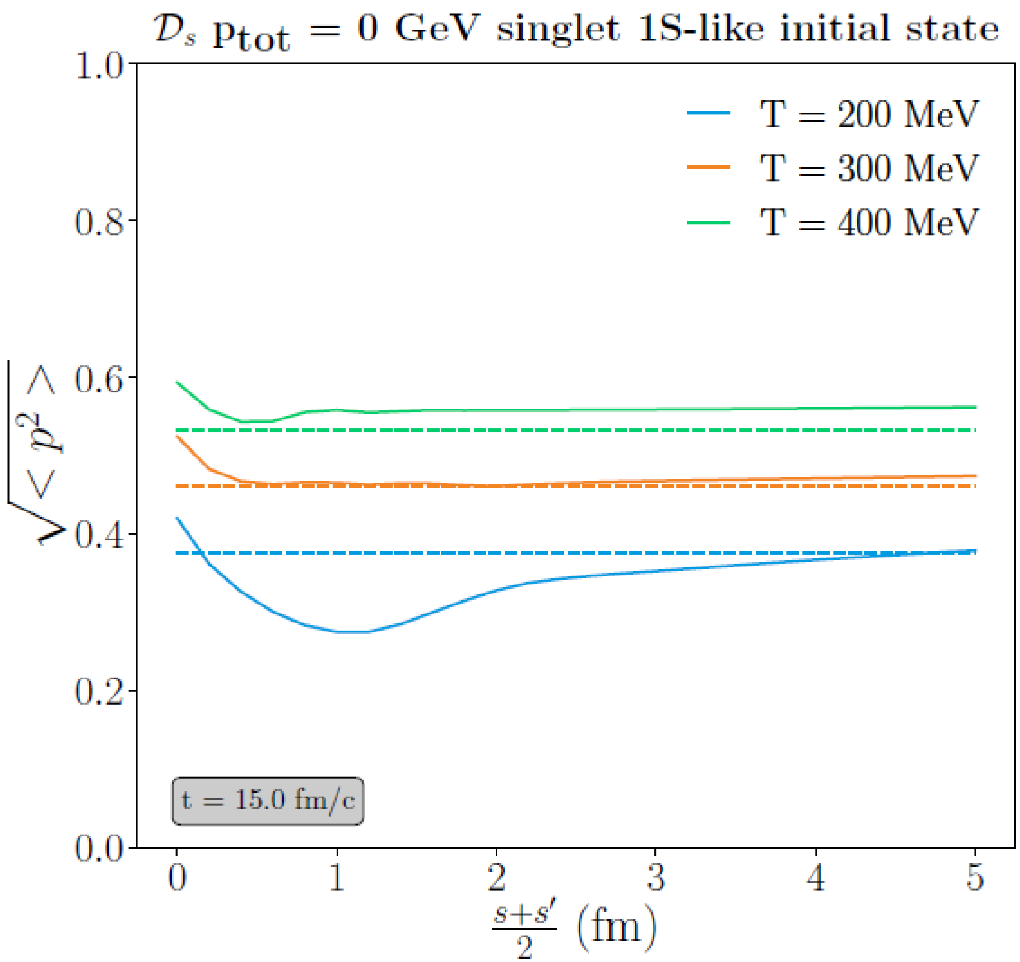


At a given T, increasing delocalisation with time

At a given time t, increasing delocalisation with T

Some c-cbar stay at intermediate distance ("recombination")

Asymptotic Behavior



Reaches values close to the statistical equipartition limit :

$$\langle p^2 \rangle \approx \frac{m_c T}{2}$$

Sanity test of the full approach

At long time, quantum coherence is localized on a scale

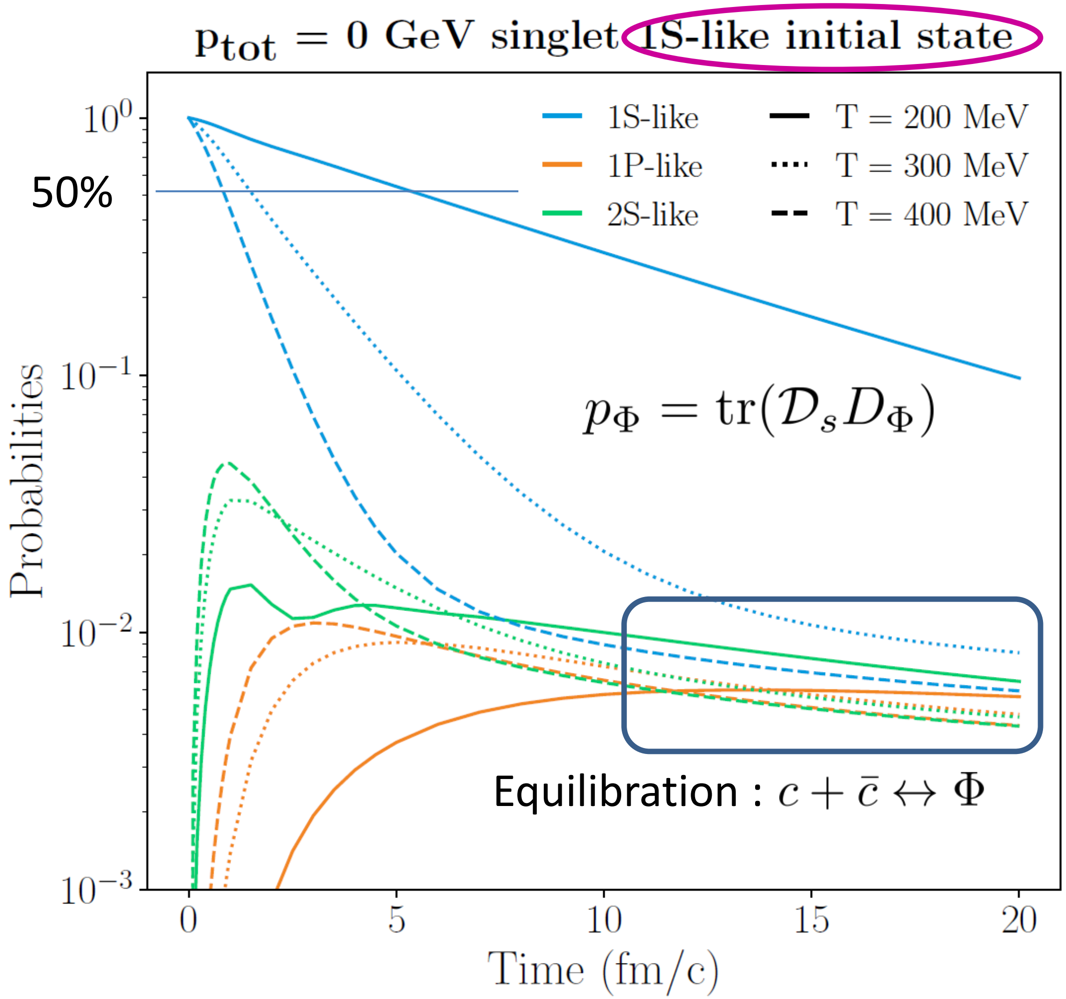
$$\lambda_{\mathrm{th}} \sim \frac{1}{\sqrt{m_Q T}}$$

Small wrt other scales => (semi-classical) limit

$$W(r,p) \propto \exp\left(-\frac{H_{Q\bar{Q}}(r=\frac{s+s'}{2},p)}{T}\right)$$

Results for the projection on vacuum states

For various « realistic » temperatures



Pretty complex interplay between binding, diffusion and transitions between states

After some "quantum regeneration" 2S decays slower than 1S.

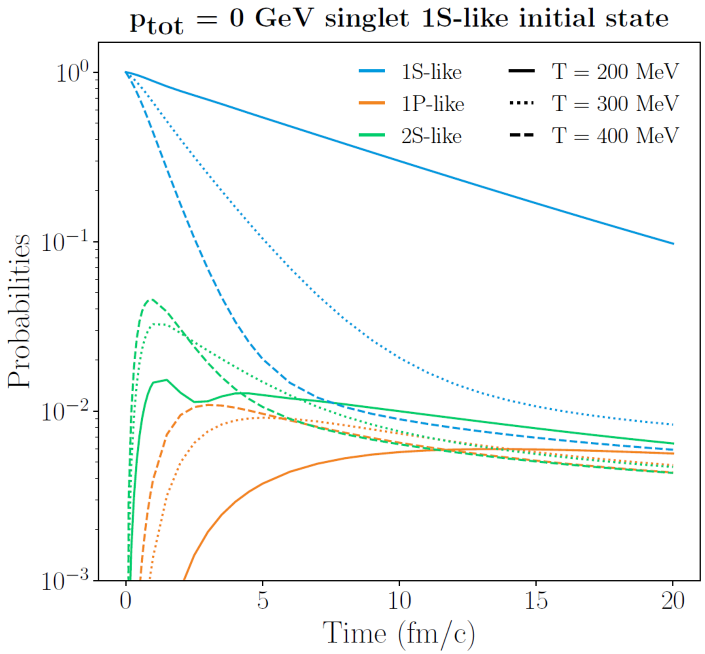
- Faster (and larger) suppression for larger QGP temperature
- Transient phase up to 5 fm/c : re-equilibration
- Common evolution (decrease) of all states at large times for T=300 and 400 MeV

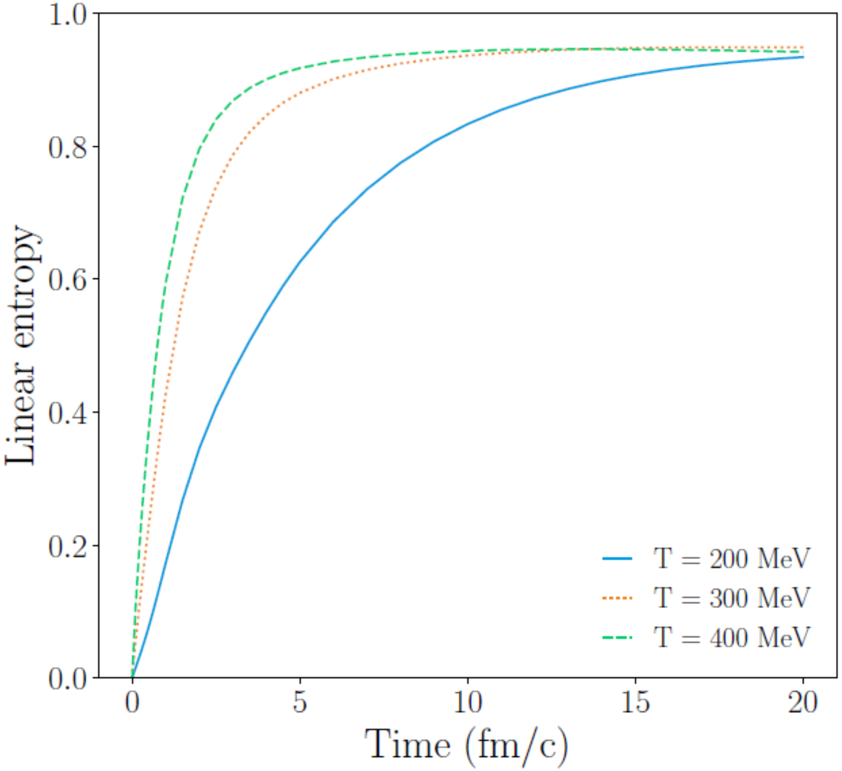
Results for Linear quantum entropy

$$S_L = \mathrm{Tr}\hat{\rho} - \mathrm{Tr}\hat{\rho}^2 = 1 - \mathrm{Tr}\hat{\rho}^2$$

De Boni, J. High Energ. Phys. (2017) 2017: 64

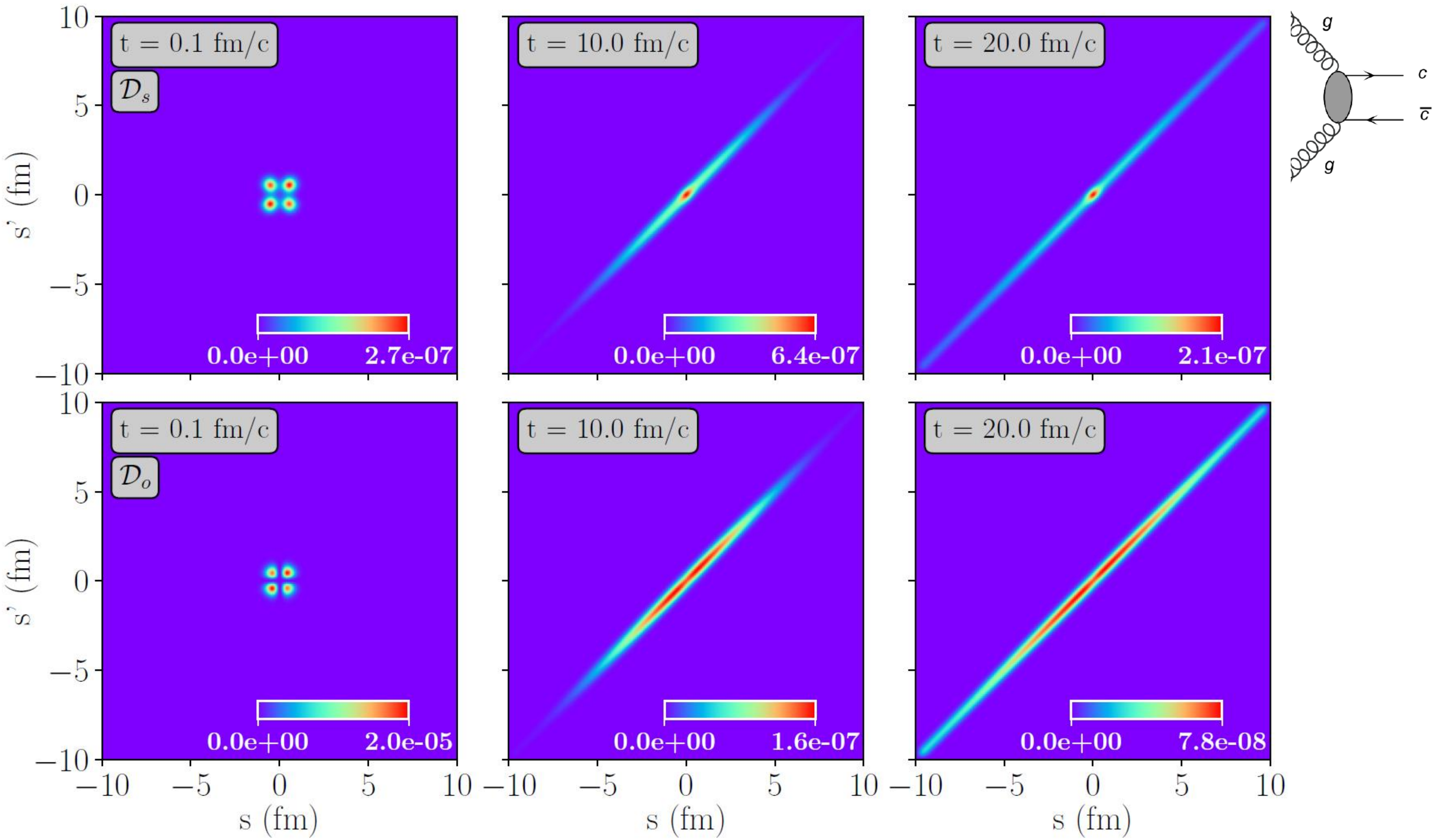
(results for QED like evolution)





- Suppression and decoherence appear to happen on the same time scale…
- … does not seem in favour of applying classical rate equations (to be investigated further)

Results for more realistic octet Initial State

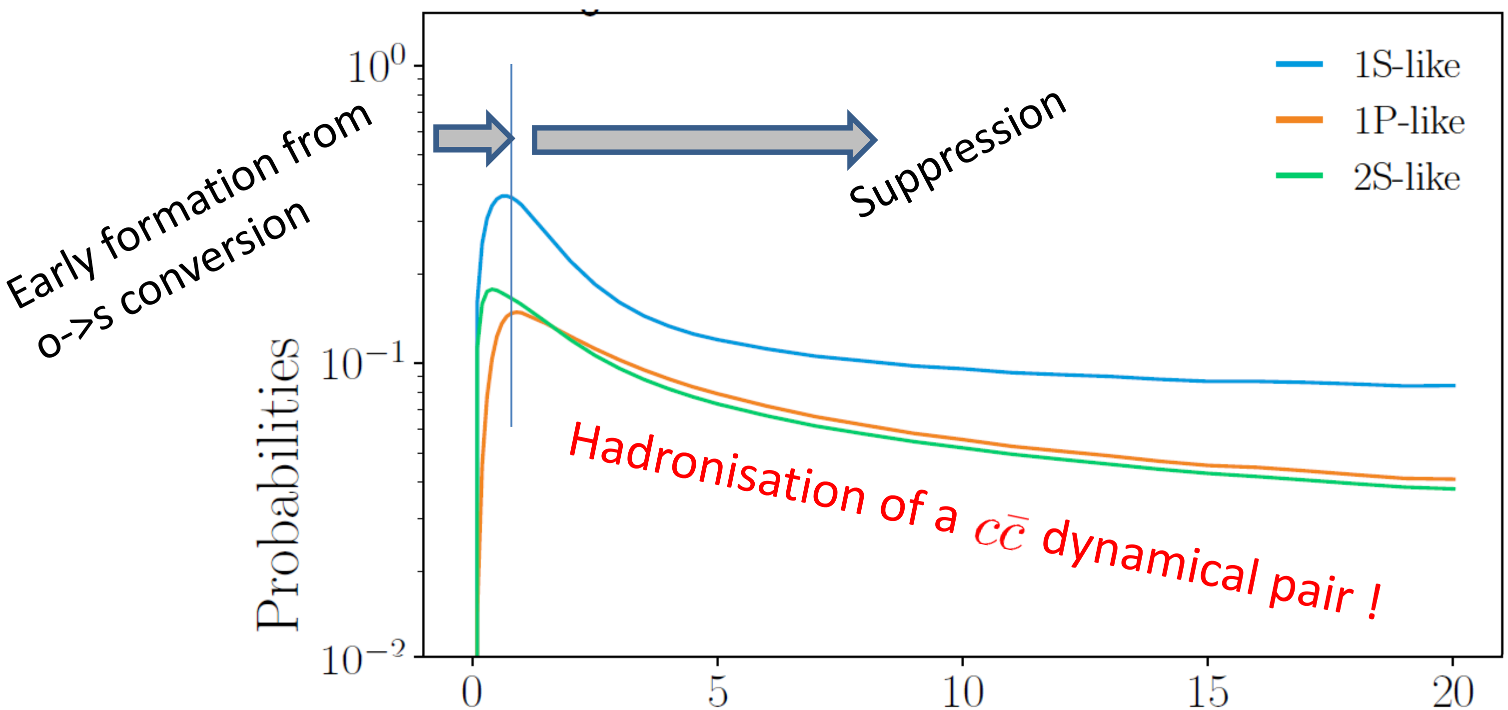


- Initial population of D_s shows a node at x=0 due to dipolar transitions… However, similar asymptotic behavior as for the singlet initial state.
- Delocalization of initial state along s = s′ axis (especially in the octet channel)

Results for more realistic octet Initial State

Now with cooling medium:

- Bjorken-like evolution of the temperature : $T(t) = T_0 \times \left(\frac{\tau_0}{t+\tau_0}\right)^{\frac{1}{3}}$ $T_0 = 600\,\mathrm{MeV}$ $\tau_0 = 1\,\mathrm{fm}$



- Bound state formation at extremely **early times**… (rather opposite to the statistical hadronization picture… however not “exogeneous” pair => to be taken with a grain of salt)
- Even moderate *repopulation* of the ground state at late times… can be understood as the cooling of the level distribution.

B-E Quantum Master Equation: SC approximation

- For the relative motion (2 body):

$$\left.\begin{array}{l}\vec{s} = \vec{x}_1 - \vec{x}_2 \\ \vec{s}\,' = \vec{x}_1' - \vec{x}_2'\end{array}\right\} \qquad \vec{r} = \frac{\vec{s}+\vec{s}\,'}{2} \quad \text{and} \quad \vec{y} = \vec{s} - \vec{s}\,'$$

- Near thermal equilibrium, Density operator is nearly diagonal => **semi-classical expansion** (power series in *y* up to 2nd order)

... However, we know from open heavy flavor analysis that it takes some finite relaxation time to reach this state

$$\frac{d}{dt}\mathcal{D}(r,y) = \mathcal{L}\mathcal{D}(r,y)$$

$$\begin{cases} \mathcal{L}_0 = \frac{2i\nabla_y\cdot\nabla_r}{M} \\ \mathcal{L}_1 = i\vec{y}\cdot\nabla V(r) \\ \mathcal{L}_2 = -\frac{1}{4}\vec{y}\cdot(\mathcal{H}(\vec{r})+\mathcal{H}(0))\cdot\vec{y} \\ \mathcal{L}_3 = -\frac{1}{2MT}\vec{y}\cdot(\mathcal{H}(\vec{r})+\mathcal{H}(0))\cdot\nabla_{\vec{y}} \end{cases}$$

$\mathcal{H}(\vec{r})$: Hessian matrix of im. pot. W

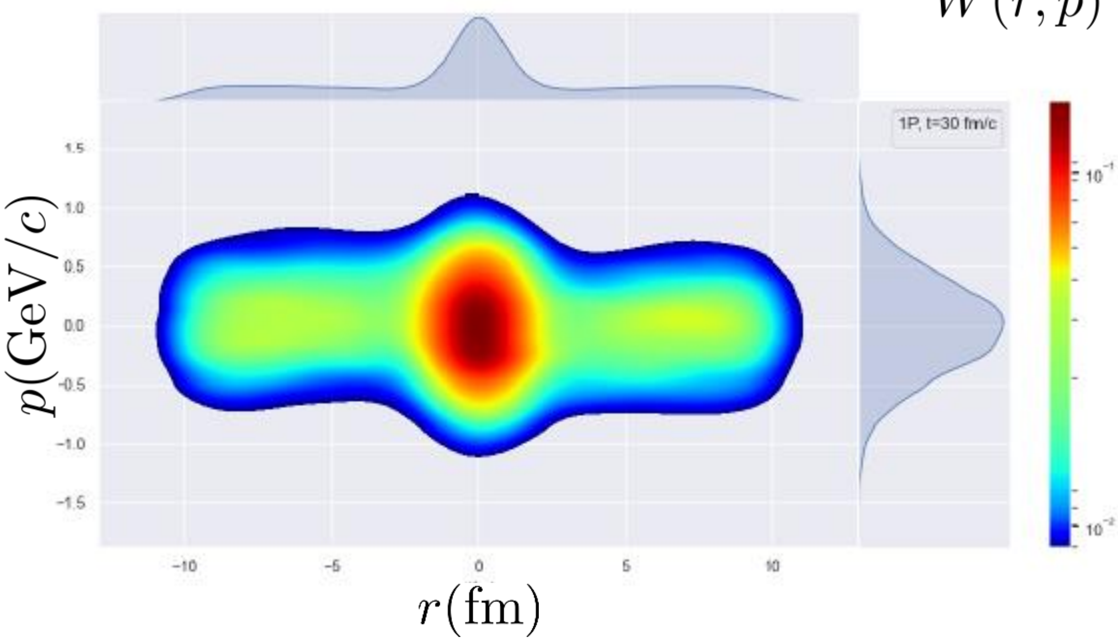
$$W(\vec{y}) = W(\vec{0}) + \frac{1}{2}\vec{y}\cdot\mathcal{H}(0)\cdot\vec{y}$$

- Wigner transform -> $\mathcal{D}(\vec{r},\vec{p})$ => $\{\vec{y},\nabla_y\} \rightarrow \{\nabla_p,\vec{p}\}$ Usual Fokker Planck eq.
- Easy MC implementation + generalization for N body system (c-cbar @ LHC)

Recent results from semi-classical approximation (A. Daddi's PhD)

- Main goal : understand the validity regime of the SC approximation by comparing it to the exact quantum solution and then extend it to the case of several ccbar pairs

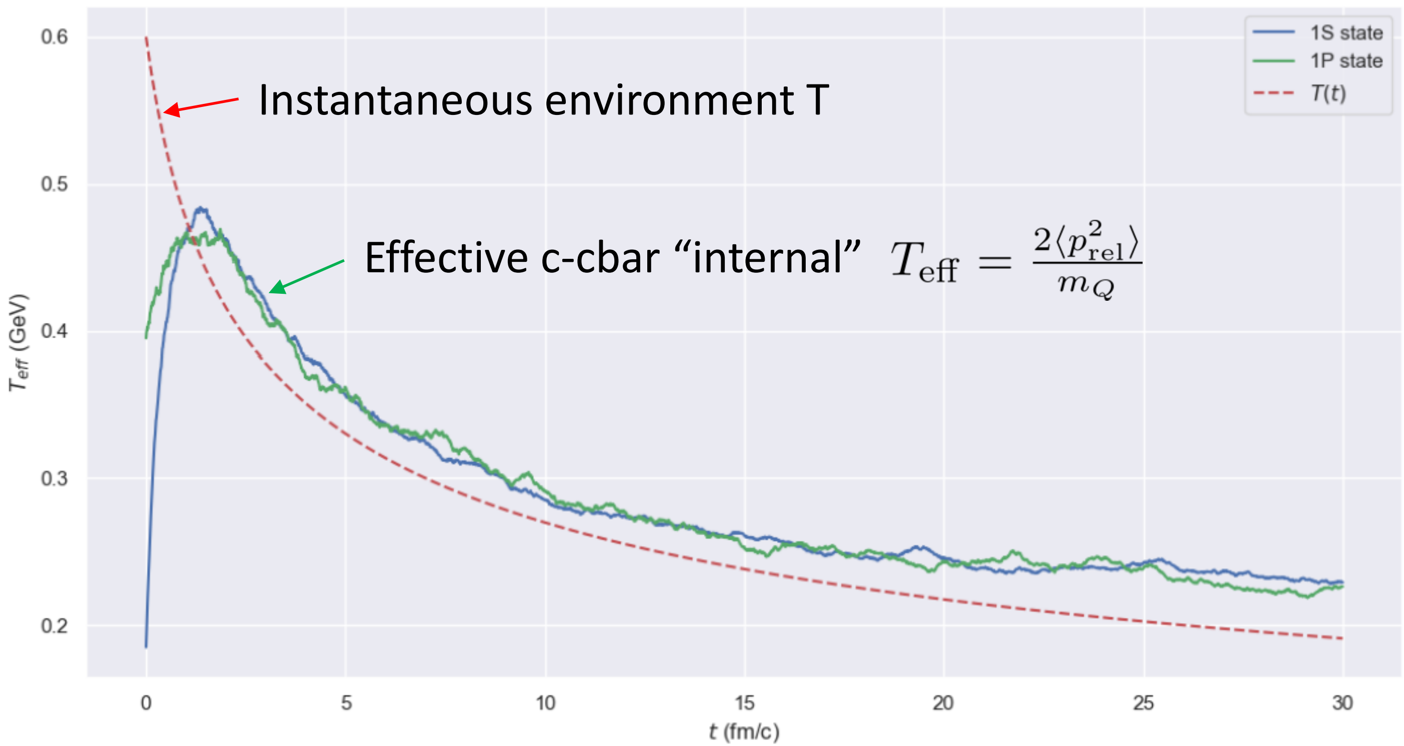
$$W(r,p) \propto \exp(-\frac{H_{Q\bar{Q}}(r,p)}{T})$$



Wigner distribution at the end of the Bjorken evolution (QED-like) : strong correlation at small distance similar to exact calculations

Recent results from semi-classical approximation (A. Daddi's PhD)

- By-product : looking with new eyes



- Delay in the "cooling" of the c-cbar system… freezes at the end of the evolution (similar feature observed in TAMU)
- If quarkonia production seen as the "percolation" from a c-cbar pair at FO, the effective temperature it > T_{FO}. Tension in the SHM for ψ' vs ψ production in ALICE

Conclusions

- Recent progress in all 3 classes of models, especially in implementing open quantum system approaches in the case of a **single pair**… either in the quantum Brownian regime or in the quantum optical regime (Yao, Mehen and Muller)

- => new predictions by various groups (Duke, TUM+Kent State) for bottomonia, in good agreement with experimental data… but still some aspects are *terra incognita* like f.i. the initial state

- Need to Investigation of the validity of the semi-classical approximation if one wants to reliably apply these methods to charmonia production (see however new microscopic model : 2206.01308)

- Need as well to better qualify and quantify the relations between OQS and transport theories

- Nantes: Link the QME with EPOS4 (T,v) profiles and prediction for bottomonia R_{AA} is coming

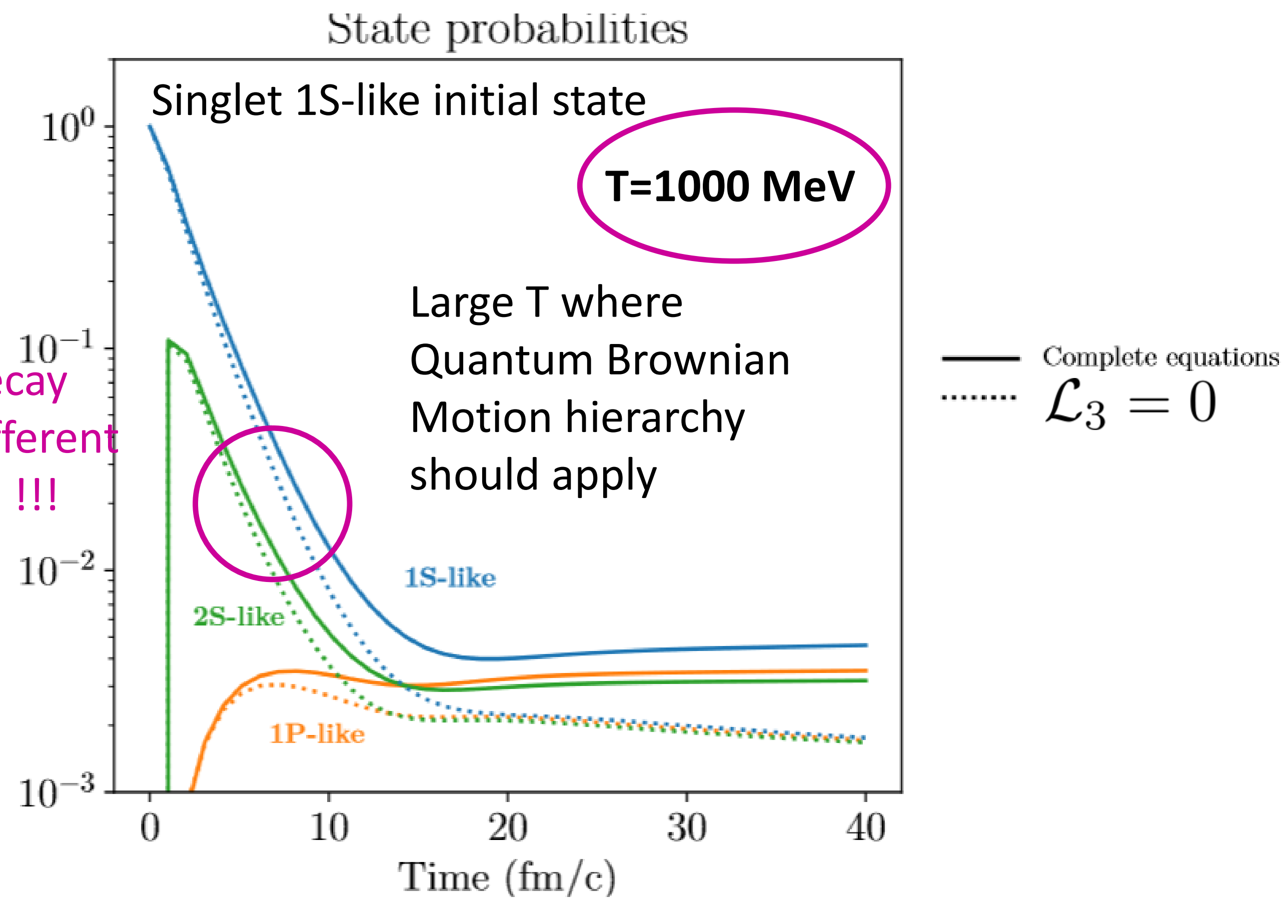
Back up

Results for projection on vacuum states

!!! Vacuum states ≠ eigenstates at local T

Gedanken experiment : instantaneous cooling down -> T=0 after t in QGP

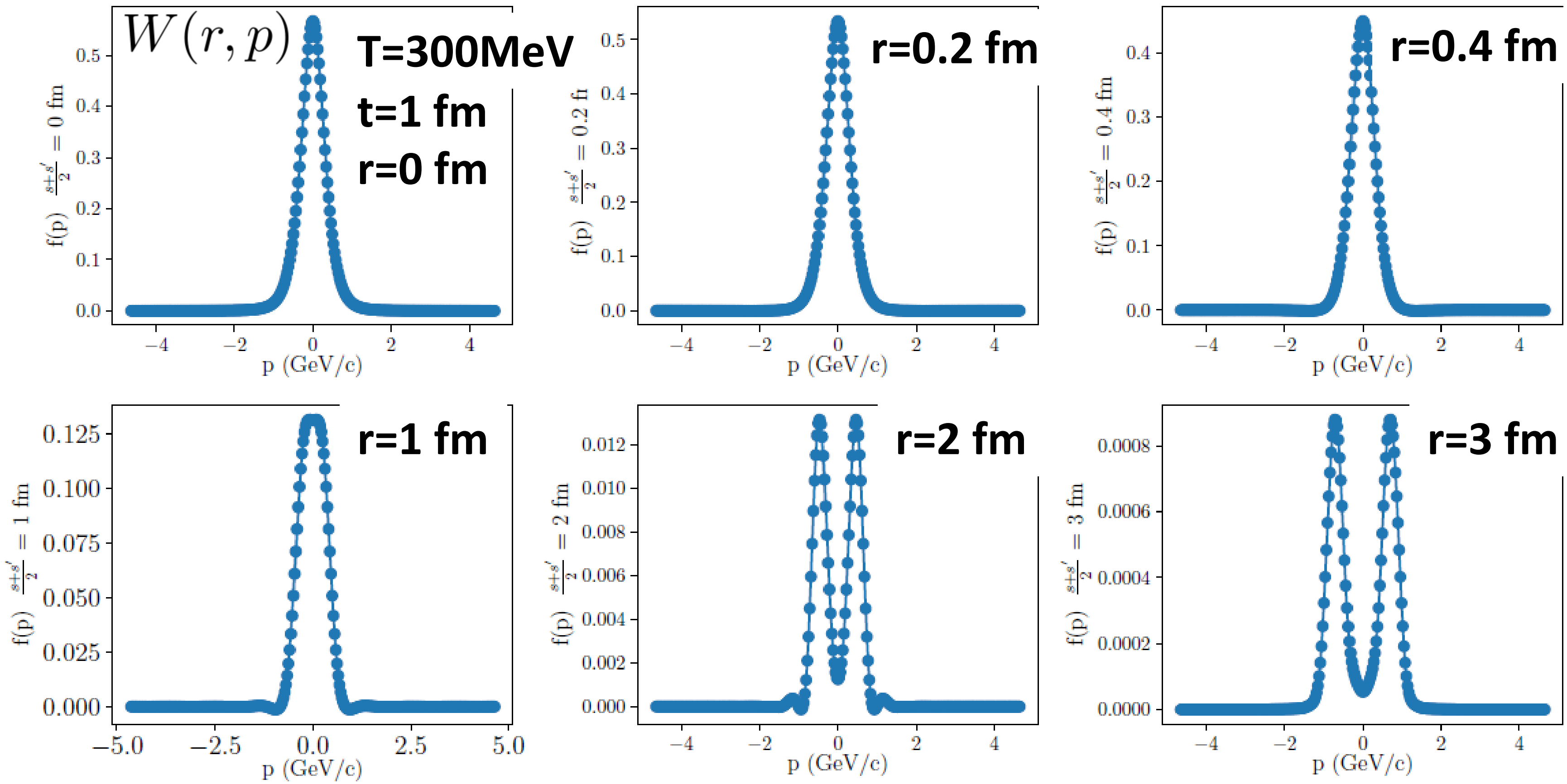
Common decay although different decay width !!!



- At small times, $\mathcal{L}_3 \ll \mathcal{L}_2$ fluctuations dominate… higher state repopulation
- At late times, $\mathcal{L}_3 \sim \mathcal{L}_2$ leading to asymptotic distribution of states. If $\mathcal{L}_3 = 0$, no dissipation => internal energy keeps rising.

Results for Density

Semi-classical analysis: computation of the discretized Wigner transform W(r,p) of D_s for different values of $r = \frac{s+s'}{2}$ (≡ position in a semi-classical approach)

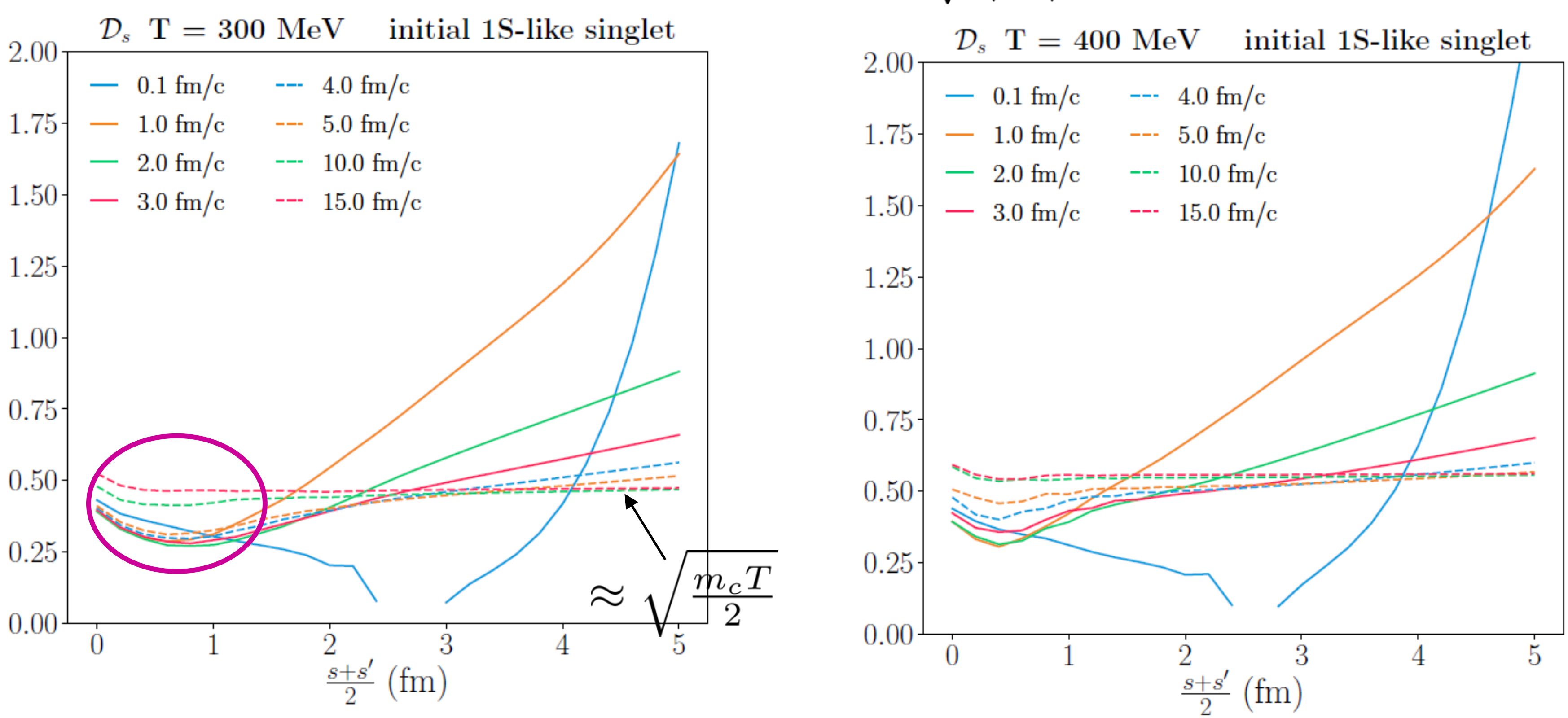


For a wide set of (t,r) : positive defined, Gaussian-like… However, some non Gaussian shapes observed as well

For large r – however supra luminous – some negative shoulders are observed.

Results for Density

Semi-classical analysis: Next compute the r.m.s. p : $\sqrt{\langle p^2 \rangle_W}$



- At asymptotic times : convergence -> thermal value whatever $c\bar{c}$ distance
- At early times : some undefined $\sqrt{\langle p^2 \rangle_W}$ due to the negative shoulders. Genuine quantum effect, however at supra luminous separations
- For intermediate times : survival of the $c\bar{c}$ correlation at small distance, with r.m.s. p < thermal value (cold state need some time to heat up)… How realistic is it described by SC equations ? Under investigation.