

Double & triple quarkonia from N-parton scatterings in p-p, p-A, A-A

Quarkonia As Tools

Aussois, Savoie, 12th Jan. 2023

David d'Enterria



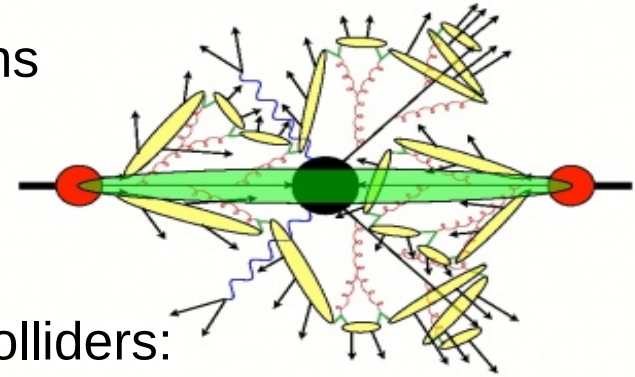
CERN

(*) Details in DPS/TPS/NPS in pp, pA, AA review:

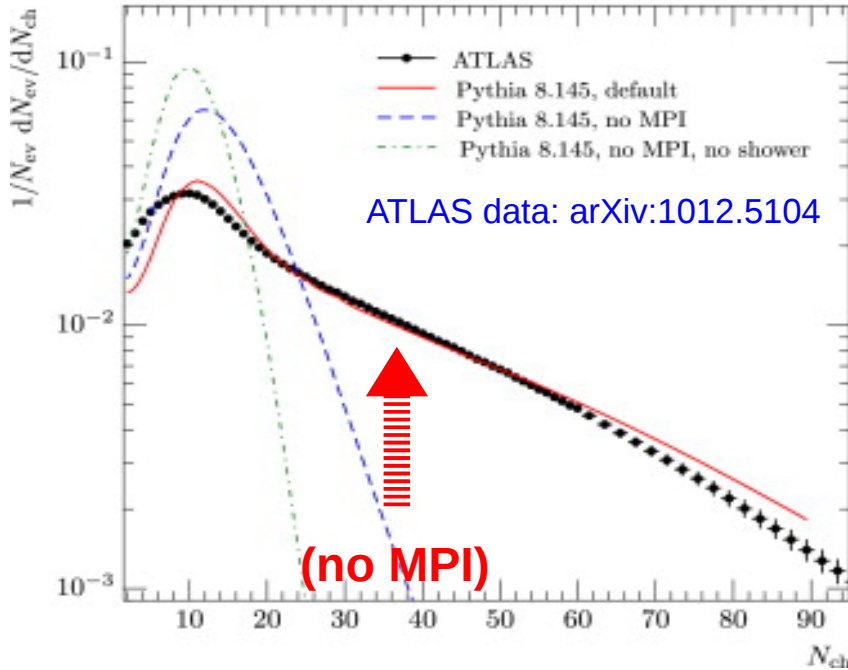
D.d'E & A.Snigirev: arXiv:1708.07519 [Adv.Ser.Direct.High.En.Phys. 29 (2018) 159]

Multi-parton interactions at the LHC

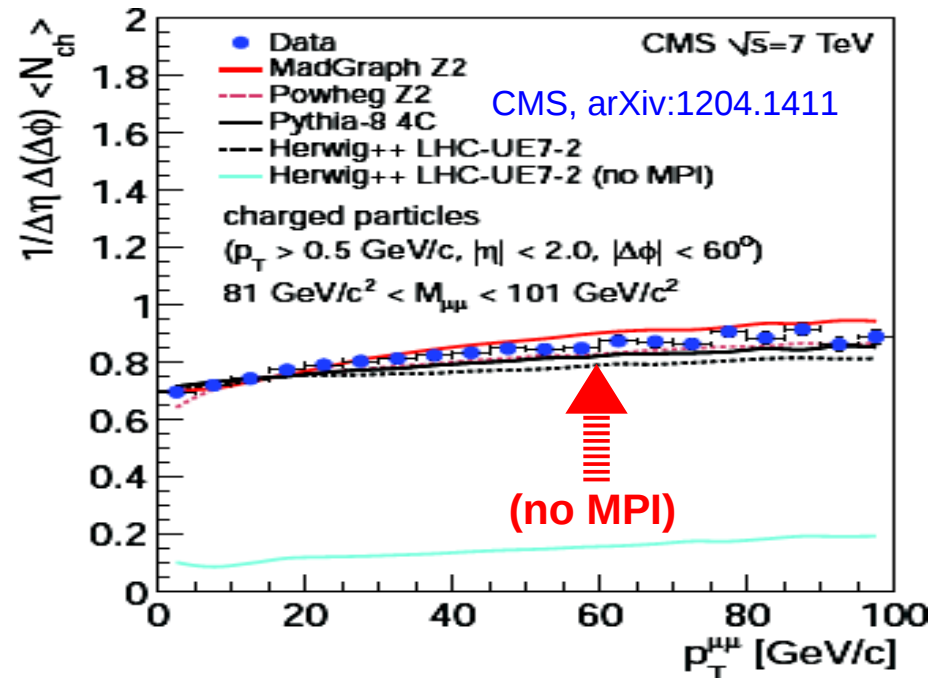
- MPI are intrinsic component of hadron collisions
(p,Pb) = non-pointlike objects with finite transverse size and increasingly larger gluon density with \sqrt{s} .



- MPI $O(1-3 \text{ GeV})$ clearly observed at hadron colliders:
~50% of total hadron production



Underlying event in hard scatterings:

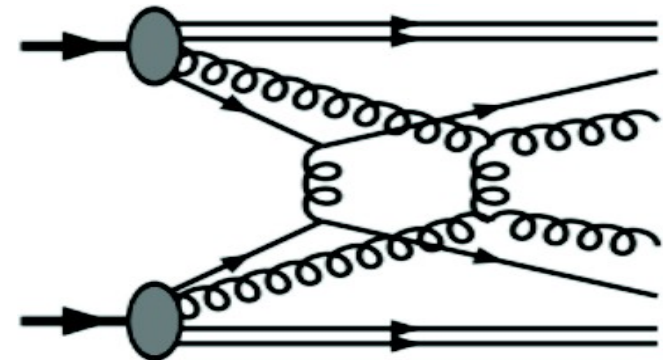


- Double hard parton scatts. ($p_T, m_X > 3 \text{ GeV}$) happen also & been observed

Double Parton Scattering x-sections (p-p)

- Assuming that the probability to produce two hard collisions is independent, one can simply write double parton scatterings (DPS) cross section as the product of two single-parton scatterings (SPS) ones:

$$\sigma_{(hh' \rightarrow ab)}^{\text{DPS}} = \left(\frac{m}{2}\right) \frac{\sigma_{(hh' \rightarrow a)}^{\text{SPS}} \cdot \sigma_{(hh' \rightarrow b)}^{\text{SPS}}}{\sigma_{\text{eff}}}$$



normalized by an effective x-section (σ_{eff}), with a trivial combinatorial factor (m) to avoid double-counting in case of same particles produced.

- How to interpret σ_{eff} ? What values one would naively expect for it?
- Let's start with the most generic expression for DPS cross section:

$$\sigma_{(hh' \rightarrow ab)}^{\text{DPS}} = \left(\frac{m}{2}\right) \sum_{i,j,k,l} \int \Gamma_h^{ij}(x_1, x_2; \mathbf{b}_1, \mathbf{b}_2; Q_1^2, Q_2^2) \times \hat{\sigma}_a^{ik}(x_1, x'_1, Q_1^2) \hat{\sigma}_b^{jl}(x_2, x'_2, Q_2^2) \\ \times \Gamma_{h'}^{kl}(x'_1, x'_2; \mathbf{b}_1 - \mathbf{b}, \mathbf{b}_2 - \mathbf{b}; Q_1^2, Q_2^2) dx_1 dx_2 dx'_1 dx'_2 d^2 b_1 d^2 b_2 d^2 b$$

Generalized PDFs = $f(x, Q^2, \mathbf{b})$

Double Parton Scattering x-sections (p-p)

- Assumption 1: Generalized PDFs factorize into longitudinal & transverse components: transv. density = $f(\mathbf{b})$

$$\Gamma_h^{ij}(x_1, x_2; \mathbf{b}_1, \mathbf{b}_2; Q_1^2, Q_2^2) = D_h^{ij}(x_1, x_2; Q_1^2, Q_2^2) f(\mathbf{b}_1) f(\mathbf{b}_2)$$

p-p transv. overlap function (mb^{-1}): $t(\mathbf{b}) = \int f(\mathbf{b}_1) f(\mathbf{b}_1 - \mathbf{b}) d^2 b_1$

- Assumption 2: The longitudinal double-PDF is the product of 2 single PDF (i.e. no parton correlations in colour, momentum, flavour, spin,...)

$$D_h^{ij}(x_1, x_2; Q_1^2, Q_2^2) = D_h^i(x_1; Q_1^2) D_h^j(x_2; Q_2^2)$$

- $\sigma_{\text{eff}} = \langle \text{Interparton transv. separation} \rangle^2$. Derivable from geometric p-p overlap with naive expected size of $\sigma_{\text{eff}} \approx 30 \text{ mb}$

$$\sigma_{\text{eff}} = \left[\int d^2 b t^2(\mathbf{b}) \right]^{-1}$$

- But experimentally:

$$\sigma_{\text{eff}}(\text{exp}) \approx 15 \text{ mb.}$$

proton “hard” radius:
 $r = 0.3\text{--}0.7 \text{ fm}$ appears smaller than e.m. one:

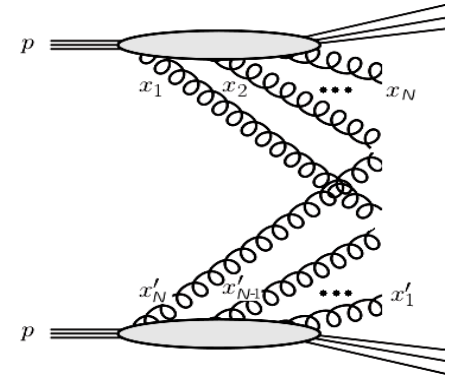
Model for density	Form of density, dN/d^3r	Predictions rms r	σ_{eff}	Measurements Scale (fm)
Solid sphere	Constant, $r < r_p$	$\sqrt{3/5} r_p$	$4\pi r_p^2/4.6$	$r_p = 0.73$
Gaussian	$e^{-r^2/2\Sigma^2}$	$\sqrt{3}\Sigma$	$4\pi\Sigma^2$	$\Sigma = 0.34$
Exponential	$e^{-r/\lambda}$	$\sqrt{12}\lambda$	$35.5\lambda^2$	$\lambda = 0.20$
Fermi, $\lambda/r_0 = 0.2$	$(e^{(r-r_0)/\lambda} + 1)^{-1}$	$1.07r_0$	$4.6r_0^2$	$r_0 = 0.56$

Understandable: Probability of 2nd scatt. is larger if 1st scatter already took place (“centrality bias”).

N-parton scattering x-sections (p-p)

- Assuming that the probabilities for N hard collisions to be independent of each other, one can write a generic pocket-formula for NPS x-section:

$$\sigma_{hh' \rightarrow a_1 \dots a_n}^{\text{NPS}} = \left(\frac{m}{n!} \right) \frac{\sigma_{hh' \rightarrow a_1}^{\text{SPS}} \cdots \sigma_{hh' \rightarrow a_n}^{\text{SPS}}}{\sigma_{\text{eff,NPS}}^{n-1}}$$



normalized by the $N^{\text{th}}-1$ power of an effective x-section ($\sigma_{\text{eff,NPS}}$) plus a trivial combinatorial factor ($m/n!$) to avoid double, triple, N-counting in case of same particles produced:

- DPS: $m = 1$ if $a_1 = a_2$; and $m = 2$ if $a_1 \neq a_2$.
- TPS: $m = 1$ if $a_1 = a_2 = a_3$; $m = 3$ if $a_1 = a_2$, or $a_1 = a_3$, or $a_2 = a_3$; and $m = 6$ if $a_1 \neq a_2 \neq a_3$.

- Ignoring all parton correlations, $\sigma_{\text{eff,NPS}}$ is the inverse $N^{\text{th}}-1$ power of the integral of the N^{th} power of the pp overlap function:

$$\sigma_{\text{eff,NPS}} = \left\{ \int d^2b T^n(\mathbf{b}) \right\}^{-1/(n-1)}$$

- A generic framework for the most economical (geometrical) expressions for N-parton scattering cross sections is available.

Double Parton Scatterings

DPS studies at the LHC

- Motivation for studies of multiple production of hard/heavy particles:
 - (1) **Generalized PDFs** (x, Q^2, b) of the proton, in particular the unknown energy **evolution of transverse proton profile**.
 - (2) Role of **partonic correlations** (in space, p, x , flavour, colour, spin,...) in hadronic wave functions.
 - (3) **Backgrounds** for rare **(B)SM** resonance decays w/ **multiple heavy particles**

- “Pocket formula” results at the LHC:

$$\sigma_{\text{DPS}}^{\text{pp} \rightarrow \psi_1 \psi_2 + X} = \left(\frac{m}{2}\right) \frac{\sigma_{\text{SPS}}^{\text{pp} \rightarrow \psi_1 + X} \sigma_{\text{SPS}}^{\text{pp} \rightarrow \psi_2 + X}}{\sigma_{\text{eff,DPS}}}$$

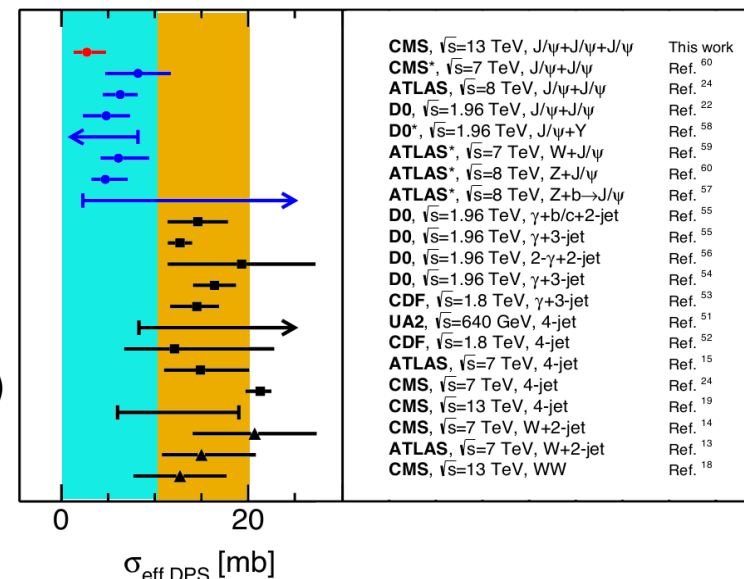
$\sigma_{\text{eff}} \sim \langle \text{Interparton transv. separation} \rangle^2$

derivable from p-p transverse overlap:

$\sigma_{\text{eff}} \sim 20\text{--}30 \text{ mb}$ (PYTHIA8/HERWIG p form-factor)

$\sigma_{\text{eff}} \sim 15 \text{ mb}$ (from DPS of jets, EWK bosons)

$\sigma_{\text{eff}} \sim 5 \text{ mb}$ (from di-quarkonia)

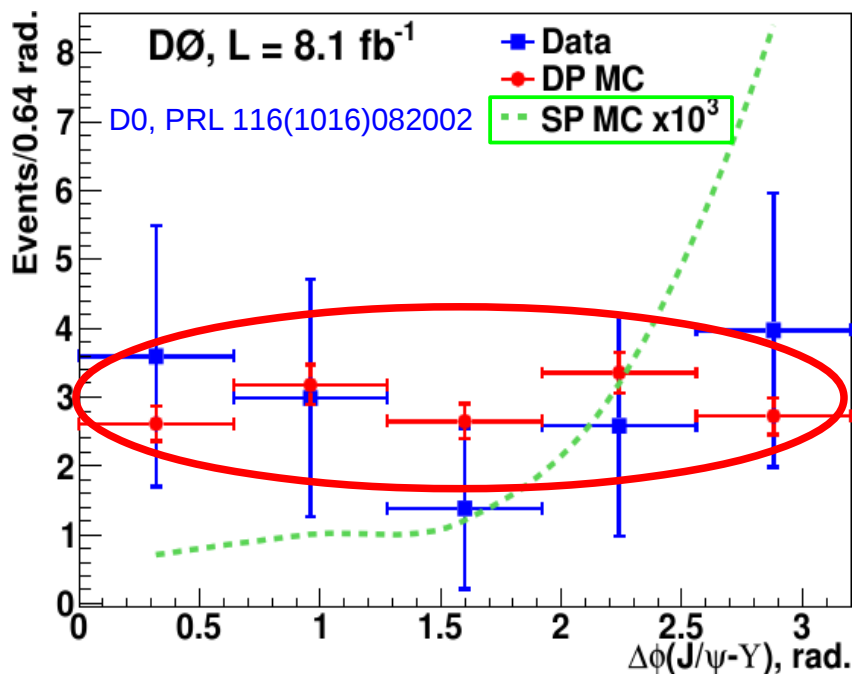


- Reasons: Parton **correlations?** x -,flavour-dependent transverse p profile?

- Novel observables: **DPS with ions, Triple-parton scatterings (TPS)**
in particular with **quarkonia final states: largest pQCD cross sections**

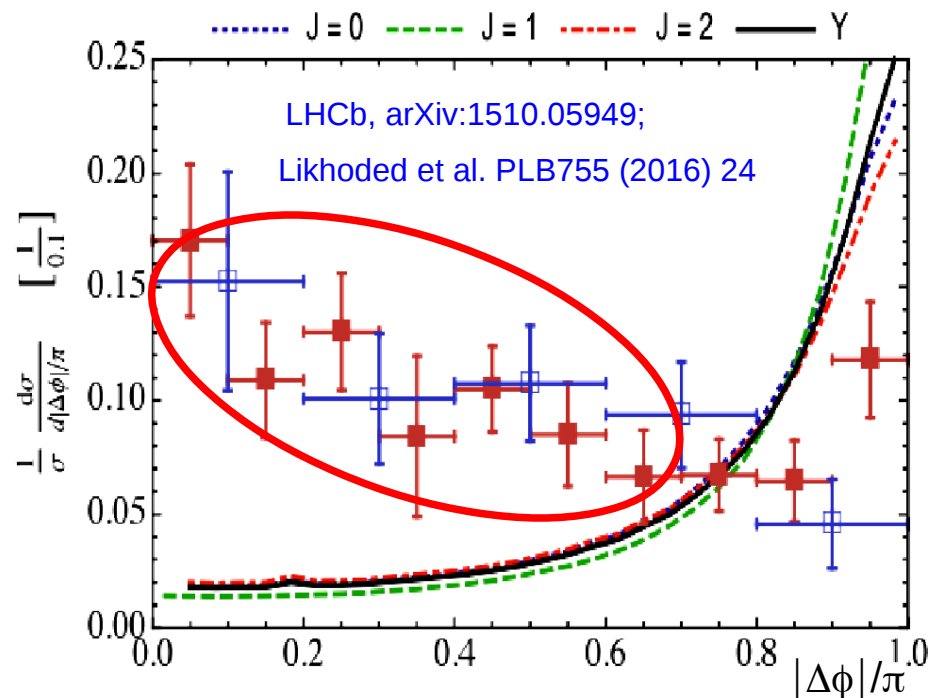
DPS studies with $Q\bar{Q}$: $p\text{-}p \rightarrow J/\psi + Y, Y + D$

- Uncorrelated $J/\psi + Y$ azimuthal production in ppbar at 1.96 TeV:



$$\sigma_{\text{eff}} = 2.2 \pm 0.7 \text{ (stat)} \pm 0.9 \text{ (syst)} \text{ mb.}$$

- Uncorrelated $Y + D$ azimuthal production in pp at 7 TeV:

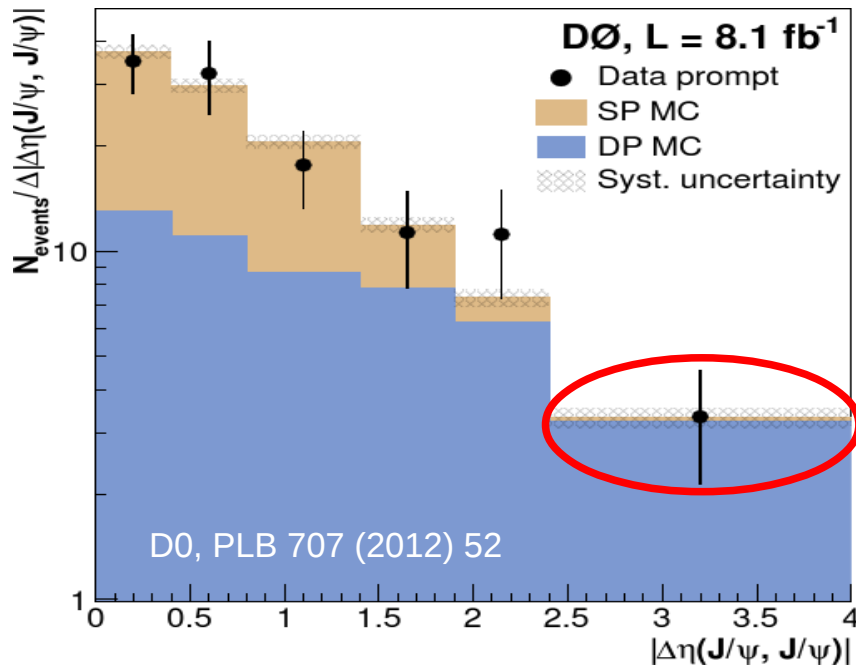


$$\sigma_{\text{eff}}|_{\Upsilon(1S)D} = 18.0 \pm 1.3 \text{ (stat)} \pm 1.2 \text{ (syst)} \text{ mb}$$

- Extracted σ_{eff} values differ by up to a factor of 8 for similar (g-induced) processes at 1.96 TeV & 7 TeV:
 - Energy-dependent parton transverse profile?
 - (Higher-order) SPS contributions under control?

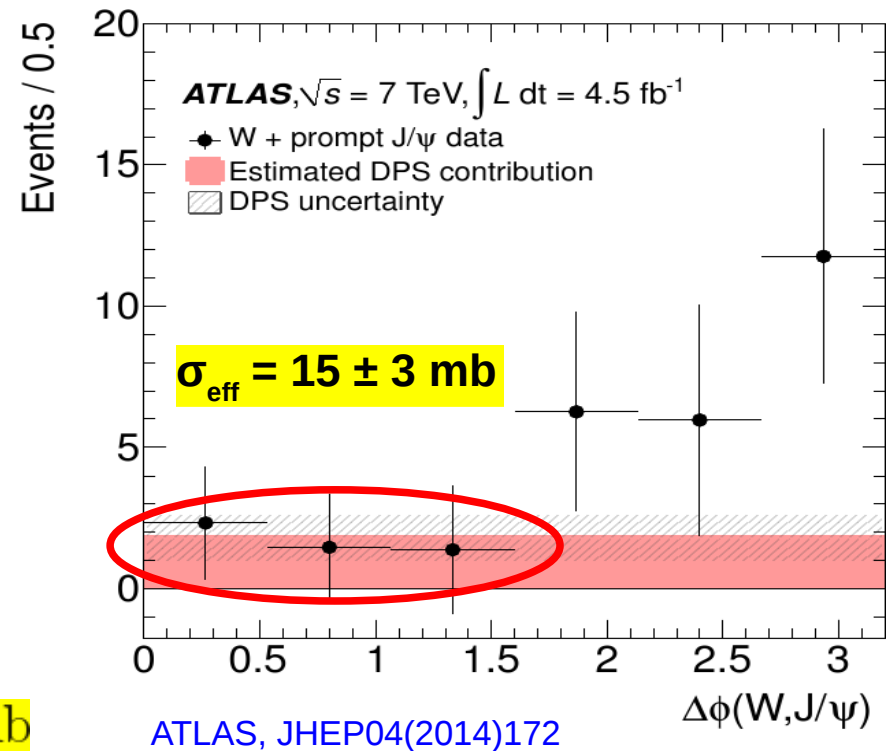
DPS studies with $Q\bar{Q}$: $p\text{-}p \rightarrow W^+ + J/\psi, J/\psi J/\psi$

- Uncorrelated $J/\psi + J/\psi$ rapidity production in ppbar at 1.96 TeV:



$$\sigma_{\text{eff}} = 4.8 \pm 0.5(\text{stat}) \pm 2.5(\text{syst}) \text{ mb}$$

- Uncorrelated $W + J/\psi$ azimuthal production in pp at 7 TeV:



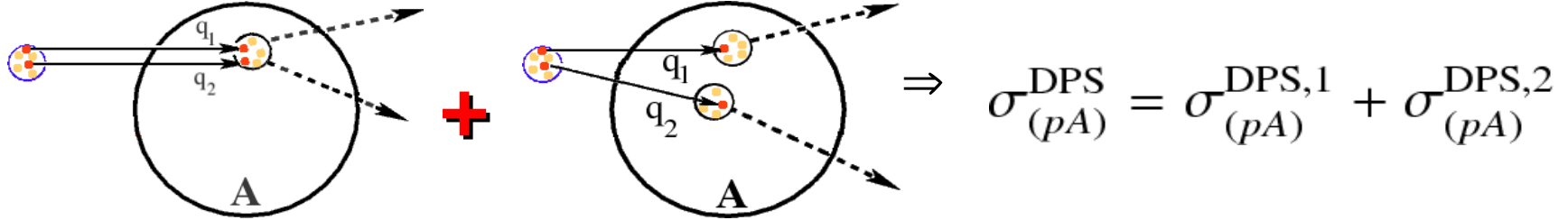
- Extracted σ_{eff} values differ at 1.96 TeV & 7 TeV:

- (Higher-order) SPS contributions under control?
- Energy-dependent parton transverse profile? (Quark vs. gluon?)

Double Parton Scattering x-sections in p-A

- Two contributions to DPS x-section in p-A:

[DdE, Snigirev, PLB 718 (2013)1395]
[Also Treleani, Strikman, Blok...]



$$\sigma_{(pA)}^{\text{DPS}} = \sigma_{(pA)}^{\text{DPS},1} + \sigma_{(pA)}^{\text{DPS},2}$$

$$\sigma_{(pA \rightarrow ab)}^{\text{DPS},1} = A \cdot \sigma_{(pN \rightarrow ab)}^{\text{DPS}} + \sigma_{(pA \rightarrow ab)}^{\text{DPS},2} = \sigma_{(pN \rightarrow ab)}^{\text{DPS}} \cdot \sigma_{\text{eff,pp}} \cdot F_{pA}$$

p-A overlap function:

$$F_{pA} = \int d^2r T_{pA}^2(\mathbf{r}) = 30.4 \text{ mb}^{-1}$$

Pb Woods-Saxon density
($r=6.62 \text{ fm}$, $a=0.546 \text{ fm}$)

Relative weight of DPS terms: $\sigma^{\text{DPS},1}:\sigma^{\text{DPS},2} = 0.7 : 0.3$ (small A), $0.33 : 0.66$ (large A)

- “Pocket” formula for DPS p-A x-section:

$$\sigma_{(pA \rightarrow ab)}^{\text{DPS}} = \left(\frac{m}{2}\right) \frac{\sigma_{(pN \rightarrow a)}^{\text{SPS}} \cdot \sigma_{(pN \rightarrow b)}^{\text{SPS}}}{\sigma_{\text{eff,pA}}}$$

$$\sigma_{\text{eff,pA}} = \frac{\sigma_{\text{eff,pp}}^{(\sigma_{\text{eff,pp}}=13 \pm 2 \text{ mb})}}{A + \sigma_{\text{eff,pp}} F_{pA}} = 21.5 \pm 1.1 \mu\text{b}$$

▶ Ratio of DPS p-Pb/p-p x-sections: $\sigma_{\text{eff,DPS}}/\sigma_{\text{eff,DPS,pA}} \approx [A + A^{4/3}/\pi]$

- DPS x-sections are large in p-A: a factor $\times 600$ (not $\times 208$) for p-Pb (!)

- Pb transverse density (F_{pA}) well known: Alternative extraction of $\sigma_{\text{eff,pp}}$

Examples: DPS x-sections in p-Pb (8.8 TeV)

[DdE, Snigirev, NPA 931 (2014) 303]

- Cross sections & rates for **DPS processes with $J/\psi, Y$ & W, Z bosons**
[Also V. Goncalves (2018): double- J/ψ ; Paukunen (2019): double-D,...]

pPb (8.8 TeV)	$J/\psi + J/\psi$	$J/\psi + \Upsilon$	$J/\psi+W$	$J/\psi+Z$
$\sigma_{pN \rightarrow a}^{\text{SPS}}, \sigma_{pN \rightarrow b}^{\text{SPS}}$	45 μb ($\times 2$)	45 μb , 2.6 μb	45 μb , 60 nb	45 μb , 35 nb
$\sigma_{pPb}^{\text{DPS}}$	45 μb	5.2 μb	120 nb	70 nb
N_{pPb}^{DPS} (1 pb^{-1})	~ 65	~ 60	~ 15	~ 3
	$\Upsilon + \Upsilon$	$\Upsilon+W$	$\Upsilon+Z$	ss WW
$\sigma_{pN \rightarrow a}^{\text{SPS}}, \sigma_{pN \rightarrow b}^{\text{SPS}}$	2.6 μb ($\times 2$)	2.6 μb , 60 nb	2.6 μb , 35 nb	60 nb ($\times 2$)
$\sigma_{pPb}^{\text{DPS}}$	150 nb	7 nb	4 nb	150 pb
N_{pPb}^{DPS} (1 pb^{-1})	~ 15	~ 8	~ 1.5	~ 4

Leptonic final states: $\text{BR}(J/\psi, Y, W, Z) = 6\%, 2.5\%, 11\%, 3.4\%$

Accept.*Effic.= 1% ($J/\psi, |y|=0,2$), 20% ($Y, |y|<2.5$), 50% ($W, Z |y|<2.4$)

- **Many double hard scatterings** processes with visible p-Pb x-sections at the LHC. (Note: J/ψ values are per unit- $|y|$).
- Useful **independent extraction of $\sigma_{\text{eff,pp}}$** !

First study of DPS in p-Pb (LHCb, 8.2 TeV)

[LHCb, PRL 125 (2020) 212001]

double charm production in proton lead collisions

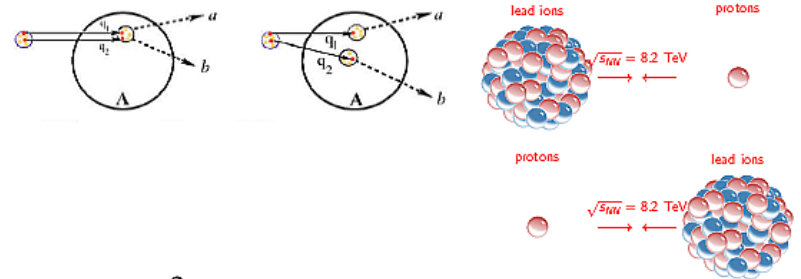
- select pairs of $D^0, \bar{D}^0, D^+, D^-, D_s^+, D_s^-$ and J/ψ
- sort them into pair production and “DPS” categories

$$\sigma_{C_1, C_2} = \alpha \frac{\sigma_{C_1} \sigma_{C_2}}{\sigma_{\text{eff}}}$$

$$R_{\text{forward}}^{D_1 D_2} = \frac{\sigma_{D_1 D_2}}{\sigma_{D_1 \bar{D}_2}} = 0.308 \pm 0.015 \pm 0.010$$

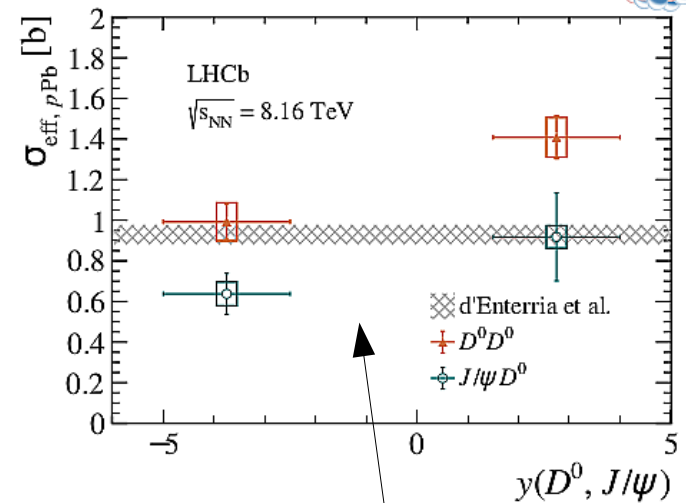
$$R_{\text{backward}}^{D_1 D_2} = 0.391 \pm 0.019 \pm 0.025$$

$$R_{pp}^{D^0 D^0} = 0.109 \pm 0.008$$



Like sign charm fraction tripled!

$$\sqrt{s_{\text{NN}}} = 8.2 \text{ TeV} \quad \text{Phys. Rev. Lett. 125 (2020) 212001}$$



Albert Bursche

charming DPS

10th October 2021

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- Useful independent extraction of $\sigma_{\text{eff,pp}}$:

$$\sigma_{\text{eff,pA}} = \frac{\sigma_{\text{eff,pp}}}{A + \sigma_{\text{eff,pp}} F_{\text{pA}}}$$

$$\sigma_{\text{eff,pp}}(D^0 D^0) = 7\text{--}16 \text{ mb}$$

$$\sigma_{\text{eff,pp}}(J/\psi D^0) = 13\text{--}40 \text{ mb}$$

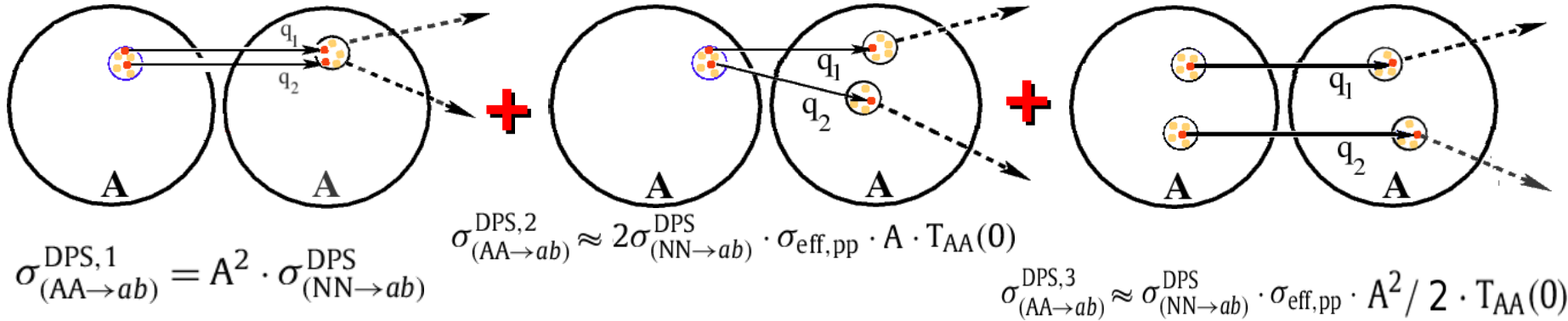
nPDF effects visible in -y/+y results.

(why LHCb does not quote the equivalent $\sigma_{\text{eff,pp}}$ values?)

Double Parton Scattering x-sections in A-A

[DdE, Snigirev, PLB727 (2013)157]

■ Three contributions to DPS x-section in A-A:



► Third “ N_{coll} term” $\propto A^2 \cdot T_{AA}(0)$, clearly dominant (1:4:200 ratio for PbPb)

“Genuine” DPS (within same nucleon): $\sim 2.5\%$ (in Pb-Pb) or $\sim 13\%$ (Ar-Ar)

■ “Pocket formula” for DPS A-A x-section:

$$\sigma_{(AA \rightarrow ab)}^{\text{DPS}} = \left(\frac{m}{2}\right) \frac{\sigma_{(NN \rightarrow a)}^{\text{SPS}} \cdot \sigma_{(NN \rightarrow b)}^{\text{SPS}}}{\sigma_{\text{eff,AA}}}$$

$$\sigma_{\text{eff,AA}} = \frac{1}{A^2 [\sigma_{\text{eff,pp}}^{-1} + \frac{2}{A} T_{AA}(0) + \frac{1}{2} T_{AA}(0)]} = 1.5 \text{ nb} \quad (\text{for Pb-Pb collisions})$$

► Ratio of DPS Pb-Pb/p-p x-sections: $\sigma_{\text{eff,pp}} / \sigma_{\text{eff,AA}} \propto A^{3.3} / 5 \simeq 9 \cdot 10^6 !$

■ Strong centrality dependence:

$$\sigma_{(AA \rightarrow ab)}^{\text{DPS}} [b_1, b_2] \approx \left(\frac{m}{2}\right) \sigma_{(NN \rightarrow a)}^{\text{SPS}} \cdot \sigma_{(NN \rightarrow b)}^{\text{SPS}} \cdot f_{\%} \sigma_{AA} \cdot \langle T_{AA}[b_1, b_2] \rangle^2$$

Examples: DPS x-sections in Pb-Pb (5.5 TeV)

[DdE, Snigirev, NPA 931 (2014)303]

- Cross sections & rates for **DPS processes with $J/\psi, Y$ & W, Z bosons:**

PbPb (5.5 TeV)	$J/\psi + J/\psi$	$J/\psi + \Upsilon$	$J/\psi+W$	$J/\psi+Z$
$\sigma_{NN \rightarrow a}^{\text{SPS}}, \sigma_{NN \rightarrow b}^{\text{SPS}}$	25 μb ($\times 2$)	25 μb , 1.7 μb	25 μb , 30 nb	25 μb , 20 nb
$\sigma_{\text{PbPb}}^{\text{DPS}}$	210 mb	28 mb	500 μb	330 μb
$N_{\text{PbPb}}^{\text{DPS}} (1 \text{ nb}^{-1})$	~ 250	~ 340	~ 65	~ 14
	$\Upsilon + \Upsilon$	$\Upsilon+W$	$\Upsilon+Z$	ss WW
$\sigma_{NN \rightarrow a}^{\text{SPS}}, \sigma_{NN \rightarrow b}^{\text{SPS}}$	1.7 μb ($\times 2$)	1.7 μb , 30 nb	1.7 μb , 20 nb	30 nb ($\times 2$)
$\sigma_{\text{PbPb}}^{\text{DPS}}$	960 μb	34 μb	23 μb	630 nb
$N_{\text{PbPb}}^{\text{DPS}} (1 \text{ nb}^{-1})$	~ 95	~ 35	~ 8	~ 15

Leptonic final states: $\text{BR}(J/\psi, Y, W, Z) = 6\%, 2.5\%, 11\%, 3.4\%$

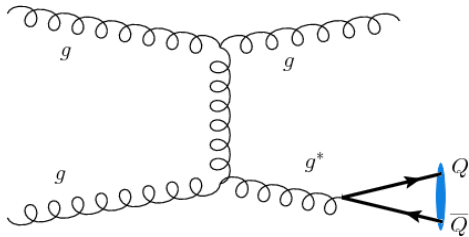
Accept.*effic.= 1% ($J/\psi, |y|=0,2$), 20% ($Y, |y|<2.5$), 50% ($W, Z |y|<2.4$)

- **Visible rates for many double hard scatterings** processes in Pb-Pb!
(Note: J/ψ values are per unit- $|y|$).

Example: Pb-Pb \rightarrow J/ ψ J/ ψ at 5.5 TeV

[DdE, Snigirev, PLB727 (2013)157]

- FONLL+CEM (R.Vogt):
Single-parton J/ ψ

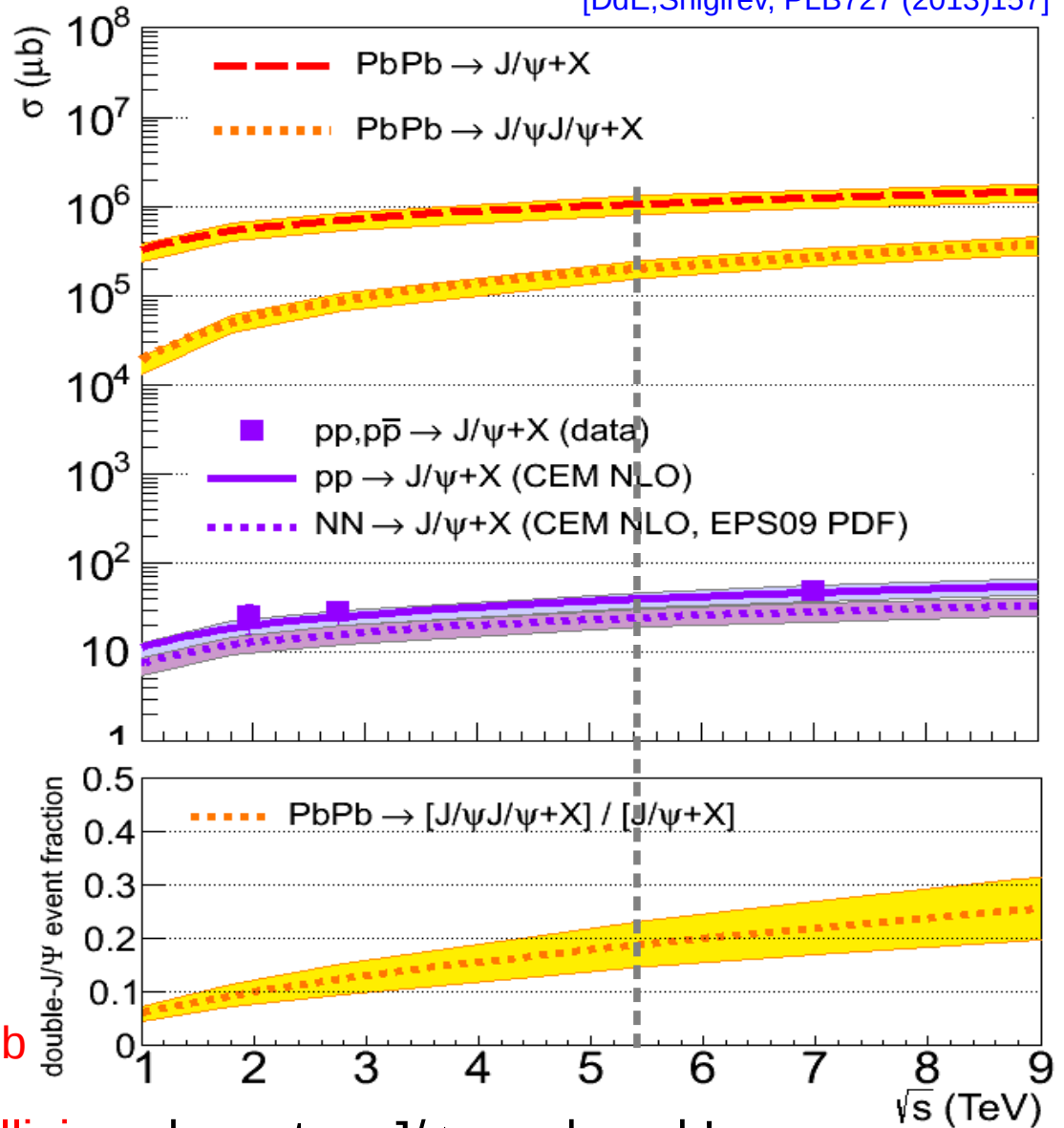


- NLO accuracy.
- Scales: $\mu_R = \mu_F = 1.5 \cdot m_c$
- Good agreement with Tevatron&LHC data

- EPS09 Pb nPDF
20–35% shadowing
x-section reduction

- At 5.5 TeV:

$$\sigma^{\text{DPS}}(\text{Pb-Pb} \rightarrow \text{J}/\psi \text{ J}/\psi \text{ X}) = 200 \pm 50 \text{ mb}$$



20% of min.bias Pb-Pb collisions have two J/ ψ produced !

Example: Pb-Pb \rightarrow J/ ψ J/ ψ at 5.5 TeV

[DdE, Snigirev, PLB727 (2013)157]

■ Visible rates:

- ▶ Fiducial x-section per unit-y: $d\sigma_{J/\psi}/dy \approx \sigma_{J/\psi}/8$
- ▶ BR(J/ $\psi \rightarrow l^+l^-$) \approx 6%
- ▶ Typical ALICE/CMS acceptance & efficiencies: $\varepsilon \approx 1/12$

■ Expected dimuon rates including yield all losses & 1 nb⁻¹ integ. luminosity:

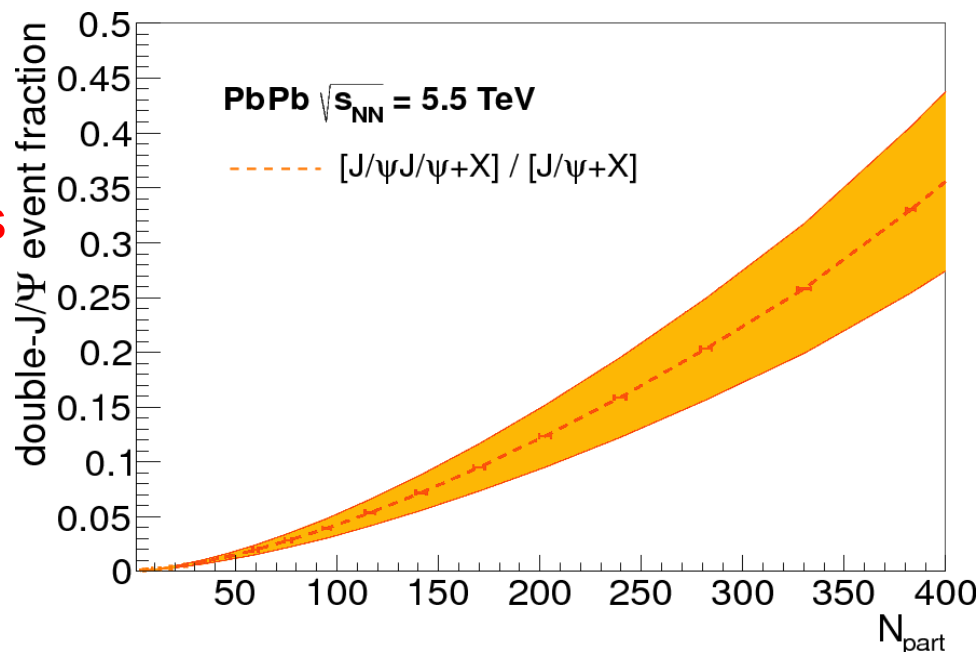
$$\mathcal{N} = \sigma_{\text{Pb-Pb} \rightarrow J/\psi J/\psi}^{\text{DPS}} / (\varepsilon \cdot \mathcal{L}_{\text{int}}) \approx \text{250 double-J}/\psi \text{ per year (per unit-|y|)}$$

(x2 less including final-state suppression)

■ Centrality dependence of double-J/ ψ fraction:

35% of central Pb-Pb collisions have two J/ ψ produced !

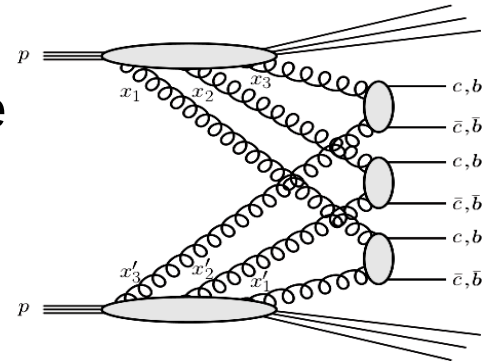
Seeing 2 J/ ψ on event-by-event basis not to be blindly taken as signal of c-cbar recombination.



Triple Parton Scatterings

Triple parton scattering x-sections (p-p)

- Assuming that the probabilities for 3 hard collisions to be independent of each other, one can again write a pocket-formula for TPS x-section:



$$\sigma_{hh' \rightarrow a_1 a_2 a_3}^{\text{TPS}} = \left(\frac{m}{3!} \right) \frac{\sigma_{hh' \rightarrow a_1}^{\text{SPS}} \cdot \sigma_{hh' \rightarrow a_2}^{\text{SPS}} \cdot \sigma_{hh' \rightarrow a_3}^{\text{SPS}}}{\sigma_{\text{eff,TPS}}^2}$$

normalized by the square of an eff. x-section ($\sigma_{\text{eff,TPS}}^2$) plus a trivial combinatorial factor ($m/3!$) to avoid triple-counting in case of same particles produced: $m = 1$ if $a_1 = a_2 = a_3$; $m = 3$ if $a_1 = a_2$, or $a_1 = a_3$, or $a_2 = a_3$; and $m = 6$ if $a_1 \neq a_2 \neq a_3$.

- How to interpret $\sigma_{\text{eff,TPS}}$? Relationship with σ_{eff} ? What values to expect?
- Most generic expression for TPS cross section:

$$\begin{aligned} \sigma_{hh' \rightarrow a_1 a_2 a_3}^{\text{TPS}} &= \left(\frac{m}{3!} \right) \sum_{i,j,k,l,m,n} \int \Gamma_h^{ijk}(x_1, x_2, x_3; \mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3; Q_1^2, Q_2^2, Q_3^2) \\ &\times \hat{\sigma}_{a_1}^{il}(x_1, x'_1, Q_1^2) \cdot \hat{\sigma}_{a_2}^{jm}(x_2, x'_2, Q_2^2) \cdot \hat{\sigma}_{a_3}^{kn}(x_3, x'_3, Q_3^2) \\ &\times \Gamma_{h'}^{lmn}(x'_1, x'_2, x'_3; \mathbf{b}_1 - \mathbf{b}, \mathbf{b}_2 - \mathbf{b}, \mathbf{b}_3 - \mathbf{b}; Q_1^2, Q_2^2, Q_3^2) \\ &\times dx_1 dx_2 dx_3 dx'_1 dx'_2 dx'_3 d^2 b_1 d^2 b_2 d^2 b_3 d^2 b. \end{aligned}$$

Generalized PDFs = $f(x, Q^2, \mathbf{b})$

Triple parton scattering x-sections (p-p)

- Assumption 1: Factorize generalized Triple-PDF into longitudinal &

transverse components:
$$\Gamma_h^{ijk}(x_1, x_2, x_3; \mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3; Q_1^2, Q_2^2, Q_3^2) = D_h^{ijk}(x_1, x_2, x_3; Q_1^2, Q_2^2, Q_3^2) f(\mathbf{b}_1) f(\mathbf{b}_2) f(\mathbf{b}_3),$$

p-p transv. overlap function (mb^{-1}):
$$T(\mathbf{b}) = \int f(\mathbf{b}_1) f(\mathbf{b}_1 - \mathbf{b}) d^2b_1, \text{ with } \int d^2b T(\mathbf{b}) = 1.$$

- Assumption 2: Longitudinal triple-PDF is the product of 3 single PDFs (i.e. no parton correlations in colour, momentum, flavour, spin,...)

$$D_h^{ijk}(x_1, x_2, x_3; Q_1^2, Q_2^2, Q_3^2) = D_h^i(x_1; Q_1^2) D_h^j(x_2; Q_2^2) D_h^k(x_3; Q_3^2)$$

- Then, $\sigma_{\text{eff,TPS}}^2$ is simply the inverse of the cube of the transv. pp overlap:

$$\sigma_{\text{eff,TPS}}^2 = \left[\int d^2b T^3(\mathbf{b}) \right]^{-1}$$

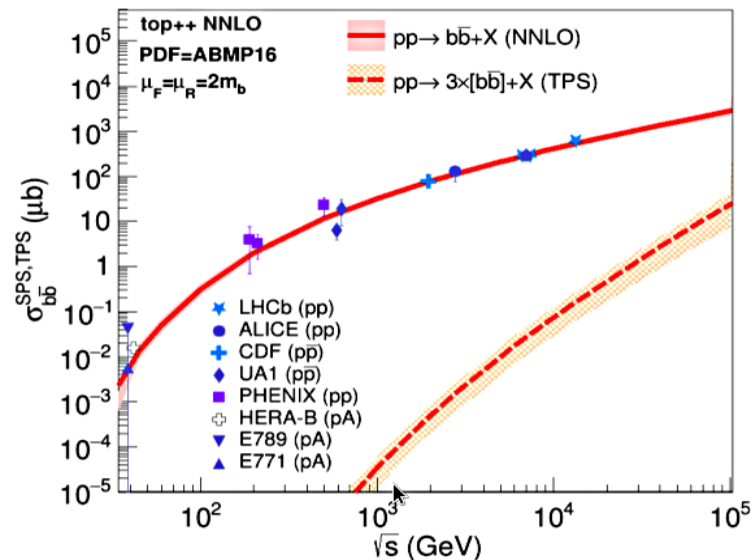
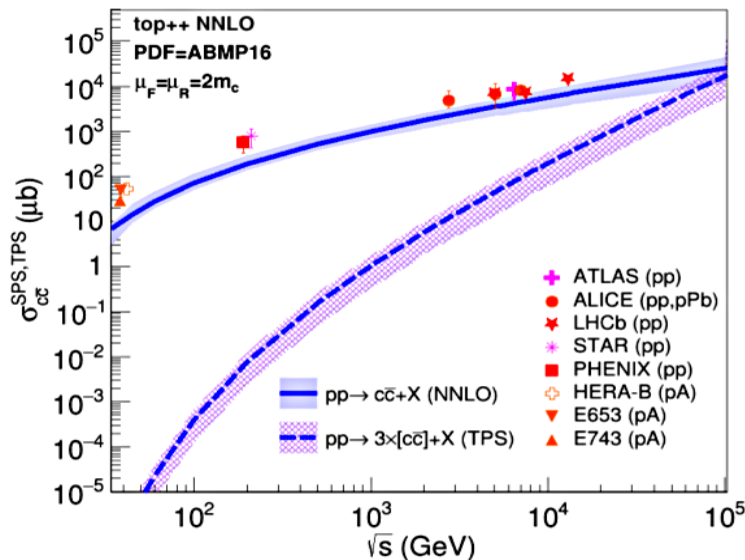
- By testing many proton overlaps/profiles (hard sphere, Gaussian, expo, dipole fit), we find a close relationship between $\sigma_{\text{eff,TPS}}$ & σ_{eff} :

$$\sigma_{\text{eff,TPS}} = k \times \sigma_{\text{eff,DPS}}, \text{ with } k = 0.82 \pm 0.11$$

- Measuring TPS provides independent info on σ_{eff} and p transv. profile.

Triple charm & beauty production (p-p)

- TPS x-sections are small: $\sigma(\text{SPS})^3/\sigma(\text{eff})^2 \approx 1 \text{ fb}$ for $\sigma(\text{SPS}) \approx 1 \mu\text{b}$, but rise fast (cube of SPS) with c.m. energy.
- Charm & beauty have large enough $\sigma(\text{SPS})$ to attempt TPS observation:

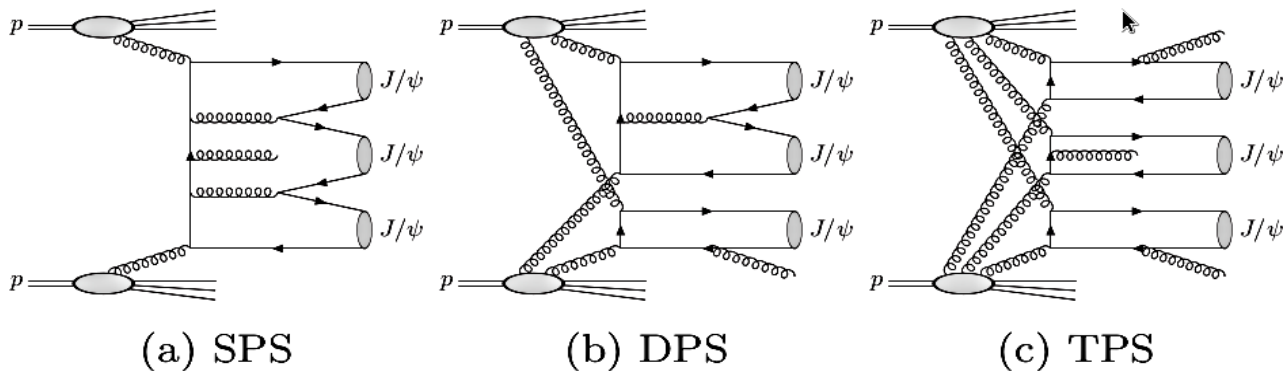


Final state	$\sqrt{s} = 14 \text{ TeV}$	$\sqrt{s} = 100 \text{ TeV}$
$\sigma_{c\bar{c}+X}^{\text{SPS}}$	$7.1 \pm 3.5_{\text{SC}} \pm 0.3_{\text{PDF}} \text{ mb}$	$25.0 \pm 16.0_{\text{SC}} \pm 1.3_{\text{PDF}} \text{ mb}$
$\sigma_{c\bar{c} c\bar{c} c\bar{c}+X}^{\text{TPS}}$	$0.39 \pm 0.28_{\text{tot}} \text{ mb}$	$16.7 \pm 11.8_{\text{tot}} \text{ mb}$
$\sigma_{b\bar{b}+X}^{\text{SPS}}$	$0.56 \pm 0.09_{\text{SC}} \pm 0.01_{\text{PDF}} \text{ mb}$	$2.8 \pm 0.6_{\text{SC}} \pm 0.1_{\text{PDF}} \text{ mb}$
$\sigma_{b\bar{b} b\bar{b} b\bar{b}+X}^{\text{TPS}}$	$0.19 \pm 0.12_{\text{tot}} \mu\text{b}$	$24 \pm 17_{\text{tot}} \mu\text{b}$

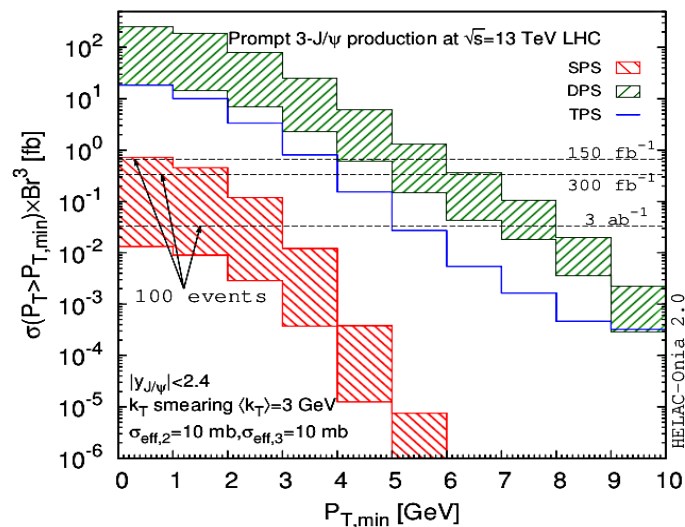
- Triple charm amounts to ~15% (50%) of inclusive charm x-sections at LHC (FCC). Contribution from triple-SPS, double-SPS processes?

Triple- J/ψ from SPS production (p-p)

- H.-S. Shao et al. [arXiv:1902.04949, PRL 122(2019)192002] computed **all triple- J/ψ x-sections with SPS HELAC-ONIA plus TPS pocket formula:**



		inclusive	$2.0 < y_{J/\psi} < 4.5$	$ y_{J/\psi} < 2.4$
13 TeV	SPS	$0.41^{+2.4}_{-0.34} \pm 0.0083$	$(1.8^{+11}_{-1.5} \pm 0.18) \times 10^{-2}$	$(8.7^{+56}_{-7.5} \pm 0.098) \times 10^{-2}$
	DPS	$(190^{+501}_{-140}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}}$	$(7.0^{+18}_{-5.1}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}}$	$(50^{+140}_{-37}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}}$
	TPS	$130 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)$	$1.3 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)$	$18 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)$
27 TeV	SPS	$0.46^{+2.9}_{-0.39} \pm 0.022$	$(3.2^{+22}_{-2.8} \pm 0.21) \times 10^{-2}$	$(5.8^{+39}_{-5.1} \pm 0.29) \times 10^{-2}$
	DPS	$(560^{+2900}_{-480}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}}$	$(19^{+97}_{-16}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}}$	$(120^{+630}_{-100}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}}$
	TPS	$570 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)$	$5.0 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)$	$57 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)$
75 TeV	SPS	$0.59^{+4.4}_{-0.52} \pm 0.016$	$(3.0^{+25}_{-2.7} \pm 0.23) \times 10^{-2}$	$(7.2^{+63}_{-6.5} \pm 0.38) \times 10^{-2}$
	DPS	$(1900^{+11000}_{-1600}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}}$	$(57^{+340}_{-50}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}}$	$(310^{+2000}_{-270}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}}$
	TPS	$3900 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)$	$27 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)$	$260 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)$
100 TeV	SPS	$1.1^{+8.4}_{-1.0} \pm 0.044$	$(4.5^{+33}_{-4.0} \pm 0.72) \times 10^{-2}$	$(36^{+290}_{-32} \pm 1.8) \times 10^{-2}$
	DPS	$(3400^{+19000}_{-2900}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}}$	$(100^{+550}_{-86}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}}$	$(490^{+3000}_{-430}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}}$
	TPS	$6500 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)$	$45 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)$	$380 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)$



- **SPS negligible, DPS (TPS) dominates at low (high) p_T .**

Clear sensitivity to σ_{eff} !

TPS in p-p collisions (13 TeV, CMS)

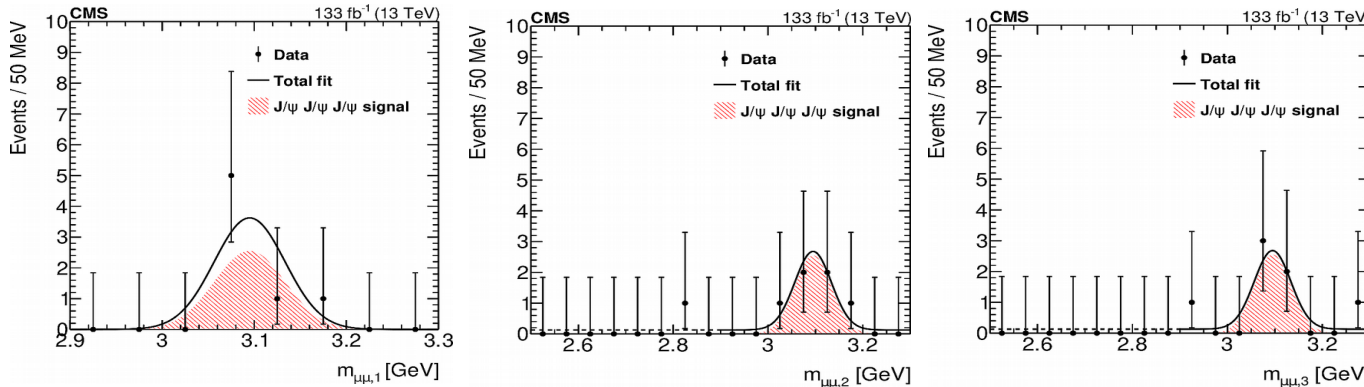
- Triple parton scatterings x-sections in p-p: alternative extraction of $\sigma_{\text{eff,DPS}}$

$$\sigma_{hh' \rightarrow a_1 a_2 a_3}^{\text{TPS}} = \left(\frac{m}{3!} \right) \frac{\sigma_{hh' \rightarrow a_1}^{\text{SPS}} \cdot \sigma_{hh' \rightarrow a_2}^{\text{SPS}} \cdot \sigma_{hh' \rightarrow a_3}^{\text{SPS}}}{\sigma_{\text{eff,TPS}}^2}$$

$$\sigma_{\text{eff,TPS}} = (0.82 \pm 0.11) \sigma_{\text{eff,DPS}}$$

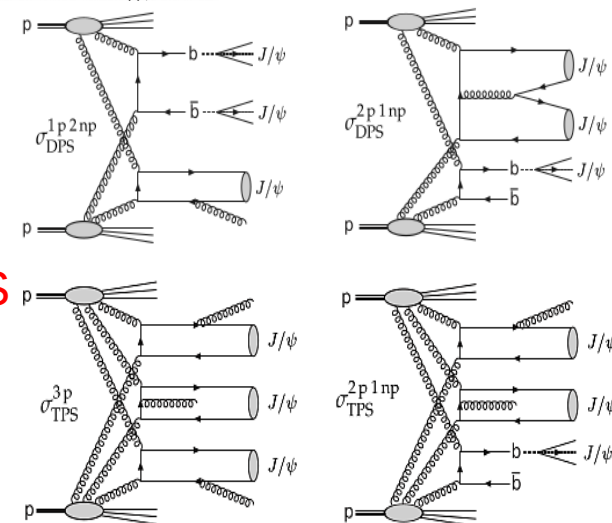
[DdE, Snigirev, PRL 118(2017)122001]

- First observation of triple- J/ψ production (CMS):



[arXiv:2111.05370
Nat. Phys. to appear]

- Measurement of fiducial cross section $\sigma(pp \rightarrow 3J/\psi) = 272_{-104}^{+141}$ (stat) ± 17 (syst) fb
- Pocket formula with (N)NLO for single-, double-, triple- J/ψ SPS x-sections:
 - Triple- J/ψ fractions: $\sim 6\%$ SPS, $\sim 74\%$ DPS, $\sim 20\%$ TPS
 - $\sigma_{\text{eff,DPS}} = 2.7_{-1.0}^{+1.4}$ (exp) $_{-1.0}^{+1.5}$ (theo) mb consistent with for di-quarkonia (lower than jet/ γ /W/Z DPS results):
 - q/g x-dependent transverse profile & correlations

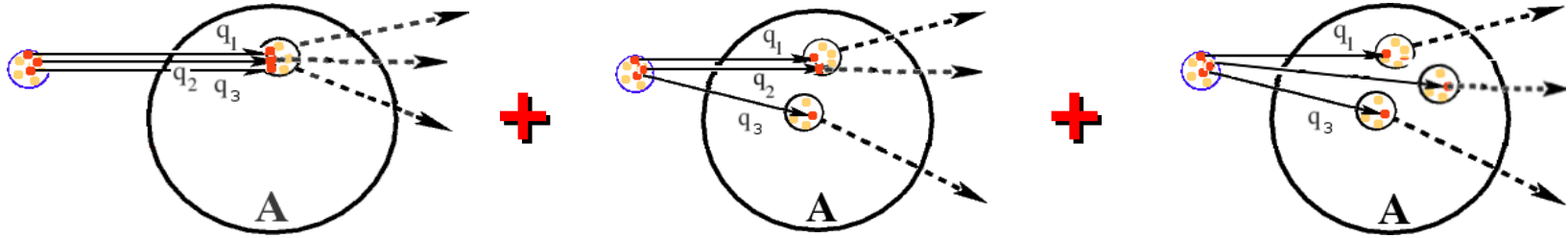


David d'Enterria (CERN)

Triple Parton Scattering x-sections in p-A

[DdE, Snigirev, EPJC 78 (2018)359]

■ Three contributions to TPS x-section in p-A:



$$\sigma_{pA \rightarrow abc}^{\text{TPS},1} = A \cdot \sigma_{pN \rightarrow abc}^{\text{TPS}}, \quad \sigma_{pA \rightarrow abc}^{\text{TPS},2} = \sigma_{pN \rightarrow abc}^{\text{TPS}} \cdot 3 \frac{\sigma_{\text{eff},\text{TPS}}^2}{\sigma_{\text{eff},\text{DPS}}} F_{pA}, \quad \sigma_{pA \rightarrow abc}^{\text{TPS},3} = \sigma_{pN \rightarrow abc}^{\text{TPS}} \cdot \sigma_{\text{eff},\text{TPS}}^2 \cdot C_{pA}, \quad \text{with}$$

$$C_{pA} = \frac{(A-1)(A-2)}{A^2} \int d^2b T_{pA}^3(\mathbf{b}),$$

Relative weight of TPS terms: $\sigma_{pA \rightarrow abc}^{\text{TPS},1} : \sigma_{pA \rightarrow abc}^{\text{TPS},2} : \sigma_{pA \rightarrow abc}^{\text{TPS},3} = 1 : 4.54 : 3.56$.

(TPS yields in pPb: 10% "genuine", 50% involve 2 nucleons, 40% involve 3 different Pb nucleons)

■ "Pocket" formula for TPS p-A x-section:

$$\sigma_{pA \rightarrow abc}^{\text{TPS}} = \left(\frac{m}{6}\right) \frac{\sigma_{pN \rightarrow a}^{\text{SPS}} \cdot \sigma_{pN \rightarrow b}^{\text{SPS}} \cdot \sigma_{pN \rightarrow c}^{\text{SPS}}}{\sigma_{\text{eff},\text{TPS},pA}^2} \quad \sigma_{\text{eff},\text{TPS},pA} = \left[\frac{A}{\sigma_{\text{eff},\text{TPS}}^2} + \frac{3 F_{pA} [\text{mb}^{-1}]}{\sigma_{\text{eff},\text{DPS}}} + C_{pA} [\text{mb}^{-2}] \right]^{-1/2}$$

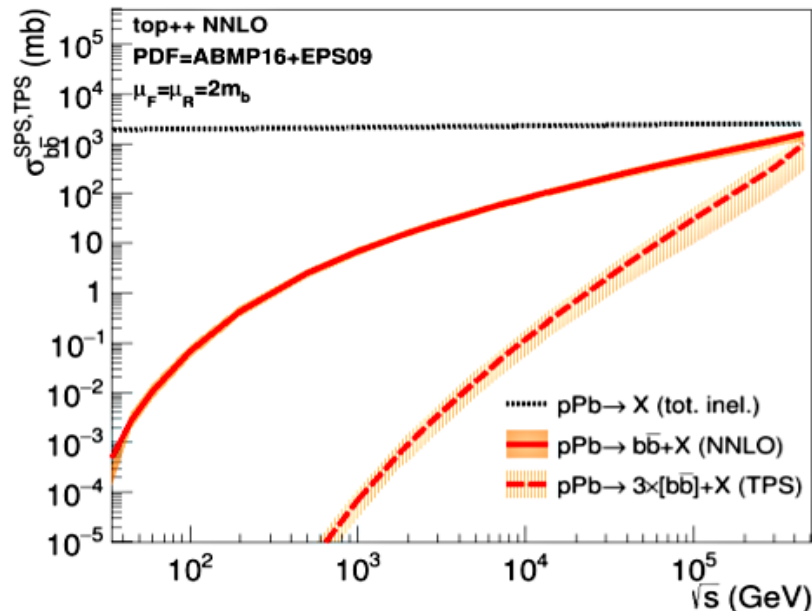
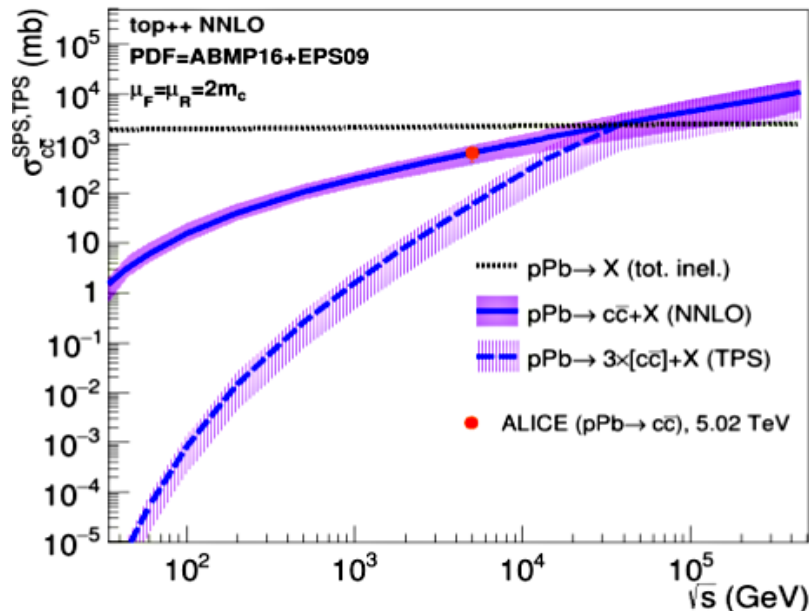
► $\sigma_{\text{eff},\text{TPS},p\text{Pb}} = 0.29 \pm 0.04 \text{ mb}$ ($\times 45$ times the p-p case with $\sigma_{\text{eff},\text{TPS}} = 12.5 \text{ mb}$)

■ TPS x-sections are large in p-A: a factor $\times 45$ for p-Pb compared to p-p

■ Pb transv. density (F_{pA} , C_{pA}) well-known: Alternative extraction of $\sigma_{\text{eff},pp}$

Example: Triple charm & beauty in p-Pb colls.

- Charm & beauty have very large TPS x-sections at the LHC & above:



Process	pPb(8.8 TeV)	pPb(63 TeV)	p-Air(430 TeV)
σ_{pA}^{inel}	2.2 ± 0.4 b	2.4 ± 0.4 mb	0.61 ± 0.10 b
$\sigma_{c\bar{c}+X}^{SPS}$	$0.96 \pm 0.45_{sc} \pm 0.10_{PDF}$ b	$3.4 \pm 1.9_{sc} \pm 0.4_{PDF}$ b	$0.75 \pm 0.5_{sc} \pm 0.1_{PDF}$ b
$\sigma_{c\bar{c}c\bar{c}c\bar{c}+X}^{TPS}$	$200 \pm 140_{tot}$ mb	$8.7^* \pm 6.2_{tot}$ b	$5.0^* \pm 3.6_{tot}$ b
$\sigma_{b\bar{b}+X}^{SPS}$	$72 \pm 12_{sc} \pm 5_{PDF}$ mb	$370 \pm 75_{sc} \pm 30_{PDF}$ mb	$110 \pm 25_{sc} \pm 5_{PDF}$ mb
$\sigma_{b\bar{b}b\bar{b}b\bar{b}+X}^{TPS}$	$0.084 \pm 0.045_{tot}$ μ b	$11 \pm 7_{tot}$ μ b	$17 \pm 11_{tot}$ μ b

- Triple charm amounts to $\sim 20\%$ ($\sim 100\%$!) of inclusive charm x-sections at LHC (FCC). Large triple J/ψ production at FCC: $\sigma(J/\psi J/\psi J/\psi + X) \approx 1$ mb
- Triple beauty amounts to $\sim 3\%$ of inclusive beauty x-sections at FCC.

Summary: DPS studies

- What's the **parton transverse density** of a proton? Its **energy evolution**? How do **partons correlate** (kinemat., quantum numbers) transversely?
- Double hard parton scatterings in **p-p collisions**:

$$\sigma_{(hh' \rightarrow ab)}^{\text{DPS}} = \left(\frac{m}{2}\right) \frac{\sigma_{(hh' \rightarrow a)}^{\text{SPS}} \cdot \sigma_{(hh' \rightarrow b)}^{\text{SPS}}}{\sigma_{\text{eff}}}$$

In absence of parton correlations:

$$\sigma_{\text{eff}} = \left[\int d^2b t^2(\mathbf{b}) \right]^{-1}$$

geom. overlap area of 2 proton transv. profiles

- $\sigma_{\text{eff}}(\text{exp}) \approx 2\text{--}20 \text{ mb}$ at Tevatron/LHC. Can HI colls. help to clarify this?

- Available DPS x-sections “**pocket formula**” for p-A and A-A:

($\sigma_{\text{eff,pp}} = 13 \pm 2 \text{ mb}$)

$$\sigma_{\text{eff,pA}} = \frac{\sigma_{\text{eff,pp}}}{A + \sigma_{\text{eff,pp}} F_{\text{pA}}} = 21.5 \pm 1.1 \mu\text{b}$$

$$\sigma_{\text{eff,AA}} = \frac{1}{A^2 [\sigma_{\text{eff,pp}}^{-1} + \frac{2}{A} T_{\text{AA}}(0) + \frac{1}{2} T_{\text{AA}}(0)]} = 1.5 \text{ nb}$$

Huge enhancements! $\sigma_{\text{eff,DPS}} / \sigma_{\text{eff,DPS,pA}} \approx 600$, $\sigma_{\text{eff,pp}} / \sigma_{\text{eff,AA}} \propto A^{3.3} / 5 \simeq 9 \cdot 10^6$

- p-Pb**: Large DPS yields in p-A (in particular with quarkonia) provide many useful independent **extractions of $\sigma_{\text{eff,pp}}$** . 1st-ever measurement by **LHCb**.

- Pb-Pb**: Large DPS but dominated by scatts. **from different nucleons**. (~16% sensitivity on $\sigma_{\text{eff,pp}}$ from DPS with lighter ions such as Ar-Ar).

Summary: TPS studies

- What's the parton transverse density of a proton? Its energy evolution? How do partons correlate (kinemat., quantum numbers) transversely?
- Derived a generic expression for NPS x-sections in p-p collisions:

$$\sigma_{hh' \rightarrow a_1 \dots a_n}^{\text{NPS}} = \left(\frac{m}{n!} \right) \frac{\sigma_{hh' \rightarrow a_1}^{\text{SPS}} \cdots \sigma_{hh' \rightarrow a_n}^{\text{SPS}}}{\sigma_{\text{eff,NPS}}^{n-1}}$$

$$\sigma_{\text{eff,NPS}} = \left\{ \int d^2b T^n(\mathbf{b}) \right\}^{-1/(n-1)}$$

- And used it to derive pocket formula for triple parton scatterings in p-p...

$$\sigma_{hh' \rightarrow a_1 a_2 a_3}^{\text{TPS}} = \left(\frac{m}{3!} \right) \frac{\sigma_{hh' \rightarrow a_1}^{\text{SPS}} \cdot \sigma_{hh' \rightarrow a_2}^{\text{SPS}} \cdot \sigma_{hh' \rightarrow a_3}^{\text{SPS}}}{\sigma_{\text{eff,TPS}}^2}$$

$$\sigma_{\text{eff,TPS}}^2 = \left[\int d^2b T^3(\mathbf{b}) \right]^{-1}$$

Summary: TPS studies

- What's the parton transverse density of a proton? Its energy evolution? How do partons correlate (kinemat., quantum numbers) transversely?

- Triple hard parton scatterings in p-p collisions:

(closely related to DPS in the absence of parton correlations):

$$\sigma_{hh' \rightarrow a_1 a_2 a_3}^{\text{TPS}} = \left(\frac{m}{3!}\right) \frac{\sigma_{hh' \rightarrow a_1}^{\text{SPS}} \cdot \sigma_{hh' \rightarrow a_2}^{\text{SPS}} \cdot \sigma_{hh' \rightarrow a_3}^{\text{SPS}}}{\sigma_{\text{eff,TPS}}^2}$$

$$\sigma_{\text{eff,TPS}} = (0.82 \pm 0.11) \sigma_{\text{eff,DPS}}$$

- Triple charm amounts to ~15% of inclusive charm x-sections in p-p collisions at the LHC. Triple-J/ψ fully dominated by DPS/TPS: “golden channel” to extract $\sigma_{\text{eff,pp}}$: 1st-ever observation by CMS.

- Derived TPS x-sections “pocket formula” for p-A:

$$\sigma_{\text{pA} \rightarrow abc}^{\text{TPS}} = \left(\frac{m}{6}\right) \frac{\sigma_{\text{pN} \rightarrow a}^{\text{SPS}} \cdot \sigma_{\text{pN} \rightarrow b}^{\text{SPS}} \cdot \sigma_{\text{pN} \rightarrow c}^{\text{SPS}}}{\sigma_{\text{eff,TPS,pA}}^2}$$

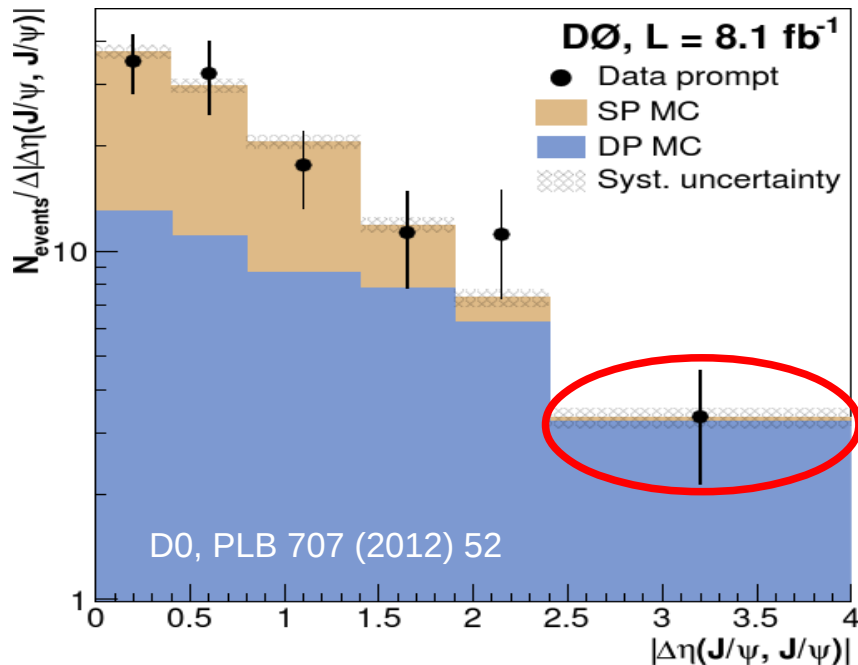
$$\sigma_{\text{eff,TPS,pA}} = \left[\frac{A}{\sigma_{\text{eff,TPS}}^2} + \frac{3 F_{\text{pA}} [\text{mb}^{-1}]}{\sigma_{\text{eff,DPS}}} + C_{\text{pA}} [\text{mb}^{-2}] \right]^{-1/2}$$

- Large TPS yields in p-Pb, e.g. σ_{TPS} (triple-c \bar{c}) = 200 mb (~20% of incl. c \bar{c} x-section): provide useful independent extractions of $\sigma_{\text{eff,pp}}$. [Don't be shy to attempt a 1st-ever measurement in p-Pb...].

Backup slides

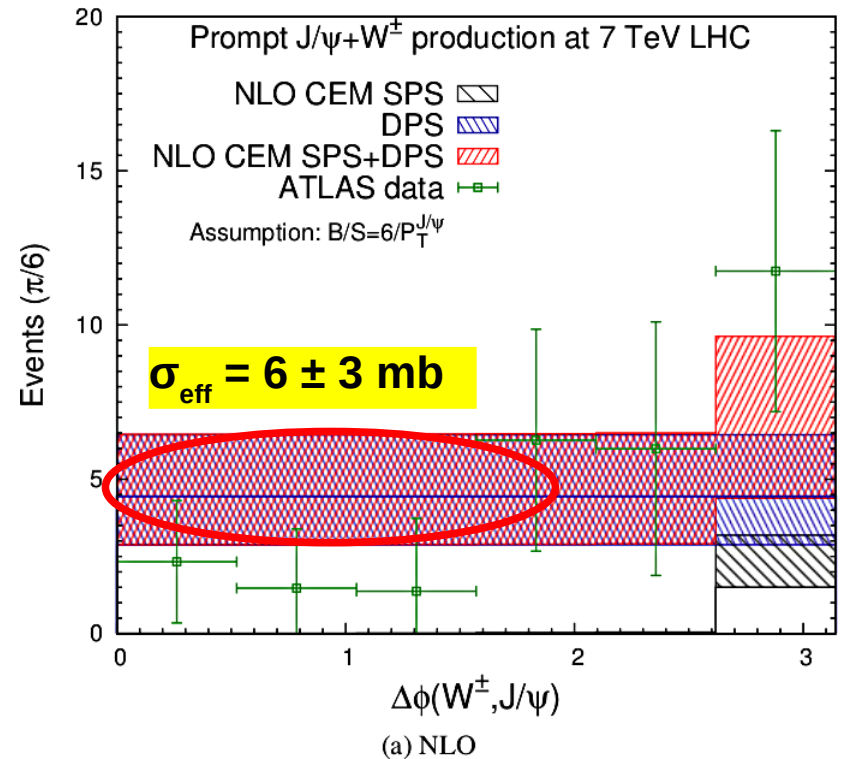
DPS studies with $Q\bar{Q}$: $p\text{-}p \rightarrow W^+ + J/\psi, J/\psi J/\psi$

- Uncorrelated $J/\psi + J/\psi$ rapidity production in ppbar at 1.96 TeV:



$$\sigma_{\text{eff}} = 4.8 \pm 0.5(\text{stat}) \pm 2.5(\text{syst}) \text{ mb}$$

- Uncorrelated $W + J/\psi$ azimuthal production in pp at 7 TeV:



Lansberg&Shao&Yamanaka,
PLB781 (2018) 485

- Extracted σ_{eff} values differ at 1.96 TeV & 7 TeV:

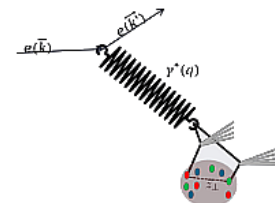
- (Higher-order) **SPS contributions** under control?
- **Energy-dependent** parton transverse profile? (Quark vs. gluon?)

DPS in Ultraperipheral p-Pb collisions?

[M.Rinaldi, et al.]

- Rinaldi&Ceccopieri (also Blok & Strikman) have proposed to study DPS from photon-proton collisions (where photon = vector meson):

6 The γ -p effective cross section

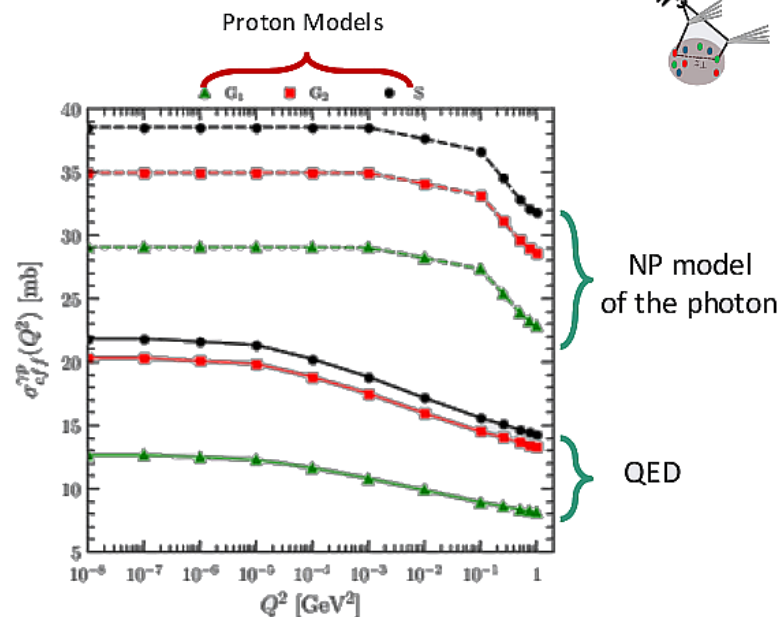


$$1 \quad [\sigma_{\text{eff}}^{\gamma p}(Q^2)]^{-1} = \int \frac{d^2 k_{\perp}}{(2\pi)^2} T_p(k_{\perp}) T_{\gamma}(k_{\perp}; Q^2)$$

2 $T_p(k_{\perp})$ proton EFF

3 ψ_{γ} Photon WF

M. R. and F. A. Ceccopieri, arXiv:2103.13480

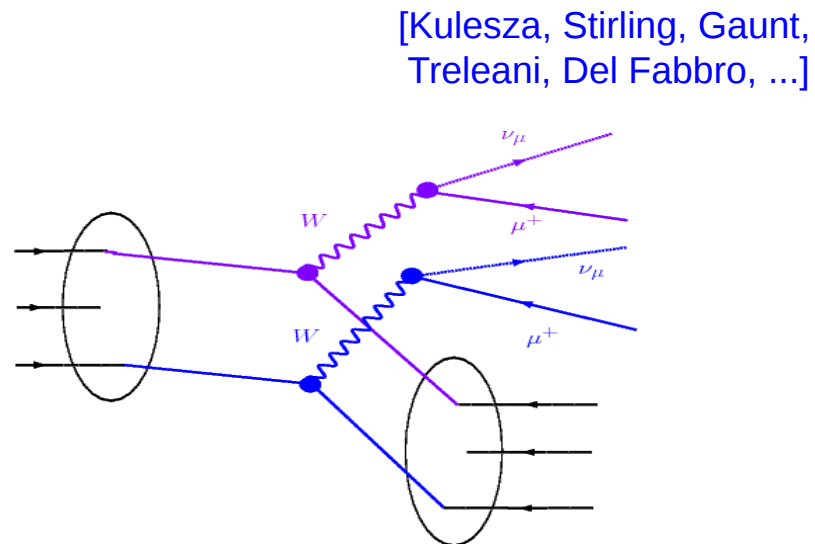


- Such studies (based on HERA data so far) could be tested with UPCs in p-Pb with the photon emitted from the Pb ion (we should go beyond searching for 'ridges' in UPCs, and extract some quantitative x-sections...)

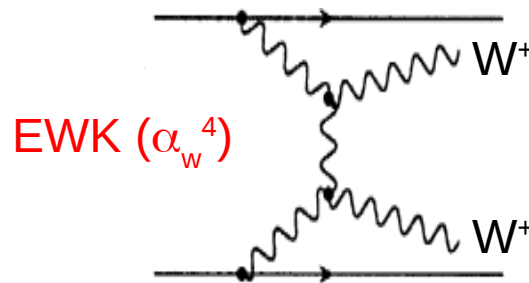
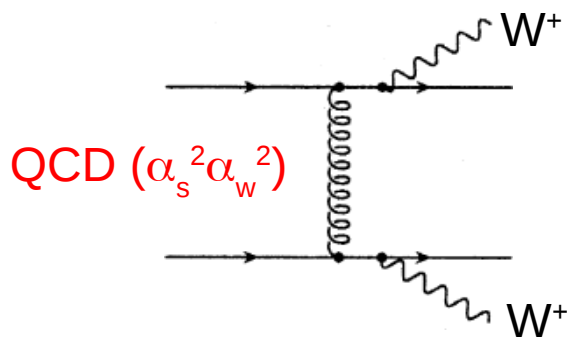
DPS “golden channel”: Same-sign WW

- Same-sign W-W production from 2 independent hard scatterings is a “golden” DPS signature:

- Well controlled pQCD x-sections.
- Clean experimental final-state: 2 like-sign leptons + missing- E_T



- Backgrounds: Same-sign W-W production in single parton scatterings (SPS) is higher-order and occurs **only with 2 extra jets**:



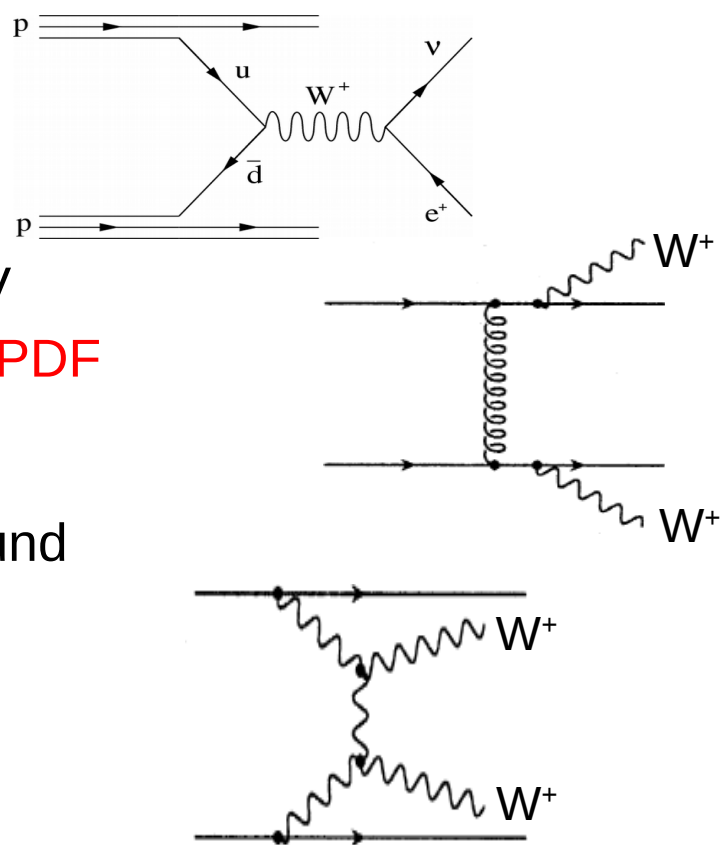
- $\sigma(WW, DPS) \sim 1/3 \cdot \sigma(WWjj, SPS)$, but SPS background reducible by more than x20 applying jet cuts.

Case study: p-Pb \rightarrow W^+W^+, W^-W^- at 8.8 TeV

[DdE, Snigirev, PLB718 (2013)1395]

Theoretical setup:

- ▶ **MCFM 6.2:** Single-parton W^+, W^-
 W^+W^+jj (QCD) background
 - **NLO** accuracy.
 - **Scales:** $\mu(W) = m_W, \mu(WW) = 150$ GeV
 - **CT10** proton PDF, **EPS09 Pb nuclear PDF**
 - Uncertainties: $\sim 10\%$
- ▶ **VBFNLO 2.6.0:** W^+W^+jj (EWK) background
 - **NLO** accuracy
 - **Scales:** $\mu^2 = t_{W,Z}$
 - **CT10** PDF
 - Uncertainties: $< 10\%$



Cross sections in pb (signal & background):

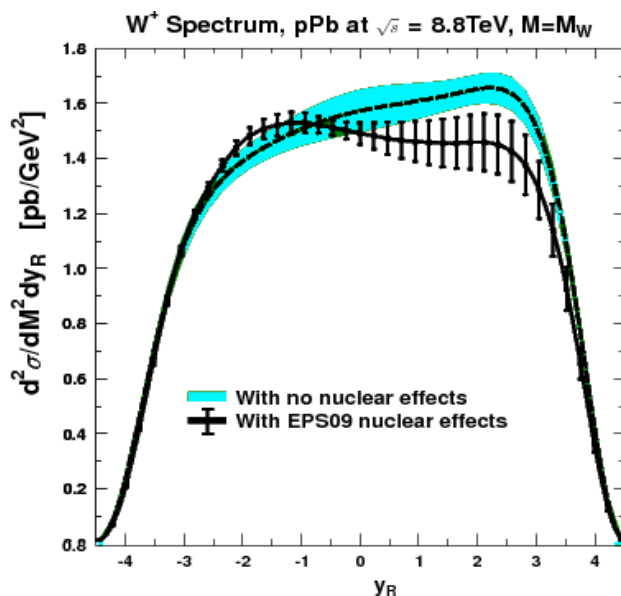
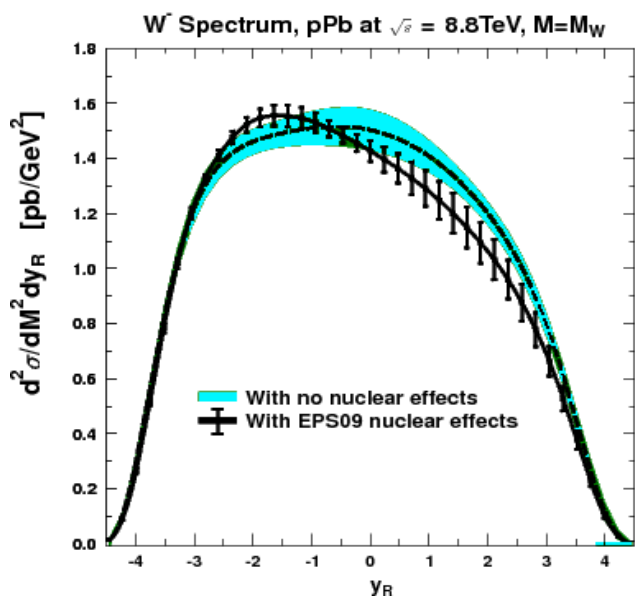
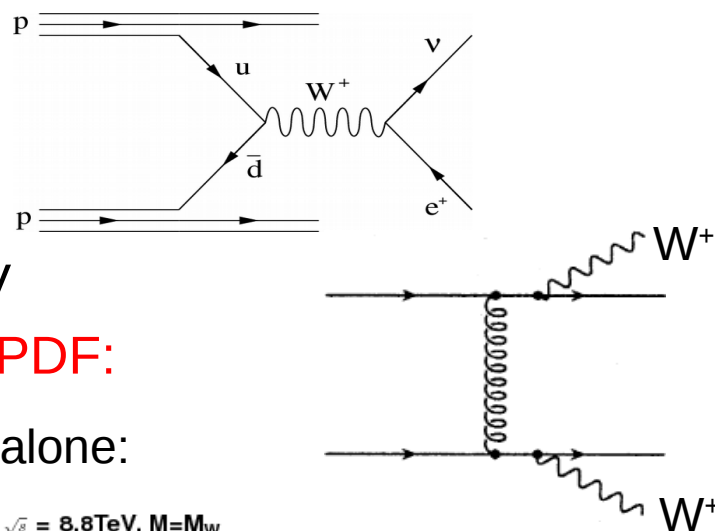
p-Pb final-state:	W^+	W^-	W^+W^-	W^+W^+jj (QCD)	W^+W^+jj (VBF)	$W^\pm W^\pm$ (DPS)
Code (process #):	MCFM (1)	MCFM (6)	MCFM (61)	MCFM (251)	VBFNLO (250)	Eq. (15)
Order (σ units):	NLO (μb)	NLO (μb)	NLO (nb)	'NLO' (pb)	NLO (pb)	(pb)
$\sqrt{s_{NN}} = 5.0$ TeV	6.85 ± 0.68	5.88 ± 0.59	5.48 ± 0.56	12.1 ± 1.2	12.4 ± 0.6	$44. \pm 8.$
$\sqrt{s_{NN}} = 8.8$ TeV	12.6 ± 1.3	11.1 ± 1.1	13.0 ± 1.3	40.4 ± 4.0	51.8 ± 2.0	$152. \pm 27.$

Case study: p-Pb \rightarrow W^+W^+, W^-W^- at 8.8 TeV

[DdE, Snigirev, PLB718 (2013)1395]

Theoretical setup:

- ▶ MCFM 6.2: Single-parton W^+, W^-
 W^+W^+jj (QCD) background
 - NLO accuracy.
 - Scales: $\mu(W) = m_W$, $\mu(WW) = 150$ GeV
 - CT10 proton PDF, EPS09 Pb nuclear PDF:
- ~10% effects due nuclear (anti)shadowing alone:



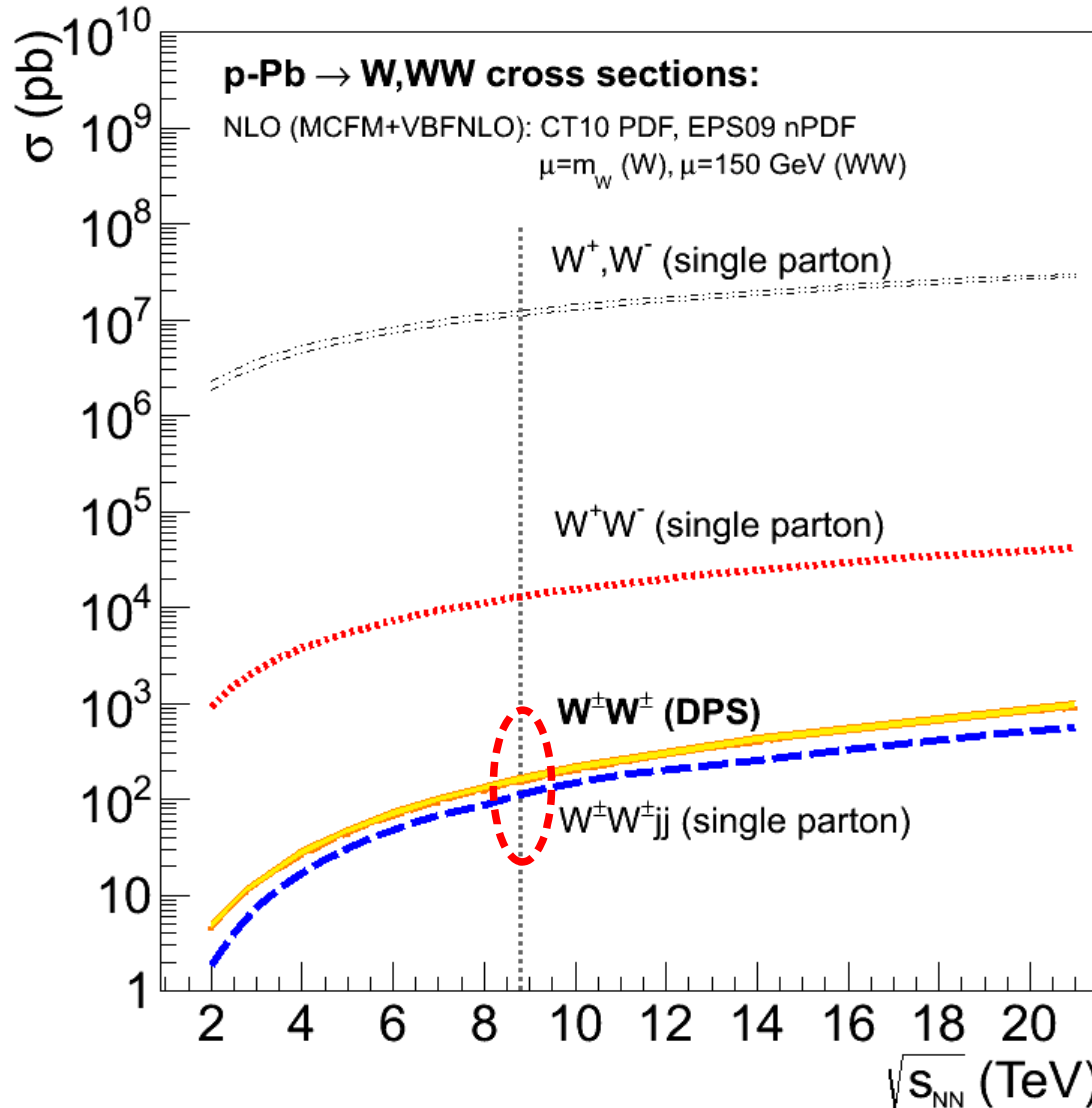
Isospin+shadow.
effects on total
inclusive x-sections:
 W^- : +7%
 W^+ : -15%
compared to p-p

[Paukkunen&Salgado JHEP 1103 (2011) 071]

Results: p-Pb \rightarrow W^+W^+, W^-W^- at 8.8 TeV

[DdE, Snigirev, PLB718 (2013)1395]

- Cross sections for all relevant SPS & DPS processes vs \sqrt{s} :



p-Pb @ 8.8 TeV:

$\sigma(WW, \text{DPS}) \approx 150$ pb

$\sigma(WWjj) \approx 100$ pb

$\pm 18\%$ uncertainties:

$\pm 15\%$ for σ_{eff}

$\pm 10\%$ for scales & PDFs

Results: p-Pb \rightarrow W^+W^+, W^-W^- at 8.8 TeV

[DdE, Snigirev, PLB718 (2013)1395]

■ Measurable final-states:

▶ W 's branching ratios:

- $BR(W \rightarrow l\nu) \approx 3 \times 1/9$, $BR(W \rightarrow qq') \approx 2/3$
- **Both leptonic**: 4 final-states ($\mu\mu, ee, e\mu, \mu e$): $4 \times (1/9)^2 \approx 1/20, 1/16$ (+ τ)
[1 leptonic + 1 hadronic (jet-charge): $2/9 \times 4/3 \approx 0.3$]

▶ Typical ATLAS/CMS acceptances & efficiencies:

- Leptons: $|y| < 2.5$, $p_T > 15$ GeV $\Rightarrow \epsilon_{WW} \approx 40\%$

■ LHC p-Pb **luminosities** (note: very small pileup):

- ▶ $\mathcal{L}_{int} = 0.2-2$ pb⁻¹ (increase to nominal p intensity, reduce beam size)

■ **Expected (purely leptonic) rates** including yield losses & luminosity:

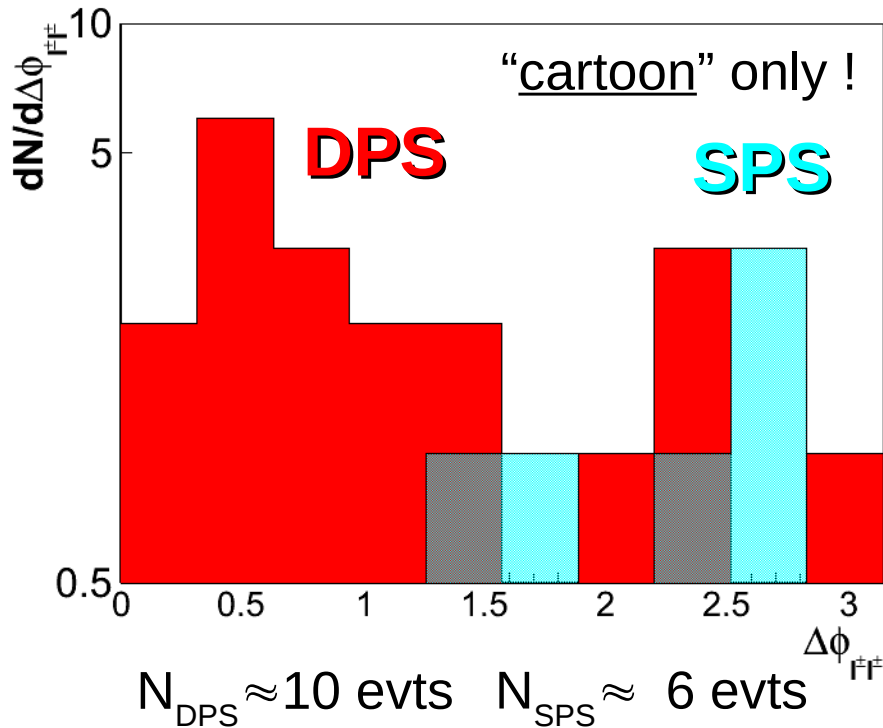
$$\mathcal{N}_{DPS} = \sigma_{pPb \rightarrow WW}^{DPS} / (\epsilon \cdot \mathcal{L}_{int}) \approx \mathbf{1-10 \text{ same-sign } WW \text{ pairs/year}}$$

(factor $\times 6$ more in 1 lepton + 1-jet channel)

Results: p-Pb \rightarrow W^+W^+, W^-W^- at 8.8 TeV

- Typical DPS-sensitive kinematical distributions for signal & background:

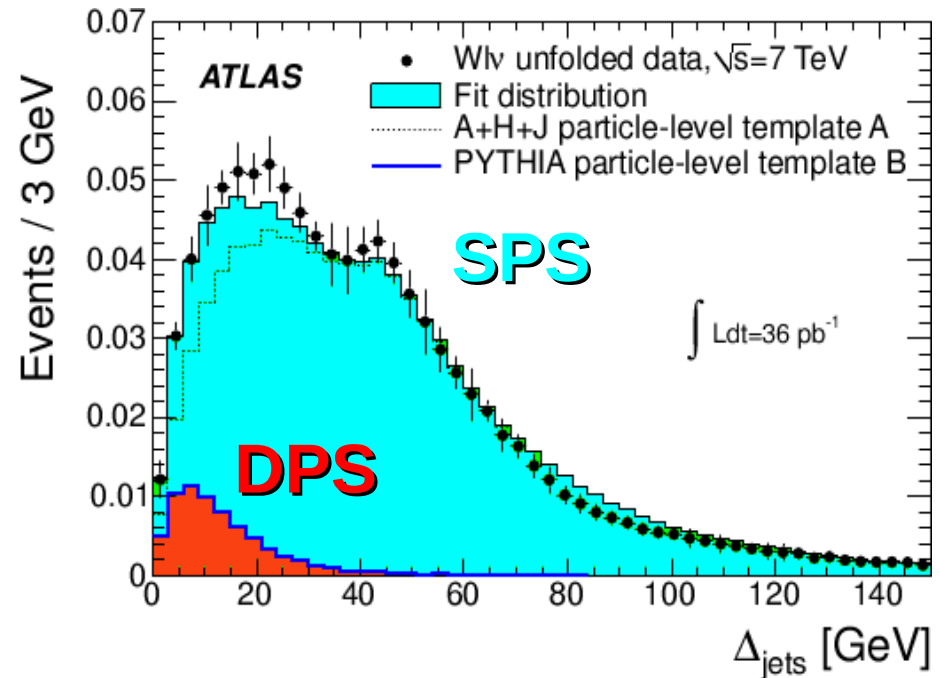
p-Pb @ 8.8 TeV (2 pb^{-1}):
Same-sign leptons
azimuthal separation:



(Other reducible bckgds: $WZ, Z^{(*)}Z^{(*)}, B^0B^0$)

Compare to:

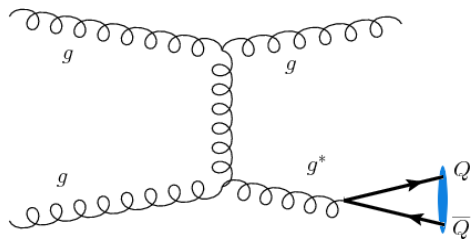
p-p \rightarrow $W+2j$ @ 7 TeV (36 pb^{-1}):
dijet azimuthal separation



Example: Pb-Pb \rightarrow J/ ψ J/ ψ at 5.5 TeV

[DdE, Snigirev, PLB727 (2013)157]

■ FONLL+CEM (R.Vogt): Single-parton J/ ψ

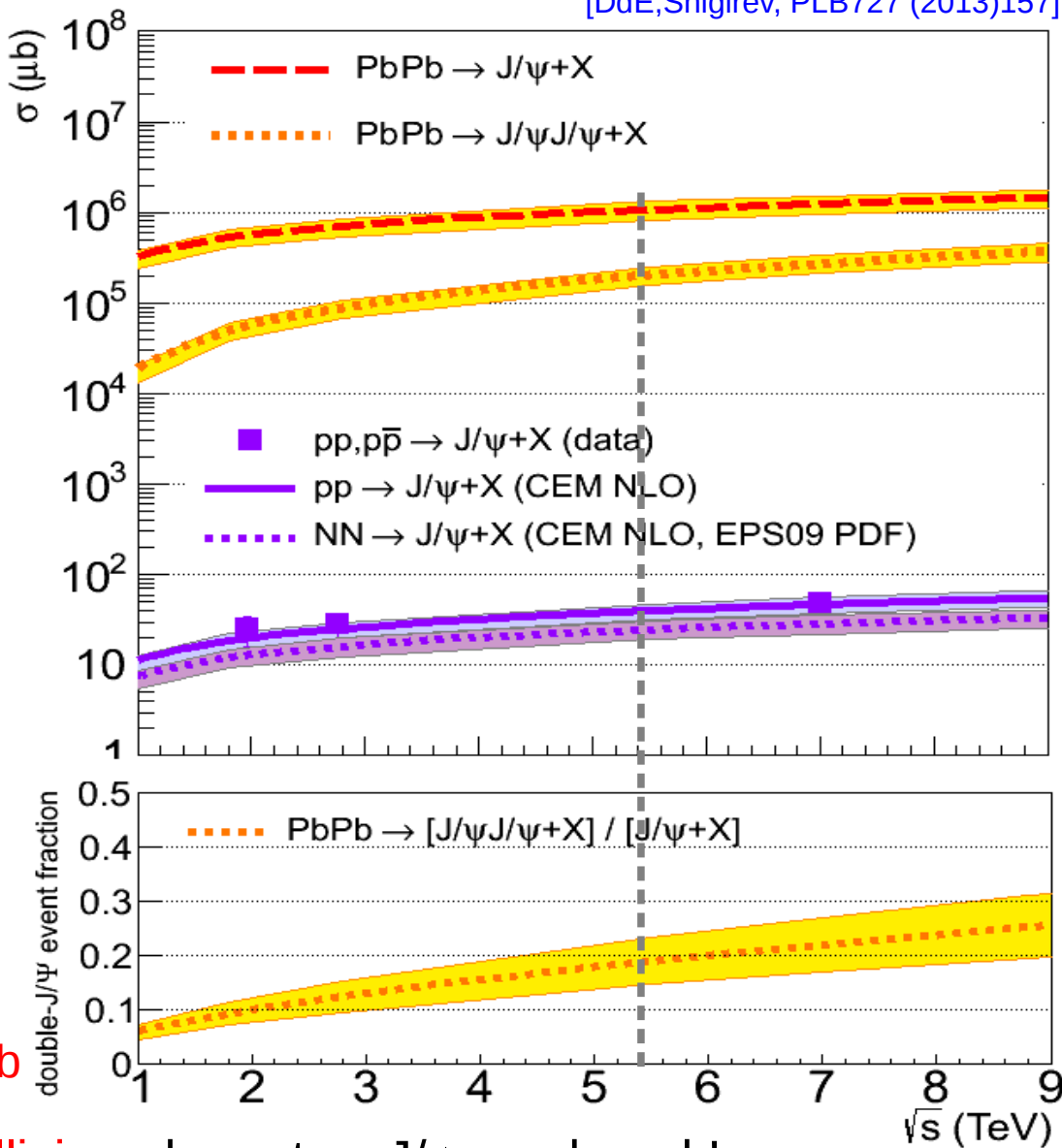


- NLO accuracy.
- Scales: $\mu_R = \mu_F = 1.5 \cdot m_c$
- Good agreement with Tevatron&LHC data

- EPS09 Pb nPDF
20–35% shadowing
x-section reduction

■ At 5.5 TeV:

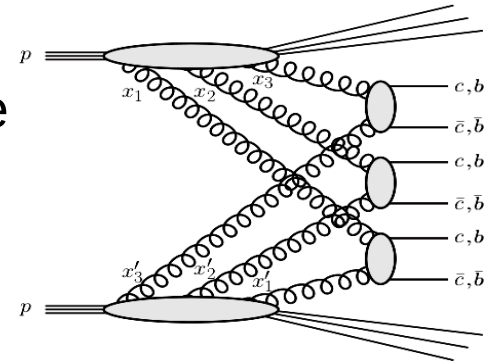
$$\sigma^{\text{DPS}}(\text{Pb-Pb} \rightarrow \text{J}/\psi \text{ J}/\psi \text{ X}) = 200 \pm 50 \text{ mb}$$



20% of min.bias Pb-Pb collisions have two J/ ψ produced !

Triple parton scattering x-sections (p-p)

- Assuming that the probabilities for 3 hard collisions to be independent of each other, one can again write a pocket-formula for TPS x-section:



$$\sigma_{hh' \rightarrow a_1 a_2 a_3}^{\text{TPS}} = \left(\frac{m}{3!} \right) \frac{\sigma_{hh' \rightarrow a_1}^{\text{SPS}} \cdot \sigma_{hh' \rightarrow a_2}^{\text{SPS}} \cdot \sigma_{hh' \rightarrow a_3}^{\text{SPS}}}{\sigma_{\text{eff,TPS}}^2}$$

normalized by the square of an eff. x-section ($\sigma_{\text{eff,TPS}}^2$) plus a trivial combinatorial factor ($m/3!$) to avoid triple-counting in case of same particles produced: $m = 1$ if $a_1 = a_2 = a_3$; $m = 3$ if $a_1 = a_2$, or $a_1 = a_3$, or $a_2 = a_3$; and $m = 6$ if $a_1 \neq a_2 \neq a_3$.

- How to interpret $\sigma_{\text{eff,TPS}}$? What values one naively expects for it?
- Most generic expression for TPS cross section:

$$\begin{aligned} \sigma_{hh' \rightarrow a_1 a_2 a_3}^{\text{TPS}} = & \left(\frac{m}{3!} \right) \sum_{i,j,k,l,m,n} \int \Gamma_h^{ijk}(x_1, x_2, x_3; \mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3; Q_1^2, Q_2^2, Q_3^2) \\ & \times \hat{\sigma}_{a_1}^{il}(x_1, x'_1, Q_1^2) \cdot \hat{\sigma}_{a_2}^{jm}(x_2, x'_2, Q_2^2) \cdot \hat{\sigma}_{a_3}^{kn}(x_3, x'_3, Q_3^2) \\ & \times \Gamma_{h'}^{lmn}(x'_1, x'_2, x'_3; \mathbf{b}_1 - \mathbf{b}, \mathbf{b}_2 - \mathbf{b}, \mathbf{b}_3 - \mathbf{b}; Q_1^2, Q_2^2, Q_3^2) \\ & \times dx_1 dx_2 dx_3 dx'_1 dx'_2 dx'_3 d^2 b_1 d^2 b_2 d^2 b_3 d^2 b. \end{aligned}$$

Generalized PDFs = $f(x, Q^2, \mathbf{b})$

Triple parton scattering x-sections (p-p)

- Assumption 1: Factorize generalized Triple-PDF into longitudinal &

transverse components:
$$\Gamma_h^{ijk}(x_1, x_2, x_3; \mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3; Q_1^2, Q_2^2, Q_3^2) = D_h^{ijk}(x_1, x_2, x_3; Q_1^2, Q_2^2, Q_3^2) f(\mathbf{b}_1) f(\mathbf{b}_2) f(\mathbf{b}_3),$$

p-p transv. overlap function (mb^{-1}):
$$T(\mathbf{b}) = \int f(\mathbf{b}_1) f(\mathbf{b}_1 - \mathbf{b}) d^2b_1, \text{ with } \int d^2b T(\mathbf{b}) = 1.$$

- Assumption 2: Longitudinal triple-PDF is the product of 3 single PDFs (i.e. no parton correlations in colour, momentum, flavour, spin,...)

$$D_h^{ijk}(x_1, x_2, x_3; Q_1^2, Q_2^2, Q_3^2) = D_h^i(x_1; Q_1^2) D_h^j(x_2; Q_2^2) D_h^k(x_3; Q_3^2)$$

- Then, $\sigma_{\text{eff,TPS}}^2$ is simply the inverse of the cube of the transv. pp overlap:

$$\sigma_{\text{eff,TPS}}^2 = \left[\int d^2b T^3(\mathbf{b}) \right]^{-1}$$

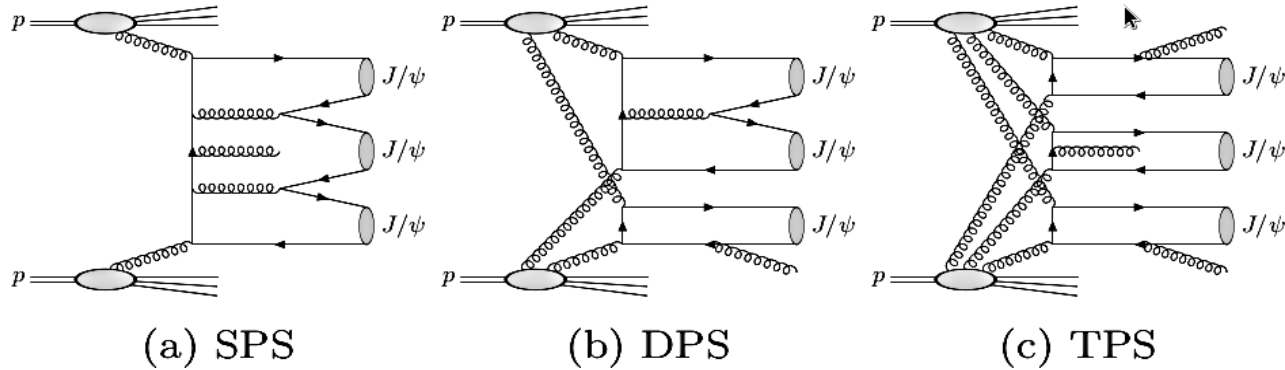
- By testing many proton overlaps/profiles (hard sphere, Gaussian, expo, dipole fit), we find a close relationship between $\sigma_{\text{eff,TPS}}$ & σ_{eff} :

$$\sigma_{\text{eff,TPS}} = k \times \sigma_{\text{eff,DPS}}, \text{ with } k = 0.82 \pm 0.11$$

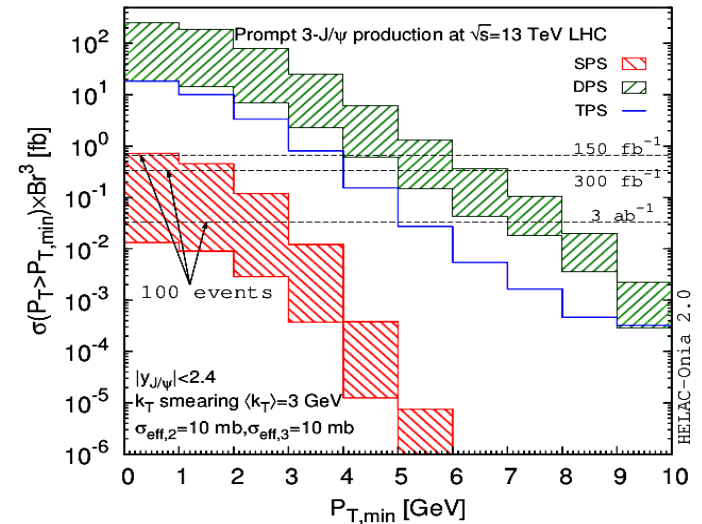
- Measuring TPS provides independent info on σ_{eff} and p transv. profile.

Triple- J/ψ from SPS production (p-p)

- H.-S. Shao et al. [arXiv:1902.04949, PRL 122(2019)192002] computed all triple- J/ψ x-sections with SPS HELAC-ONIA plus our pocket formulas:



		inclusive	$2.0 < y_{J/\psi} < 4.5$	$ y_{J/\psi} < 2.4$
13 TeV	SPS	$0.41^{+2.4}_{-0.34} \pm 0.0083$	$(1.8^{+11}_{-1.5} \pm 0.18) \times 10^{-2}$	$(8.7^{+56}_{-7.5} \pm 0.098) \times 10^{-2}$
	DPS	$(190^{+501}_{-140}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}}$	$(7.0^{+18}_{-5.1}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}}$	$(50^{+140}_{-37}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}}$
	TPS	$130 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)$	$1.3 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)$	$18 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)$
27 TeV	SPS	$0.46^{+2.9}_{-0.39} \pm 0.022$	$(3.2^{+22}_{-2.8} \pm 0.21) \times 10^{-2}$	$(5.8^{+39}_{-5.1} \pm 0.29) \times 10^{-2}$
	DPS	$(560^{+2900}_{-480}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}}$	$(19^{+97}_{-16}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}}$	$(120^{+630}_{-100}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}}$
	TPS	$570 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)$	$5.0 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)$	$57 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)$
75 TeV	SPS	$0.59^{+4.4}_{-0.52} \pm 0.016$	$(3.0^{+25}_{-2.7} \pm 0.23) \times 10^{-2}$	$(7.2^{+63}_{-6.5} \pm 0.38) \times 10^{-2}$
	DPS	$(1900^{+11000}_{-1600}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}}$	$(57^{+340}_{-50}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}}$	$(310^{+2000}_{-270}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}}$
	TPS	$3900 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)$	$27 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)$	$260 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)$
100 TeV	SPS	$1.1^{+8.4}_{-1.0} \pm 0.044$	$(4.5^{+33}_{-4.0} \pm 0.72) \times 10^{-2}$	$(36^{+290}_{-32} \pm 1.8) \times 10^{-2}$
	DPS	$(3400^{+19000}_{-2900}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}}$	$(100^{+550}_{-86}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}}$	$(490^{+3000}_{-430}) \times \frac{10 \text{ mb}}{\sigma_{\text{eff},2}}$
	TPS	$6500 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)$	$45 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)$	$380 \times \left(\frac{10 \text{ mb}}{\sigma_{\text{eff},3}}\right)$



- SPS negligible, DPS (TPS) dominates at low (high) p_T .

Clear sensitivity to σ_{eff} !