Double Parton Scattering in photon-induced reactions

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in collaboration with

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STRONG 2020

Outline

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1. Double Parton Scattering at the LHC and the hadron structure
2. Definition and properties of double parton distributions
3. Recent data and interpretations
4. DPS at the future EIC
5. Nuclear DPS at the LHC (EIC?)
6. Conclusions

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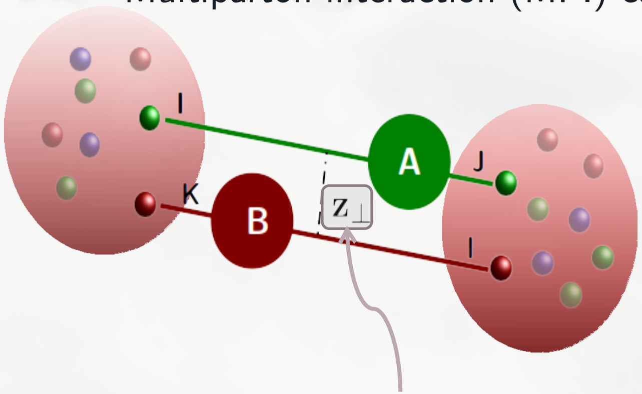
Double Parton Scattering

Multiparton interaction (MPI) can contribute to the, pp and pA, cross section @ the LHC:



Transverse distance between partons

The cross section for a double parton scattering (DPS) event can be written in the following way:

N. Paver, D. Treleani, Nuovo Cimento 70A, 215 (1982)
Mekhfi, PRD 32 (1985) 2371
M. Diehl et all, JHEP 03 (2012) 089

double parton distribution (DPD)

$$d\sigma \propto \int d^2 z_\perp \underbrace{F_{ik}(x_1, x_2, \vec{z}_\perp; \mu_A, \mu_B) \cdot F_{jl}(x_3, x_4, \vec{z}_\perp; \mu_A, \mu_B)}_{\text{double parton distribution (DPD)}}$$

Momentum scales

Momentum fractions carried by the parton inside the proton

DPS processes are important for fundamental studies, e.g. the background for the research of new physics and to grasp information on the **3D PARTONIC STRUCTURE OF THE PROTON**

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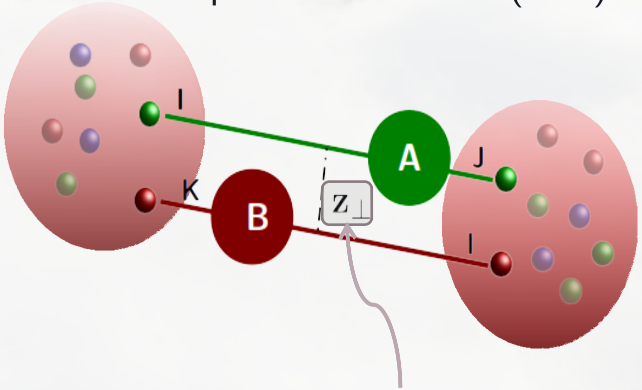

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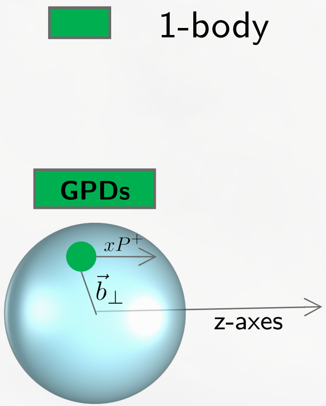
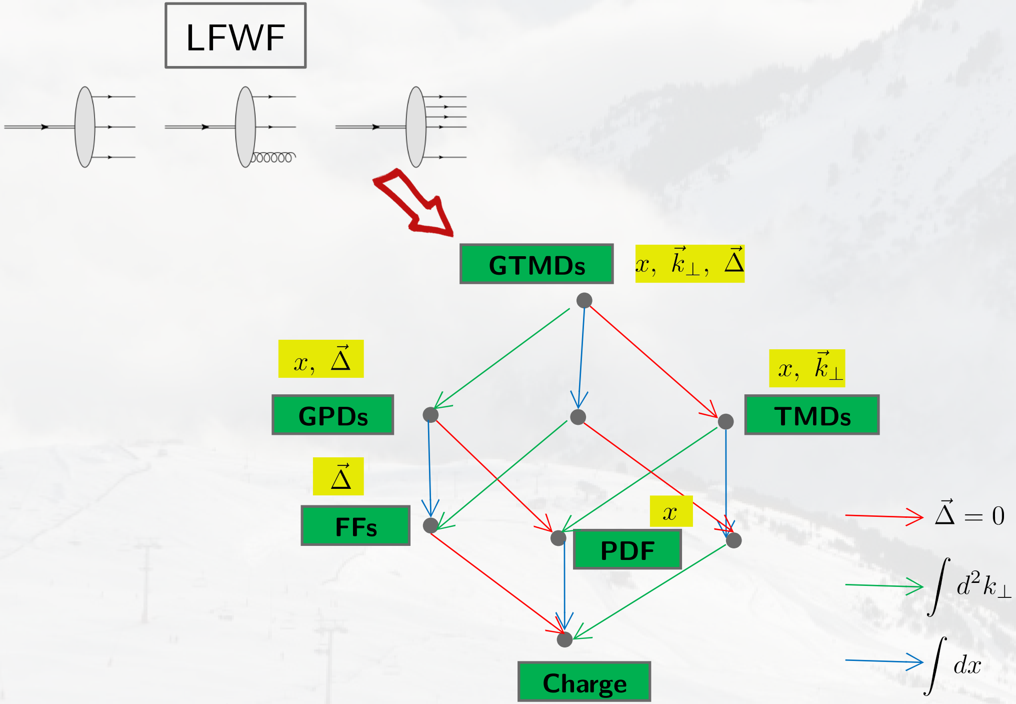
A **formal all-order proof** of the factorization formulae in perturbative QCD **has been achieved for DPS** in the case of a **colorless final state**, both for the TMD and the collinear case. Current status is at the **same level as for the SPS** counterpart.

Nagar's slides MPI 2021

Diehl et al. JHEP 03 (2012) 089, JHEP 01 (2016) 076
Vladimirov JHEP 04 (2018) 045
Buffing et al. JHEP 01 (2018) 044
Diehl, RN JHEP 04 (2019) 124

Multidimensional Pictures of Hadron





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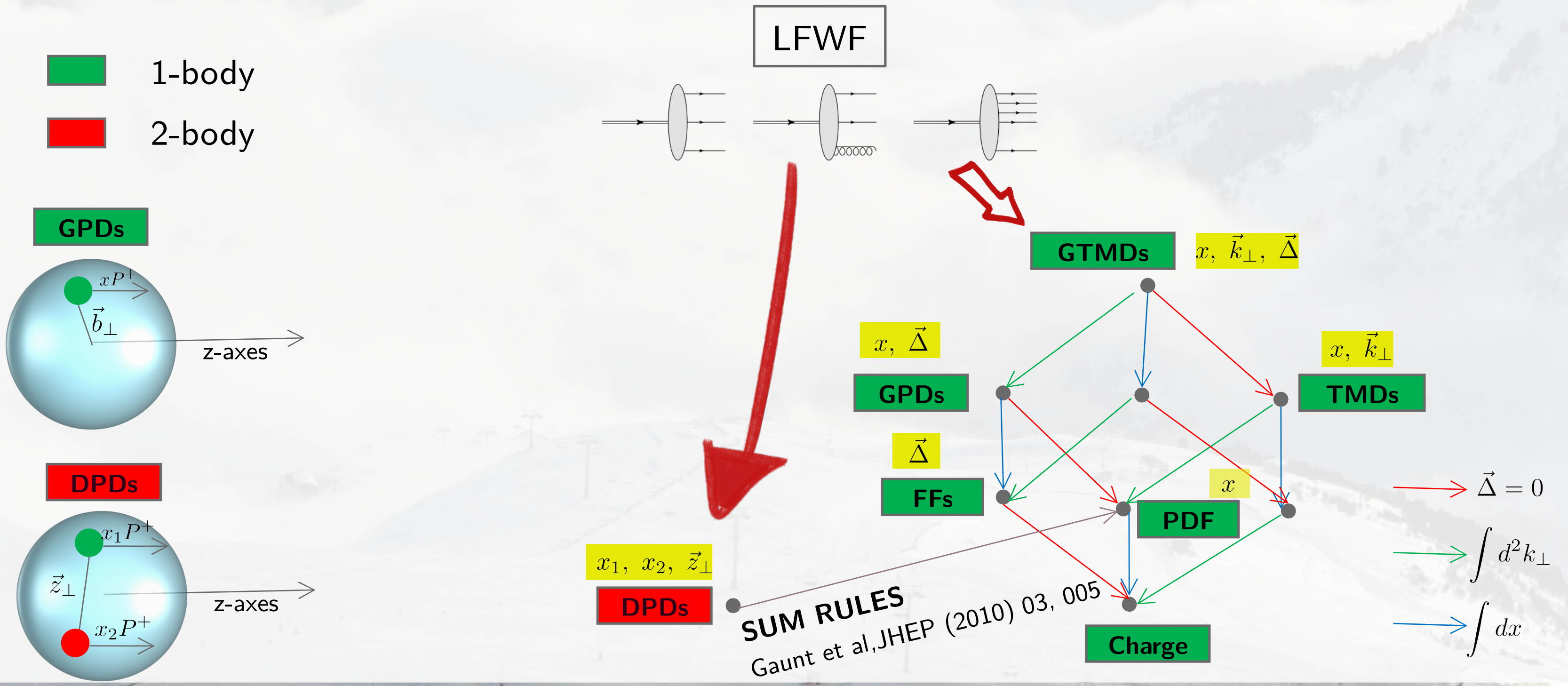

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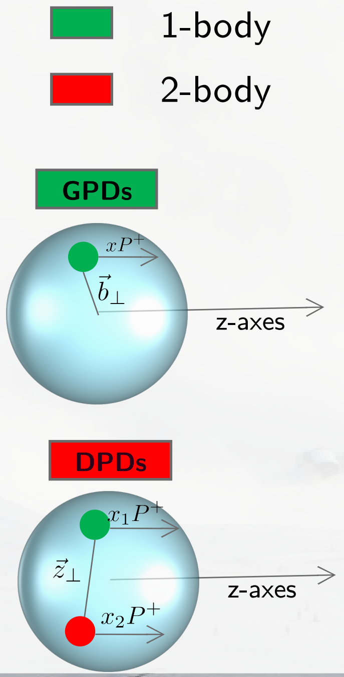
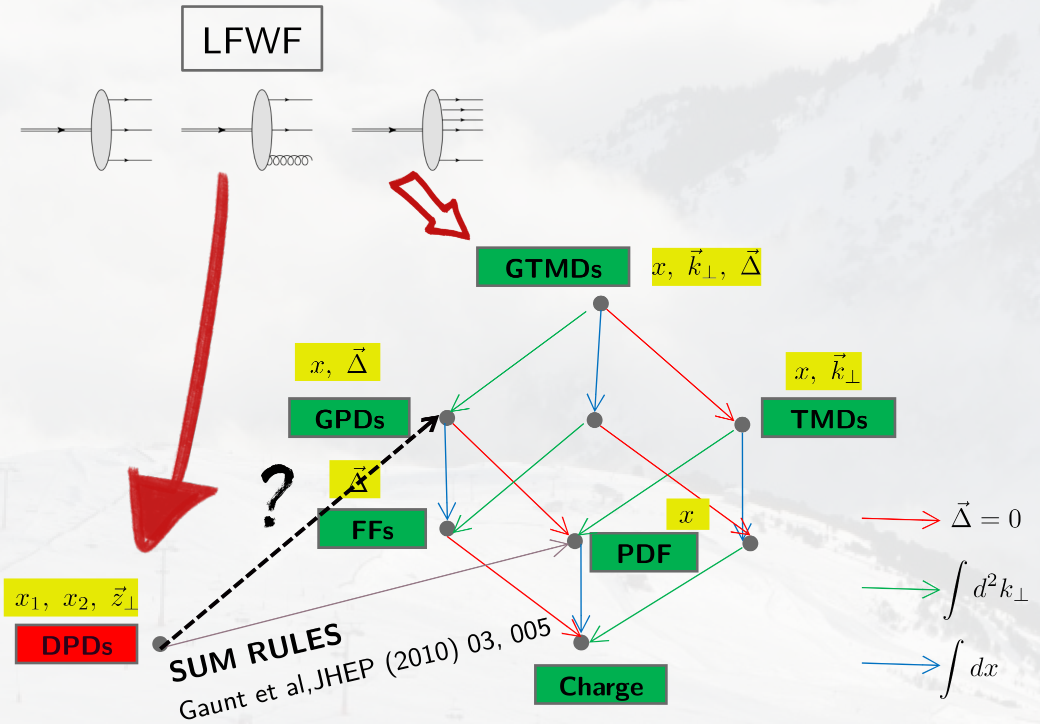
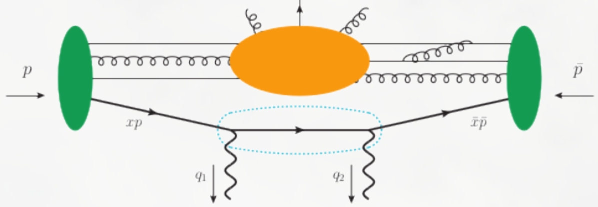


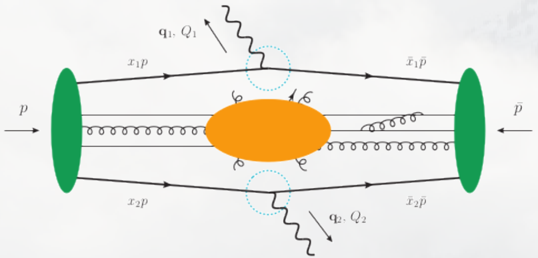


Double Parton Scattering scale

Scale analysis of SPS and DPS processes



$$|\mathbf{q_1^\perp} + \mathbf{q_2^\perp}| \sim \Lambda \ll Q$$



$$|\mathbf{q_1^\perp}| \sim \Lambda \ll Q$$
$$|\mathbf{q_2^\perp}| \sim \Lambda \ll Q$$

where:
- $Q = min(Q_1, Q_2)$
- Λ *transverse momentum scale*
- $\Lambda_{QCD} << \Lambda << Q$

Usually:

$$\frac{\sigma_{DPS}}{\sigma_{SPS}} \sim \mathcal{O}\left(\frac{\Lambda^2}{Q^2}\right)$$

$$\frac{d^2\sigma_{SPS}}{d^2q_1\ d^2q_2} \sim \frac{d^2\sigma_{DPS}}{d^2q_1\ d^2q_2}$$

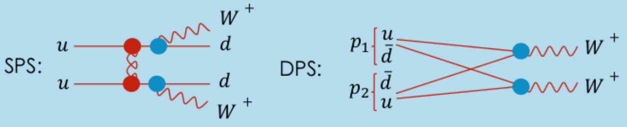
First appearance in theory studies:
Politzer
Paver, Treleani
Mekhfi

Other ground-setting works:
Gaunt, Stirling
Blok et al.
Diehl et al.
Manohar, Waalewijn
Ryskin, Snigierev
...

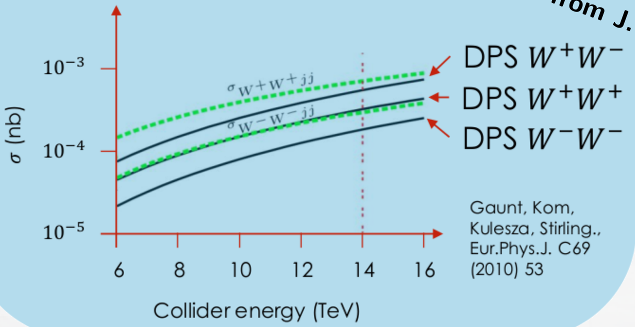
Nagar's slides MPI 2021

Where and why DPS?

DPS can give a significant contribution to processes where SPS is suppressed by small/multiple coupling constants:

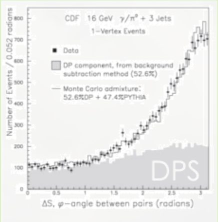


Slide from J. Gaunt



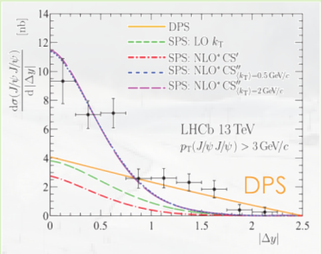
Gaunt, Kom, Kulesza, Stirling., Eur.Phys.J. C69 (2010) 53

...or in certain phase space regions

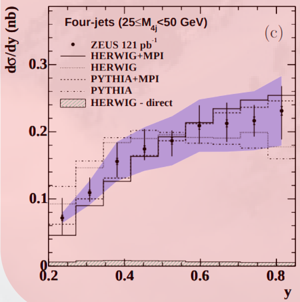


CDF, $\gamma + 3j$, Phys.Rev. D56 (1997) 3811-3832

LHCb, double J/ψ, JHEP 06, 047, (2017)



..or in ep colliders!



HERA data **ZEUS, nucl. phys. B 792,1 (2008)**

Access to:
- double parton correlations
- the transverse distance distribution of partons!!

all **UNKNOWN**

Intrinsically interesting: tells us about **correlations** between partons!

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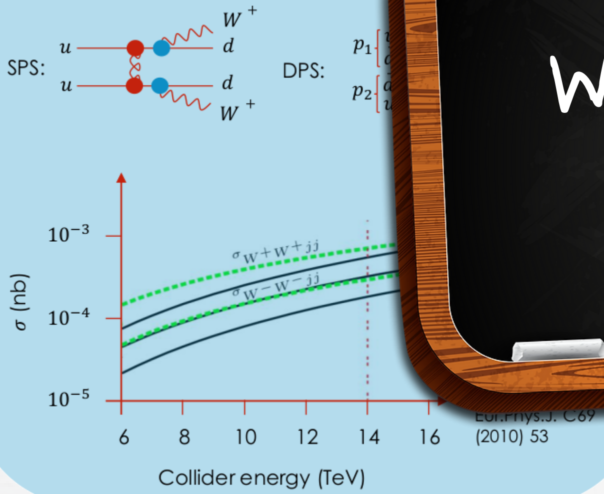

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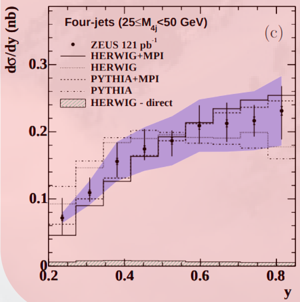

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Some help? Sum Rules

J. R. Gaunt and W. J. Stirling, JHEP 03 (2010) 005
O. Fedkevych and J.R. Gaunt, arXiv:2208.08197
M. Diehl et al, EPJC 80 (2020) 5, 468

Definition of (unpolarized) DPD:

$$F_{a_1a_2}(x_1, x_2, \mathbf{y}_\perp) = 2p^+ \int \frac{dz_1^-}{2\pi}\frac{dz_2^-}{2\pi} dy^- e^{i(x_1z_1^- + x_2z_2^-)p^+} \langle p|\mathcal{O}_{a_2}(0, z_2)\mathcal{O}_{a_1}(y, z_1)|p\rangle$$

with

$$\mathcal{O}_a(y, z) = \bar{\Psi}_a\left(y - \frac{1}{2}z\right)\gamma^+\Psi_a\left(y + \frac{1}{2}z\right)\Big|_{z^+=y^+=\mathbf{z}_\perp=0}$$

Light-Front coordinates are: $v^\pm = v^0 \pm v^3$

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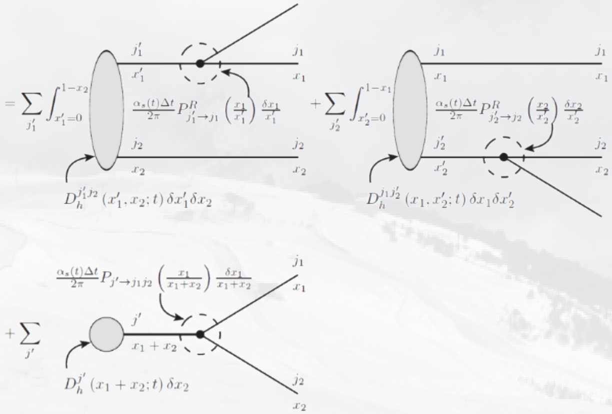
MOMENTUM SUM RULE:

$$\sum_{j_2} \int dx_2 x_2 \, F_{j_1 j_2}(x_1, x_2, Q) = (1 - x_1) f_{j_1}(x_1, Q)$$

DPD integrated over distance — PDF

NUMBER SUM RULE:

$$\int dx_2 \, F_{j_1 j_{2v}}(x_1, x_2, Q) = (N_{j_{2v}} - \delta_{j_1 j_2} + \delta_{j_1 j_2}) f_{j_1}(x_1, Q)$$



Useful to test models or to build up models.

See recent results in O. Fedkevych and J.R. Gaunt, arXiv:2208.08197

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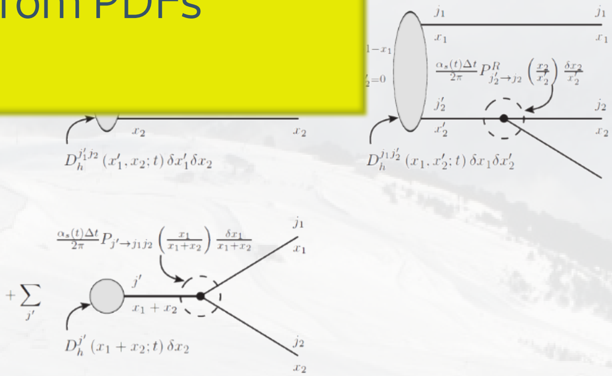
NUMBER SU … $_1, Q)$

We can try to build up DPD from PDFs



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Some help? Can we use PDFs?

$F_{ik}(x_1, x_2, \vec{z}_\perp)$ is unknown. However @LHC kinematics (small x and many partons produced)

$$F_{ik}(x_1, x_2, \vec{z}_\perp) \sim \overbrace{g(x_1, x_2)\tilde{T}(\vec{z}_\perp)}^{\text{1st uncorrelated scenario}}$$

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$g(x_1, x_2)$:

- SUM RULES AVAILABLE
- MODELS BASED ON SUM RULES
- PDF(x_1)*PDF(x_2)
- [PDF(x_1)*PDF(x_2)] $\otimes$ pQCD EVOLUTION

The last two items: **2nd uncorrelated scenario**

PERTURBATIVE CORRELATIONS
GS09 Model

Some help? Can we Constituent Quark models?

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$\tilde{T}(\vec{z}_\perp)$:
- PROBABILITY DISTRIBUTION OF FINDING TWO PARTONS WITH GIVEN TRANSVERSE DISTANCE
- UNKNOWN: ONLY MODELS

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(2nd uncorrelated scenario)

PERTURBATIVE CORRELATIONS
GS09 Model

$F_{ik}(x_1, x_2, \vec{z}_\perp)$ → Constituent quark models used to grasp basic **NON PERTURBATIVE** features

M.R., S. Scopetta et *al*, PRD 87 (2013) 114021
M.R., S, Scopetta et *al*, JHEP 12 (2014) 028
A. V. Manohar et al, PRD 87 (2013) 3, 034009

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1st uncorrelated scenario

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2nd uncorrelated scenario

PERTURBATIVE CORRELATIONS
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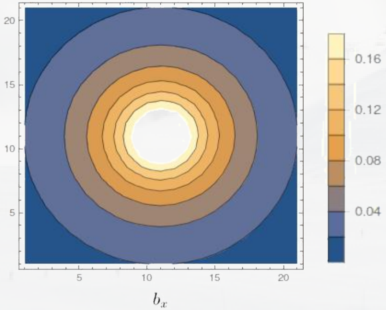
$F_{ik}(x_1, x_2, \vec{z}_\perp)$ → Constituent quark models used to grasp basic **NON PERTURBATIVE** features

$z_\perp = b_\perp$

Distance Distribution of 2 gluons



M.R. and F. A. Ceccopieri, JHEP 09 (2019) 097

Some help? Can we Lattice QCD?

$F_{ik}(x_1, x_2, \vec{z}_\perp)$ is unknown. However @LHC kinematics (small x and many partons produced)

1st uncorrelated scenario

$$F_{ik}(x_1, x_2, \vec{z}_\perp) \sim g(x_1, x_2)\tilde{T}(\vec{z}_\perp)$$

$\tilde{T}(\vec{z}_\perp)$:
- PROBABILITY DISTRIBUTION OF FINDING TWO PARTONS WITH GIVEN TRANSVERSE DISTANCE
- UNKNOWN: ONLY MODELS
- Recently some results have been obtained
  G. S. Bali et al, JHEP 09 (2021) 121

$g(x_1, x_2)$:
- SUM RULES AVAILABLE
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(last two) 2nd uncorrelated scenario

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Some help? Link to Generalized Parton Distributions ?

$$F_{qq}(x_1, x_2, \vec{k}_\perp, Q^2) \approx H^q(x_1, \xi = 0, -k_\perp^2, Q^2) H^q(x_2, \xi = 0, -k_\perp^2, Q^2) +$$
$$+ \frac{k_\perp^2}{4M_p^2} E^q(x_1, \xi = 0, -k_\perp^2, Q^2) E^q(x_2, \xi = 0, -k_\perp^2, Q^2)$$

DPD in k space

Generalized parton distributions (GPDs)
from exclusive processes

M. R., et al, JHEP 10, 063 (2016)
M. Diehl et al, JHEP 03, 089 (2012)
B. Blok et al, EPJC 72 (2012) 1963

We do not know if it is valid!

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DPD in k space

Generalized parton

from exclus

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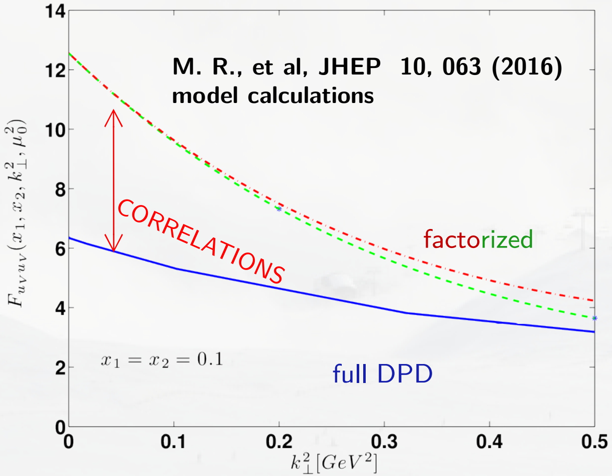

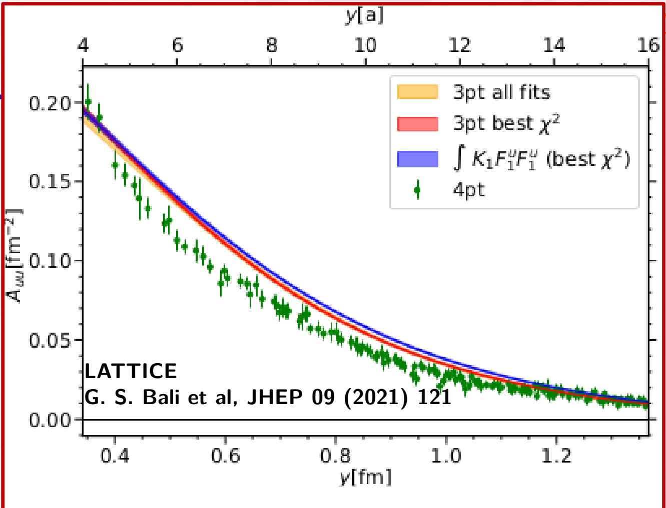

(a) A_{uu} vs $\int F_u F_u$, $py = 0$

Double PDFs contributions:

From pQCD (double DGLAP + inhomogeneus term) analyses we can build the following decomposition:

$$F(z_\perp) = F_{\text{int}}(z_\perp) + F_{\text{sp}}(z_\perp)$$

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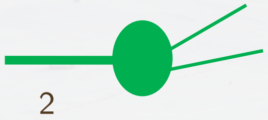

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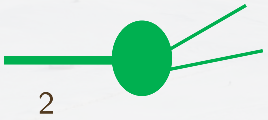

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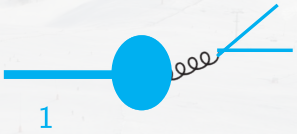

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Diehl et al, JHEP 03 (2012) 089
SciPost, Phys, 7 (2019), 017
JHEP 08 (2021) 040

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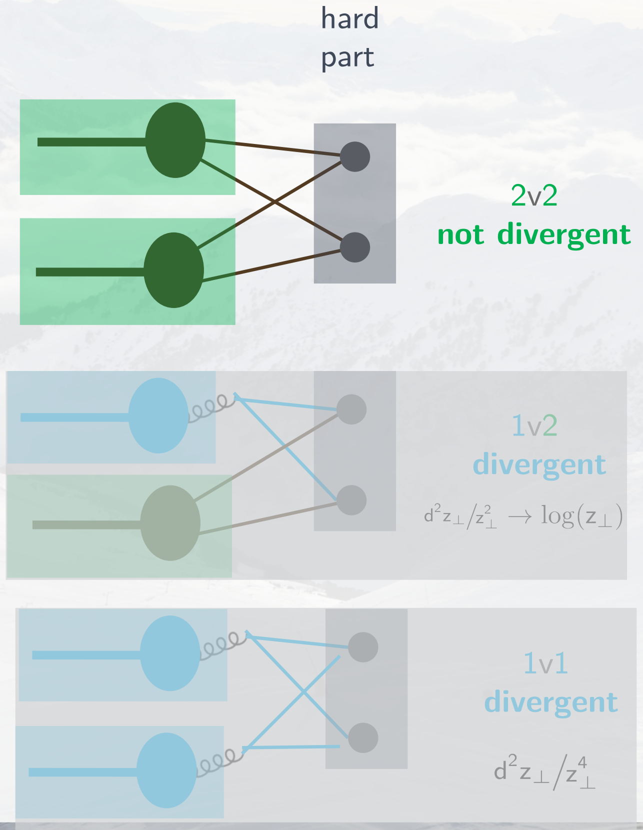

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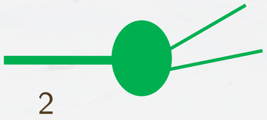

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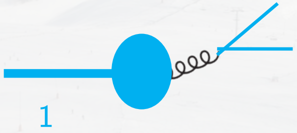

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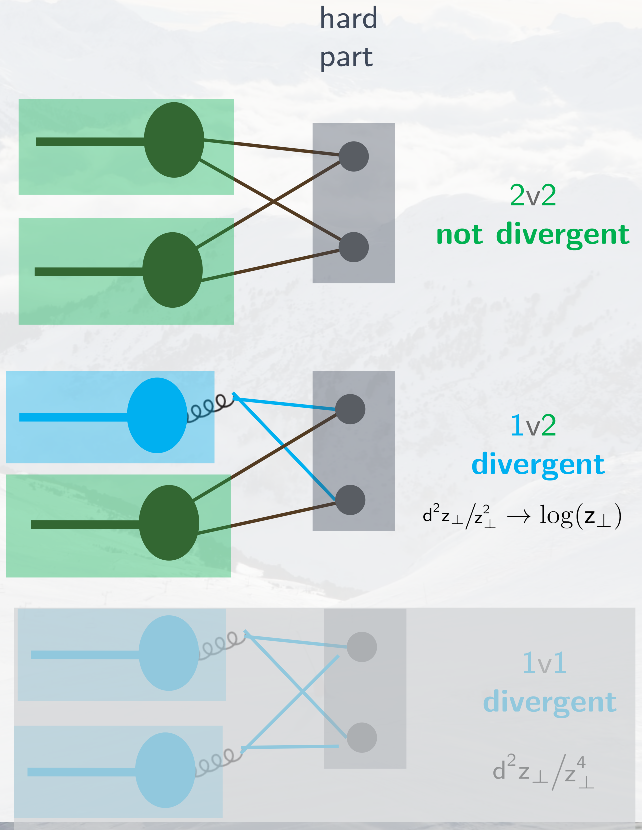

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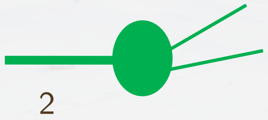

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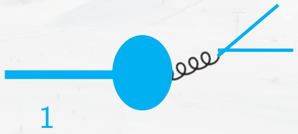

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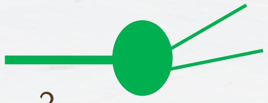

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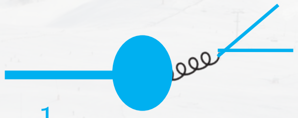
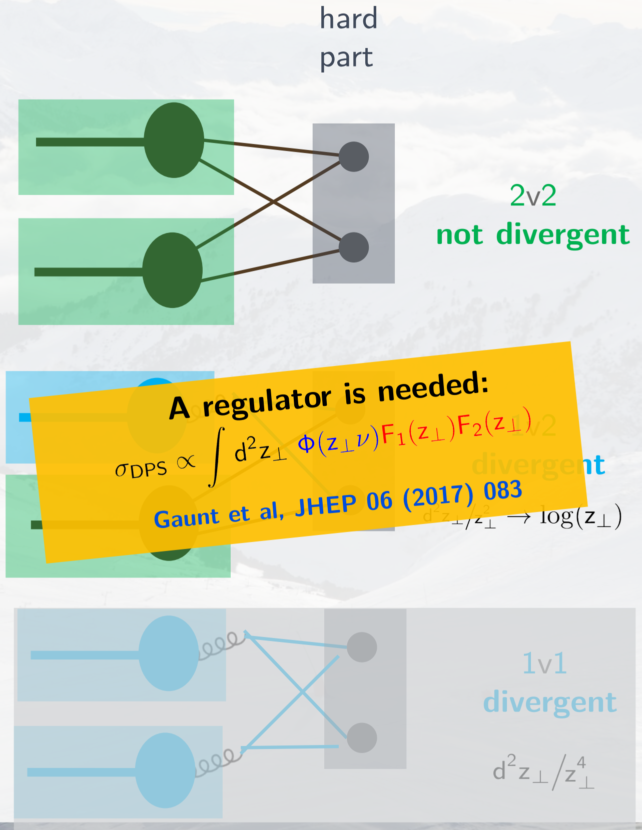


- divergent $1/z_\perp^2$

Diehl et al, JHEP 03 (2012) 089
SciPost, Phys, 7 (2019), 017
JHEP 08 (2021) 040



A regulator is needed:

$\sigma_{\text{DPS}} \propto \int d^2z_\perp\, \Phi(z_\perp\nu) F_1(z_\perp) F_2(z_\perp)$

Gaunt et al, JHEP 06 (2017) 083

Double PDFs contributions:

From pQCD analyses we can build the following decomposition:

$$F(z_\perp) = F_{int}(z_\perp) + F_{sp}(z_\perp)$$

in principle we have the following terms:

The first term:

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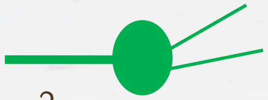
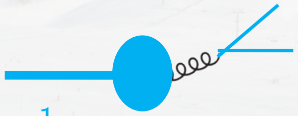


2

- not divergent for $z_\perp \to 0$

The second term:

- it corresponds to:



1

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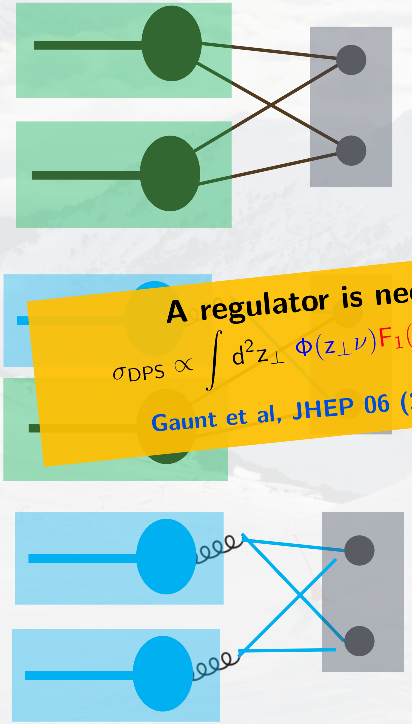
Diehl et al, JHEP 03 (2012) 089
SciPost, Phys, 7 (2019), 017
JHEP 08 (2021) 040



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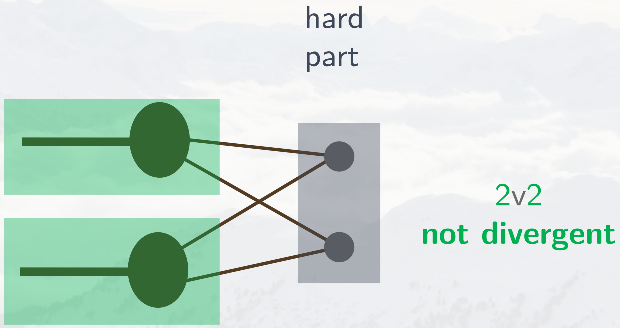

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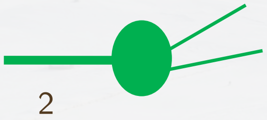

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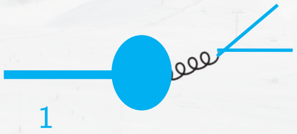

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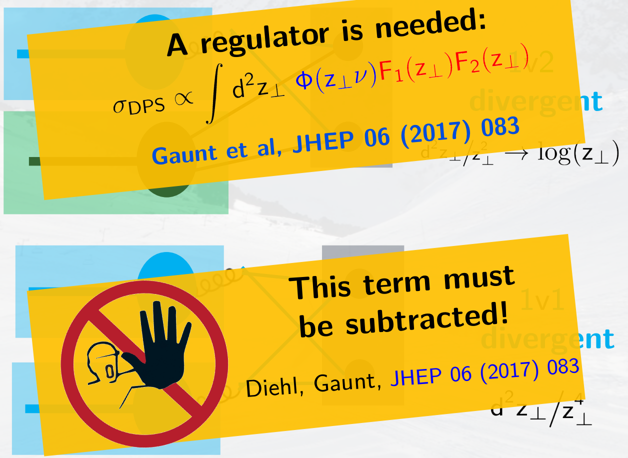

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Gaunt et al, JHEP 06 (2017) 083

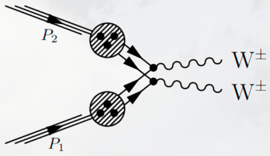
This term must be subtracted!

Diehl, Gaunt, JHEP 06 (2017) 083

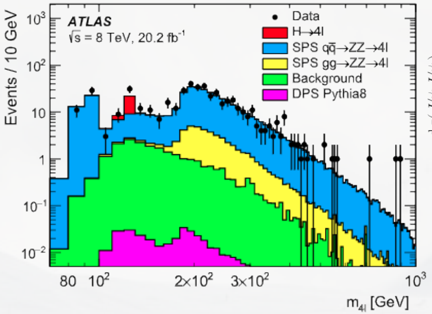
Some Data

Here some experimental and phenomenological analyses. Usually relevant final states are:

WW (same sign are very promising), W+J/Ψ ,J/Ψ+J/Ψ, W+jets, 4 jets, ϒ+3 jets, ZZ....

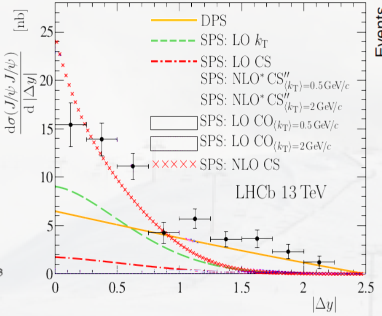


4l



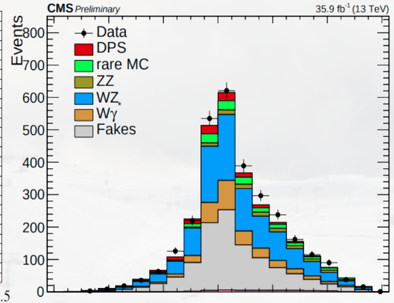
[ATLAS], PLB 790 (2019), 595-614

J/ Ψ pair



[LHCb] JHEP 06, 046 (2017)

Same sign WW



CMS PAS FSQ-16-009

Gaunt et al, EPJC 69 (2010) 53
first pheno. predictions

M.R. et al PRD 95 (2017) 3, 034040
Same sign WW= golden process to access double parton correlations!

T. Kasemets et al, JHEP 10 (2020) 214
Golden process to access spin correlations!

Data and Effective Cross Section

A tool for the comprehension of the role of DPS in hadron-hadron collisions is the so called "effective X-section".

$$\sigma_{\text{eff}}^{pp} = \frac{m}{2} \frac{\sigma_A^{pp} \sigma_B^{pp}}{\sigma_{DPS}^{pp}}$$

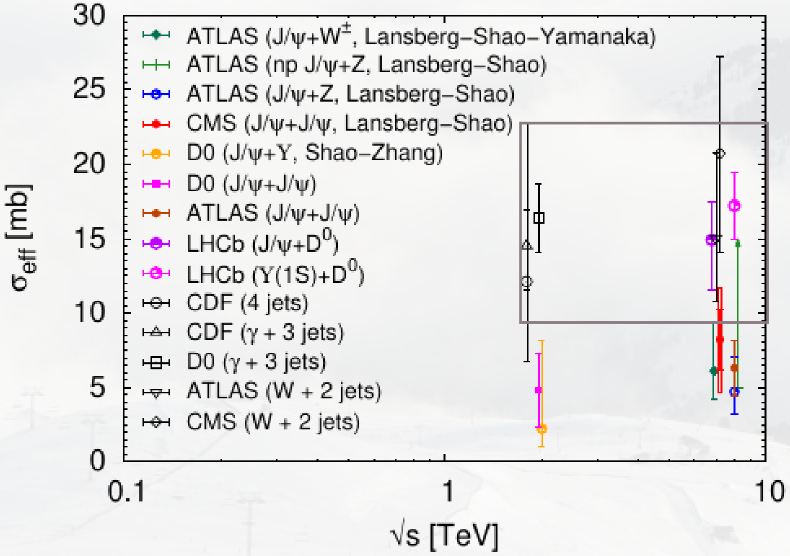
POCKET FORMULA

Differential cross section for the process:
$pp \longrightarrow A(B) + X$

Differential cross section for a DPS event:
$pp \longrightarrow A + B + X$

Data and Effective Cross Section

$$\sigma_{\text{eff}}^{pp} = \frac{m}{2} \frac{\sigma_A^{pp} \sigma_B^{pp}}{\sigma_{DPS}^{pp}}$$



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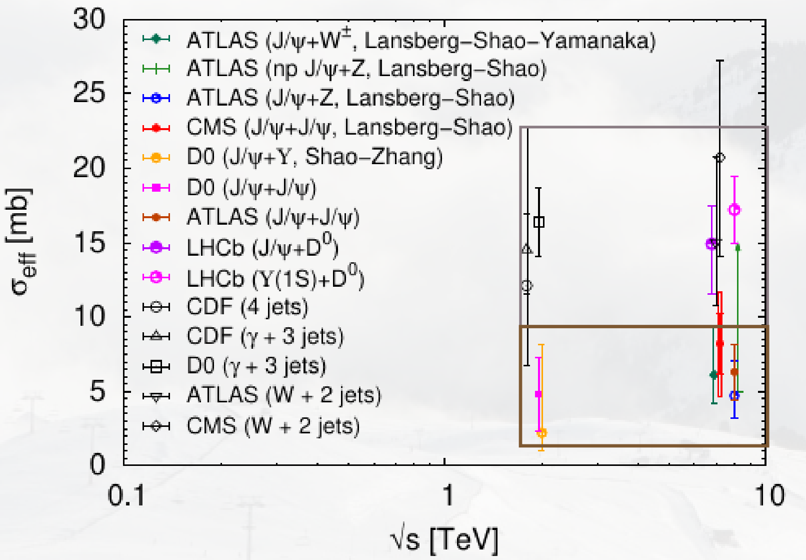
- SENSITIVE TO CORRELATIONS
- PROCESS DEPENDENT?
- SENSITIVE TO INFORMATION ON THE PROTON STRUCTURE?
  As predicted by quark models

M.R. et al PLB 752,40 (2016)
M. Traini, M. R. et al, PLB 768, 270 (2017)
M. R. et al, Phys.Rev. D95 (2017) no.11, 114030



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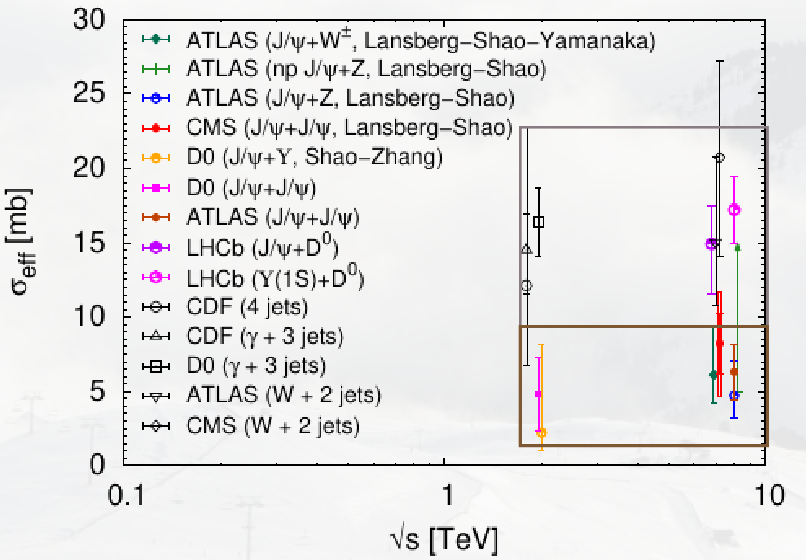

First observation of same sign WW via DPS:

$$\sigma_{\text{eff}} = 12.2^{+2.9}_{-2.2}\ \text{mb}$$

[CMS coll.], arXiv:2206.02681 accepted in PRL

$$\sigma^{\text{DPS}} \sim 6.28\ \text{fb}$$

Clues from data?

M. R. and F. A. Ceccopieri, PRD 97, no. 7, 071501 (2018)

If dPDFs factorize in terms of PDFs then $\sigma_{eff}^{-1} = \int \frac{d^2 k_\perp}{(2\pi)^2} T(k_\perp)^2$ → Effective form factor (EFF)

EFF can be formally defined as **FIRST MOMENT** of DPDs in momentum space

$$T(k_\perp) \propto \int dx_1 dx_2 \, \tilde{F}(x_1, x_2, k_\perp)$$

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$$\langle z_\perp^2 \rangle \propto \frac{d}{k_\perp dk_\perp} T(k_\perp) \Big|_{k_\perp=0}$$

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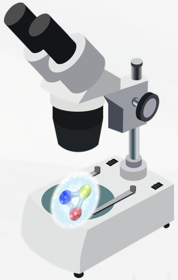
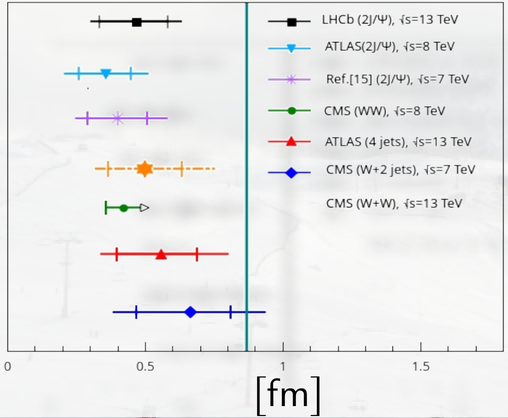


DPS processes:

The vertical line stands for the transverse proton radius



$$\frac{\sigma_{eff}}{3\pi} \leq \langle z_\perp^2 \rangle \leq \frac{\sigma_{eff}}{\pi}$$

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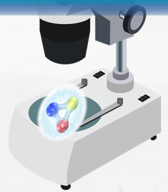
DATA SUGGESTS THAT THE DISTANCE OF THESE PARTONS IS SMALLER THEN THE PROTON RADIUS



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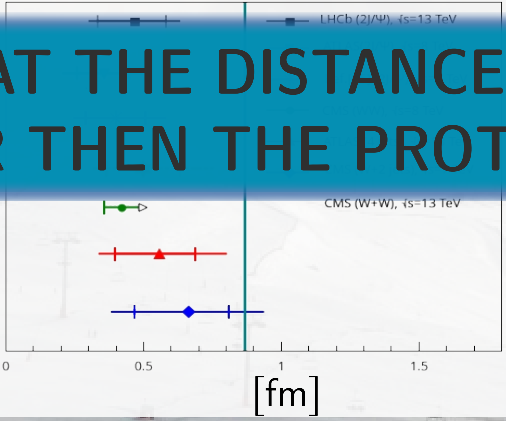

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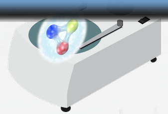
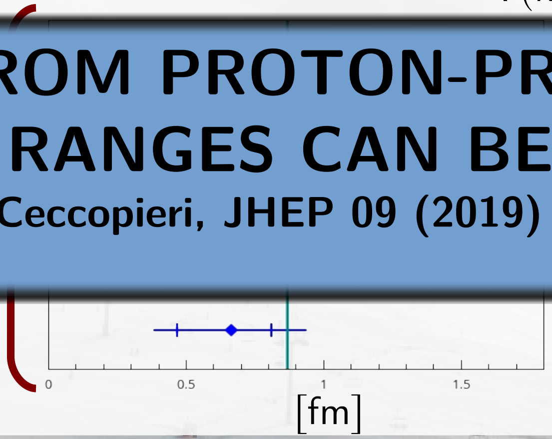


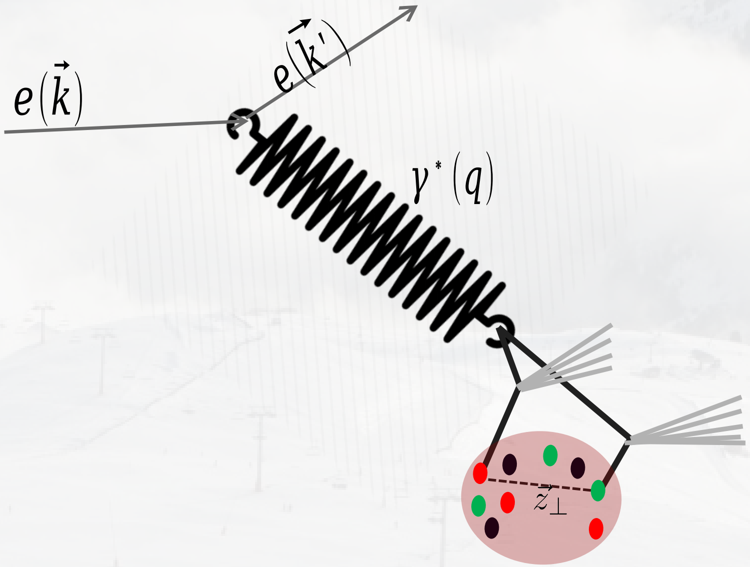


HOWEVER FROM PROTON-PROTON COLLISIONS ONLY RANGES CAN BE ACCESSED

M.R. and F. A. Ceccopieri, JHEP 09 (2019) 097

New Idea: DPS via γ-p interaction

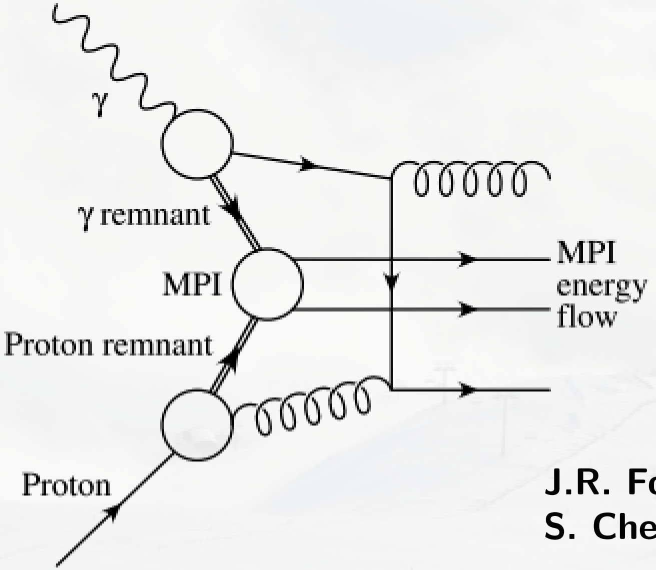
We consider the possibility offered by a DPS process involving a photon FLACTUATING in a quark-antiquark pair interacting with a proton:

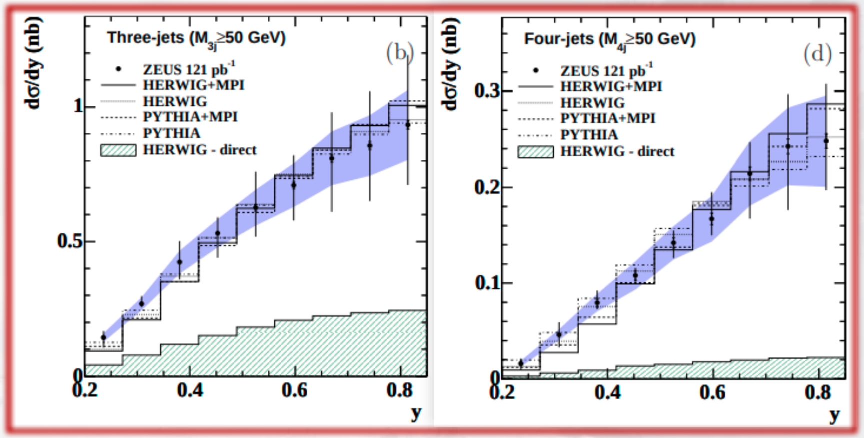


M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

Some Data

We just mention here the importance of MPI for the **3,4 jets photo-production** at HERA:

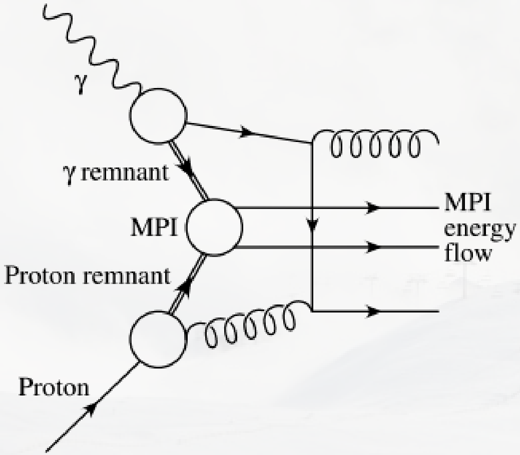




J.R. Forshaw et al, Z phys. C 72, 637
S. Chekanov et al [ZEUS coll.], Nucl. Phys B 792,1 (2008)

New Idea: DPS via γ-p interaction

In order to study the impact of the DPS contribution to a process initiated via photon-proton interactions we evaluated the 4-JET photo-production at HERA **(S. Checkanov et al. (ZEUS), Nucl. Phys B792, 1 (2008))**



*Single Parton Scattering (SPS)

For this first investigation, we make use of the POCKET FORMULA:

$$d\sigma^{4j}_{DPS} = \frac{1}{2}\sum_{ab,cd}\int dy\, dQ^2 \frac{f_{\gamma/e}(y,Q^2)}{\sigma_{eff}^{\gamma p}(Q^2)} \times$$

$$\times \int dx_{p_a} dx_{\gamma_b} f_{a/p}(x_{p_a}) f_{b/\gamma}(x_{\gamma_b}) d\hat{\sigma}^{2j}_{ab}(x_{p_a}, x_{\gamma_b}) \quad \text{SPS*}$$

$$\times$$

$$\times \int dx_{p_c} dx_{\gamma_d} \underbrace{f_{c/p}(x_{p_c})}_{\text{p-PDF}} \underbrace{f_{d/\gamma}(x_{\gamma_d})}_{\gamma\text{-PDF}} d\hat{\sigma}^{2j}_{cd}(x_{p_c}, x_{\gamma_d}) \quad \text{SPS}$$

Flux Factor: P. Nason et al, PLB319 339 (1993)

p-PDF **(J. Pumplin et al. JHEP 07, 012 (2002))**

γ-PDF **(M. Gluck et al. PRD46, 1973 (1992))**

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

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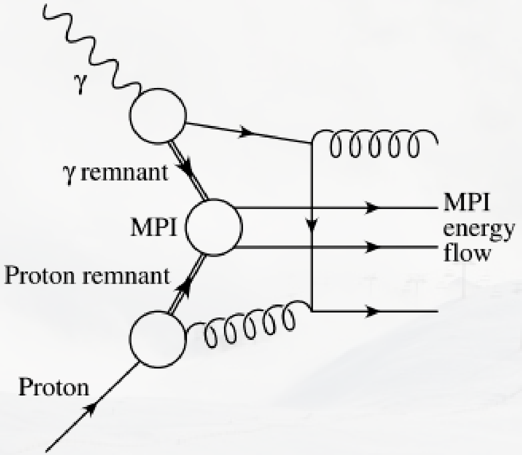

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$$\times$$

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Flux Factor
P. Nason et al,PLB319 339 (1993)

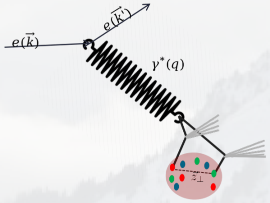
…PDF (M. Gluck et al. PRD46, 1973 (1992))
… (2002))

The main quantity we have to evaluate is:

$$\sigma_{eff}^{\gamma p}(Q^2)$$

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

The γ-p effective cross section



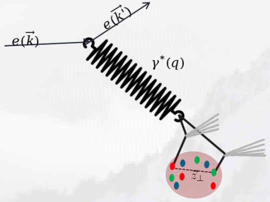
The expression of this quantity is very similar to the proton-proton collision case and can be formally derived by comparing the product of SPS cross sections and the DPS one obtained in **Gaunt, JHEP 01, 042 (2013)** and describing a DPS from a vector bosons splitting with given Q^2 virtuality

Proton EFF

$$\left[\sigma_{\text{eff}}^{\gamma p}(Q^2)\right]^{-1} = \int \frac{d^2 k_\perp}{(2\pi^2)} \boxed{T_p(k_\perp)} T_\gamma(k_\perp; Q^2)$$

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

The γ-p effective cross section



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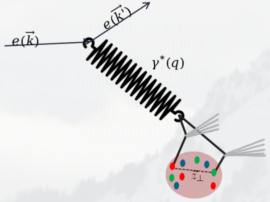
This quantity is similar to an EFF

The full DPS cross section depends on the amplitude of the splitting photon in a $q-\bar{q}$ pair. The latter can be formally described within a Light-Front (LF) approach in terms of LF wave functions.

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

The γ-p effective cross section



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2 $T_p(k_\perp)$ proton EFF

3 ψ^γ Photon WF

For the proton EFF use has been made of three choices:

1) G1: $e^{-\alpha_1 k_\perp^2}$, $\alpha_1 = 1.53\ \text{GeV}^{-2} \Longrightarrow \sigma_{\text{eff}}^{pp} = 15\ \text{mb}$

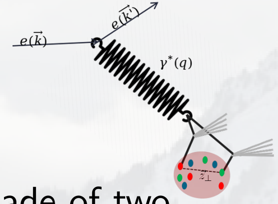
2) G2: $e^{-\alpha_2 k_\perp^2}$, $\alpha_2 = 2.56\ \text{GeV}^{-2} \Longrightarrow \sigma_{\text{eff}}^{pp} = 25\ \text{mb}$

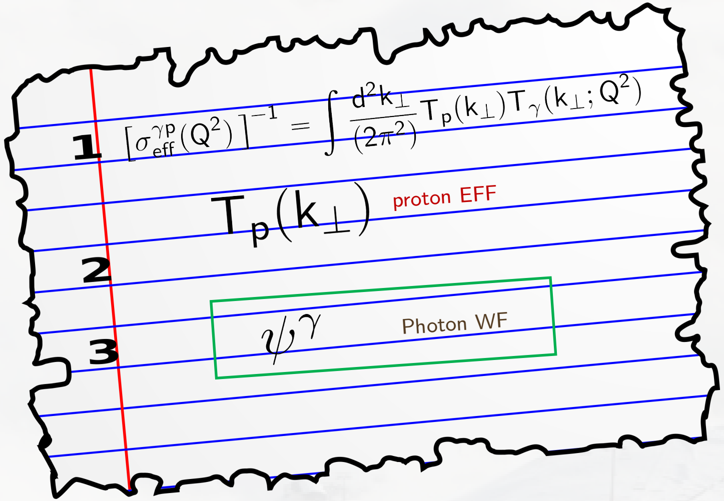
3) S: $\left(1 + \frac{k_\perp^2}{m_g^2}\right)^{-4}$, $m_g^2 = 1.1\ \text{GeV}^2 \Longrightarrow \sigma_{\text{eff}}^{pp} = 30\ \text{mb}$

B. Blok et al, EPJC74, 2926 (2014)

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

The γ-p effective cross section





For the photon W.F. use has been made of two choices representing two extreme cases:

1) QED at LO (S.J. Brodsky et al. PRD50, 3134 (1994)):

$$\psi^{\lambda=\pm}_{q,\bar{q}}(x, k_{1\perp}; Q^2) = -e_f \frac{\bar{u}_q(k)\ \gamma \cdot \varepsilon^\lambda\ v_{\bar{q}}(q-k)}{\sqrt{x(1-x)}\left[Q^2 + \frac{k^2_{1\perp} + m^2}{x(1-x)}\right]}$$

2) Non-Pertubative (NP) effects (E.R.Arriola et al, PRD74,054023 (2006))

$$\psi^\gamma_A(x, k_{\perp 1}; Q^2) = \frac{6(1 + Q^2/m^2_\rho)}{m^2_\rho\left(1 + 4\frac{k^2_{\perp 1} + Q^2 x(1-x)}{m^2_\rho}\right)^{5/2}}$$

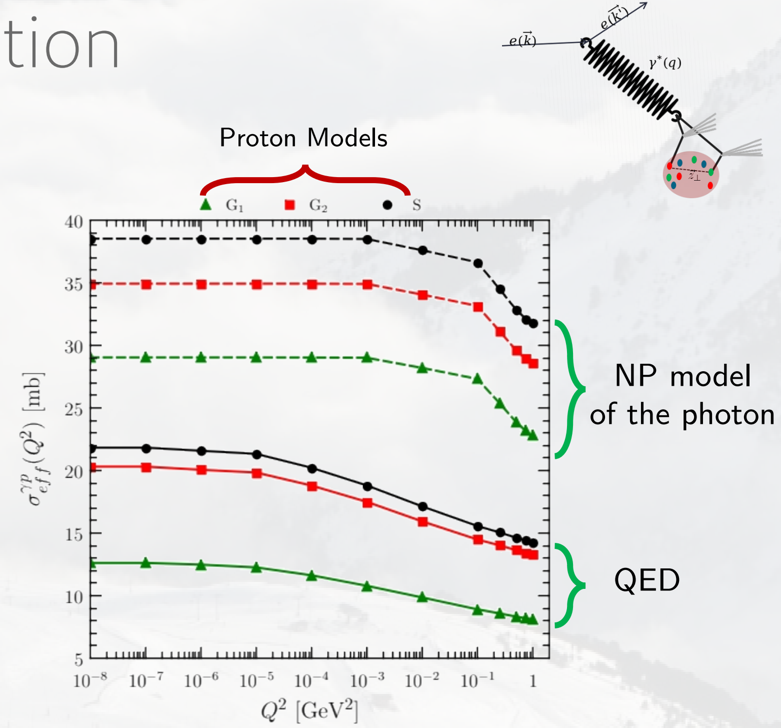
M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

The γ-p effective cross section

1 $\left[\sigma_{\text{eff}}^{\gamma p}(Q^2)\right]^{-1} = \int \frac{d^2k_\perp}{(2\pi^2)} T_p(k_\perp) T_\gamma(k_\perp;Q^2)$

2 $T_p(k_\perp)$ proton EFF

3 ψ_γ Photon WF



M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

The 4-jet DPS cross section

KINEMATICS:

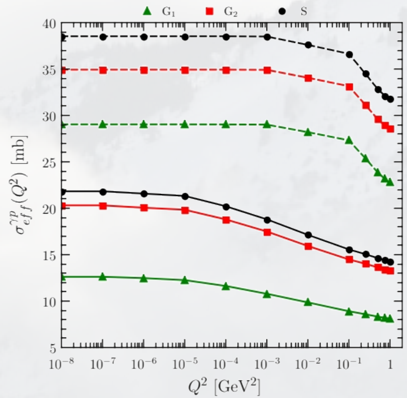
$E_T^{jet} > 6$ GeV

$|\eta_{jet}| < 2.4$

$Q^2 < 1$ GeV^2

$0.2 \leqslant y \leqslant 0.85$

$$d\sigma^{4j}_{DPS} = \frac{1}{2}\sum_{ab,cd}\int dy\, dQ^2\, \frac{f_{\gamma/e}(y,Q^2)}{\sigma_{eff}^{\gamma p}(Q^2)} \times$$
$$\times \int dx_{p_a} dx_{\gamma_b} f_{a/p}(x_{p_a}) f_{b/\gamma}(x_{\gamma_b}) d\hat{\sigma}^{2j}_{ab}(x_{p_a}, x_{\gamma_b})$$
$$\times \int dx_{p_c} dx_{\gamma_d} f_{c/p}(x_{p_c}) f_{d/\gamma}(x_{\gamma_d}) d\hat{\sigma}^{2j}_{cd}(x_{p_c}, x_{\gamma_d})$$



The ZEUS collaboration quoted an integrated total 4-jet cross section of 136 pb

S. Checkanov et al. (ZEUS), Nucl. Phys B792, 1 (2008)

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

The 4-jet DPS cross section

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

KINEMATICS

$E_T^{jet} > 6$ Ge

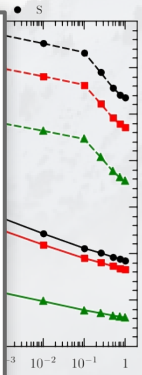
$|\eta_{jet}| < 2.4$

$Q^2 < 1$ GeV

$0.2 \leqslant y \leqslant 0$

	photon	σ_{DPS} [pb] $Q^2 \leq 10^{-2}$ [GeV^2]	$10^{-2} \leq Q^2 \leq 1$ [GeV^2]	$Q^2 \leq 1$ [GeV^2]	$\frac{\sigma_{DPS}}{\sigma_{tot}}$ [%]
NP model	G_1	35.1	18.6	53.7	40
	G_2	29.1	15.2	44.3	33
	S	26.4	13.7	40.1	30
QED	G_1	87.8	54.3	142.1	101
	G_2	54.3	33.4	87.7	65
	S	50.5	31.1	81.6	60

proton



The 4-jet DPS cross section

KINEMATICS

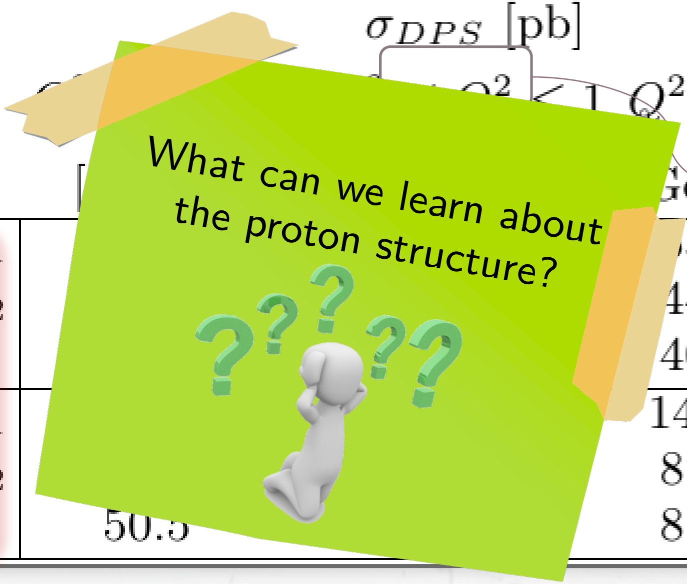
$E_T^{jet} > 6$ GeV

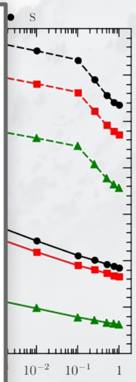
$|\eta_{jet}| < 2.4$

$Q^2 < 1$ GeV

$0.2 \leqslant y \leqslant 0.$

photon	proton		$Q^2 \leq 1$ [GeV^2]	$\frac{\sigma_{DPS}}{\sigma_{tot}}$ [%]
		σ_{DPS} [pb]		
NP model	G_1		3.7	40
	G_2		44.3	33
	S		40.1	30
QED	G_1		142.1	101
	G_2		87.7	65
	S	50.5	81.6	60





The effective cross section: a key for the proton structure

The effective cross section can be also written in terms of Fourier Transform of the EFF:

$$\tilde{F}_2^{\gamma}(z_\perp; Q^2) = \sum_n C_n(Q^2) z_\perp^n$$

$$\left[\sigma_{eff}^{\gamma p}(Q^2)\right]^{-1} = \int d^2 z_\perp \; \tilde{F}_2^{p}(z_\perp) \tilde{F}_2^{\gamma}(z_\perp; Q^2)$$

$$= \sum_n \underbrace{C_n(Q^2)} \boxed{\langle (z_\perp)^n \rangle_p}$$

This coefficient can be determined from the structure of the photon described in a given approach

If we can measure the dependence of the effective-cross section on the photon VIRTUALITY

We could access for the first time the <u>mean transverse distance between partons in the proton</u>

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

The effective cross section: a key for the proton structure

The effective cross section can be also written in terms of Fourier Transform of the EFF:

$\left[\sigma_{\text{eff}}^{\gamma p}(\right.$

We estimated that with an integrated luminosity of 200 pb^{-1} Q^2 effects can be observed

M. R. and F. A. Ceccopieri, PRD 105 (2022) 1, L011501

If we can measure the dependence of the effective-cross section on the photon VIRTUALITY

We could access for the first time the mean transverse distance between partons in the proton

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

Di-J/Ψ photo-production

F. A. Ceccopieri, H. S. Shao, J. P. Lansberg, M. R. and R. Sangem in prep.

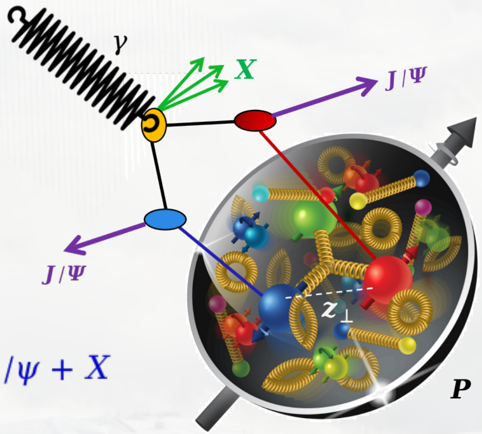


Illustration of DPS for $\gamma + p \to J/\psi + J/\psi + X$

We consider the possibility of **resolved** photon to estimate the DPS cross section in quarkonium-pair photoproduction at the EIC

Di-J/Ψ photo-production

F. A. Ceccopieri, H. S. Shao, J. P. Lansberg, M. R. and R. Sangem in prep.

*Slide from R. Sangem

$$\sigma^{(J/\psi,J/\psi)}_{SPS} \propto \sum_{a=g,q} \int dx_{p_a}\, f_{a/p}(x_{p_a},\mu)\, d\hat{\sigma}^{\gamma a \to J/\psi + J/\psi + a}$$

(Unresolved/direct)

$$\sigma^{(J/\psi,J/\psi)}_{SPS} \propto \sum_{a,b=g,q} \int dx_{\gamma_a}\, dx_{p_b}\, f_{a/\gamma}(x_{\gamma_a},\mu)\, f_{b/p}(x_{p_b},\mu)\, d\hat{\sigma}^{ab \to J/\psi + J/\psi}$$

(Resolved)

Photon PDF

Proton PDF

Partonic X-section

$$\sigma^{(J/\psi,J/\psi)}_{DPS} \propto \frac{1}{2}\frac{1}{\sigma^{\gamma p}_{eff}} \sum_{a,b,c,d} \int dx_{\gamma_a}\, dx_{p_b}\, f_{a/\gamma}(x_{\gamma_a},\mu)\, f_{b/p}(x_{p_b},\mu)\, d\hat{\sigma}^{ab \to J/\psi}_{SPS}(x_{\gamma_a},x_{p_b})$$
$$\times\, dx_{\gamma_c}\, dx_{p_d}\, f_{c/\gamma}(x_{\gamma_c},\mu)\, f_{d/p}(x_{p_d},\mu)\, d\hat{\sigma}^{cd \to J/\psi}_{SPS}(x_{\gamma_c},x_{p_d})$$

Single SPS resolved (namely same partonic cross section as hadroproduction)

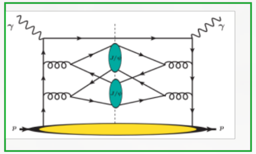
Di-J/Ψ photo-production

F. A. Ceccopieri, H. S. Shao, J. P. Lansberg, M. R. and R. Sangem in prep.

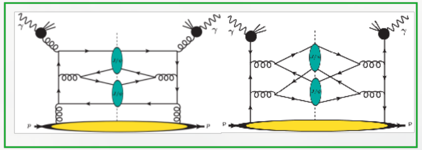
*Slide from R. Sangem

SPS → **Unresolved**, **Resolved**

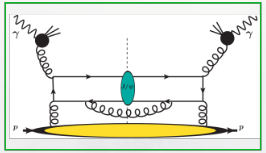
DPS → **Resolved**



$\gamma + q \to J/\psi + J/\psi + q$



$i + j \to J/\psi + J/\psi$ $\quad i, j = g, q$



$|g + g \to J/\psi + g|^2$

- GRV photon PDF is used PRD 46, 1973 (1992) , while CT18NLO PDF for proton T.J. Hou et al., PRD 103, 014013 (2021)
- HELAC-Onia latest version is used for generating matrix elements HS Shao, CPC 184, 2562 (2013), 198, 238 (2016)
- CO LDMEs are taken from M. Butenschoen and B. A. Kniehl, PRD 84, 051501 (2011)
- We expect at least 600 four-muon events with 100 fb^{-1} luminosity

Di-J/Ψ photo-production

F. A. Ceccopieri, H. S. Shao, J. P. Lansberg, M. R. and R. Sangem in prep.

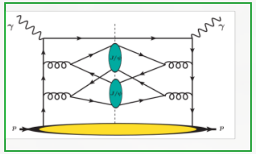
*Slide from R. Sangem

SPS

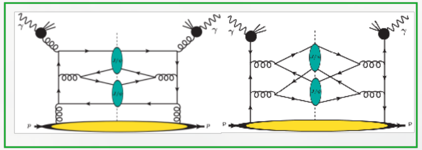
- **Unresolved**
- **Resolved**

DPS

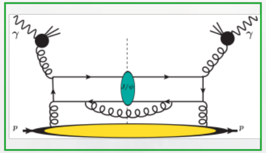
- **Resolved**



$\gamma + q \to J/\psi + J/\psi + q$



$i + j \to J/\psi + J/\psi$ $\quad i, j = g, q$



$|g + g \to J/\psi + g|^2$

Range of cross sections in CSM =100 GeV

$$\sigma_{SPS}^{(J/\psi,J/\psi)} \times Br^2 = 4 - 30 \text{ fb}$$

$$\sigma_{DPS}^{(J/\psi,J/\psi)} \times Br^2 = 0.2 - 5 \text{ fb}$$

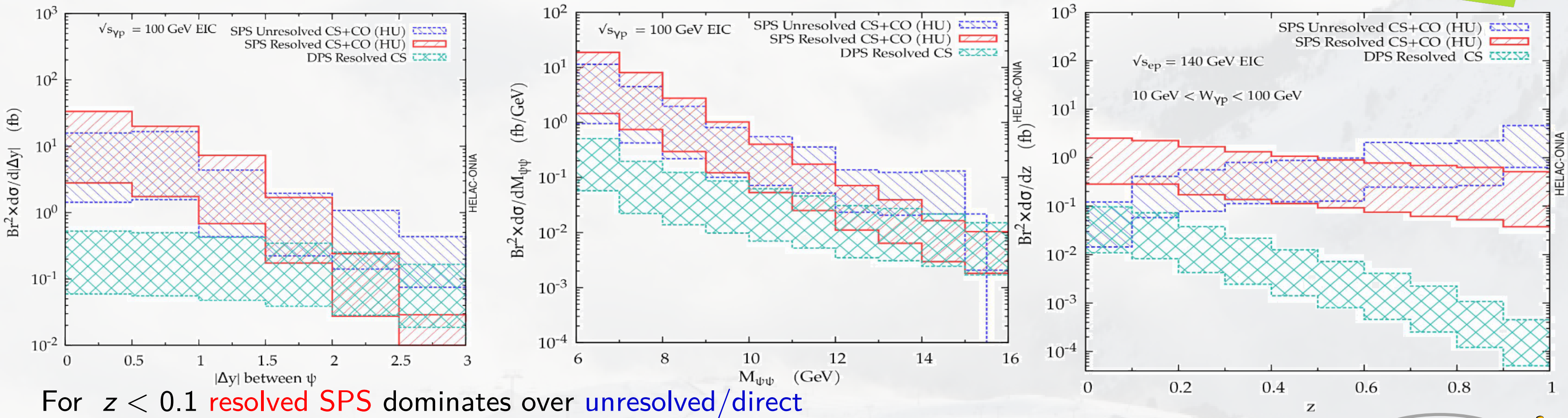
(Resolved) $\sigma_{eff}^{\gamma p} = 10$ mb for DPS

$$\sigma_{SPS}^{(J/\psi,J/\psi)} \times Br^2 = 2 - 12 \text{ fb}$$

(Unresolved)

Di-J/Ψ photo-production (first predictions)

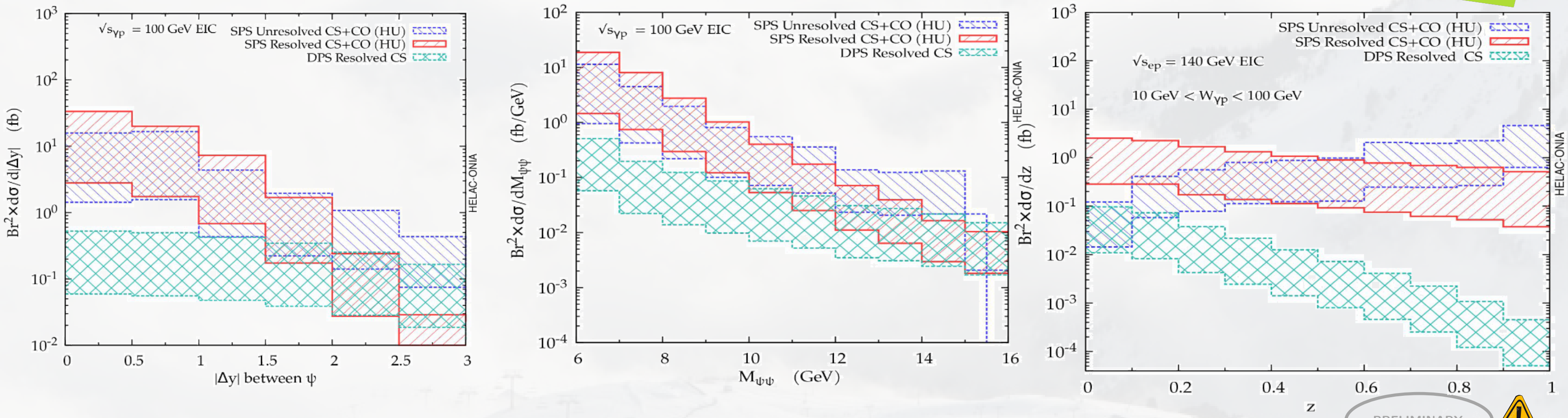
F. A. Ceccopieri, H. S. Shao, J. P. Lansberg, M. R. and R. Sangem in prep.



- For $z < 0.1$ resolved SPS dominates over unresolved/direct
- Unique opportunity to study the photon structure
- At larger z one can test quarkonium production mechanism via direct photoproduction
- Resolved case: gluon channel dominates in the low z region, and quark channel at high z
- CS and CO states are considered: CO states contribution is only significant (for some LDMEs) in unresolved but not in the resolved case

Di-J/ψ photo-production (first predictions)

F. A. Ceccopieri, H. S. Shao, J. P. Lansberg, M. R. and R. Sangem in prep.



PRELIMINARY

DPS COULD BE NOT NEGLIGIBLE BUT FURTHER ANALYSES OF THE ERRORS ON THE DPS CONTRIBUTION ARE ON GOING

DPS in pA collisions

Discussed e.g. in:
M. Strikman and D. Treleani, PRL (2002) 3, 88
E. Cattaruzza, A. D. Fabbro and D. Treleani, PRD 70, 034022 (2004)
D. Treleani and G. Calucci PRD 86, 036003 (2012)

1)Same-sign WW production in proton-nucleus collisions at the LHC as a signal for double parton scattering
D. d'E. & A. Snigirev, PLB 718 (2013) 1395-1400

2)Enhanced J/ΨJ/\PsiJ/Ψ-pair production from double parton scatterings in nucleus-nucleus collisions at the Large Hadron Collider
D. d'E. & A. Snigirev, PLB 727 (2013) 157-162

3)Pair production of quarkonia and electroweak bosons from double-parton scatterings in nuclear collisions at the LHC
D. d'E. & A. Snigirev, Nucl. Phys. A 931 (2014) 303-308

and for TPS:

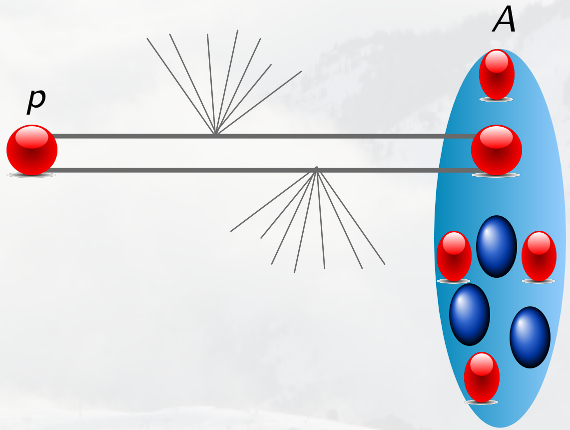
Triple-parton scatterings in proton–nucleus collisions at high energies
D. d'E. & A. Snigirev, EPJC 78 (2018) 5, 359

DPS in pA collisions

In this case we have two mechanisms that contribute:

DPS 1

$$F_{a_1a_2}(x_1, x_2, \mathbf{y}_\perp) = 2p^+ \int \frac{dz_1^-}{2\pi}\frac{dz_2^-}{2\pi} dy^- e^{i(x_1z_1^- + x_2z_2^-)p^+} \times \langle A|\mathcal{O}_{a_2}(0, z_2)\mathcal{O}_{a_1}(y, z_1)|A\rangle$$



The two partons belong to the same nucleon inside the nucleus!

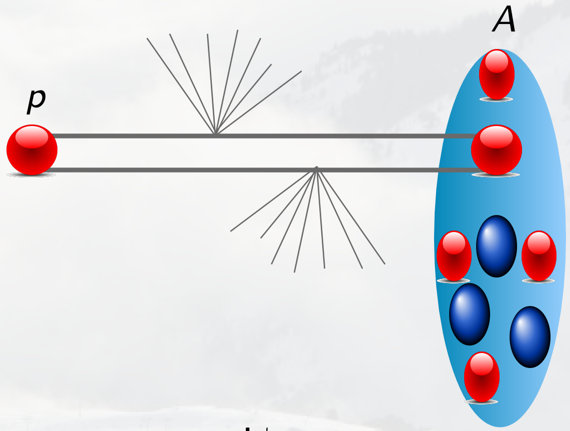
DPS in pA collisions

In this case we have two mechanisms that contribute:

B. Blok et al, EPJC (2013) 73:2422

DPS 1

$$F_{a_1 a_2}(x_1, x_2, \mathbf{y}_\perp) = 2p^+ \int \frac{dz_1^-}{2\pi} \frac{dz_2^-}{2\pi} dy^- e^{i(x_1 z_1^- + x_2 z_2^-)p^+} \times \langle A | \mathcal{O}_{a_2}(0, z_2) \mathcal{O}_{a_1}(y, z_1) | A \rangle$$



The two partons belong to the same nucleon inside the nucleus!

$$\tilde{F}^1_{a_1 a_2}(x_1, x_2, k_\perp) = \sum_{N=p,n} \int \frac{1}{\xi^2} \tilde{F}^N_{a_1 a_2}\left(\frac{x_1}{\xi}, \frac{x_2}{\xi}, k_\perp\right) \rho_A^N(\xi, p_{t,N}) \frac{d\xi}{\xi} d^2 p_{t,N}$$

Conjugate variable to $\mathbf{y}_\perp$

long. momentum fraction carried from the nucleon

Light-cone momentum Nucleon density

Transverse momentum of the nucleon

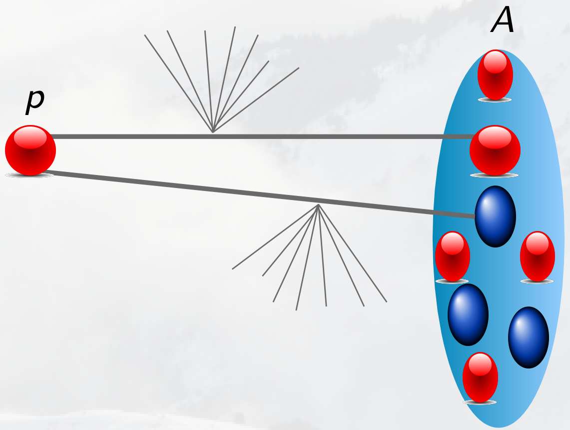
DPS in pA collisions

In this case we have two mechanisms that contribute:

B. Blok et al, EPJC (2013) 73:2422

DPS 2

$$F_{a_1a_2}(x_1,x_2,\mathbf{y}_\perp) = 2p^+\int \frac{dz_1^-}{2\pi}\frac{dz_2^-}{2\pi}dy^- e^{i(x_1z_1^-+x_2z_2^-)p^+}$$
$$\times\langle A|\mathcal{O}_{a_2}(0,z_2)\mathcal{O}_{a_1}(y,z_1)|A\rangle$$



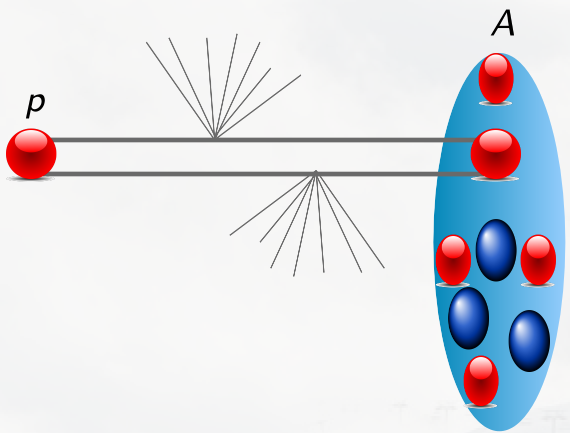
The two partons belong
To 2 different nucleons inside the
nucleus!

$$\tilde{F}^2_{a_1a_2}(x_1,x_2,\vec{k}_\perp) \propto \int \frac{1}{\xi_1\xi_2}\prod_{i=1}^{i=A}\frac{d\xi_i d^2p_{ti}}{\xi_i}\delta\left(\sum_i \xi_i - A\right)\delta^{(2)}\left(\sum_i \mathbf{p}_{ti}\right)\psi_A^*(\xi_1,\xi_2,p_{t1},p_{t2},\ldots)$$ Nucleus wave-function

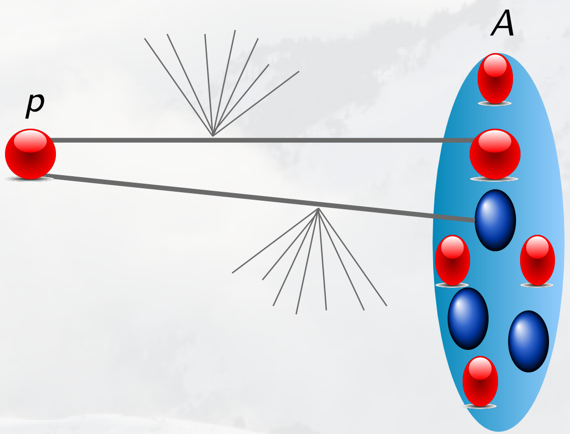
$$\times\psi_A\left(\xi_1,\xi_2,p_{t1}+\vec{k}_\perp,p_{t2}-\vec{k}_\perp,\ldots\right)G_{a_1}^{N_1}\left(x_1/\xi_1,|\vec{k}_\perp|\right)G_{a_2}^{N_2}\left(x_2/\xi_2,|\vec{k}_\perp|\right)$$ Nucleon GPDs

DPS in pA collisions

In this case we have two mechanisms that contribute:



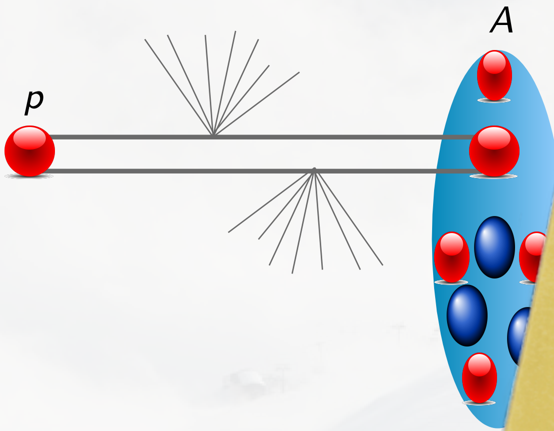
DPS 1



DPS 2

DPS in pA collisions

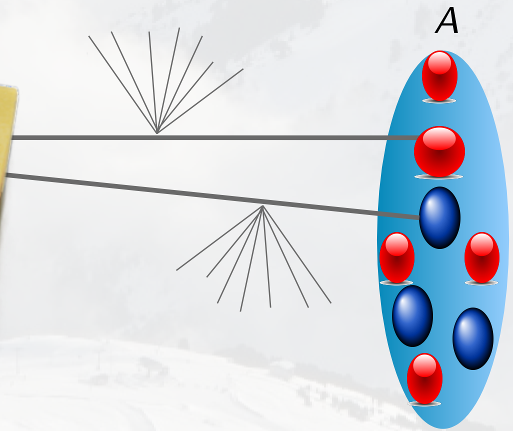
In this case we have two mechanisms that contribute:



DPS 1

$$\sigma_{DPS2} \sim A^{1/3}\sigma_{DPS1}$$

$$\sigma_{DPS1} \sim A\sigma^{pp}_{DPS}$$



DPS 2

DPS in pA collisions

Slides from:
- **Boris Blok**
- **Federico Alberto Ceccopieri**

The DPS cross-section

$$d\sigma^{ML}_{DPS} = \frac{m}{2} \sum_{i,j,k,l} d\hat{\sigma}^{M}_{ik} d\hat{\sigma}^{L}_{jl} \int d^2 b_\perp \, F^{ij}_{p}(x_1, x_2, \vec{b}_\perp) \int d^2 B \Bigg\{$$

The ingredients:

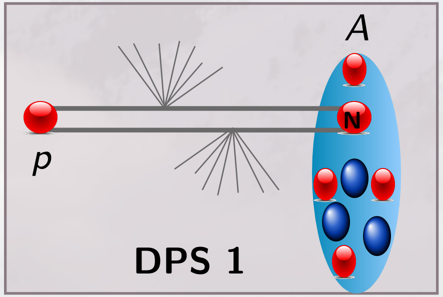
- the nucleon dPDFs= PDF x PDF x $\tilde{T}(b_\perp)$

$$\int d^2 b_\perp \, \tilde{T}(b_\perp) = 1 \quad \int d^2 b_\perp \, \tilde{T}(b_\perp)^2 = 1/\sigma^{pp}_{eff}$$

$$\sigma^{pp}_{eff} \sim 18 \pm 6 \text{ mb}$$

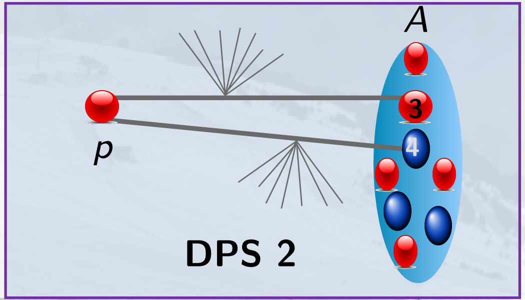
- the contribution of nucleon to the nuclear PDF

$$\sum_{N=p,n} F^{kl}_{N}(x_3, x_4, \vec{b}_\perp) \bar{T}_N(B)$$



$$+$$

$$\sum_{N_3,N_4=p,n} \boxed{f^k_{N_3/A}(x_3)} f^l_{N_4/A}(x_4) \bar{T}_{N_3}(B) \bar{T}_{N_4}(B)$$



$$\Bigg\}$$

DPS in pA collisions

Slides from:
- Boris Blok
- Federico Alberto Ceccopieri

The DPS cross-section

$$d\sigma^{ML}_{DPS} = \frac{m}{2}\sum_{i,j,k,l} d\hat{\sigma}^M_{ik} d\hat{\sigma}^L_{jl} \int d^2b_\perp \, F^{ij}_p(x_1, x_2, \vec{b}_\perp) \int d^2B \Bigg\{ \sum_{N=p,n} F^{kl}_N(x_3, x_4, \vec{b}_\perp) \bar{T}_N(B) + \sum_{N_3,N_4=p,n} f^k_{N_3/A}(x_3) f^l_{N_4/A}(x_4) \bar{T}_{N_3}(B) \bar{T}_{N_4}(B) \Bigg\}$$

The ingredients:
- the nucleon dPDFs= PDF x PDF x $\tilde{T}(b_\perp)$

$$\int d^2b_\perp \, \tilde{T}(b_\perp) = 1 \qquad \int d^2b_\perp \, \tilde{T}(b_\perp)^2 = 1/\sigma^{pp}_{eff}$$

$$\sigma^{pp}_{eff} \sim 18 \pm 6 \text{ mb}$$

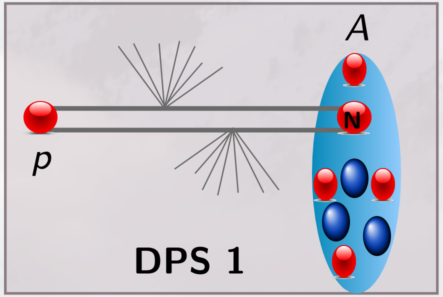
- the contribution of nucleon to the nuclear PDF

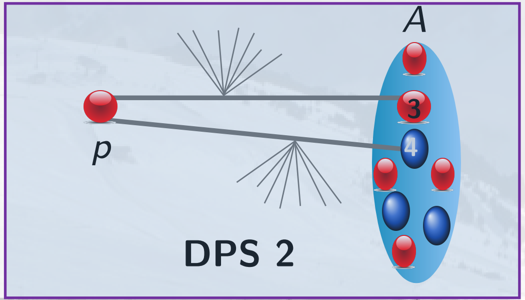
- the thickness function as a function of the impact parameter *B*:

$$\bar{T}(\vec{b}_\perp + \vec{B}) \sim \bar{T}(\vec{B})$$

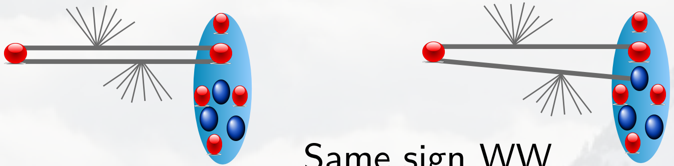
$$\bar{T}_N(B) = \int dz \, \underbrace{\rho_N}(\sqrt{B^2 + z^2})$$

Wood-Saxon distribution for pb normalized to A



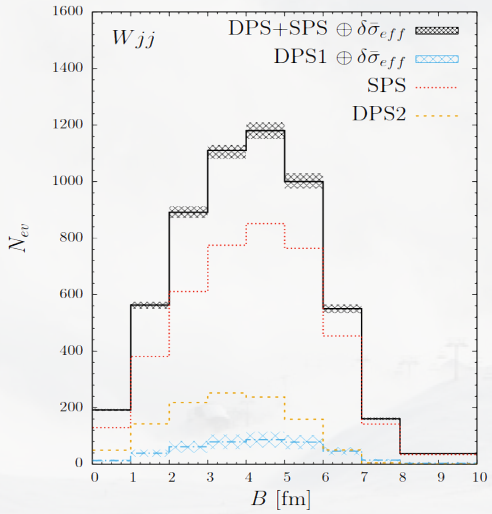


DPS in pA collisions



W+di-jets

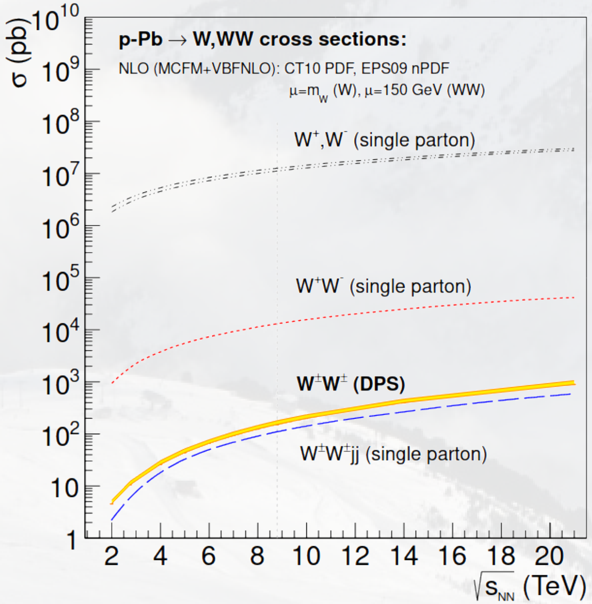
B. Blok and F. A. Ceccopieri EPJC (2020) 80, 278



- SPS dominant
- DPS2 bigger then DPS1 has expected

Same sign WW

D. D'Enterria and Snigirev, PLB 718 (2013) 1395-1400



What about DPS in γA collisions with light nuclei?

In p-Pb collisions there are some difficulties:

1) both cross-sections (DPS1 and DPS2) depends on proton DPD (still almost unknown) therefore both mechanisms are very important → could be difficult to extract some information on the proton DPD

2) for heavy nuclei is difficult to perform calculation with wave-function obtained from realistic potentials

POSSIBLE SOLUTION?

1) In γA the DPS2 will not contain any DPD of the proton → this mechanism can now be viewed as a background that can be evaluated if we properly treat the photon (as previously discussed) and the Nuclear geometry

↓

2) For light nuclei these calculations can be done starting from realistic wave-function (Av18 or chiral potential)

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⬇

2) For light nuclei these calculations can be done starting from realistic wave-function (Av18 or chiral potential)!

What about DPS in γA collisions with light nuclei?

For example in DPS1:

$$\tilde{F}^{1}_{a_1 a_2}(x_1, x_2, k_\perp) = \sum_{N=p,n} \int \frac{1}{\xi^2} \tilde{F}^{N}_{a_1 a_2}\left(\frac{x_1}{\xi}, \frac{x_2}{\xi}, k_\perp\right) \rho^{N}_{A}(\xi, p_{t,N}) \frac{d\xi}{\xi} d^2 p_{t,N}$$

The nuclear light-cone distribution can be evaluated with realistic wave-function
In a fully relativistic and Poincaré covariant approach for:

1) H^2 in E. Pace and G. Salmé, TNPI2000 (2001), arXiv:nucl-th/0106004

2) He^3 in e.g. A. Del Dotto et al, PRC 95, 014001 (2017)

3) He^4 work in progress

What about DPS in γA collisions with light nuclei?

For example in DPS2:

$$\tilde{F}^2_{a_1a_2}(x_1, x_2, \vec{k}_\perp) \propto \int \frac{1}{\xi_1\xi_2} \prod_{i=1}^{i=A} \frac{d\xi_i d^2 p_{ti}}{\xi_i} \delta\left(\sum_i \xi_i - A\right) \delta^{(2)}\left(\sum_i \mathbf{p}_{ti}\right) \psi^*_A(\xi_1, \xi_2, p_{t1}, p_{t2}) \psi_A\left(\xi_1, \xi_2, p_{t1} + \vec{k}_\perp, p_{t2} - \vec{k}_\perp\right)$$
$$\times G^{N_1}_{a_1}\left(\frac{x_1}{\xi_1}, |\vec{k}_\perp|\right) G^{N_2}_{a_2}\left(\frac{x_2}{\xi_2}, |\vec{k}_\perp|\right);$$

B. Blok et al, EPJC (2013) 73:2422

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$$\times\, G_{a_1}^{N_1}\left(\frac{x_1}{\xi_1},|\vec{k}_\perp|\right)G_{a_2}^{N_2}\left(\frac{x_2}{\xi_2},|\vec{k}_\perp|\right);$$

$$\underset{\xi_i\sim 1}{\sim} G_{a_1}^{N_1}\left(x_1,|\vec{k}_\perp|\right)G_{a_2}^{N_2}\left(x_2,|\vec{k}_\perp|\right)$$

$$\times\left[\int \frac{1}{\xi_1\xi_2}\prod_{i=1}^{i=A}\frac{d\xi_i d^2p_{ti}}{\xi_i}\delta\left(\sum_i \xi_i - A\right)\delta^{(2)}\left(\sum_i \mathbf{p}_{ti}\right)\psi_A^*(\xi_1,\xi_2,p_{t1},p_{t2})\psi_A\left(\xi_1,\xi_2,p_{t1}+\vec{k}_\perp,p_{t2}-\vec{k}_\perp\right)\right]$$

B. Blok et al, EPJC (2013) 73:2422

2-body form factor: $F_2(\vec{k}_\perp, -\vec{k}_\perp)$

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2-body form factor: $F_2(\vec{k}_\perp,-\vec{k}_\perp)$

Calculated $F_2(\vec{k}_1,\vec{k}_2)$ for He^3 and He^4 in

V. Guzey, M.R., S. Scopetta, M. Strilman and M. Viviani et al, "*Coherent J/Ψ electroproduction on He⁴ and He³ at the EIC: probing Nuclear shadowing one nucleon at a time*", PRL 129 (2022) 24, 242503

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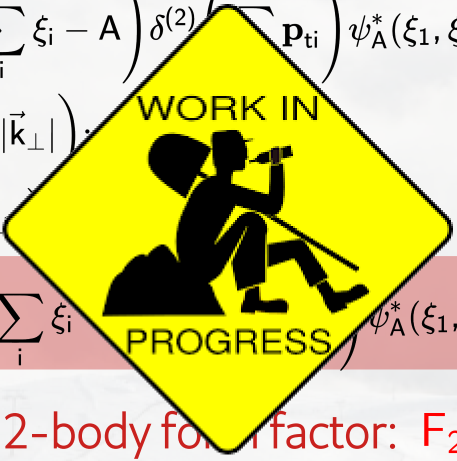
$$\times\, G^{N_1}_{a_1}\left(\frac{x_1}{\xi_1}, |\vec{k}_\perp|\right) G^{N_2}_{a_2}\left(\frac{x_2}{\xi_2}, |\vec{k}_\perp|\right) \dots$$

$$\underset{\xi_i \sim 1}{\sim} G^{N_1}_{a_1}\left(x_1, |\vec{k}_\perp|\right) G^{N_2}_{a_2}\left(x_2, |\vec{k} \dots$$

$$\times \left[\int \frac{1}{\xi_1 \xi_2} \prod_{i=1}^{i=A} \frac{d\xi_i d^2 p_{ti}}{\xi_i} \delta\left(\sum_i \xi_i \dots\right) \dots \psi^*_A(\xi_1, \xi_2, p_{t1}, p_{t2}) \psi_A\left(\xi_1, \xi_2, p_{t1} + \vec{k}_\perp, p_{t2} - \vec{k}_\perp\right)\right]$$



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CONCLUSIONS

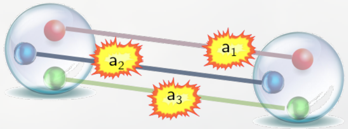
1) We demonstrated DPS represents a new way to access new information of hadrons

2) Several experimental analyses and theoretical developments are on going

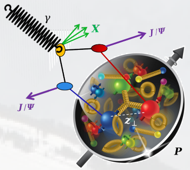
3) New exciting possibilities are represented by

a) Nuclear DPS (also at EIC?)

b) Triple Parton Scattering (see David's talk)



2) We proposed to consider DPS initiated via photon-proton interactions:



a) DPS contributes, in particular in the 4-jets photoproduction
b) We have estimated SPS and DPS cross sections for quarkonium-pair photoproduction at the EIC using the NRQCD framework
c) The dependence of $\sigma_{\text{eff}}^{\gamma p}$ on the Q^2 can unveil the mean distance of partons in the proton
d) Quarkonium-pair photoproduction is a promising channel to probe the gluonic content of the photon structure

Triple Parton Scattering: Tr-*J/Ψ production*

$$\sigma_{TPS} \propto \frac{\sigma^{SPS}_{a_1}\sigma^{SPS}_{a_2}\sigma^{SPS}_{a_3}}{\boxed{\sigma^2_{eff,TPS}}}$$

Extension of the Pocket Formula

D. d' E. et al PRL 118 (2017) 122001

here also relation with $\sigma_{eff,DPS}$ Is discussed

We can hope to study and access:

1) $\sigma_{eff,TPS}$ parameter which depends on the geometrical distribution of the 3 interacting partons!

2) Triple parton correlations?

Triple Parton Scattering: Tr-*J/Ψ production*

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Extension of the Pocket Formula
D. d' E. et al PRL 118 (2017) 122001

Sum rules relating tPDFs and dPDFs shown by **O. Fedkevych and J. R. Gaunt**, **arXiv:2208.08197**:

$$\sum_{j_3}\int_0^{1-x_1-x_2} dx_3 x_3 T^B_{j_1j_2j_3}(x_1,x_2,x_3) = (1-x_1-x_2)D^B_{j_1j_2}(x_1,x_2)$$

Momentum Sum Rule

tPDF　　dPDF

$$\int_0^{1-x_1-x_2} dx_3 T^B_{j_1j_2j_{3v}}(x_1,x_2,x_3) = \left(N_{j_{3v}} - \delta_{j_3j_1} - \delta_{j_3j_2} + \delta_{\bar{j}_3j_1} + \delta_{\bar{j}_3j_2}\right) D^B_{j_1j_2}(x_1,x_2)$$

Number Sum Rule

Distribution integrated on transverse dependence

see Gaunt's talk

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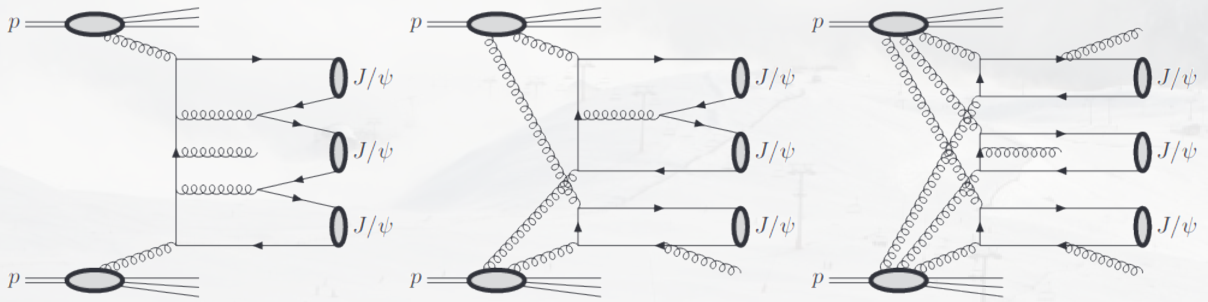
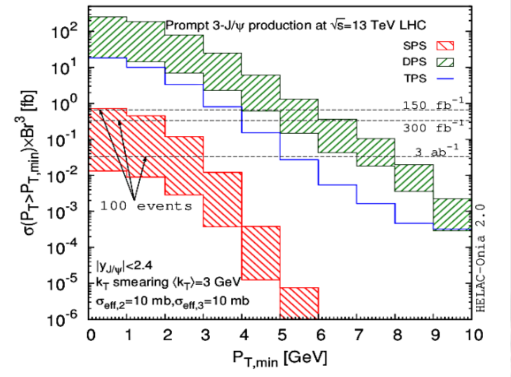
Proposed in e.g. **H. S. Shao et al PRL 122 (2019) 192002**, **D. d'E. et al PRL 118 (2017) 122001, EPJC 78 (2018) 359**





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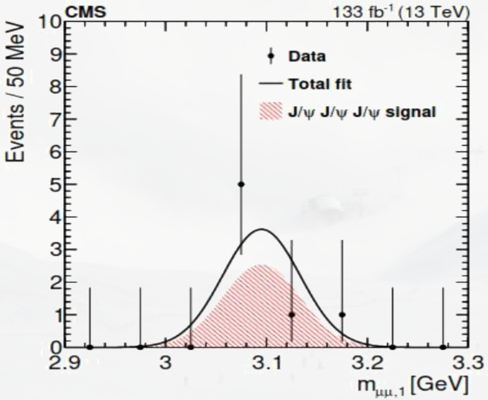
Extension of the Pocket Formula

D. d' E. et al PRL 118 (2017) 122001

First observation from the CMS collaboration **arXiv:2111.05370**



$$\sigma_{pp\to 3\,J/\psi} = 272^{+141}_{-104}(\text{ stat }) \pm 17\ (\text{syst})\ \text{fb}$$

Contribution: ~6% SPS, ~20% TPS and DPS~74%

Pocket formula and extended pocket formula used!

Triple Parton Scattering: Tr-*J/Ψ production*

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Extension of the Pocket Formula

D. d' E. et al PRL 118 (2017) 122001

We remark the seminal papers: **D. d'E. et al: PRL 118 (2017) 122001 and EPJC 78 (2018) 359**. The authors also studied TPS in:

$$pp \to c\bar{c} + c\bar{c} + c\bar{c} + X$$

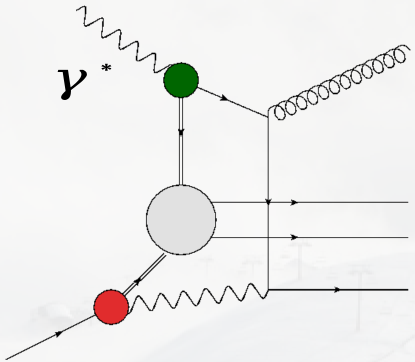
$$pp \to b\bar{b} + b\bar{b} + b\bar{b} + X$$

In particular, in **EPJC 78 (2018) 359** the same authors studied TPS in pA collisions

New Idea: DPS via γ-p interaction

6

In order to study the impact of the DPS contribution to a process initiated via photon-proton interactions we evaluated the 4-JET photoproduction at HERA (**S. Checkanov et al. (ZEUS), Nucl. Phys B792, 1 (2008)**)



In

1) **G. Abbiend et al, Phys. Commun 67, 465 (1992)**
2) **J.R. Forshaw et al, Z. Phys. C 72, 637 (1992)**

It has been shown that the agreement with data improves if MPI are included in the Monte Carlo

WE EVALUATE THE DPS CONTRIBUTION TO THIS PROCESS

The effective cross section: a key for the proton structure

6

The effective cross section can be also written in terms of Fourier Transform of the EFF:

$$\tilde{F}_2^{\gamma}(z_\perp; Q^2) = \sum_n C_n(Q^2) z_\perp^n$$

$$\left[\sigma_{eff}^{\gamma p}(Q^2)\right]^{-1} = \int d^2 z_\perp \; \tilde{F}_2^p(z_\perp)\tilde{F}_2^{\gamma}(z_\perp; Q^2)$$

$$= \sum_n C_n(Q^2) \langle (z_\perp)^n \rangle_p$$

If we can measure the dependence of the effective-cross section on the photon VIRTUALITY

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

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$$= \sum_n \underbrace{C_n(Q^2)}\langle (z_\perp)^n \rangle_p$$

This coefficient can be determined from the structure of the photon described in a given approach

If we can measure the dependence of the effective-cross section on the photon VIRTUALITY

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

The effective cross section: a key for the proton structure

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The effective cross section can be also written in terms of Fourier Transform of the EFF:

$$\tilde{F}(z_\perp)$$

The probability of finding a parton pair at distance

$$z_\perp$$

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

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$$= \sum_n C_n(Q^2) \langle z_\perp^n \rangle_p$$

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

The effective cross section: a key for the proton structure

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To test if in future a dependence of the effective cross section on the photon virtuality could be observed, we considered again the 4 JET photoproduction:

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The effective cross section: a key for the proton structure

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To test if in future a dependence of the effective cross section on the photon virtuality could be observed, we considered again the 4 JET photoproduction:

1) We divided the integral of the cross section on Q^2 in two intervals:

$$Q^2 \leqslant 10^{-2} \quad \text{and} \quad 10^{-2} \leqslant Q^2 \leqslant 1 \quad \text{GeV}^2$$

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2) We have estimated for each photon and proton models a constant effective cross section $\bar{\sigma}_{\text{eff}}^{\gamma p}$ (with respect to Q^2) such that the total integral of the cross section on Q^2 reproduce the full calculation obtained by means of $\sigma_{\text{eff}}^{\gamma p}(Q^2)$

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3) We estimate the minimum luminosity to distinguish the two cases

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The effective cross section: a key for the proton structure

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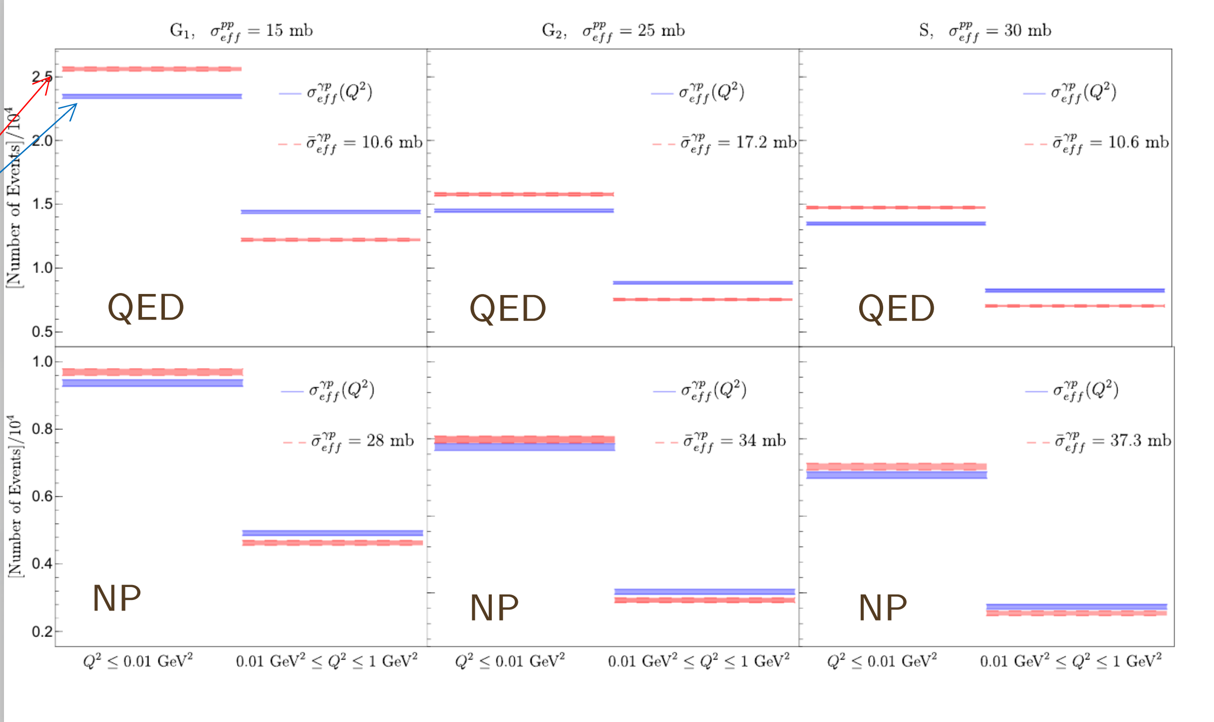
With an integrated luminosity of 200 pb^{-1} we can separate:



Relativistic effects

6

- Almost model independence
- Almost scale independence

SUGGEST: *parametrize the impact of Melosh effects in dPDFs to encode* ***some*** *general correlations between x_i and $k_\perp$*

$$\overbrace{F_{ij}(x_1, x_2, k_\perp; Q^2)}^{\text{pheno}} = \underbrace{q_i(x_1; Q^2) q_j(x_2; Q^2)}_{\text{phenomenology from PDFs}} \overbrace{\theta(1 - x_1 - x_2)}^{\text{good support}} \underbrace{f()}_{\text{sum rules}} \overbrace{R(x_1, x_2, k_\perp)}^{\text{Melosh effects}} F(k_\perp)$$

correlations!

} To be modeled: CQMs, GPDs...

$$R(x_1, x_2, k_\perp) \equiv \frac{F^{HO}_{[L]}(x_1, x_2, k_\perp; Q^2)}{F^{HO}_{[I]}(x_1, x_2, k_\perp; Q^2)} = w(k_\perp) \left[x_1 x_2\right]^{t(k_\perp)} (1 - x_2 - x_2)^{|x_1 - x_2| e(k_\perp)} e^{-(1 - x_1 - x_2) h(k_\perp)}$$

Relativistic effects

Let us consider the LF expression of the dPDF with its non relativistic (NR) limit:

$$F_{[I]}(x_1, x_2, k_\perp) = \int d\vec{k}_1 d\vec{k}_2 f(\vec{k}_1, \vec{k}_2, k_\perp)\delta\left(x_1 - \frac{k_1^+}{M_P}\right)\delta\left(x_2 - \frac{k_2^+}{M_P}\right) \quad \text{NR}$$

$$\left.\begin{aligned} F_{[L]}(x_1, x_2, k_\perp) &= \int d\vec{k}_1 d\vec{k}_2 f(\vec{k}_1, \vec{k}_2, k_\perp)\langle SPIN | O_1(\vec{k}_1, \vec{k}_2, k_\perp) O_2(\vec{k}_1, \vec{k}_2, k_\perp) | SPIN\rangle \\ &\times \delta\left(x_1 - \frac{k_1^+}{M_0}\right)\delta\left(x_2 - \frac{k_2^+}{M_0}\right) \end{aligned}\right\} \text{LF}$$

$f(\vec{k}_1, \vec{k}_2, k_\perp)$ = product of canonical w.f. (momentum space)

Melosh Operators!

They rotate canonical spin into LF ones

In the small x region, the main difference between $F_{[I]}$ and $F_{[L]}$ is given by the Melosh op

More on the LO QED photon EFF and effective x-section

1) Since the photon starts to be a **small** system, the effective-form factor must be similar to a constant (to be properly related to the FT of the probability distribution)

2) as a conseguence, the effective cross section should be of the same order of that for pp collissions.

3) why this two effective x-section are similar if the system are different?

4) a possible explanation can be obtained by considering:

$$\frac{\sigma_{eff}}{3\pi} \leq \langle b^2 \rangle \leq \frac{\sigma_{eff}}{\pi}$$

(proven for pp collisions)

Inverting this inequality one gets:

$$\pi\langle b^2\rangle \leq \sigma_{\rm eff}^{\rm pp} \leq 3\pi\langle b^2\rangle$$

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$$\pi\langle b^2\rangle \leq \sigma_{eff}^{pp} \leq 3\pi\langle b^2\rangle \Rightarrow$$

(proven for pp collisions)

therefore, similar effective x-sections can be related to different **distances,** i.e. **different gemetrical structures!**

(Proton) Model Independent conlcusions

1) in arXiv:2103.1340 we show that high virtual behavior of the effective cross sections correctly follows the result **in J.R. Gaunt JHEP 01, 042 (2013)**, i.e.:

$$\sigma_{eff}^{\gamma p}(Q^2 \to \infty) = \sigma_{1v2}^{pp} = \left[\int \frac{d^2 k_\perp}{(2\pi)^2} T_p(k_\perp)\right]^{-1}$$

2) In Ref. **M.Rinaldi and F.A: Ceccopieri JHEP 09, 097 (2019),** we prove, in a general framework:

$$\frac{\sigma_{\mathrm{eff},2\mathrm{v}1}}{2\pi} \leq \langle b^2 \rangle \leq \frac{2\ \sigma_{\mathrm{eff},2\mathrm{v}1}}{\pi}$$

therefore, by inverting this relation one gets:

$$\frac{\pi}{2}\langle b^2 \rangle \leq \sigma_{eff}^{\gamma p}(Q^2 \to \infty) \leq 2\pi \langle b^2 \rangle$$

(Proton) Model Independent conlcusions

$$\frac{\pi}{2}\langle b^2\rangle \leq \sigma_{eff}^{\gamma p}(Q^2 \to \infty) \leq 2\pi\langle b^2\rangle$$

3) in arXiv:2103.1340, for the moment being we considered proton model producing a (2v2) effective cross section of 15-30 mb (**in new analysis we can relax this condition**).
Now in **M. Rinaldi and F. A. Ceccopieri PRD 97 (2018) 7, 071501**, we prove:

$$\frac{\sigma_{eff}^{pp}}{3\pi} \leq \langle b^2\rangle \leq \frac{\sigma_{eff}^{pp}}{\pi}$$

combining everything:

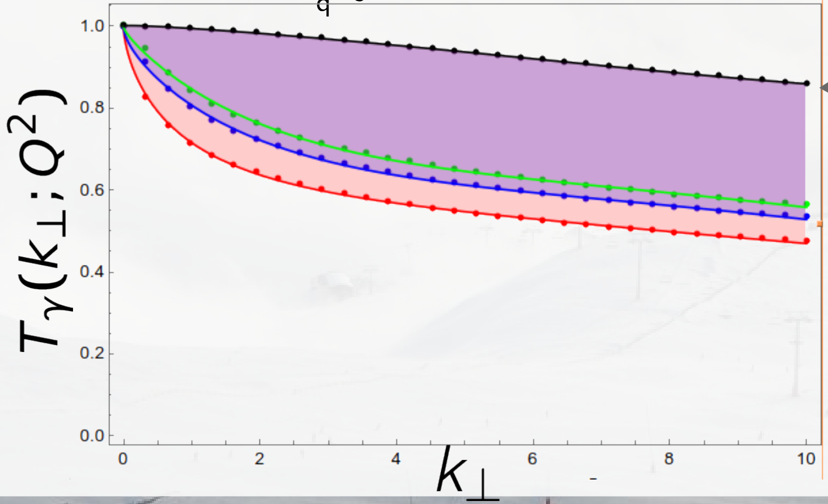
$$\frac{\sigma_{eff}^{pp}}{6} \leq \sigma_{eff}^{\gamma p}(Q^2 \to \infty) \leq 2\sigma_{eff}^{pp}$$

VERIFIED!!

More on the LO QED photon EFF and effective x-section

$$T_\gamma(k_\perp; Q^2) = \frac{\sum_q \int dx\, f^\gamma_{q,\bar{q}}(x, k_\perp; Q^2)}{\sum_q \int dx\, f^\gamma_{q,\bar{q}}(x, k_\perp = 0; Q^2)}$$

$$f^\gamma_{q,\bar{q}}(x, \tilde{k}_\perp; Q^2) = \int d^2k_{\perp,1}\, \psi^{\dagger\gamma}_{q\bar{q}}(x, \vec{k}_{\perp,1}; Q^2) \times \psi^\gamma_{q\bar{q}}(x, \vec{k}_{\perp,1} + \vec{k}_\perp; Q^2)$$



$Q^2 = 10\ \text{GeV}^2$

$\langle z_\perp^2 \rangle_\gamma \propto \frac{1}{Q^2}$

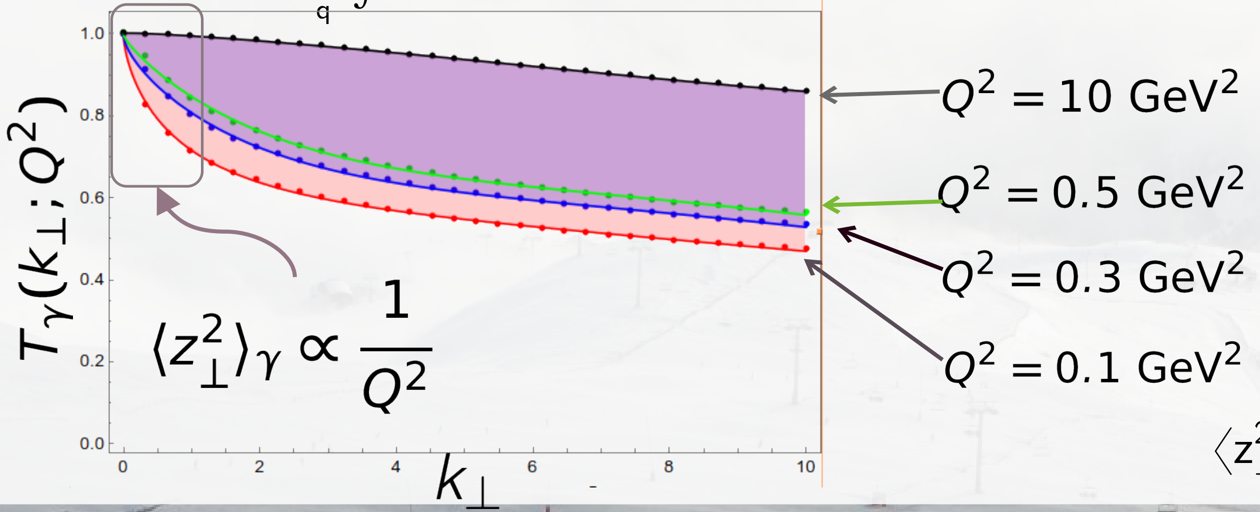
Small system → EFF **SLOWLY** decreasing:

$$T_\gamma(k_\perp; Q^2 >> 1\ \text{GeV}^2) \sim 1$$

More on the LO QED photon EFF and effective x-section

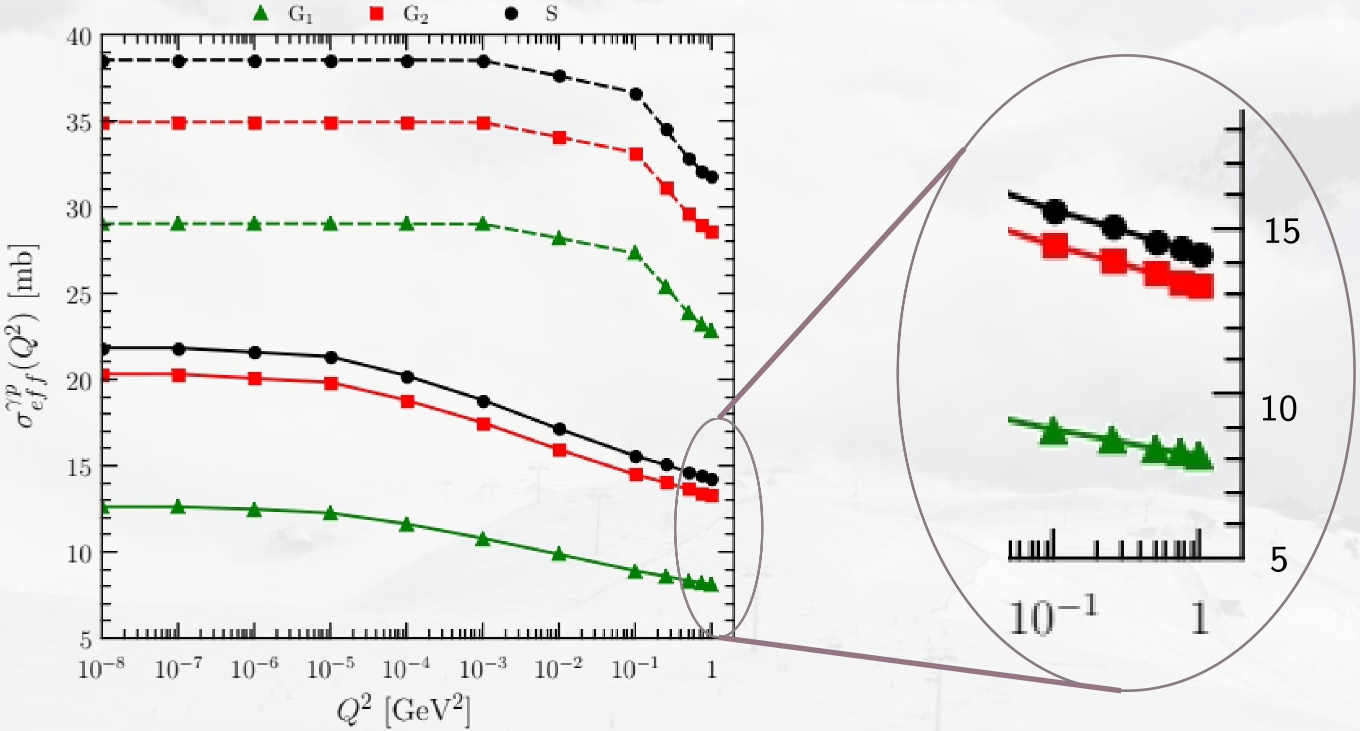
$$T_\gamma(k_\perp; Q^2) = \frac{\sum_q \int dx\, f^\gamma_{q,\bar q}(x, k_\perp; Q^2)}{\sum_q \int dx\, f^\gamma_{q,\bar q}(x, k_\perp = 0; Q^2)}$$

$$f^\gamma_{q,\bar q}(x, \tilde k_\perp; Q^2) = \int d^2k_{\perp,1}\, \psi^{\dagger\gamma}_{q\bar q}(x, \vec k_{\perp,1}; Q^2) \times \psi^\gamma_{q\bar q}(x, \vec k_{\perp,1} + \vec k_\perp; Q^2)$$



$$\langle z_\perp^2\rangle \propto \frac{d}{k_\perp dk_\perp} T(k_\perp)\Big|_{k_\perp=0}$$

More on the LO QED photon EFF and effective x-section

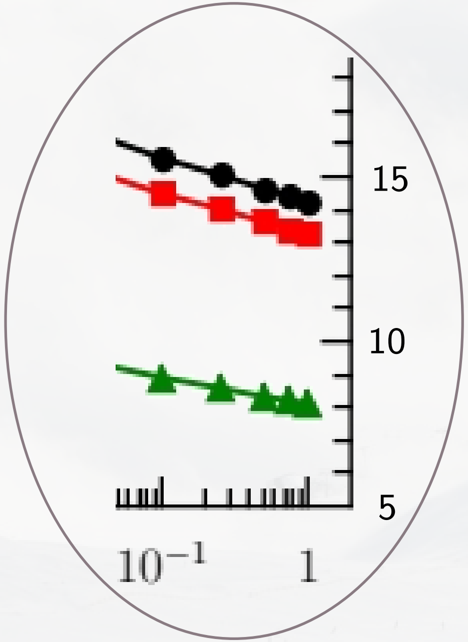


~ 30/2 mb

~ 25/2 mb

~ 15/2 mb

More on the LO QED photon EFF and effective x-section



~ 30/2 mb
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~ 15/2 mb

$$[\sigma_{eff}^{\gamma p}(Q^2)]^{-1} = \int \frac{d^2k_\perp}{(2\pi)^2} T_p(k_\perp) T_\gamma(k_\perp; Q^2)$$

$$[\sigma_{eff}^{\gamma p}(Q^2)]^{-1} \underset{Q^2>>1}{\sim} \int \frac{d^2k_\perp}{(2\pi)^2} T_p(k_\perp) \times 1$$

For the proton models we have used:

$$\int \frac{d^2k_\perp}{(2\pi)^2} T_p(k_\perp) \sim 2 \int \frac{d^2k_\perp}{(2\pi)^2} T_p(k_\perp)^2$$

$$\sigma_{eff}^{\gamma p}(Q^2 >> 1\ \text{GeV}^2) \sim \sigma_{eff}^{pp}/2$$

Double PDFs (intrinsic) of the proton

$F_{ik}(x_1, x_2, \vec{z}_\perp)$ is unknown. However @LHC kinematics (small x and many partons produced)

$$F_{ik}(x_1, x_2, \vec{z}_\perp) \sim \overbrace{g(x_1, x_2)\tilde{T}(\vec{z}_\perp)}^{\text{1st uncorrelated scenario}}$$

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$F_{ik}(x_1, x_2, \vec{z}_\perp)$ is unknown. However @LHC kinematics (small x and many partons produced)

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DPD → SUM RULES AVAILABLE

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DPD

→ SUM RULES AVAILABLE

→ MODELS BASED ON SUM RULES

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↓

1st uncorrelated scenario

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DPD

→ SUM RULES AVAILABLE

→ MODELS BASED ON SUM RULES

→ PDF(x_1)*PDF(x_2)

→ [PDF(x_1)*PDF(x_2)] ⊗ pQCD EVOLUTION

2nd uncorrelated scenario

J. R. Gaunt and W. J. Stirling, JHEP 03, 005 (2010)

Double PDFs (intrinsic) of the proton

$F_{ik}(x_1, x_2, \vec{z}_\perp)$ is unknown. However @LHC kinematics (small x and many partons produced)

1st uncorrelated scenario

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DPD

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2nd uncorrelated scenario

PERTURBATIVE CORRELATIONS

Double PDFs (intrinsic) of the proton

$F_{ik}(x_1, x_2, \vec{z}_\perp)$ is unknown. However @LHC kinematics (small x and many partons produced)

1st uncorrelated scenario

$$F_{ik}(x_1, x_2, \vec{z}_\perp) \sim g(x_1, x_2)\tilde{T}(\vec{z}_\perp)$$

Constituent quark models used to grasp basic NON PERTURBATIVE features

M.R., S. Scopetta et *al*, PRD 87 (2013) 114021
M.R., S, Scopetta et *al*, JHEP 12 (2014) 028

DPD

- SUM RULES AVAILABLE
- MODELS BASED ON SUM RULES
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2nd uncorrelated scenario

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PROBABILITY DISTRIBUTION OF FINDING TWO PARTONS WITH GIVEN TRANSVERSE DISTANCE

Constituent quark models used to grasp basic NON PERTURBATIVE features

M.R., S. Scopetta et *al*, PRD 87 (2013) 114021
M.R., S, Scopetta et *al*, JHEP 12 (2014) 028

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- SUM RULES AVAILABLE
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2nd uncorrelated scenario

PERTURBATIVE CORRELATIONS

Double PDFs (intrinsic) of the proton

$F_{ik}(x_1, x_2, \vec{z}_\perp)$ is unknown. However @LHC kinematics (small x and many partons produced)

$$F_{ik}(x_1, x_2, \vec{z}_\perp) \sim g(x_1, x_2)\tilde{T}(\vec{z}_\perp)$$

$\tilde{T}(\vec{z}_\perp)$:
- PROBABILITY DISTRIBUTION OF FINDING TWO PARTONS WITH GIVEN TRANSVERSE DISTANCE
- UNKNOWN: ONLY MODELS, LATTICE?

$g(x_1, x_2)$ — DPD:
- SUM RULES AVAILABLE
- MODELS BASED ON SUM RULES
- PDF(x_1)*PDF(x_2)
- [PDF(x_1)*PDF(x_2)] ⊗ pQCD EVOLUTION

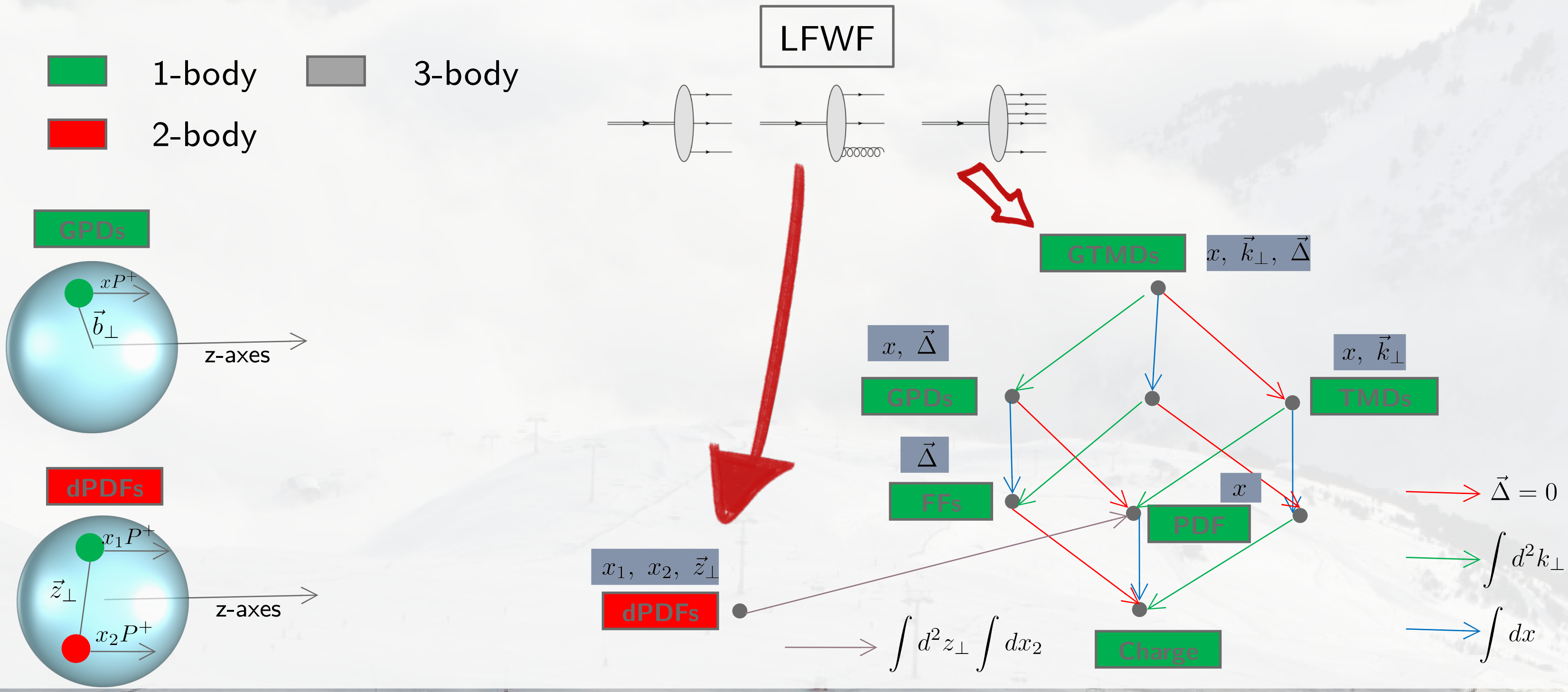
(last two: 2nd uncorrelated scenario)

PERTURBATIVE CORRELATIONS

F_{ik}: Constituent quark models used to grasp basic non perturbative feature

M.R., S. Scopetta et *al*, PRD 87 (2013) 114021
M.R., S, Scopetta et *al*, JHEP 12 (2014) 028

Multidimensional Pictures of Hadron



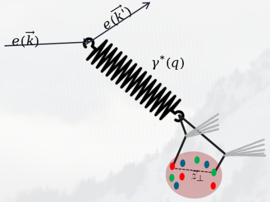
The γ-p effective cross section

6

The expression of this quantity is very similar t… collision case and can be formall… eri… …ss sections and the DPS one… …a DPS from a

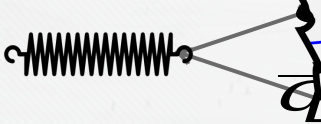
$$\left[\sigma_{eff}^{\gamma p}(Q^2)\right]^{-1} = \int \frac{d}{(2\ldots} \ldots \int dx\, f^{\gamma}_{q,\bar{q}}(x, k_\perp; Q^2) \Big/ \ldots dx\, f^{\gamma}_{q,\bar{q}}(x, k_\perp = 0; Q^2)$$

$$f^{\gamma}_{q,\bar{q}}(x, \tilde{k}_\perp; Q^2) = \int d^2 \ldots$$



INGREDIENTS OF THE CALCULATION:

1. $T_p(k_\perp)$ proton EFF
2. ψ^γ Photon WF
3.

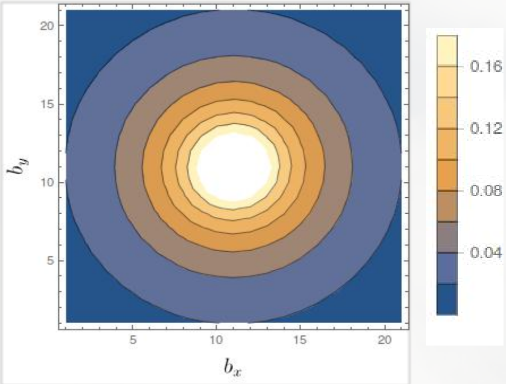


$\bar{q} : (1-x, -k_{\perp,1})$

M. R. and F. A. Ceccopieri, arXiv:2103.13480

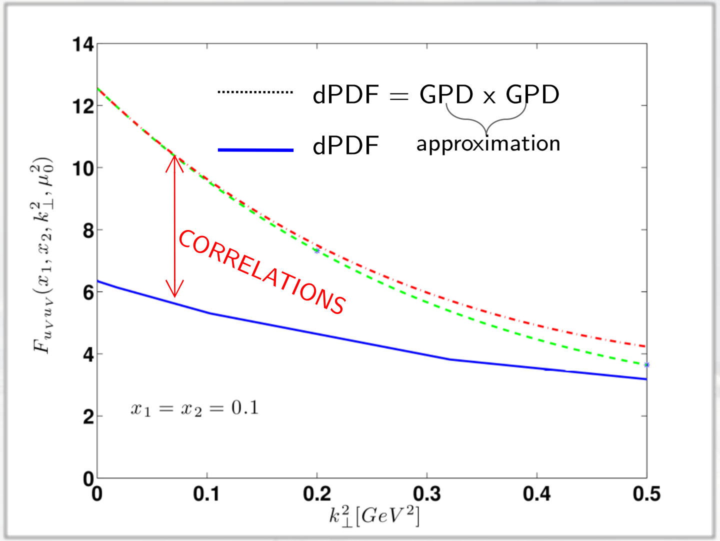
Information from Quark Models

2



M.R. and F. A. Ceccopieri, JHEP 09 (2019) 097

1) e.g. the distance distribution of
two gluons in the proton



2) Correlations
are important

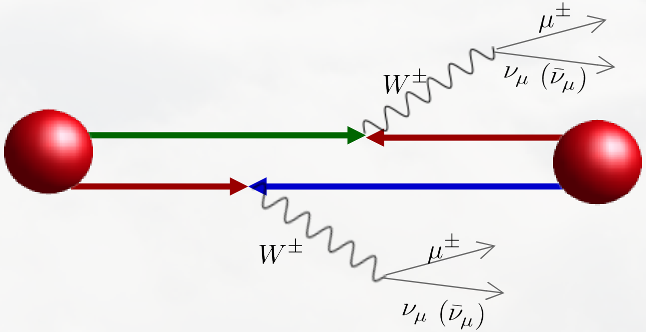
M.R., S. Scopetta et al, JHEP 10 (2016) 063

M.R. and F. A. Ceccopieri PRD 95 (2017) 034040

Same sign W's production at the LHC

4

M. R. et al, Phys.Rev. D95 (2017) no.11, 114030

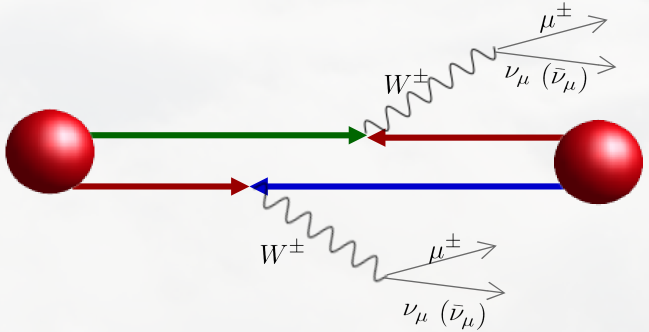


In this channel, the single parton scattering (usually dominant w.r.t to the double one) starts to contribute to higher order in strong coupling constant.

"Same-sign W boson pairs production is a promising channel to look for signature of double Parton interactions at the LHC."

Same sign W's production at the LHC

M. R. et al, Phys.Rev. D95 (2017) no.11, 114030



In this channel, the single parton scattering (usually dominant w.r.t to the double one) starts to contribute to higher order in strong coupling constant.

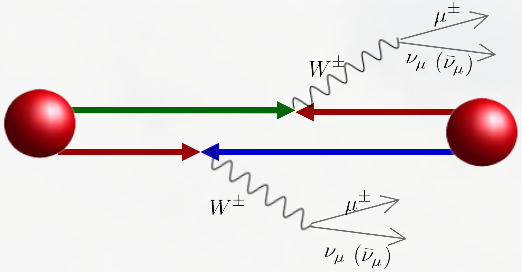
↓

Can double parton correlations be observed for the first time in the next LHC run ?

Same sign W's production at the LHC

4

M. R. et al, Phys.Rev. D95 (2017) no.11, 114030



Kinematical cuts

$pp,\ \sqrt{s} = 13$ TeV
$p_{T,\mu}^{leading} > 20$ GeV, $\quad p_{T,\mu}^{subleading} > 10$ GeV
$\lvert p_{T,\mu}^{leading}\rvert + \lvert p_{T,\mu}^{subleading}\rvert > 45$ GeV
$\lvert\eta_\mu\rvert < 2.4$
20 GeV $< M_{inv} <$ 75 GeV or $M_{inv} >$ 105 GeV

DPS cross section:

$$\frac{d^4\sigma^{pp\to\mu^\pm\mu^\pm X}}{d\eta_1 dp_{T,1} d\eta_2 dp_{T,2}} = \sum_{i,k,j,l} \frac{1}{2} \int d^2\vec{b}_\perp F_{ij}(x_1, x_2, \vec{b}_\perp, M_W) F_{kl}(x_3, x_4, \vec{b}_\perp, M_W) \frac{d^2\sigma_{ik}^{pp\to\mu^\pm X}}{d\eta_1 dp_{T,1}} \frac{d^2\sigma_{jl}^{pp\to\mu^\pm X}}{d\eta_2 dp_{T,2}} \mathcal{I}(\eta_i, p_{T,i})$$

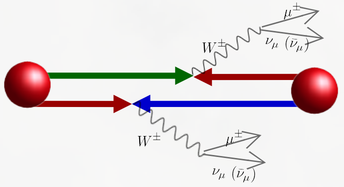
In order to estimate the role of double parton correlations we have used as input of dPDFs:

1) Longitudinal and transverse correlations arise from the relativistic CQM model describing three valence quarks

2) These correlations propagate to sea quarks and gluons through pQCD evolution

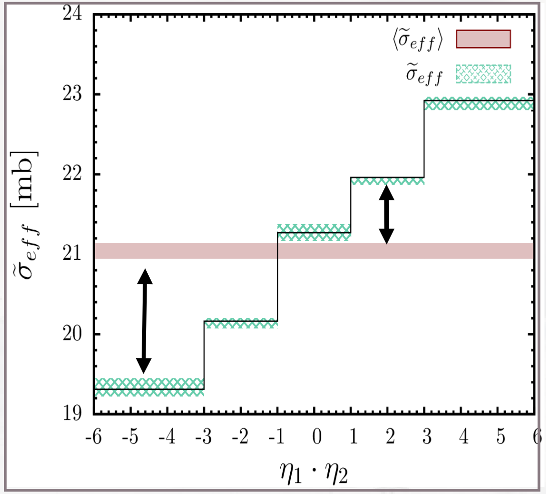
Same sign W's production at the LHC

M. R. et al,
Phys.Rev. D95 (2017)
no.11, 114030



$$\eta_1 \cdot \eta_2 \simeq \frac{1}{4} ln\frac{x_1}{x_3} ln\frac{x_2}{x_4}$$

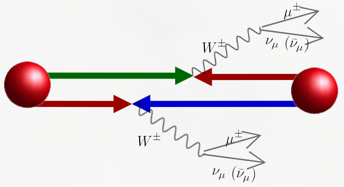
$$\langle\widetilde{\sigma}_{eff}\rangle = 21.04 \,^{+0.07}_{-0.07}\,(\delta Q_0)\,^{+0.06}_{-0.07}(\delta\mu_F)\text{ mb}\ .$$



Difference [↕] between green and red line is due to correlations effects

Same sign W's production at the LHC

M. R. et al, Phys.Rev. D95 (2017) no.11, 114030



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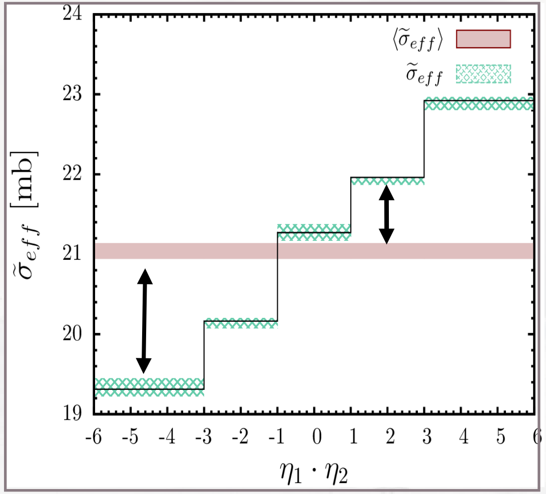


x- dependence of effective x-section

M.Rinaldi et al PLB 752,40 (2016)

M. Traini, M. R. et al, PLB 768, 270 (2017)

Assuming that the results of the first and the last bins can be distinguished if they differ by 1 sigma, we estimated that:

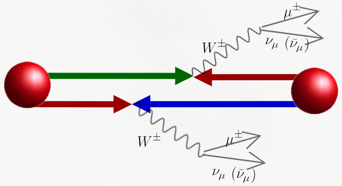
$$\mathcal{L} = 1000\ \text{fb}^{-1}$$

is necessary to observe correlations

* to be updated to new CMS cuts

Same sign W's production at the LHC

4

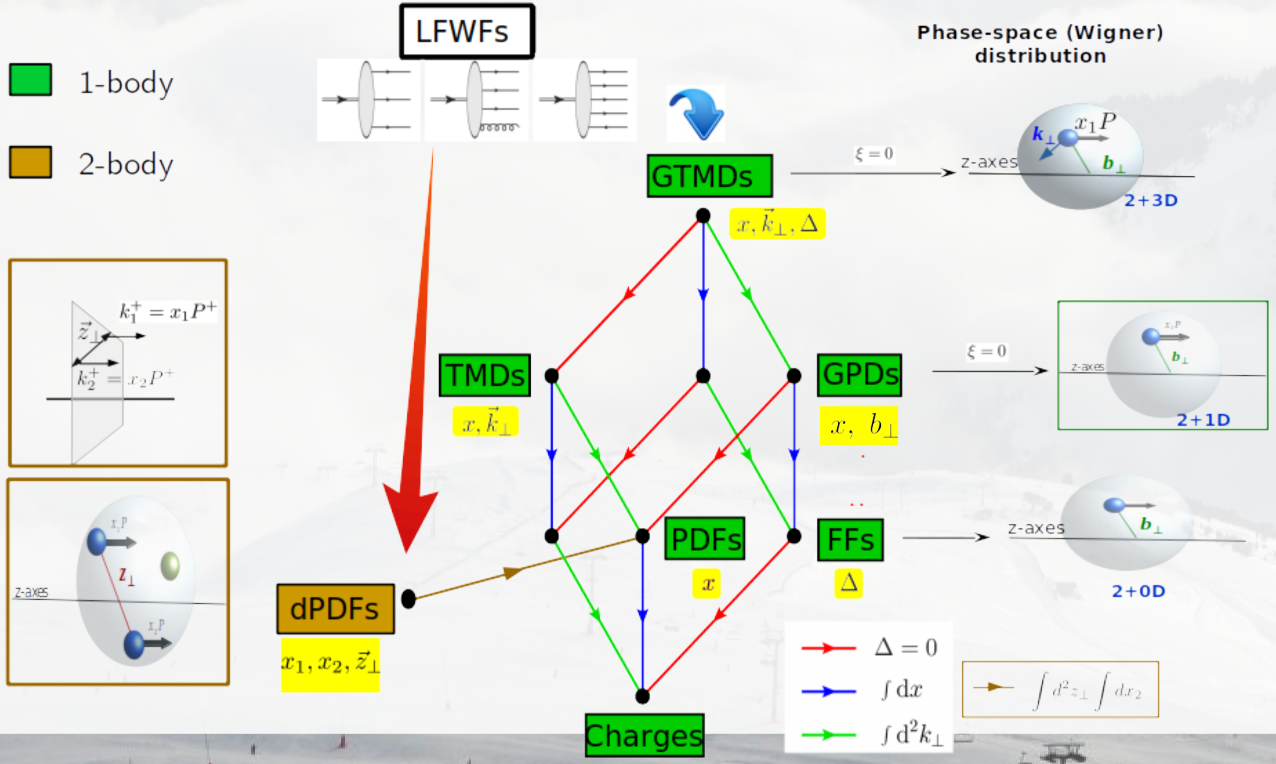


In Ref. **S. Cotogno et al, JHEP 10 (2020) 214**, it has been shown that several experimental observable are sensitive to **double spin correlations**.

The LHC has the potential to access these new information!

IN THIS CHANNEL, WE ESTABLISHED THE POSSIBILITY TO OBSERVE, FOR THE FIRST TIME, TWO-PARTON CORRELATIONS IN THE NEXT LHC RUN!

Multidimensional Pictures of Hadron 2

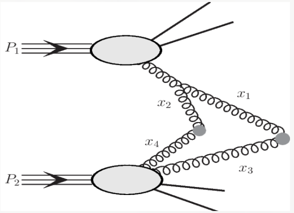


Further implementations

4

Relevant for processes involving heavy particles is the splitting term (1v2 mechanism) i.e.:

$^{*}D_{j_1,j_2}(x_1,x_2) = \int d^2 b_\perp \tilde{F}_{j_1,j_2}(x_1,x_2,b_\perp)$

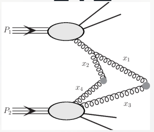


In pQCD evolution: $$\frac{dD_{j_1 j_2}(x_1,x_2;t)}{dt} = \begin{cases} \text{Homogeneous term (double DGLAP)} \\ + \\ \sum_{j'} F_{j'}(x_1+x_2;t) \underbrace{\frac{1}{x_1+x_2} P_{j' \to j_1 j_2}}\left(\frac{x_1}{x_1+x_2}\right) \end{cases}$$

Gaunt J.R. and Stirling W. J., JHEP 03 (2010)

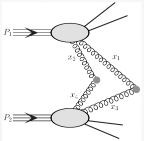
SPLITTING TERM

1v2



$+$

2v2



$$\frac{\sigma_{eff}}{3\pi}\left(1+\frac{3}{2}r_v\right) \le \langle b^2 \rangle \le \frac{\sigma_{eff}}{\pi}\left(1+2r_v\right)$$

$$r_v \sim \frac{F^{splitting}_{j_1 j_2}(x_1,x_2,k_\perp=0;t)}{F_{j_1 j_2}(x_1,x_2,k_\perp=0;t)}$$

with:
$0 \le r_v \le 1$

J.R. Gaunt, R. Maciula and A. Szczurek,
PRD 90 (2014) 054017

Due to the difficulty in the estimate of the 2 contributions:

Absolute minimum $r_v = 0$ $$\frac{\sigma_{eff}}{3\pi} \le \langle b^2 \rangle \le \frac{3\,\sigma_{eff}}{\pi}$$ Absolute maximum $\mathbf{r_v = 1}$

Further implementations

4

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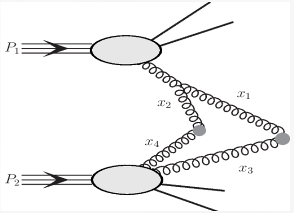


In p

Gaur

$) = \int d^2b_\perp \tilde{F}_{j_1,j_2}(x_1, x_2, b_\perp)$

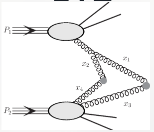
e DGLAP)

$_{j_1 j_2}\left(\frac{x_1}{x_1+x_2}\right)$

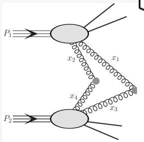
M

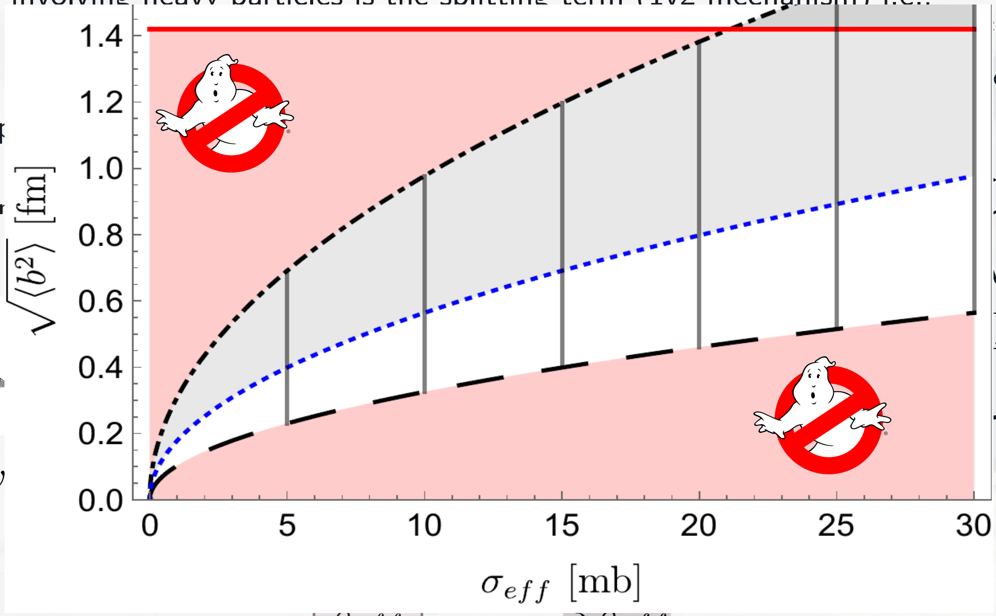
$\frac{{}^{splitting}_{j_1 j_2}(x_1, x_2, k_\perp = 0; t)}{F_{j_1 j_2}(x_1, x_2, k_\perp = 0; t)}$

1v2



$+$





with:

$0 \le r_v \le 1$

1) Minimum as function of r_v

$m(r_v)$

2) Maximum as function of r_v

$M(r_v)$

Absolute minimum $\quad \frac{\sigma_{eff}}{3\pi} \le \langle b^2 \rangle \le \frac{3\,\sigma_{eff}}{\pi} \quad$ Absolute maximum

$r_v = 0$ $\qquad$ $\mathbf{r_v = 1}$

Double PDFs within the Light-Front

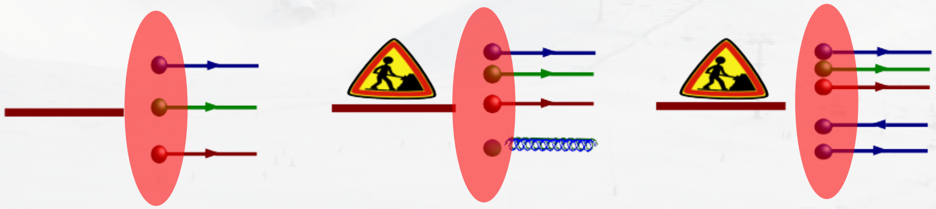
Extending the procedure developed in **S. Boffi, B. Pasquini and M. Traini, Nucl. Phys. B 649, 243 (2003)** for GPDs, we obtained the following expression of the dPDF in momentum space, often called $_2$GPDs:

$$F_{ij}(x_1,x_2,k_\perp) = 3(\sqrt{3})^3 \int \prod_{i=1}^{3} d\vec{k}_i \, \delta\left(\sum_{i=1}^{3} \vec{k}_i\right) \Phi^*(\{\vec{k}_i\},k_\perp)\Phi(\{\vec{k}_i\},-k_\perp)$$

$$\times \; \delta\left(x_1 - \frac{k_1^+}{P_+}\right)\delta\left(x_2 - \frac{k_2^+}{P_+}\right)$$

Conjugate to $z_\perp$

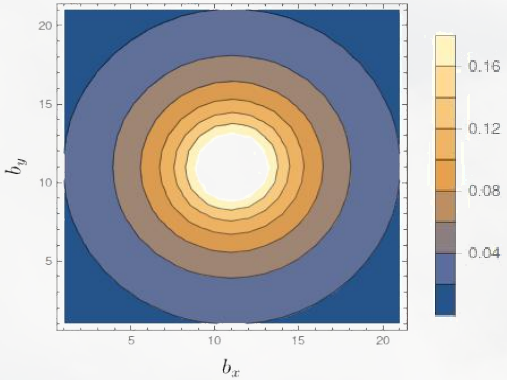
LF wave-function



$$\Phi(\{\vec{k}_i\},\pm k_\perp) = \Phi\left(\vec{k}_1 \pm \frac{\vec{k}_\perp}{2}, \vec{k}_2 \mp \frac{\vec{k}_\perp}{2}, \vec{k}_3\right)$$

Information from Quark Models 2

$z_\perp = b_\perp$



M.R. and F. A. Ceccopieri, JHEP 09 (2019) 097

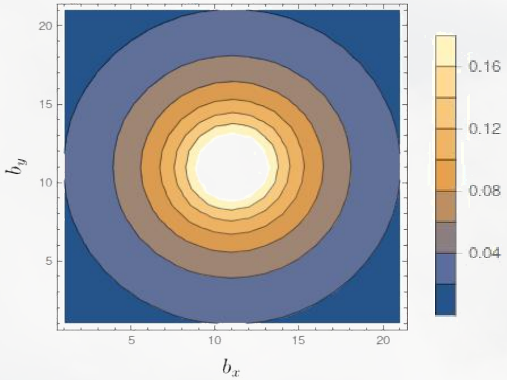
1) e.g. the distance distribution of **two gluons** in the proton

$$\langle z_\perp^2 \rangle_{x_1,x_2}^{ij} = \frac{\int d^2 z_\perp \; z_\perp^2 F_{ij}(x_1, x_2, z_\perp)}{\int d^2 z_\perp \; F_{ij}(x_1, x_2, z_\perp)}$$

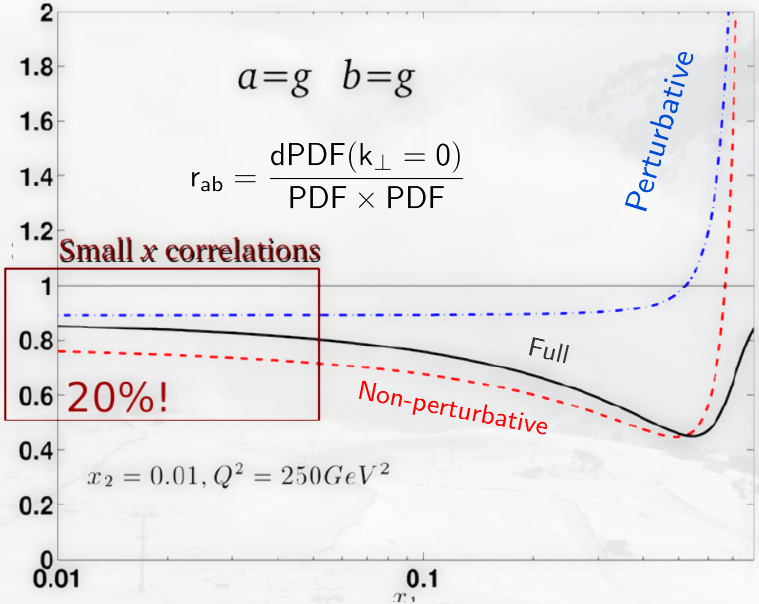
Information from Quark Models

2

$z_\perp = b_\perp$



M.R. and F. A. Ceccopieri, JHEP 09 (2019) 097



2) Correlations are important

M.R., S. Scopetta et al, JHEP 10 (2016) 063

M.R. and F. A. Ceccopieri PRD 95 (2017) 034040

Further implementations

4

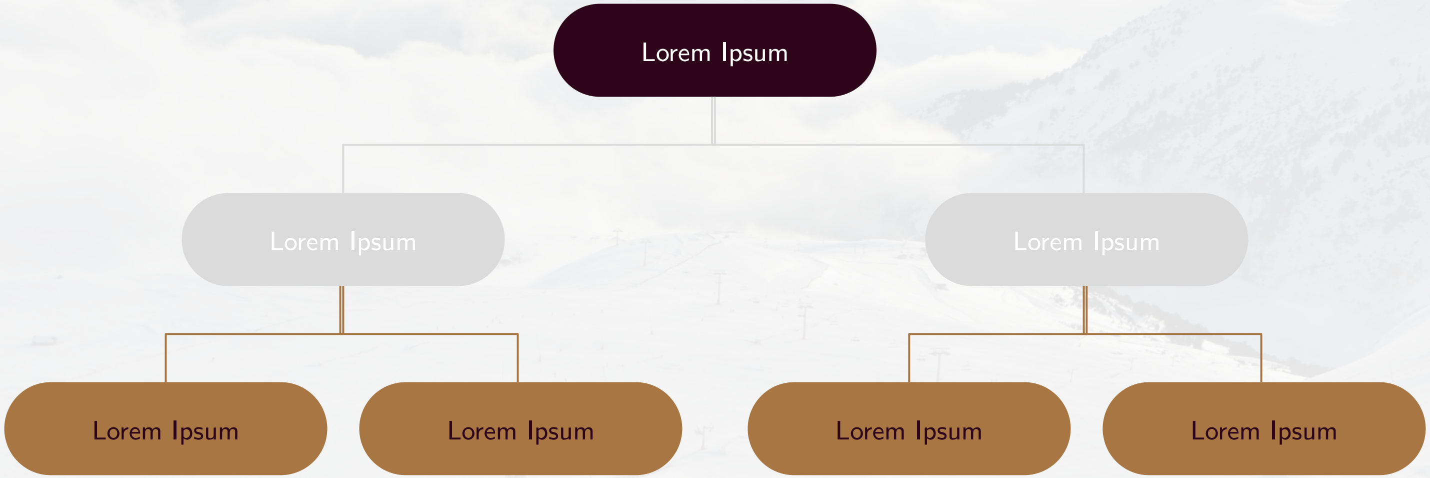
IF WE DO NOT CONSIDER ANY FACTORIZATION ANSATZ IN DOUBLE PDFs:

$$\frac{\sigma_{eff}(x_1,x_2)}{3\pi}\left[r^{2v2}(x_1,x_2)^2+\frac{3}{2}r^{2v1}(x_1,x_2)^2\ r_v\right]\leq\langle b^2\rangle_{x_1,x_2}\leq\frac{\sigma_{eff}(x_1,x_2)}{\pi}\left[r^{2v2}(x_1,x_2)^2+2r^{2v1}(x_1,x_2)^2\ r_v\right]$$

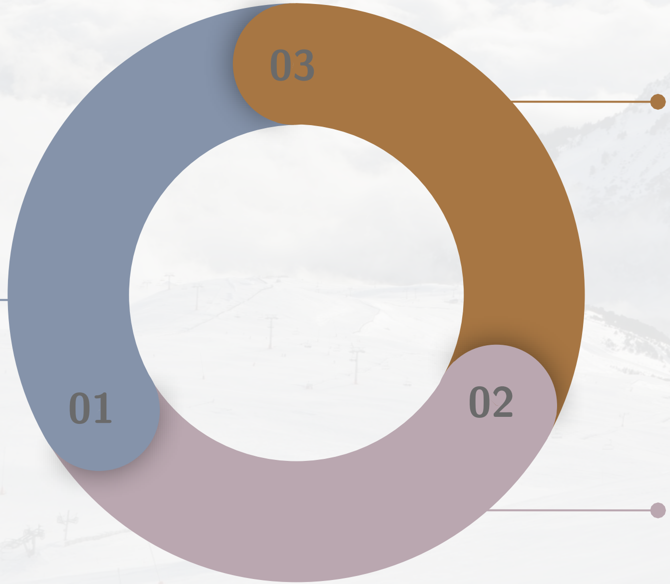
$$r^{2v2}(x_1,x_2)=\frac{F(x_1,x_2,k_\perp=0;t)}{F(x_1;t)F(x_2;t)}$$

$$r^{2v1}(x_1,x_2)=\frac{F^{splitting}(x_1,x_2,k_\perp=0;t)}{F(x_1;t)F(x_2;t)}$$

Use diagrams to explain your ideas



Our process is easy



Vestibulum congue tempus

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Vestibulum congue tempus

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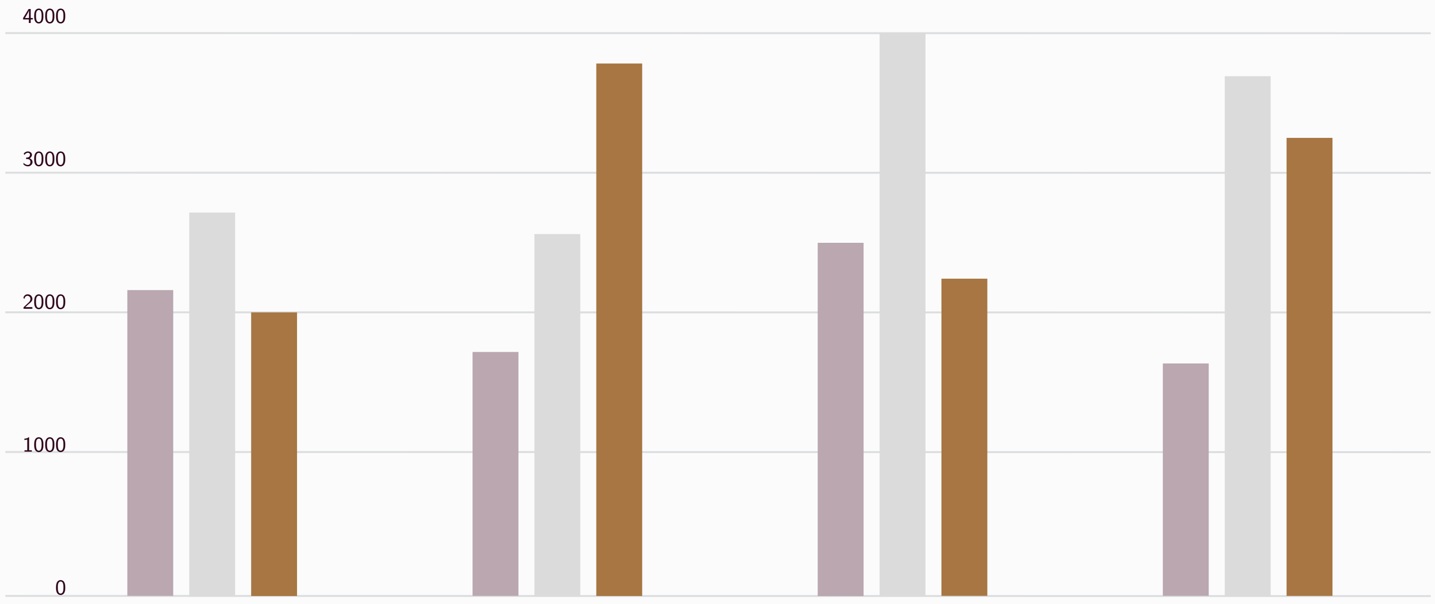


You can insert graphs from Excel or Google Sheets

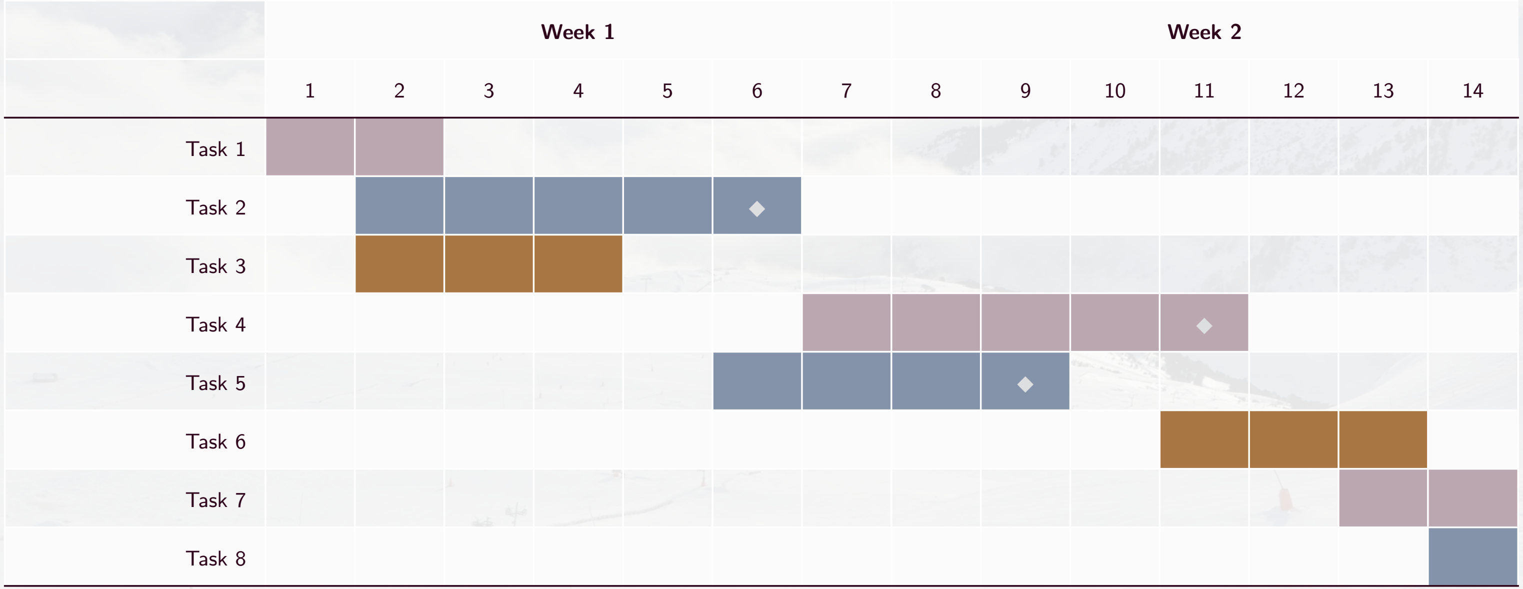
Desktop project

Show and explain your web, app or software projects using these gadget templates.

Timeline

- JAN — Blue is the colour of the clear sky and the deep sea
- FEB — Yellow is the color of gold, butter and ripe lemons
- MAR — Red is the colour of danger and courage
- APR — White is the color of milk and fresh snow
- MAY — Black is the color of ebony and of outer space
- JUN — Blue is the colour of the clear sky and the deep sea
- JUL — Yellow is the color of gold, butter and ripe lemons
- AUG — Red is the colour of danger and courage
- SEP — White is the color of milk and fresh snow
- OCT — Black is the color of ebony and of outer space
- NOV — Blue is the colour of the clear sky and the deep sea
- DEC — Yellow is the color of gold, butter and ripe lemons

Gantt chart


Week 1
Week 2
1
2
3
4
5
6
7
8
9
10
11
12
13
14
Task 1
Task 2
Task 3
Task 4
Task 5
Task 6
Task 7
Task 8

SWOT Analysis

STRENGTHS

Blue is the colour of the clear sky and the deep sea

WEAKNESSES

Yellow is the color of gold, butter and ripe lemons

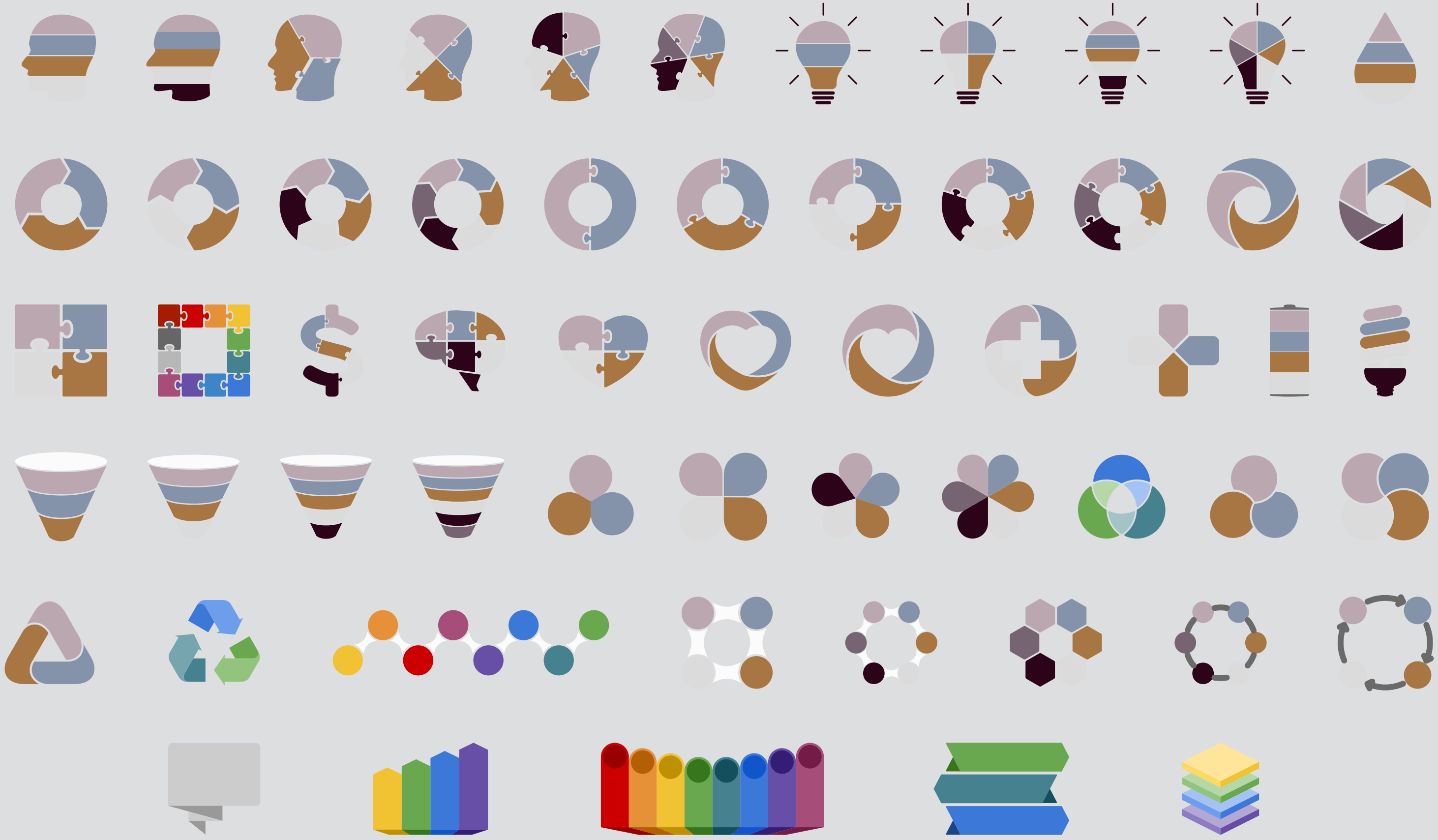
Black is the color of ebony and of outer space

OPPORTUNITIES

White is the color of milk and fresh snow

THREATS

Diagrams and infographics



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