

Constraining gluon TMDs with quarkonia

QaT23+ - Aussois, 14th January 2023

Francesco Giovanni Celiberto

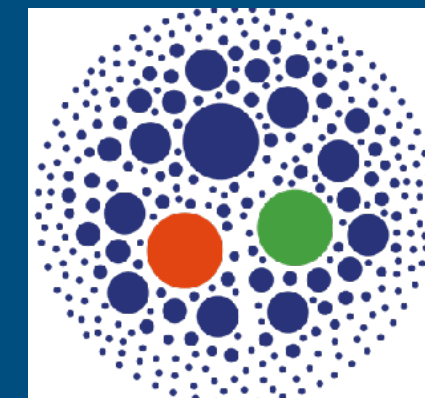
ECT*/FBK Trento & INFN-TIFPA

ECT*

EUROPEAN CENTRE FOR THEORETICAL STUDIES
IN NUCLEAR PHYSICS AND RELATED AREAS



Trento Institute for
Fundamental Physics
and Applications



HAS QCD
HADRONIC STRUCTURE AND
QUANTUM CHROMODYNAMICS

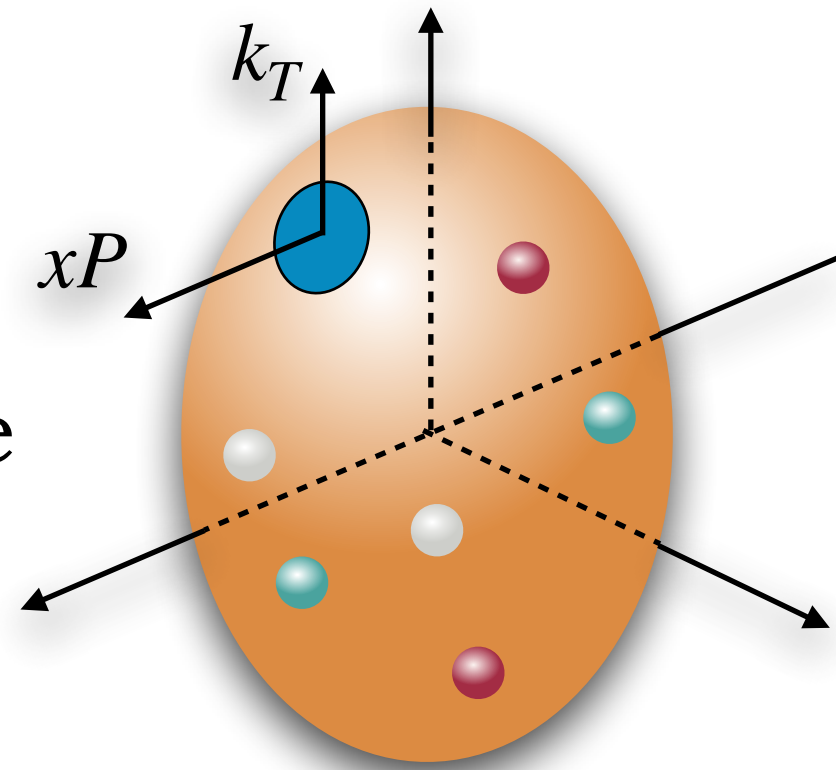


1. Introductory remarks

Parton densities: an incomplete family tree

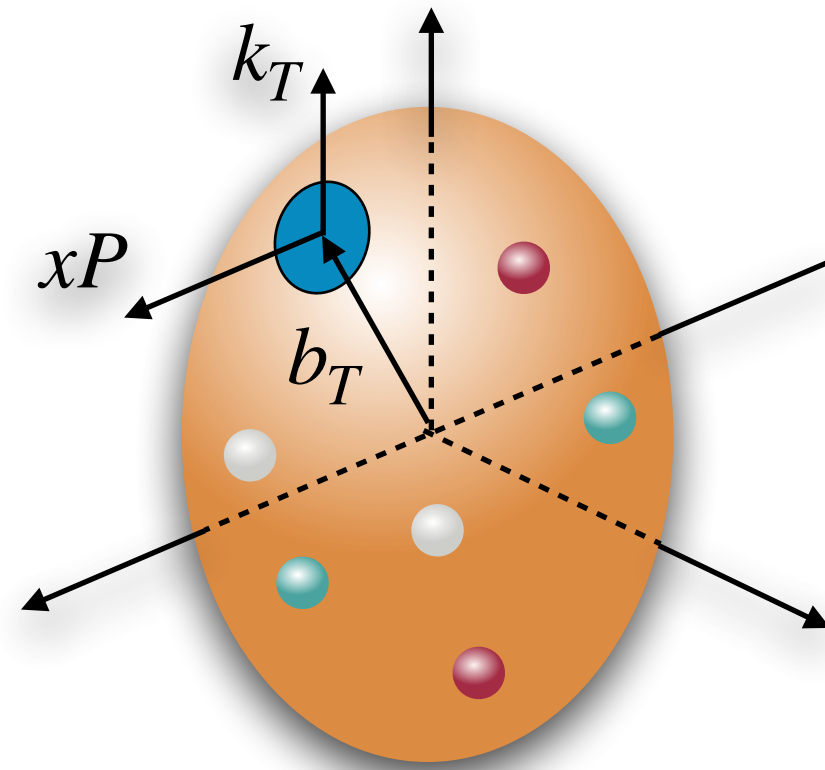
3D

TMDs
(semi-)inclusive



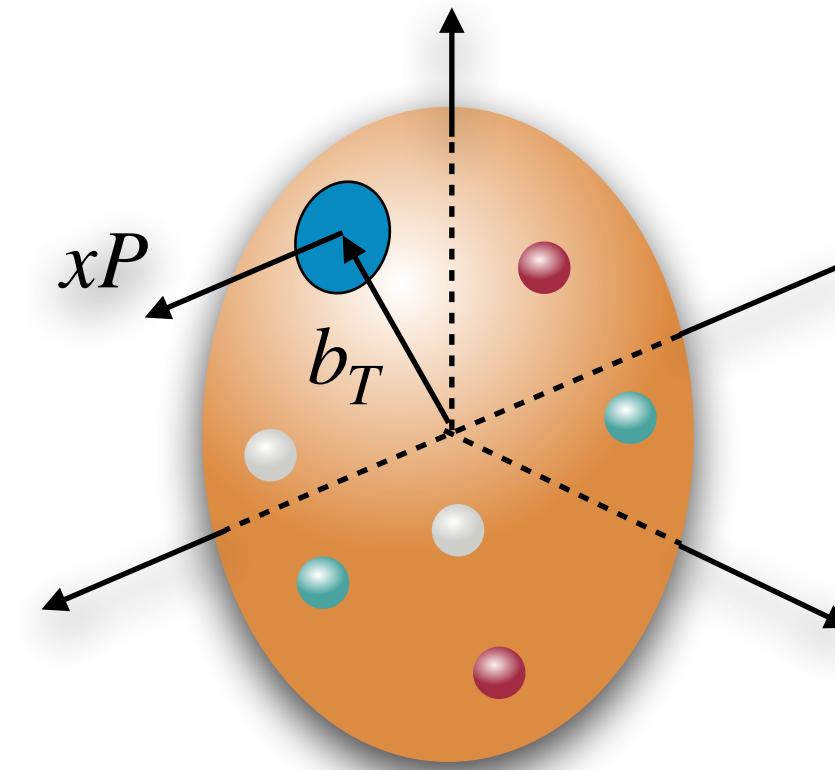
5D

Wigner distributions

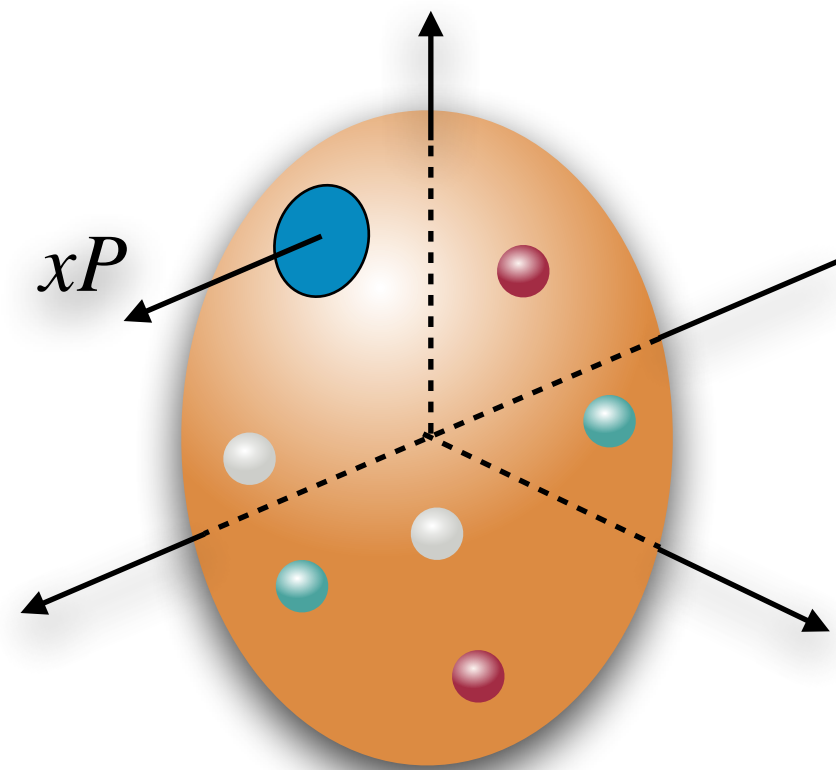


3D

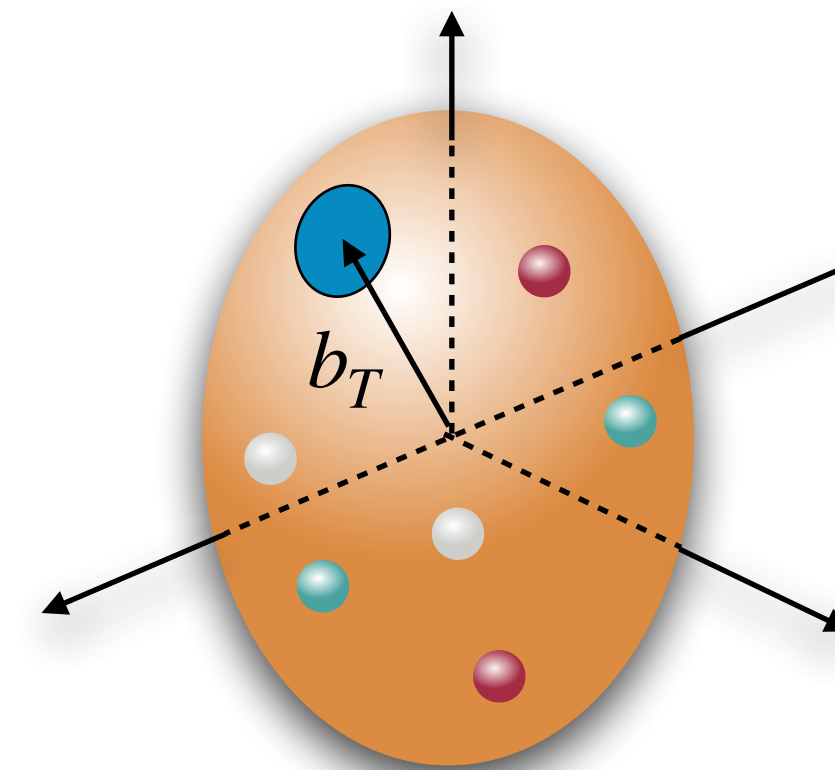
FT of GPDs
exclusive



PDFs
(semi-)inclusive



FT of Form Factors



1D

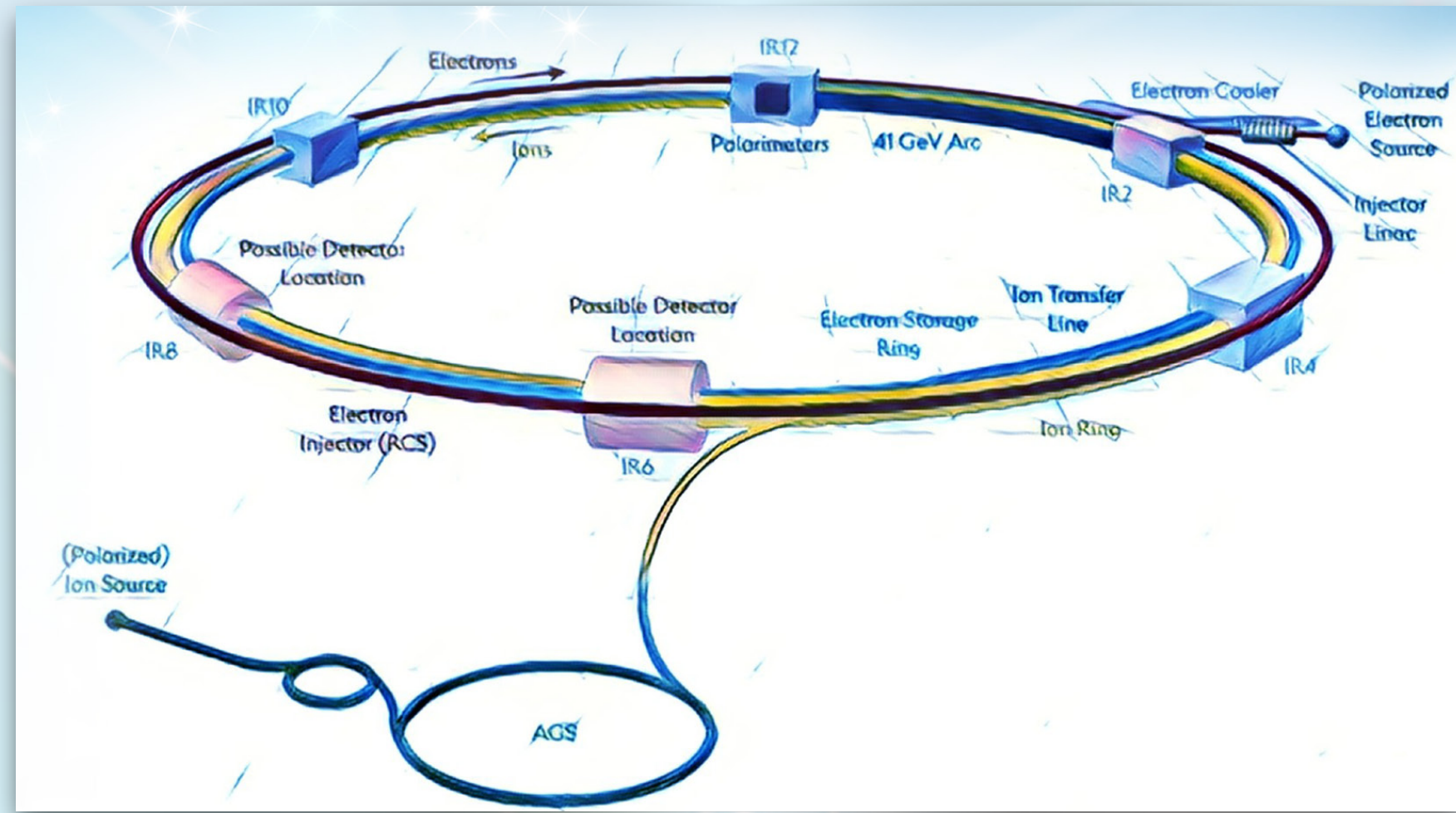
2D

→ \vec{b}_\perp dependence
→ \vec{k}_\perp dependence



these two variables are NOT Fourier conjugate

3D proton tomography at new-generation colliders

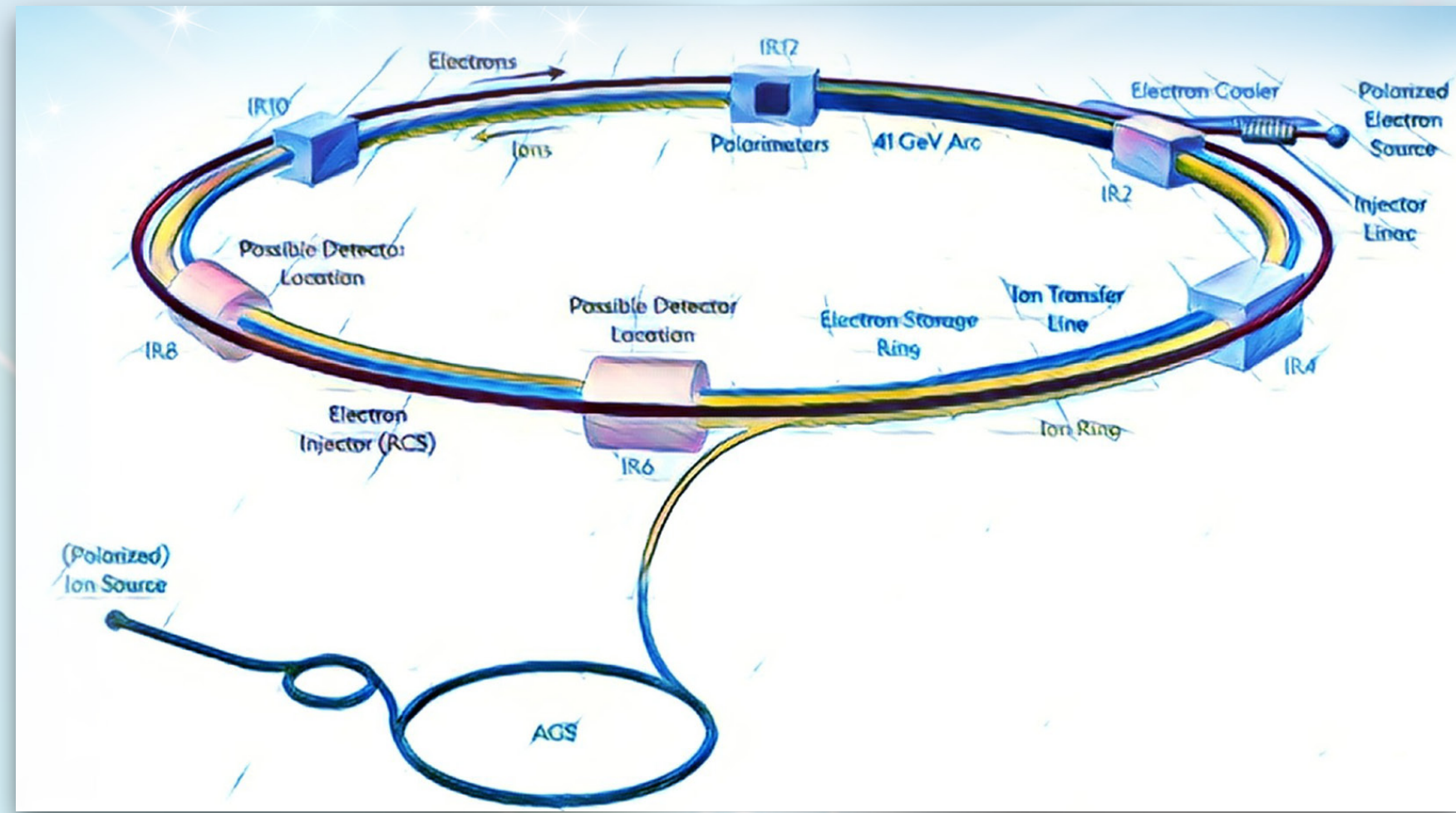


EIC

EIC Yellow Report  [EICUG [arXiv:2103.05419]]

Accessing the proton content

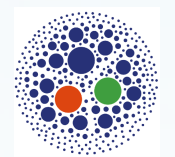
3D proton tomography at new-generation colliders



EIC

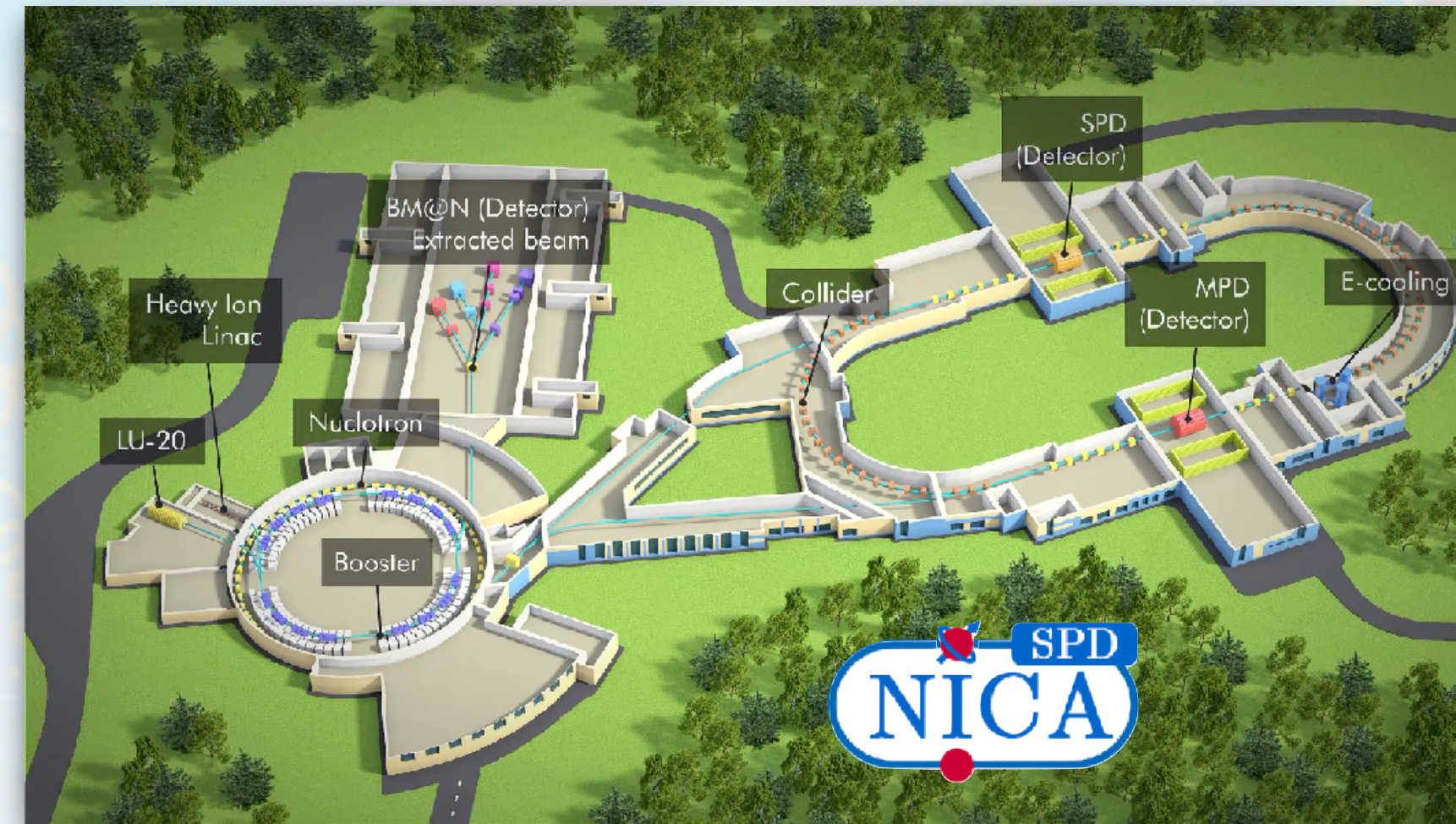
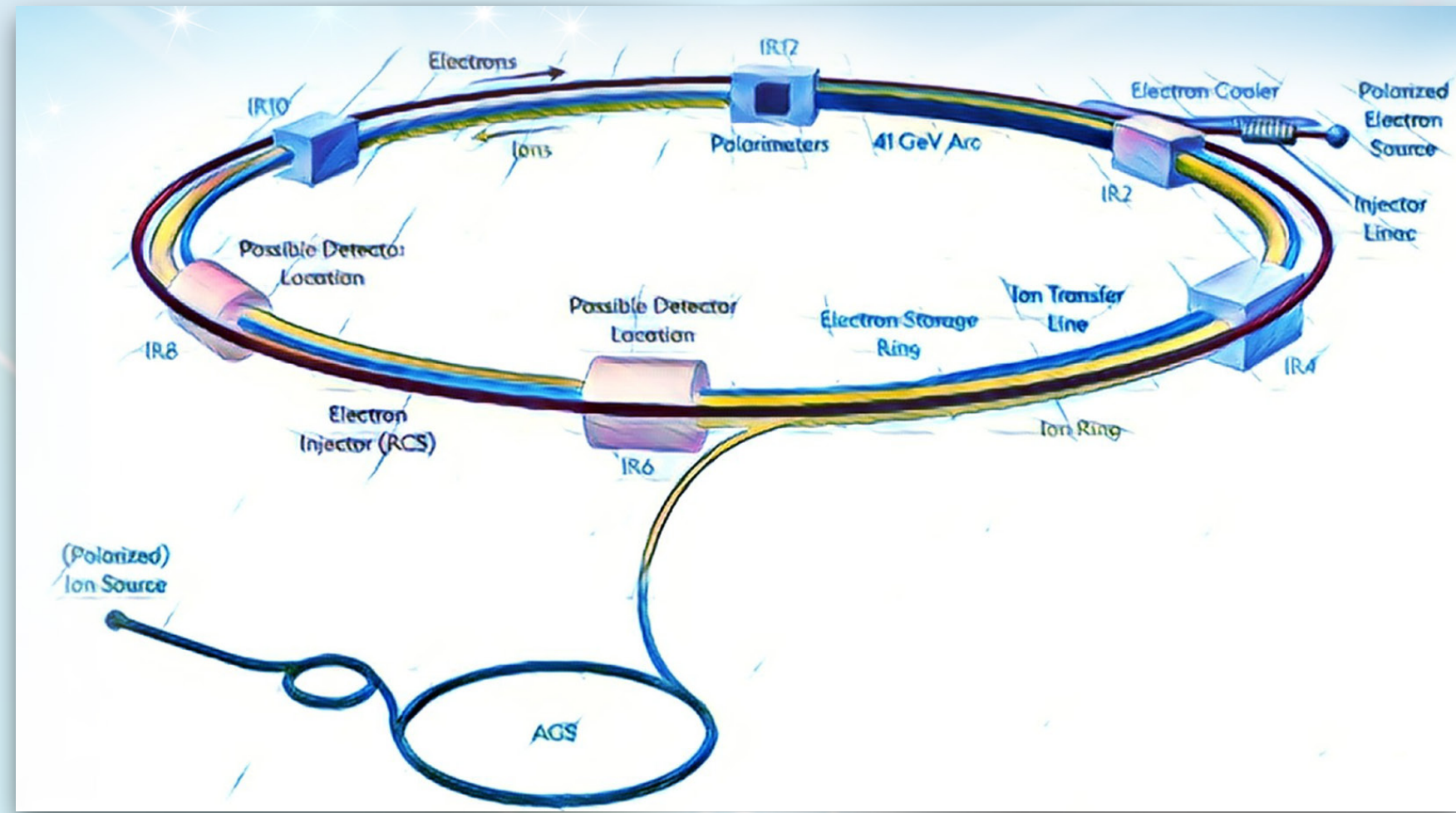
EIC Yellow Report  [EICUG [arXiv:2103.05419]]

Accessing the proton content



Core sector of EIC analyses

3D proton tomography at new-generation colliders



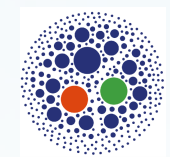
EIC

NICA-SPD

EIC Yellow Report  [EICUG [arXiv:2103.05419]]

Glucos at NICA-SPD  [Prog. Part. Nucl. Phys. 119 (2021) 103858]

Accessing the proton content

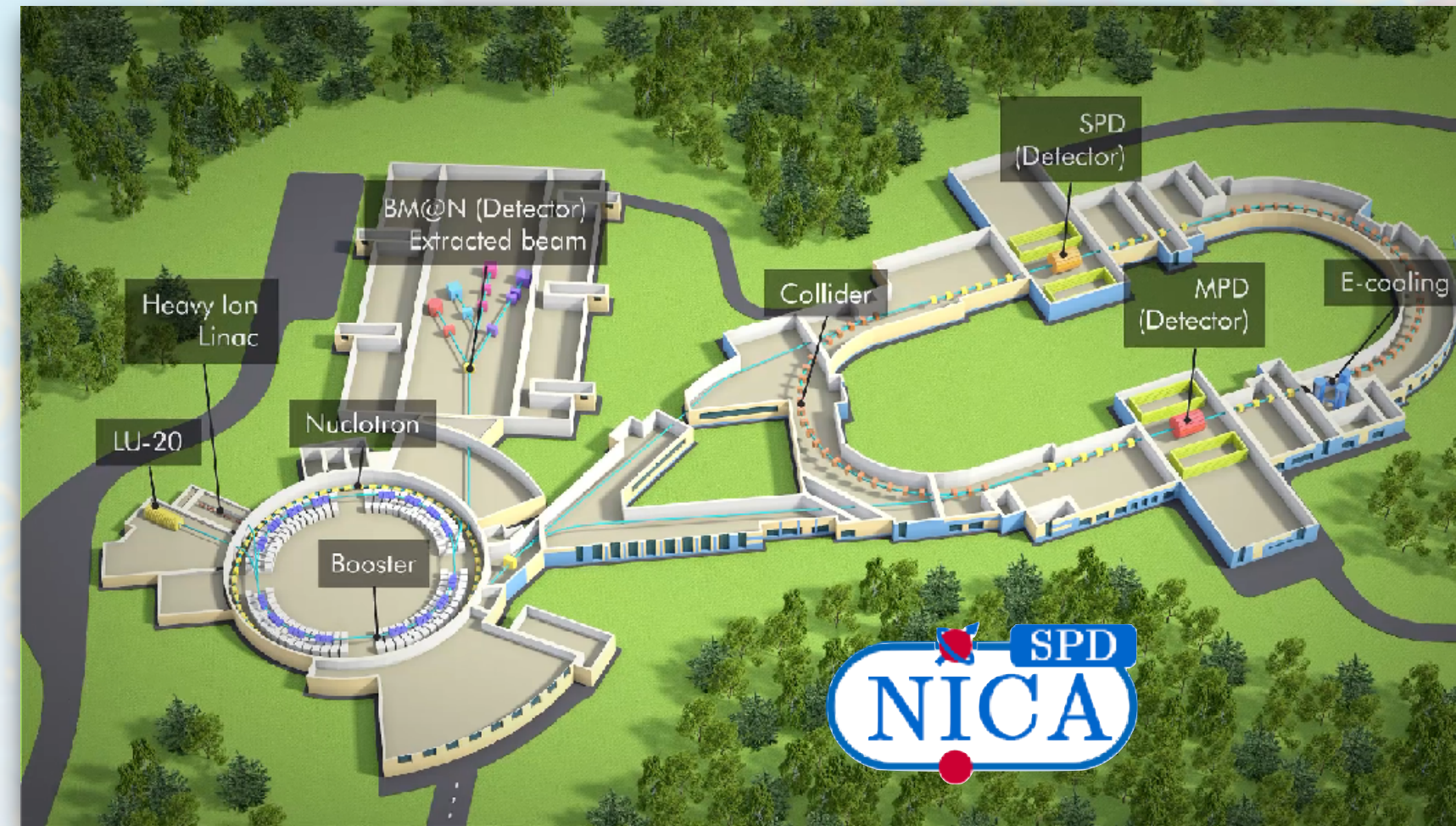
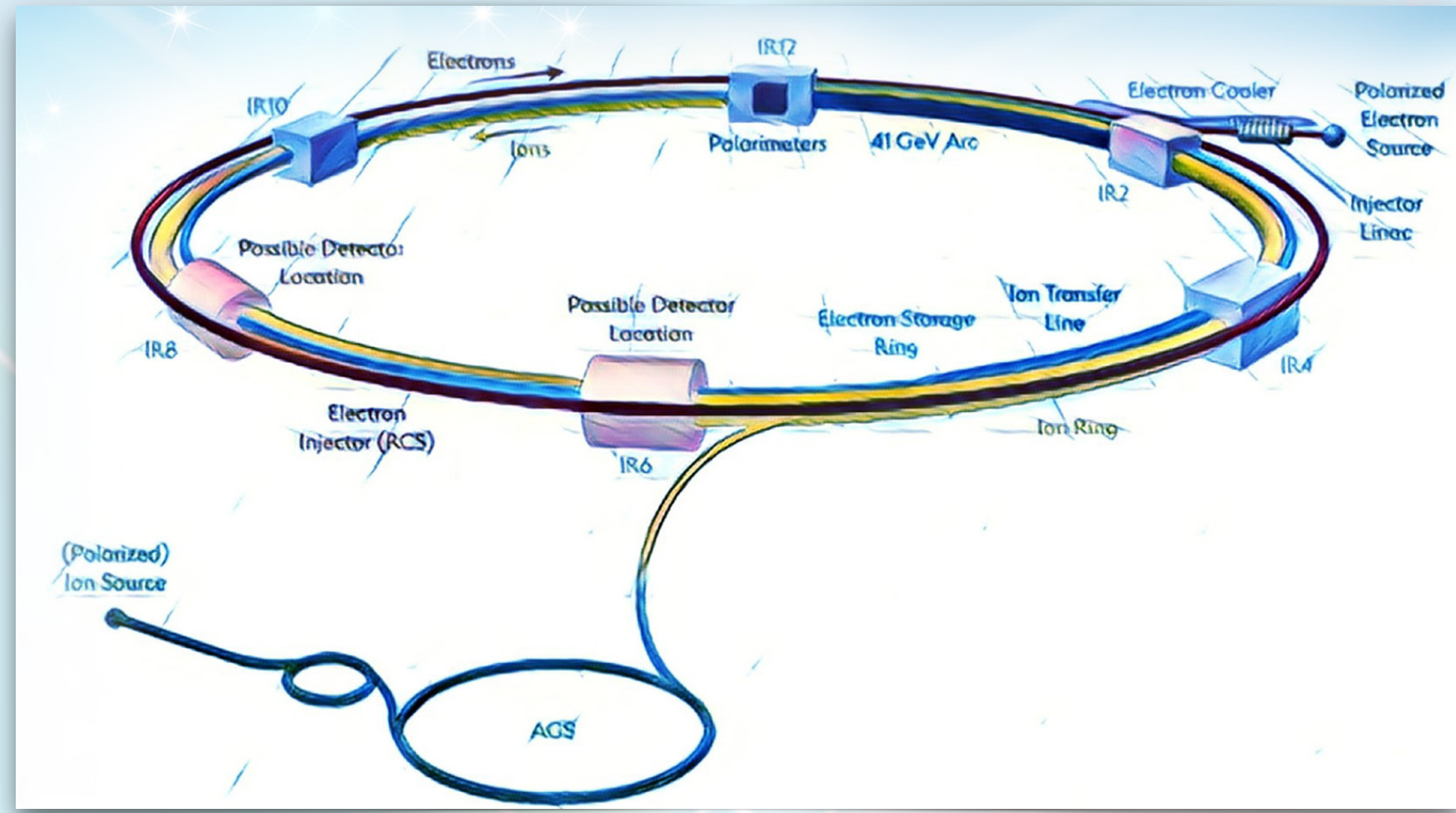


Core sector of EIC analyses



Significance of large-x studies at NICA-SPD

3D proton tomography at new-generation colliders



EIC

NICA-SPD

HL-LHC, FPF

EIC Yellow Report [\[EICUG \[arXiv:2103.05419\]\]](#)

Glucos at NICA-SPD [\[Prog. Part. Nucl. Phys. 119 \(2021\) 103858\]](#)

Accessing the proton content



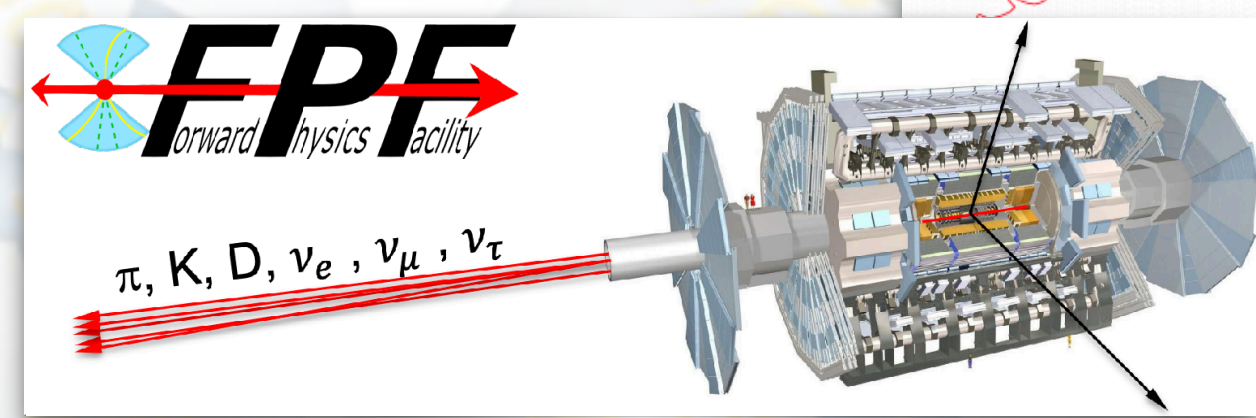
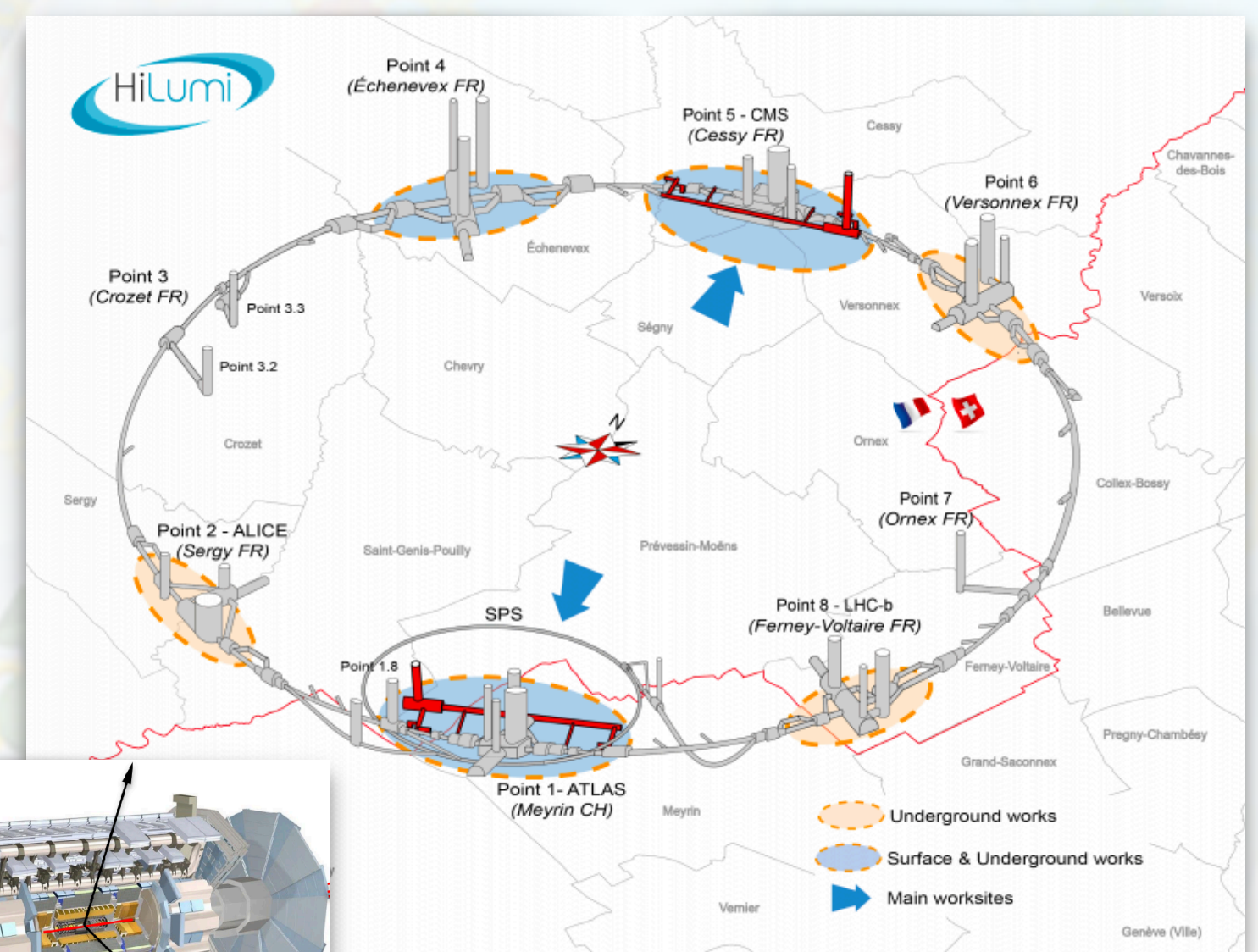
Core sector of EIC analyses



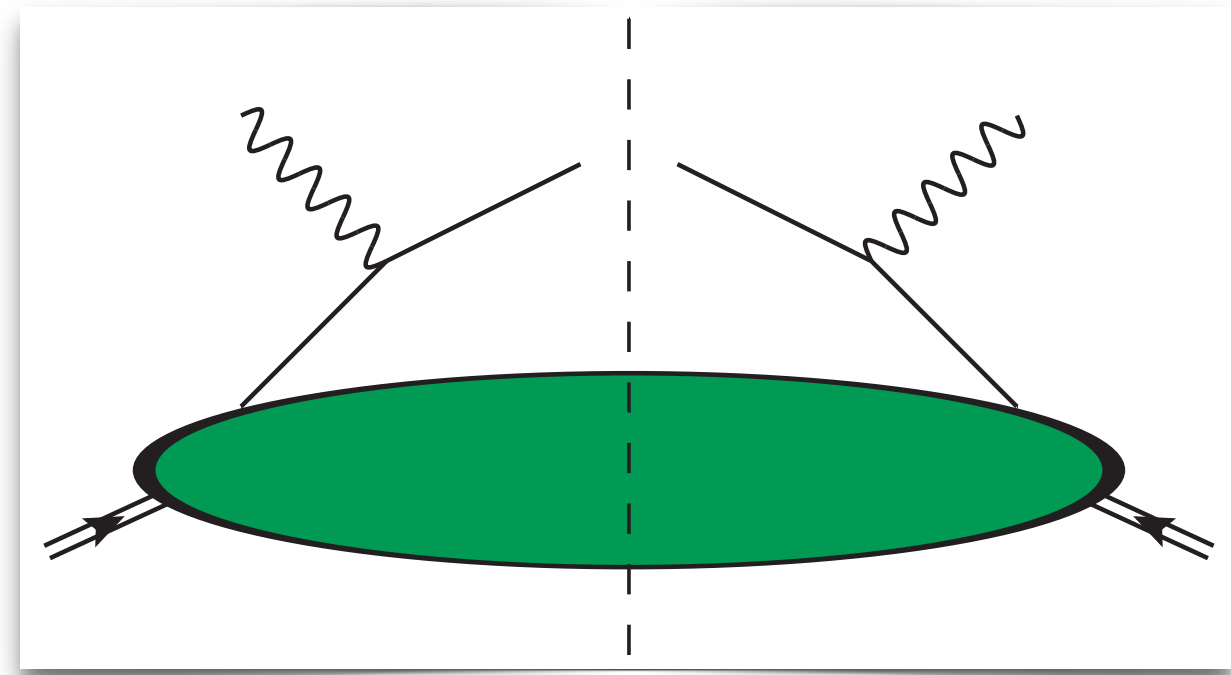
Significance of large-x studies at NICA-SPD



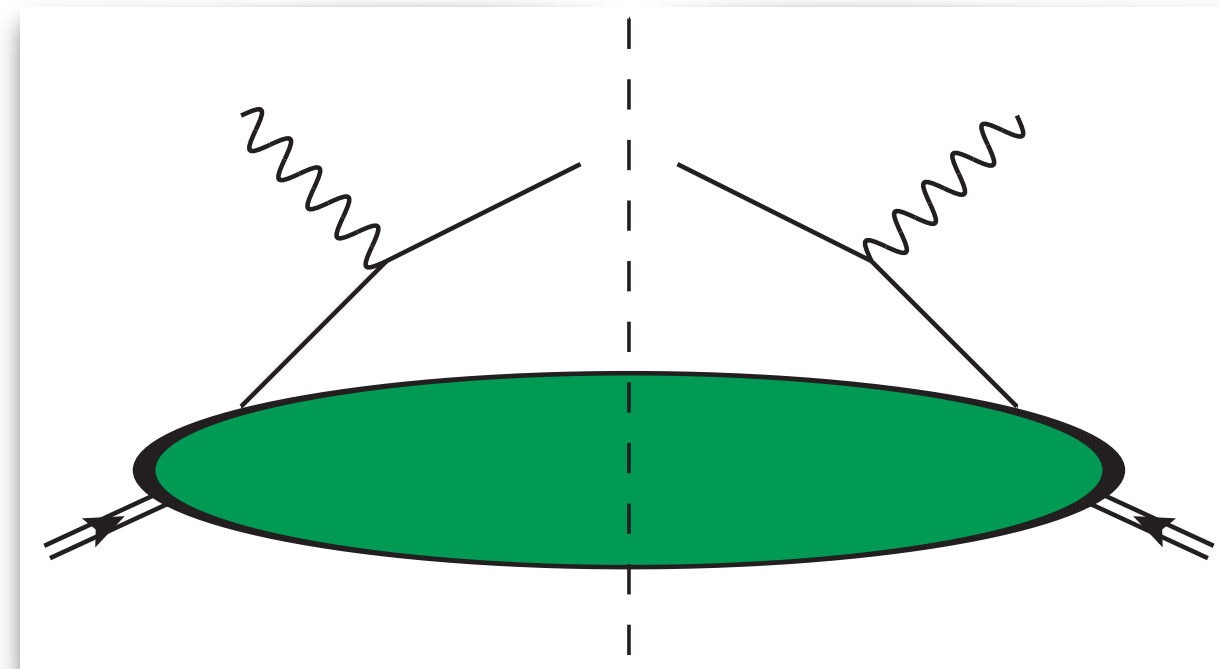
Intrinsic gluon polarization at HL-LHC (+ FPF)



Gauge links and processes dependence



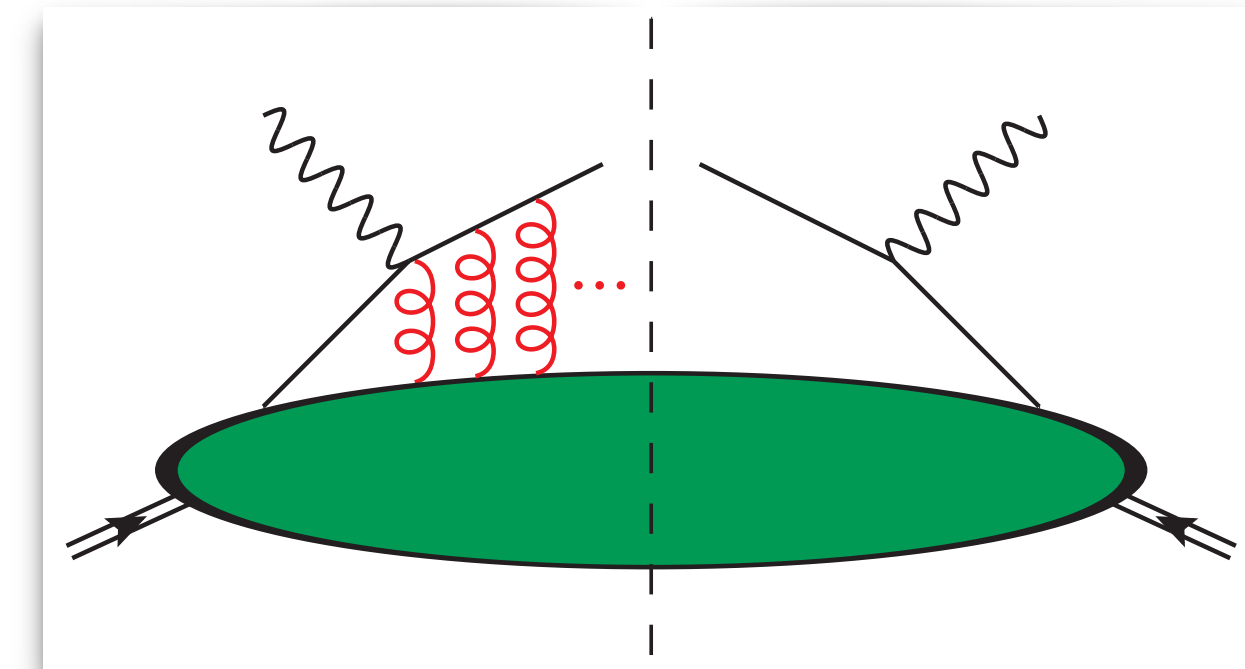
Gauge links and processes dependence



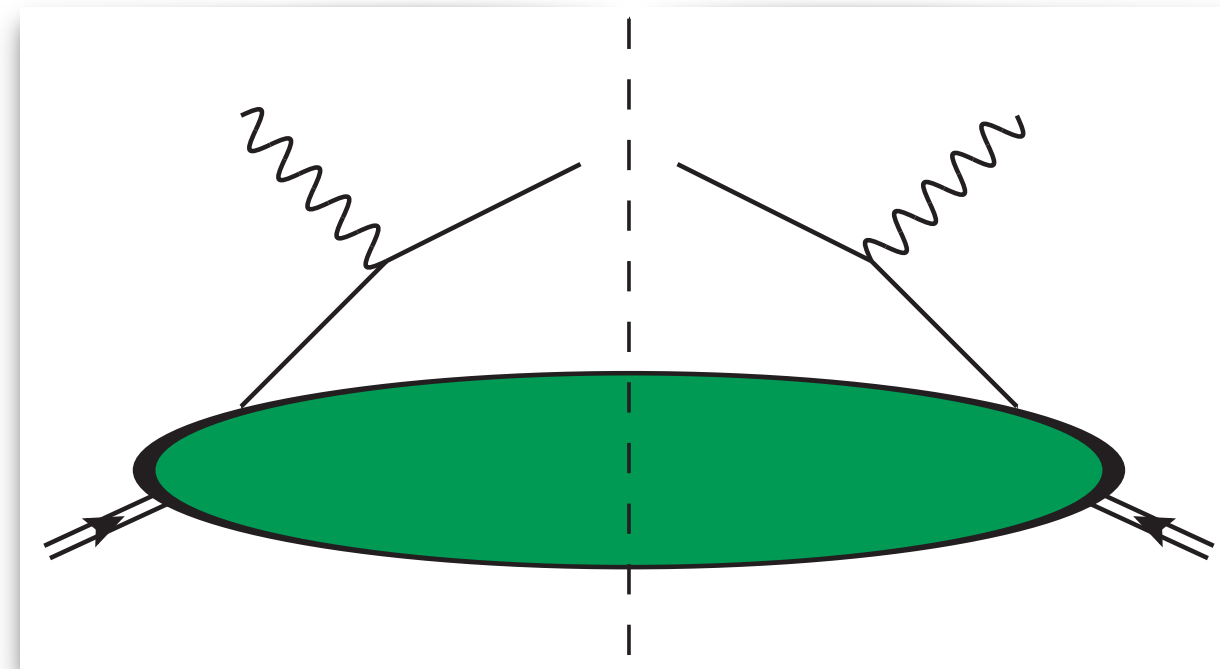
$$\bar{\psi}_j(0) U(0, \xi) \psi_i(\xi)$$



Gauge link (Wilson line)
Resummation of (calculable)
infinite gluon emissions



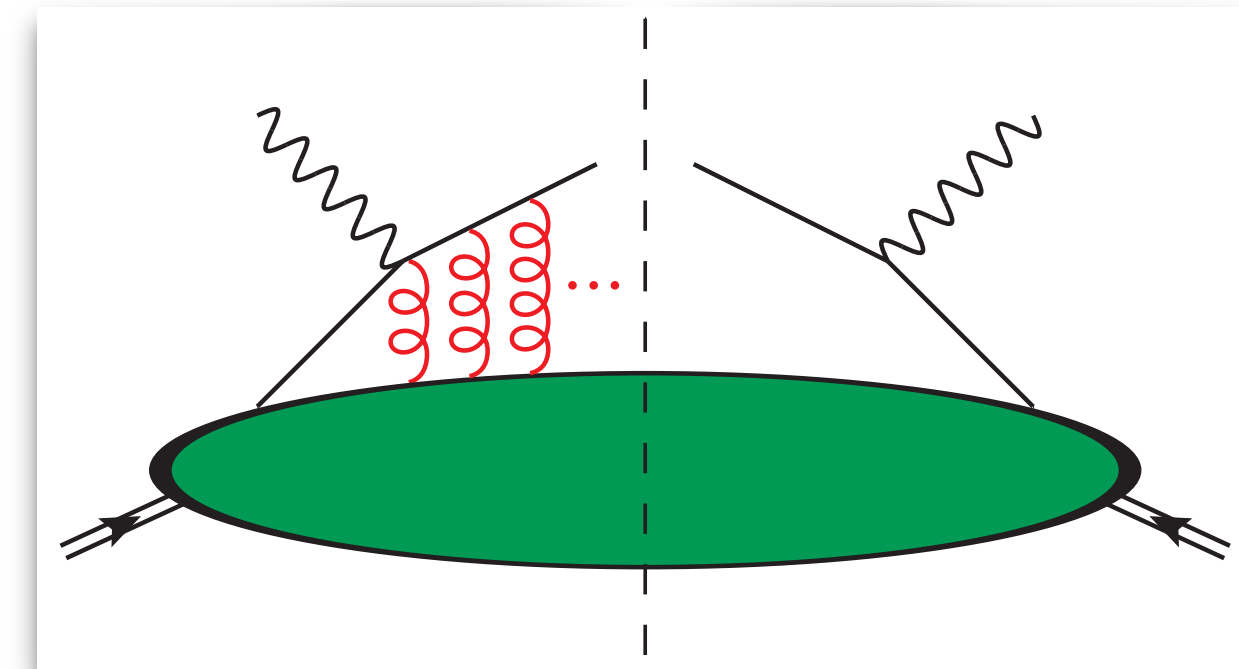
Gauge links and processes dependence



$$\bar{\psi}_j(0) U(0, \xi) \psi_i(\xi)$$

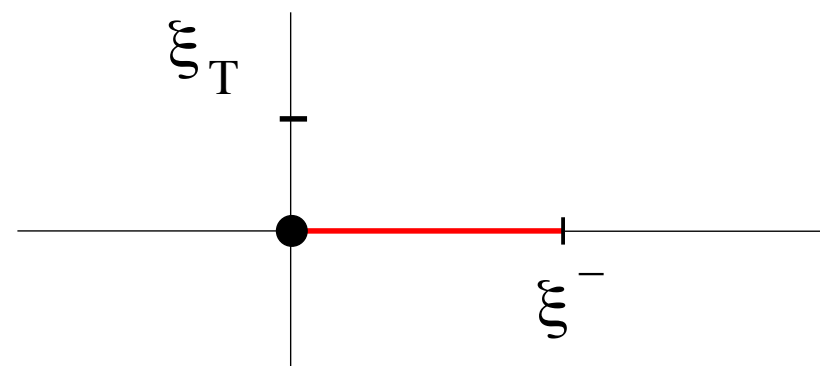


Gauge link (Wilson line)
Resummation of (calculable)
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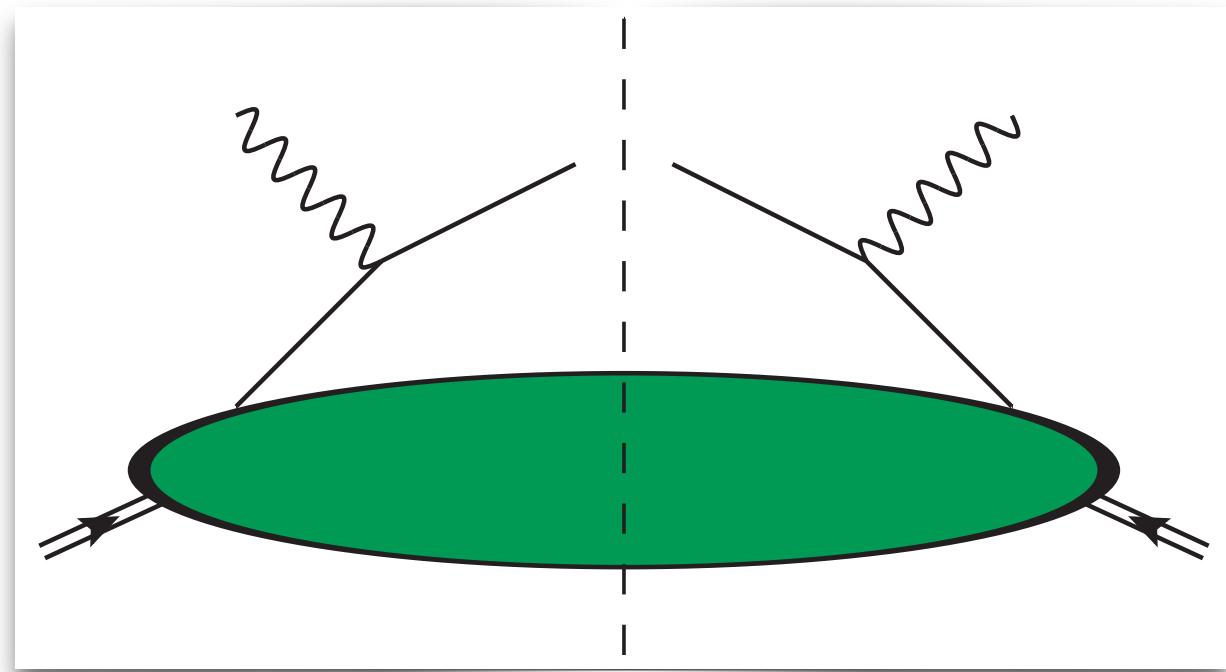
Collinear PDFs

$$\Phi_{ij}(x) \doteq \int d^2 \mathbf{p}_T \Phi_{ij}(x, \mathbf{p}_T) = \int \frac{d\xi^-}{2\pi} e^{ip \cdot \xi} \langle P | \bar{\psi}_j(0) \psi_i(\xi) | P \rangle |_{\xi^+ = 0, \xi_T = 0}$$



- Light-cone: $\xi^+ = 0, \xi = 0$
- **Straight** gauge link (unique!)
- ($A^+ = 0$) light-cone: WL = $\hat{1}$
- ✓ **Universality warranted**

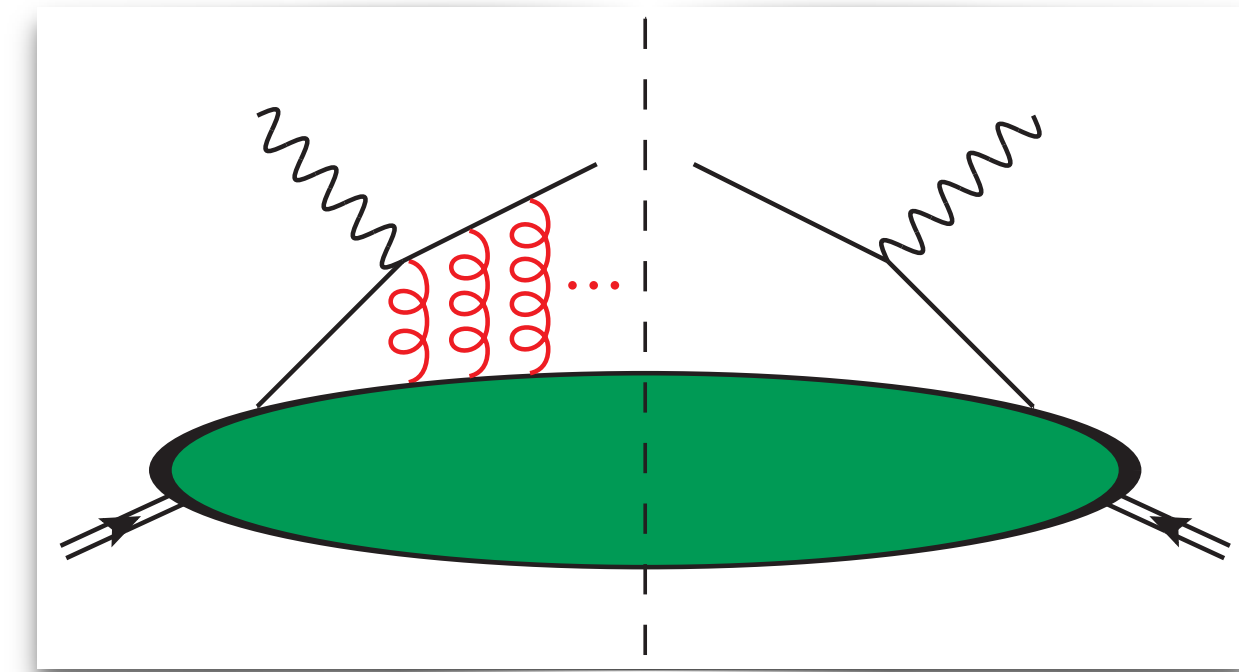
Gauge links and processes dependence



$$\bar{\psi}_j(0) U(0, \xi) \psi_i(\xi)$$

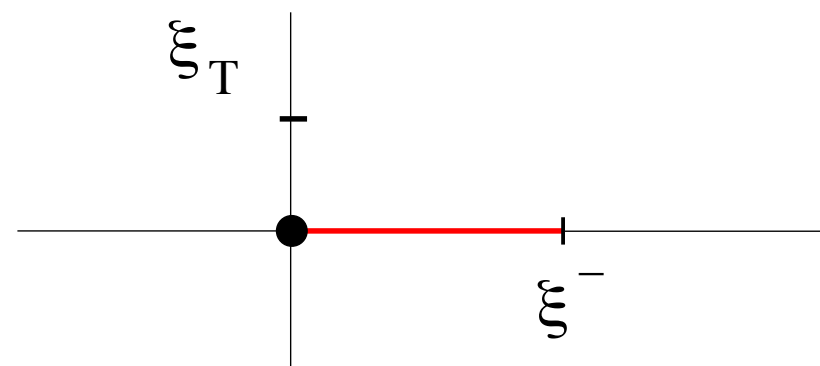


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Collinear PDFs

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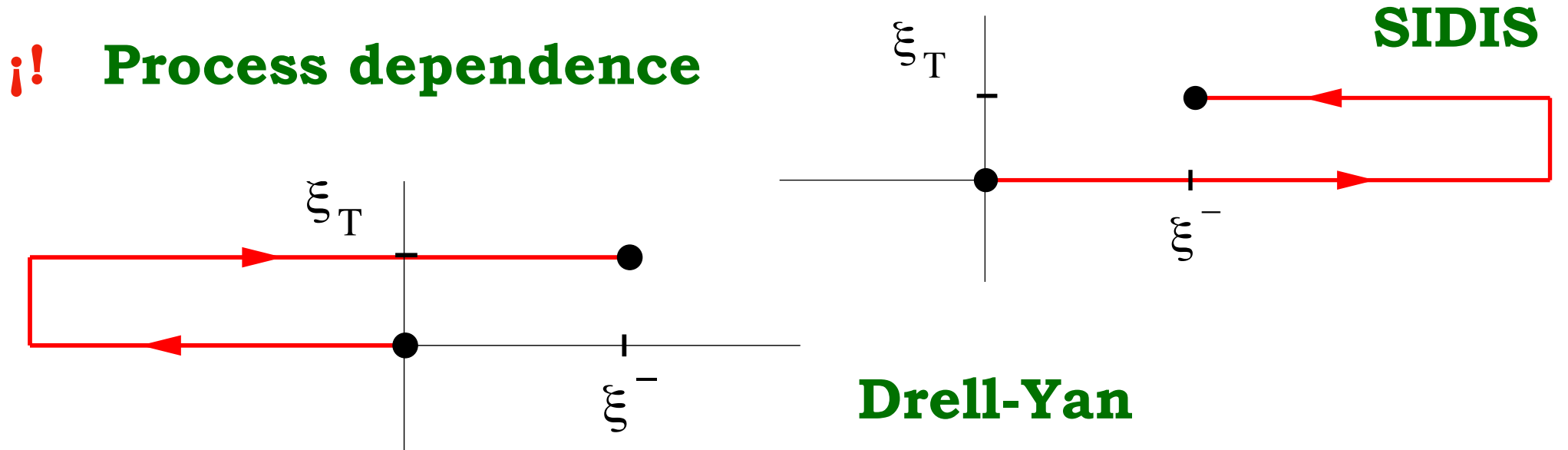


- Light-cone: $\xi^+ = 0, \xi = 0$
- **Straight** gauge link (unique!)
- $(A^+ = 0)$ light-cone: $WL = \hat{1}$
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TMD PDFs

- *Transverse* gauge link not eliminated by gauge choice
- **Staple-like** gauge link (not unique!)

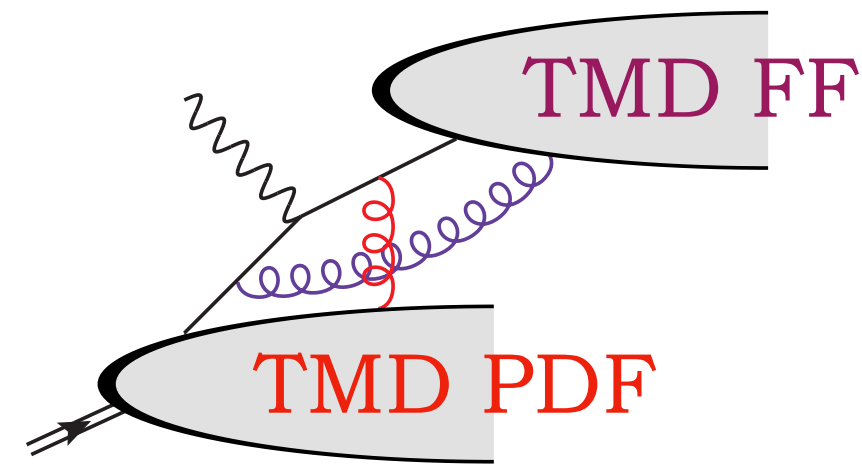
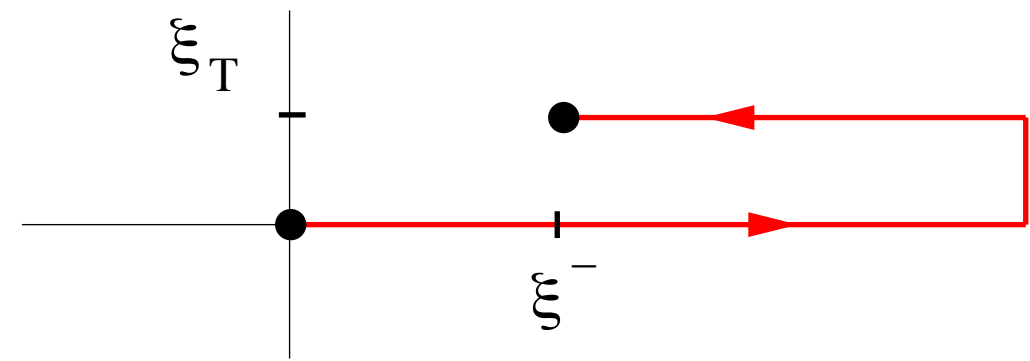
⚠ **Process dependence**



Process dependence of quark TMD PDFs & FFs

SIDIS

[+] staple link



- * PDF \rightarrow color flow annihilated within final state
- * FF \rightarrow color flow from final to initial state

Process dependence of quark TMD PDFs & FFs

SIDIS

[+] staple link



- * PDF \rightarrow color flow annihilated within final state
- * FF \rightarrow color flow from final to initial state

Drell-Yan

[-] staple link

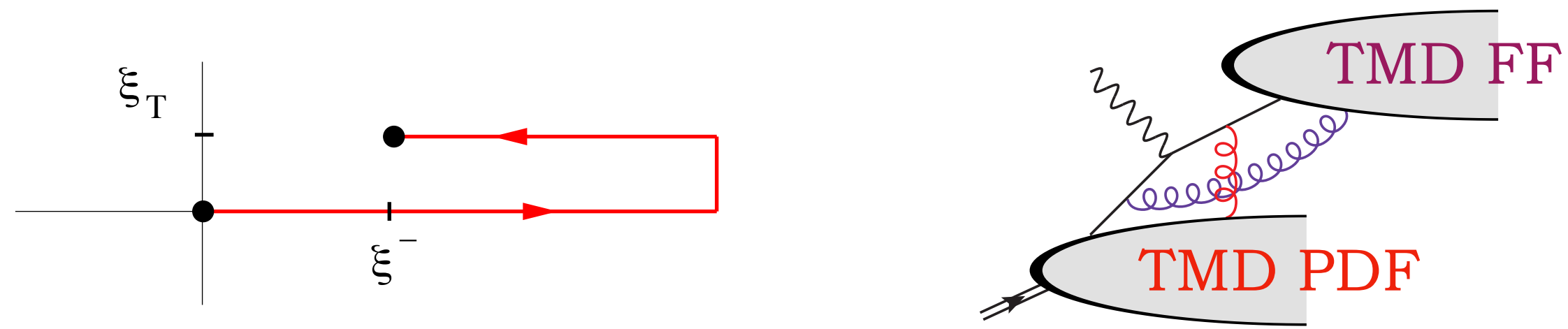


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Process dependence of quark TMD PDFs & FFs

SIDIS

[+] staple link



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Drell-Yan

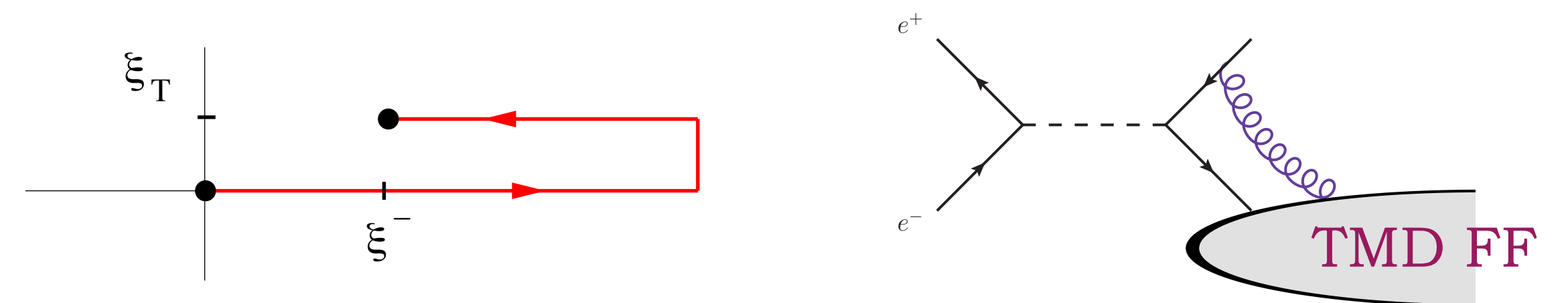
[-] staple link



- * PDF \rightarrow color flow from final to initial state

$e^+ + e^- \rightarrow$ hadrons

[+] staple link



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Process dependence of quark TMD PDFs & FFs

SIDIS

[+] staple link



- * PDF \rightarrow color flow annihilated within final state
- * FF \rightarrow color flow from final to initial state

Drell-Yan

[-] staple link



- * PDF \rightarrow color flow from final to initial state

Modified universality

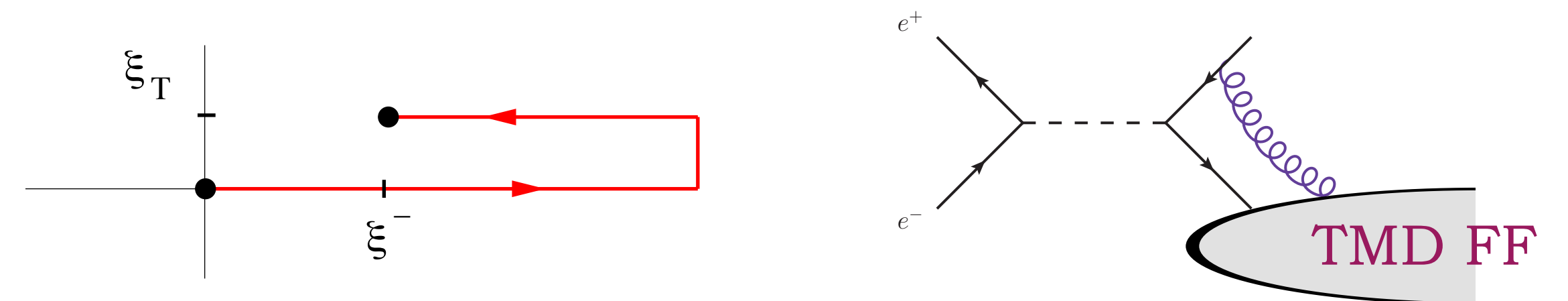
- * PDFs \rightarrow change of sign in T-odd densities

$$f_{1T}^\perp [\text{SIDIS}] \equiv f_{1T}^\perp [+] = -f_{1T}^\perp [-] \equiv -f_{1T}^\perp [\text{DY}]$$

- * FFs \rightarrow standard universality preserved

$e^+ + e^- \rightarrow \text{hadrons}$

[+] staple link



- * FF \rightarrow color flow annihilated within final state

Gluon TMD PDFs: Gauge links & modified universality

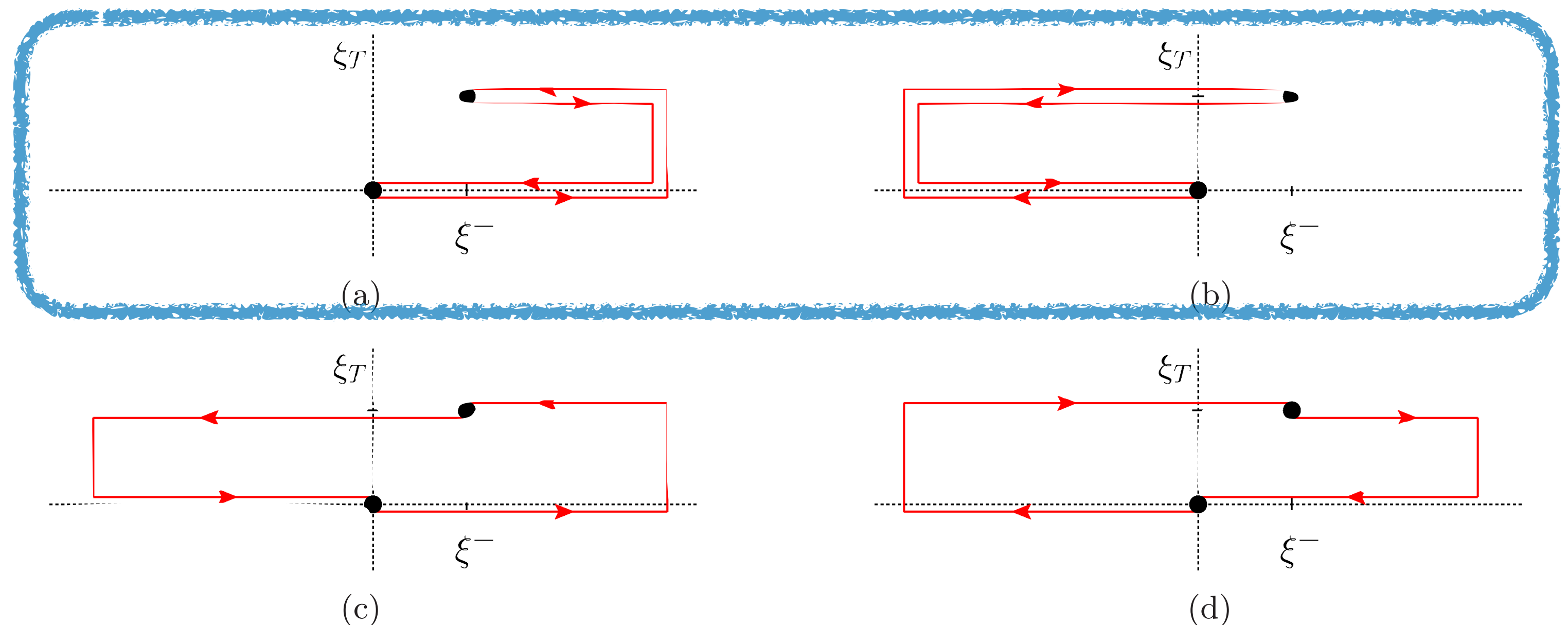
- * Gluon TMDs → more complicated structure with respect to quark staple links
- * Factorization-preserving processes → two main kinds of modified universality
- * Different classes of processes → distinct gluon TMDs, not related to each other

Gluon TMD PDFs: Gauge links & modified universality

- * Gluon TMDs \rightarrow more complicated structure with respect to quark staple links
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f-type (WW)

(a) [+ , +] or (b) [- , -]

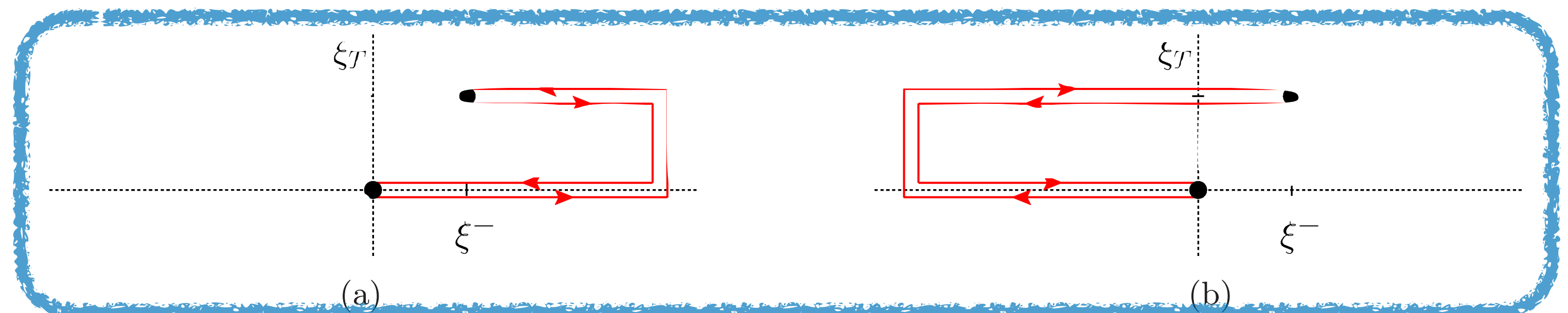


Gluon TMD PDFs: Gauge links & modified universality

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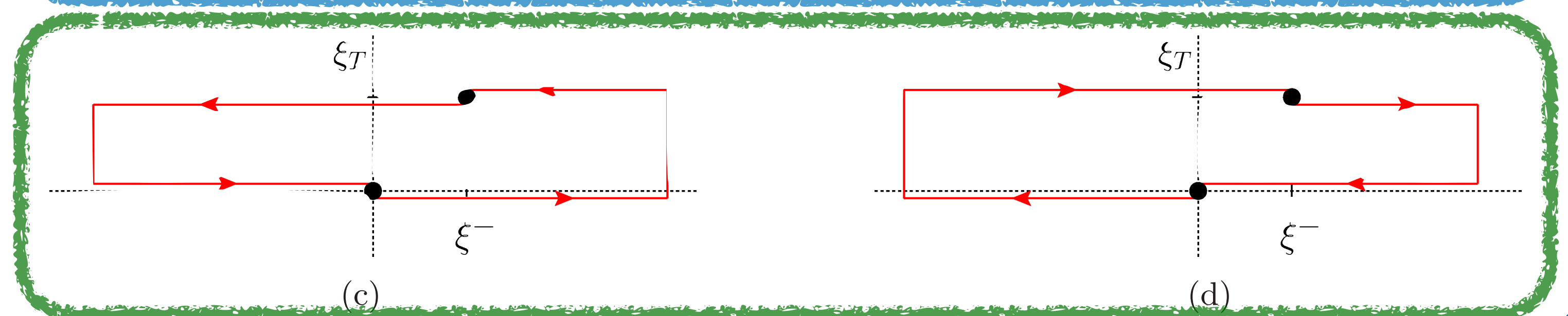
f-type (WW)

(a) $[+, +]$ or (b) $[-, -]$



d-type (dipole)

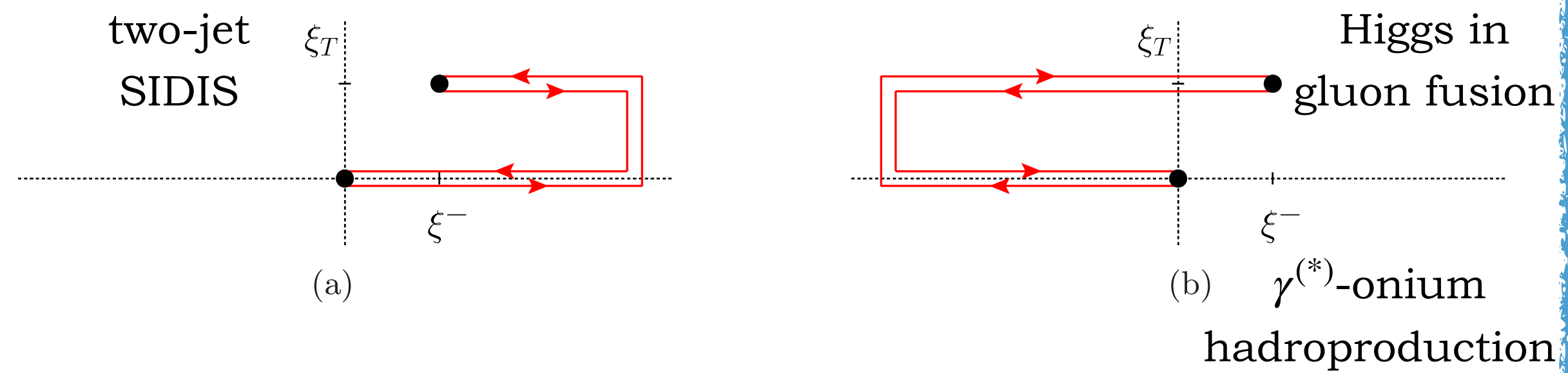
(c) $[+, -]$ or (d) $[-, +]$



Accessing f-type and d-type gluon TMD PDFs

f-type (WW)

(a) [+ , +] or (b) [- , -]

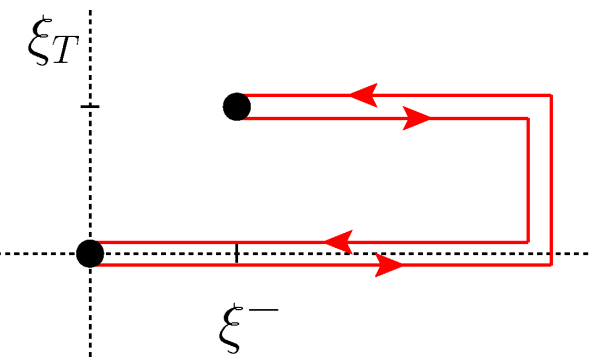


Accessing f-type and d-type gluon TMD PDFs

f-type (WW)

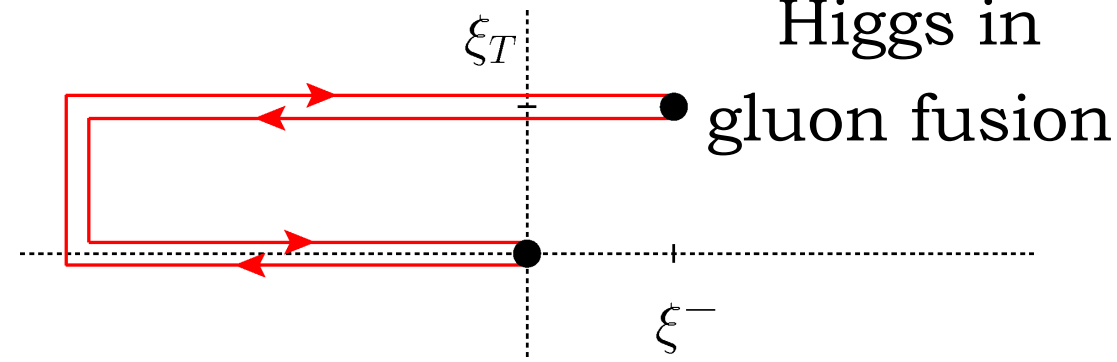
(a) [+ , +] or (b) [- , -]

two-jet
SIDIS



(a)

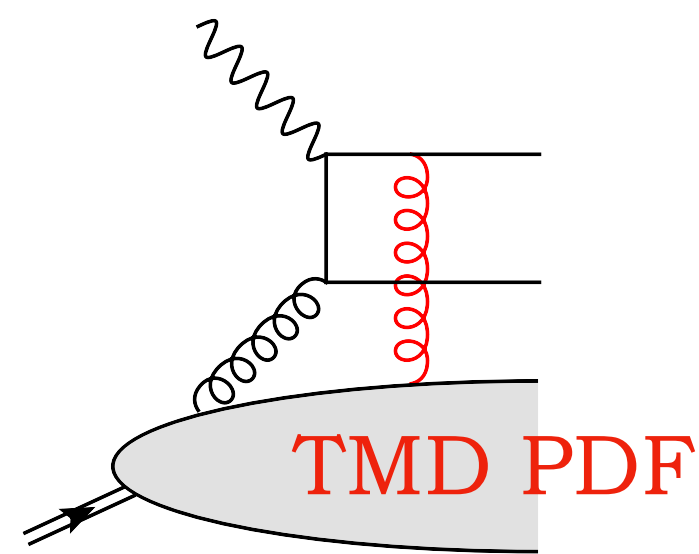
Higgs in
gluon fusion



(b)

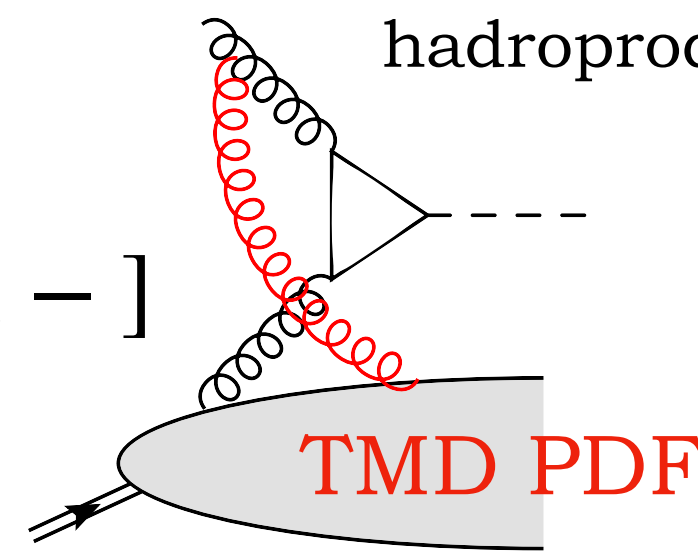
$\gamma^{(*)}$ -onium
hadroproduction

[+



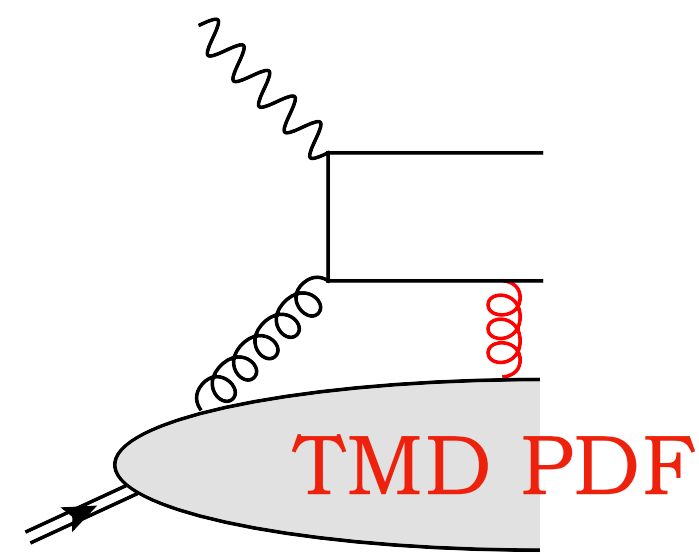
TMD PDF

[- , -]



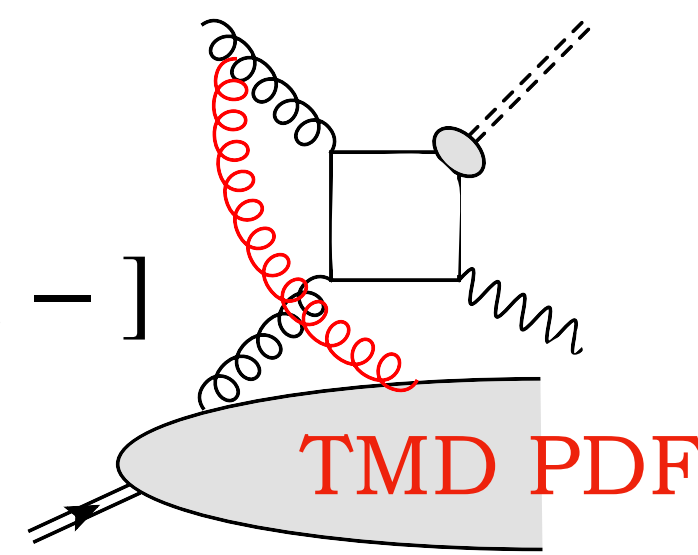
TMD PDF

+]



TMD PDF

[- , -]



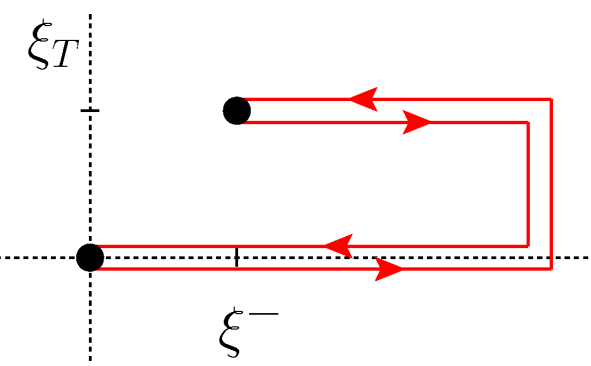
TMD PDF

Accessing f-type and d-type gluon TMD PDFs

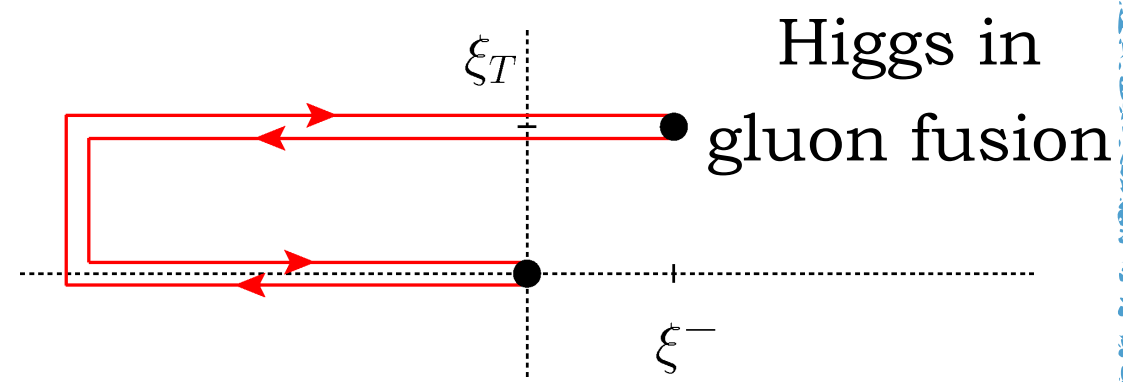
f-type (WW)

(a) $[+, +]$ or (b) $[-, -]$

two-jet
SIDIS



(a)

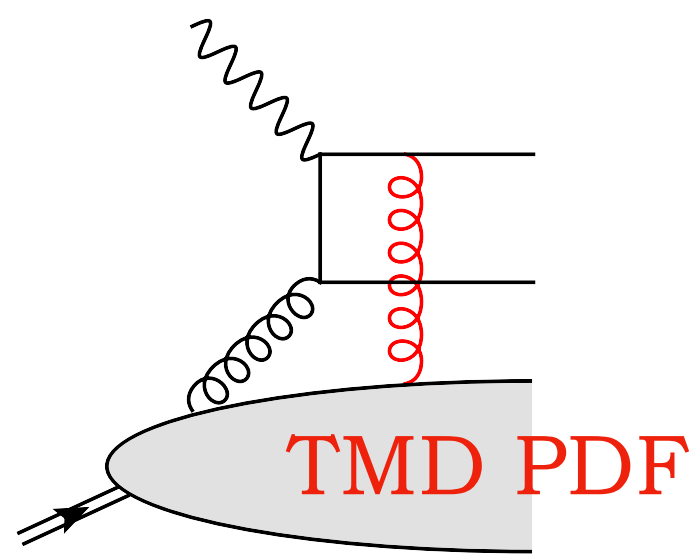


(b)

Higgs in
gluon fusion

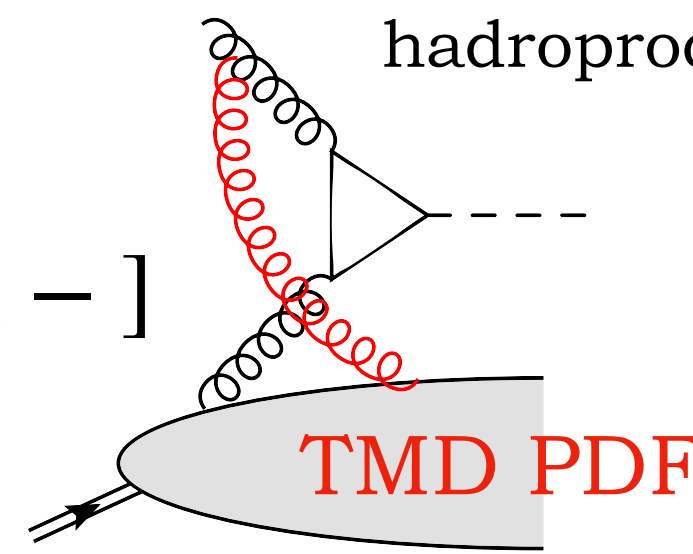
$\gamma^{(*)}$ -onium
hadroproduction

[+]



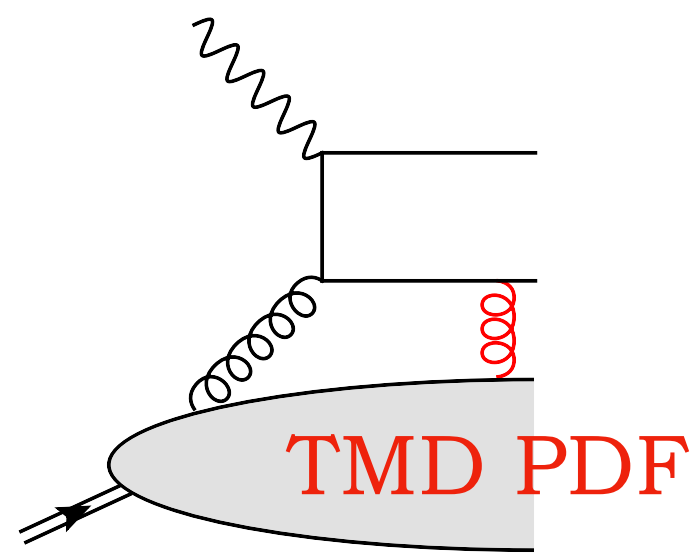
TMD PDF

[-, -]



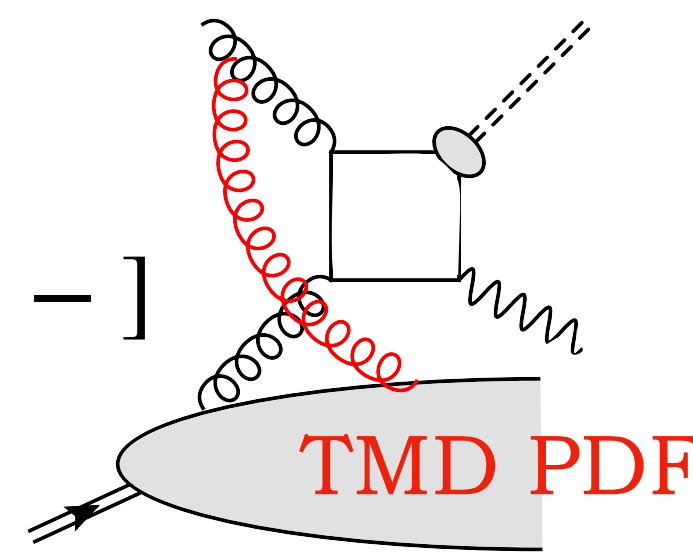
TMD PDF

[+]



TMD PDF

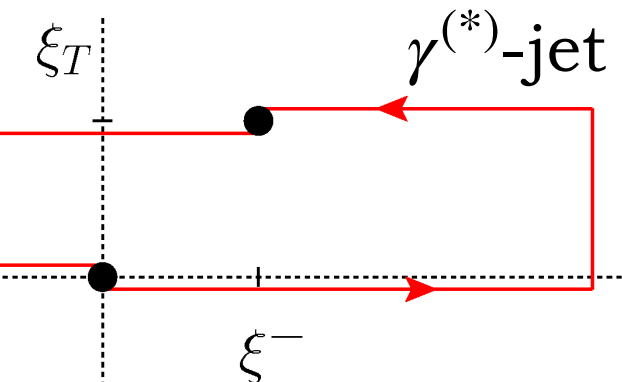
[-, -]



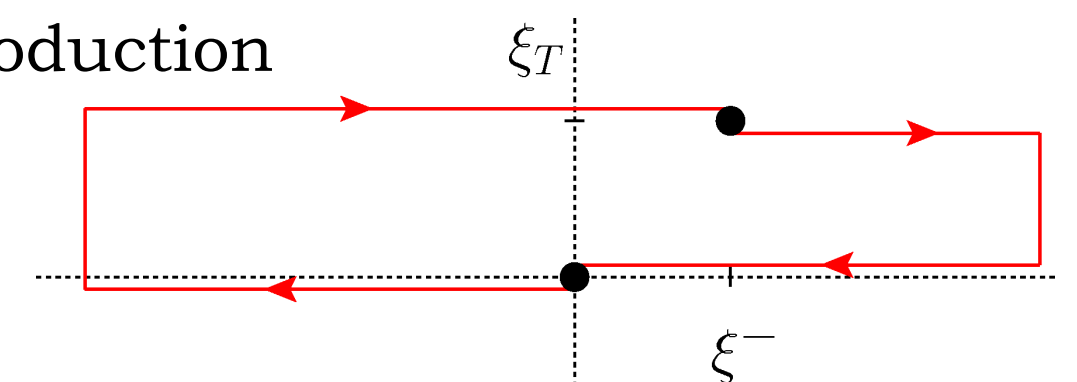
TMD PDF

d-type (DP)

(c) $[+, -]$ or (d) $[-, +]$



(c)



(d)

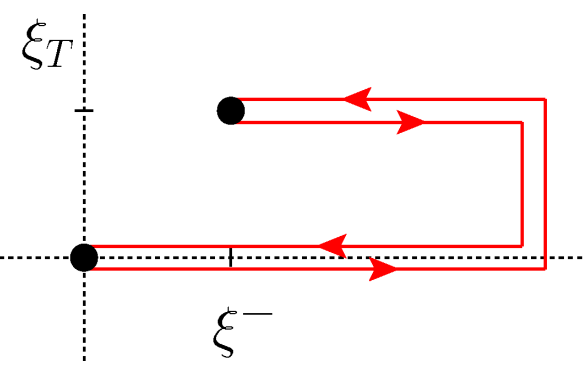
$\gamma^{(*)}$ -jet hadroproduction

Accessing f-type and d-type gluon TMD PDFs

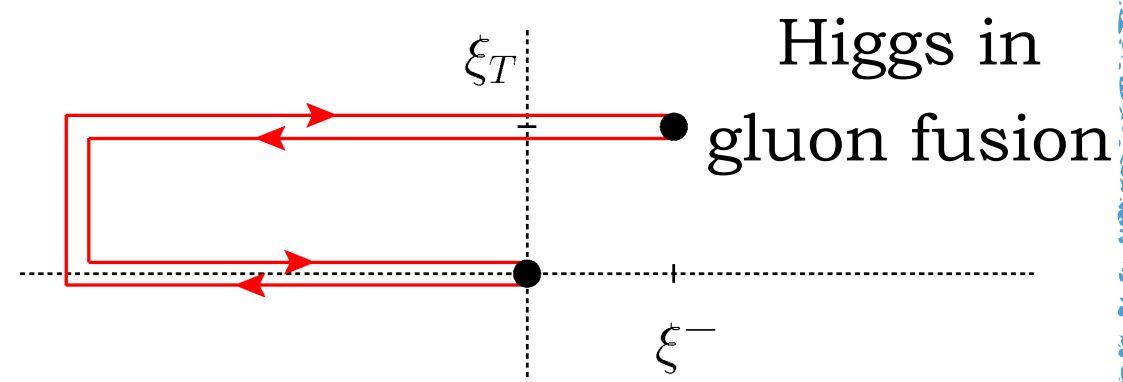
f-type (WW)

(a) $[+, +]$ or (b) $[-, -]$

two-jet
SIDIS



(a)

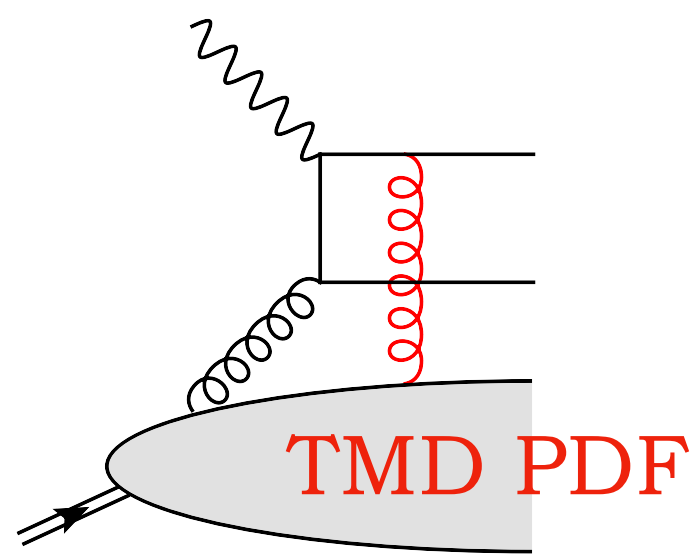


(b)

Higgs in
gluon fusion

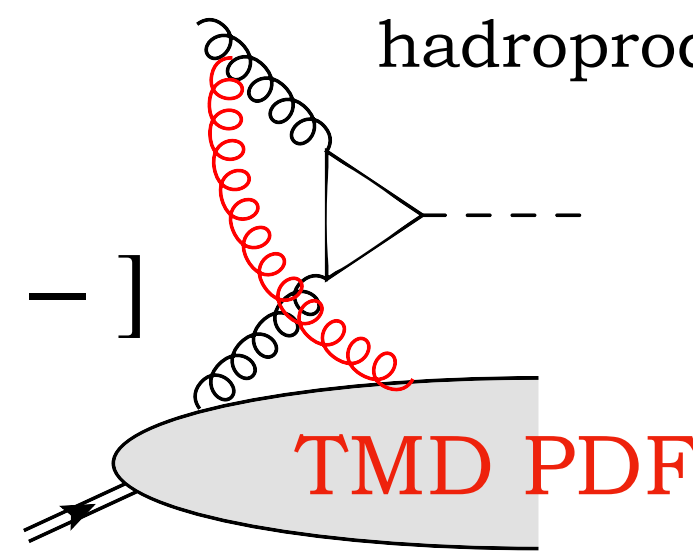
$\gamma^{(*)}$ -onium
hadroproduction

[+]



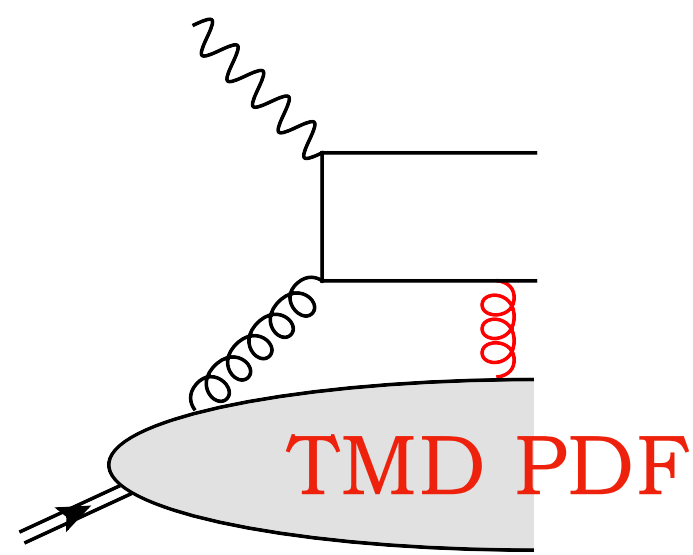
TMD PDF

[-, -]



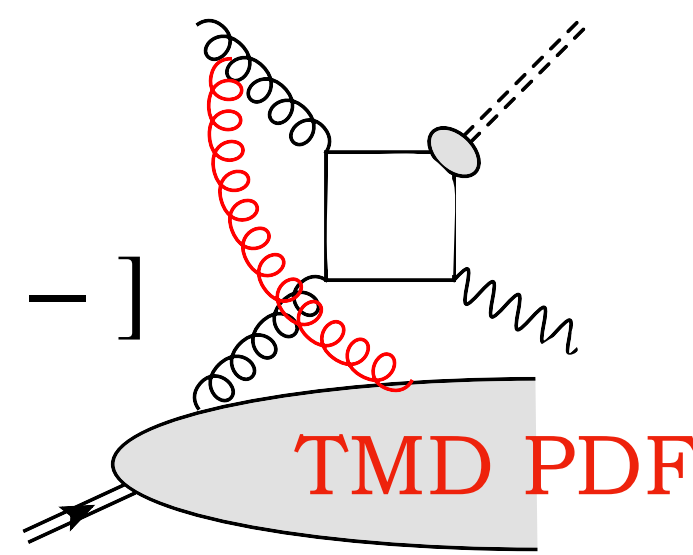
TMD PDF

[+]



TMD PDF

[-, -]



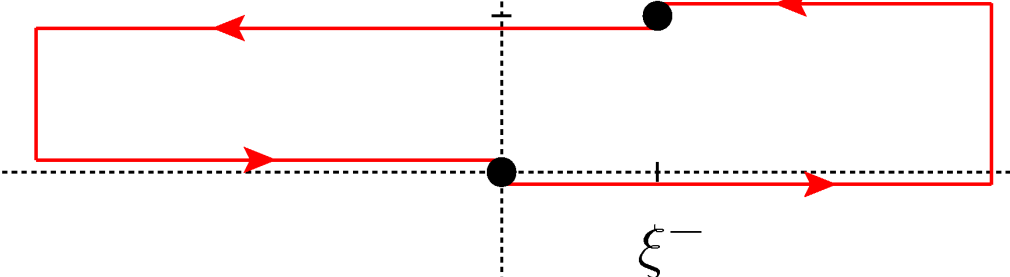
TMD PDF

d-type (DP)

(c) $[+, -]$ or (d) $[-, +]$

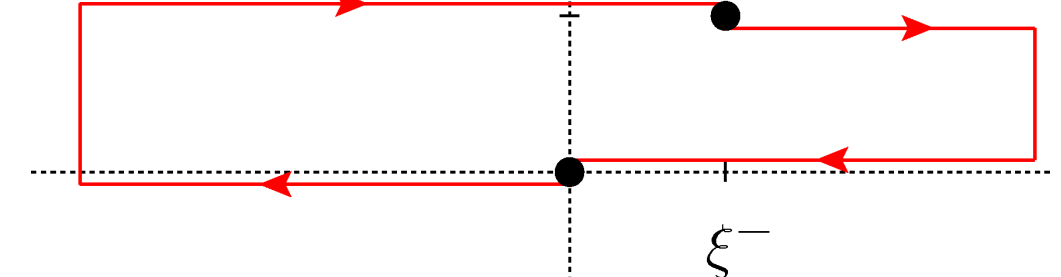
ξ_T

$\gamma^{(*)}$ -jet hadroproduction



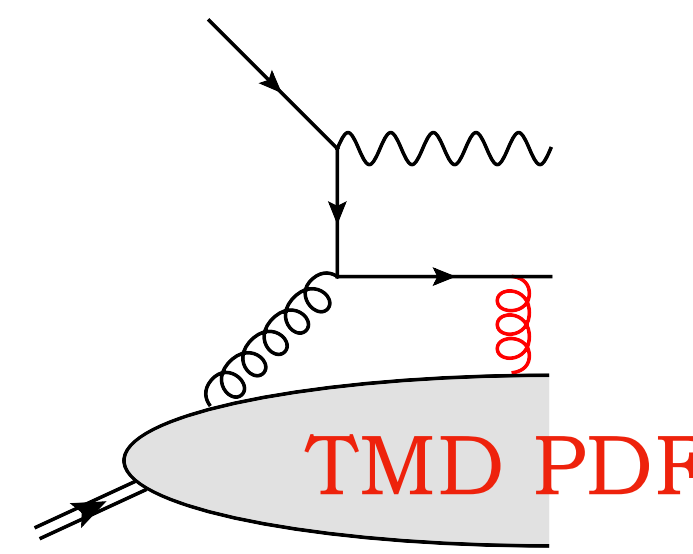
(c)

ξ_T



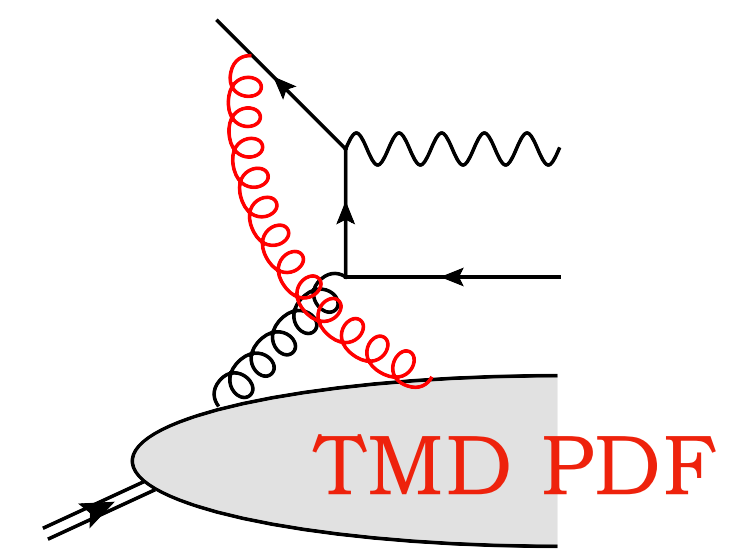
(d)

[+]



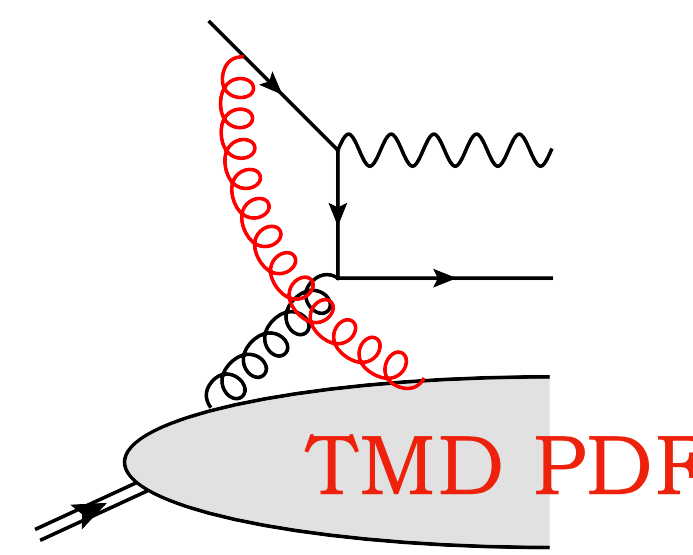
TMD PDF

[-]



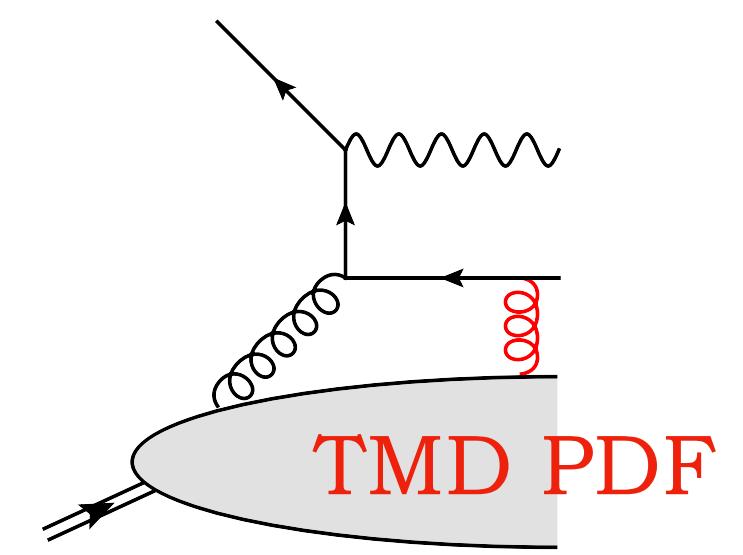
TMD PDF

-]



TMD PDF

[+]

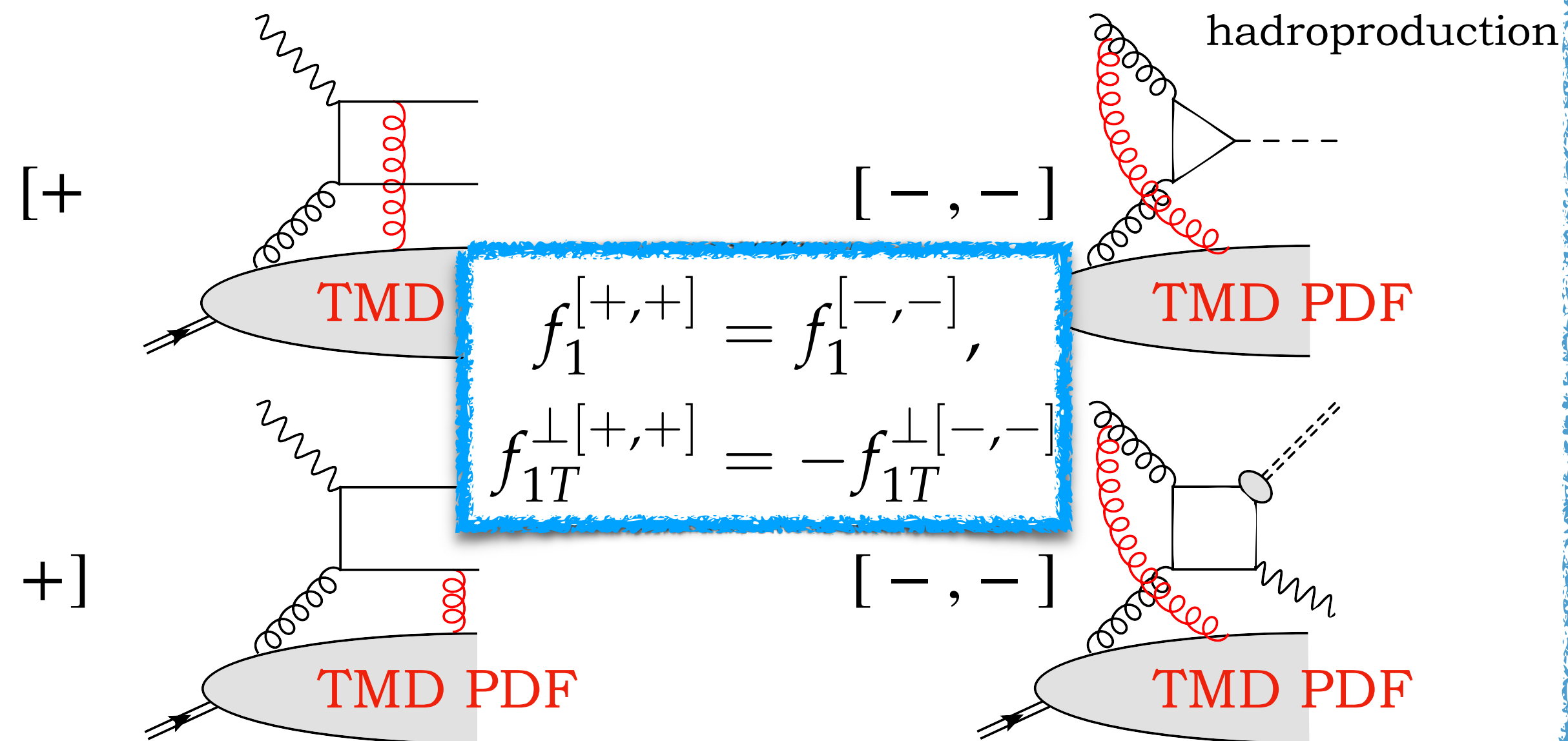
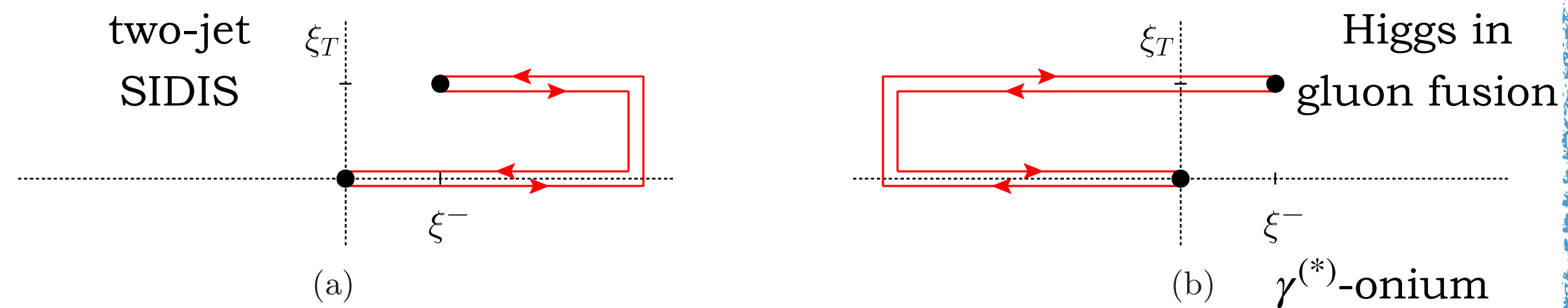


TMD PDF

Accessing f-type and d-type gluon TMD PDFs

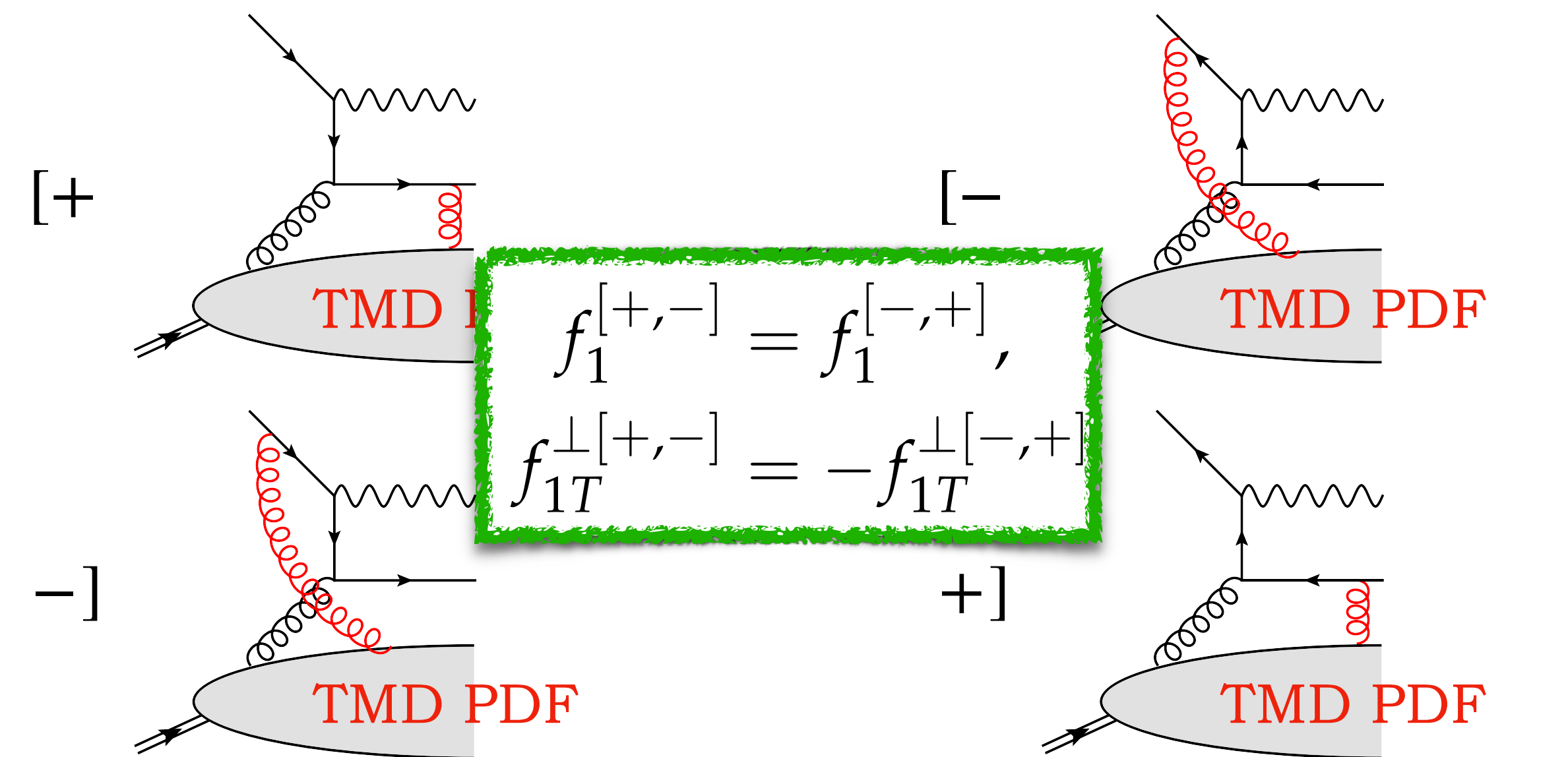
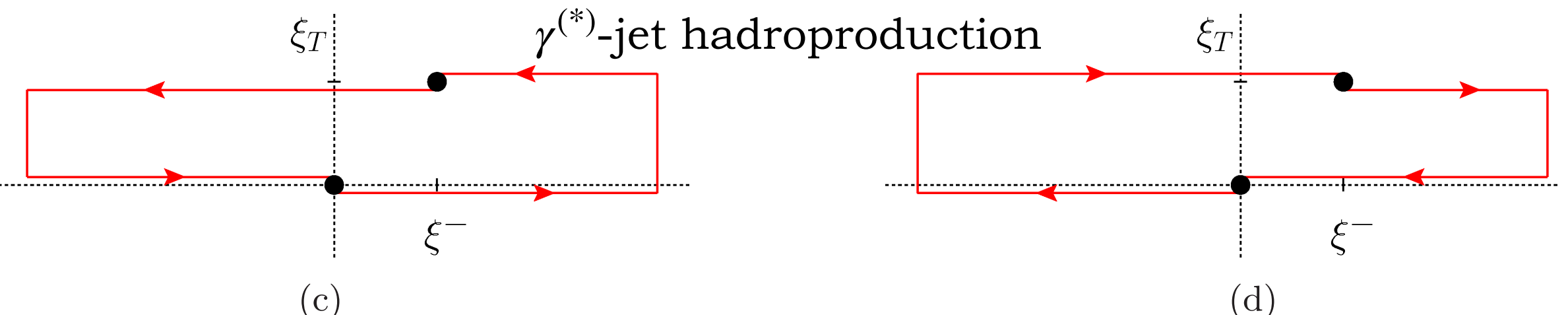
f-type (WW)

(a) $[+, +]$ or (b) $[-, -]$



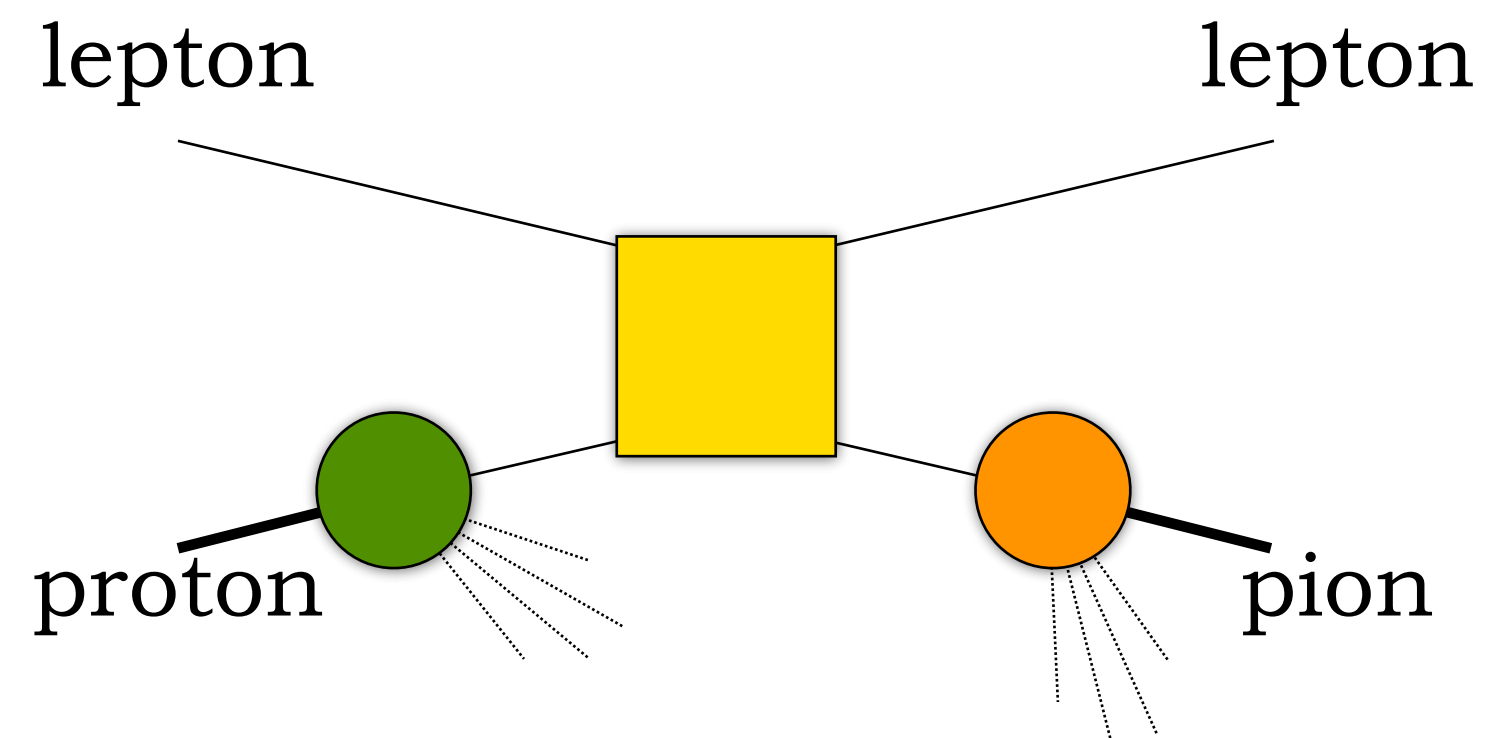
d-type (DP)

(c) $[+, -]$ or (d) $[-, +]$



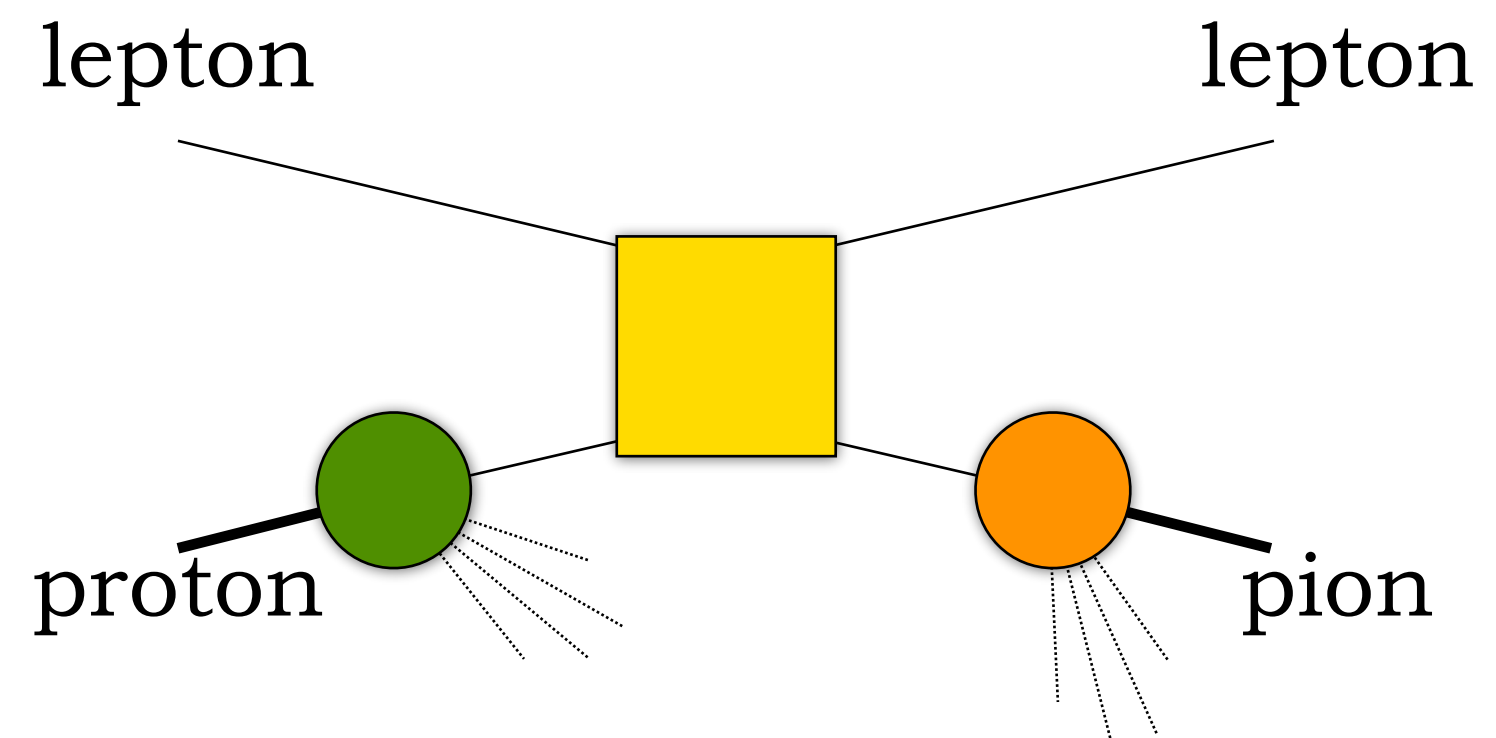
! Gauge link \rightarrow two main independent sets of TMDs, **not related** to each other !

SIDIS

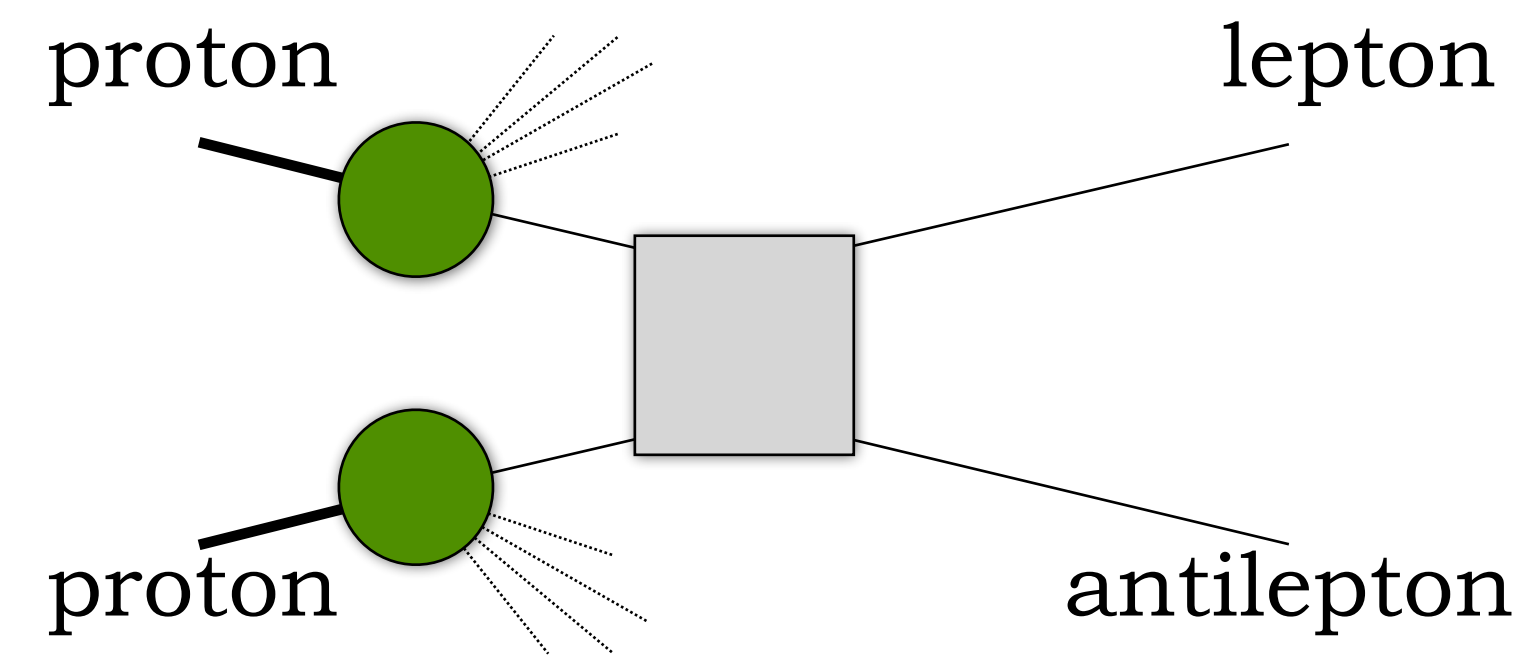


Factorization and universality

SIDIS

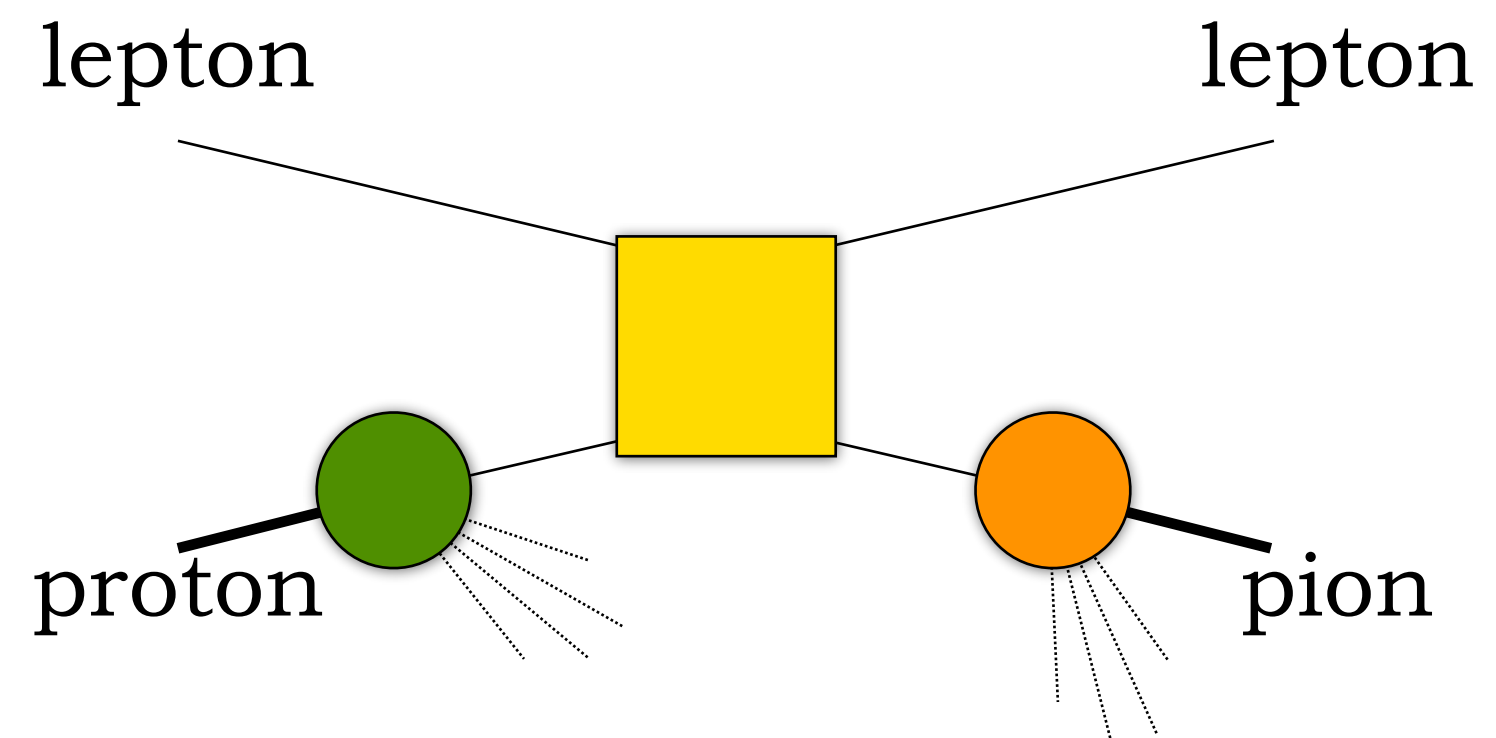


Drell-Yan

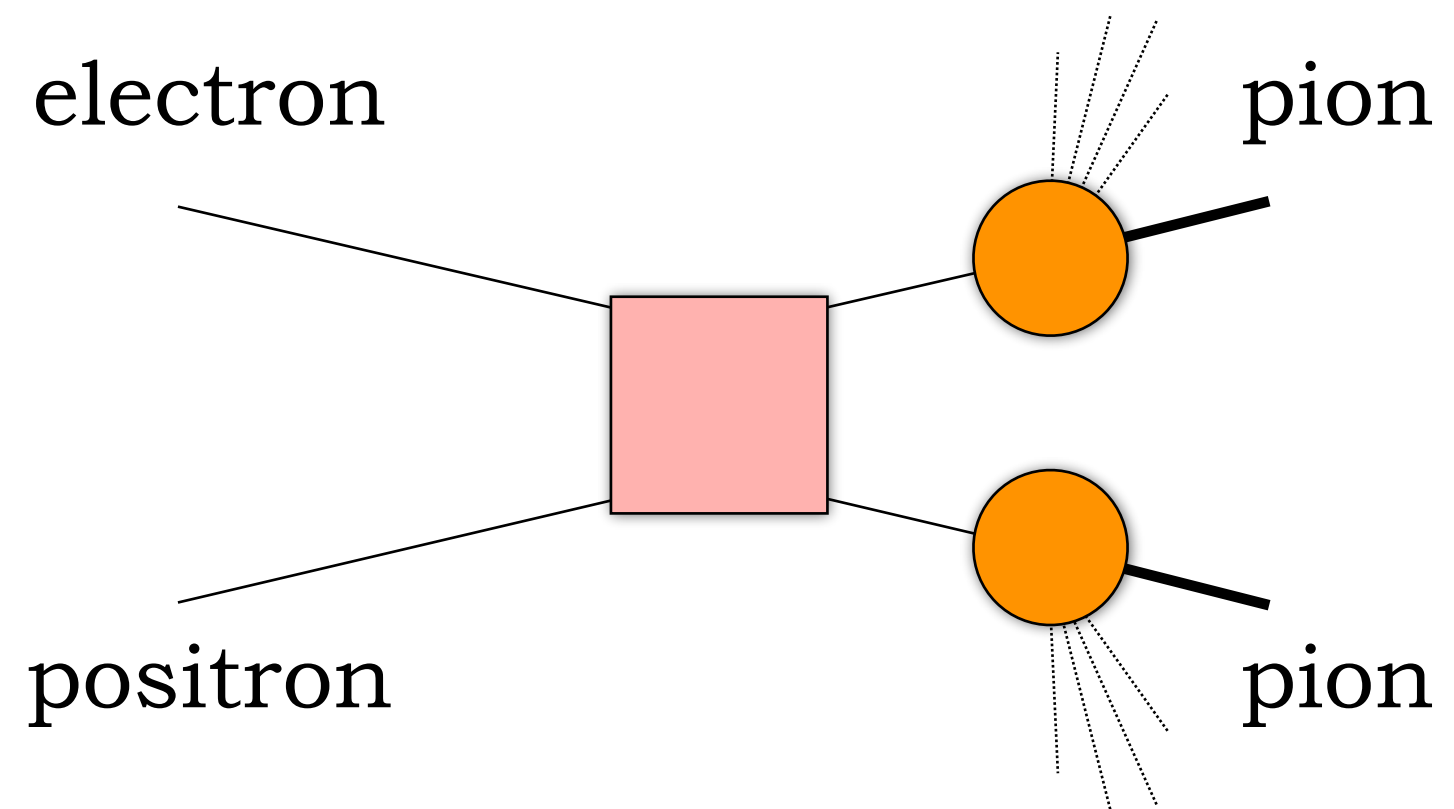
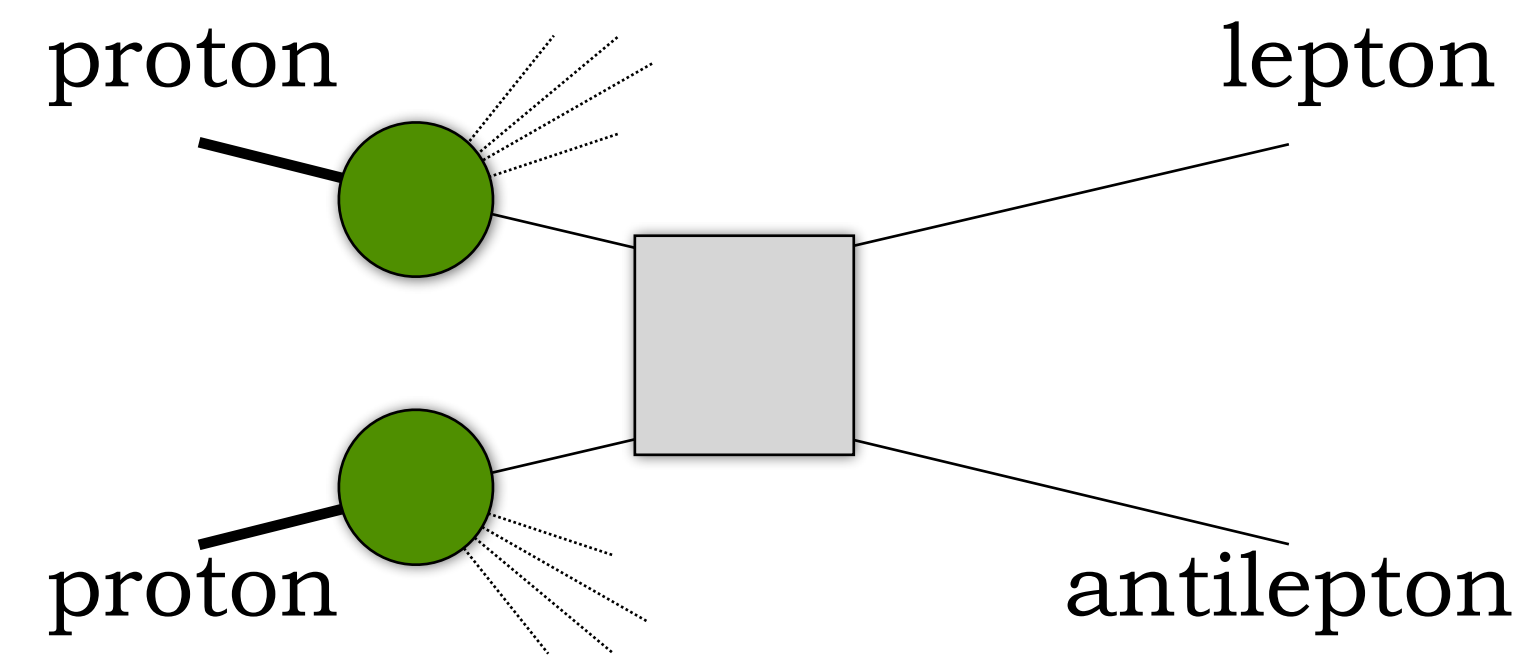


Factorization and universality

SIDIS



Drell-Yan

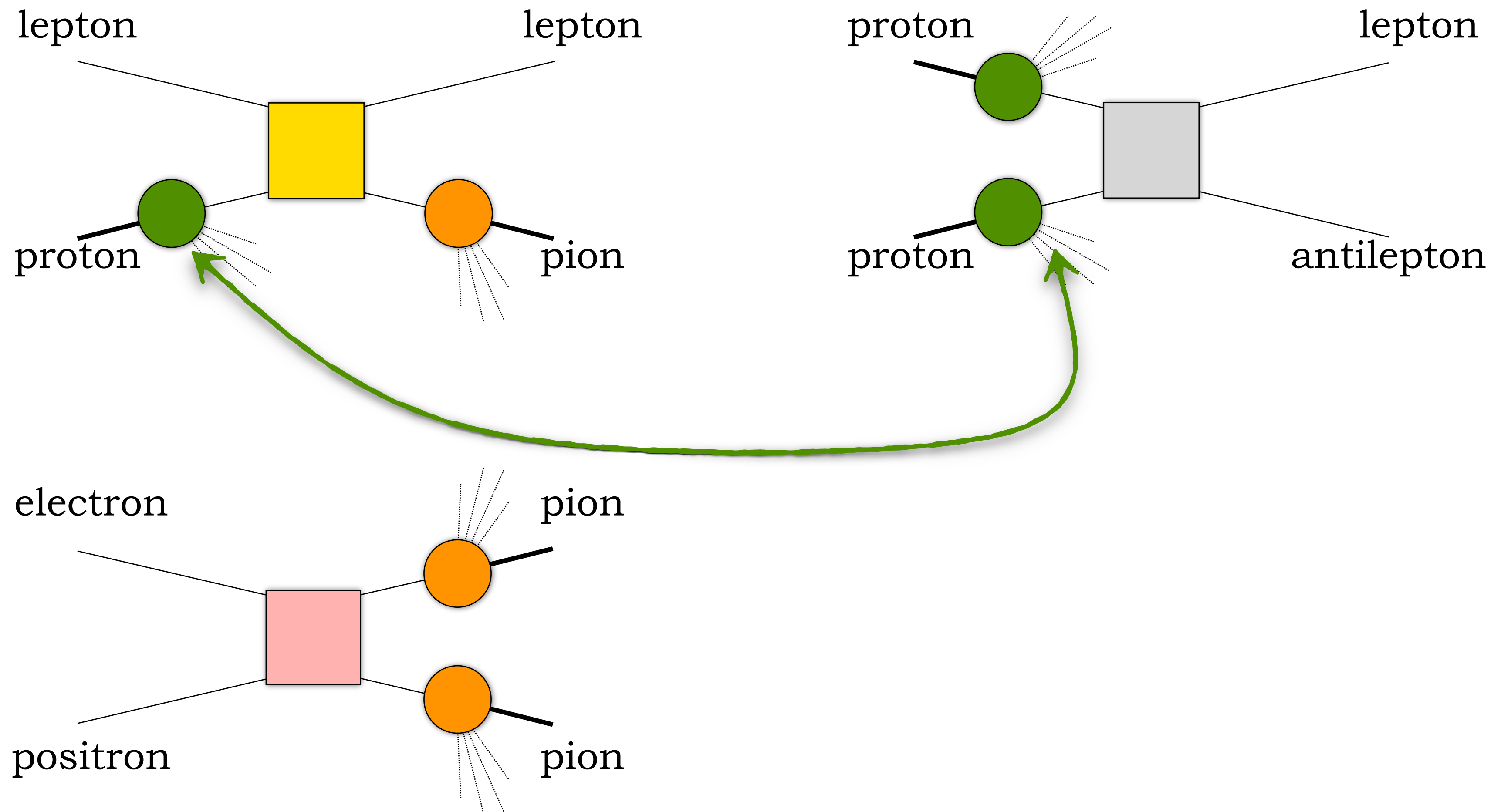


$$e^+ + e^- \rightarrow \mathbf{hadrons}$$

Factorization and universality

SIDIS

Drell-Yan

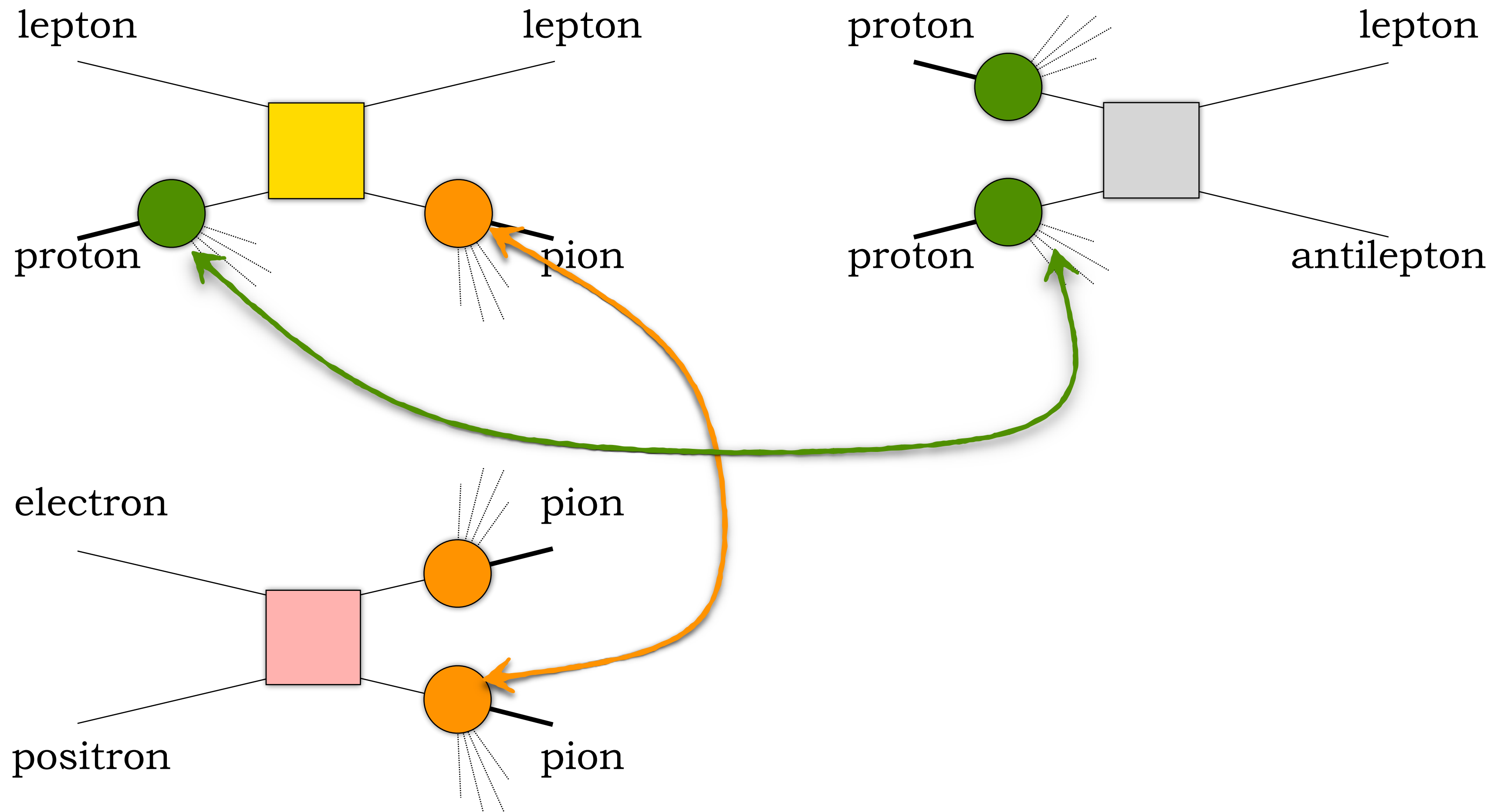


$$e^+ + e^- \rightarrow \text{hadrons}$$

Factorization and universality

SIDIS

Drell-Yan

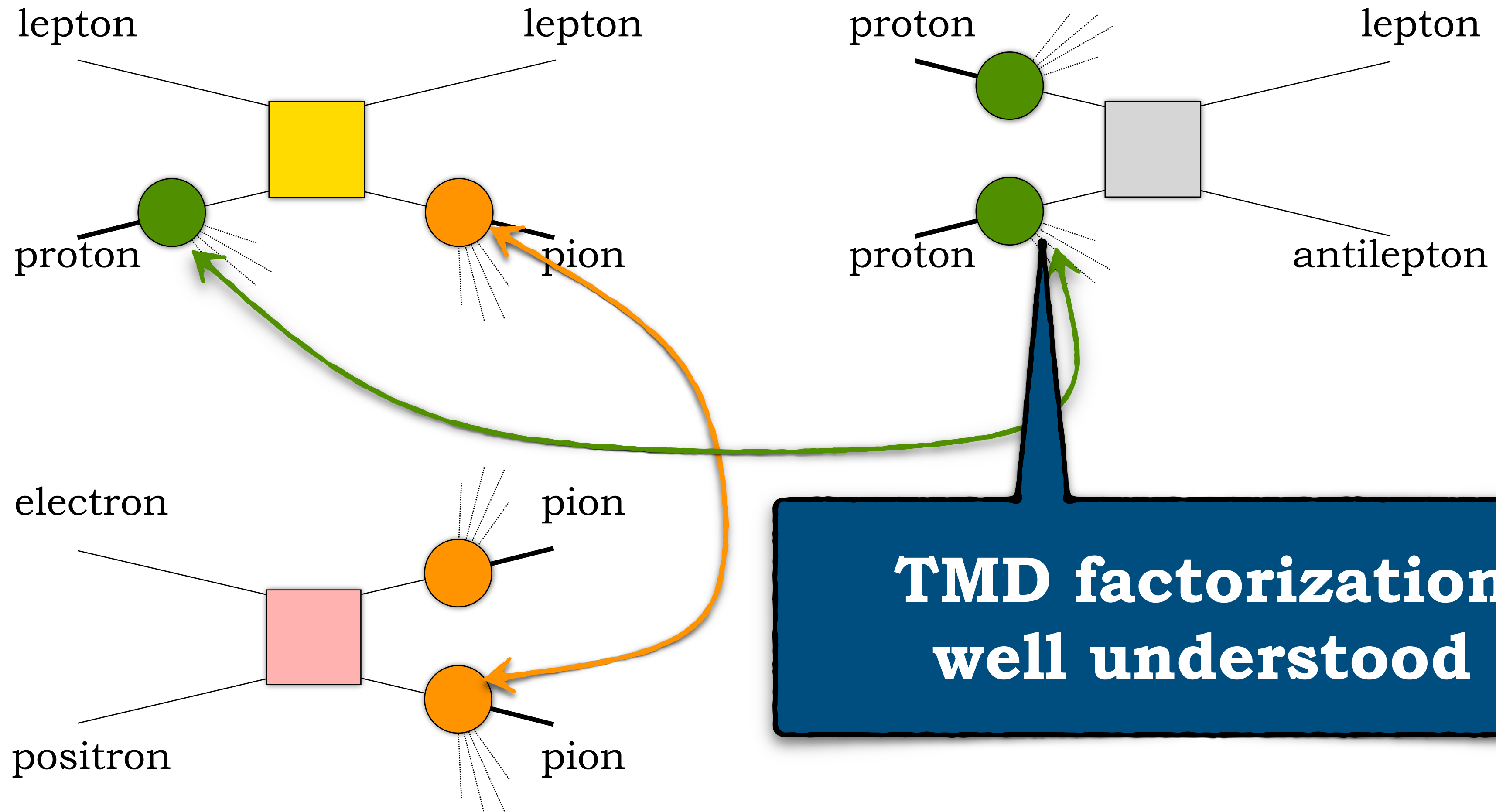


$$e^+ + e^- \rightarrow \text{hadrons}$$

Factorization and universality

SIDIS

Drell-Yan



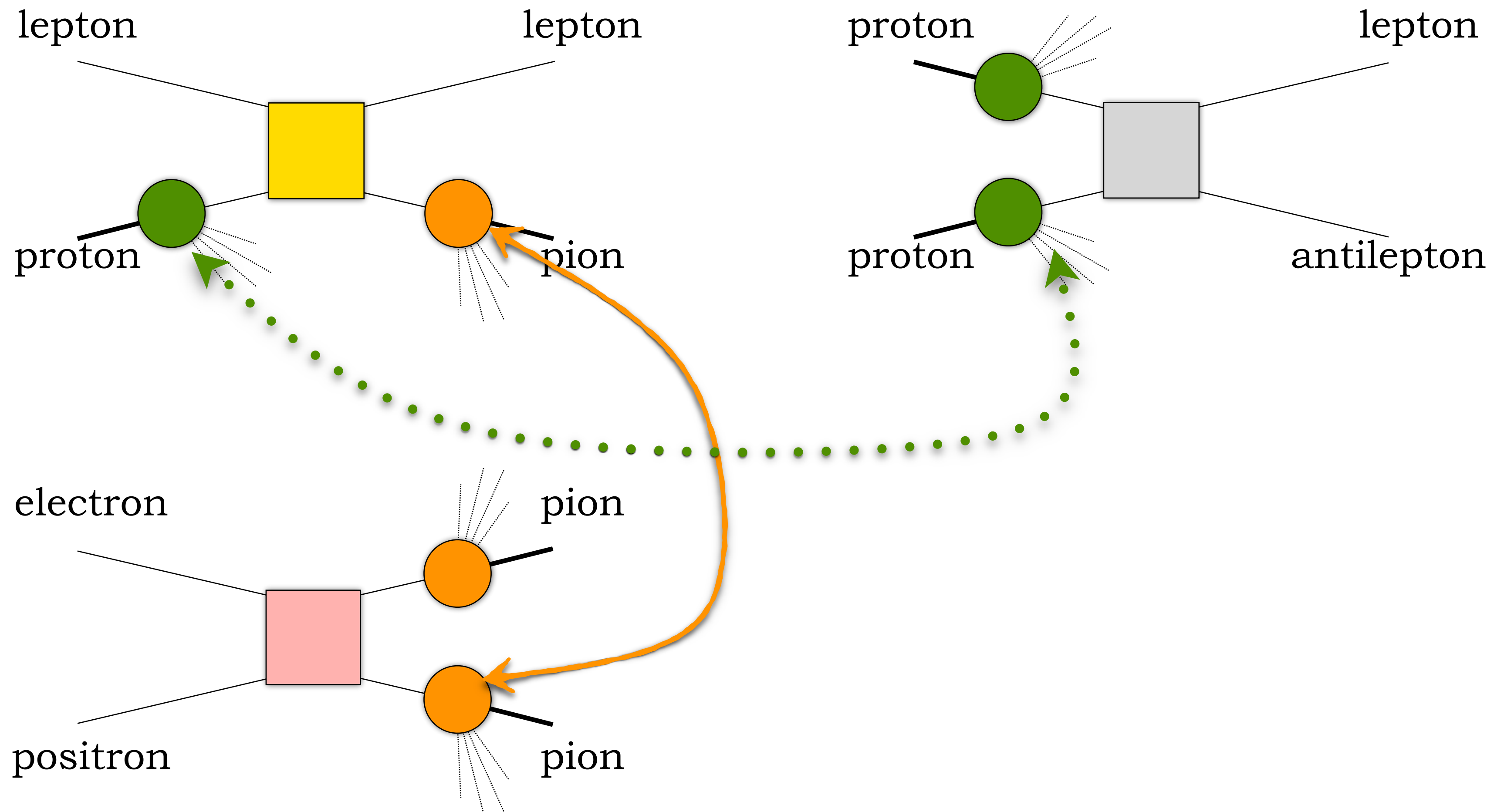
**TMD factorization
well understood**

$$e^+ + e^- \rightarrow \text{hadrons}$$

Factorization and universality

SIDIS

Drell-Yan

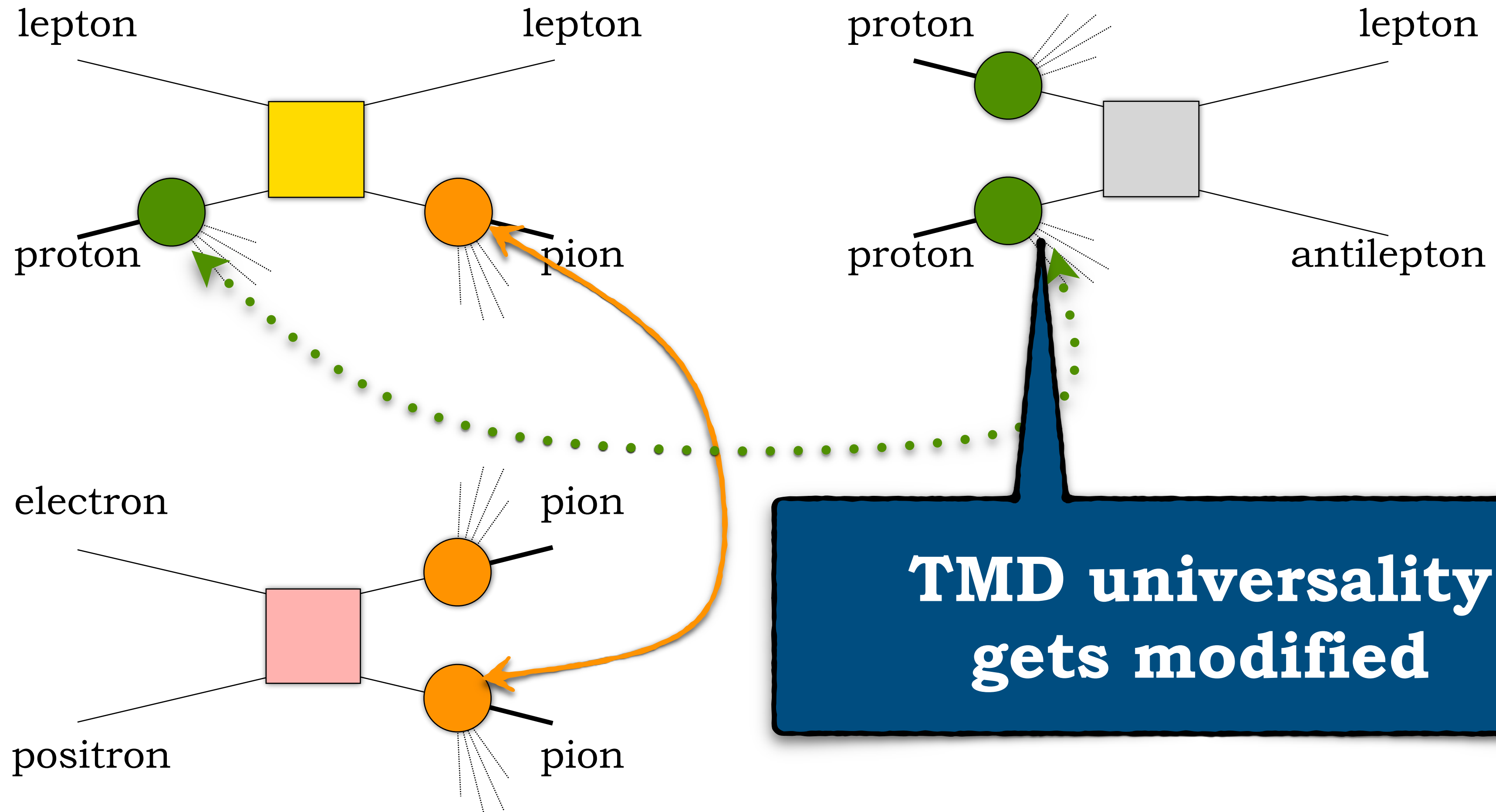


$$e^+ + e^- \rightarrow \text{hadrons}$$

Factorization and universality

SIDIS

Drell-Yan

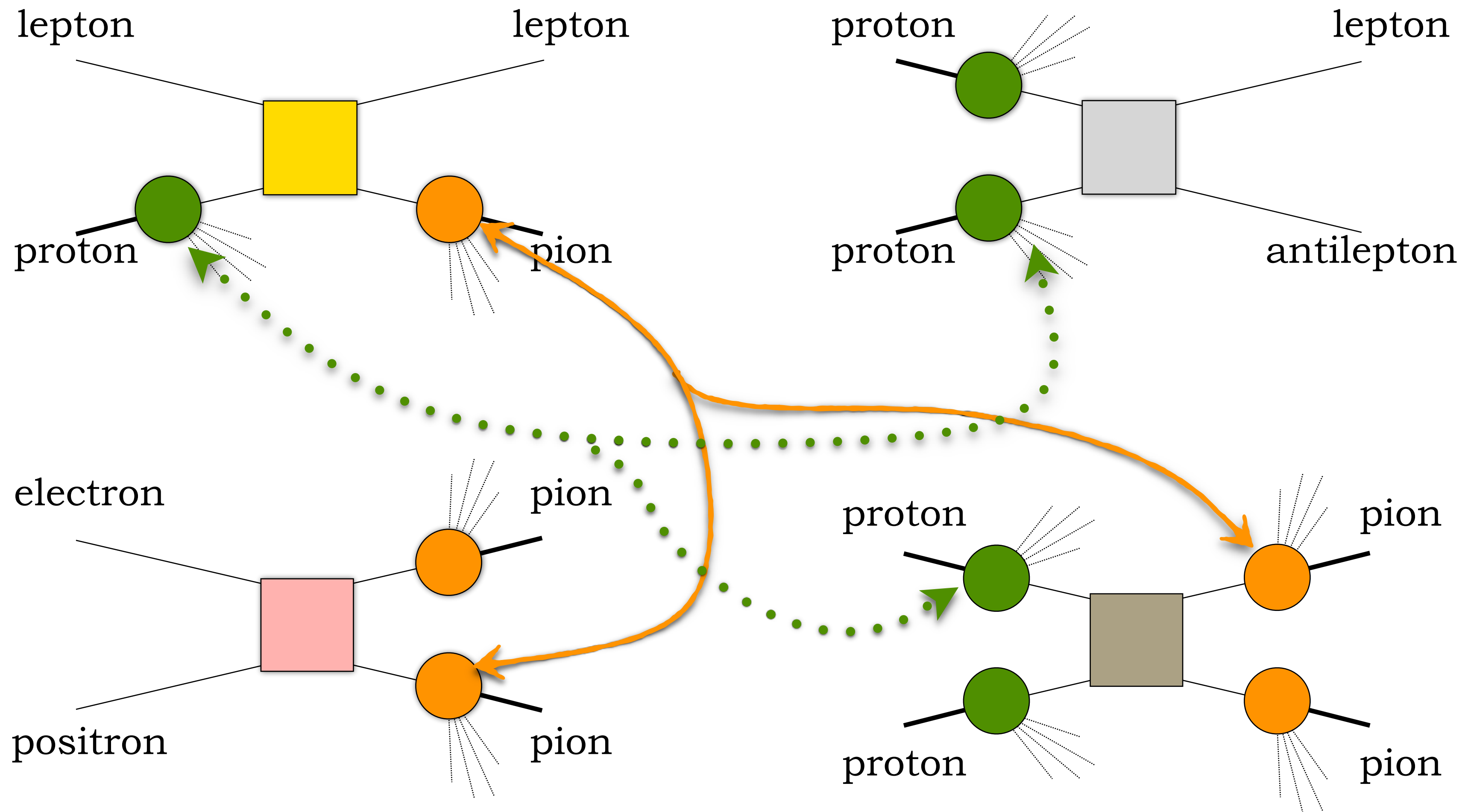


$$e^+ + e^- \rightarrow \mathbf{hadrons}$$

Factorization and universality

SIDIS

Drell-Yan



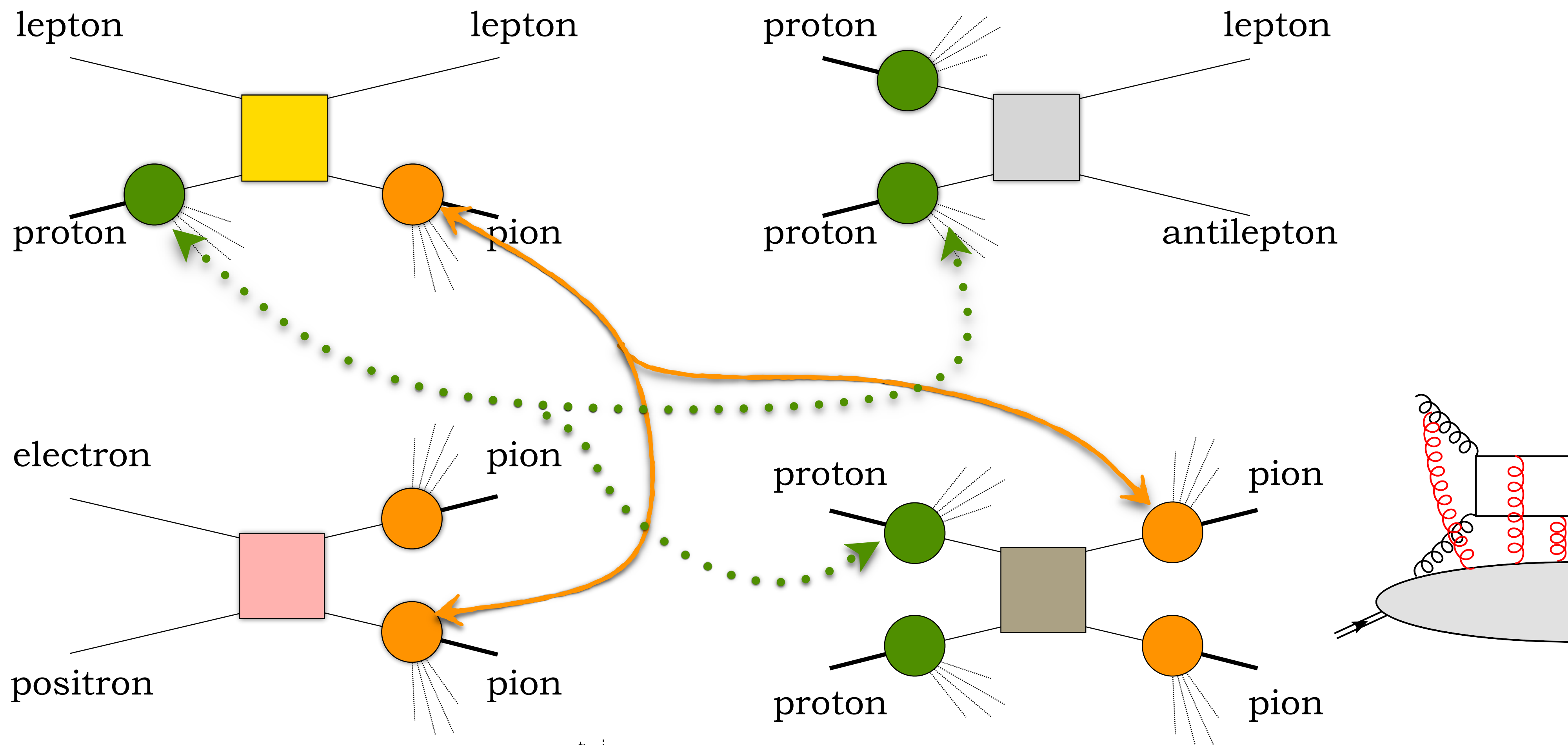
$e^+ + e^- \rightarrow \text{hadrons}$

$p + p \rightarrow \text{hadrons}$

Factorization and universality

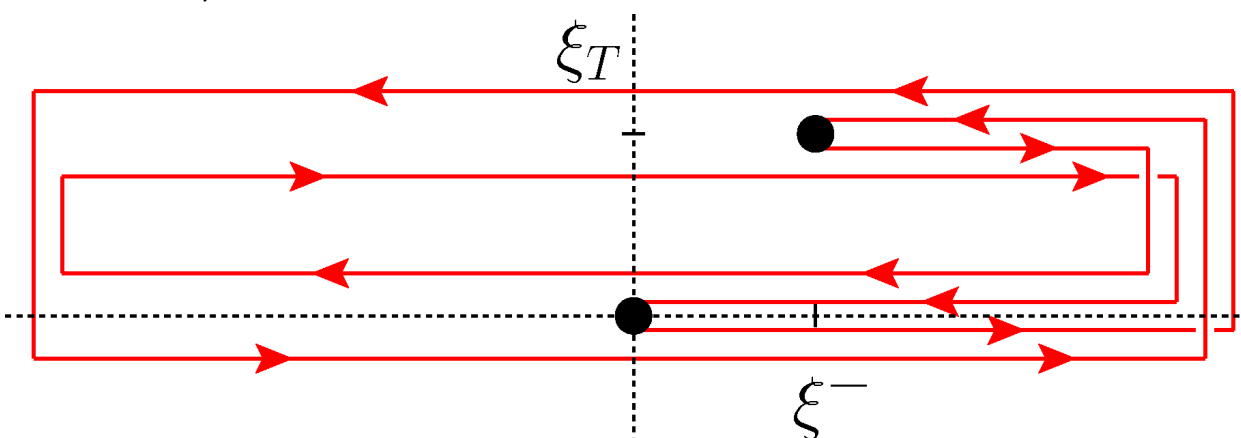
SIDIS

Drell-Yan



$e^+ + e^- \rightarrow \mathbf{hadrons}$

$p + p \rightarrow \mathbf{hadrons}$

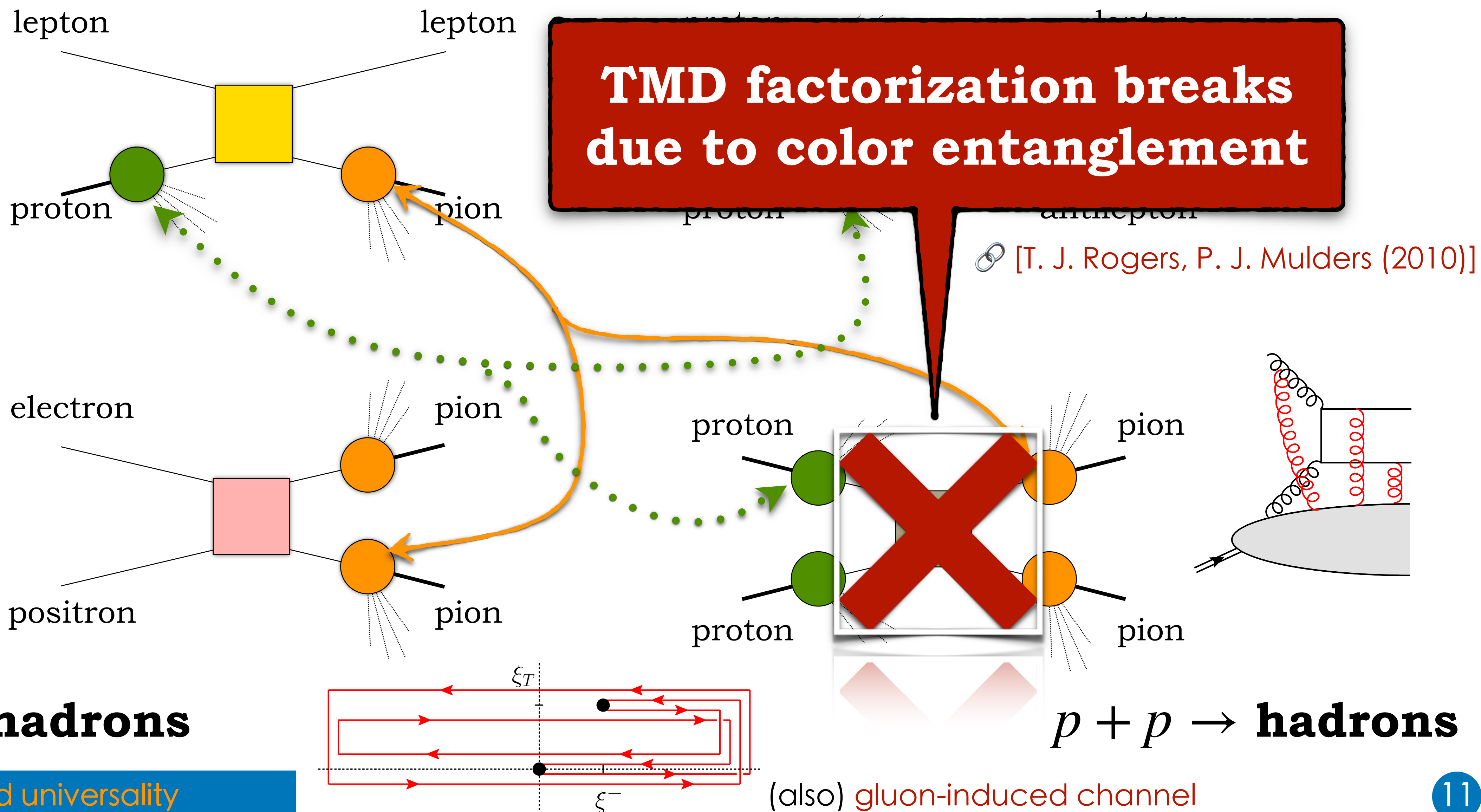


(also) gluon-induced channel

Factorization and universality

SIDIS

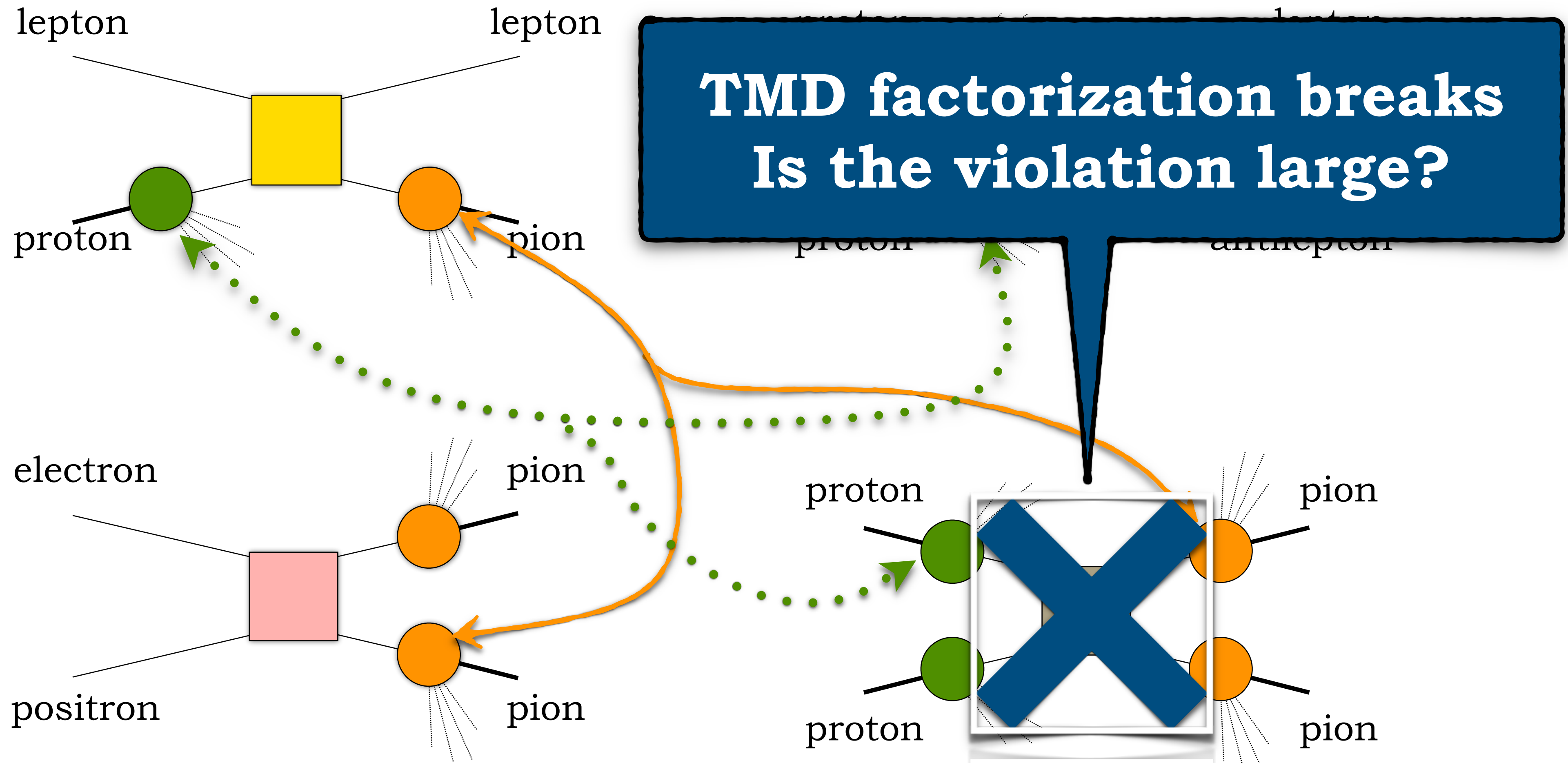
Drell-Yan



Factorization and universality

SIDIS

Drell-Yan



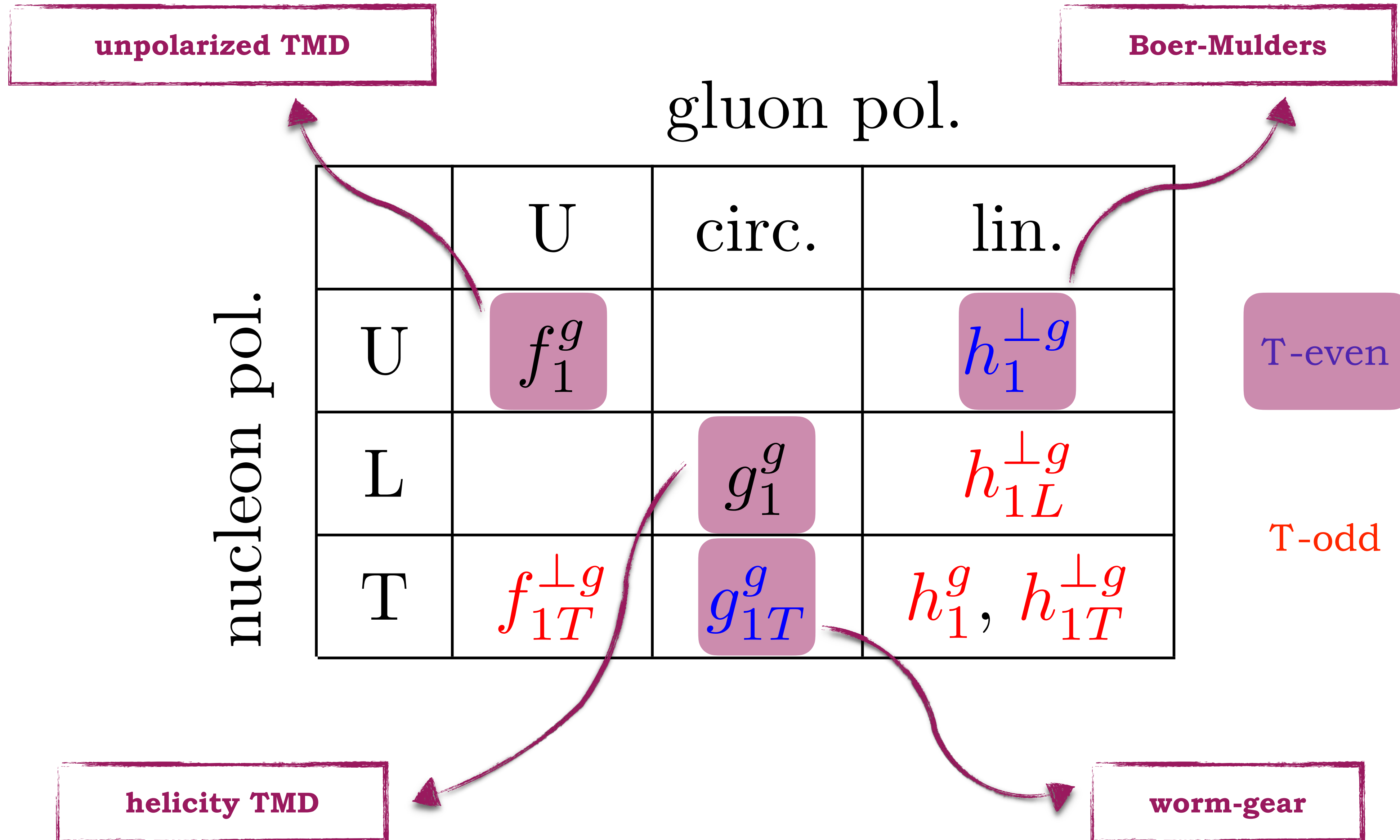
$$e^+ + e^- \rightarrow \mathbf{hadrons}$$

$$p + p \rightarrow \mathbf{hadrons}$$

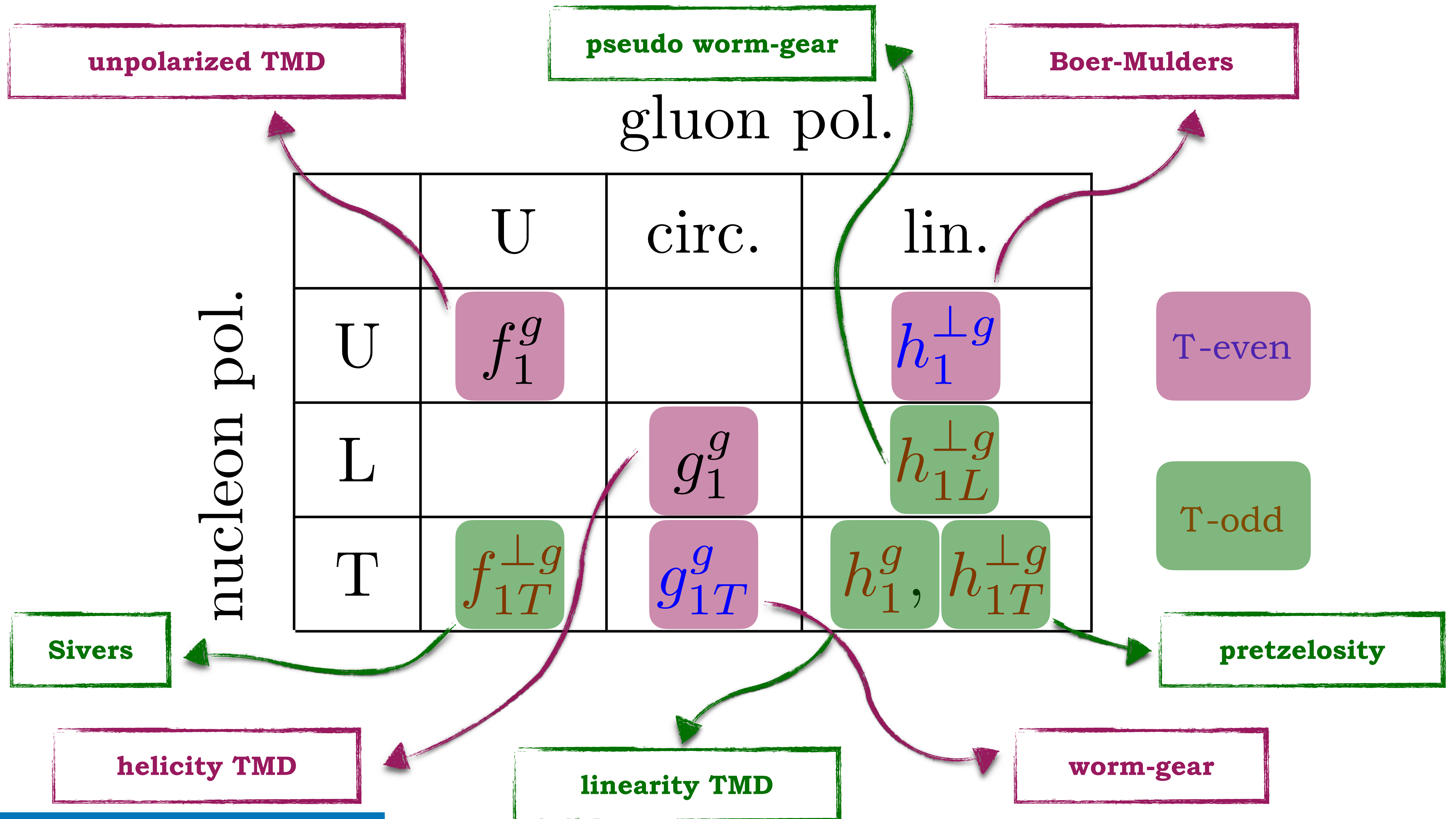
Gluon TMD PDFs at leading twist

		gluon pol.			
		U	circ.	lin.	
nucleon pol.	U	f_1^g		$h_1^{\perp g}$	T-even
	L		g_1^g	$h_{1L}^{\perp g}$	T-odd
	T	$f_{1T}^{\perp g}$	g_{1T}^g	$h_1^g, h_{1T}^{\perp g}$	

Gluon TMD PDFs at leading twist



Gluon TMD PDFs at leading twist

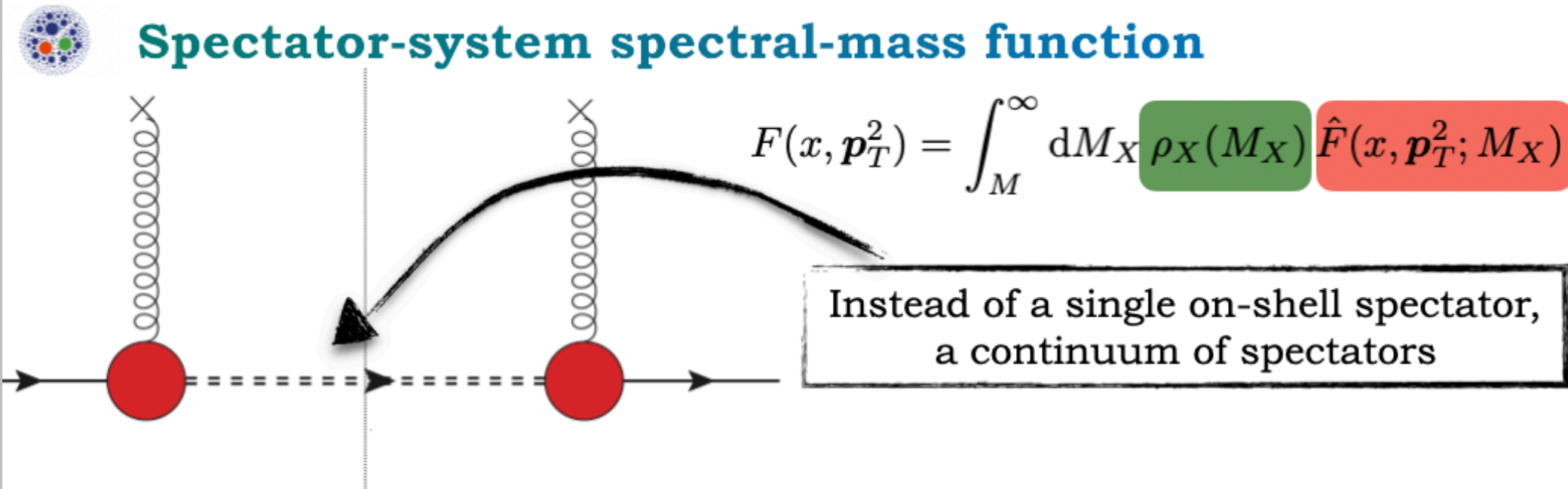




2. Modeling gluon TMD PDFs

Spectator-model gluon TMDs

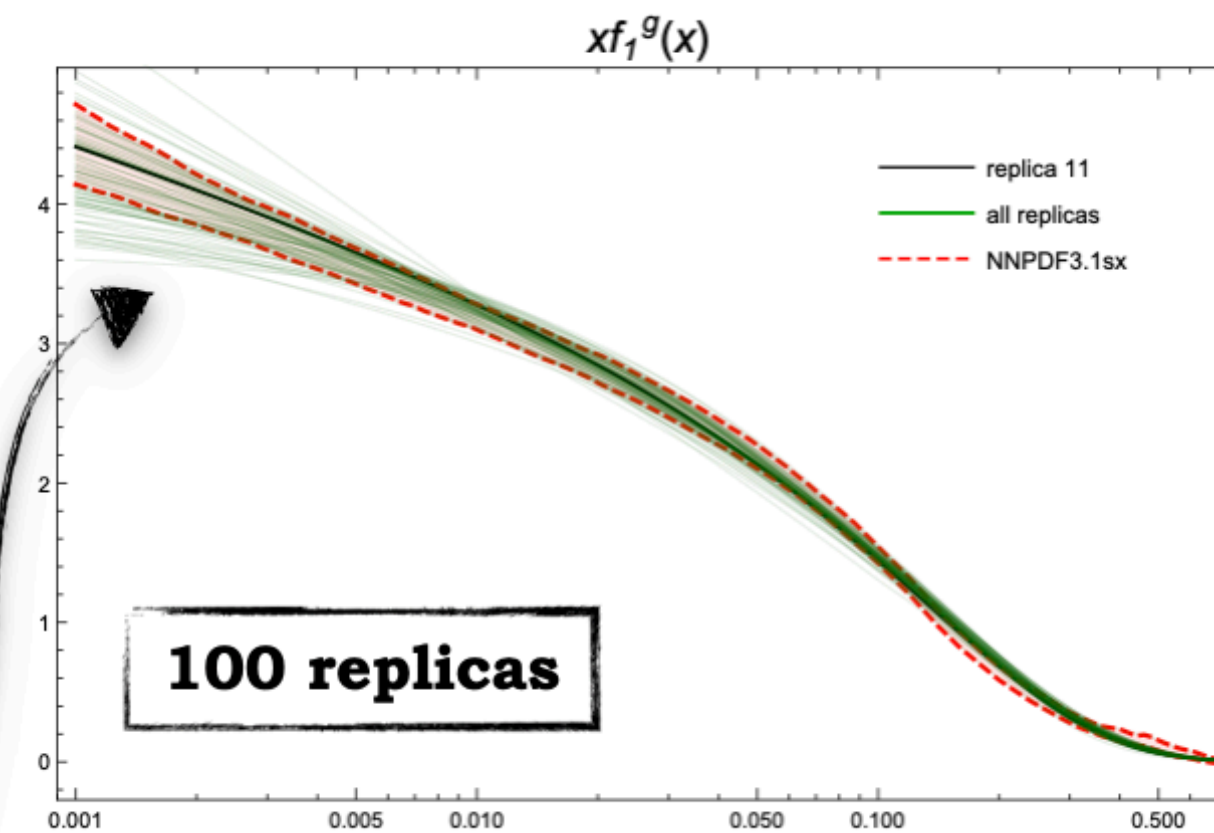
Our model at a glance



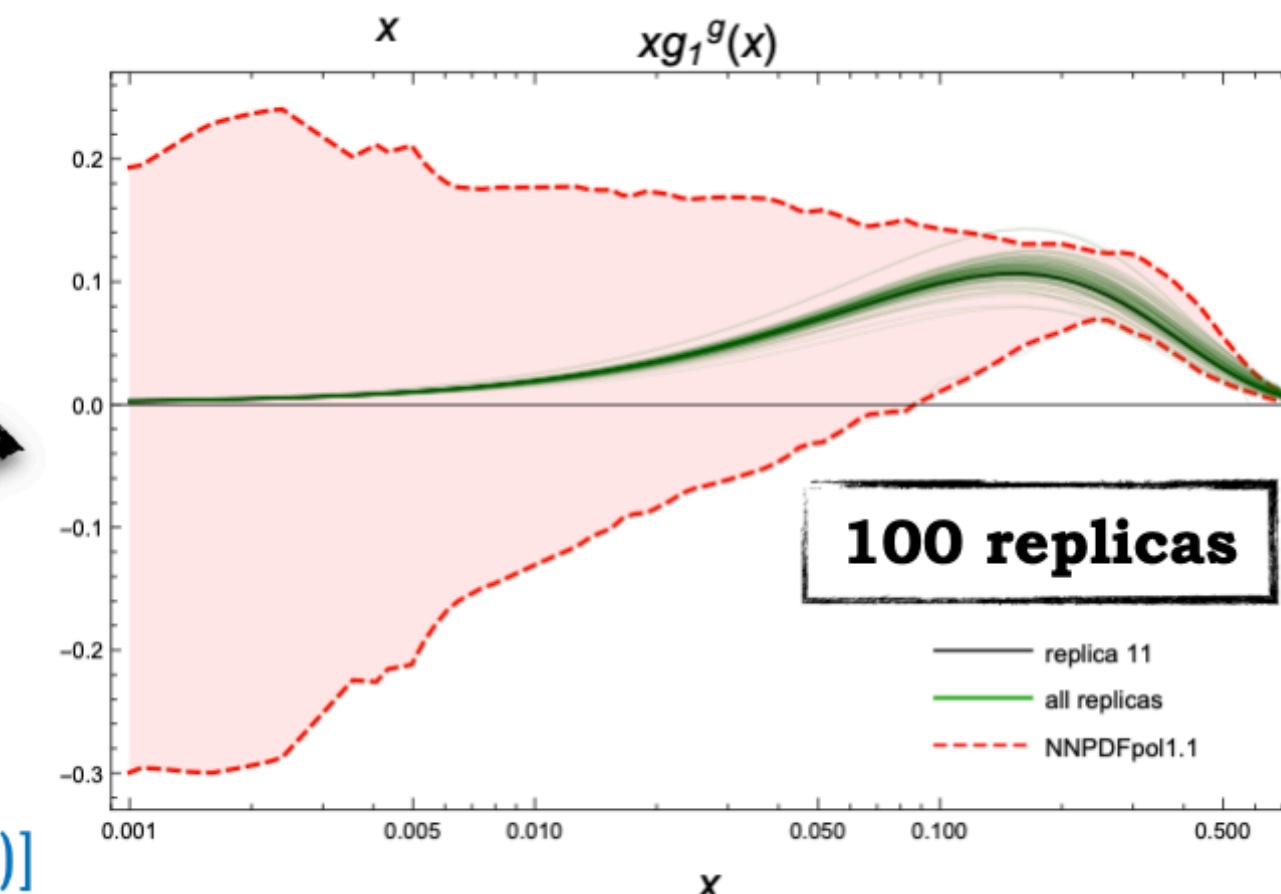
Link with collinear factorization

p_T -integrated TMDs **have to** reproduce PDFs at the lowest scale (Q_0) *before* evolution

Spectral function **learns** small- and moderate- x info encoded in **NNPDF** collinear parametrizations (NNPDF3.1sx + NNPDFpol1.1)



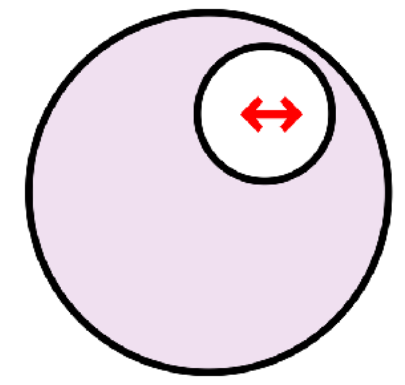
- ✓ Simultaneous fit of f_1 and g_1 PDFs
- ✓ Inclusion of small- x resummation effects (**BFKL**)
- ✓ Calculation of all leading-twist T-even gluon TMDs



2.3 Modeling gluon TMDs [A. Bacchetta, F.G. C., M. Radici, P. Taels (2020)]

14

Boer-Mulders effect in unpolarized pp collisions

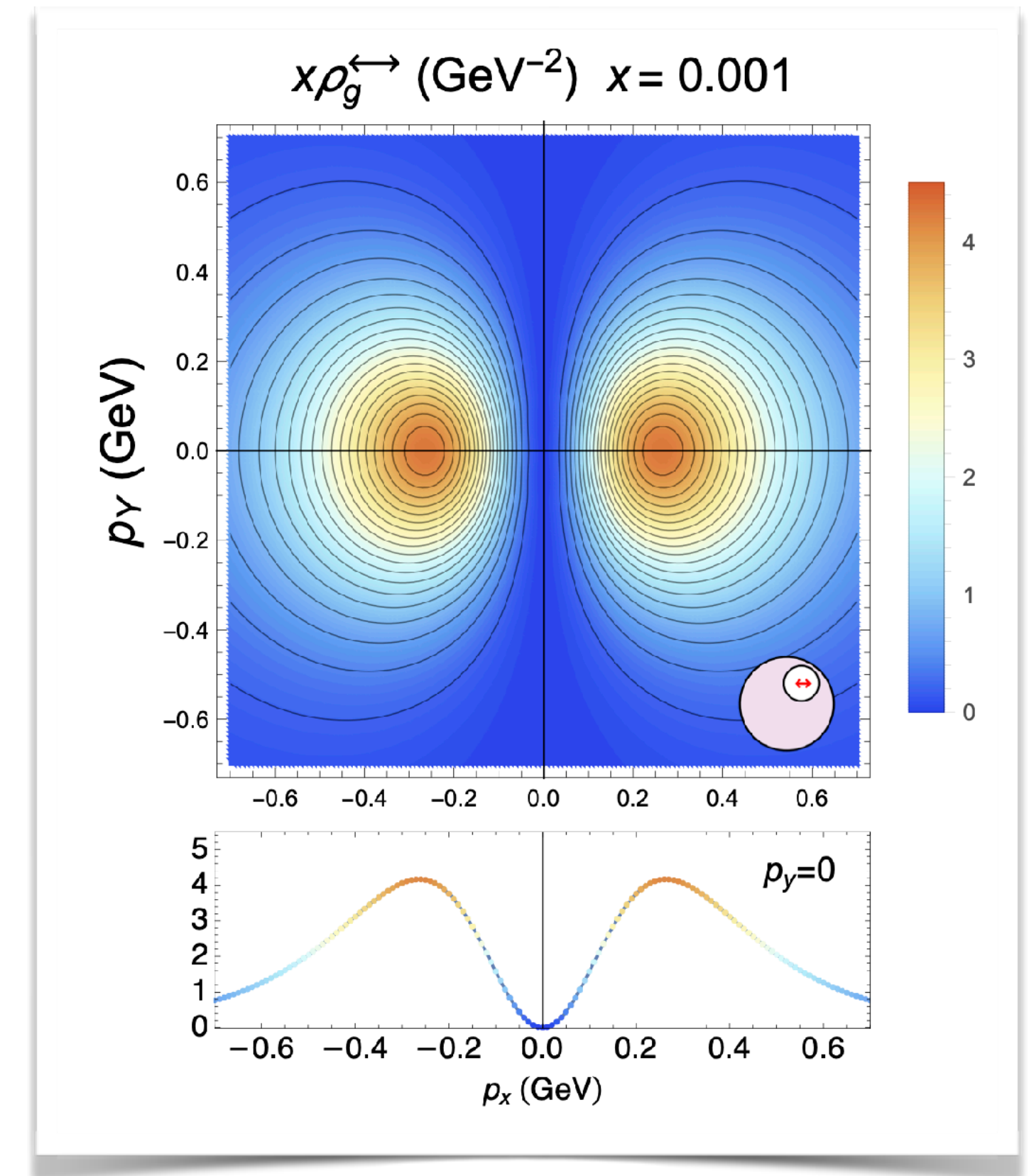


$[\leftrightarrow / \mathbf{u}]$

$$f_1(x, p_x, p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^\perp(x, p_x, p_y)$$

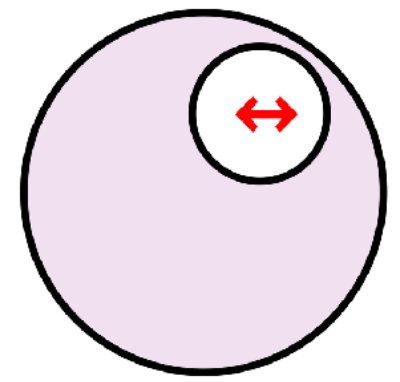
unpol.

Boer-Mulders



[A. Bacchetta, F.G.C., M. Radici, P. Taelis (2020)]

Boer-Mulders effect in unpolarized pp collisions



$[\leftrightarrow / \mathbf{u}]$

$$f_1(x, p_x, p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^\perp(x, p_x, p_y)$$

unpol.

Boer-Mulders



(Pseudo)scalar p_T -distributions: Higgs, $\eta_{c,b}$

$$\frac{d\sigma}{dq_T} \sim \Phi_A^U \Phi_B^U |\mathcal{M}|^2$$

at low transverse momentum
for (pseudo)scalar state

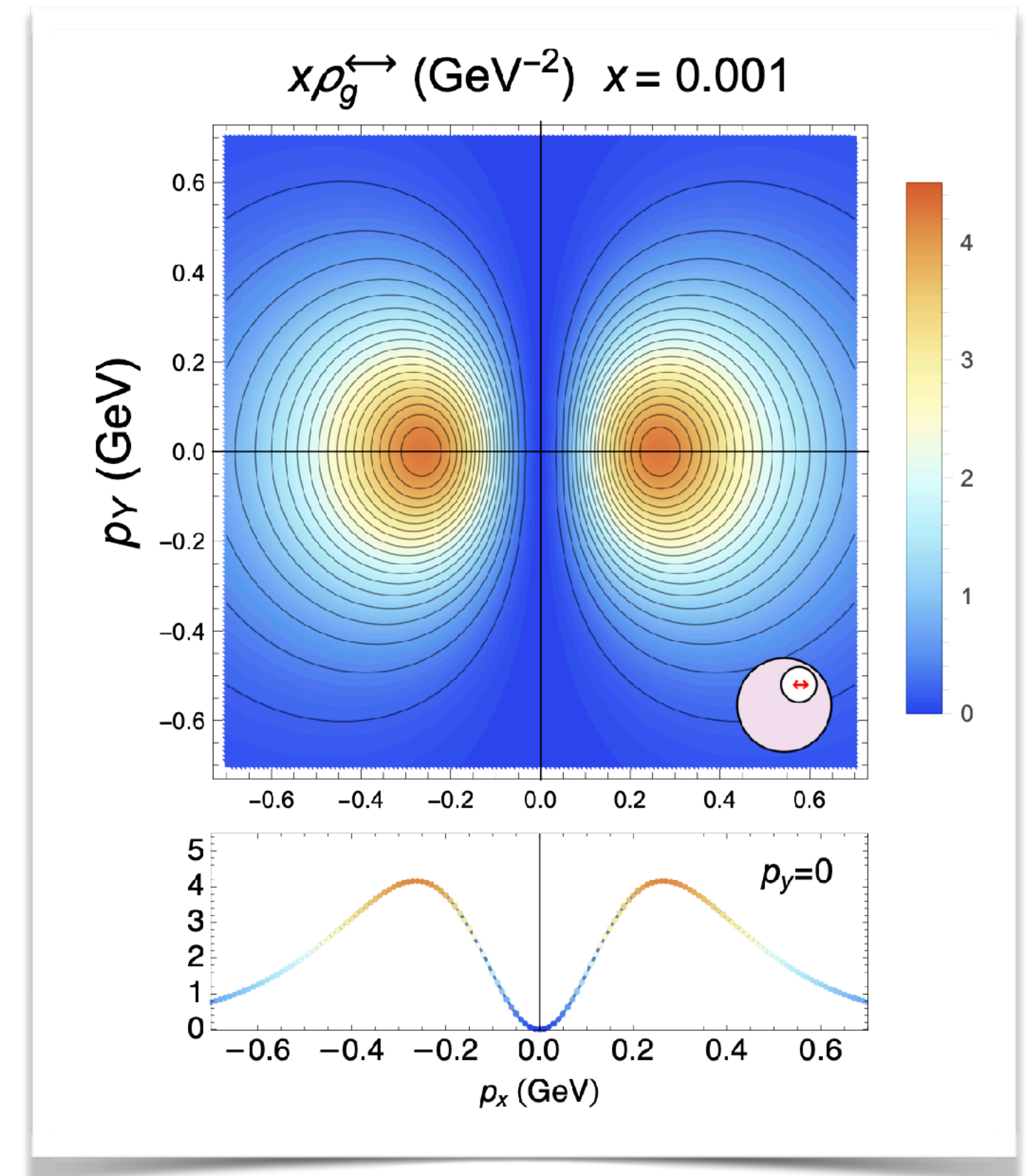
$$\sim \mathcal{C} \left[\begin{array}{cc} f_1^{g/A} & f_1^{g/B} \end{array} \right] \pm \mathcal{C} \left[\begin{array}{cc} h_1^{\perp g/A} & h_1^{\perp g/B} \end{array} \right]$$

unpolarized gluons

lin. polarized gluons

NRQCD

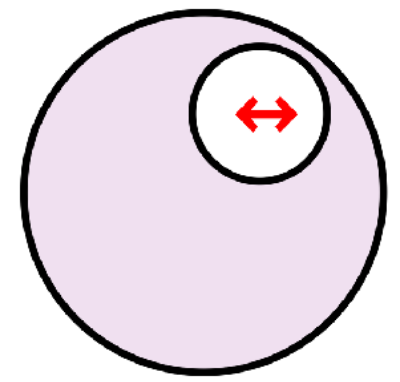
$$\frac{CS}{CO} \sim \frac{1}{v^4}$$



[D. Boer, W.J. den Dunnen, C. Pisano, M. Schlegel, W. Vogelsang (2012)]
(Higgs+jet angular distributions) [D. Boer, C. Pisano (2015)]

[A. Bacchetta, F.G.C., M. Radici, P. Taelis (2020)]

Boer-Mulders effect in unpolarized pp collisions



$[\leftrightarrow / \mathbf{u}]$

$$f_1(x, p_x, p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^\perp(x, p_x, p_y)$$

unpol.

Boer-Mulders



(Pseudo)scalar p_T -distributions: Higgs, $\eta_{c,b}$

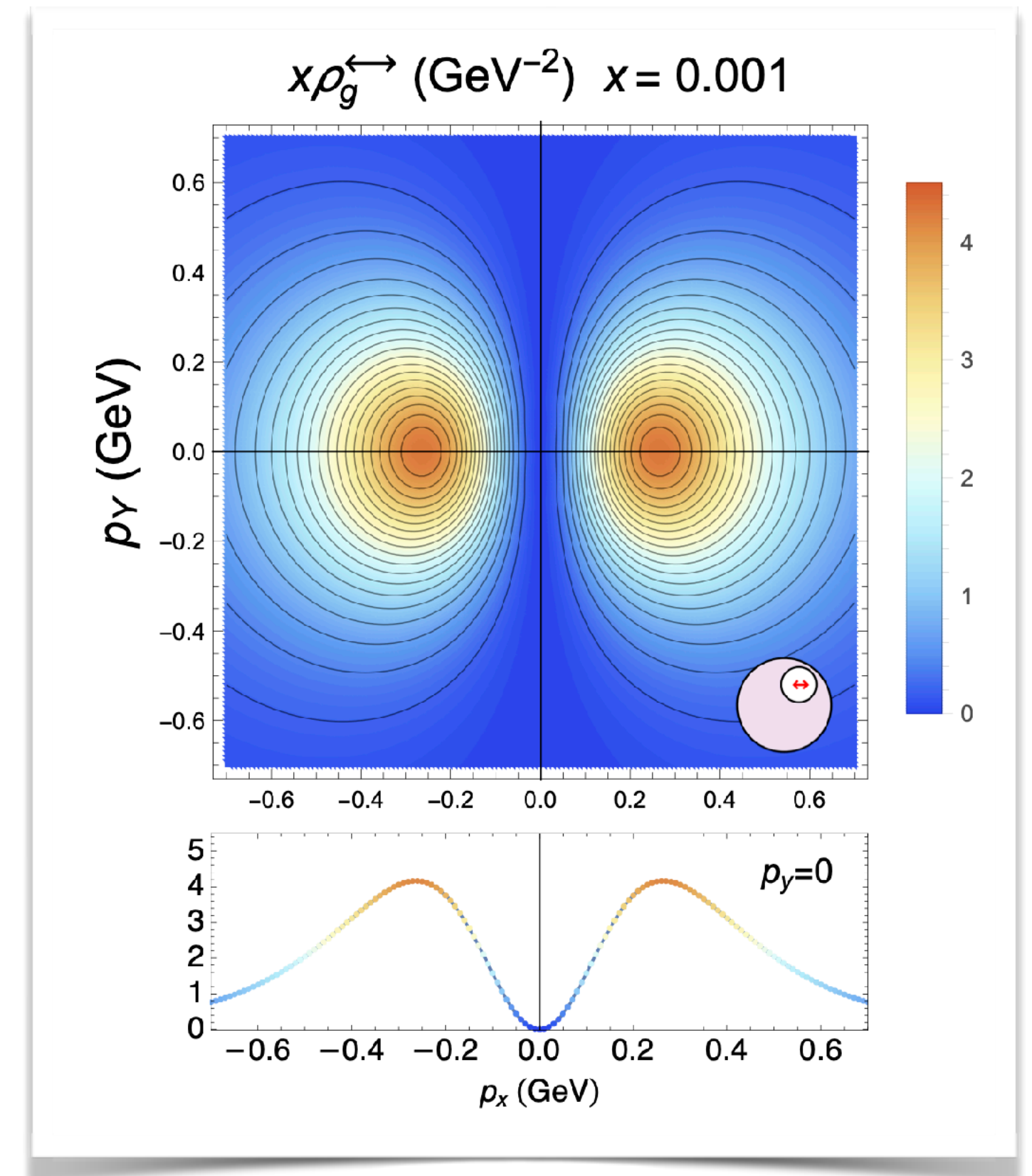
$$\frac{d\sigma}{dq_T} \sim \Phi_A^U \Phi_B^U |\mathcal{M}|^2$$

at low transverse momentum
for (pseudo)scalar state

$$\sim \mathcal{C} \left[\begin{matrix} f_1^{g/A} & f_1^{g/B} \\ \text{unpolarized gluons} & \end{matrix} \right] \pm \mathcal{C} \left[\begin{matrix} h_1^{\perp g/A} & h_1^{\perp g/B} \\ \text{lin. polarized gluons} & \end{matrix} \right]$$

NRQCD

$$\frac{CS}{CO} \sim \frac{1}{v^4}$$



[D. Boer, W.J. den Dunnen, C. Pisano, M. Schlegel, W. Vogelsang (2012)]
(Higgs+jet angular distributions) [D. Boer, C. Pisano (2015)]

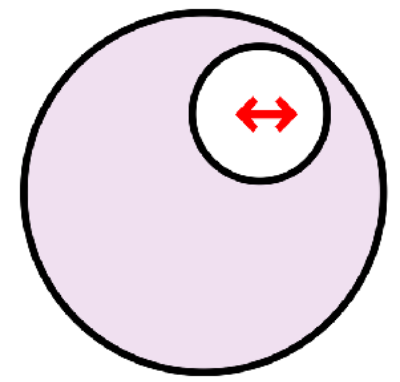
[A. Bacchetta, F.G.C., M. Radici, P. Taelis (2020)]



Model prediction at low x

$$\frac{f_1^g(x, p_T^2)}{h_1^{\perp g}(x, p_T^2)} \underset{x \rightarrow 0^+}{\sim} \text{constant}$$

Boer-Mulders effect in unpolarized pp collisions



$[\leftrightarrow / \mathbf{u}]$

$$f_1(x, p_x, p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^\perp(x, p_x, p_y)$$

unpol.

Boer-Mulders



(Pseudo)scalar p_T -distributions: Higgs, $\eta_{c,b}$

$$\frac{d\sigma}{dq_T} \sim \Phi_A^U \Phi_B^U |\mathcal{M}|^2$$

at low transverse momentum
for (pseudo)scalar state

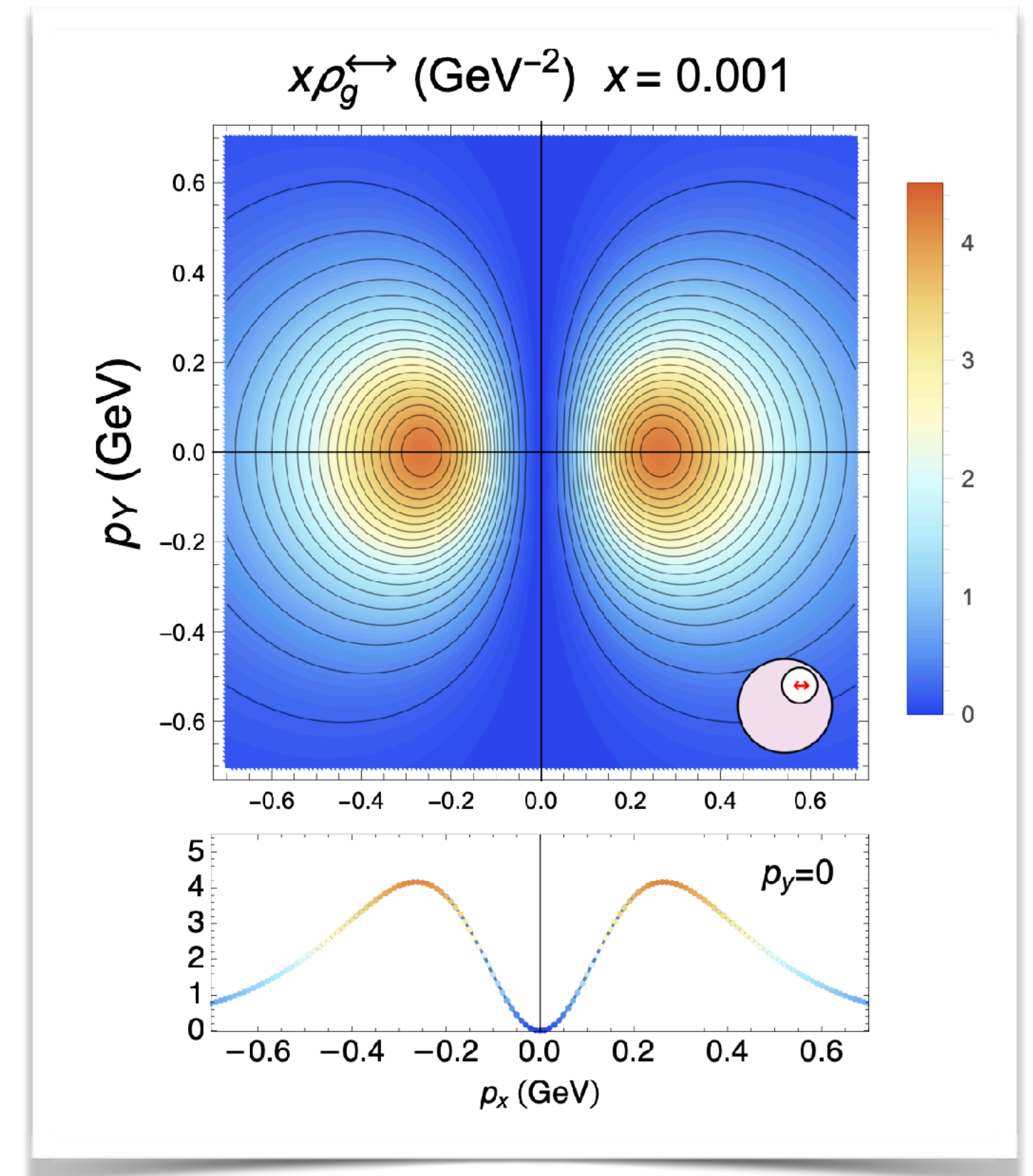
$$\sim \mathcal{C} \left[\begin{array}{cc} f_1^{g/A} & f_1^{g/B} \end{array} \right] \pm \mathcal{C} \left[\begin{array}{cc} h_1^{\perp g/A} & h_1^{\perp g/B} \end{array} \right]$$

unpolarized gluons

lin. polarized gluons

NRQCD

$$\frac{CS}{CO} \sim \frac{1}{v^4}$$



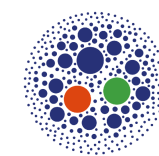
[D. Boer, W.J. den Dunnen, C. Pisano, M. Schlegel, W. Vogelsang (2012)]
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[A. Bacchetta, F.G.C., M. Radici, P. Taelis (2020)]



Model prediction at low x

$$\frac{f_1^g(x, p_T^2)}{h_1^{\perp g}(x, p_T^2)} \underset{x \rightarrow 0^+}{\sim} \text{constant}$$



BFKL regime (linear low-x evolution)



$$f_1^g(x, p_T^2) = h_1^{\perp g}(x, p_T^2) + \text{higher twist}$$

Anatomy of gluon TMDs

$$F(x, \mathbf{b}; \mu, \zeta) = \sum_j \left(C_j^{(F)} \otimes F^j \right) (x, b_*; \mu_b) e^{S(b_*; \mu_b, \mu, \zeta)} e^{S_{\text{NP}}(b)} F_{\text{NP}}(x, b)$$

matching coefficients collinear PDF nonperturbative Sudakov nonperturbative TMD function

perturbative expansion in $\alpha_s(\mu)$

perturbative Sudakov

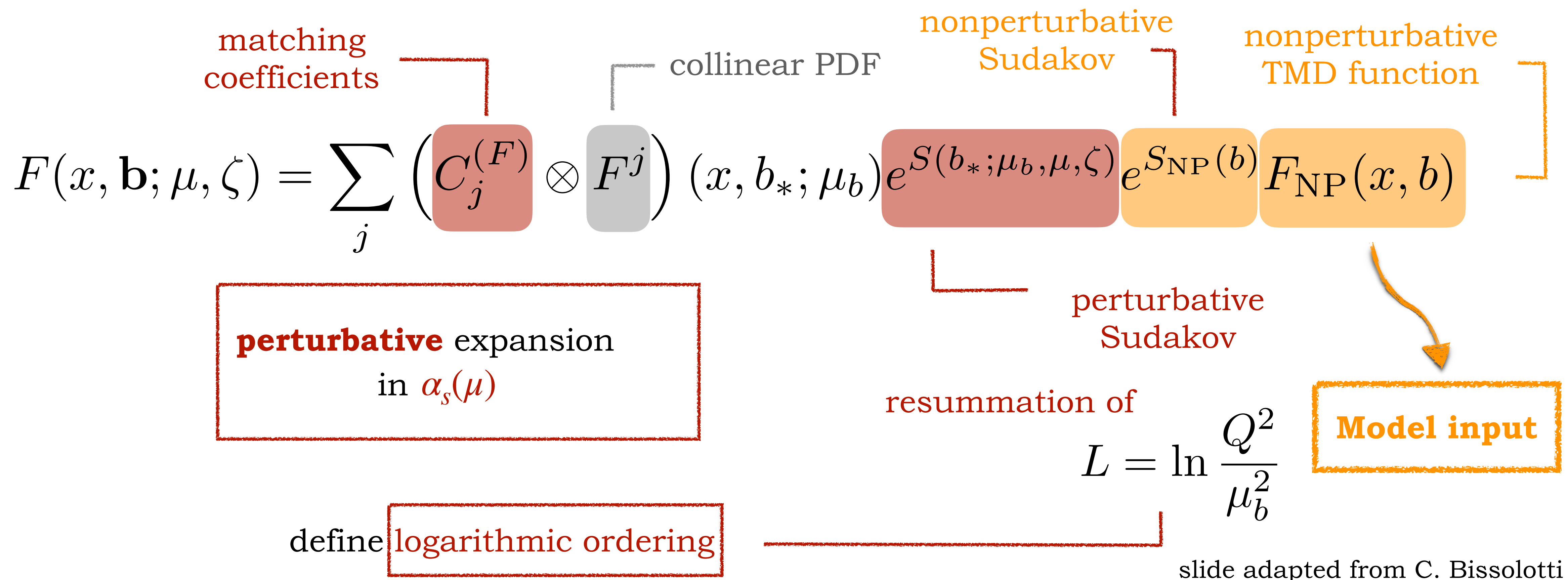
resummation of

$$L = \ln \frac{Q^2}{\mu_b^2}$$

define **logarithmic ordering**

slide adapted from C. Bissolotti

Anatomy of gluon TMDs



Anatomy of gluon TMDs

$$F(x, \mathbf{b}; \mu, \zeta) = \sum_j \left(C_j^{(F)} \otimes F^j \right) (x, b_*; \mu_b) e^{S(b_*; \mu_b, \mu, \zeta)} e^{S_{\text{NP}}(b)} F_{\text{NP}}(x, b)$$

matching coefficients $C_j^{(F)}$ (red box)
 collinear PDF F^j (grey box)
 nonperturbative Sudakov $e^{S(b_*; \mu_b, \mu, \zeta)}$ (red box)
 nonperturbative TMD function $F_{\text{NP}}(x, b)$ (orange box)
 perturbative Sudakov $e^{S_{\text{NP}}(b)}$ (orange box)

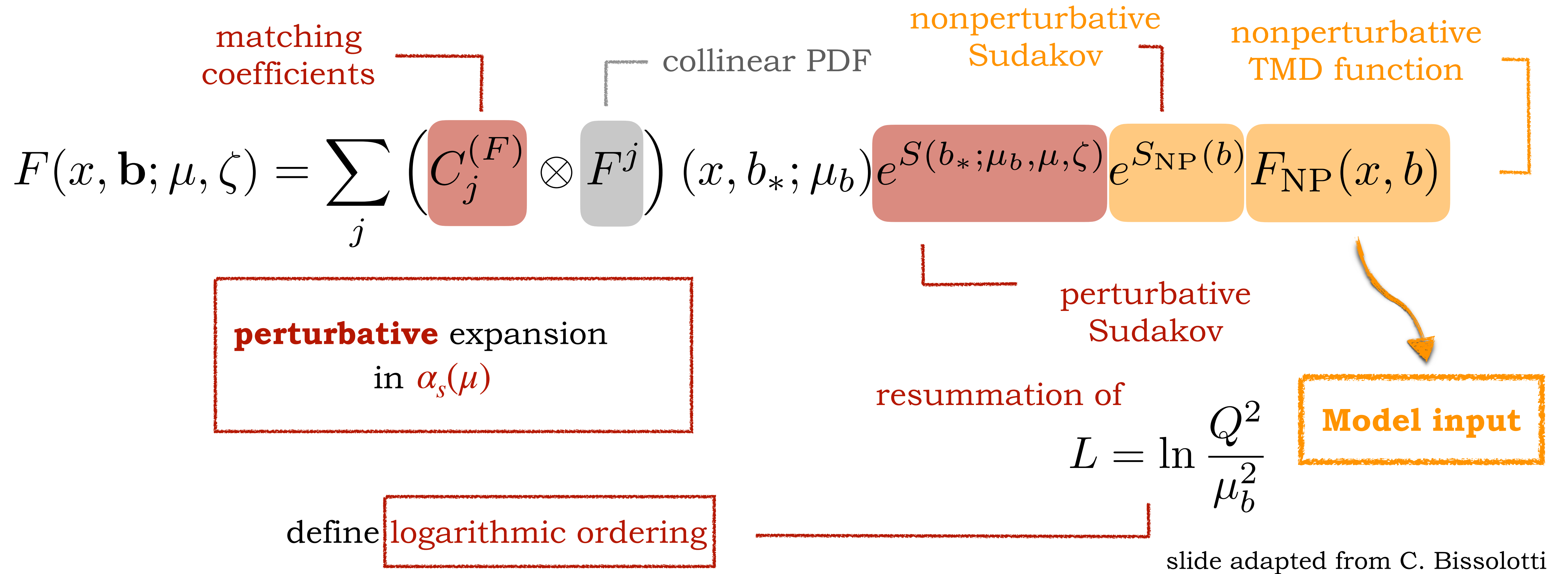
perturbative expansion in $\alpha_s(\mu)$ (red box)
 define **logarithmic ordering** (red box)

resummation of $L = \ln \frac{Q^2}{\mu_b^2}$ (red text)
Model input (orange box)

slide adapted from C. Bissolotti

$$f_1(x, \mathbf{b}, \mu, \zeta) \rightarrow C_j^{(f_1)} \otimes f_1^j = [1 + \mathcal{O}(\alpha_s)]_j \otimes f_1^j$$

Anatomy of gluon TMDs





$$f_1(x, \mathbf{b}, \mu, \zeta) \rightarrow C_j^{(f_1)} \otimes f_1^j = [1 + \mathcal{O}(\alpha_s)]_j \otimes f_1^j$$

$$h_1^\perp(x, \mathbf{b}, \mu, \zeta) \rightarrow C_j^{(h_1^\perp)} \otimes f_1^j = [\mathcal{O}(\alpha_s)]_j \otimes f_1^j$$

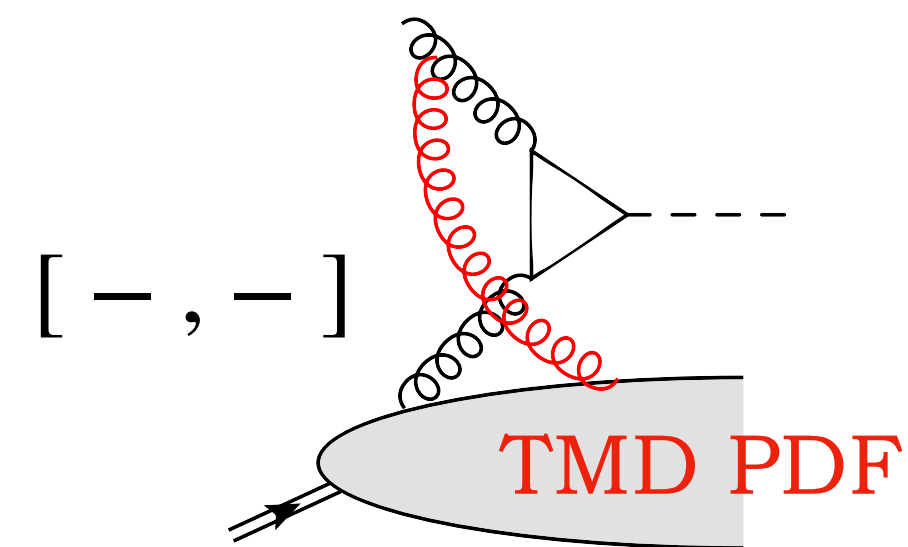
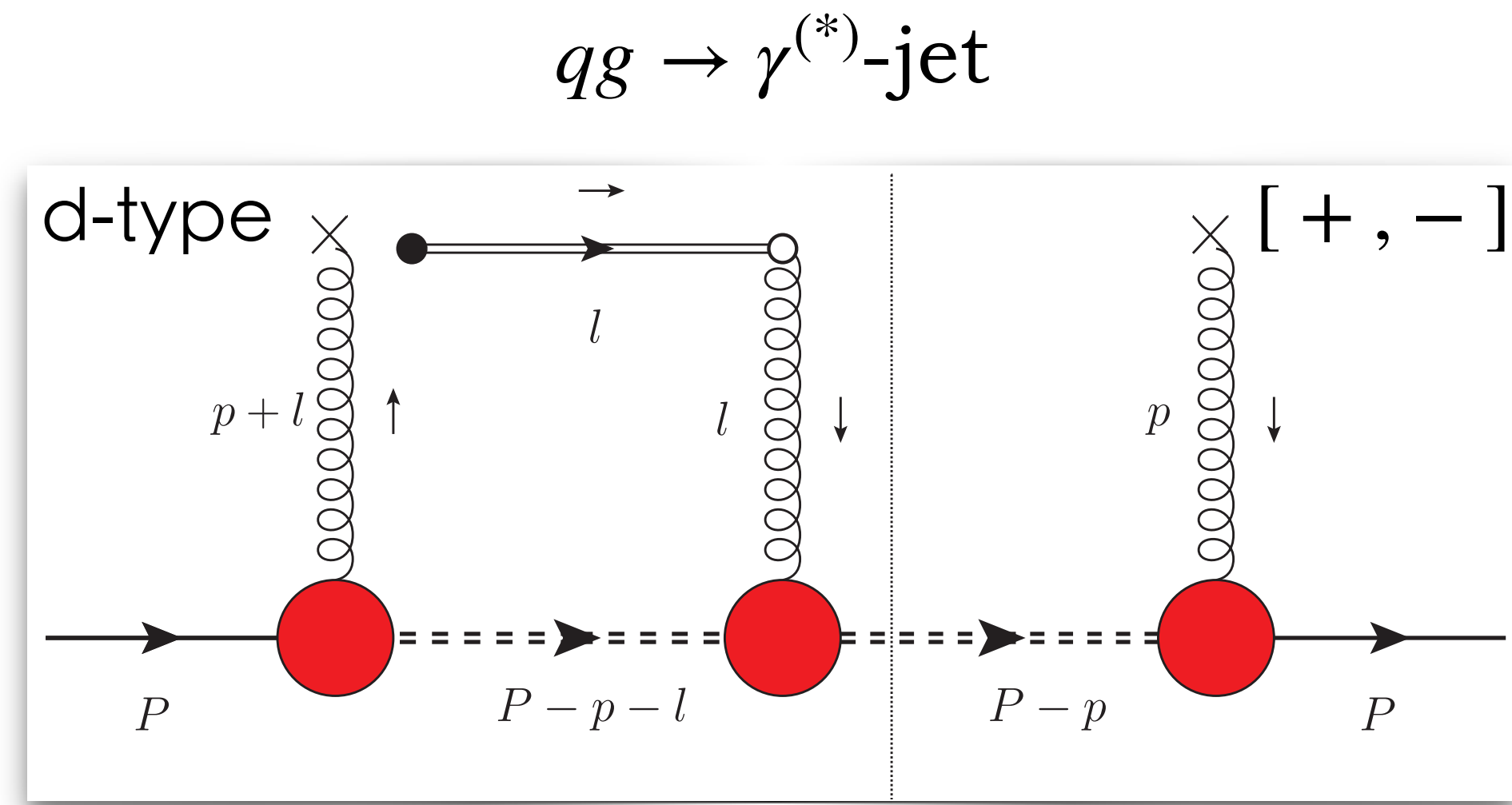
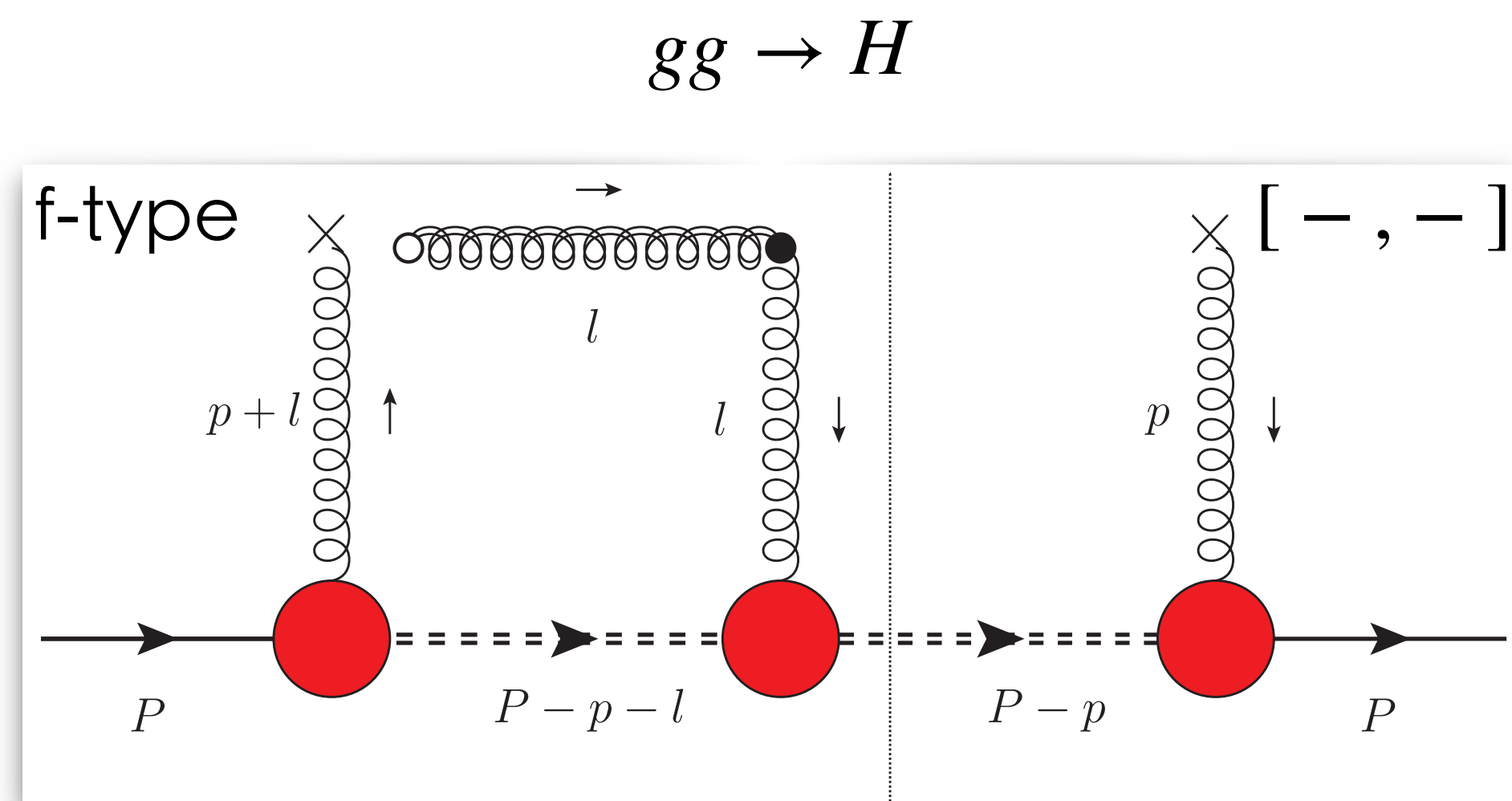
¿ **Suppression of genuine NP effects ?** ←

T-odd gluon TMDs in a spectator model

-  No residual gluon-spectator interaction at tree level
-  Interference with one-gluon exchange (eikonal)

T-odd gluon TMDs in a spectator model

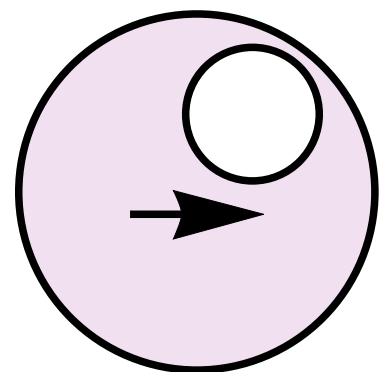
- No residual gluon-spectator interaction at tree level
- Interference with one-gluon exchange (eikonal)



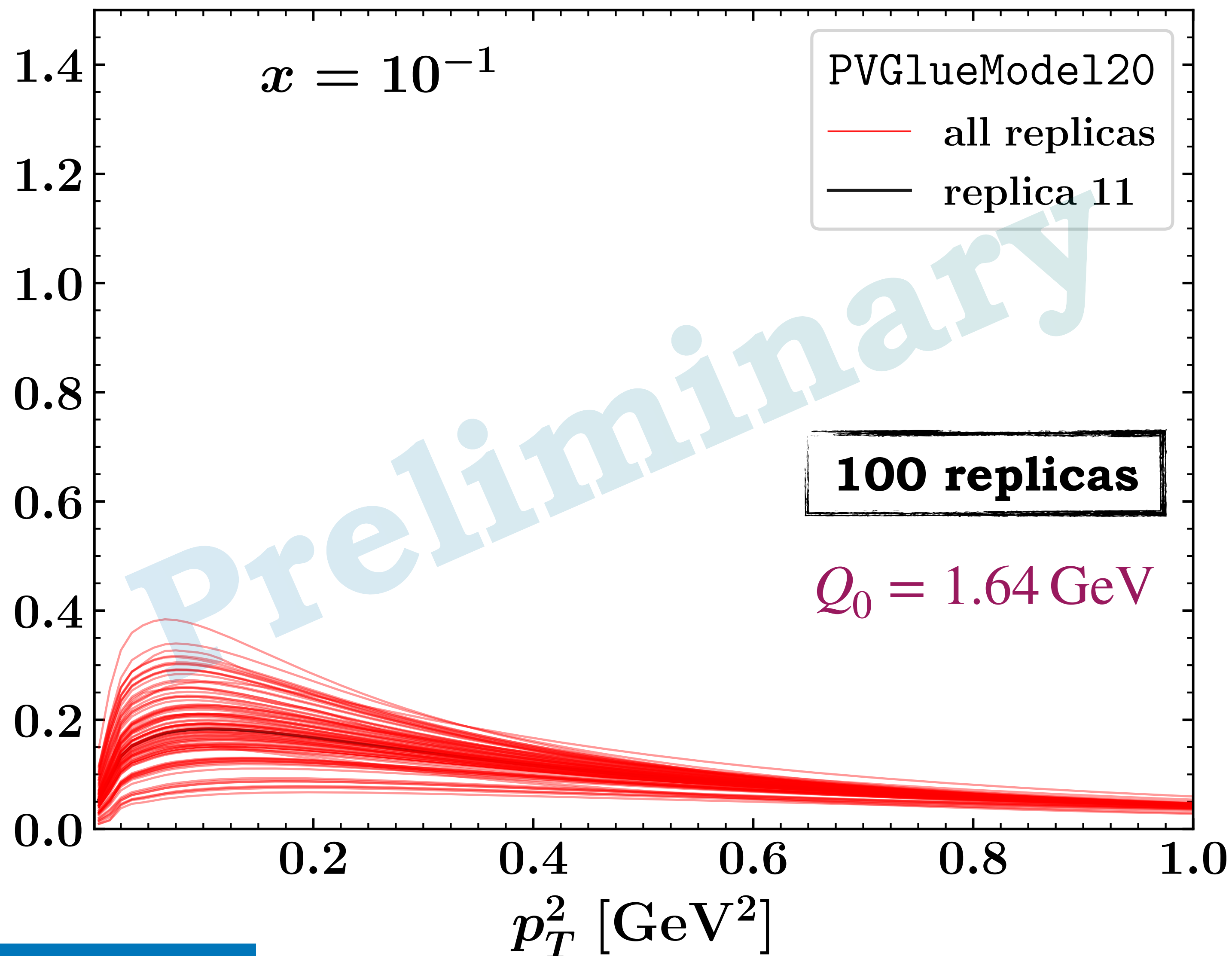
f-type (WW) structure

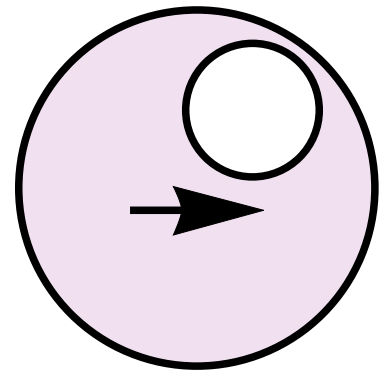


d-type (dipole) structure

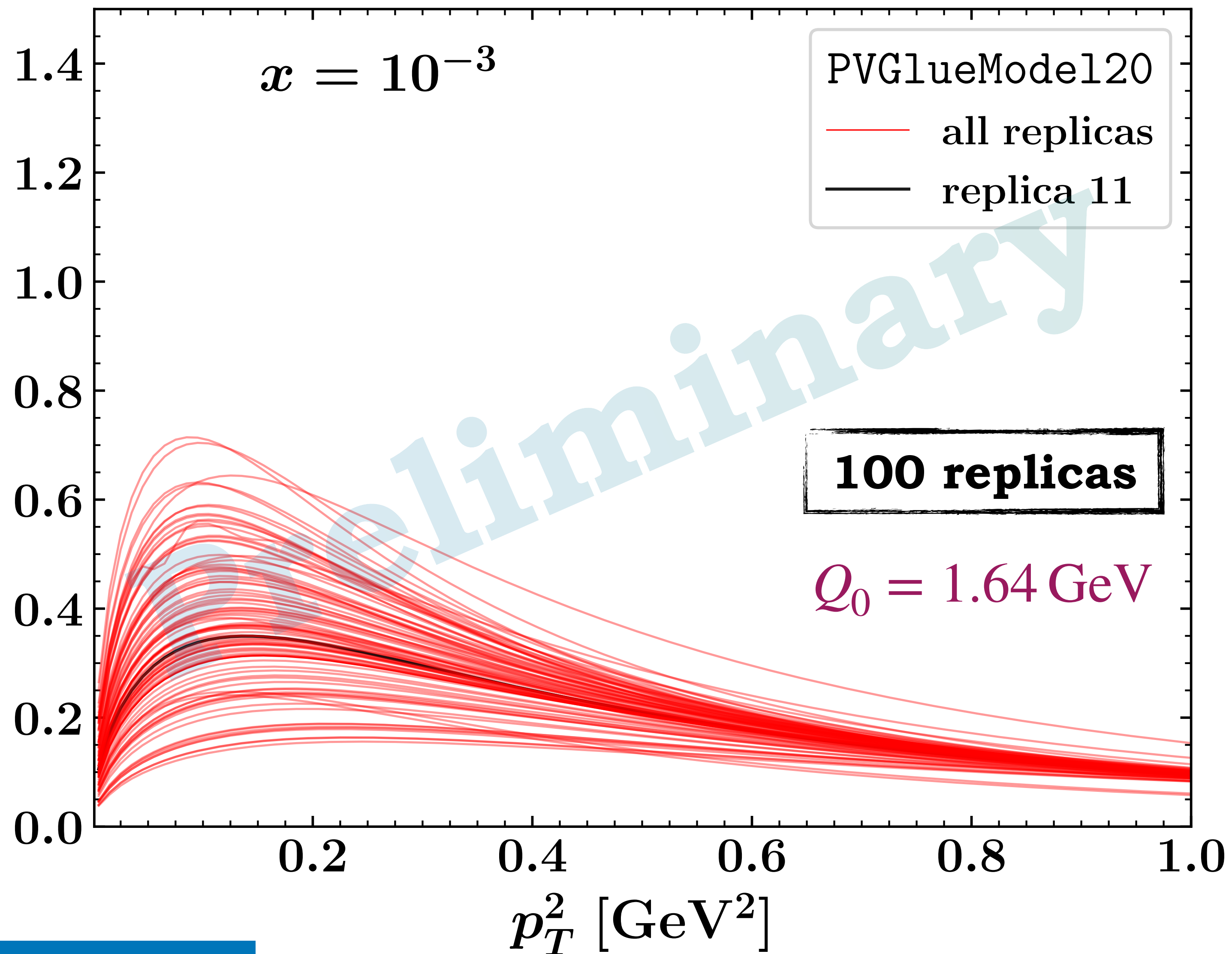


$$x \frac{p_T}{M} f_{1T}^{\perp[+,+]}(x, p_T^2)$$





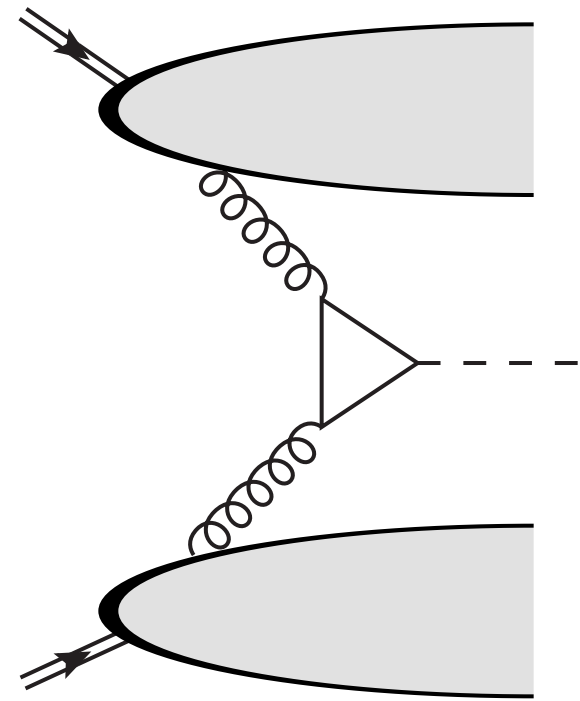
$$x \frac{p_T}{M} f_{1T}^{\perp[+,+]}(x, p_T^2)$$



3. Accessing gluon TMDs @LHC

The background features a repeating pattern of circular diagrams, each representing a gluon Transverse Momentum Distribution (TMD). These diagrams are rendered in a semi-transparent, light blue style. Each diagram shows a central gluon (represented by a red sphere with a red arrow) interacting with a quark (represented by a blue sphere with a blue arrow). The interaction is depicted by a yellow wavy line (representing a gluon) that forms a loop around the quark. The entire scene is set against a light blue background with a subtle grid of white lines and a soft, glowing effect.

Higgs in gluon fusion



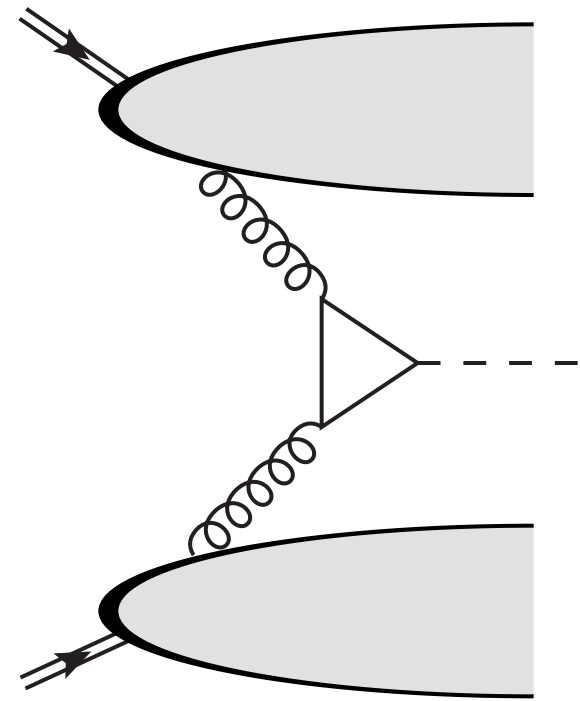
No color entanglement
TMD factorization



Large low- p_T bin @CMS
More data @HL-LHC

Golden channels for gluon TMD PDFs @LHC

Higgs in gluon fusion

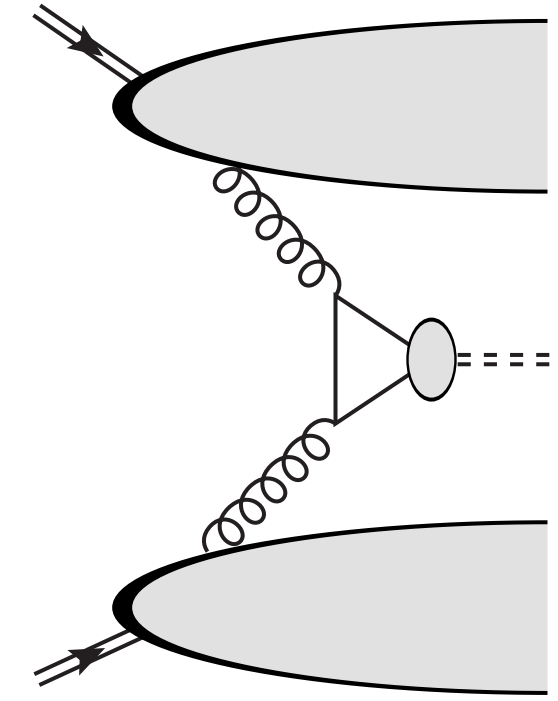


No color entanglement
TMD factorization



Large low- p_T bin @CMS
More data @HL-LHC

Single quarkonium



$\eta_{c,b}$



$J/\psi, \Upsilon$

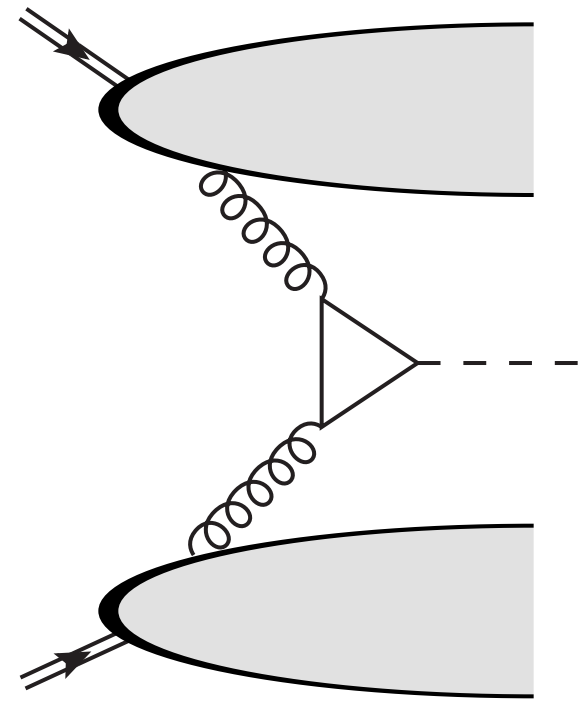
TMD factorization
C-parity selection rules



Large- p_T data @LHCb
More data @FT-LHC

Golden channels for gluon TMD PDFs @LHC

Higgs in gluon fusion

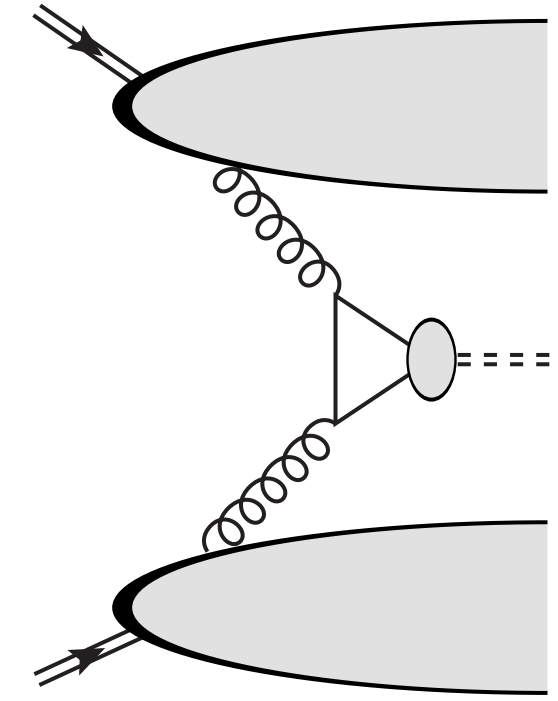


No color entanglement
TMD factorization



Large low- p_T bin @CMS
More data @HL-LHC

Single quarkonium



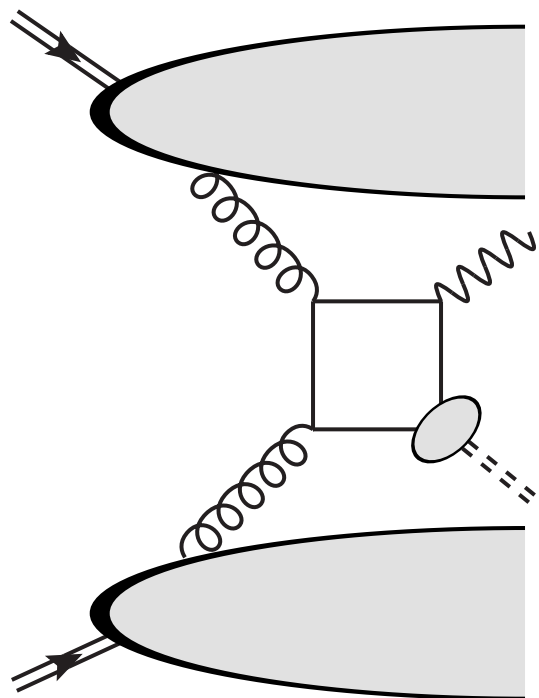
$\eta_{c,b}$ $J/\psi, \Upsilon$

TMD factorization
C-parity selection rules



Large- p_T data @LHCb
More data @FT-LHC

$J/\psi + \gamma^{(*)}$



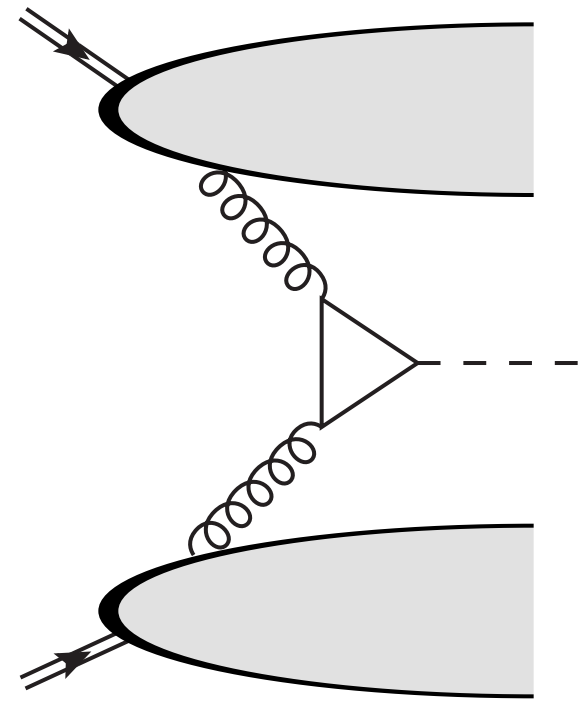
Color entanglement
Potential TMD violation (CO)
Back-to-back suppresses CO



Possible studies @HL-LHC
Currently no low- p_T data

Golden channels for gluon TMD PDFs @LHC

Higgs in gluon fusion

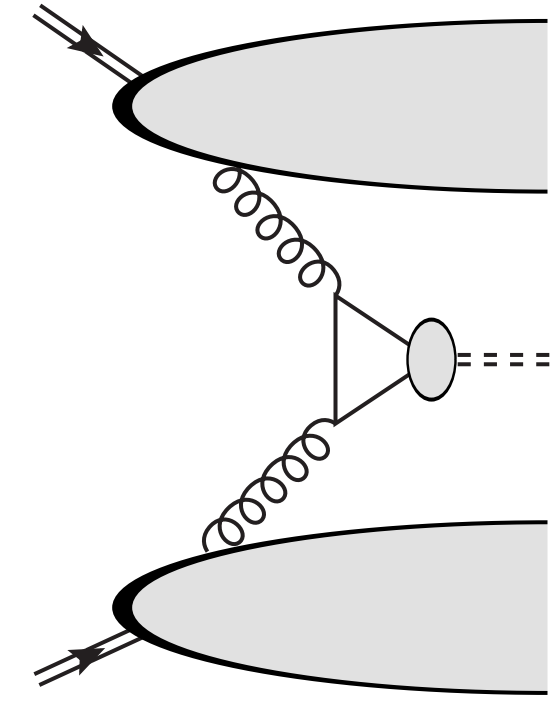


No color entanglement
TMD factorization



Large low- p_T bin @CMS
More data @HL-LHC

Single quarkonium



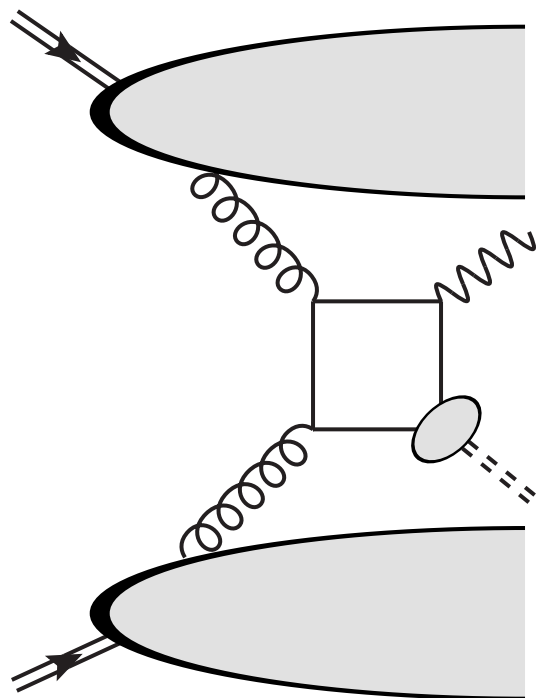
TMD factorization
C-parity selection rules

$\eta_{c,b}$ $J/\psi, \Upsilon$



Large- p_T data @LHCb
More data @FT-LHC

$J/\psi + \gamma^{(*)}$

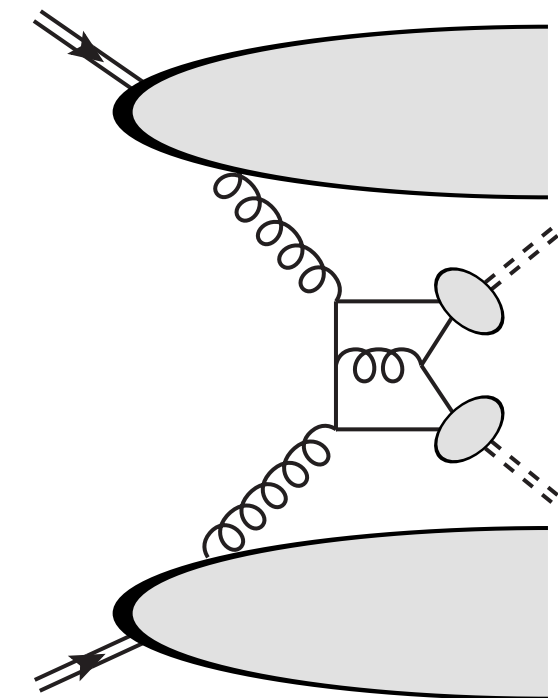


Color entanglement
Potential TMD violation (CO)
Back-to-back suppresses CO



Possible studies @HL-LHC
Currently no low- p_T data

$J/\psi + J/\psi$



No color entanglement
TMD factorization (CSM)

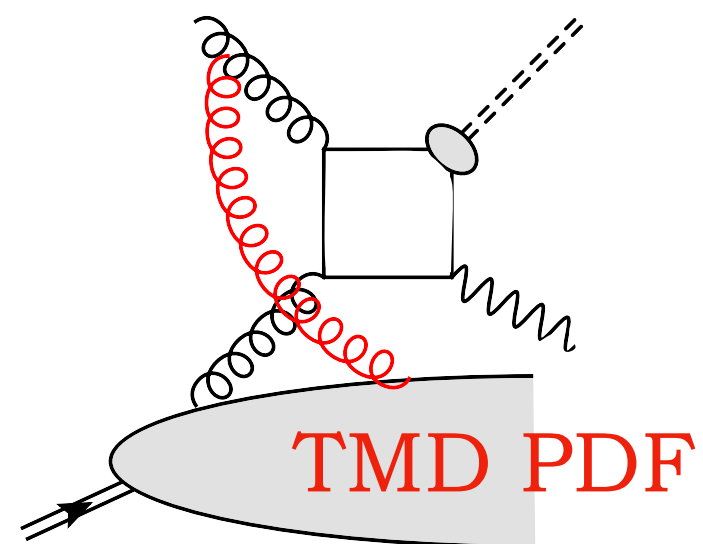


Low- p_T data @LHCb
Opportunities @HL- & @FT-LHC

Quarkonia: Assets & challenges

Assets

 Onia \Rightarrow clean channels of f-type gluon TMDs



Initial-state color flow \Rightarrow $[-, -]$ gauge link

(overview)  [\[D. Boer \(2017\)\]](#)

Sivers	$ep^\dagger \rightarrow e' Q \bar{Q} X$ $ep^\dagger \rightarrow e' j_1 j_2 X$
$f_{1T}^{\perp g[-,-]}$	✓
$f_{1T}^{\perp g[+,-]}$	×

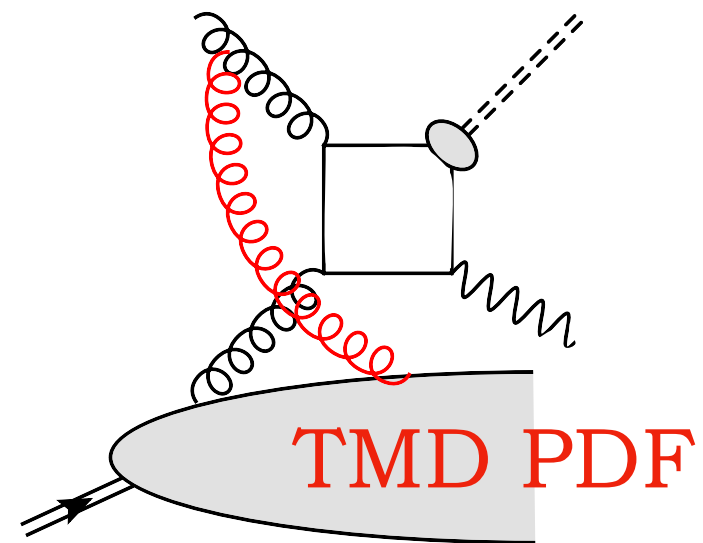
Boer-Mulders	$ep \rightarrow e' Q \bar{Q} X$ $ep \rightarrow e' j_1 j_2 X$
$h_1^{\perp g[-,-]}(WW)$	✓
$h_1^{\perp g[+,-]}(DP)$	×

Challenges

Quarkonia: Assets & challenges

Assets

Onia \Rightarrow clean channels of f-type gluon TMDs



Initial-state color flow \Rightarrow $[-, -]$ gauge link

(overview) [\[D. Boer \(2017\)\]](#)

Sivers	$ep^\uparrow \rightarrow e' Q \bar{Q} X$ $ep^\uparrow \rightarrow e' j_1 j_2 X$	Boer-Mulders	$ep \rightarrow e' Q \bar{Q} X$ $ep \rightarrow e' j_1 j_2 X$
$f_{1T}^{\perp g[-,-]}$	✓	$h_1^{\perp g[-,-]}(WW)$	✓
$f_{1T}^{\perp g[+,-]}$	×	$h_1^{\perp g[+,-]}(DP)$	×

$\eta_{c,b}$ \Rightarrow LHC complementarity, TMD factorization

$$\frac{d\sigma}{dq_T} \sim \text{at low transverse momentum for (pseudo)scalar state}$$

$$\sim \mathcal{C} [f_1^{g/A} f_1^{g/B}] \pm \mathcal{C} [h_1^{\perp g/A} h_1^{\perp g/B}]$$

unpolarized gluons
lin. polarized gluons

(factorization) [\[M. García Echevaría \(2019\)\]](#)

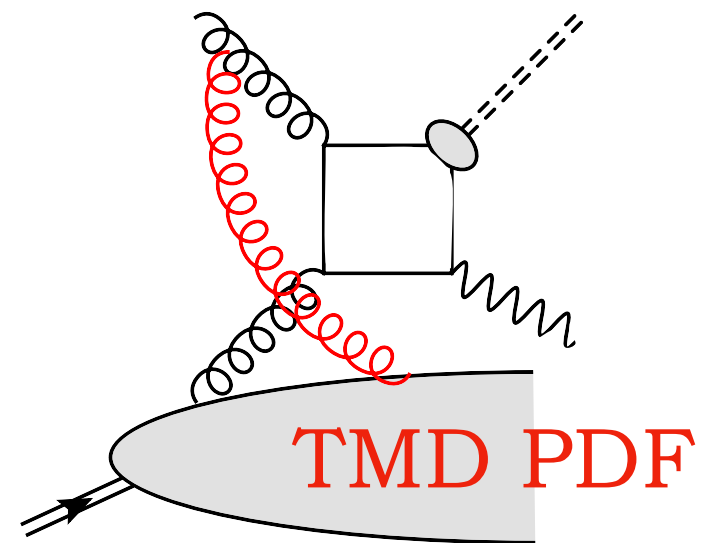
(pheno) [\[A. Bacchetta, F.G.C., J.-P. Lansberg, M. Radici, et al. \(in progress\)\]](#)

Challenges

Quarkonia: Assets & challenges

Assets

 Onia \Rightarrow clean channels of f-type gluon TMDs



Initial-state color flow \Rightarrow $[-, -]$ gauge link

Sivers	$ep^\dagger \rightarrow e' Q \bar{Q} X$ $ep^\dagger \rightarrow e' j_1 j_2 X$
$f_{1T}^{\perp g[-,-]}$	✓
$f_{1T}^{\perp g[+,-]}$	×

(overview)  [D. Boer (2017)]

Boer-Mulders	$ep \rightarrow e' Q \bar{Q} X$ $ep \rightarrow e' j_1 j_2 X$
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(factorization)  [M. García Echevaría (2019)]

(pheno) [A. Bacchetta, F.G.C., J.-P. Lansberg, M. Radici, et al. (in progress)]

Challenges

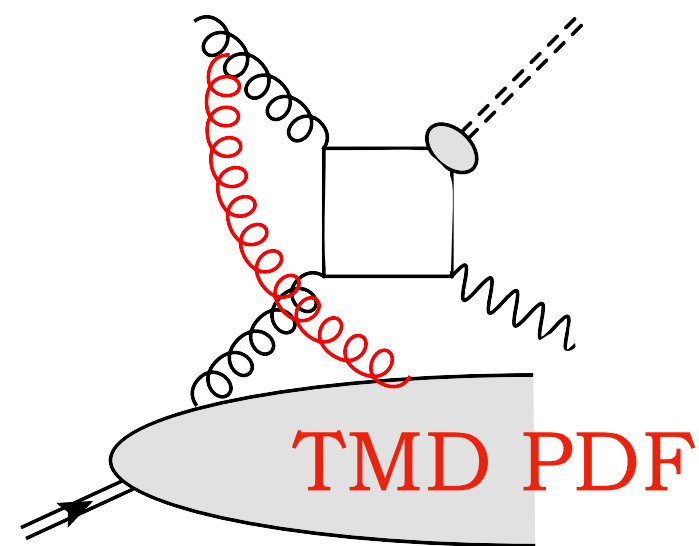
 Precision TMD \Leftrightarrow production mechanism(s)

(production mechanisms, LHC)  [J.-P. Lansberg (2020)]

Quarkonia: Assets & challenges

Assets

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(overview) [\[D. Boer \(2017\)\]](#)

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(factorization) [\[M. García Echevarría \(2019\)\]](#)

(pheno) [\[A. Bacchetta, F.G.C., J.-P. Lansberg, M. Radici, et al. \(in progress\)\]](#)

Challenges

Precision TMD \Leftrightarrow production mechanism(s)

(production mechanisms, LHC) [\[J.-P. Lansberg \(2020\)\]](#)

Color Evaporation Model

$(Q\bar{Q})$ decorrelated from onium, semi-soft gluon emissions
Overshoots data at large p_T

Color Singlet Model


$(Q\bar{Q})$ to onium, no gluon emissions
Fails at large p_T , improves at NLO

NRQCD and Color Octet

Higher Fock states, soft gluon emissions
Problems at low p_T , fails on polarization

TMD & shape functions

 NRQCD \Rightarrow double expansion: $\alpha_s \oplus v$


 NRQCD \Rightarrow $d\sigma(|Q\rangle) \propto \mathcal{H} \otimes \text{LDME}$

$$\begin{aligned} |Q\rangle = & \mathcal{O}(1) |Q\bar{Q}[^3S_1^{(1)}]\rangle + \mathcal{O}(v) |Q\bar{Q}[^3P_J^{(8)}g]\rangle + \mathcal{O}(v^2) |Q\bar{Q}[^1S_0^{(8)}g]\rangle \\ & + \mathcal{O}(v^2) |Q\bar{Q}[^3S_1^{(1,8)}gg]\rangle + \mathcal{O}(v^2) |Q\bar{Q}[^3D_J^{(1,8)}gg]\rangle + \dots \end{aligned}$$

S-wave quarkonium wave function

TMD & shape functions

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S-wave quarkonium wave function

 TMD \Rightarrow from LDMEs to shape functions (ShFs)

 2 mechanisms: bound state + soft-gluon

(factorization)  [M. Garcia Echevarria (2019)]

(SCET)  [S. Fleming, Y. Makris, T. Mehen (2020)]

(unpol. J/ψ)  [D. Boer, U. D'Alesio, F. Murgia, C. Pisano, P. Taelis (2020)]


(pol. J/ψ)  [U. D'Alesio, L. Maxia, F. Murgia, C. Pisano, R. Sangem (2022)]

(unpol. J/ψ) [D. Boer, J. Bor, L. Maxia, C. Pisano (in preparation)]

Quarkonia: A path toward precision

TMD & shape functions

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Revised TMD shape function in SIDIS

$$\Delta^{[n]}(\kappa_T^2) \propto \frac{\alpha_s}{2\pi^2 \kappa_T^2} C_A \left(1 + \ln \frac{M_Q^2}{M_Q^2 + Q^2} \right) \langle O[n] \rangle$$

Quarkonia: A path toward precision

TMD & shape functions

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(unpol. J/ψ)  [D. Boer, U. D'Alesio, F. Murgia, C. Pisano, P. Taelis (2020)]

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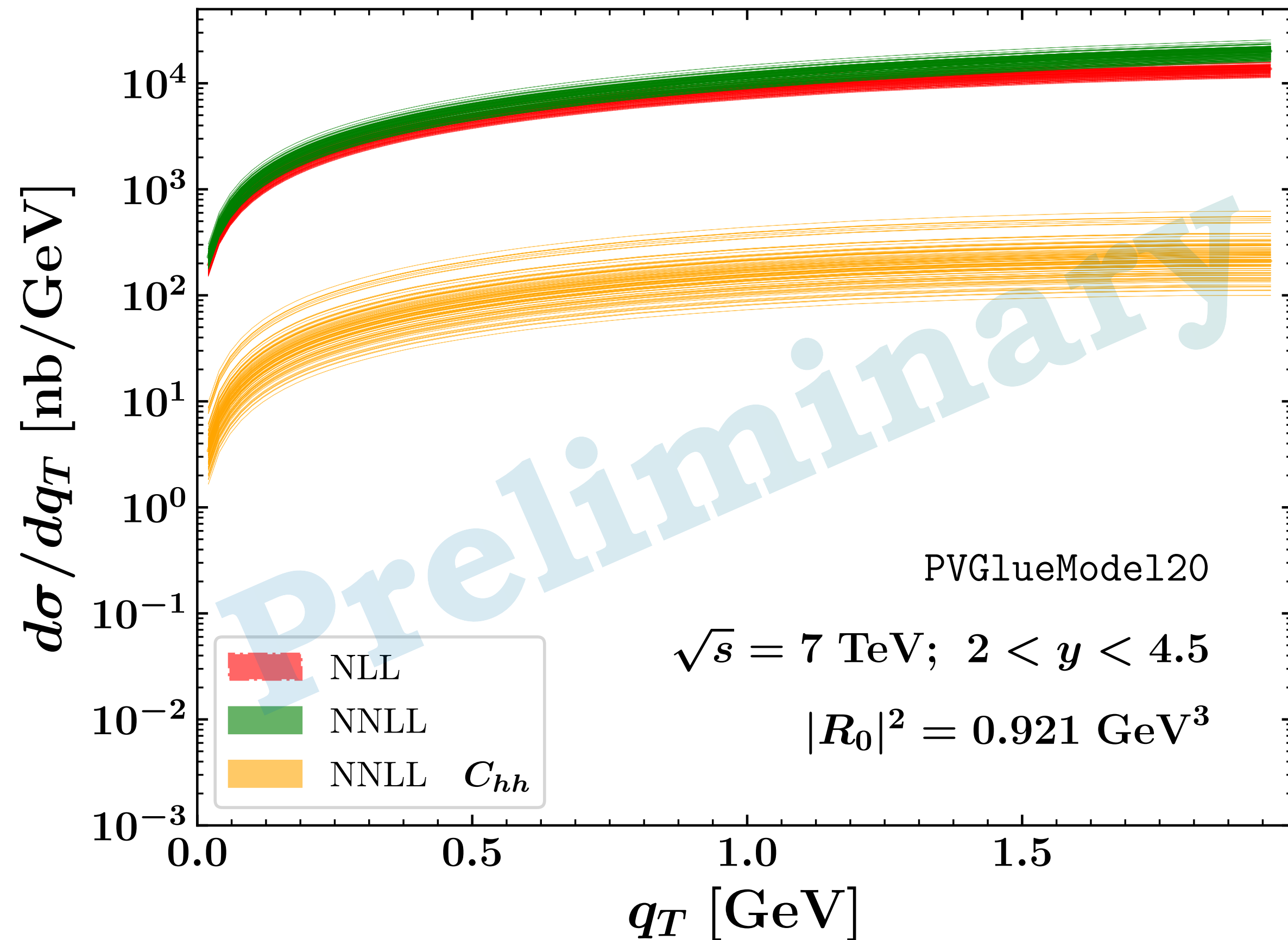
 Perturbative tail \otimes LDME

 [Today's talk by Luca]

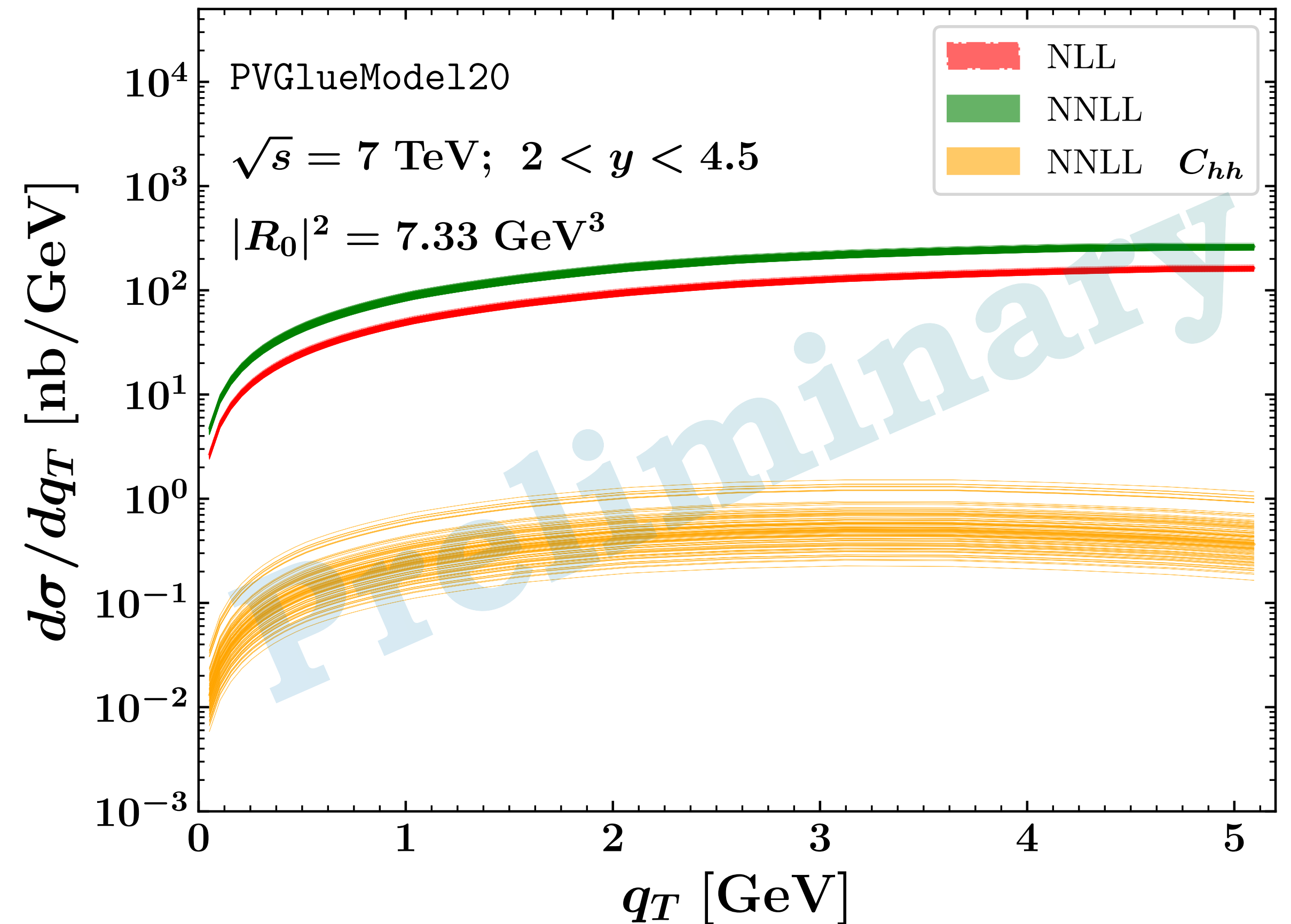
$\eta_{c,b}$ production @ 7TeV LHCb


 Perturbative scales fixed, NP-evolution parameters fixed, TMD 100-replica analysis, [NRQCD](#) w/o [ShFs](#)

$$p(P_1) + p(P_2) \rightarrow \eta_c(q_T)$$



$$p(P_1) + p(P_2) \rightarrow \eta_b(q_T)$$

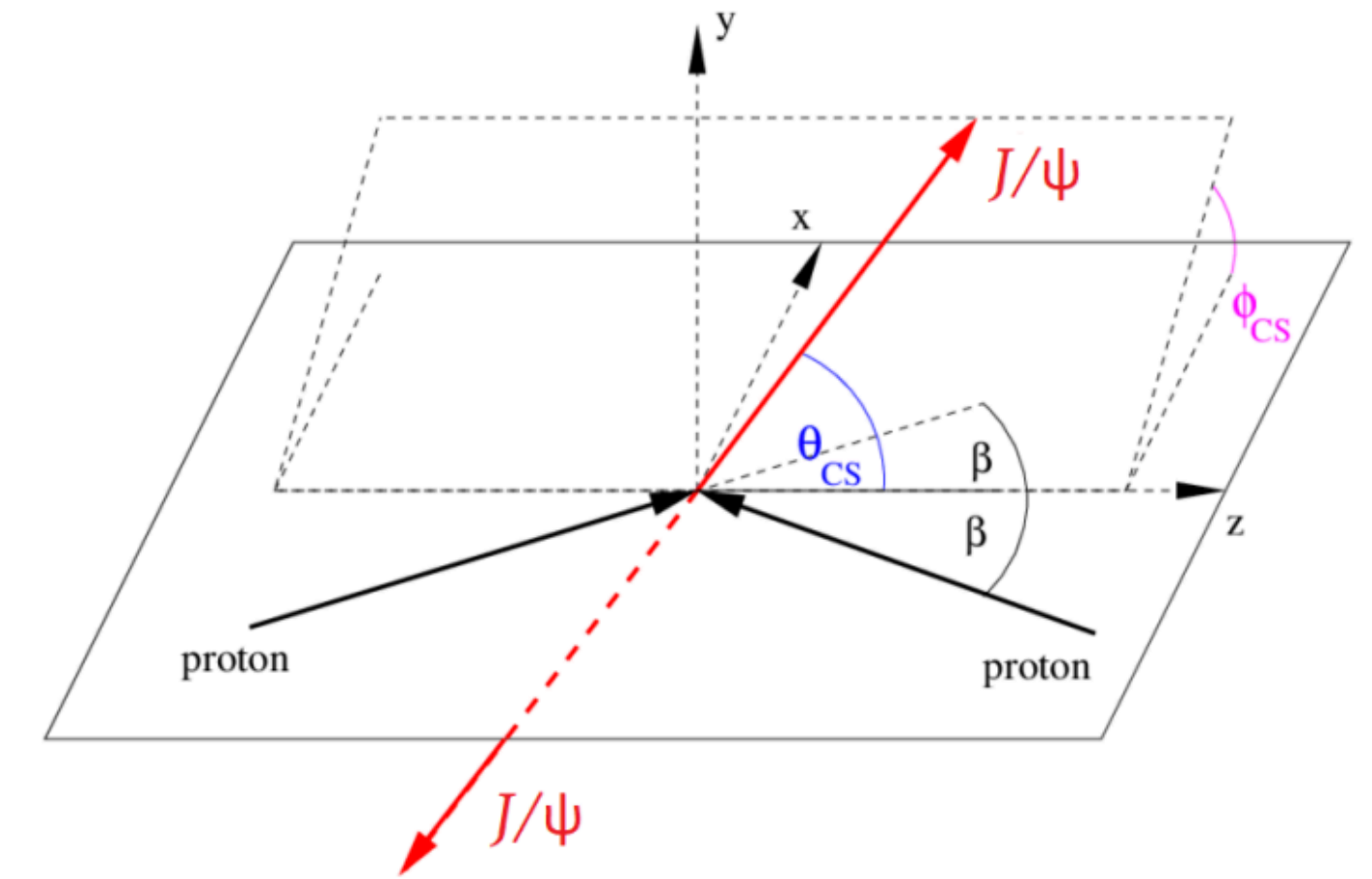


[A. Bacchetta, F.G.C., M.G. Echevarria, J.-P. Lansberg, M. Ozelik, M. Radici, A. Signori (in preparation)]

Double J/ψ production @ (HL-)LHC

More spin asymmetries, measurable @HL-LHC

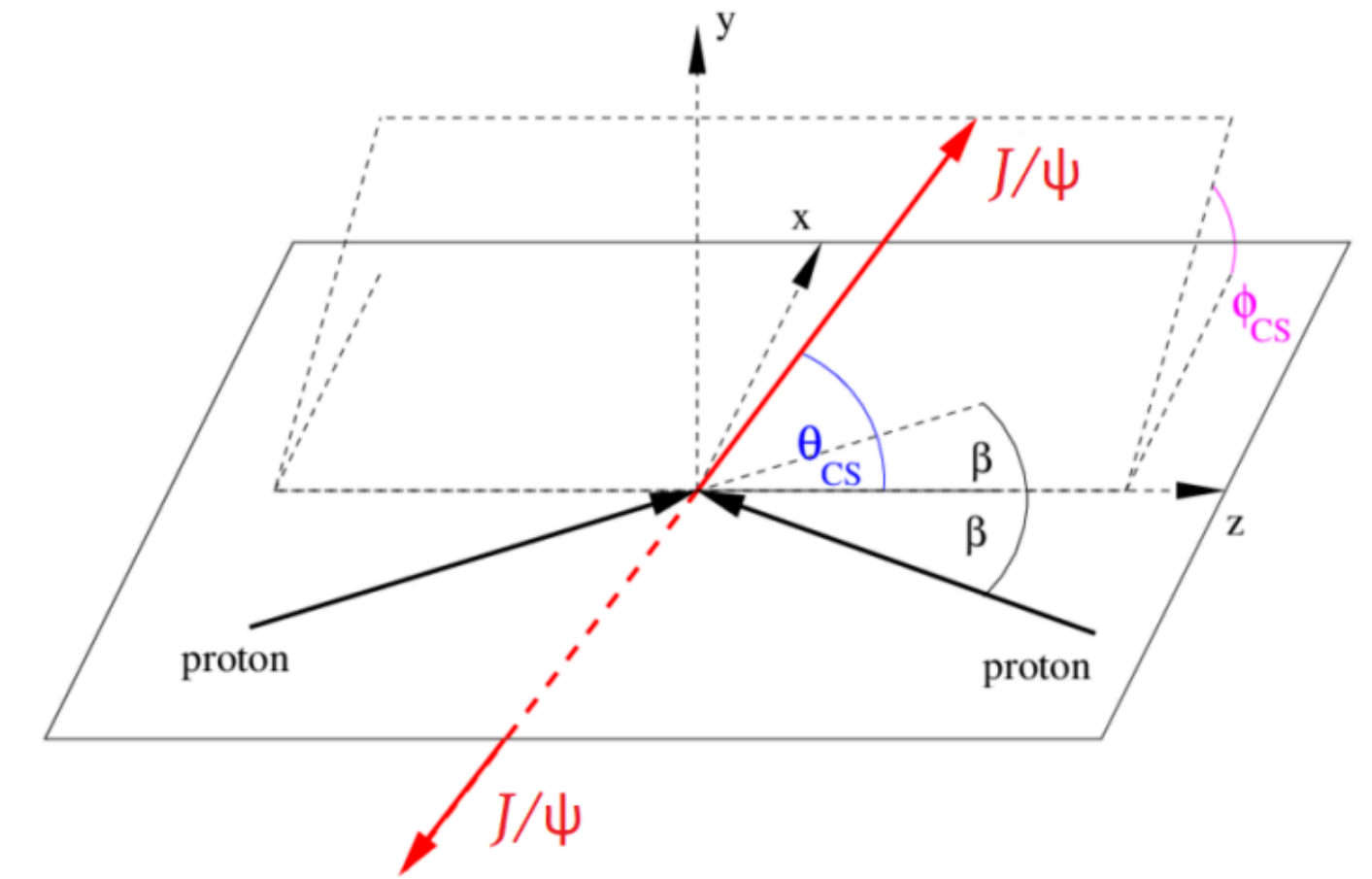
$$\frac{d\sigma}{dM_{QQ}dY_{QQ}d^2P_{QQT}d\Omega} = \frac{\sqrt{Q^2 - 4M_Q^2}}{(2\pi)^2 8s Q^2} \left\{ F_1 C[f_1^g f_1^g] \right. \\ \left. + F_2 C[w_2 h_1^{\perp g} h_1^{\perp g}] + \cos 2\phi_{CS} \left(F_3 C[w_3 f_1^g h_1^{\perp g}] \right. \right. \\ \left. \left. + F'_3 C[w'_3 h_1^{\perp g} f_1^g] \right) + \cos 4\phi_{CS} F_4 C[w_4 h_1^{\perp g} h_1^{\perp g}] \right\},$$



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TMD Models [\[J.-P. Lansberg et al. \(2018\)\]](#)

$$f_1^g(x, \mathbf{k}_T^2, \mu) = \frac{g(x, \mu)}{\pi \langle k_T^2 \rangle} \exp\left(-\frac{\mathbf{k}_T^2}{\langle k_T^2 \rangle}\right)$$

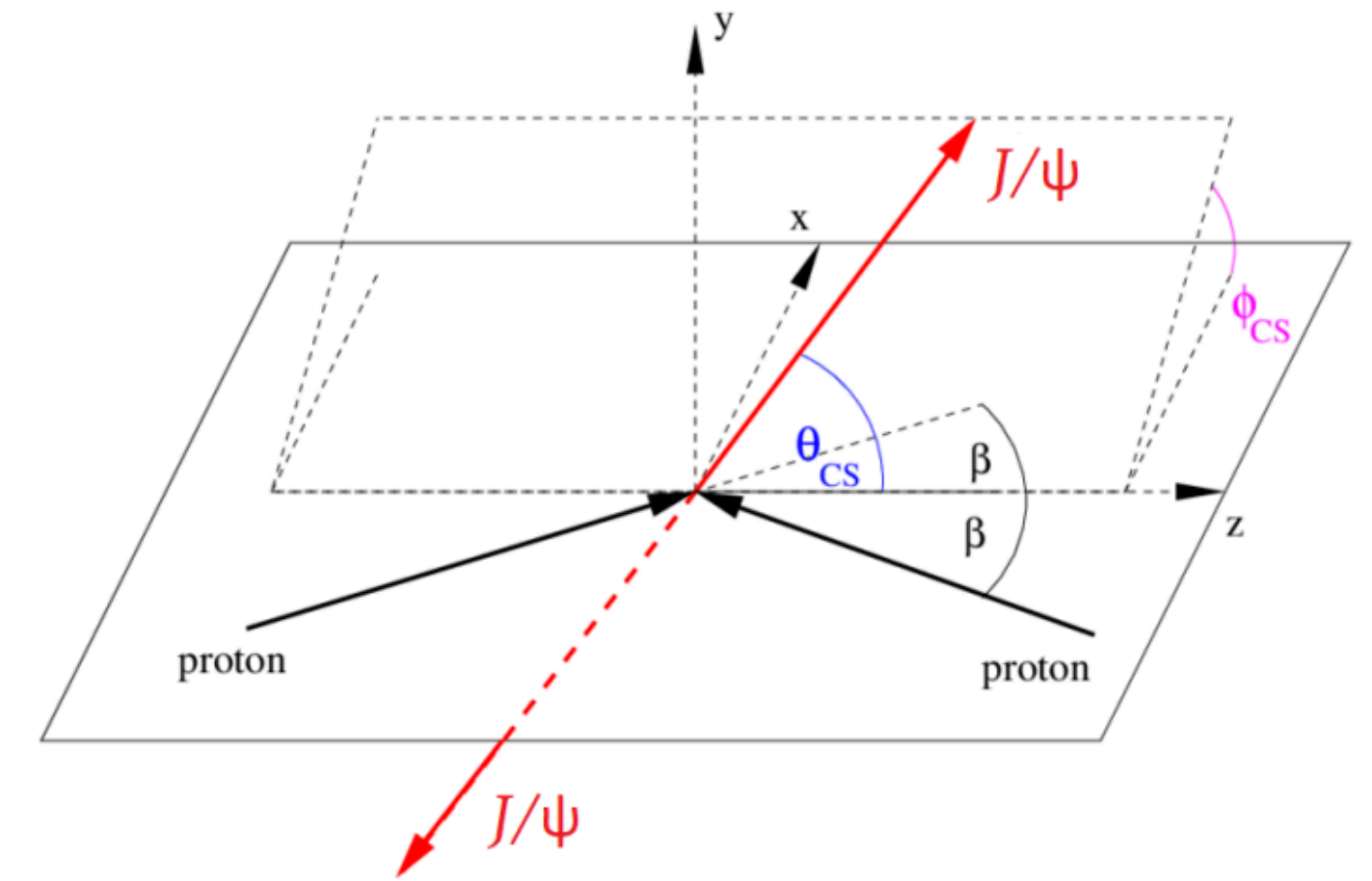
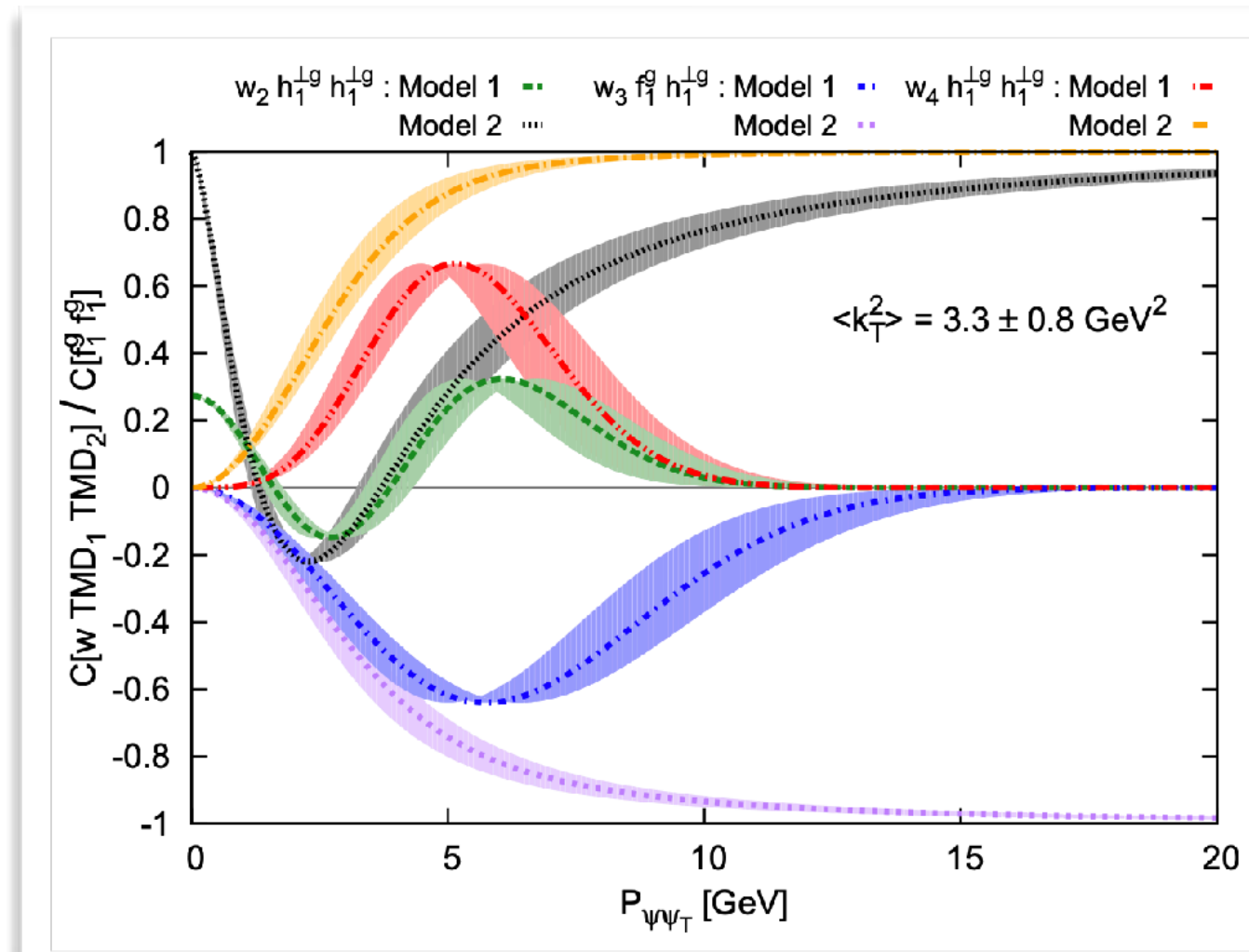
$$\text{? } f_1^g / h_1^{\perp g} (p_T \rightarrow 0) \text{ ?} \quad \Rightarrow \quad |h_1^{\perp g}| \leq f_1^g$$

$$h_1^{\perp g}(x, \mathbf{k}_T^2, \mu) = \frac{2M_p^2}{\langle k_T^2 \rangle} \frac{(1-r)}{r} \frac{g(x, \mu)}{\pi \langle k_T^2 \rangle} \exp\left(1 - \frac{\mathbf{k}_T^2}{r \langle k_T^2 \rangle}\right)$$

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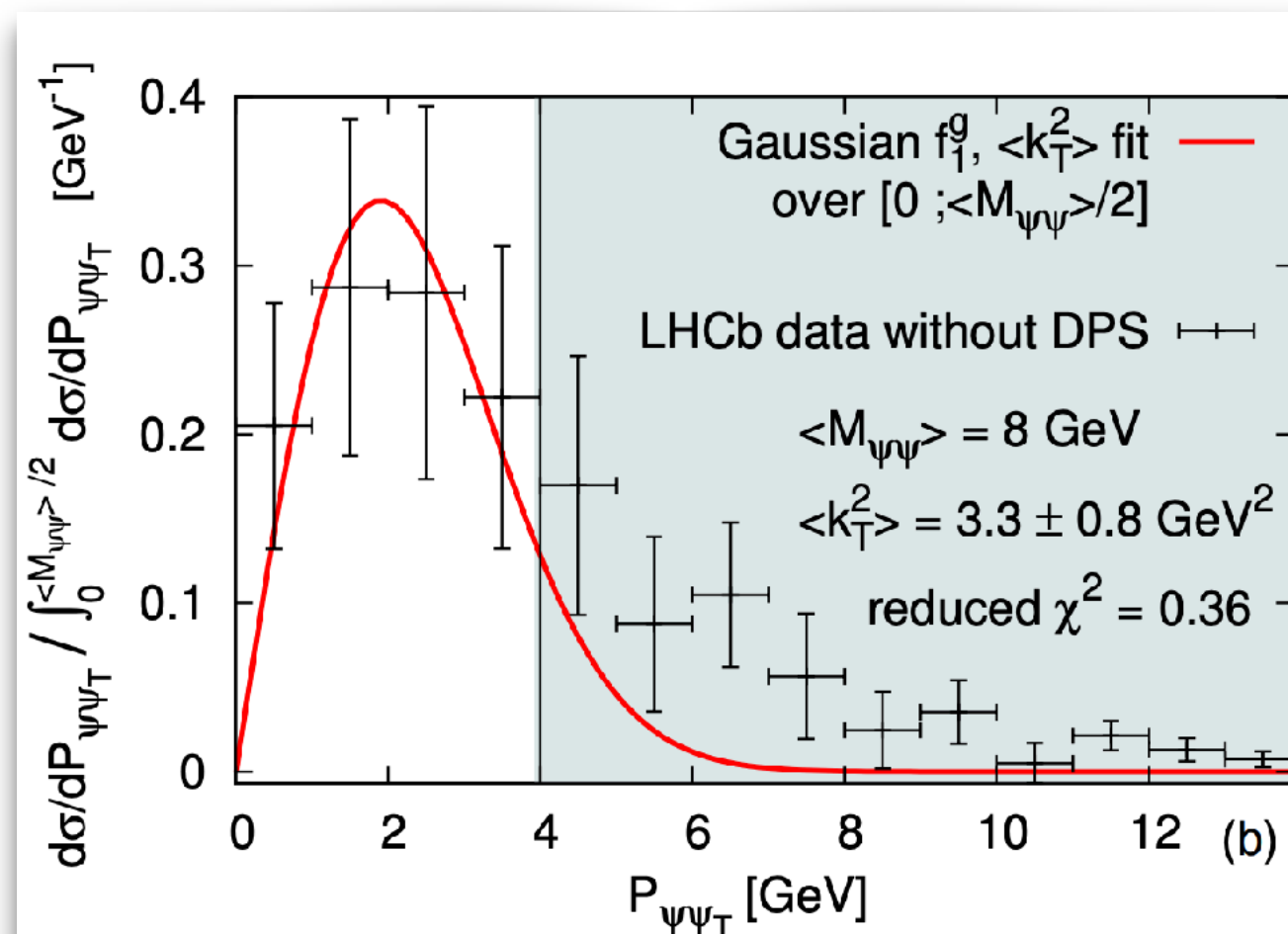


TMD Models [\[J.-P. Lansberg et al. \(2018\)\]](#)

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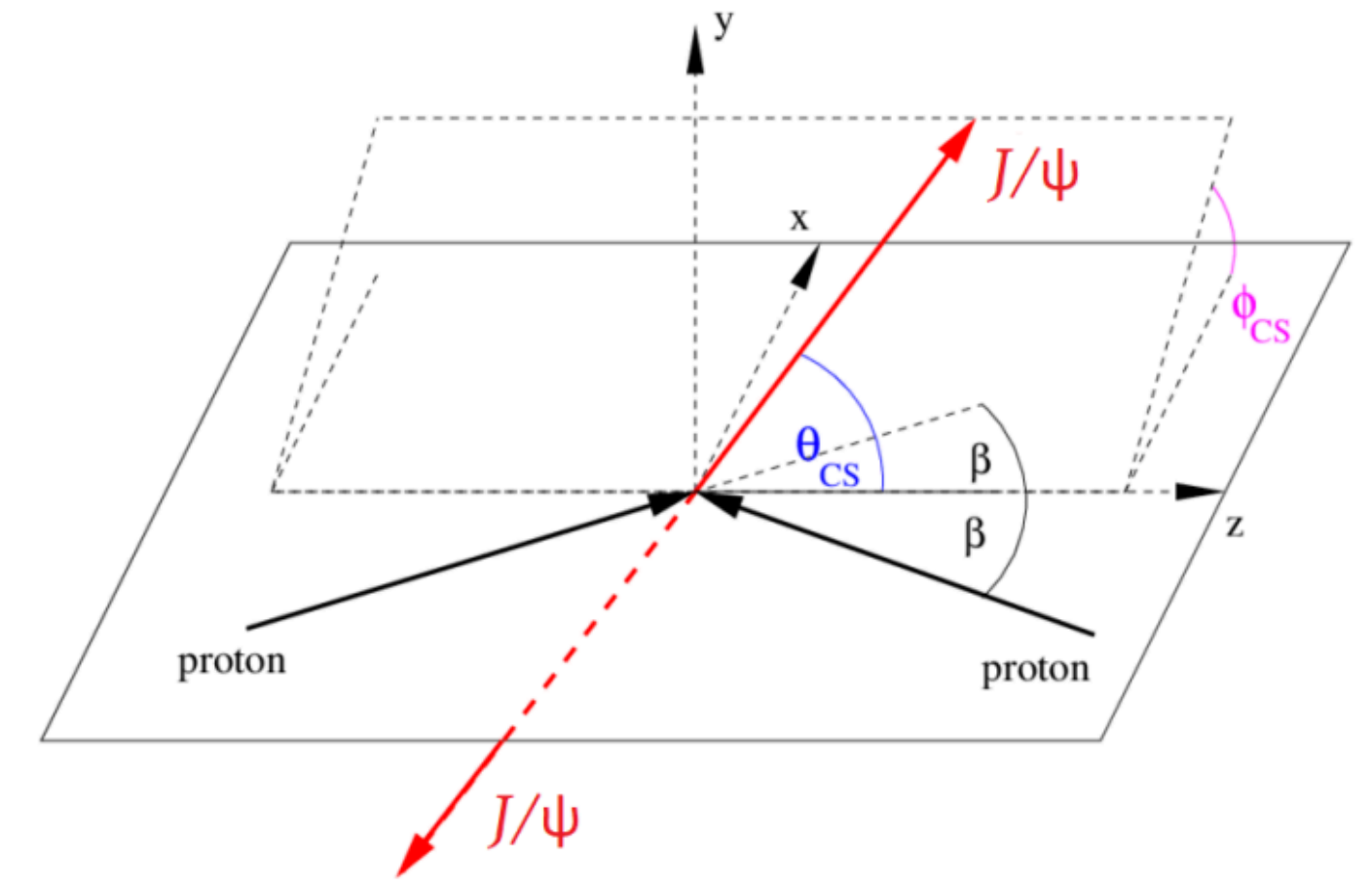
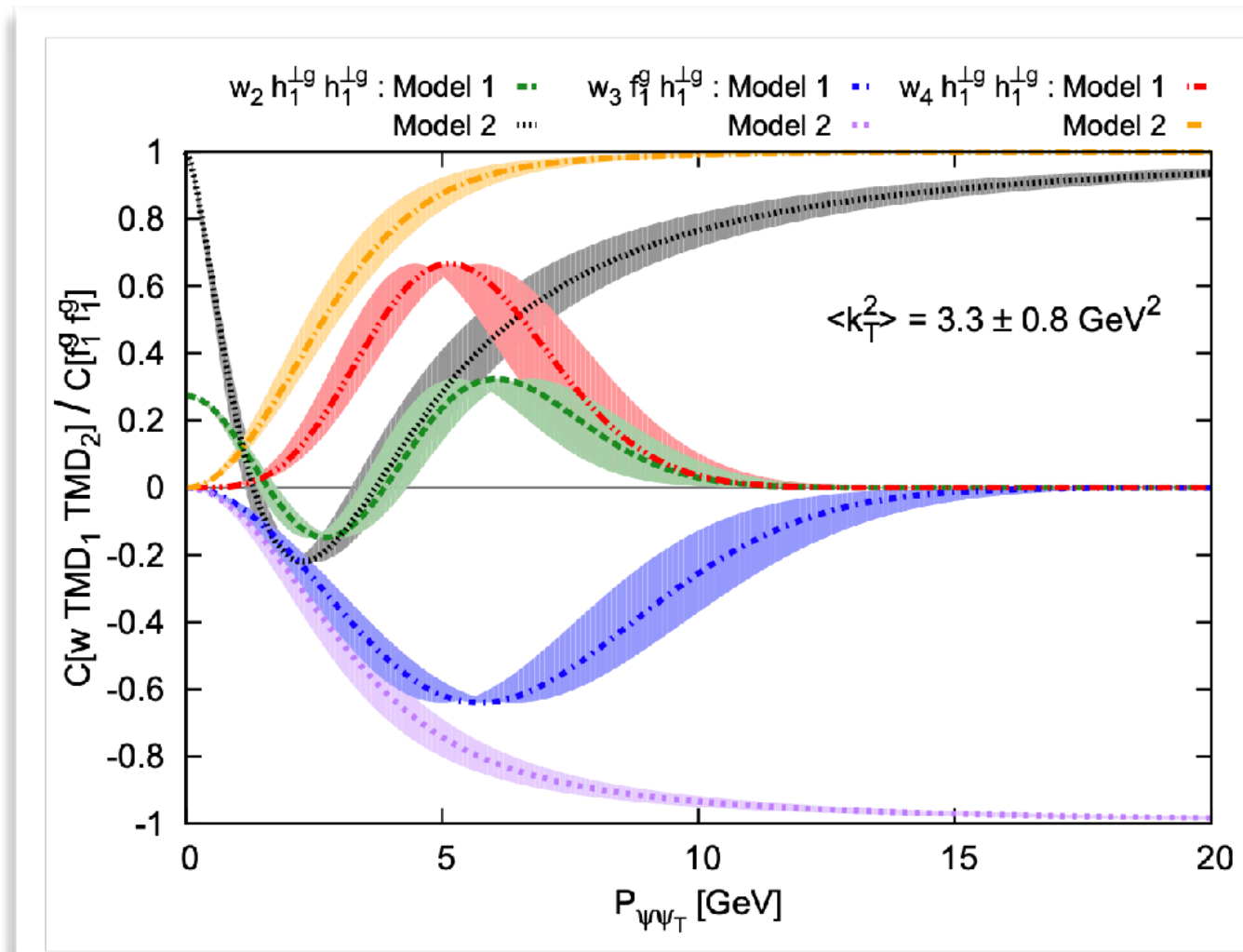


[Model-dependent fit on 13 TeV LHCb data]

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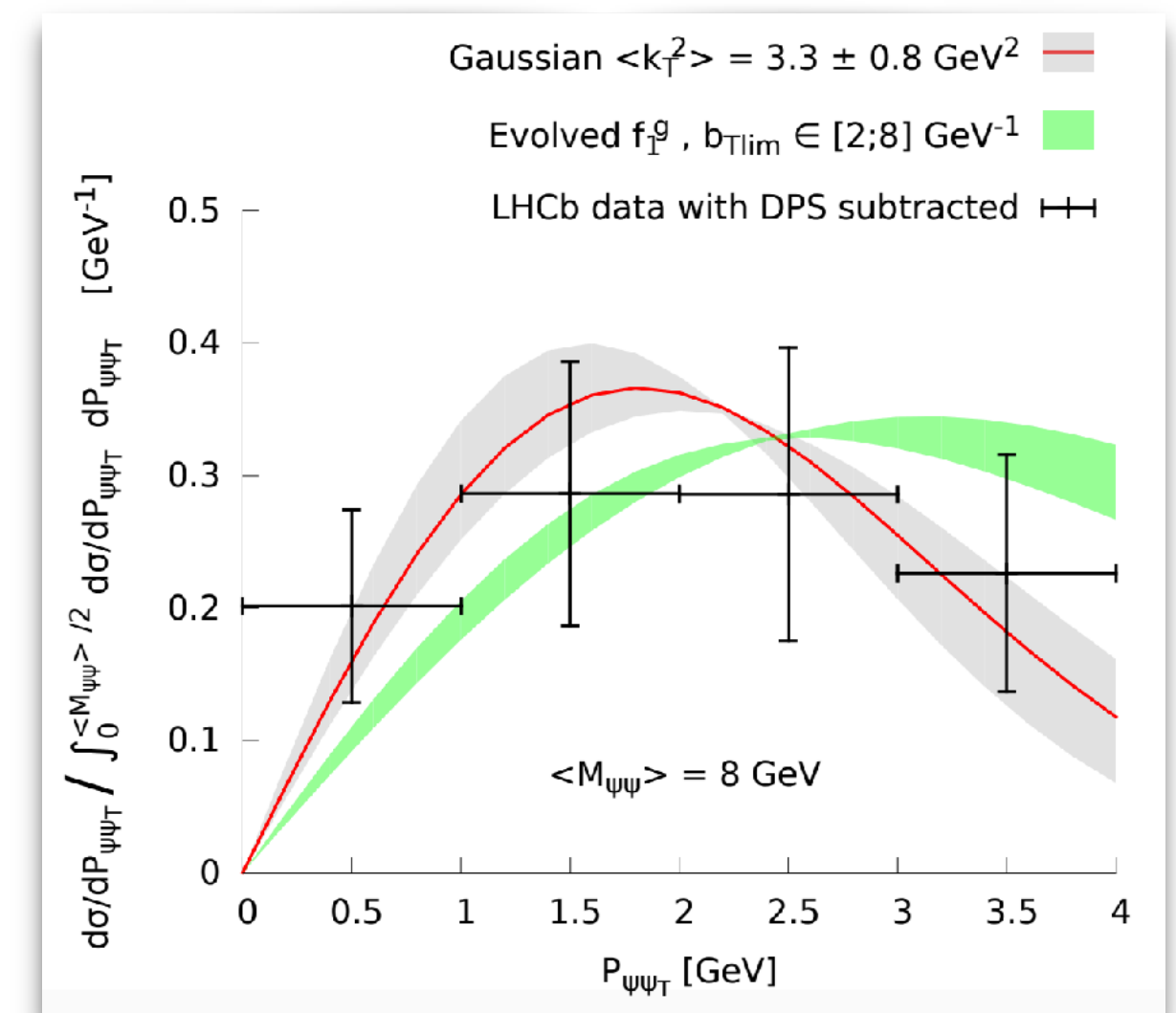
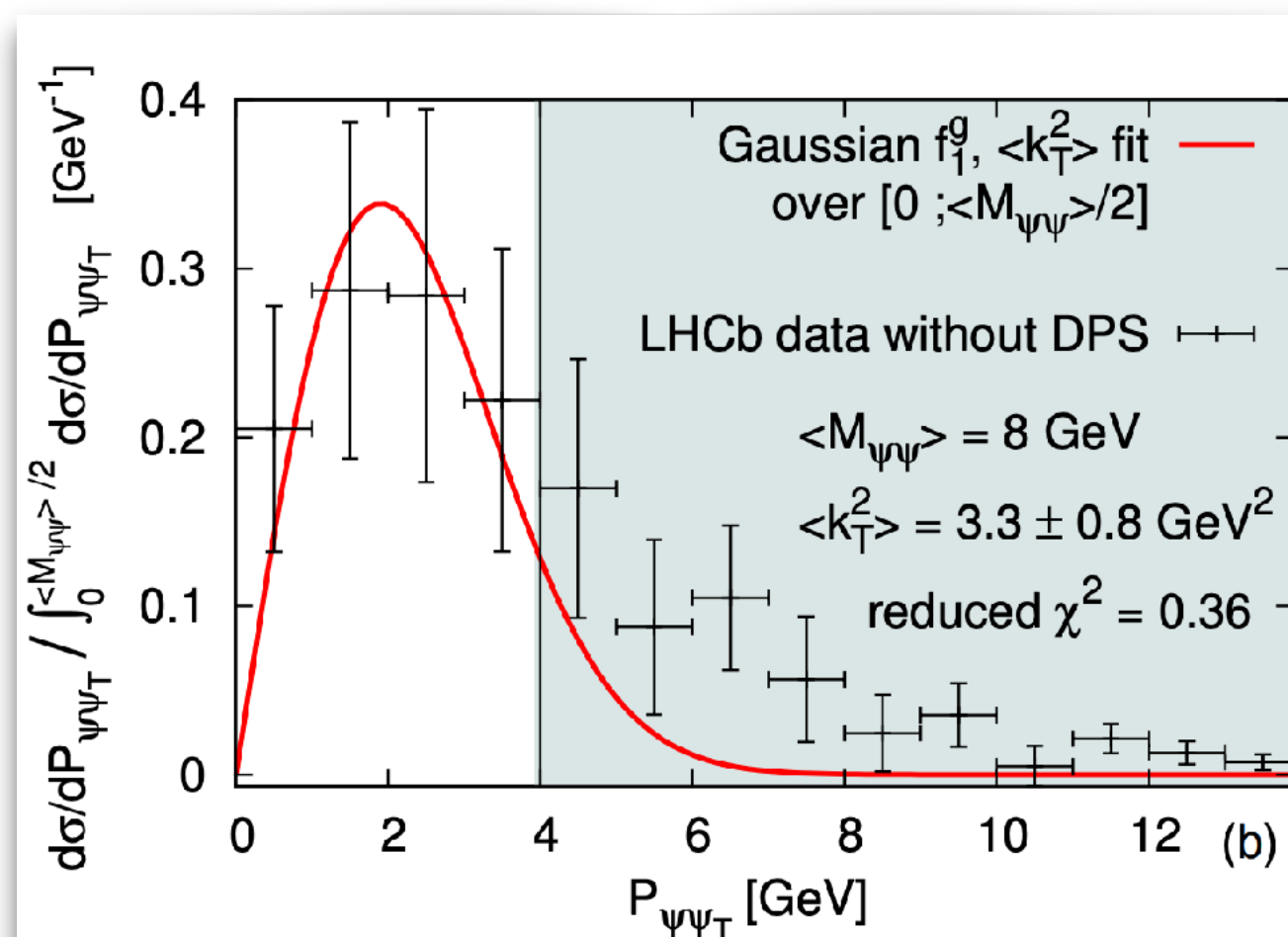
NP + TM resummation

TMD Models [J.-P. Lansberg et al. (2018)]

$$f_1^g(x, k_T^2, \mu) = \frac{g(x, \mu)}{\pi \langle k_T^2 \rangle} \exp\left(-\frac{k_T^2}{\langle k_T^2 \rangle}\right)$$

$f_1^g / h_1^{\perp g} (p_T \rightarrow 0) ? \Rightarrow |h_1^{\perp g}| \leq f_1^g$

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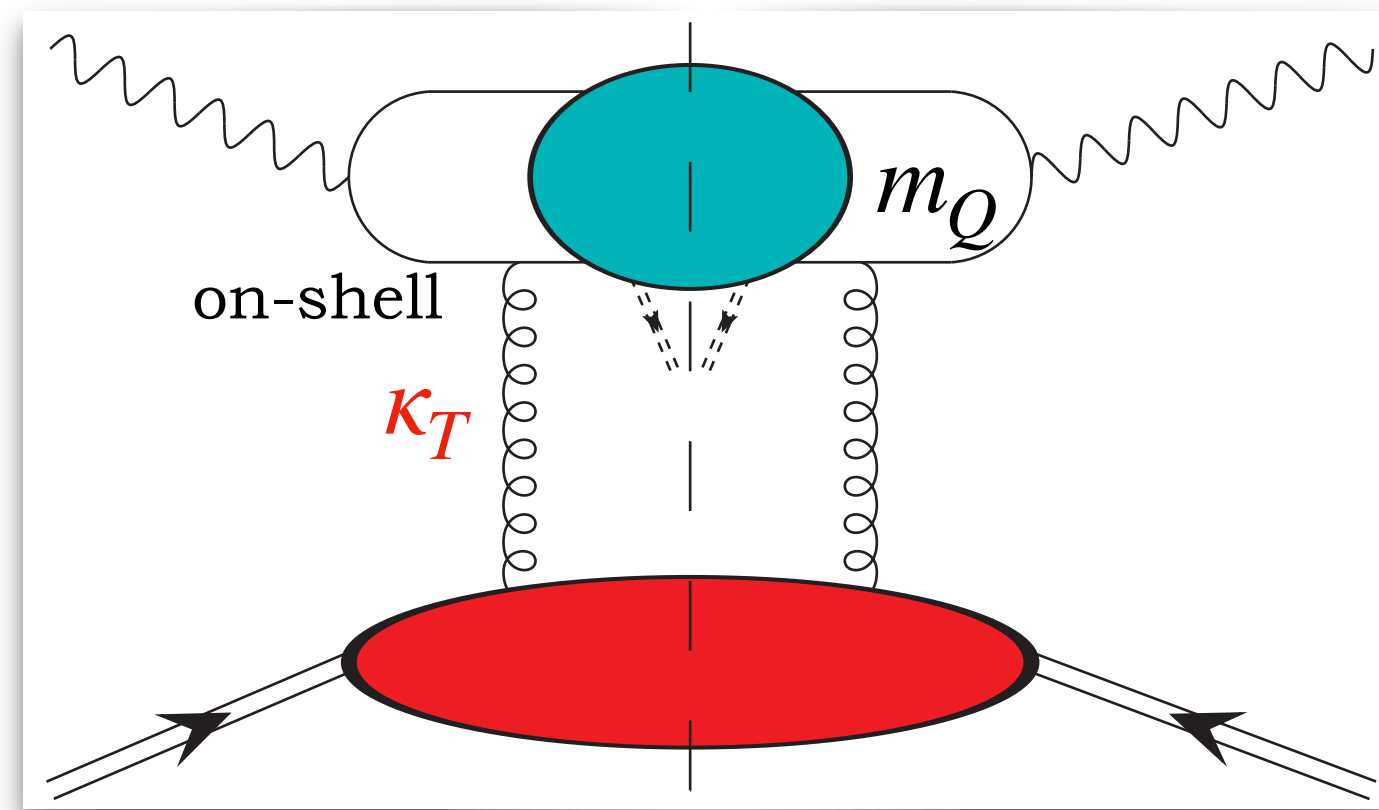
[F. Scarpa et al. (2020)]

[Today's talk by Alice]

[Model-dependent fit on 13 TeV LHCb data]

Inclusive quarkonium production mechanisms

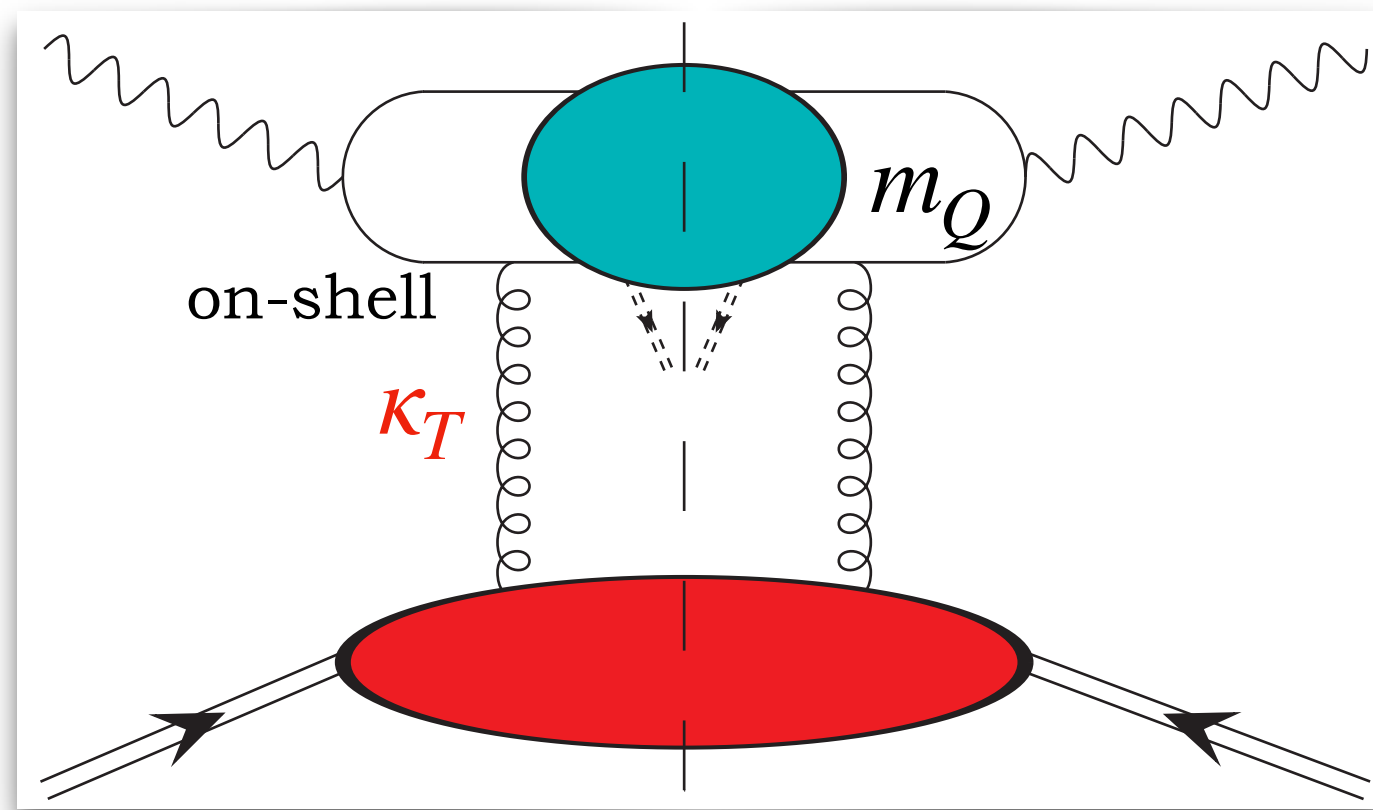
$$\kappa_T \ll Q$$



- * Gluon TMD PDF
- * Short-distance ($Q\bar{Q}$) + ShFs

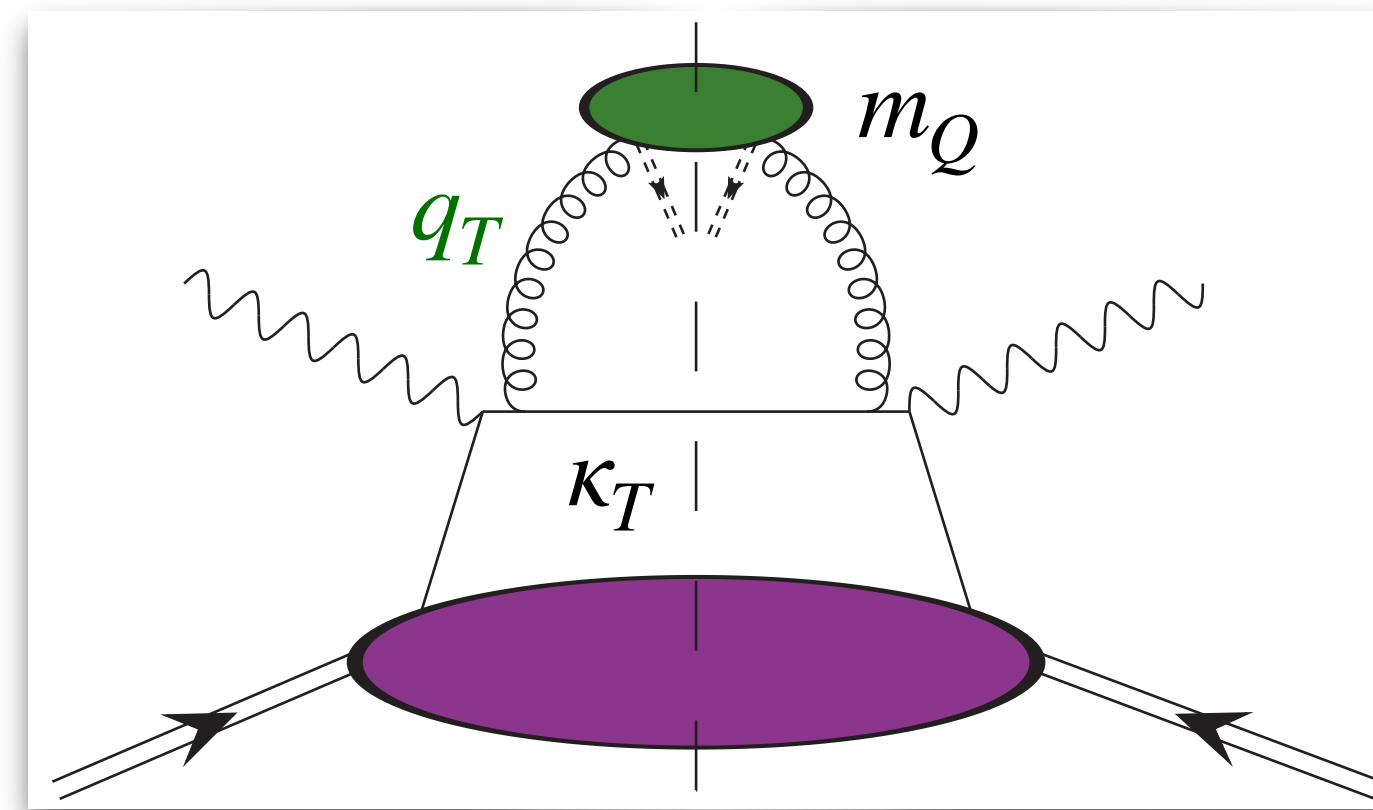
Inclusive quarkonium production mechanisms

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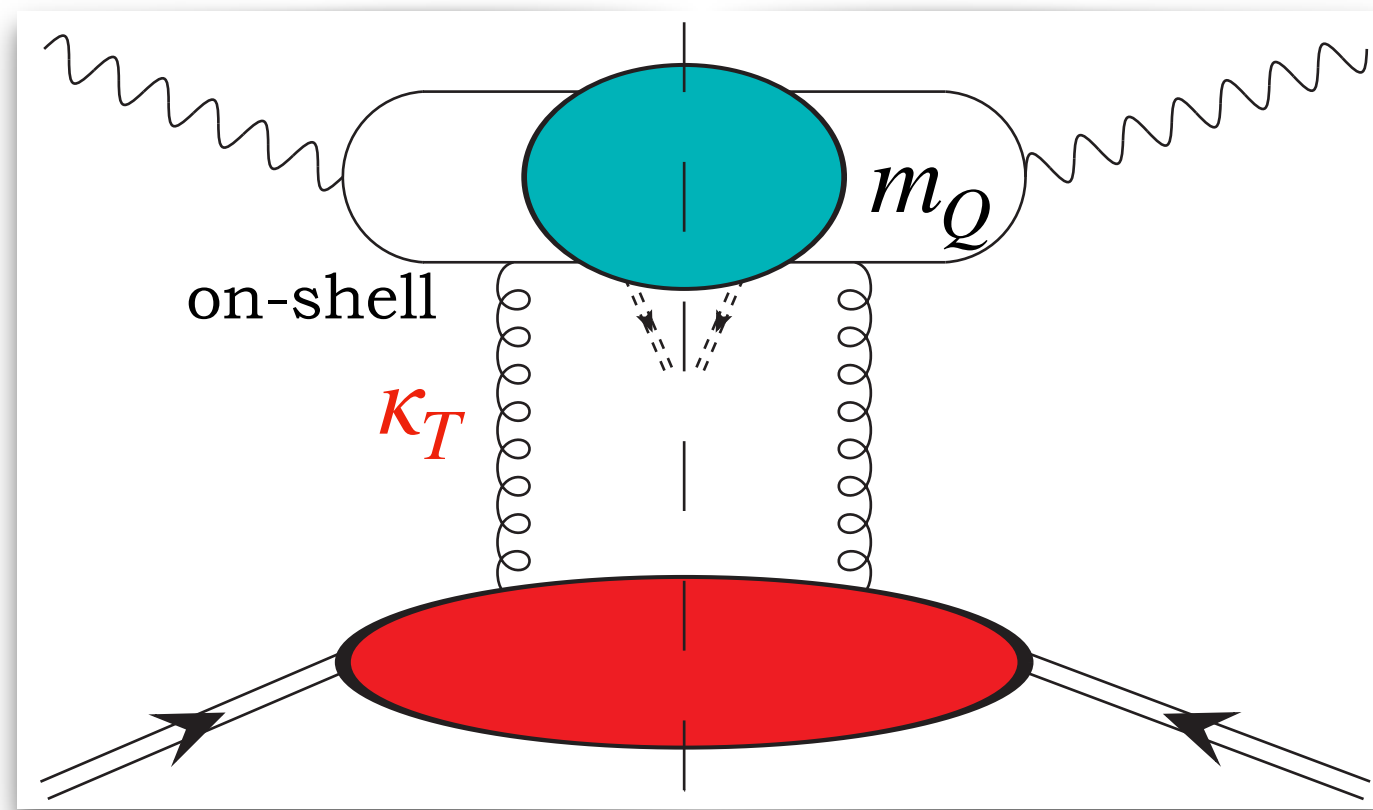
$$\kappa_T \gg m_Q$$



- * Quark collinear PDF
- * Onium in jet
- * Single-quark TMD FF

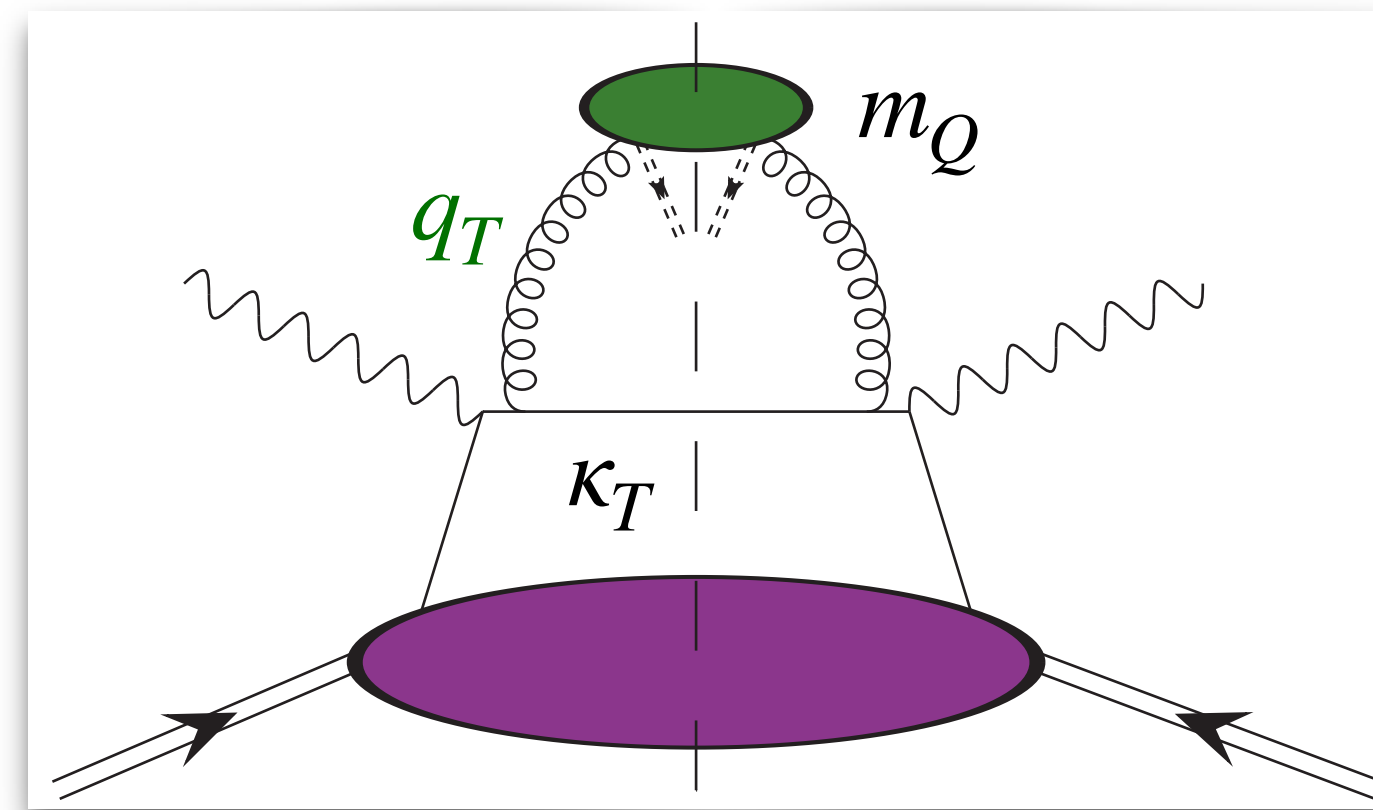
Inclusive quarkonium production mechanisms

$$\kappa_T \ll Q$$



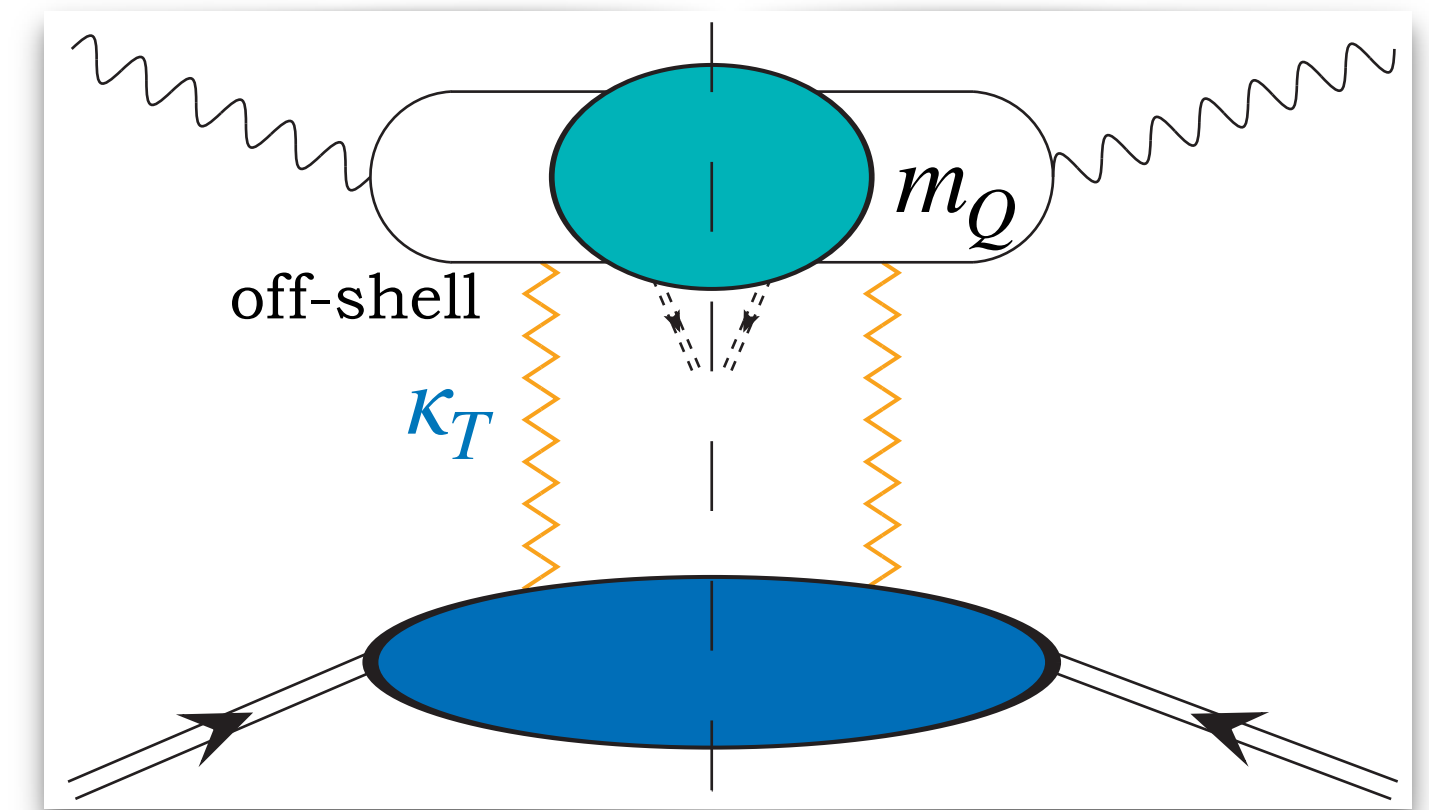
- * Gluon TMD PDF
- * Short-distance ($Q\bar{Q}$) + ShFs

$$\kappa_T \gg m_Q$$



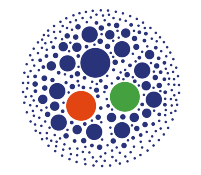
- * Quark collinear PDF
- * Onium in jet
- * Single-quark TMD FF

HEF

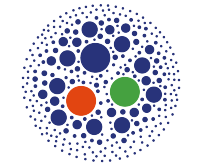


- * BFKL UGD
- * Reggeized gluons
- * Dipole mechanism

Gluon TMD PDFs @new-gen colliders: A win-win strategy

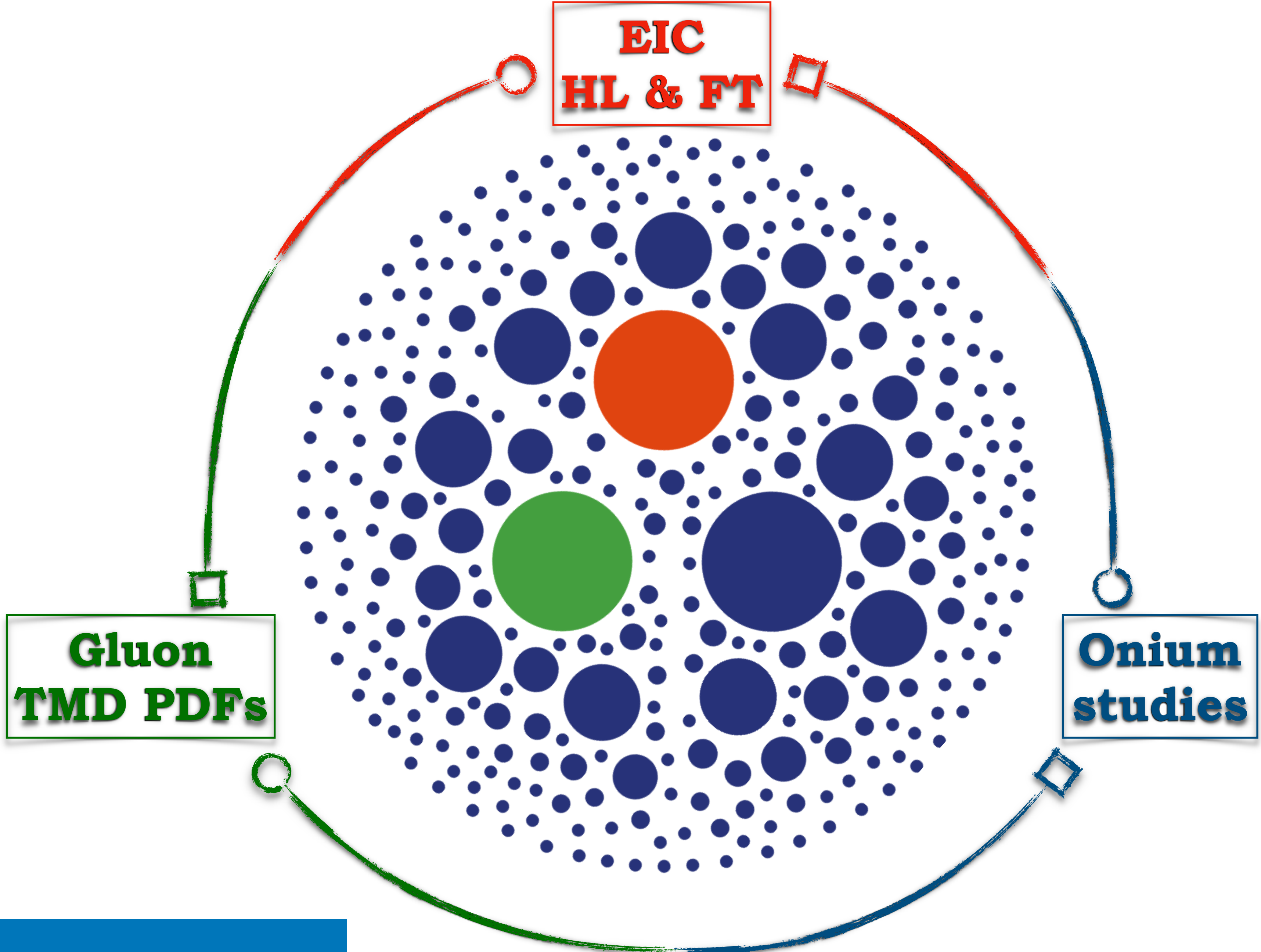


Onia as Tools \Rightarrow 3D proton imaging via gluon TMD PDFs

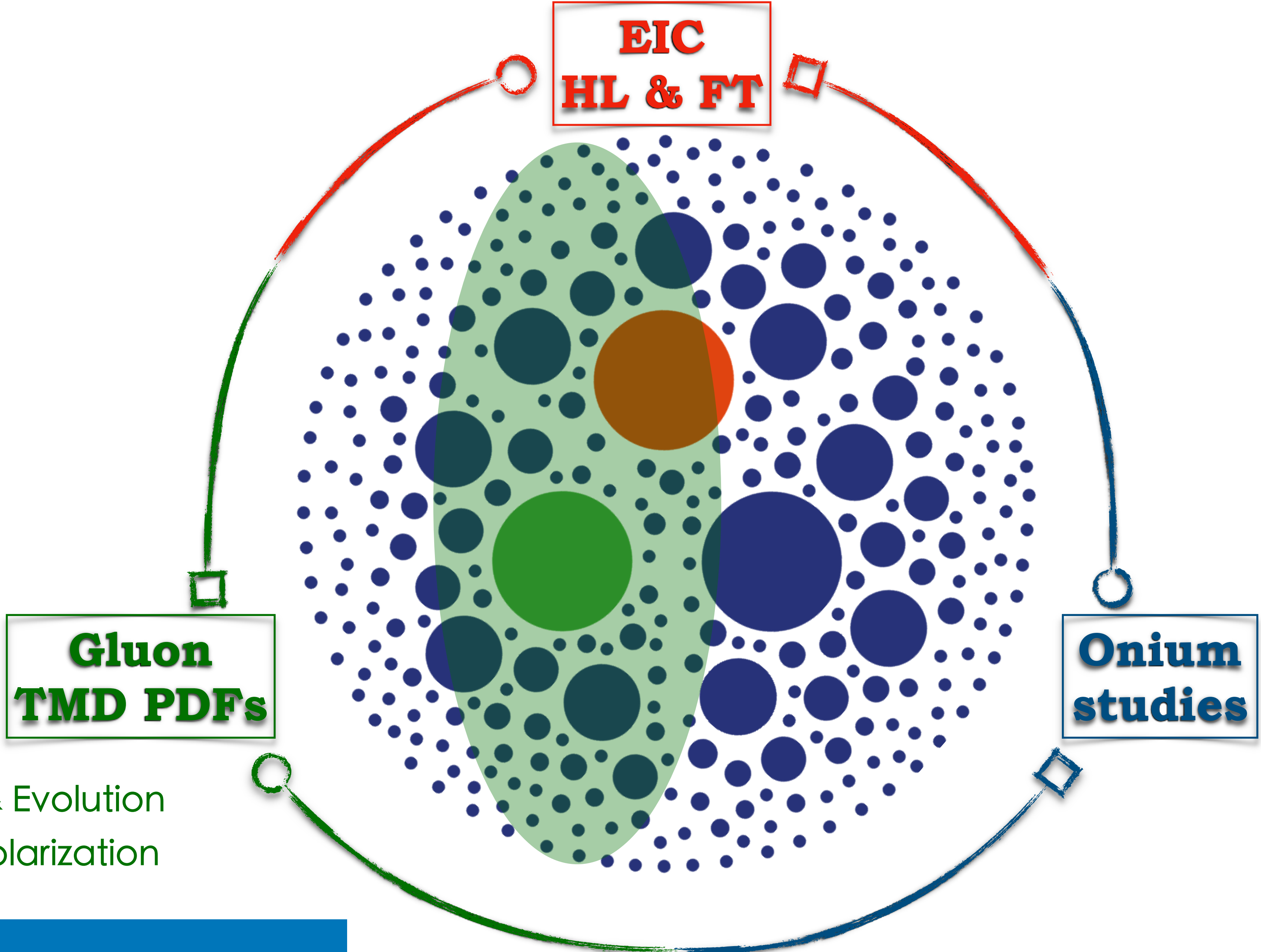


Gluon TMD PDFs as Tools \Rightarrow unveil onium production mechanisms

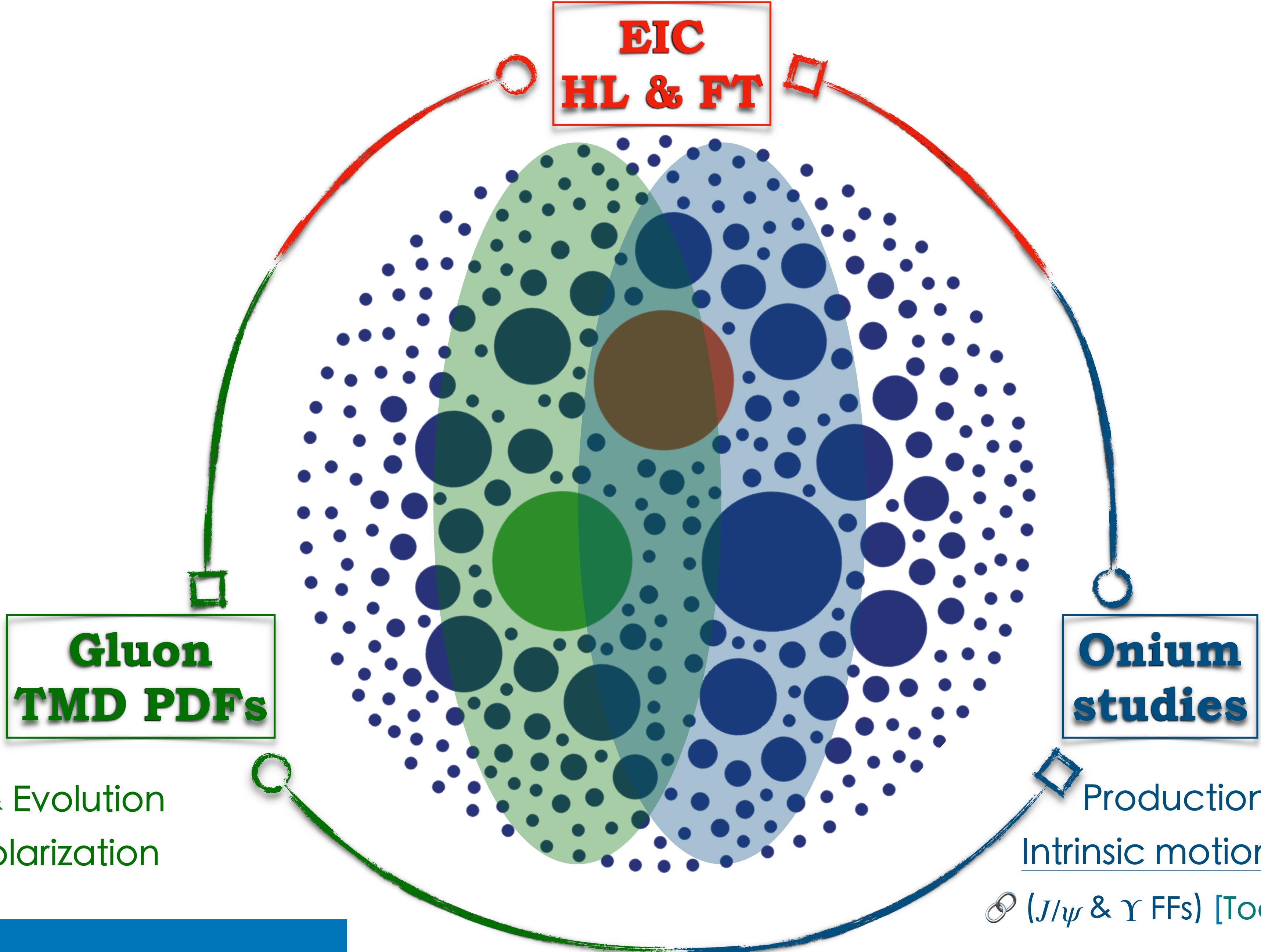
Gluon TMD PDFs @new-gen colliders: A win-win strategy



Gluon TMD PDFs @new-gen colliders: A win-win strategy



Gluon TMD PDFs @new-gen colliders: A win-win strategy



**EIC
HL & FT**

**Gluon
TMD PDFs**

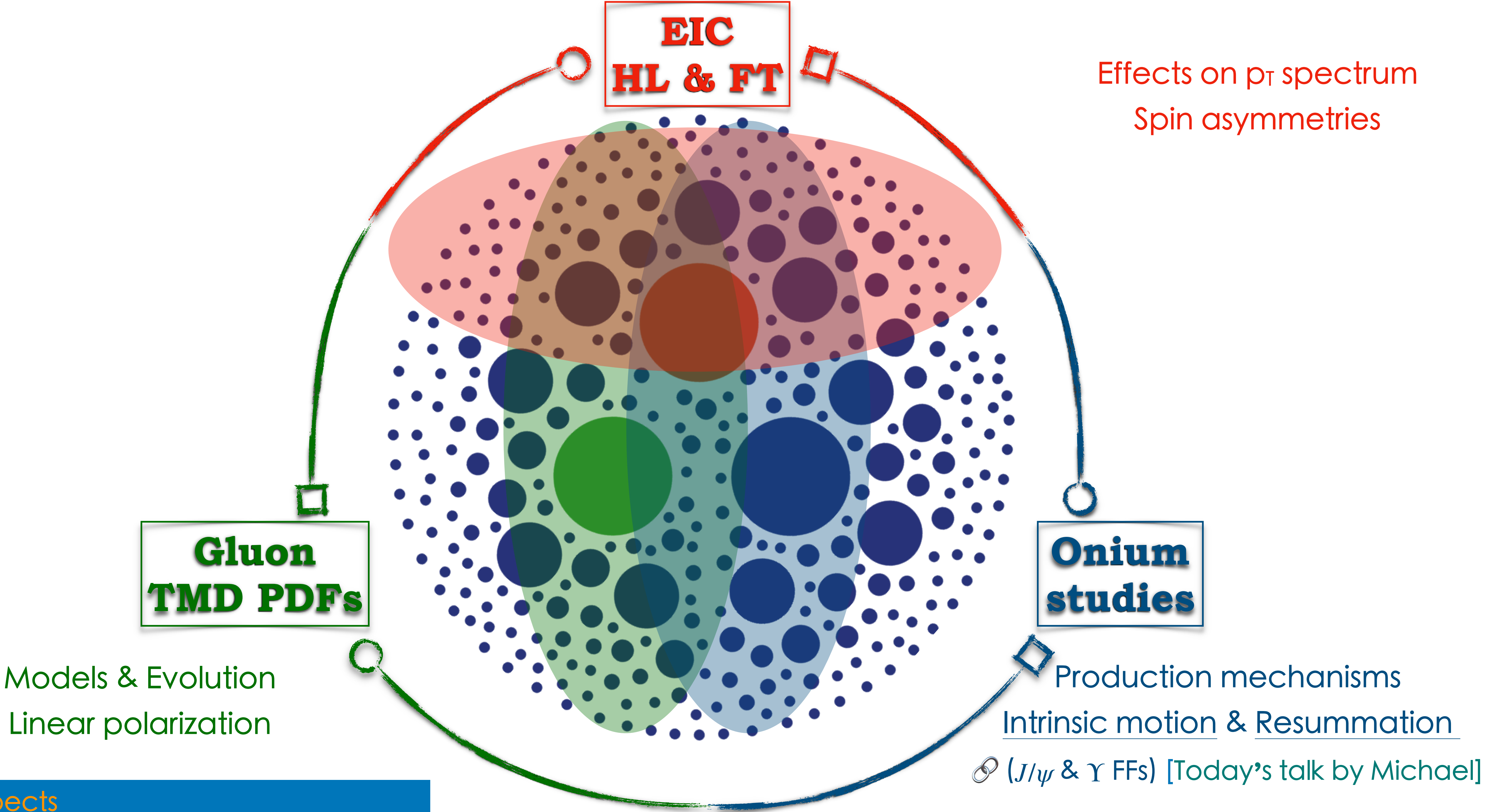
**Onium
studies**

Models & Evolution
Linear polarization

Production mechanisms
Intrinsic motion & Resummation

⊗ (J/ψ & Υ FFs) [Today's talk by Michael]

Gluon TMD PDFs @new-gen colliders: A win-win strategy

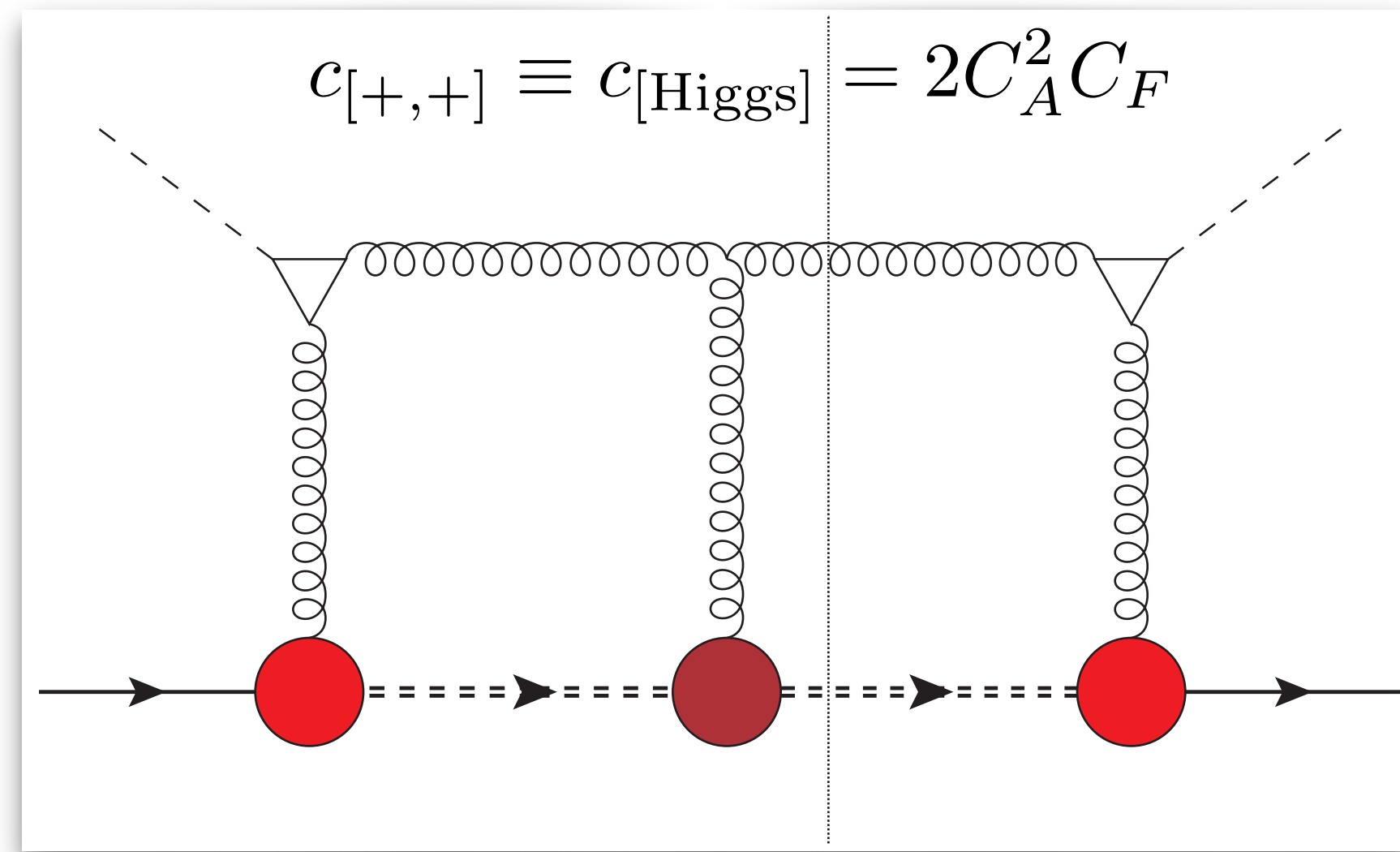




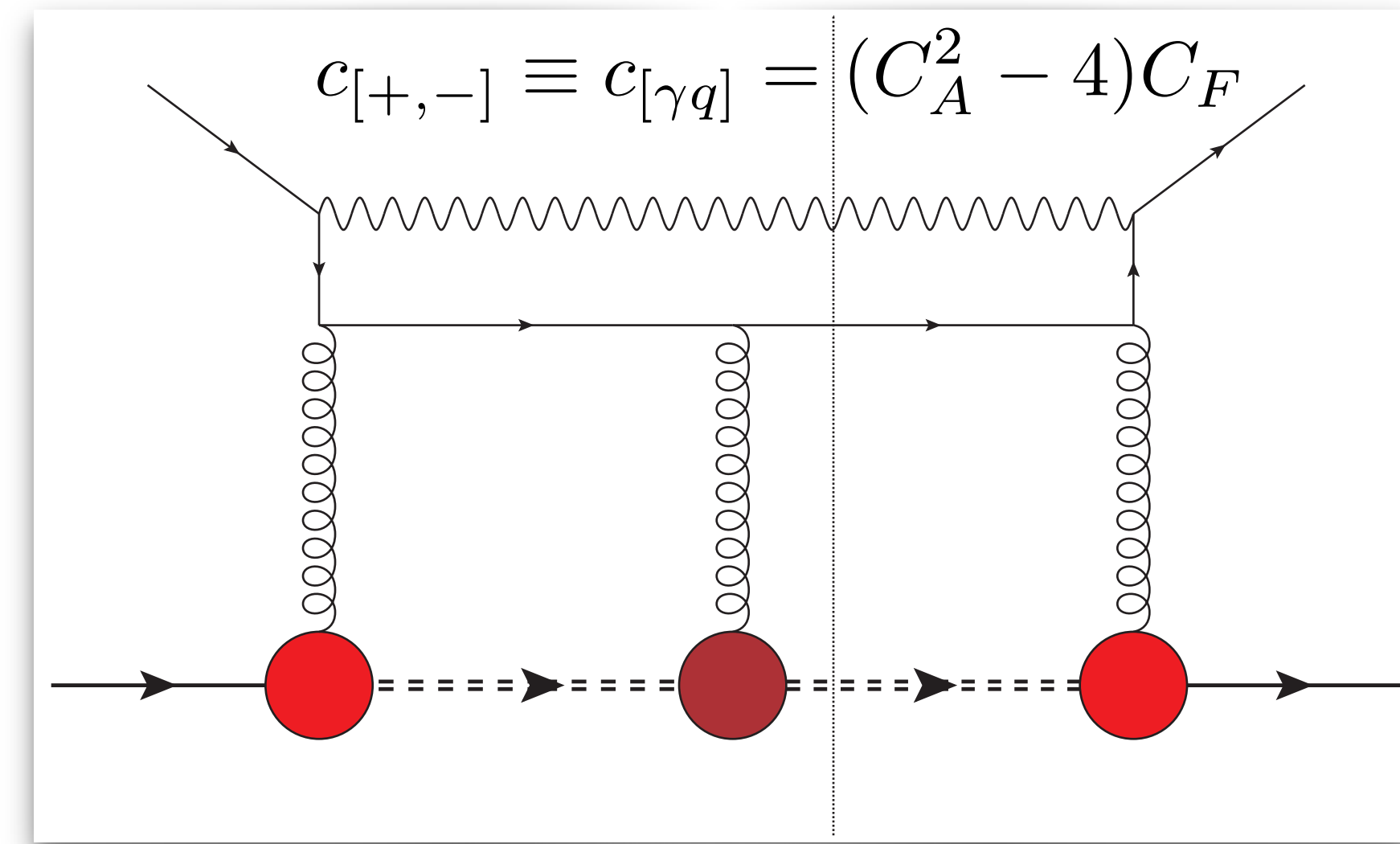
Extras

T-odd gluon TMDs in a spectator model

Higgs-gluon fusion \Rightarrow f-type $[+, +]$



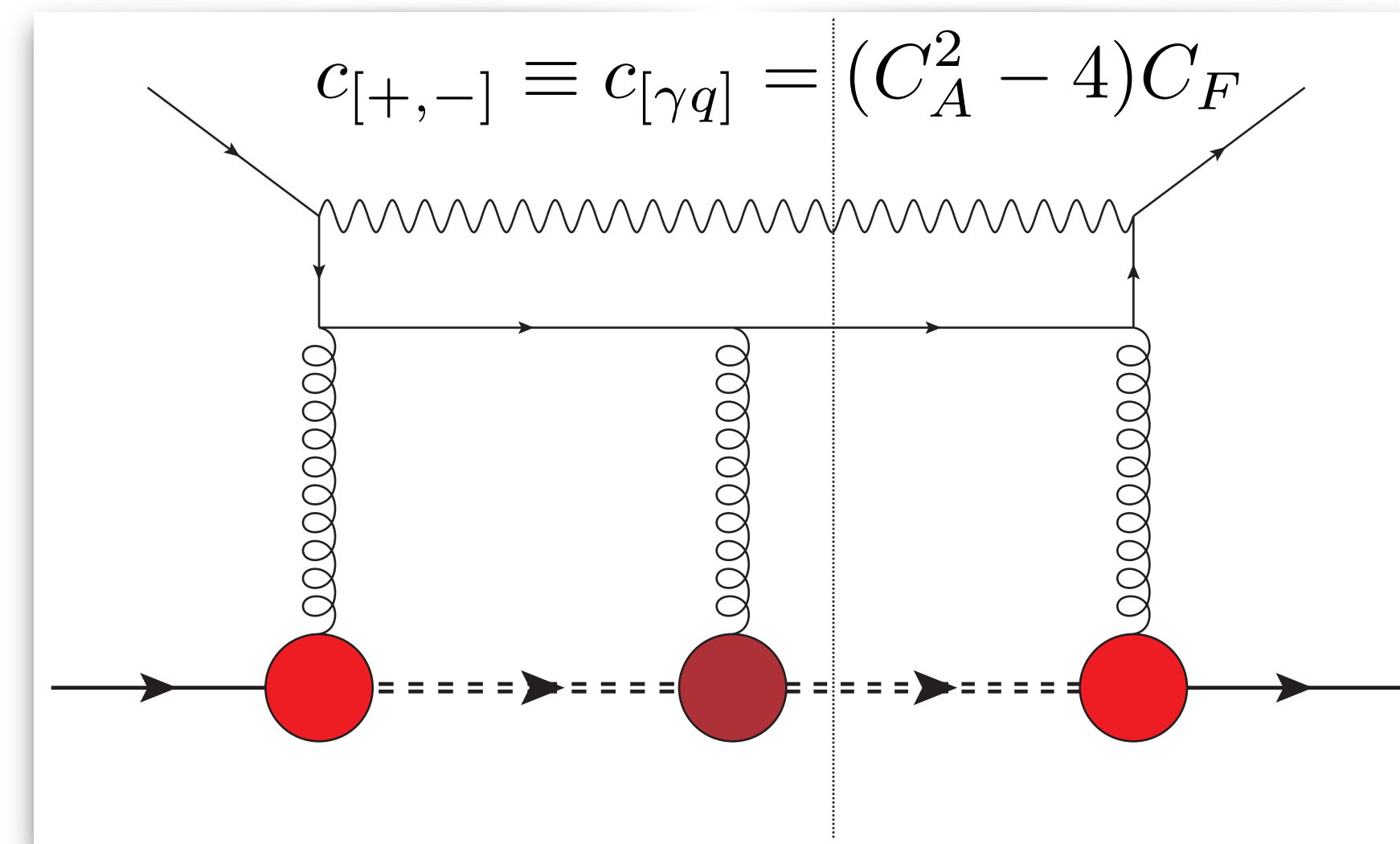
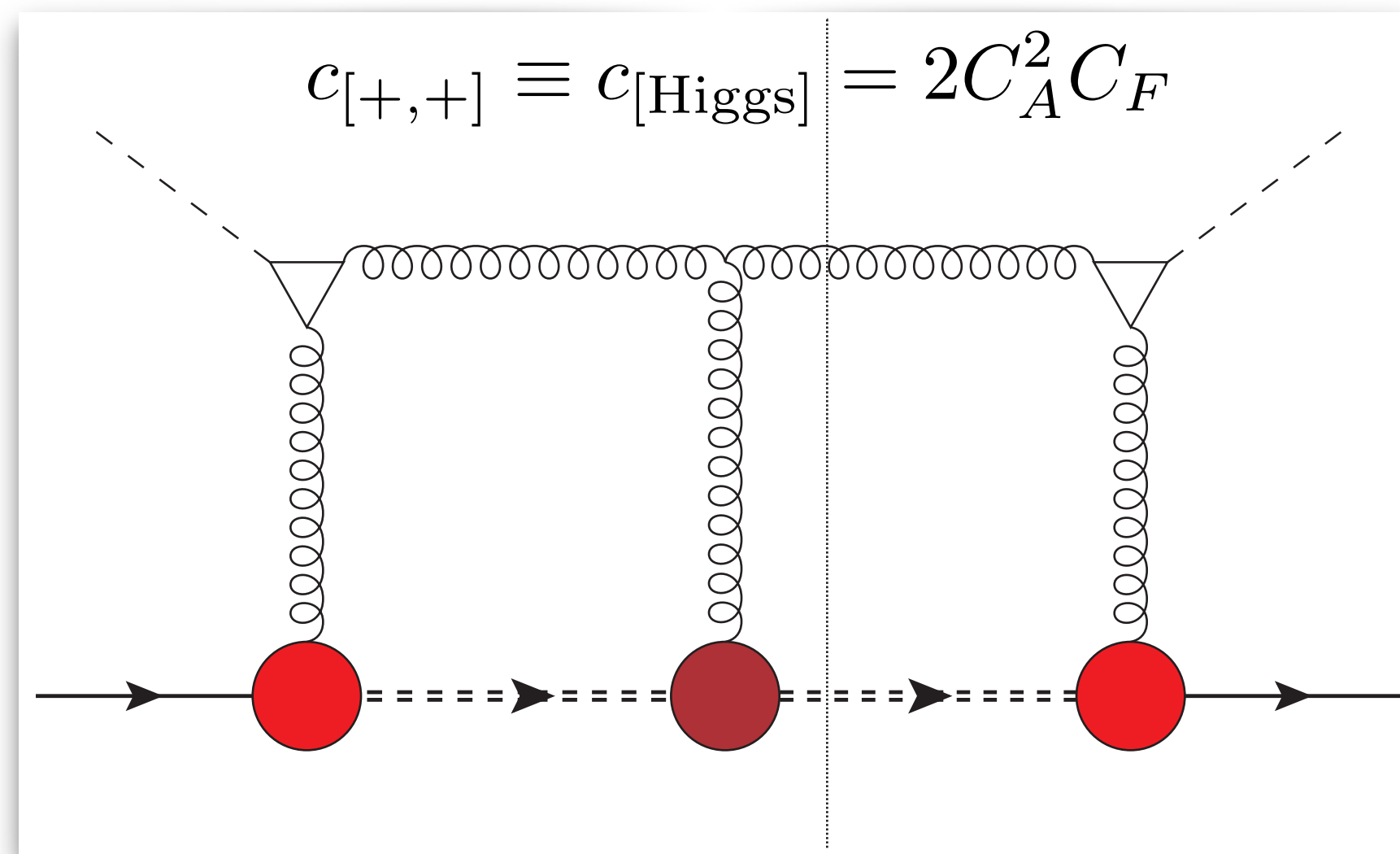
Photon-jet emission \Rightarrow d-type $[+, -]$

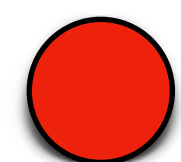


T-odd gluon TMDs in a spectator model

Higgs-gluon fusion \Rightarrow f-type [+ , +]

Photon-jet emission \Rightarrow d-type [+ , -]



 nucleon-gluon-spectator

 spectator-gluon-spectator

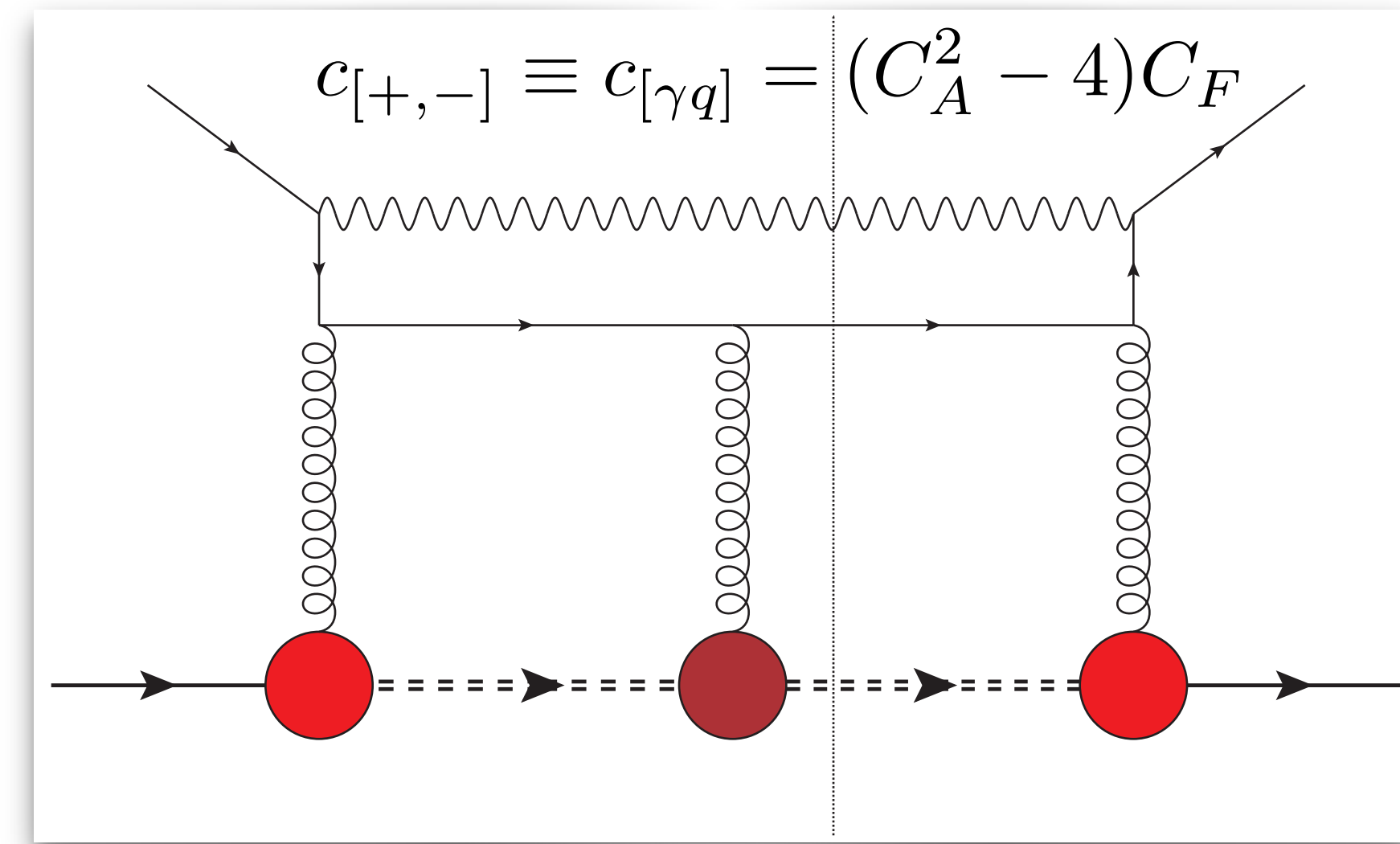
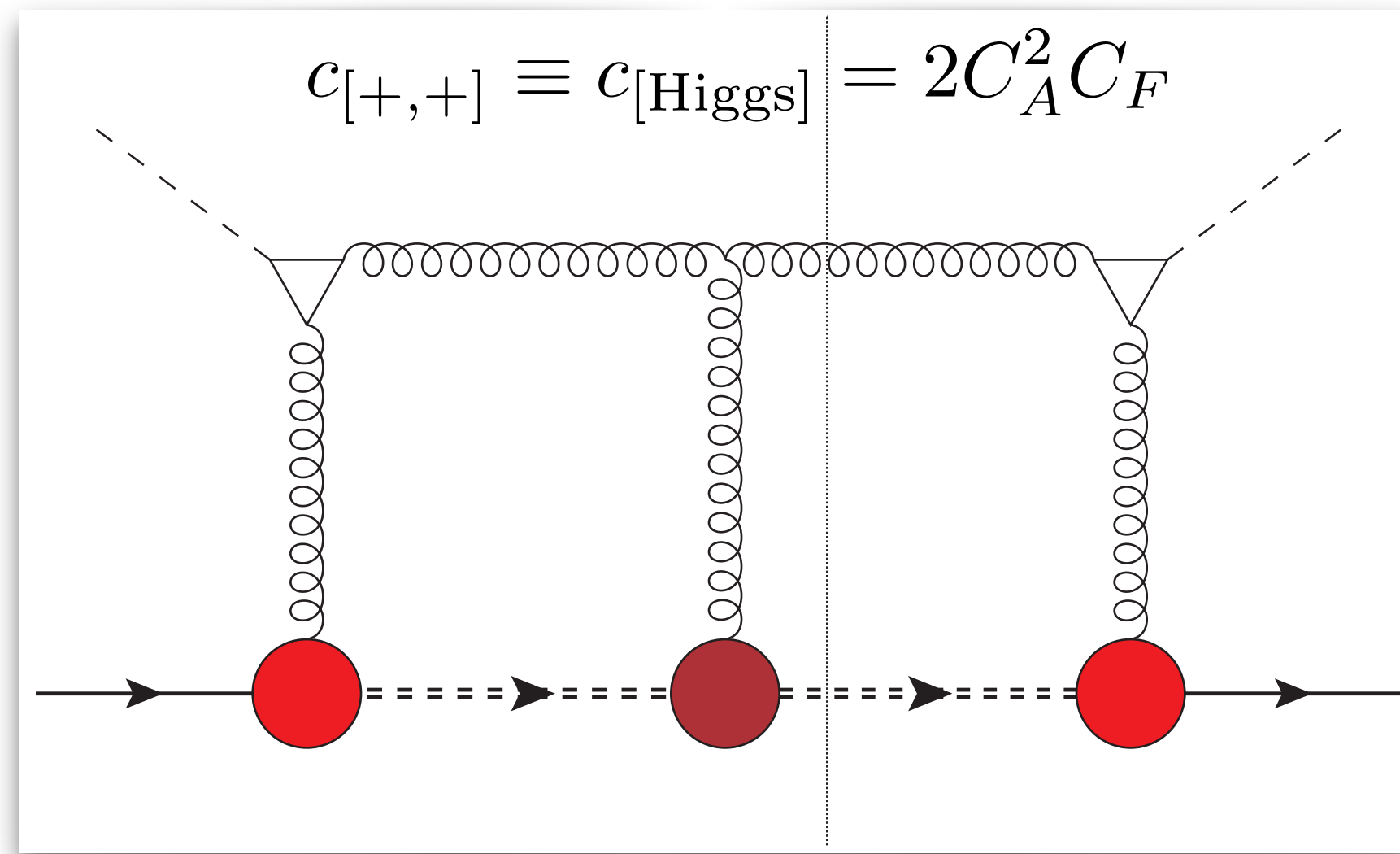
$$\mathcal{Y}_{bc}^\mu(p^2) = \delta_{bc} \left[g_1(p^2) \gamma_\mu + g_2(p^2) \frac{i}{2M} \sigma^{\mu\nu} p_\nu \right]$$

$$\mathcal{X}_{abc}^\mu(p^2) = f^{abc} \left[g_1^f(p^2) \gamma^\mu + g_2^f(p^2) \frac{i}{2M} \sigma^{\mu\nu} p_\nu \right] - i d^{abc} \left[g_1^d(p^2) \gamma^\mu + g_2^d(p^2) \frac{i}{2M} \sigma^{\mu\nu} p_\nu \right]$$

T-odd gluon TMDs in a spectator model

Higgs-gluon fusion \Rightarrow f-type $[+, +]$

Photon-jet emission \Rightarrow d-type $[+, -]$



 nucleon-gluon-spectator

 spectator-gluon-spectator

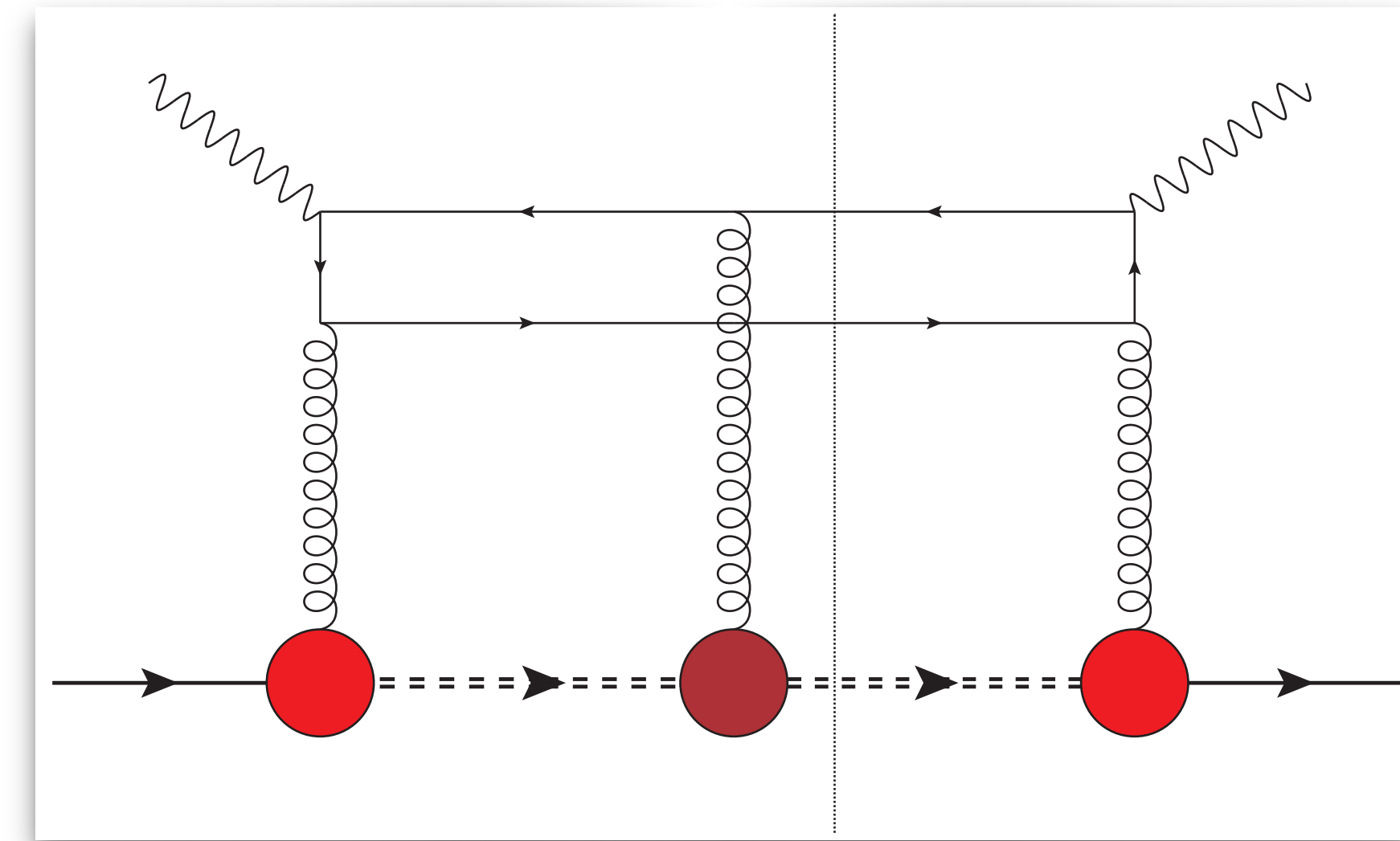
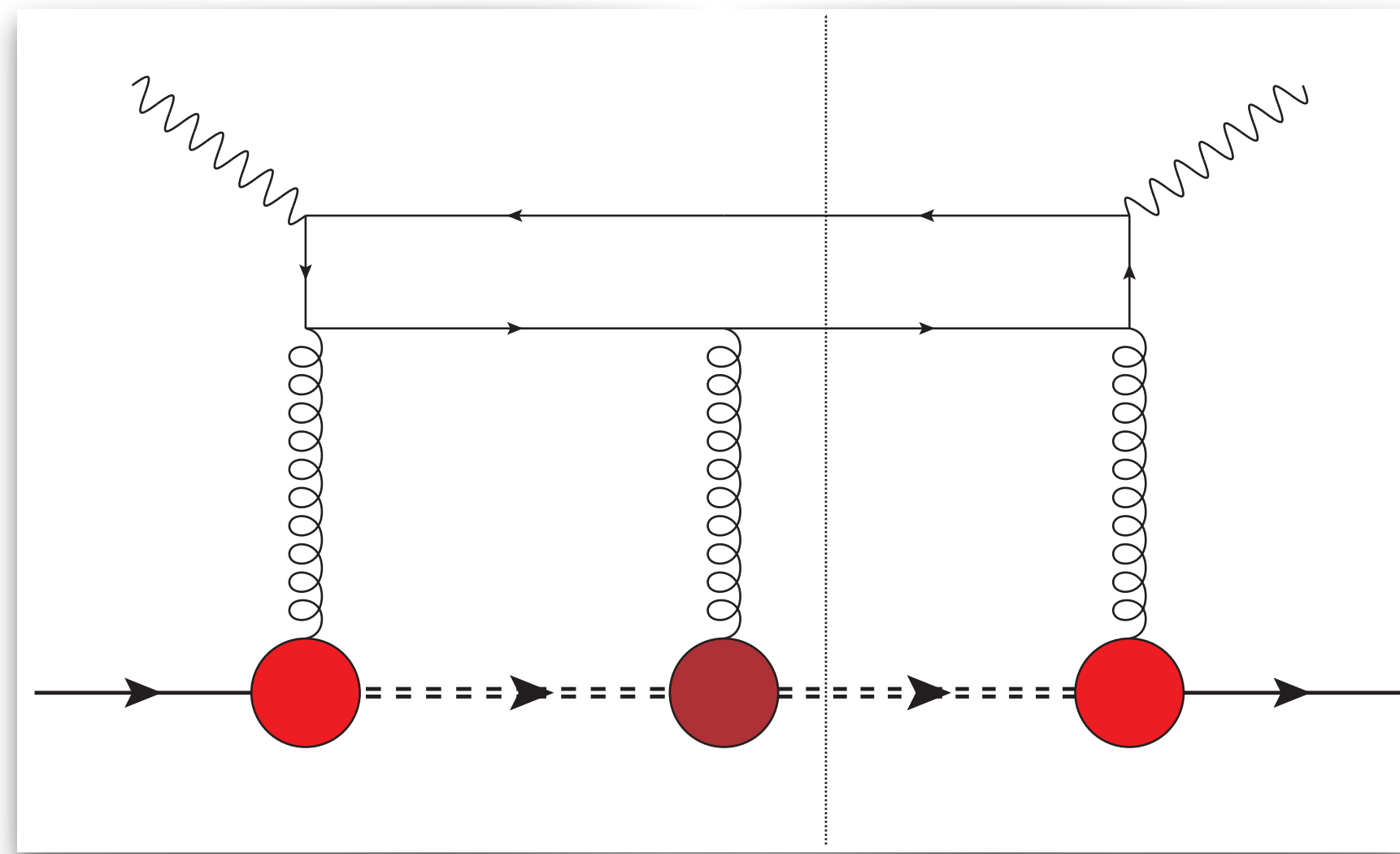
$$\mathcal{Y}_{bc}^\mu(p^2) = \delta_{bc} \left[g_1(p^2) \gamma_\mu + g_2(p^2) \frac{i}{2M} \sigma^{\mu\nu} p_\nu \right]$$

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Assumption: $g_{1,2}^d(p^2) = g_{1,2}^f(p^2) \equiv g_{1,2}(p^2) \quad \Leftrightarrow \quad f_{1T}^\perp[+,-] = \frac{c_{[+,-]}}{c_{[+,+]}} f_{1T}^\perp[+,+] \equiv \frac{5}{18} f_{1T}^\perp[+,+]$

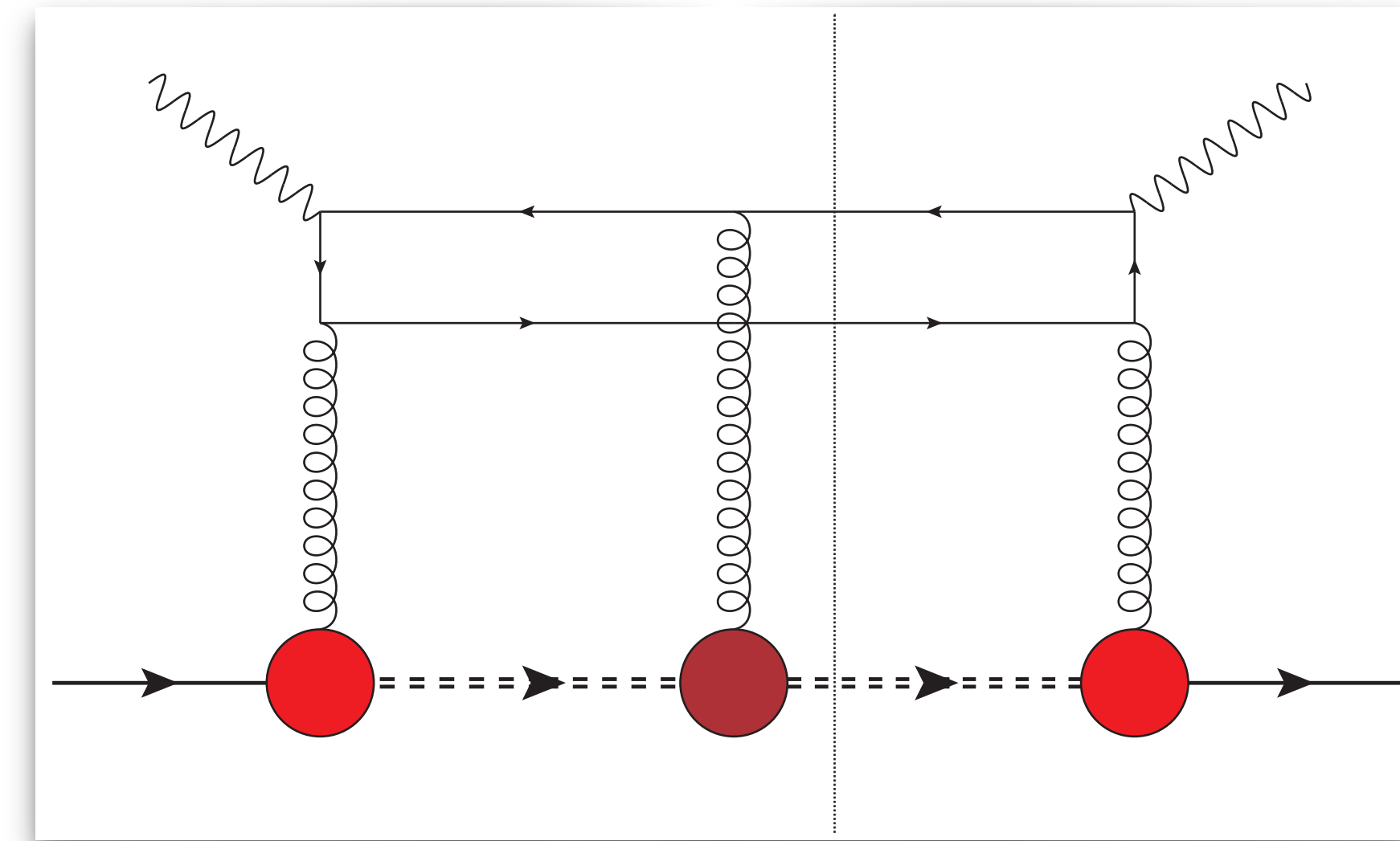
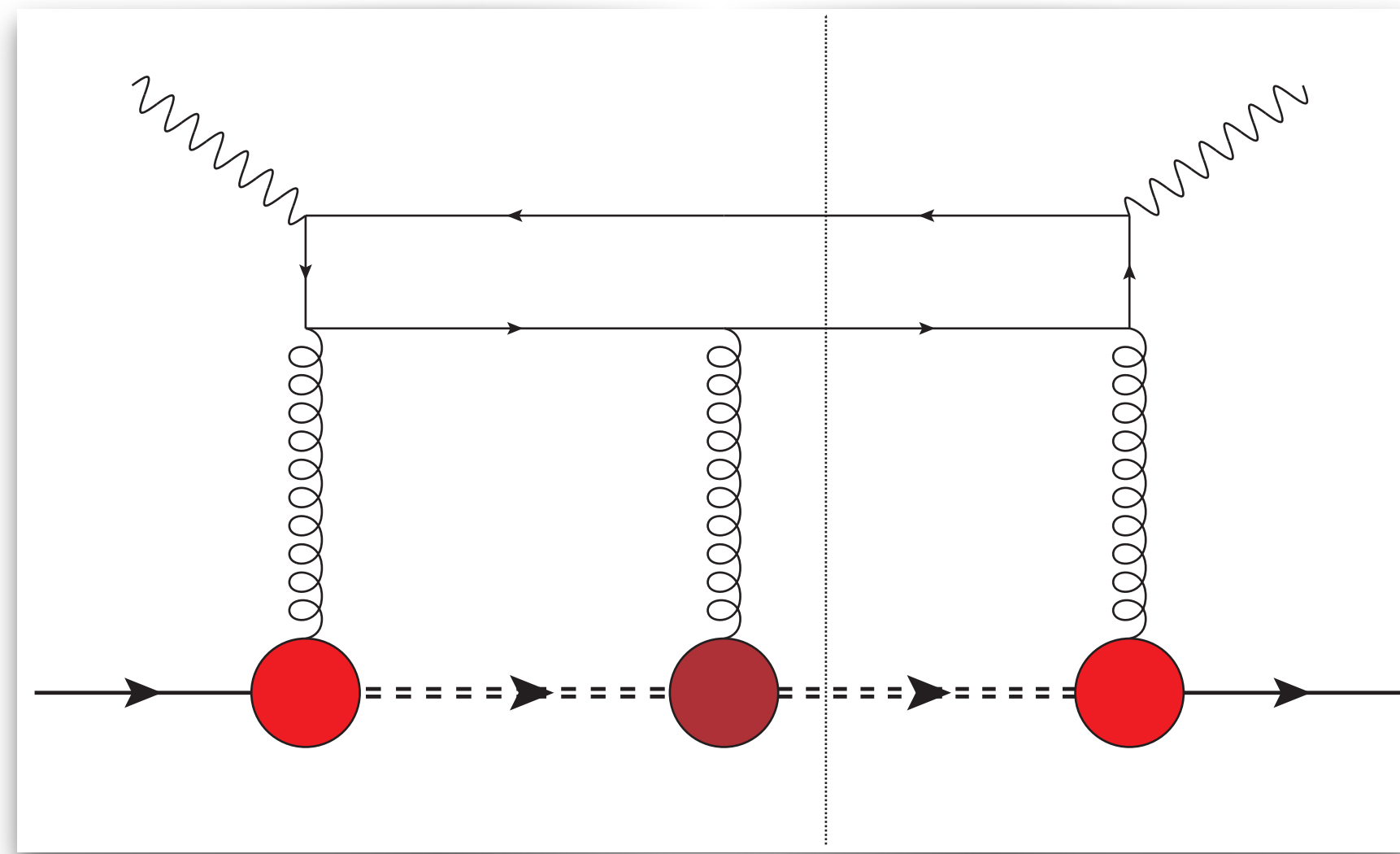
Analytic structure of T-odd gluon TMDs

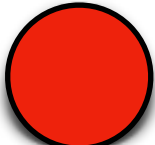
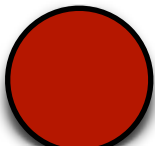
Two-jet SIDIS \Rightarrow f -type $[+, +]$



Analytic structure of T-odd gluon TMDs

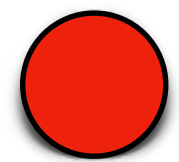
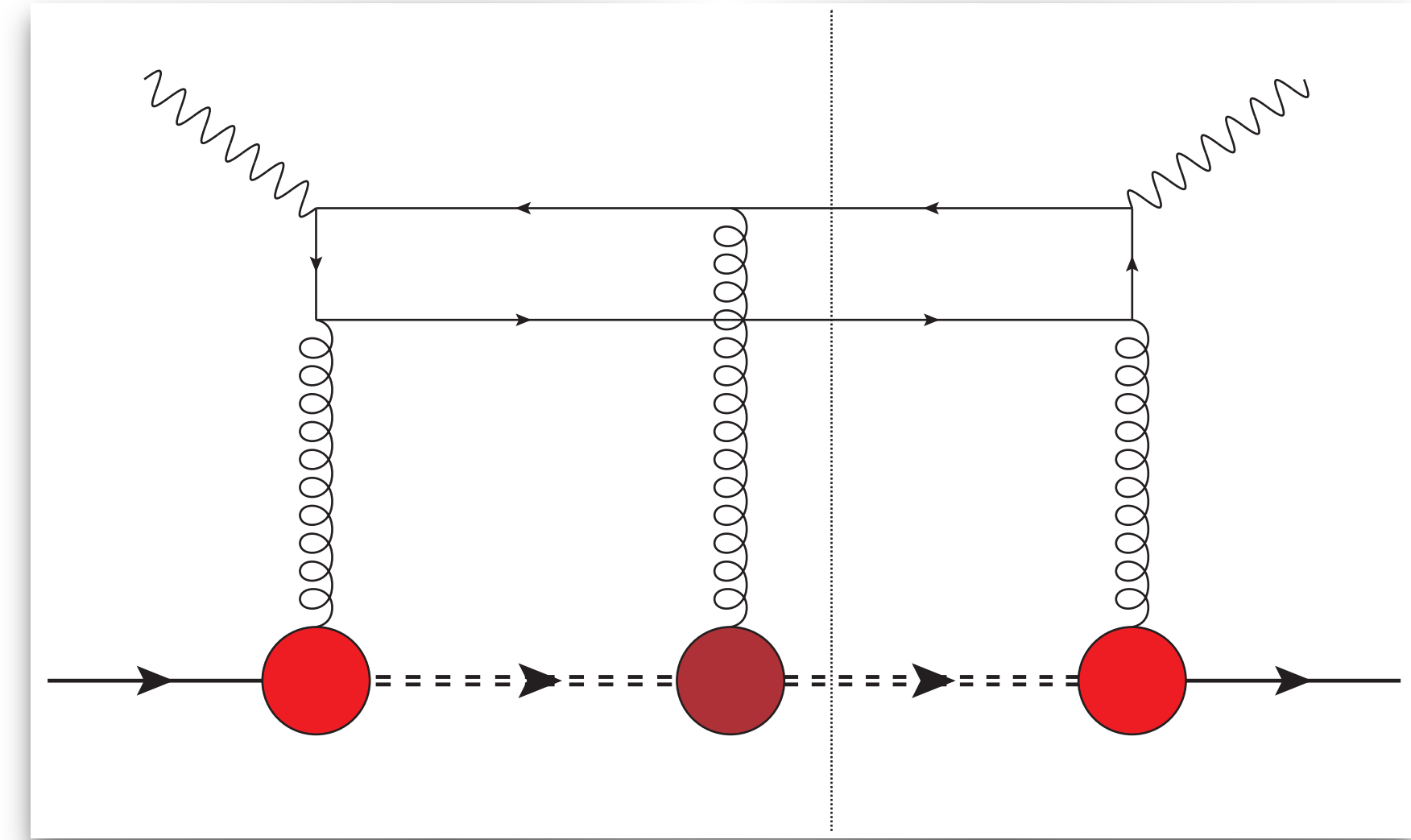
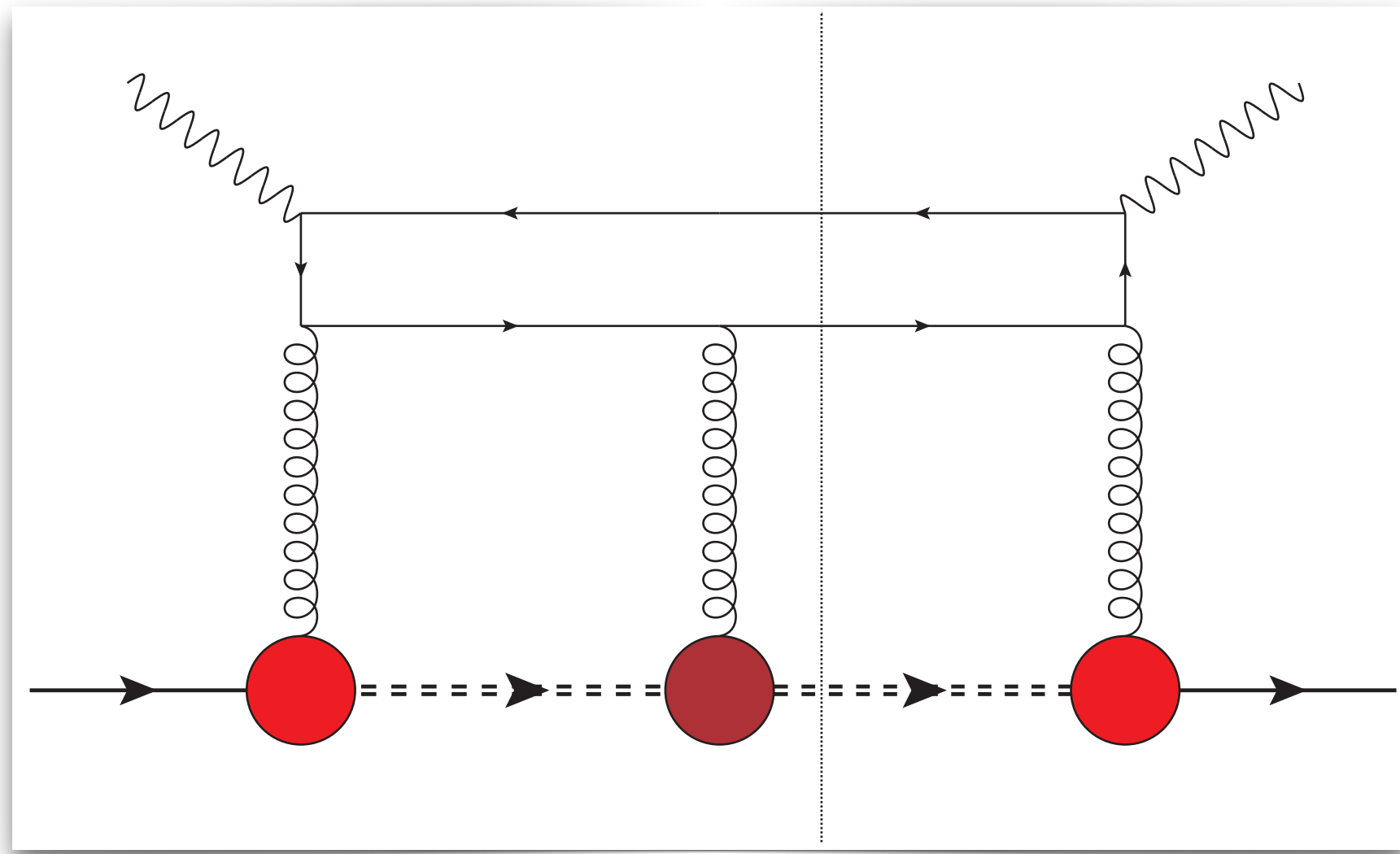
Two-jet SIDIS $\Rightarrow f$ -type [+ , +]



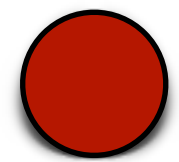
-  nucleon-gluon-spectator
-  spectator-gluon-spectator

Analytic structure of T-odd gluon TMDs

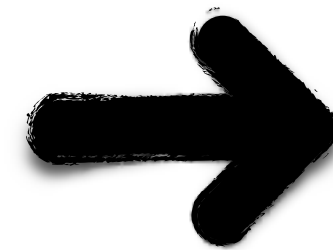
Two-jet SIDIS $\Rightarrow f$ -type [+ , +]



nucleon-gluon-spectator



spectator-gluon-spectator



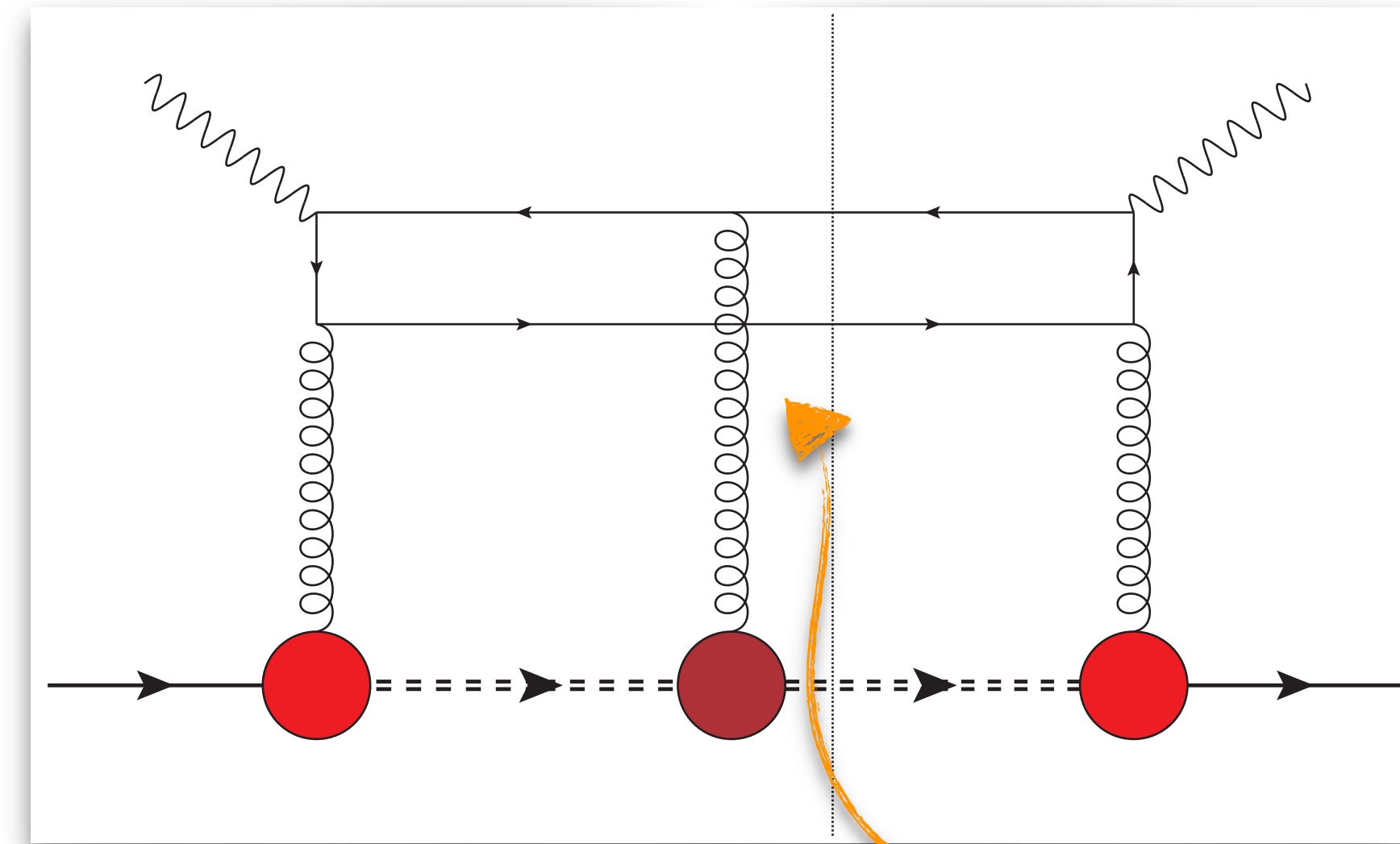
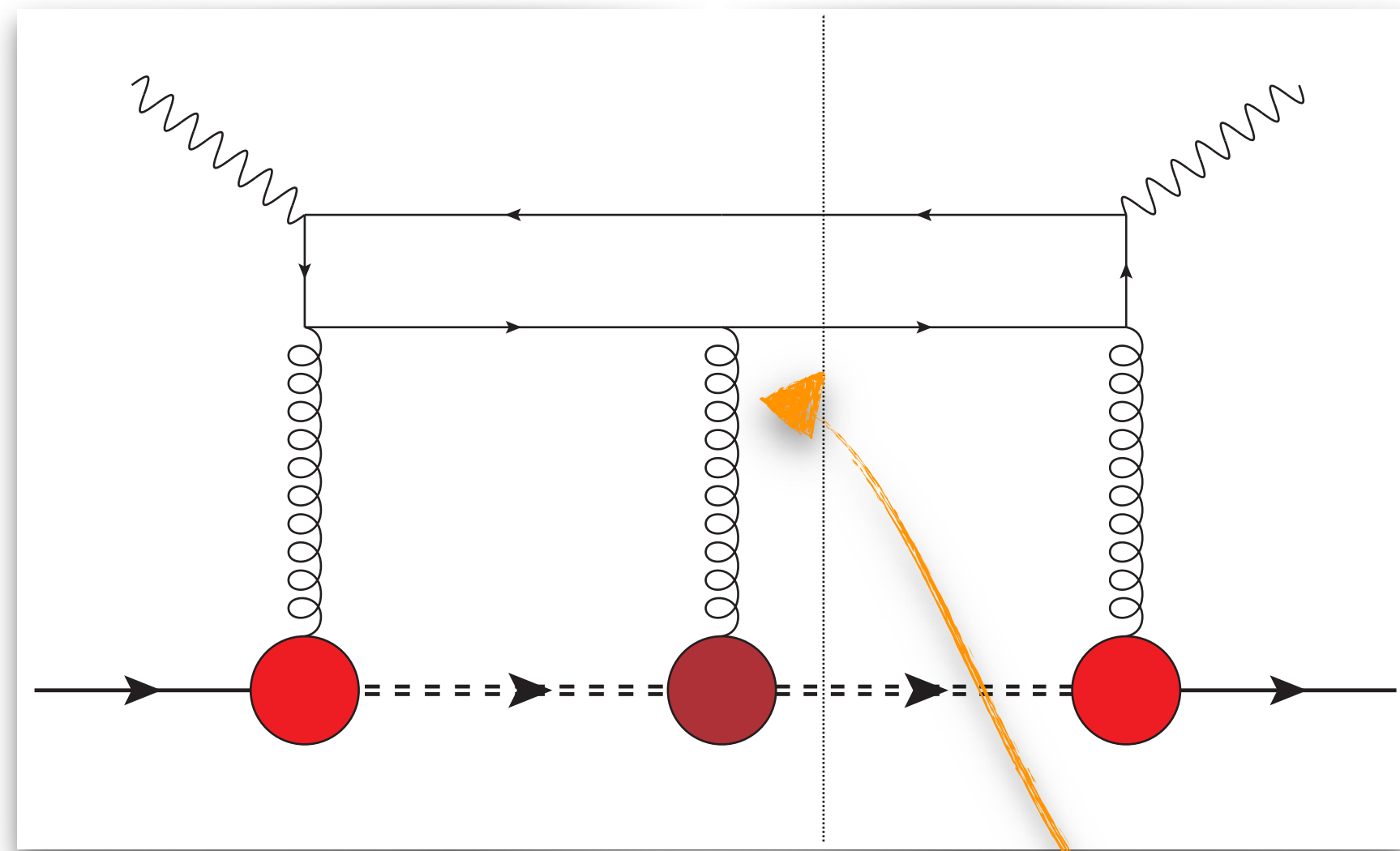
$$8 \times 7 \times 4$$

$$F(x, \mathbf{p}_T^2) = \sum_{i,j,k}^{1,2} C_{ijk}^{(F)}(x, \mathbf{p}_T^2) g_i(\mathbf{p}_T^2) g_j(\mathbf{p}_T^2) g_k(\mathbf{p}_T^2)$$

$$C_{ijk}^{(F)}(x, \mathbf{p}_T^2) = \sum_{l=1}^7 C_{ijk}^{(F),l}(x, \mathbf{p}_T^2) \mathcal{D}_l(x, \mathbf{p}_T^2)$$

Analytic structure of T-odd gluon TMDs

Two-jet SIDIS $\Rightarrow f$ -type [+ , +]



- nucleon-gluon-spectator
- spectator-gluon-spectator

➔

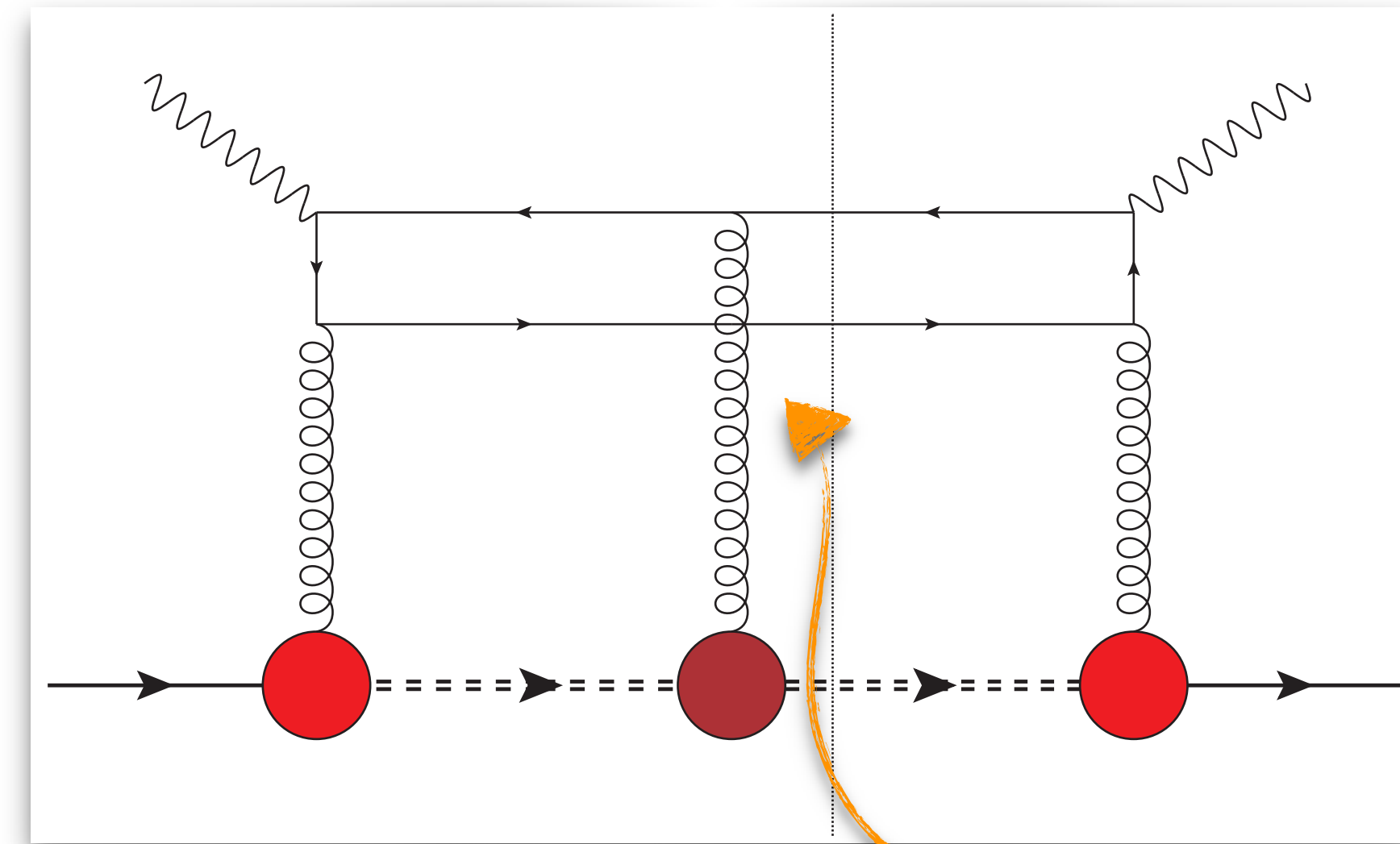
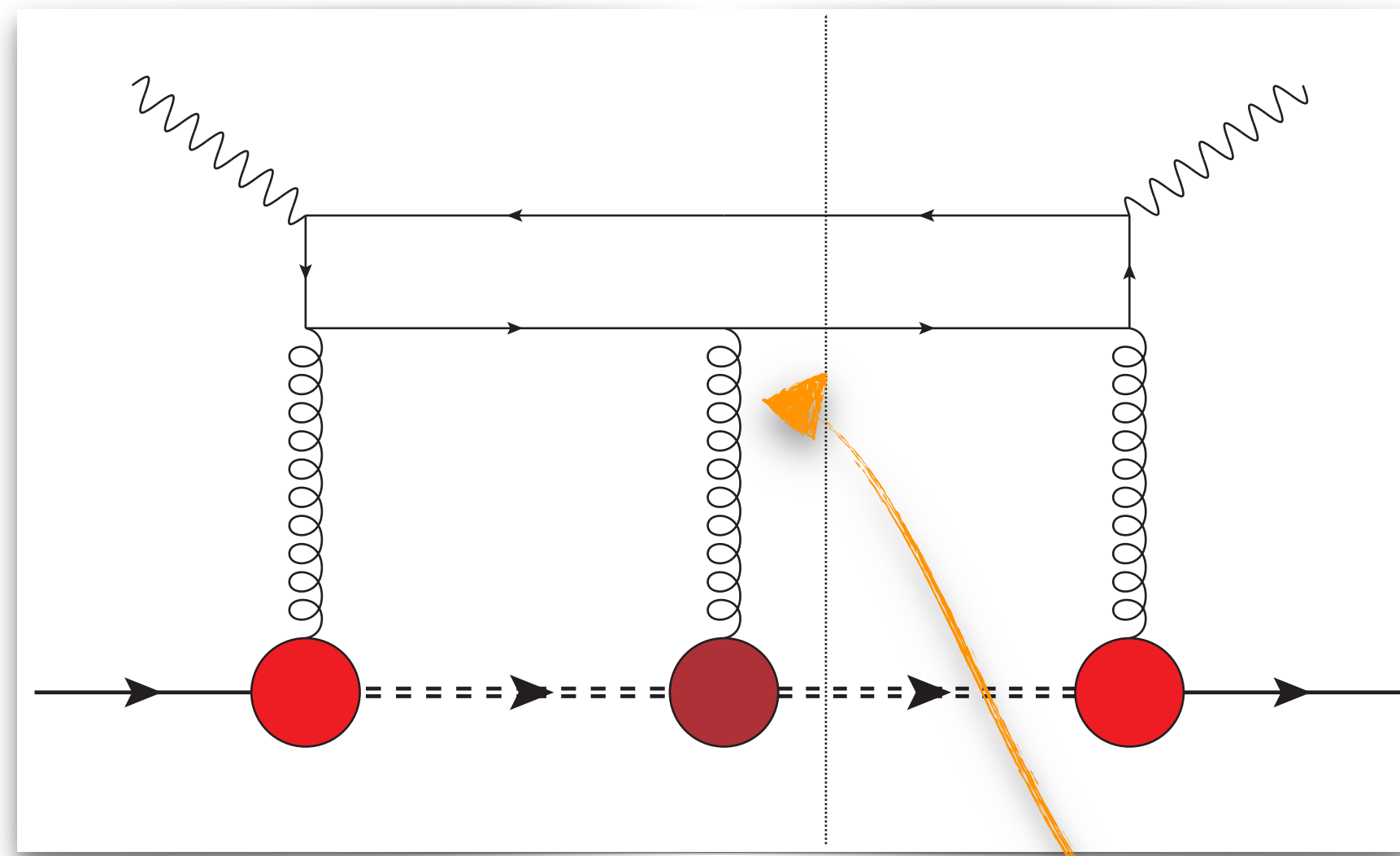
8 × 7 × 4

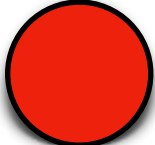
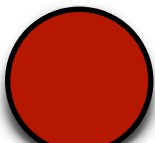
$$F(x, \mathbf{p}_T^2) = \sum_{i,j,k}^{1,2} C_{ijk}^{(F)}(x, \mathbf{p}_T^2) g_i(\mathbf{p}_T^2) g_j(\mathbf{p}_T^2) g_k(\mathbf{p}_T^2)$$

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Analytic structure of T-odd gluon TMDs

Two-jet SIDIS $\Rightarrow f$ -type [+ , +]



-  nucleon-gluon-spectator
-  spectator-gluon-spectator


8 × 7 × 4

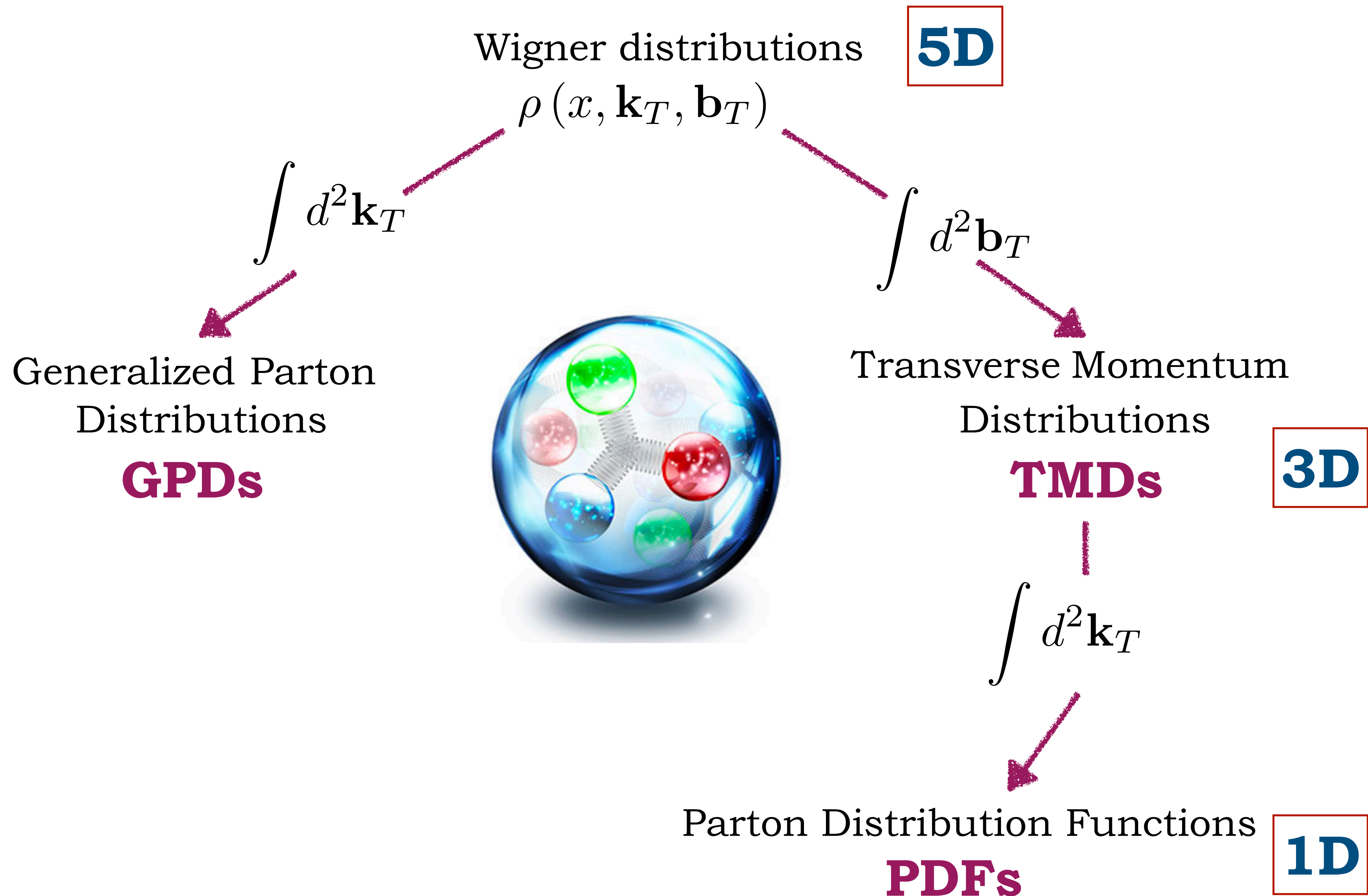
$$F(x, \mathbf{p}_T^2) = \sum_{i,j,k}^{1,2} C_{ijk}^{(F)}(x, \mathbf{p}_T^2) g_i(\mathbf{p}_T^2) g_j(\mathbf{p}_T^2) g_k(\mathbf{p}_T^2)$$

$$C_{ijk}^{(F)}(x, \mathbf{p}_T^2) = \sum_{l=1}^7 C_{ijk}^{(F),l}(x, \mathbf{p}_T^2) \mathcal{D}_l(x, \mathbf{p}_T^2)$$

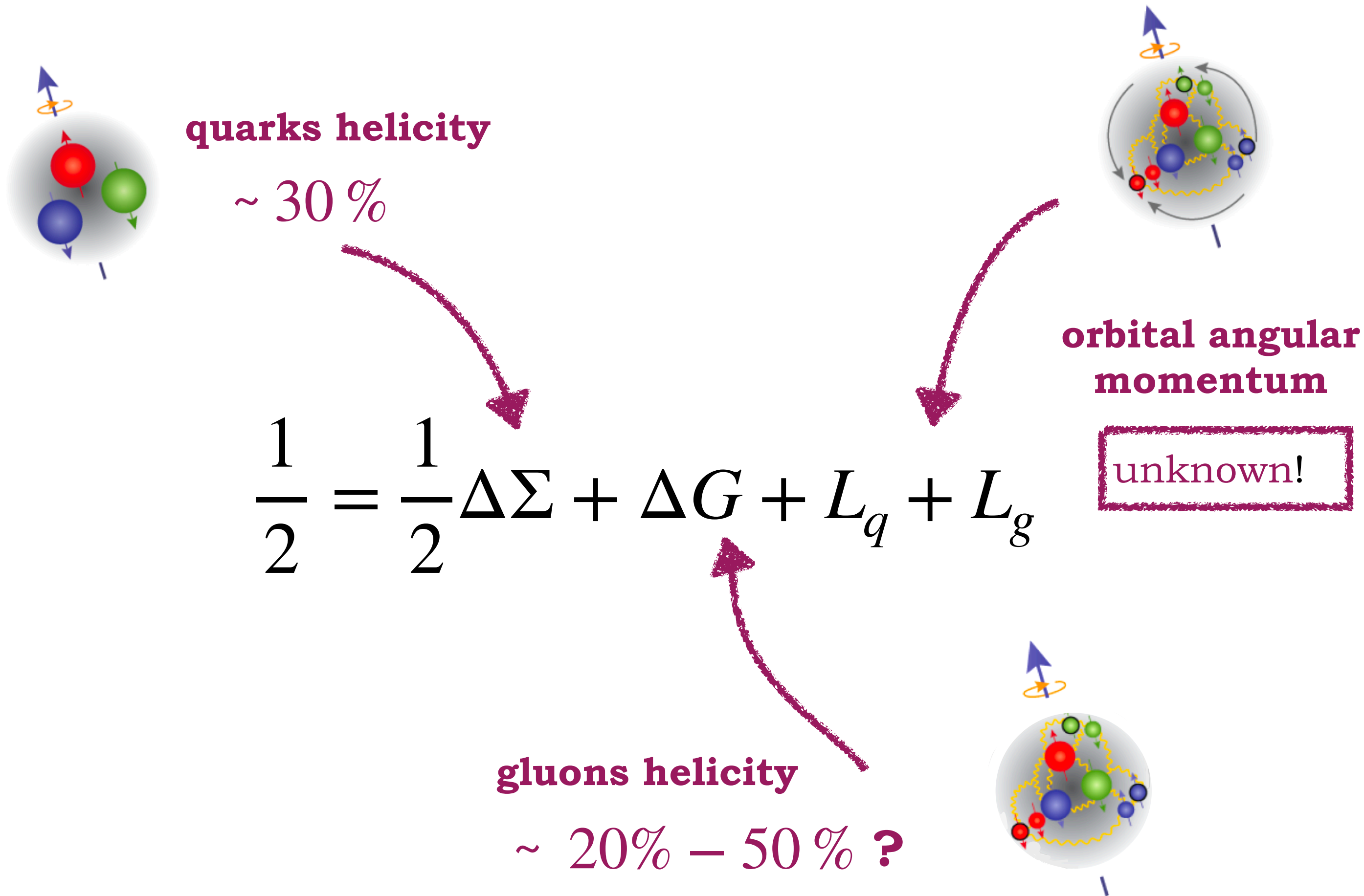


TMD factorization

Parton densities: an incomplete family tree



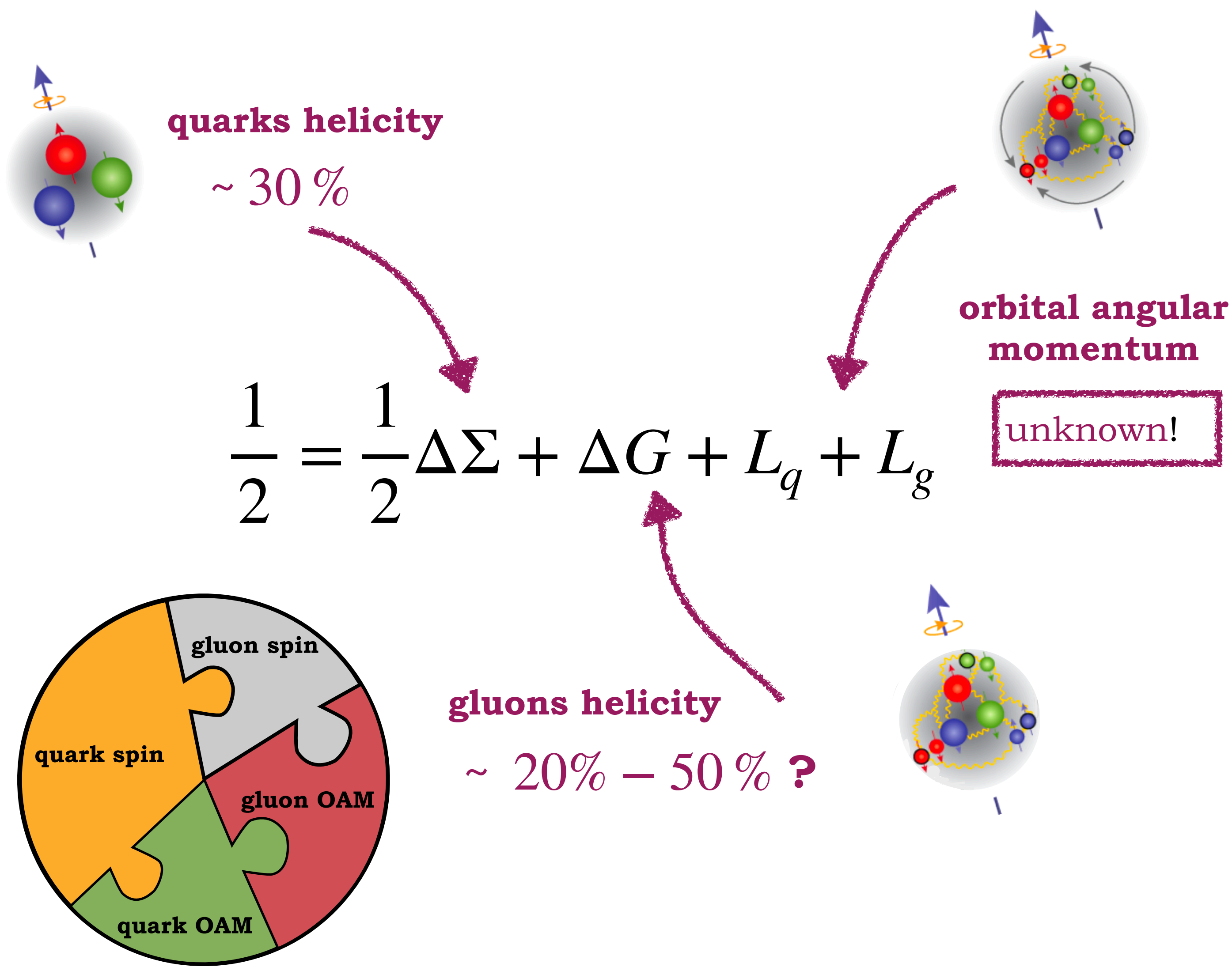
The proton spin crisis



Total spin carried by quarks and gluons does not amount to 1/2, one needs orbital angular momentum, then a 3D description...

(proton spin crisis) [EMC Collaboration, CERN (1987)]

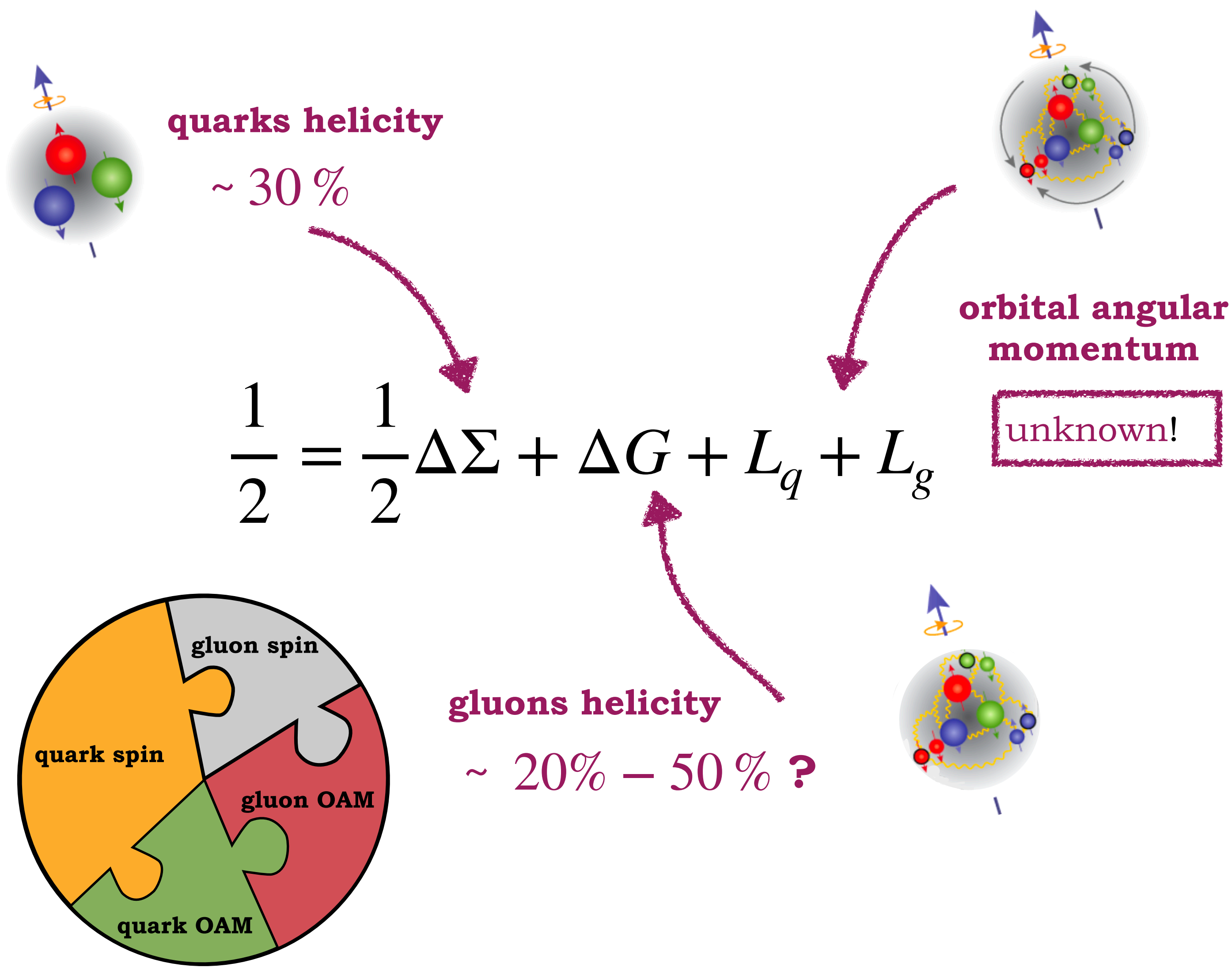
The proton spin crisis



Total spin carried by quarks and gluons does not amount to 1/2, one needs orbital angular momentum, then a 3D description...

(proton spin crisis) [EMC Collaboration, CERN (1987)]

The proton spin crisis



...many other effects in hadronic interactions cannot be understood in the purely collinear approach

Total spin carried by quarks and gluons does not amount to 1/2, one needs orbital angular momentum, then a 3D description...

(proton spin crisis) [EMC Collaboration, CERN (1987)]

Dihadron hadroproduction and factorization breaking

* Proof of factorization violation [\[T. J. Rogers, P. J. Mulders \(2010\)\]](#)

* Assumed factorization in SCET and CGC

* Significance of low- x studies

* Size of factorization-breaking effects small?

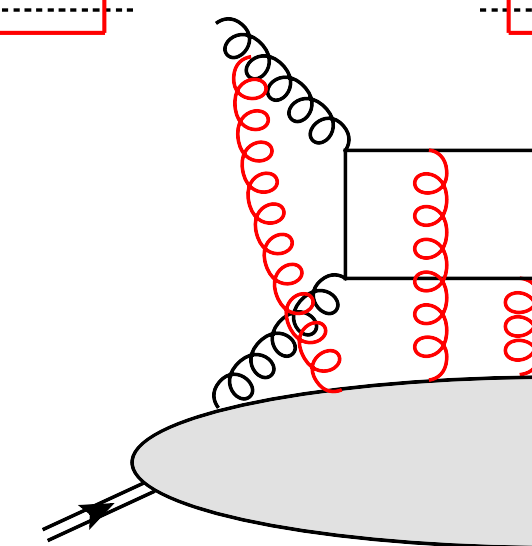
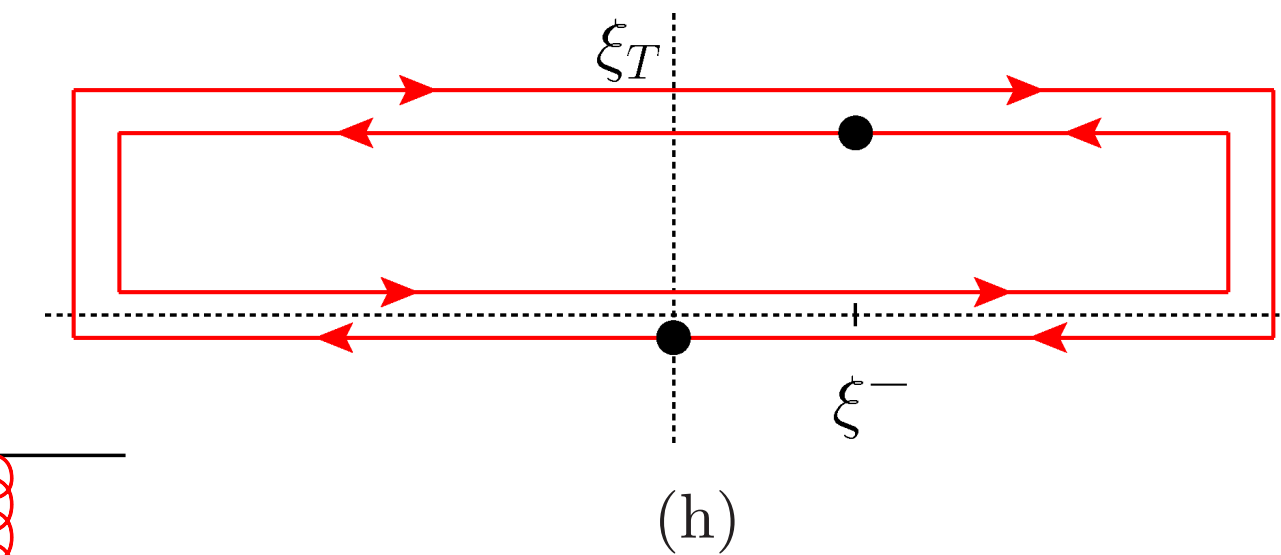
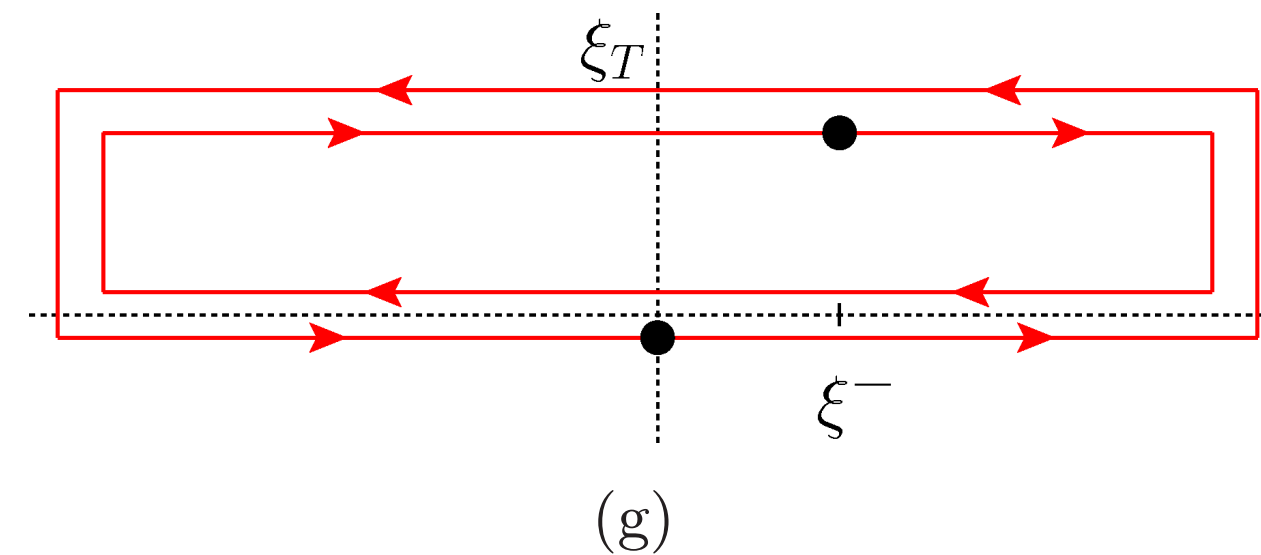
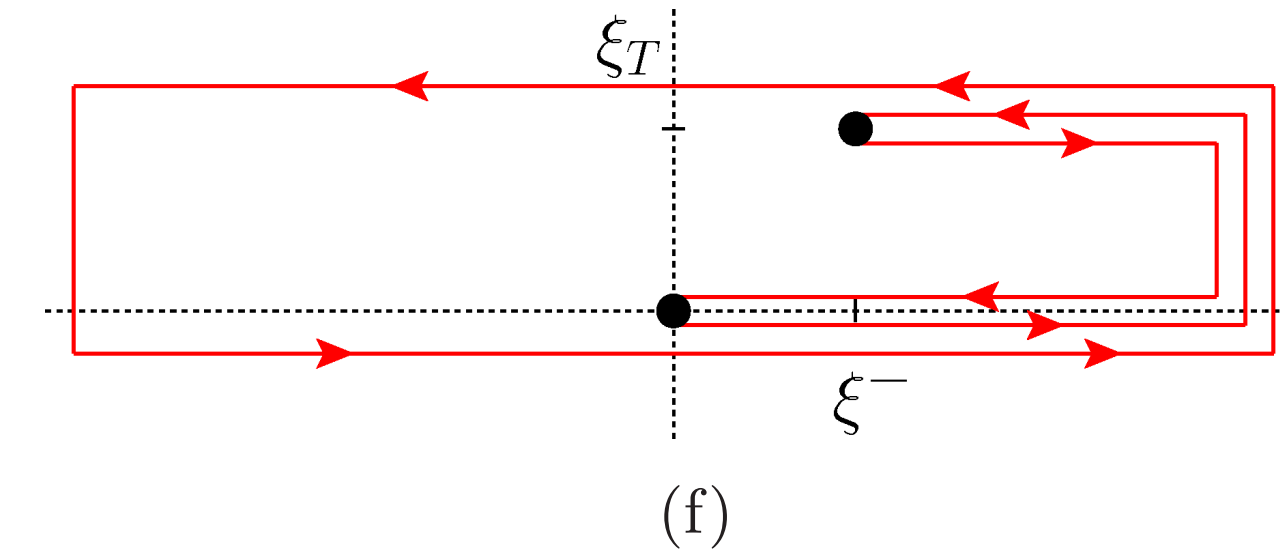
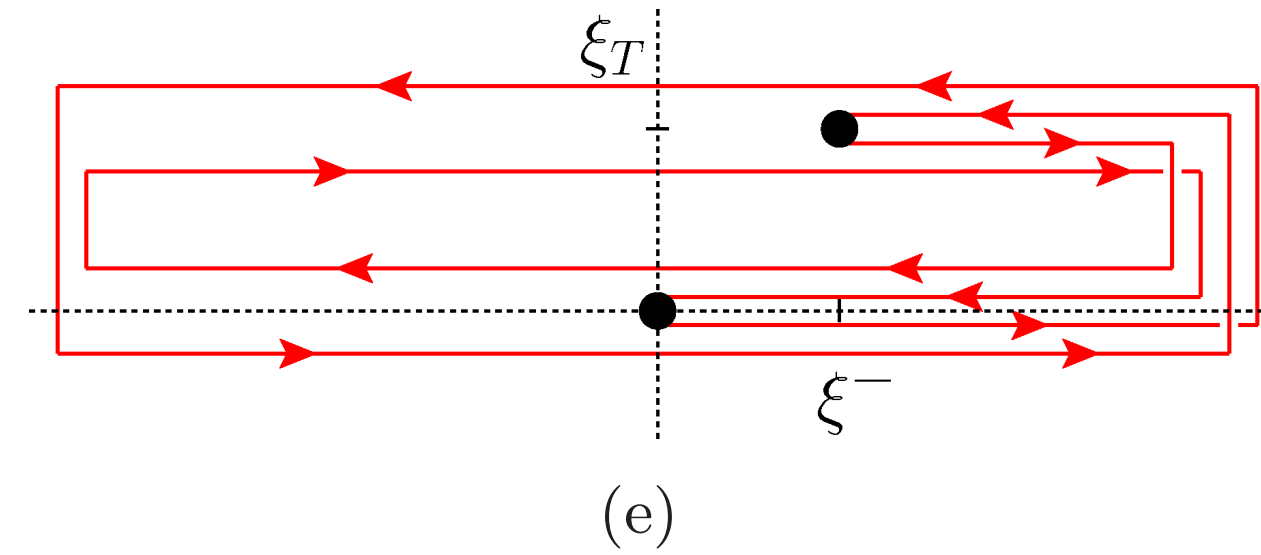
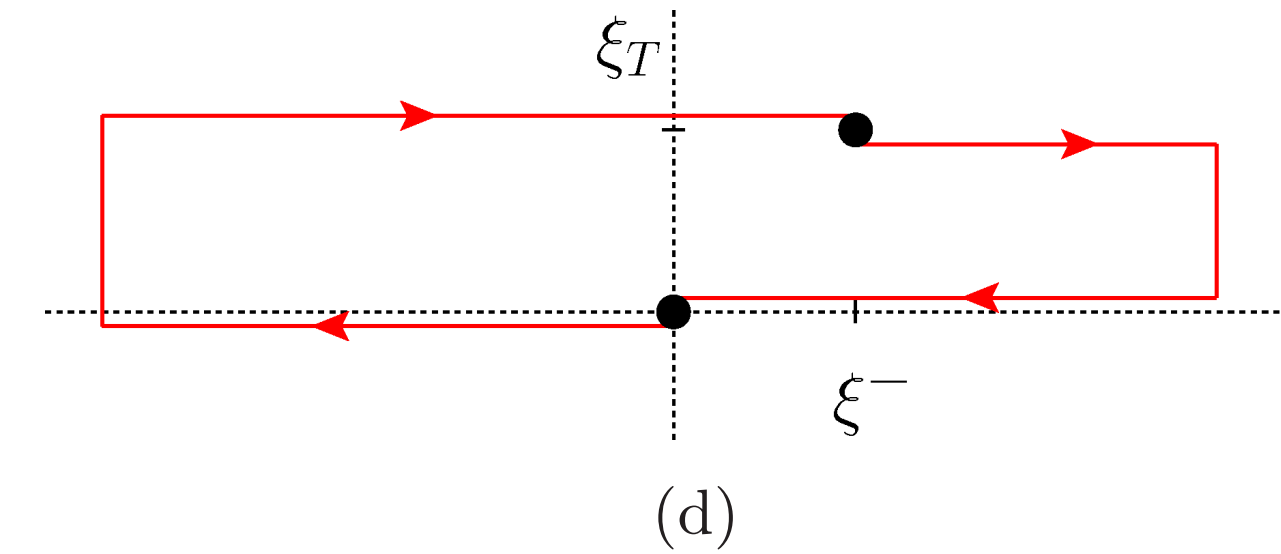
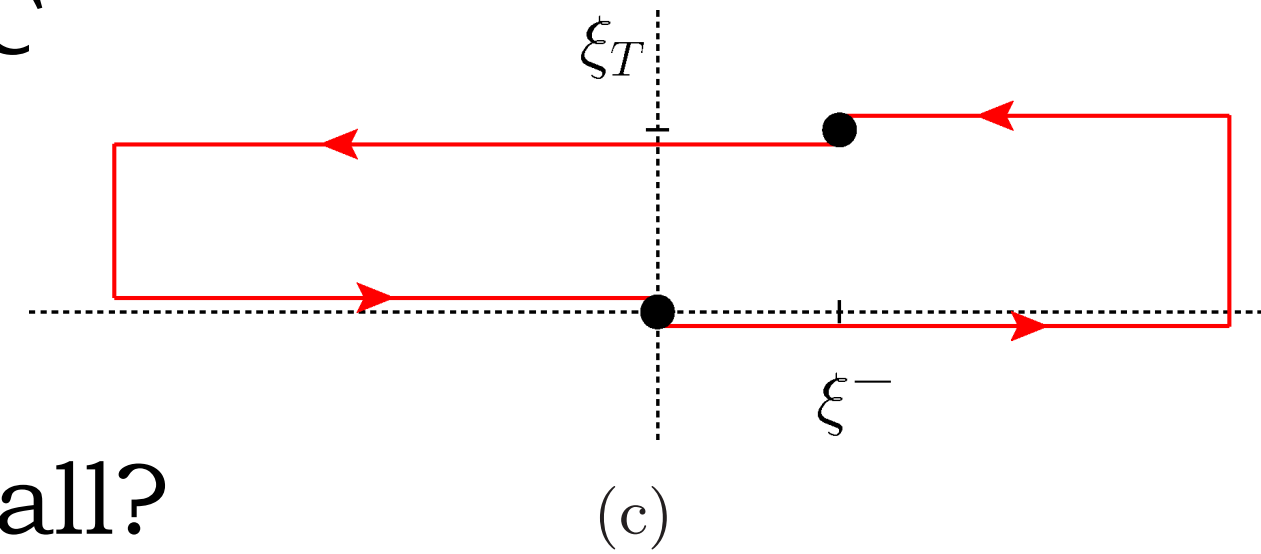
* DP TMDs:

(c) $[+, -]$ and (d) $[-, +]$

* Appearance of new gauge **loop links**:

(e) $[+\square, +\square]$, (f) $[+, +\square]$,

(g) $[\square, \square]$, and (h) $[\square, \square]$



The background features a repeating pattern of circular diagrams illustrating gluon Transverse Momentum Distributions (TMDs). Each diagram shows a central gluon (represented by a red sphere) interacting with a quark (represented by a blue sphere) within a nucleon. The gluon's transverse momentum is indicated by a red arrow, and the quark's transverse momentum is indicated by a blue arrow. The diagrams are set against a light blue and green background with a subtle grid and a bright light source in the upper left corner.

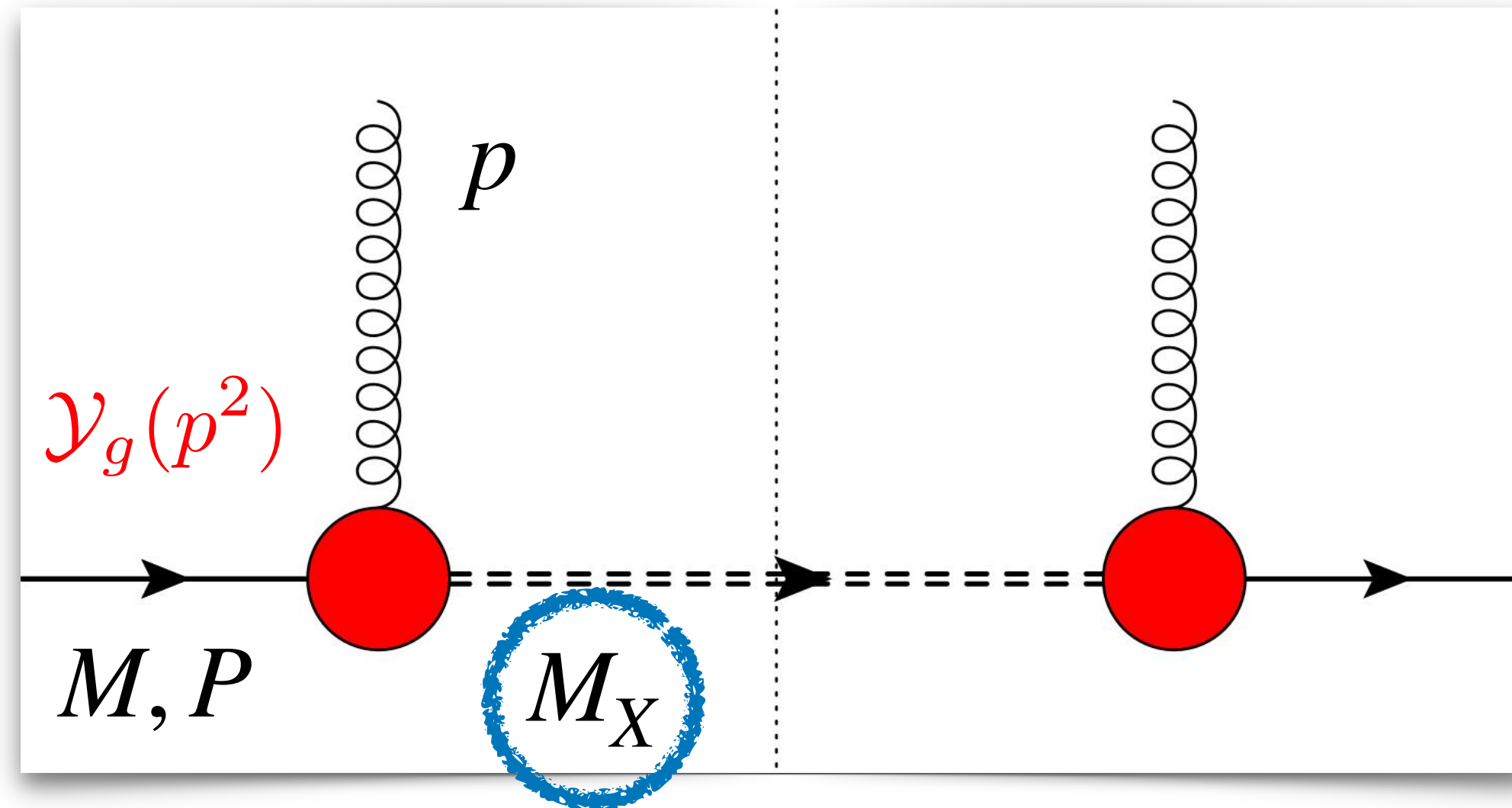
Small-x improved gluon TMDs

Assumptions of the model



Spin-1/2 spectator

Lowest Fock state:
tri-quark spectator
on-shell and
with mass M_X

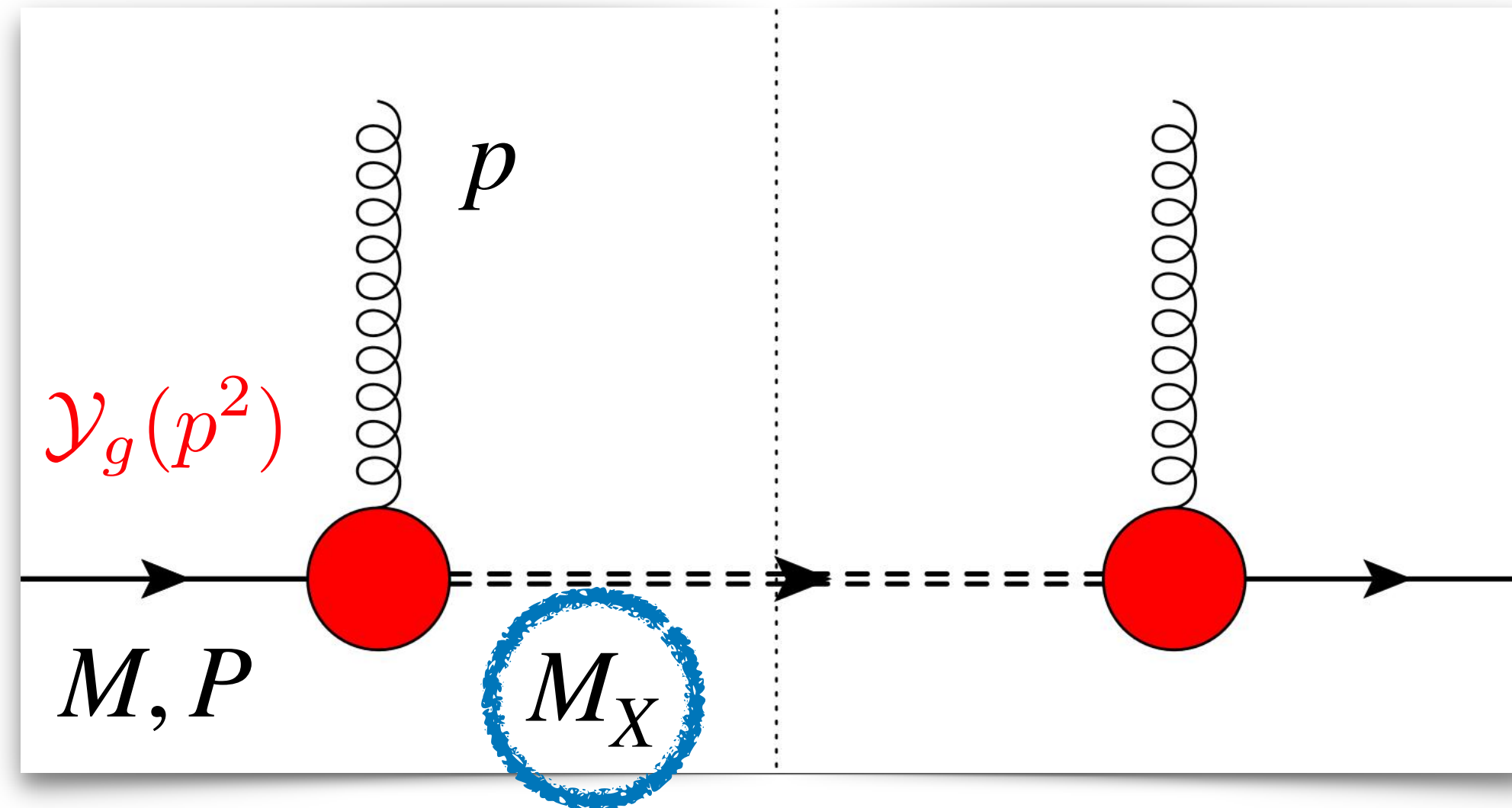


Assumptions of the model



Spin-1/2 spectator

Lowest Fock state:
tri-quark spectator
 on-shell and
 with mass M_X



Nucleon-gluon-spectator vertex

$$\Phi_g = \frac{1}{2(2\pi)^3(1-x)P^+} \text{Tr} \left[(\not{P} + M) \frac{1 + \gamma^5 \not{\not{p}}}{2} G_{\mu\rho}^*(p) G^{\nu\sigma}(p) \mathcal{Y}_g^{\rho*} \mathcal{Y}_{g\sigma} (\not{P} - \not{p} + M) \right]$$

$$\mathcal{Y}_g^\mu = g_1(p^2) \gamma^\mu + i \frac{g_2(p^2)}{2M} \sigma^{\mu\nu} p_\nu$$



mimics proton form factors
 (conserved EM current
 of a free nucleon)



Link with collinear factorization

1. p_T -integrated TMDs **have to** reproduce PDFs at the lowest scale (Q_0) *before* evolution
2. TMDs and PDFs *decouple* due to evolution

Assumptions of the model



Link with collinear factorization

1. p_T -integrated TMDs **have to** reproduce PDFs at the lowest scale (Q_0) *before* evolution
2. TMDs and PDFs *decouple* due to evolution

$$g_{1,2}(p^2) = \kappa_{1,2} \frac{p^2}{|p^2 - \Lambda_X^2|^2}$$



Dipolar form factor(s)

1. Cancels singularity of gluon propagator
2. Suppresses effects of high p_T
3. Compensates log divergences arising from p_T -integration
4. Adds three more parameters: $\kappa_{1,2}$ and Λ_X

Assumptions of the model



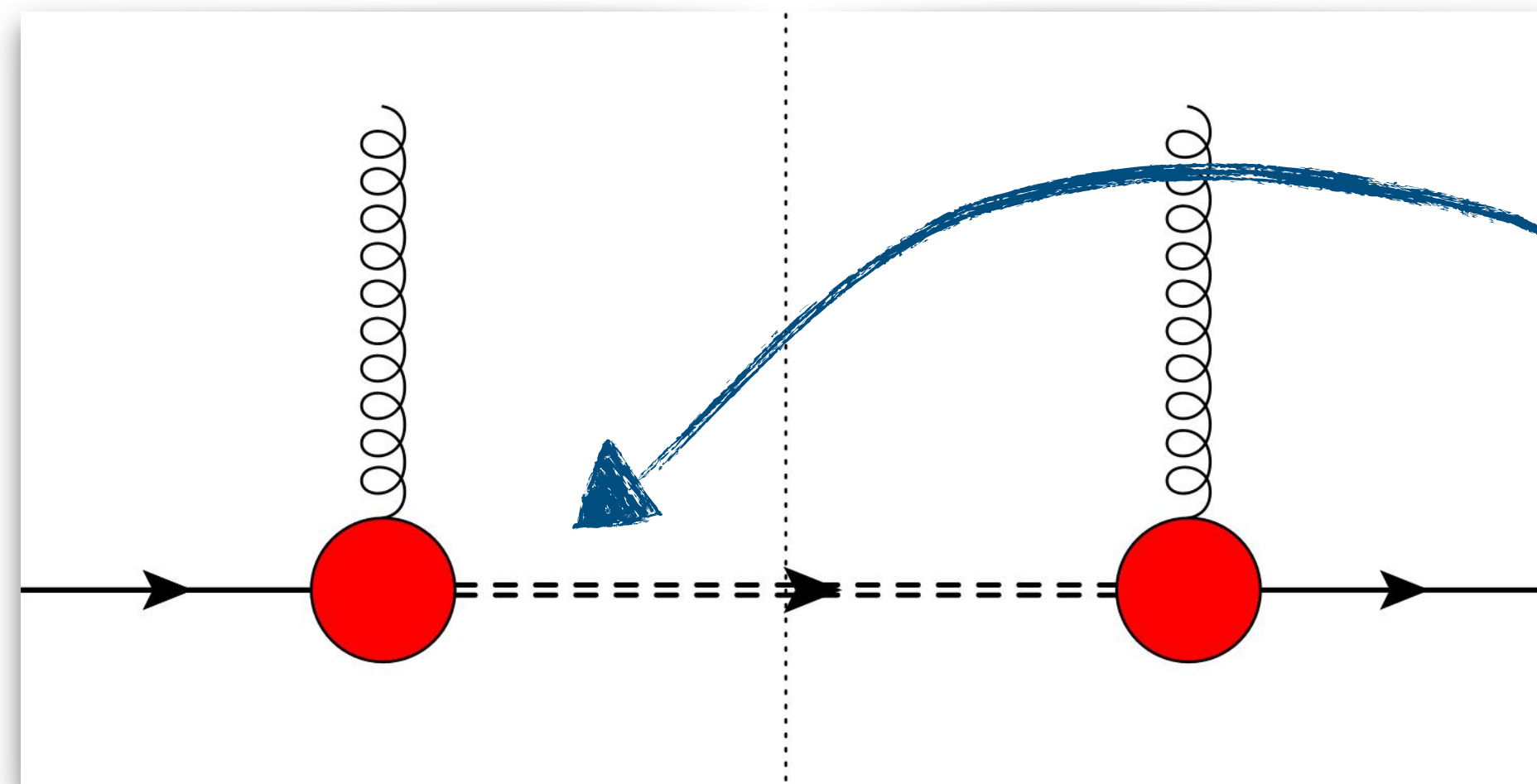
Spectator-system spectral-mass function

spectral-mass function

$$F(x, \mathbf{p}_T^2) = \int_M^\infty dM_X \rho_X(M_X) \hat{F}(x, \mathbf{p}_T^2; M_X)$$

spectator-model TMD

[Inspired by G.R. Goldstein, J.O.G. Hernandez, S. Liuti (2011)]



$\mathcal{V}_g(p^2)$

Instead of a single on-shell spectator, a continuum of spectators

Assumptions of the model



Spectator-system spectral-mass function

spectral-mass function

$$F(x, \mathbf{p}_T^2) = \int_M^\infty dM_X \rho_X(M_X) \hat{F}(x, \mathbf{p}_T^2; M_X)$$

spectator-model TMD

ⓘ [Inspired by G.R. Goldstein, J.O.G. Hernandez, S. Liuti (2011)]

$$\rho_X \left(M_X; \{X^{(\text{pars})}\} \equiv \{A, B, a, b, C, D, \sigma\} \right) = \mu^{2a} \left[\frac{A}{B + \mu^{2b}} + \frac{C}{\pi\sigma} e^{-\frac{(M_X - D)^2}{\sigma^2}} \right]$$

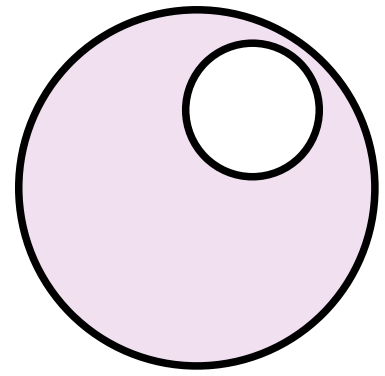
low- x (high- μ^2) tail $\propto (a - b)$

$q\bar{q}$ contributions energetically available at large M_X

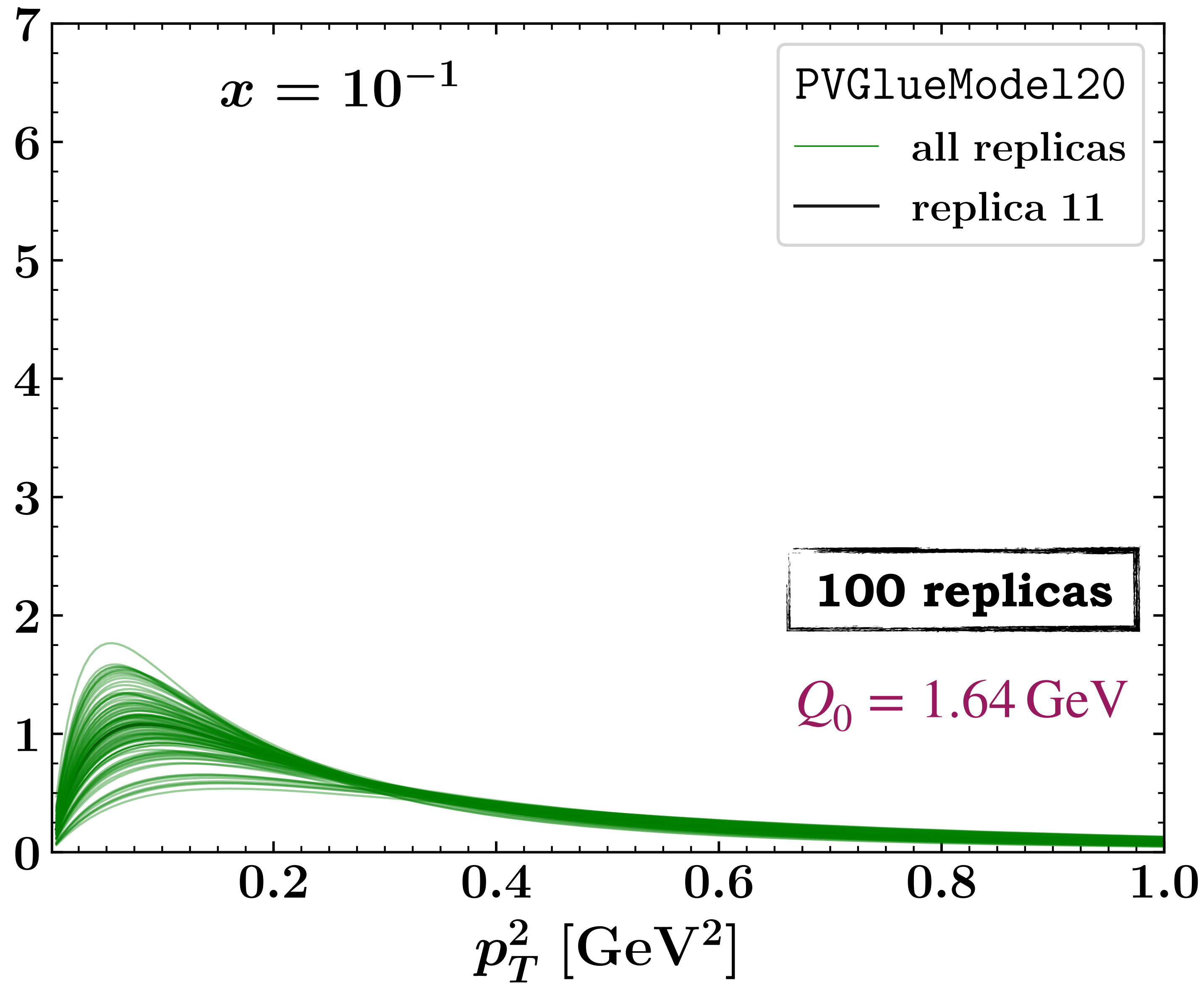
$$\mu^2 = M_X^2 - M^2$$

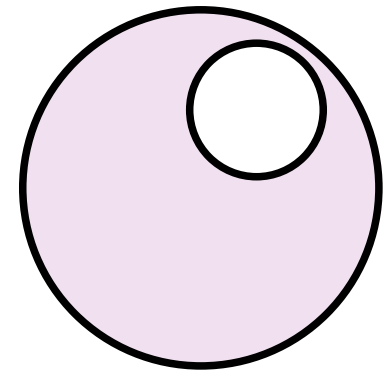
moderate- x trend

pure tri-quark contribution at low M_X

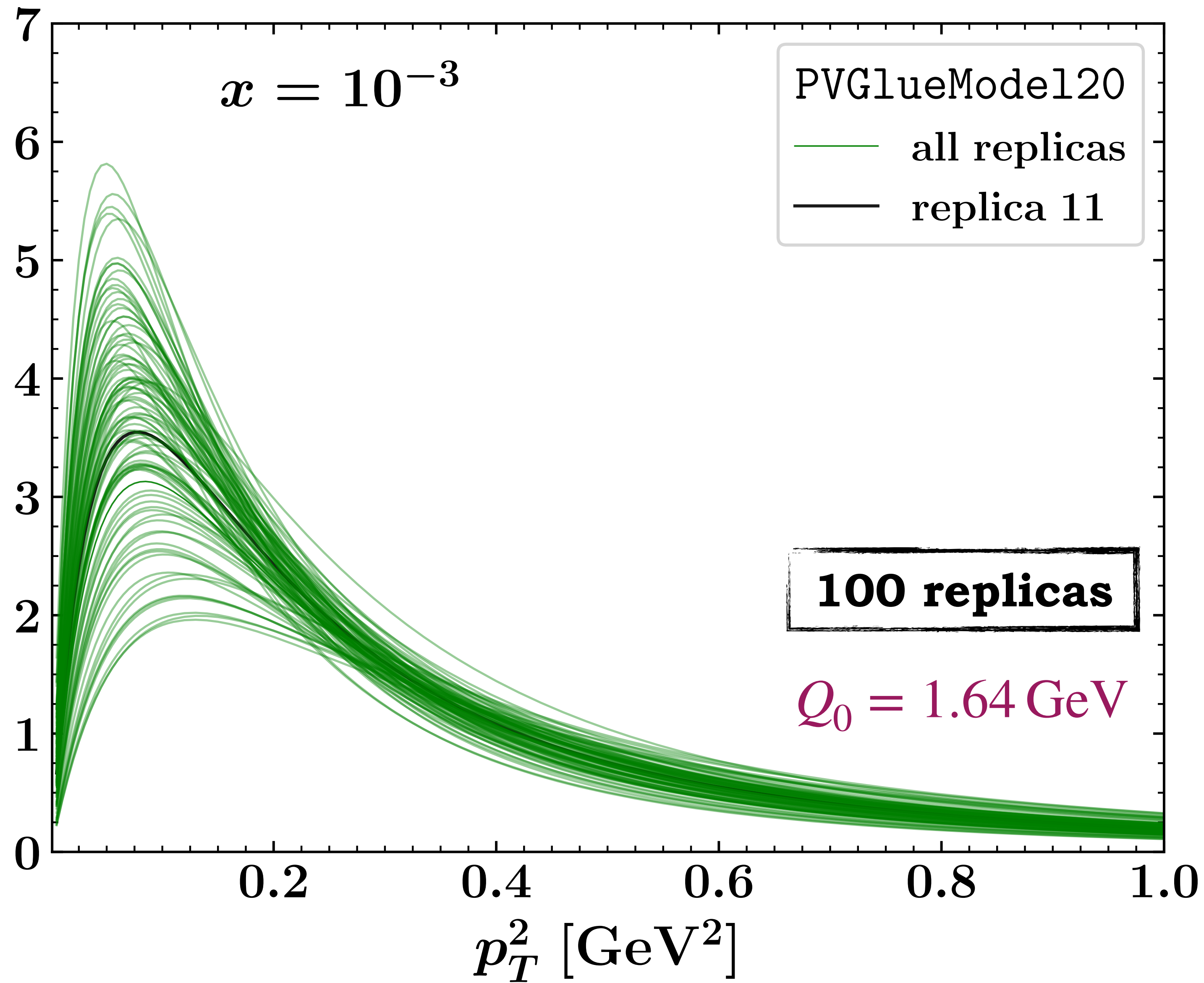


$$x f_1(x, p_T^2)$$



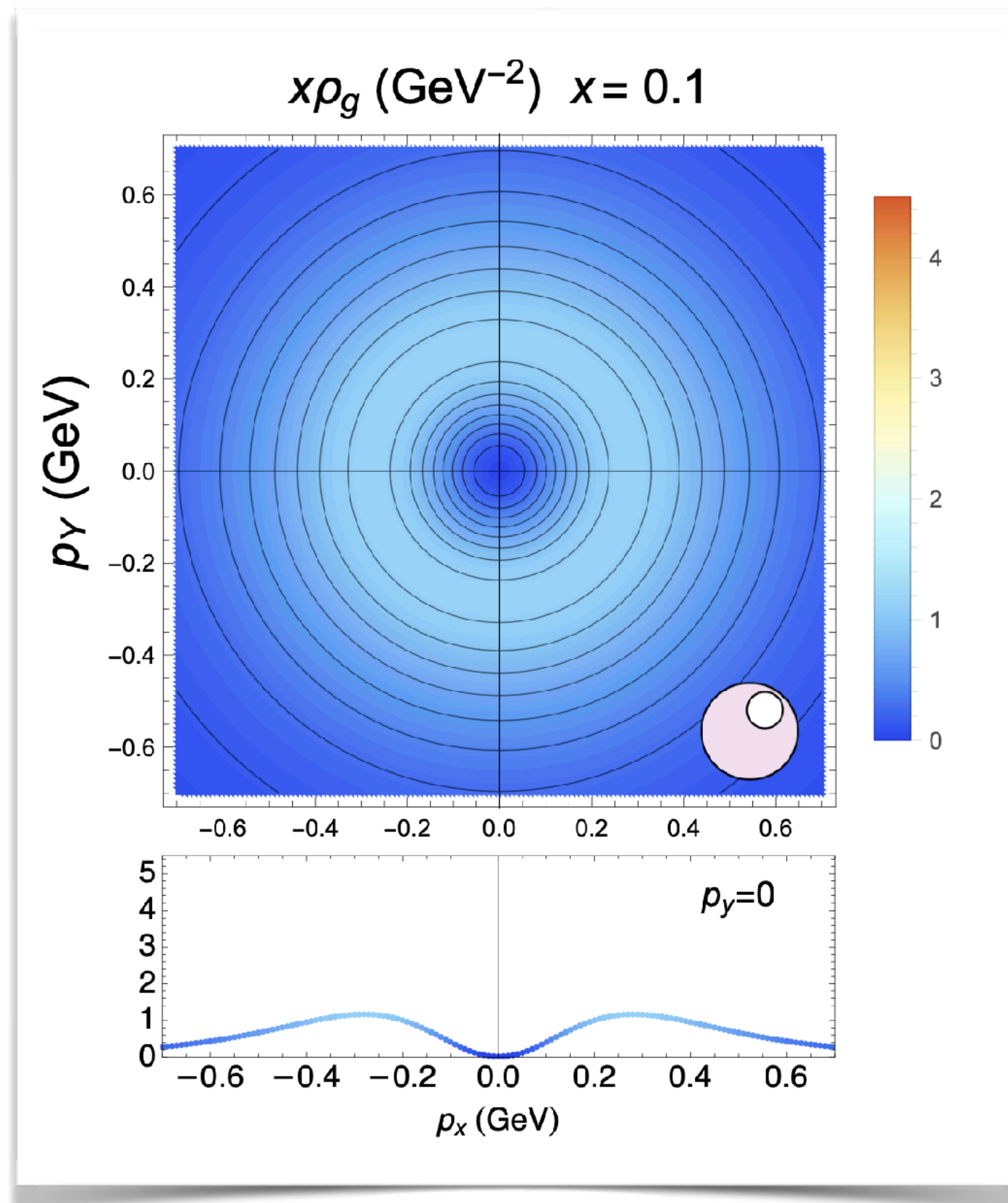


$$x f_1(x, p_T^2)$$



3D proton tomography: T-even gluon TMD PDFs

unpolarized TMD



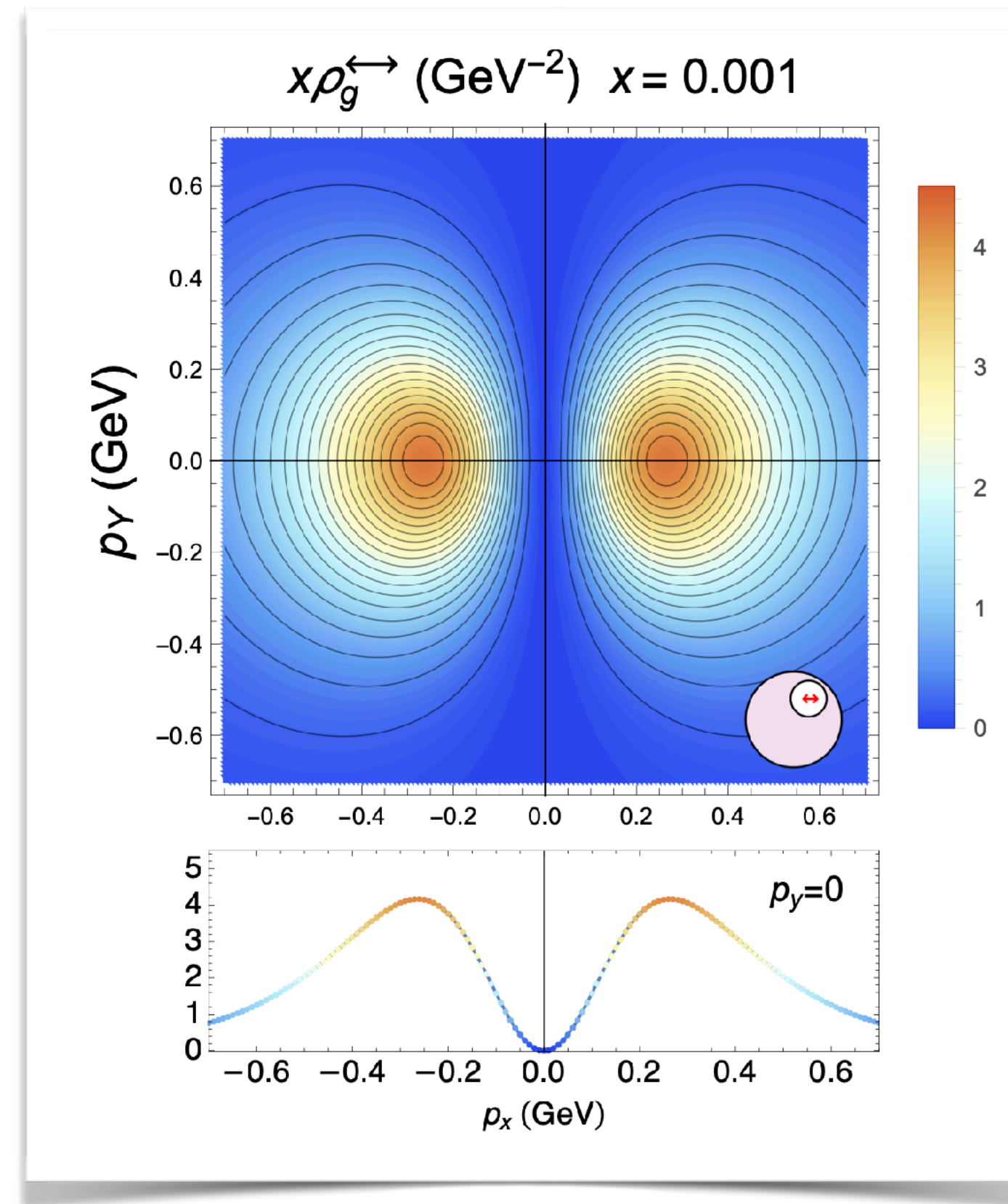
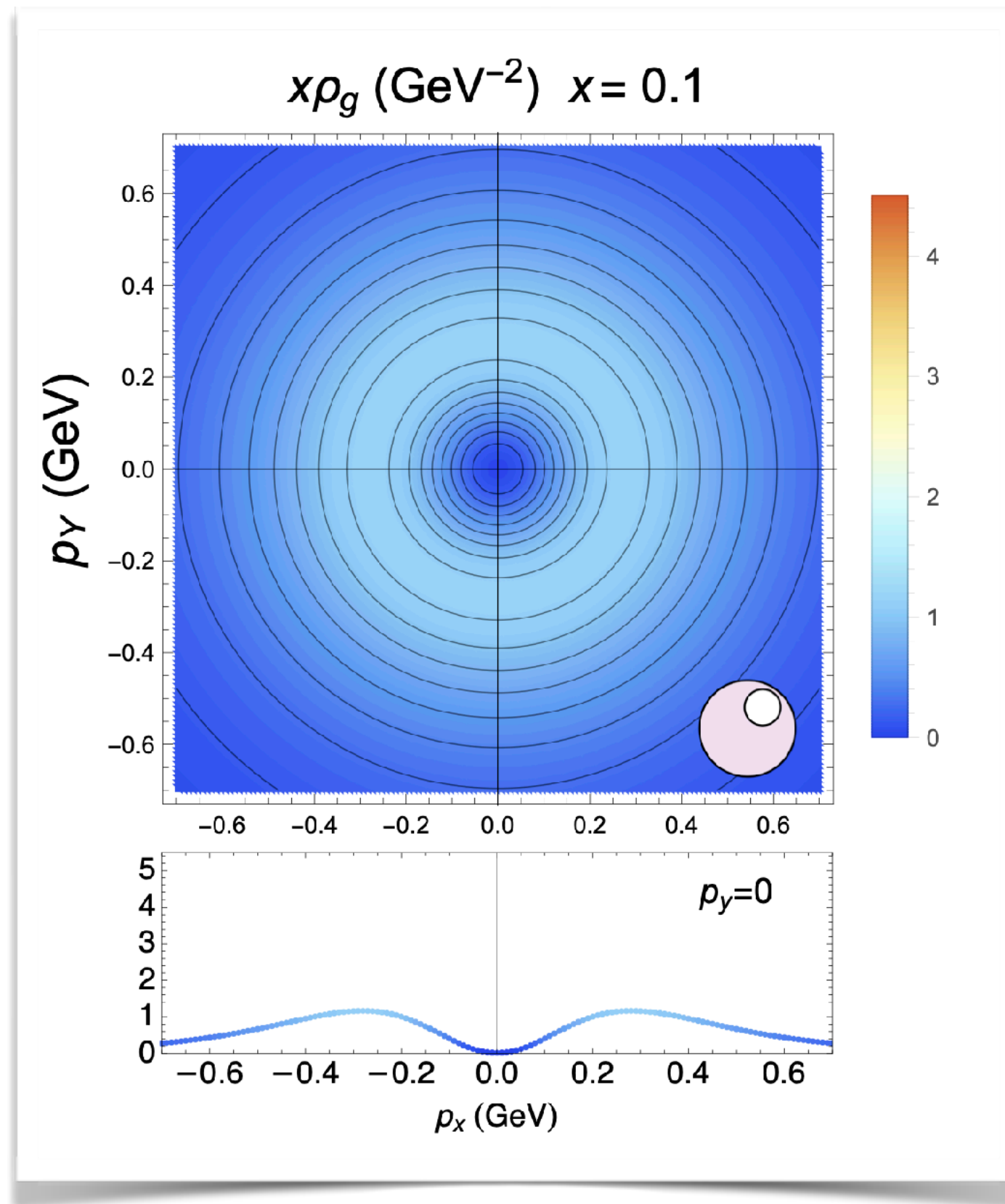
 [A. Bacchetta, F.G.C., M. Radici, P. Tael, Eur. Phys. J. C 80 (2020) no.8 [[arXiv:2005.02288](https://arxiv.org/abs/2005.02288)]]

Backup

3D proton tomography: T-even gluon TMD PDFs

unpolarized TMD

Boer-Mulders



 [A. Bacchetta, F.G.C., M. Radici, P. Tael, Eur. Phys. J. C 80 (2020) no.8 [[arXiv:2005.02288](https://arxiv.org/abs/2005.02288)]]

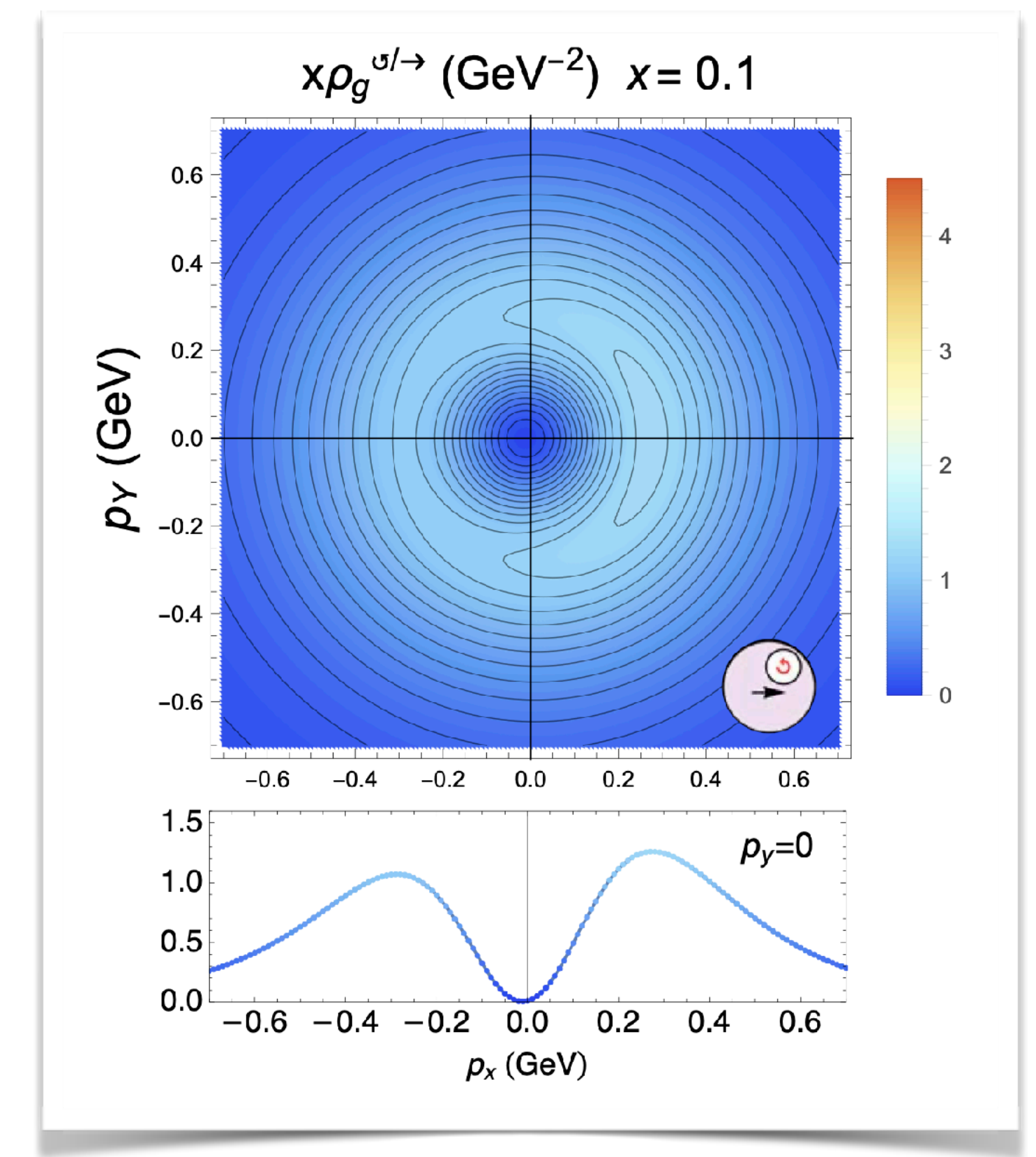
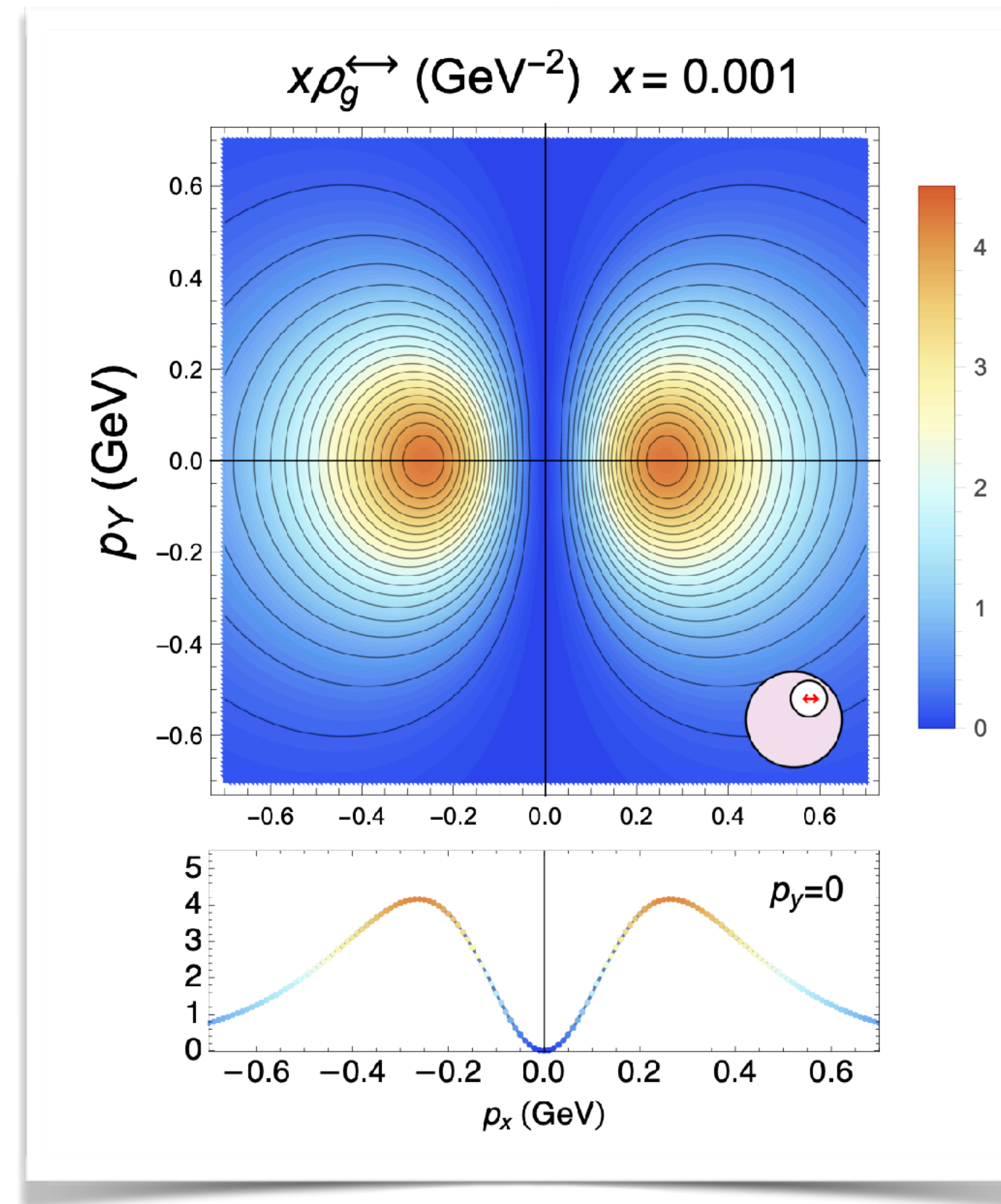
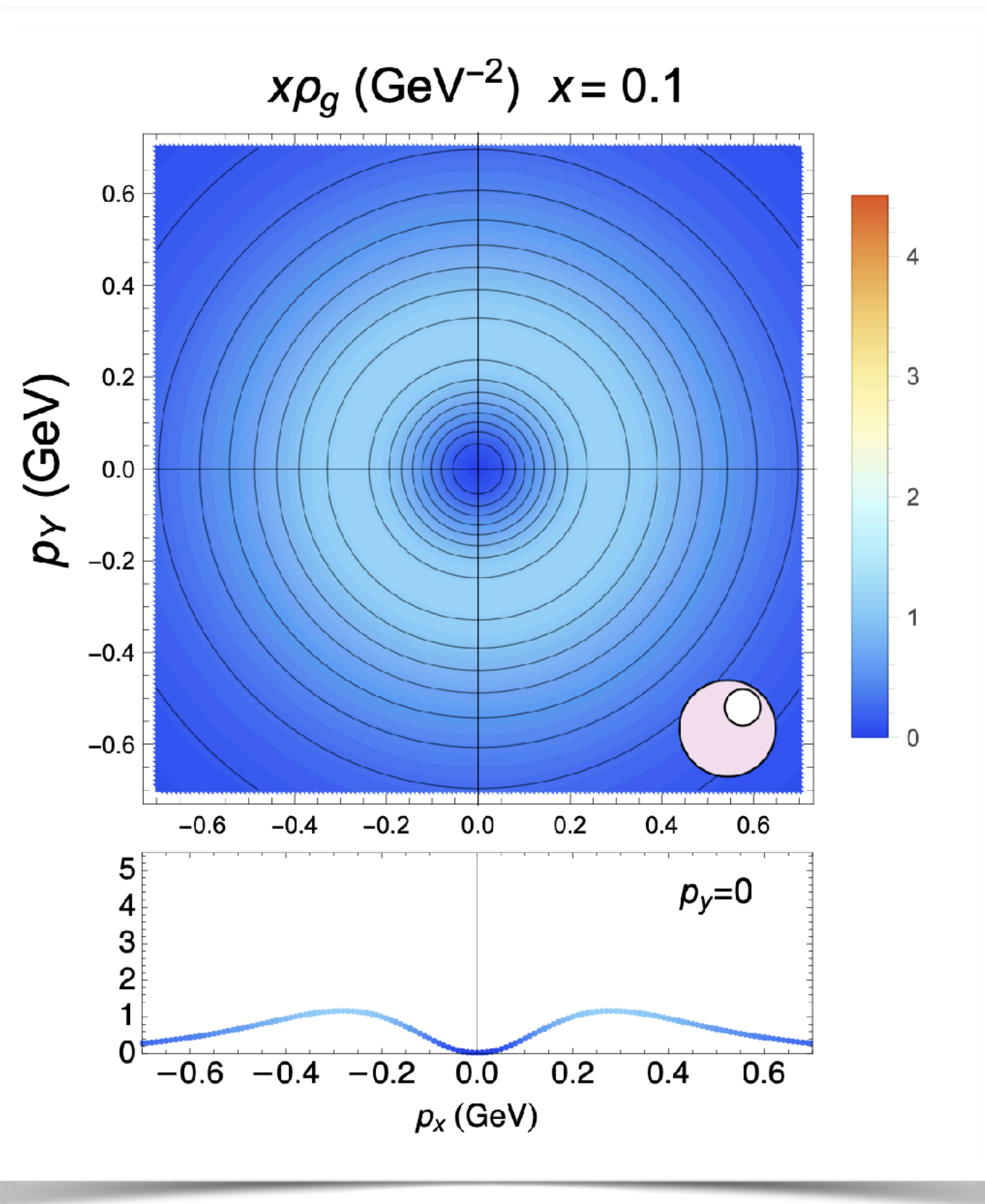
Backup

3D proton tomography: T-even gluon TMD PDFs

unpolarized TMD

Boer-Mulders

worm-gear

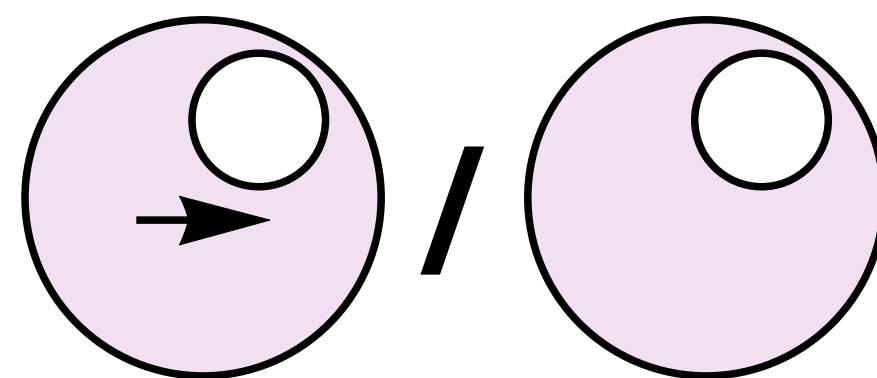


[A. Bacchetta, F.G.C., M. Radici, P. Tael, Eur. Phys. J. C 80 (2020) no.8 [[arXiv:2005.02288](https://arxiv.org/abs/2005.02288)]]

Backup

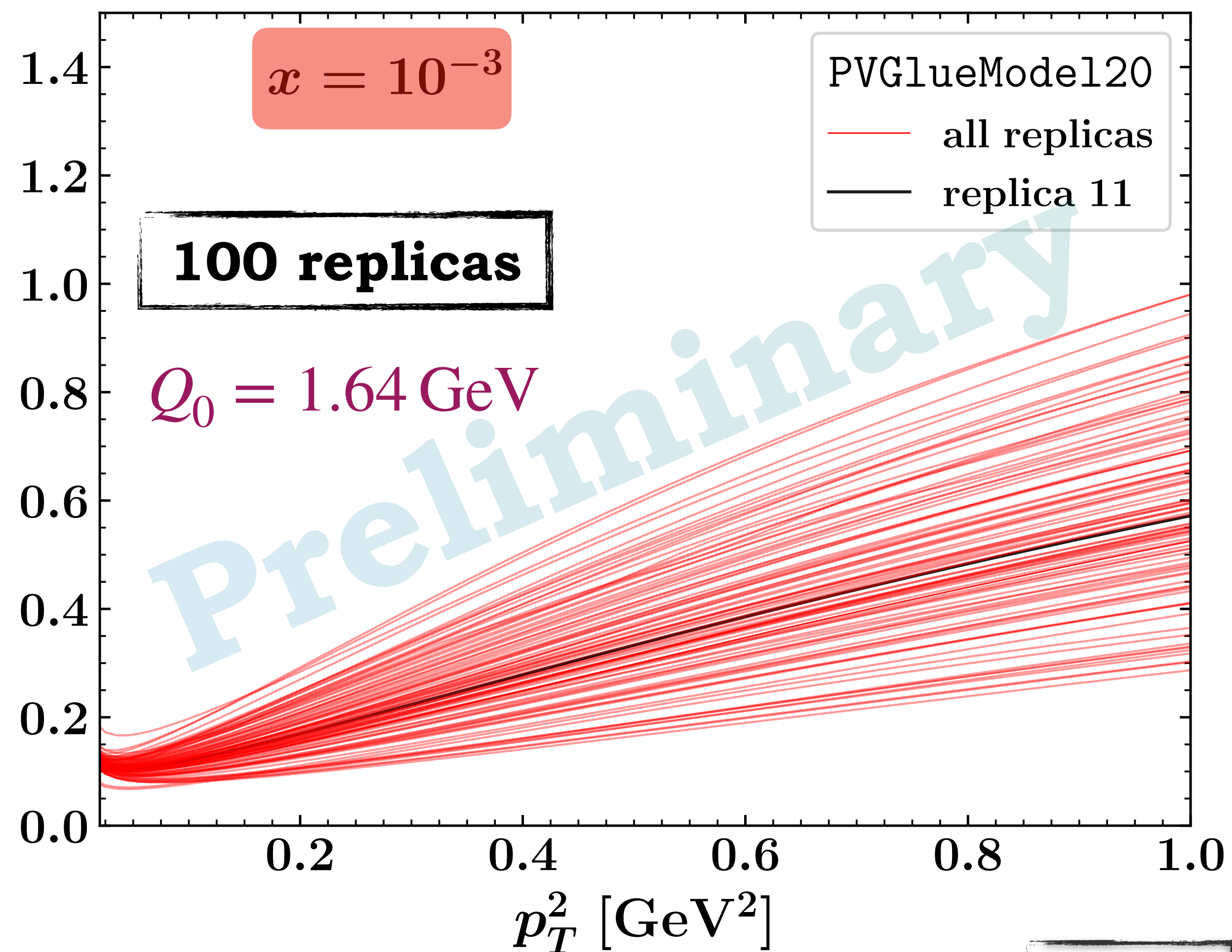
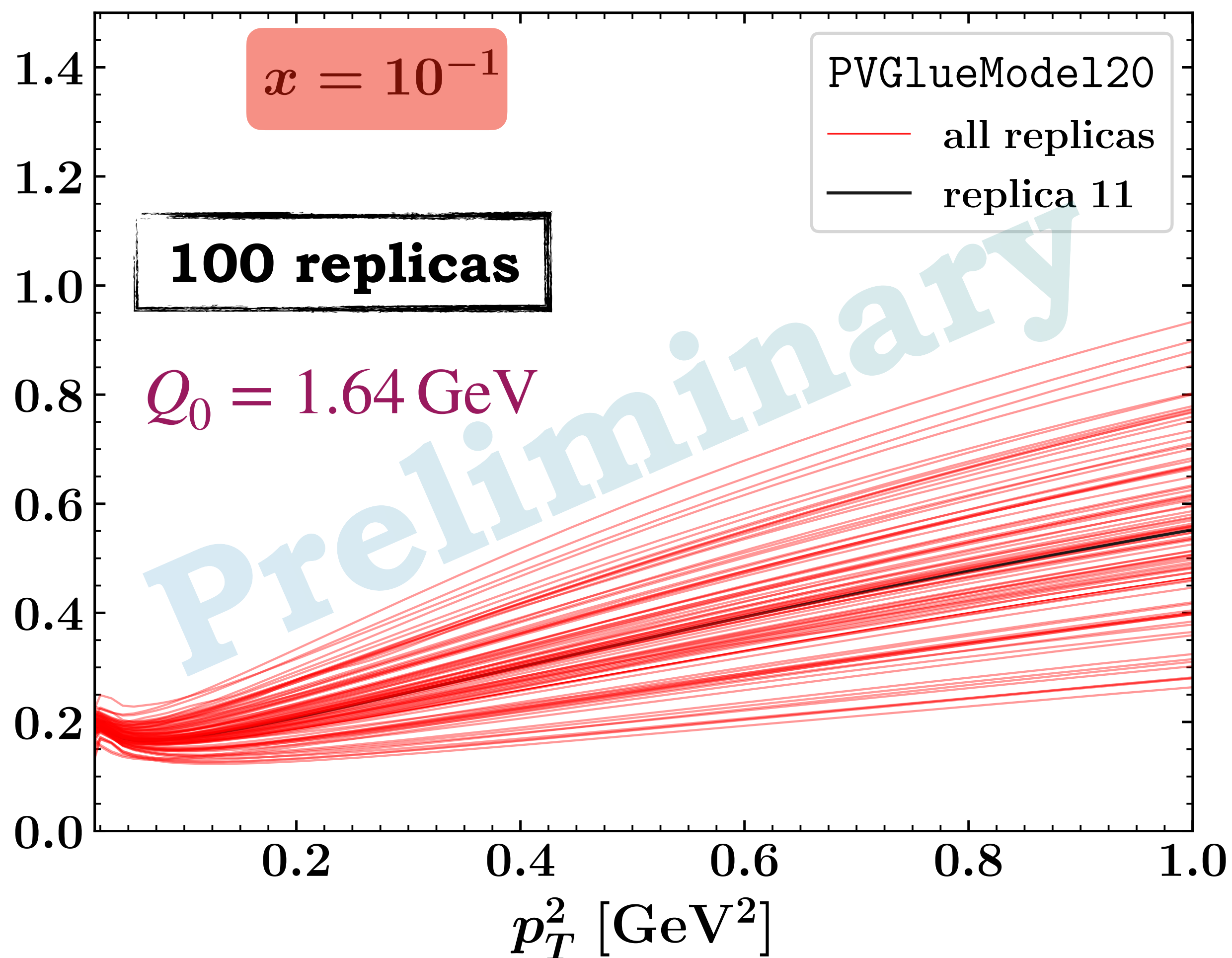
f -type Sivers/unpol.

A. Bacchetta, F.G. C., M. Radici (in preparation)



$$\frac{p_T}{M} f_{1T}^{\perp[+,+]} / f_1$$

$$\frac{p_T}{M} f_{1T}^{\perp[+,+]} / f_1$$



Backup

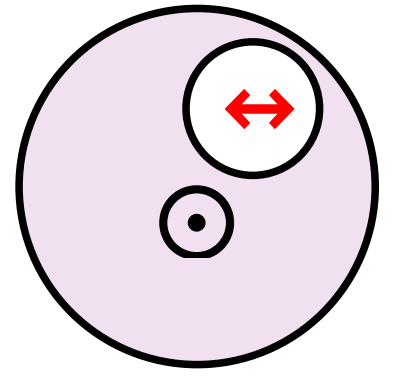
Glueon TMD correlator and T-odd glueon densities

$$\Gamma_U^{ij}(x, \mathbf{k}) = x \left[\delta_T^{ij} f_1(x, \mathbf{k}^2) + \frac{k_T^{ij}}{M^2} h_1^\perp(x, \mathbf{k}^2) \right]$$

$$\Gamma_L^{ij}(x, \mathbf{k}) = x \left[i\epsilon_T^{ij} S_L g_1(x, \mathbf{k}^2) + \frac{\epsilon_T^{\{i} k_T^{j\}\alpha} S_L}{2M^2} h_{1L}^\perp(x, \mathbf{k}^2) \right]$$

$$\Gamma_T^{ij}(x, \mathbf{k}) = x \left[\frac{\delta_T^{ij} \epsilon_T^{S_T k_T}}{M} f_{1T}^\perp(x, \mathbf{k}^2) + \frac{i\epsilon_T^{ij} \mathbf{k} \cdot \mathbf{S}_T}{M} g_{1T}(x, \mathbf{k}^2) \right. \\ \left. - \frac{\epsilon_T^{k_T \{i} S_T^{j\}} + \epsilon_T^{S_T \{i} k_T^{j\}}}{4M} h_1(x, \mathbf{k}^2) - \frac{\epsilon_T^{\{i} k_T^{j\}\alpha} S_T}{2M^3} h_{1T}^\perp(x, \mathbf{k}^2) \right]$$

Gluon TMD correlator and T-odd gluon densities

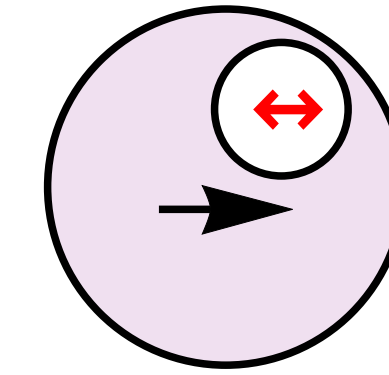


$$\Gamma_U^{ij}(x, \mathbf{k}) = x \left[\delta_T^{ij} f_1(x, \mathbf{k}^2) + \frac{k_T^{ij}}{M^2} h_1^\perp(x, \mathbf{k}^2) \right]$$

$$\Gamma_L^{ij}(x, \mathbf{k}) = x \left[i\epsilon_T^{ij} S_L g_1(x, \mathbf{k}^2) + \frac{\epsilon_T^{\{i} k_T^{j\}\alpha} S_L}{2M^2} h_{1L}^\perp(x, \mathbf{k}^2) \right]$$

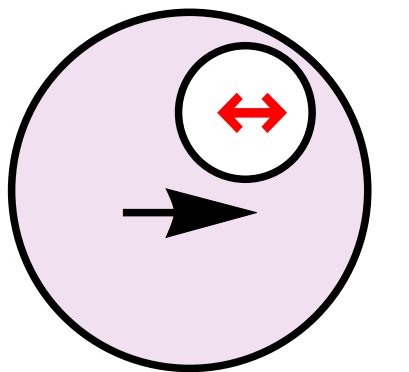
$$\Gamma_T^{ij}(x, \mathbf{k}) = x \left[\frac{\delta_T^{ij} \epsilon_T^{S_T k_T}}{M} f_{1T}^\perp(x, \mathbf{k}^2) + \frac{i\epsilon_T^{ij} \mathbf{k} \cdot \mathbf{S}_T}{M} g_{1T}(x, \mathbf{k}^2) - \frac{\epsilon_T^{k_T \{i} S_T^{j\}} + \epsilon_T^{S_T \{i} k_T^{j\}}}{4M} h_1(x, \mathbf{k}^2) - \frac{\epsilon_T^{\{i} k_T^{j\}\alpha} S_T}{2M^3} h_{1T}^\perp(x, \mathbf{k}^2) \right]$$

pseudo worm-gear

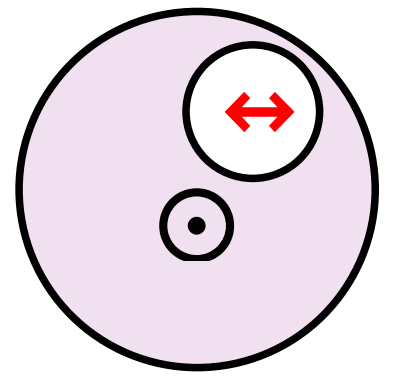


linearity TMD

pretzelosity



Gluon TMD correlator and T-odd gluon densities



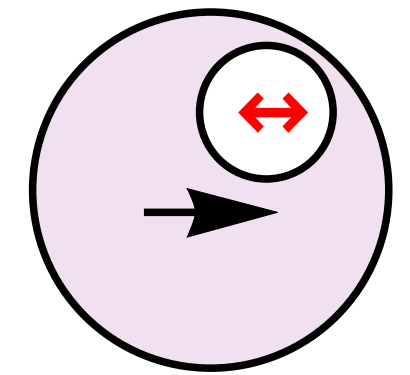
pseudo worm-gear

$$\Gamma_U^{ij}(x, k) = x \left[\delta_T^{ij} f_1(x, k^2) + \frac{k_T^{ij}}{M^2} h_1^\perp(x, k^2) \right]$$

$$\Gamma_L^{ij}(x, k) = x \left[i\epsilon_T^{ij} S_L g_1(x, k^2) + \frac{\epsilon_T^{\{i} k_T^{j\}\alpha} S_L}{2M^2} h_{1L}^\perp(x, k^2) \right]$$

$$\Gamma_T^{ij}(x, k) = x \left[\frac{\delta_T^{ij} \epsilon_T^{S_T k_T}}{M} f_{1T}^\perp(x, k^2) + \frac{i\epsilon_T^{ij} \mathbf{k} \cdot \mathbf{S}_T}{M} g_{1T}(x, k^2) \right.$$

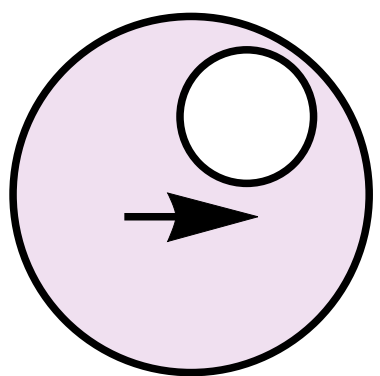
$$\left. - \frac{\epsilon_T^{k_T \{i} S_T^{j\}} + \epsilon_T^{S_T \{i} k_T^{j\}}}{4M} h_1(x, k^2) - \frac{\epsilon_T^{\{i} k_T^{j\}\alpha} S_T}{2M^3} h_{1T}^\perp(x, k^2) \right]$$



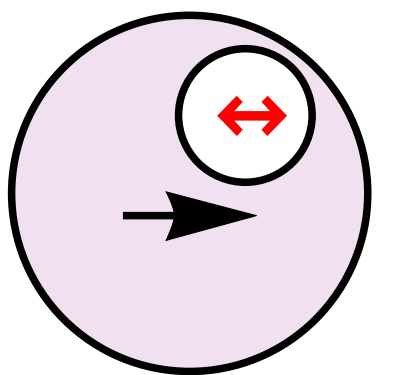
linearity TMD

pretzelosity

Sivers



$$\frac{\epsilon_T^{S_T k_T}}{M} f_{1T}^\perp(x, k^2) = \frac{1}{2} \delta_{Tij} \Gamma_T^{ij}(x, k)$$



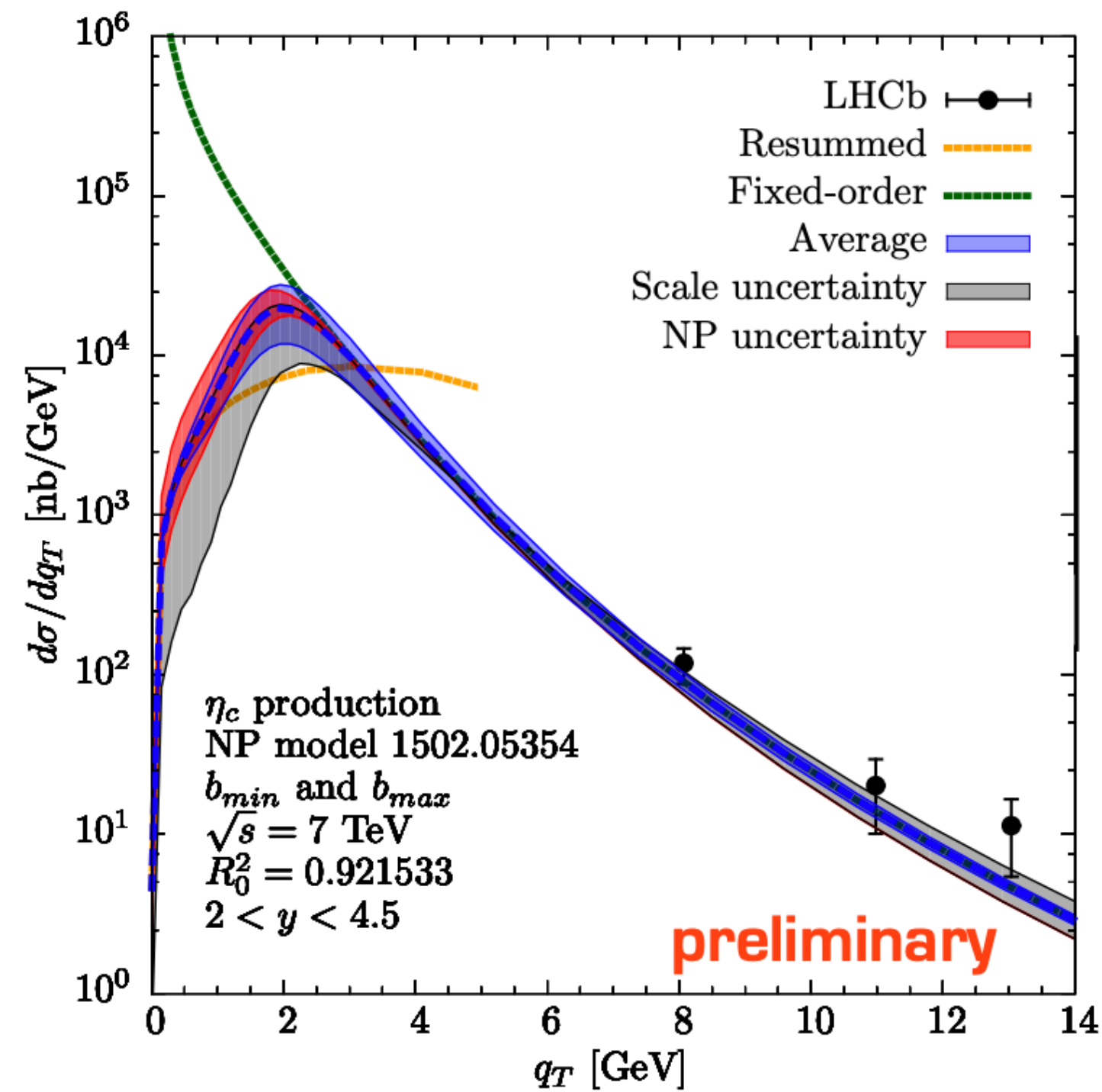
Backup

Gluon TMD phenomenology

The background features a complex, multi-layered illustration of gluon transverse momentum distributions (TMDs). It consists of several overlapping, semi-transparent circular regions. Each region contains a network of yellow wavy lines representing gluons, with various colored spheres (red, blue, green) and arrows indicating interactions and spin. The overall color palette is light blue and cyan, with a subtle grid pattern and a bright light source in the upper left corner.

η_c production at LHC

full transverse momentum spectrum:
low q_T matched with high q_T region



blue band: uncertainty from matching

grey band: scale uncertainty

red band: nonpert. uncertainty

$$S_{NP}(\bar{b}_T) = - \left[\frac{a_1}{2} + \frac{a_2}{2} \ln Q^2 \right] \bar{b}_T^2$$

$a_i = 0.5 \text{ GeV}^2$, var. 50%, envelope

both for unpolarized and
linearly polarized distributions

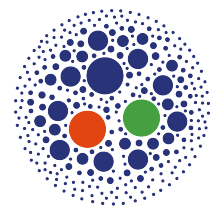
the formalism is in good shape!

we need the data at low q_T



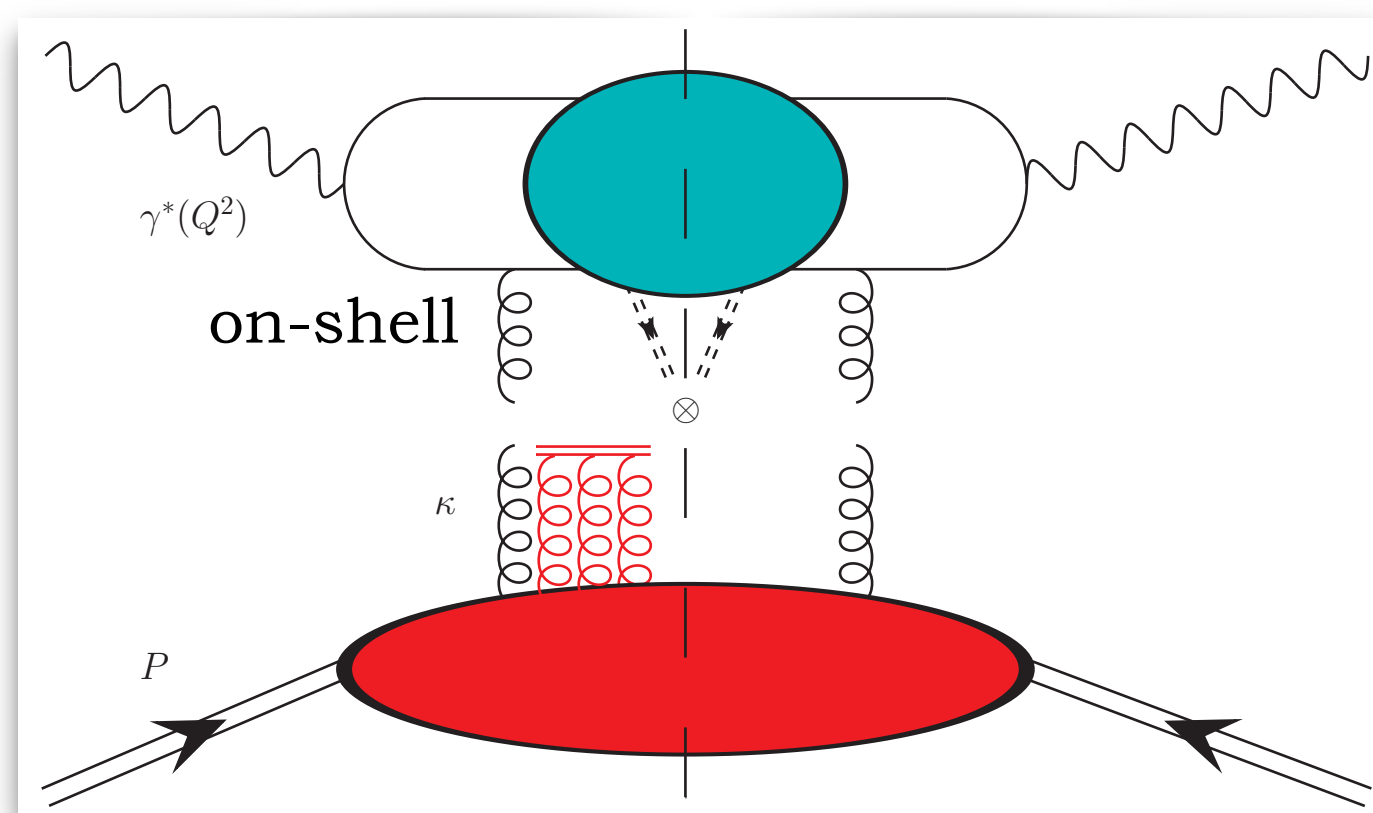
HEF and the UGD

TMD versus high-energy factorization



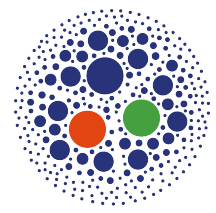
TMD

- * Semi-inclusive processes
- * $\kappa_T \ll$ hardest scale
- * Language of **parton correlators**
- * Diagram: SIDIS onium



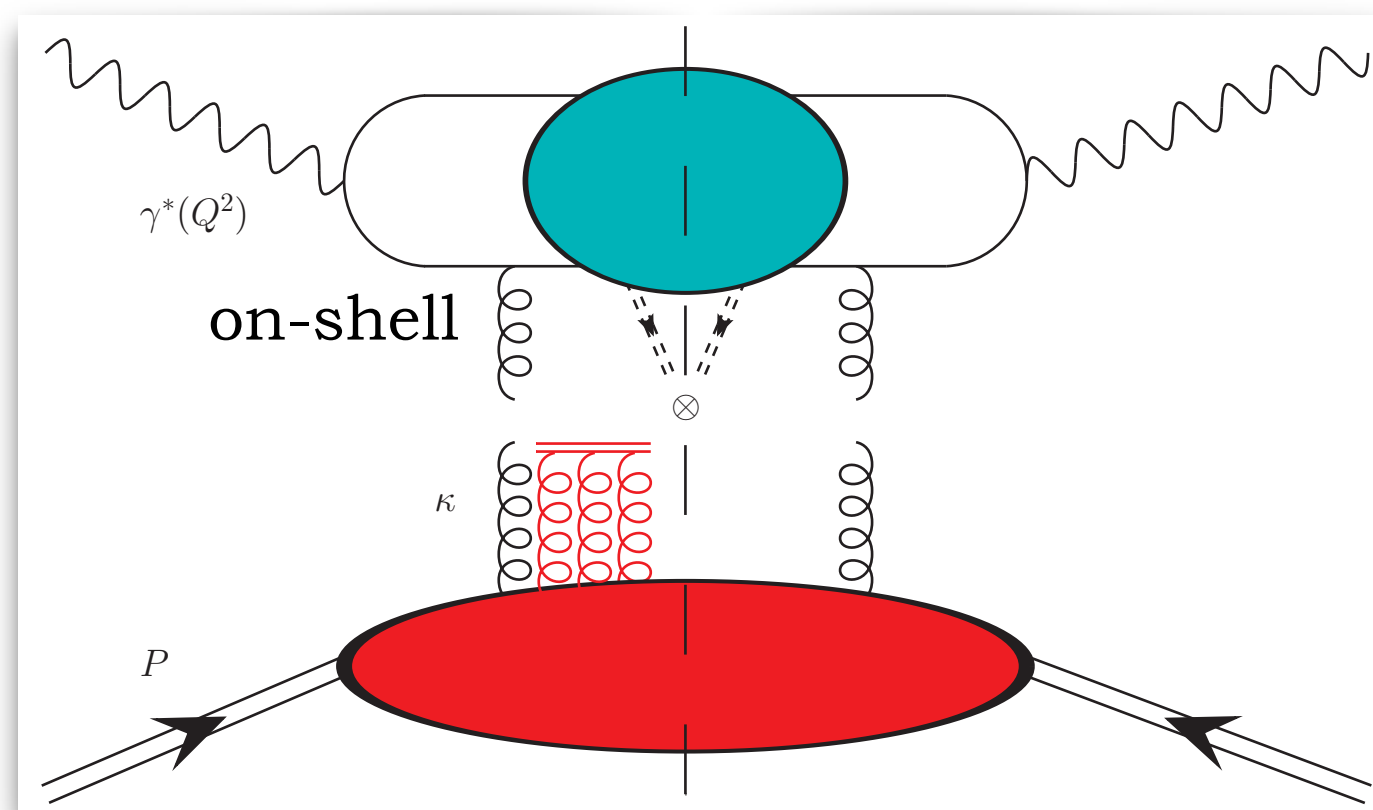
TMD
PDF

TMD versus high-energy factorization

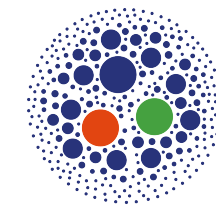


TMD

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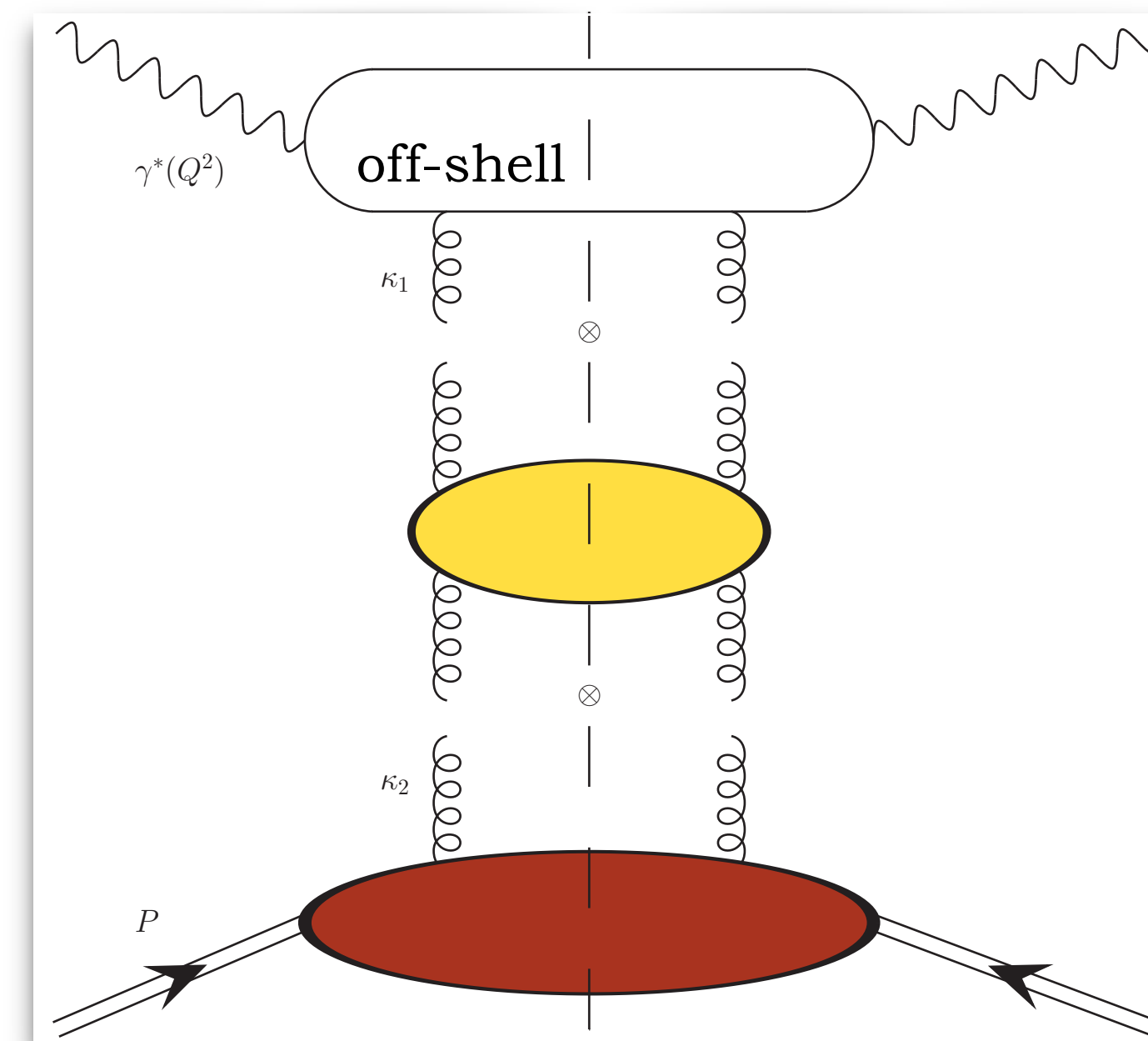


TMD
PDF



HEF

- * Inclusive or exclusive processes (!)
- * Small x , large κ_T
- * Language of **Reggeized gluons**
- * Diagram: DIS



$\Phi \gamma^* \rightarrow \gamma^*$



$\mathcal{G}_{\text{BFKL}}$



$\Phi^P_{[\text{NP}]}$

TMD versus high-energy factorization

TMD

IR-safe colorless $\{\Phi^{i \rightarrow 0}\}$

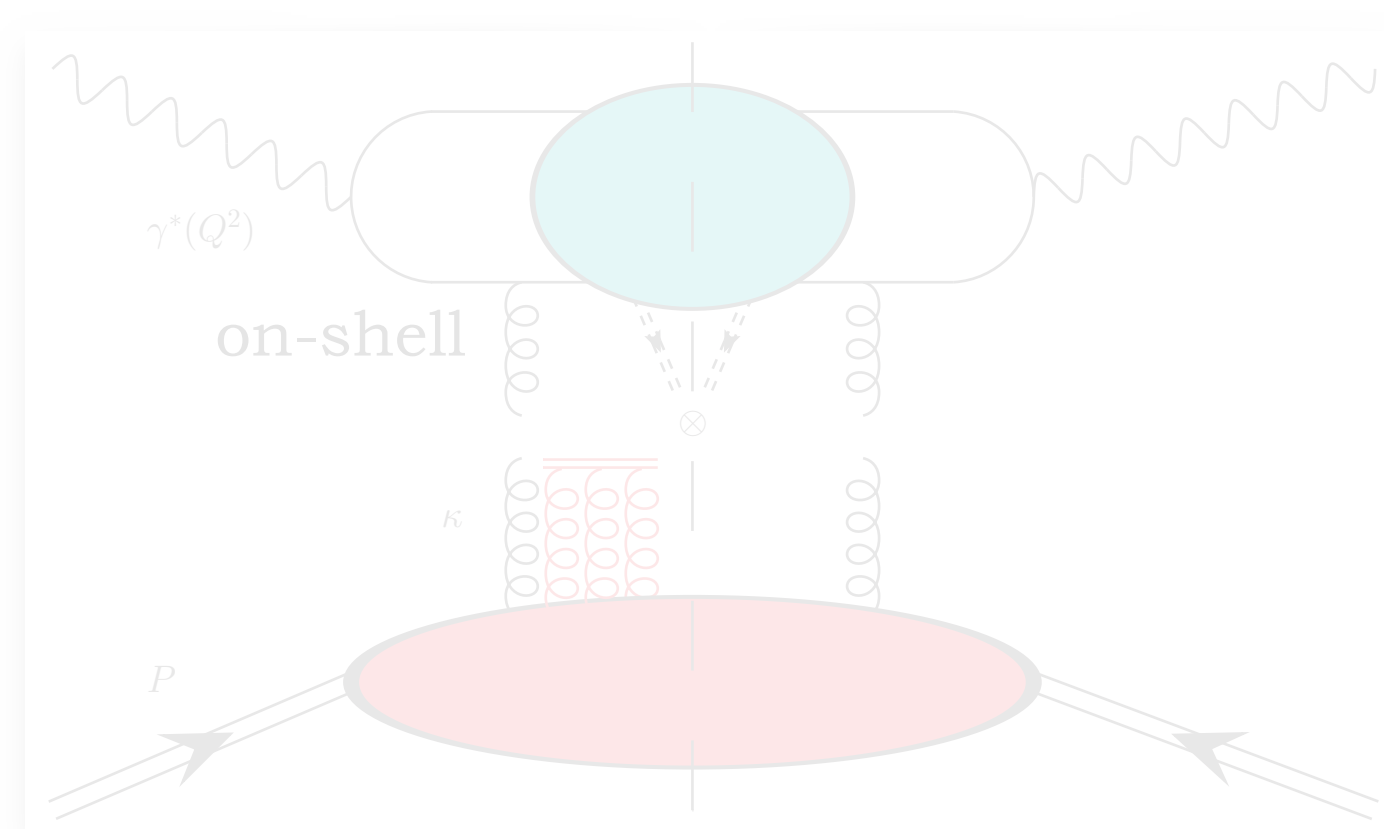
(Fadin-Martin theorem)

* Semi-inclusive processes
 [V.S. Fadin, A.D. Martin (1999)]

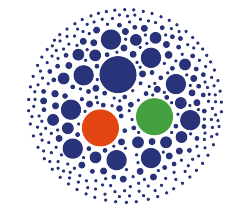
* $\kappa_T \ll$ hardest scale

* Language of parton correlators

* Diagram: SIDIS onium



TMD
PDF



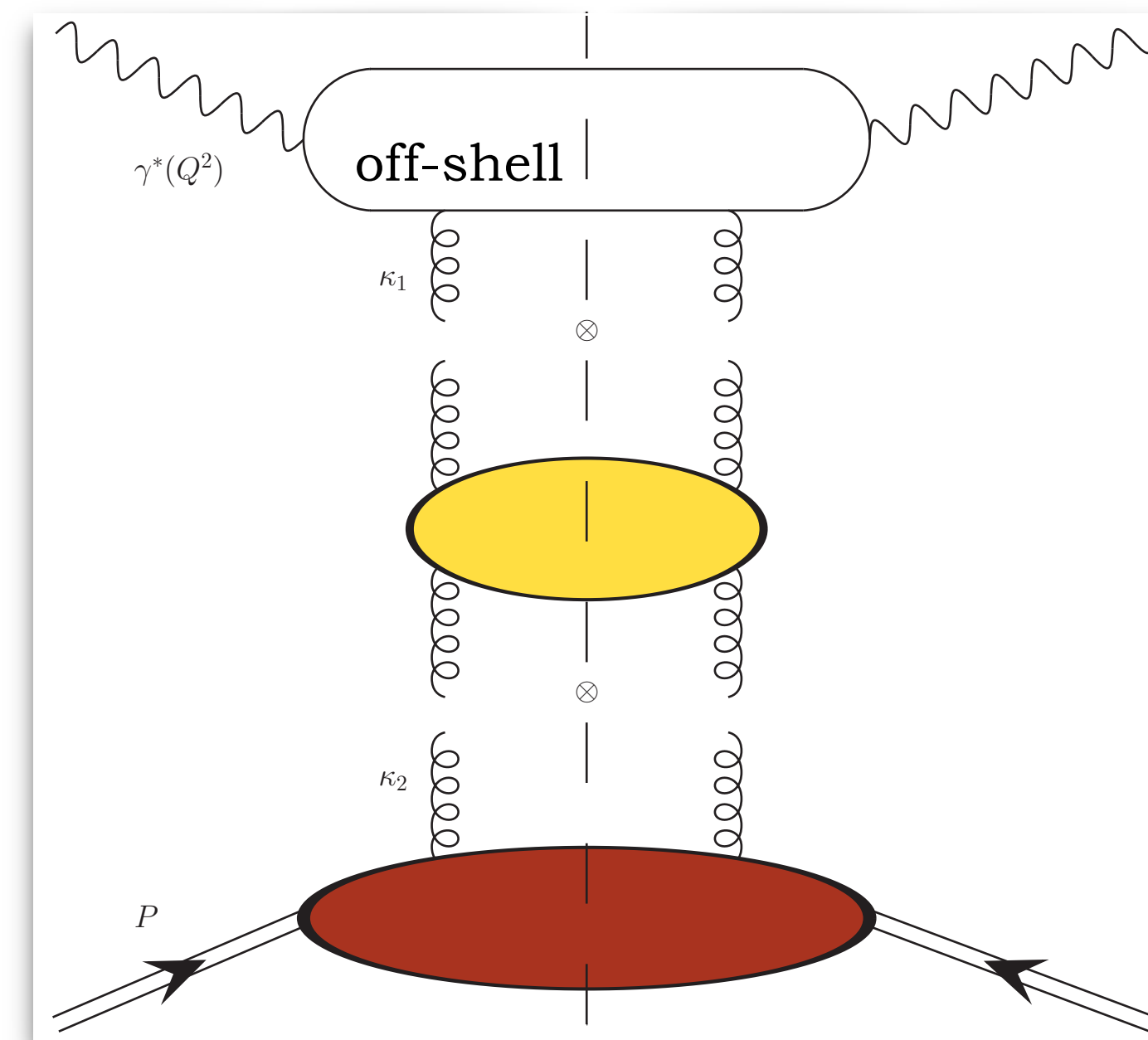
HEF

* Inclusive or exclusive processes (!)

* Small x , large κ_T

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* Diagram: DIS



$\Phi^{\gamma^* \rightarrow \gamma^*}$



$\mathcal{G}_{\text{BFKL}}$



$\Phi^P_{[\text{NP}]}$

Backup

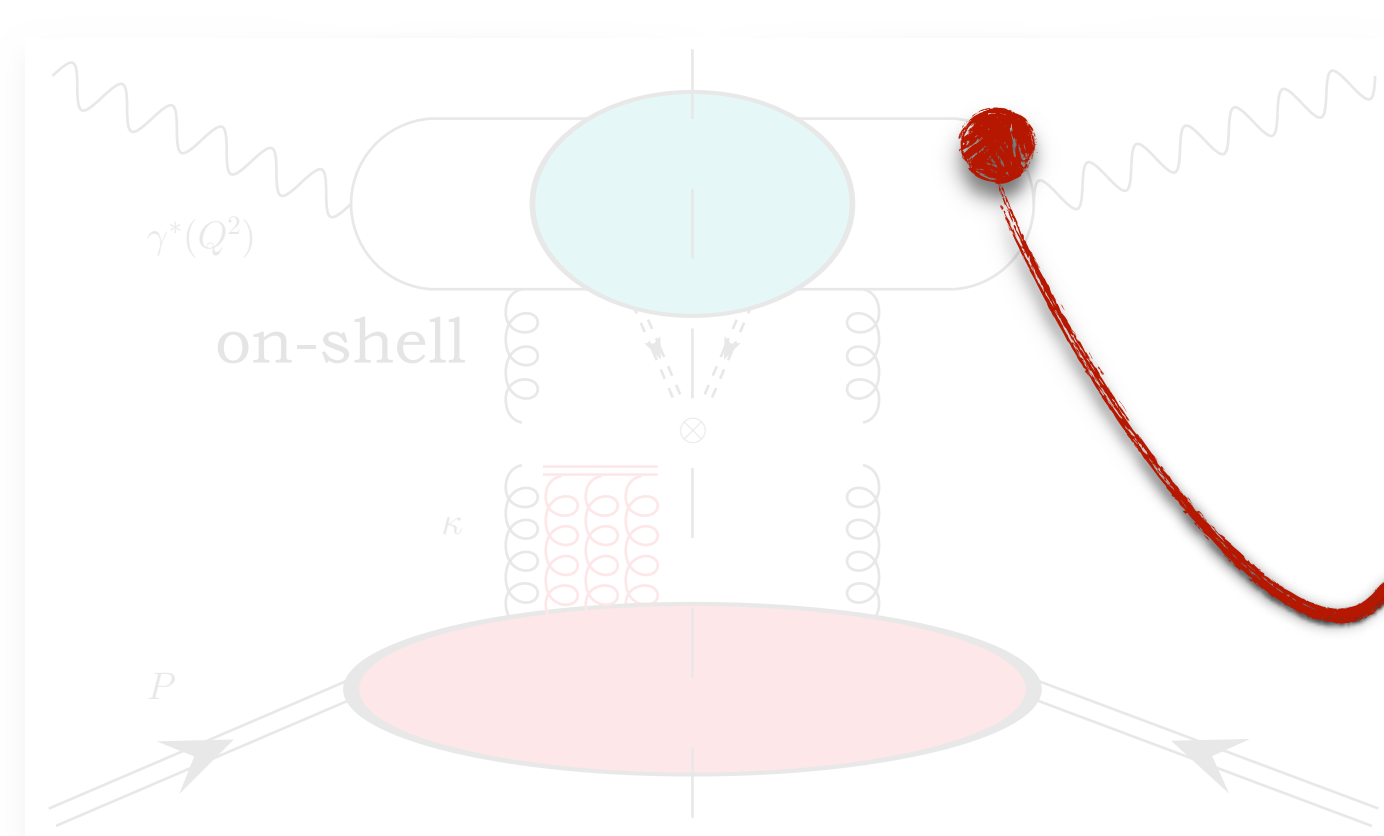
TMD versus high-energy factorization

TMD

IR-safe colorless $\{\Phi^{i \rightarrow 0}\}$
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 * Semi-inclusive processes
 [V.S. Fadin, A.D. Martin (1999)]

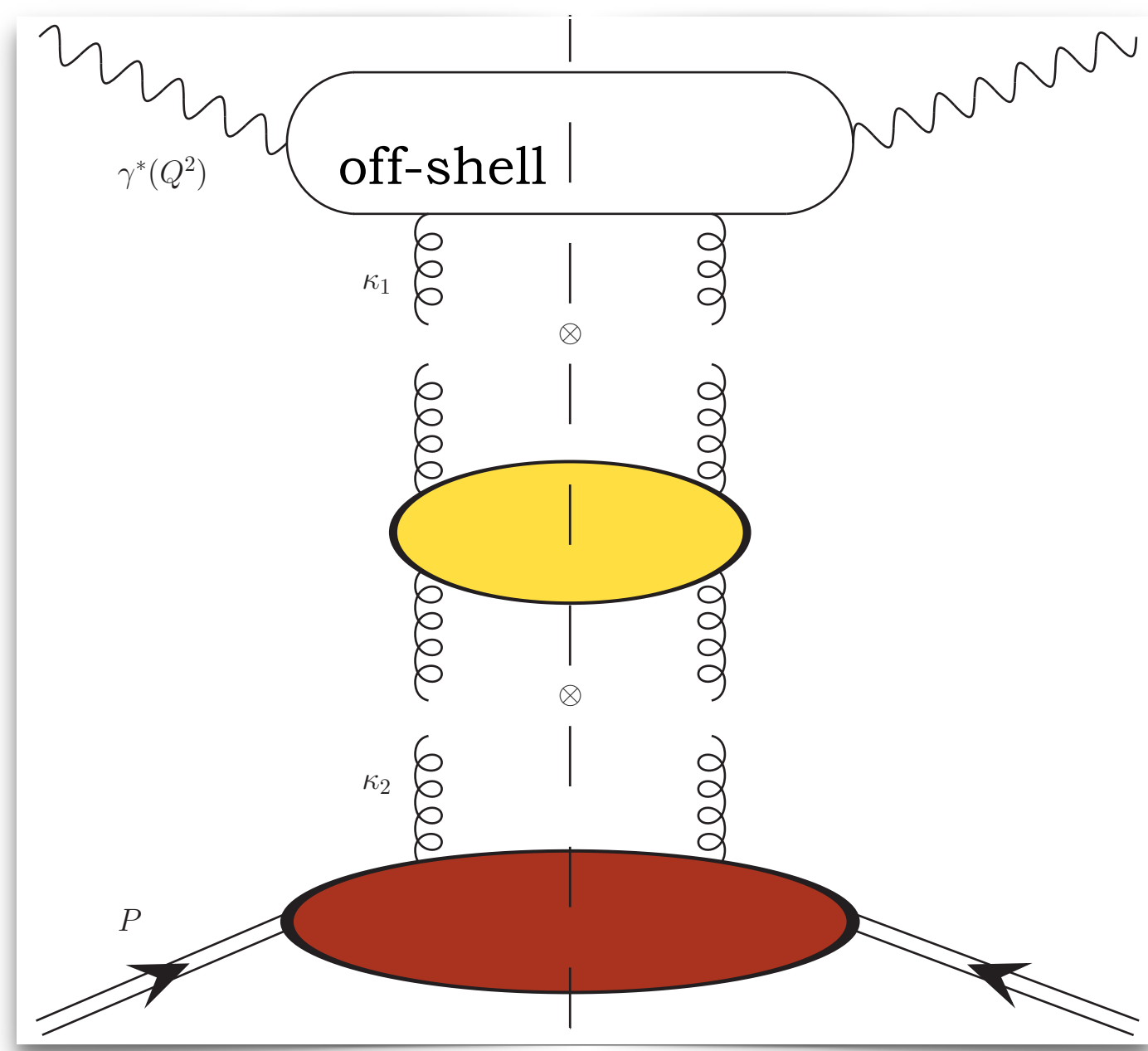
* $\kappa_T \ll$ hardest scale
 * Language of parton correlators
 * Diagram: SIDISonium

IR diffusion pattern
 (Bartels' cigar)
 [J. Bartels, H. Lotter (1993)]



HEF

- * Inclusive or exclusive processes (!)
- * Small x , large κ_T
- * Language of Reggeized gluons
- * Diagram: DIS



$\Phi^{\gamma^* \rightarrow \gamma^*}$
 \otimes
 $\mathcal{G}_{\text{BFKL}}$
 \otimes
 $\Phi^P_{[\text{NP}]}$

Q^2
 Q_0^2

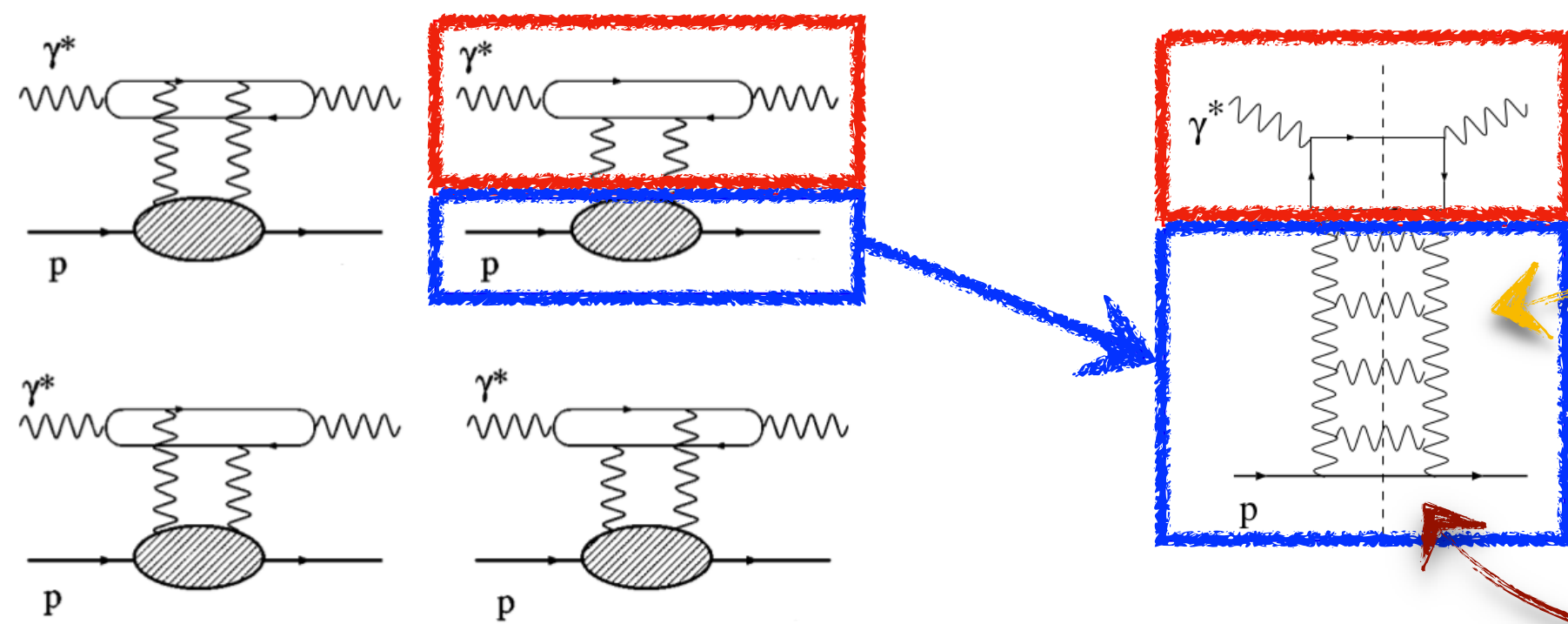
Backup

High-energy factorization and the UGD

- example: **virtual photoabsorption** in **high-energy factorization**

$$\sigma_{\text{tot}}(\gamma^* p \rightarrow X) \propto \text{Im}_s \{ \mathcal{A}(\gamma^* p \rightarrow \gamma^* p) \} \equiv \Phi_{\gamma^* \rightarrow \gamma^*} \circledast \mathcal{F}(x, \kappa^2)$$

- ◇ $\mathcal{F}(x, \kappa^2)$ is the **unintegrated gluon distribution (UGD)** in the proton

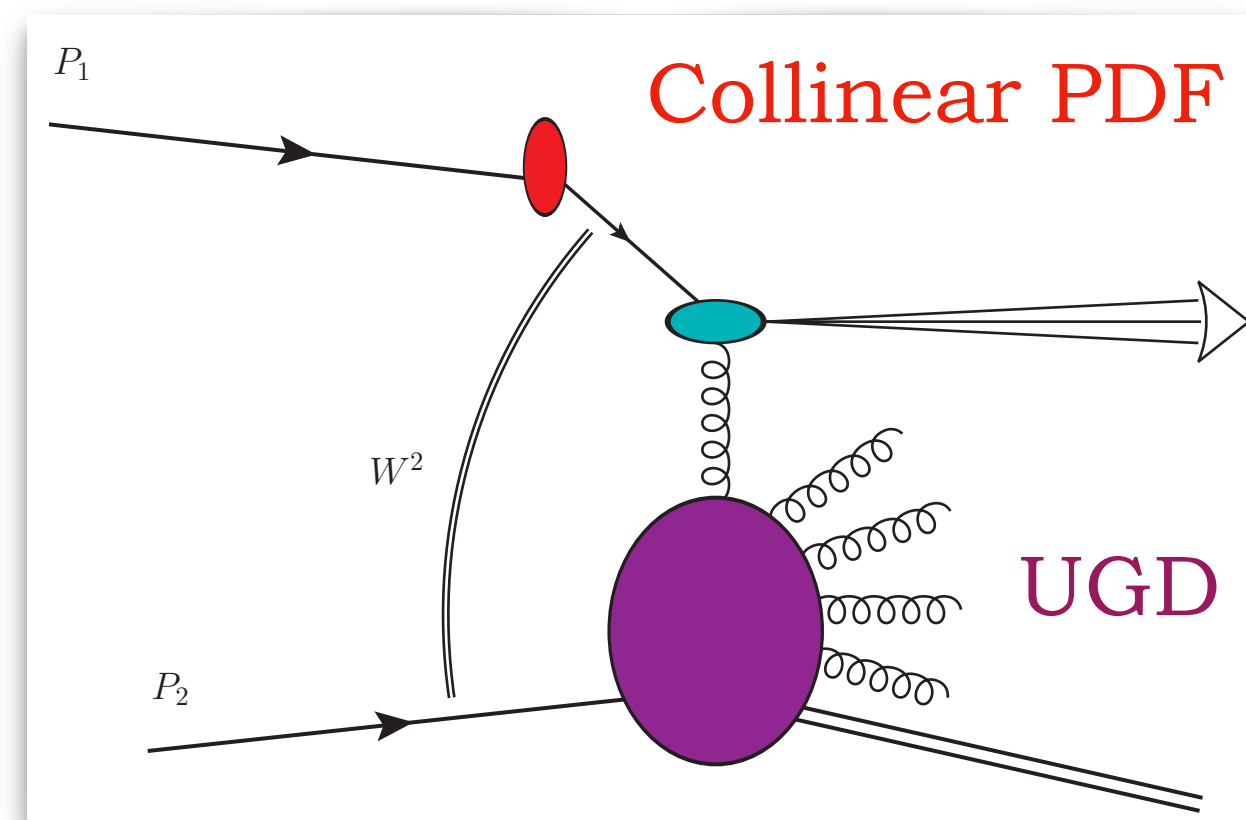


- ▶ Small- x limit: **UGD** = [**BFKL gluon ladder**] \circledast [**proton impact factor**]
 - ◇ Takes into account the **resummation** of **high-energy logs**
 - ◇ Describes the **coupling** of the gluon Green's function to the **proton**
- ▶ Proton impact factor is non-perturbative \implies UGD needs to be modeled!

Hybrid or pure factorization?

Forward emissions

- * *Asymmetric* config. \leftrightarrow fast parton + small- x gluon
- * Hybrid **high-energy** / **collinear** factorization

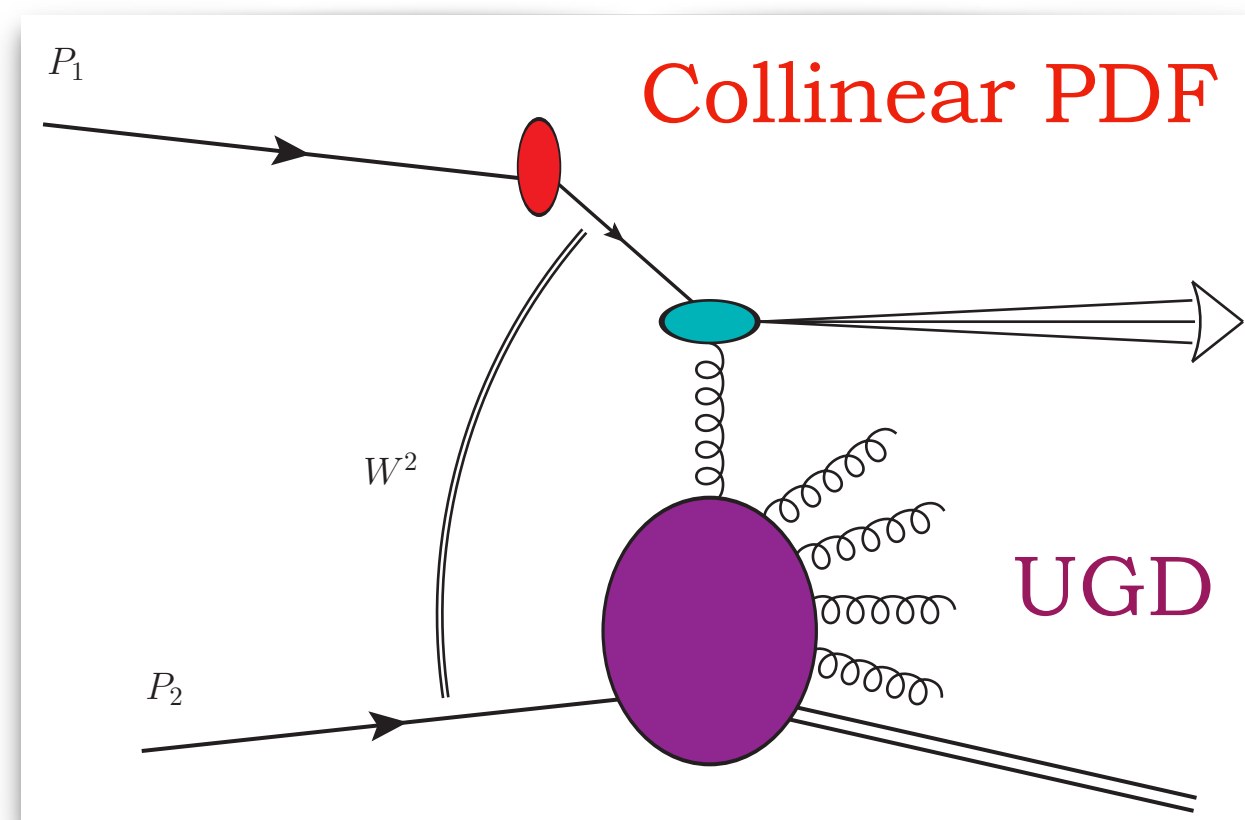


- * *Distinctive signals* of small- x dynamics **expected**
- * Phenomenology:
forward jet, Drell-Yan, Higgs or vector meson

Hybrid or pure factorization?

Forward emissions

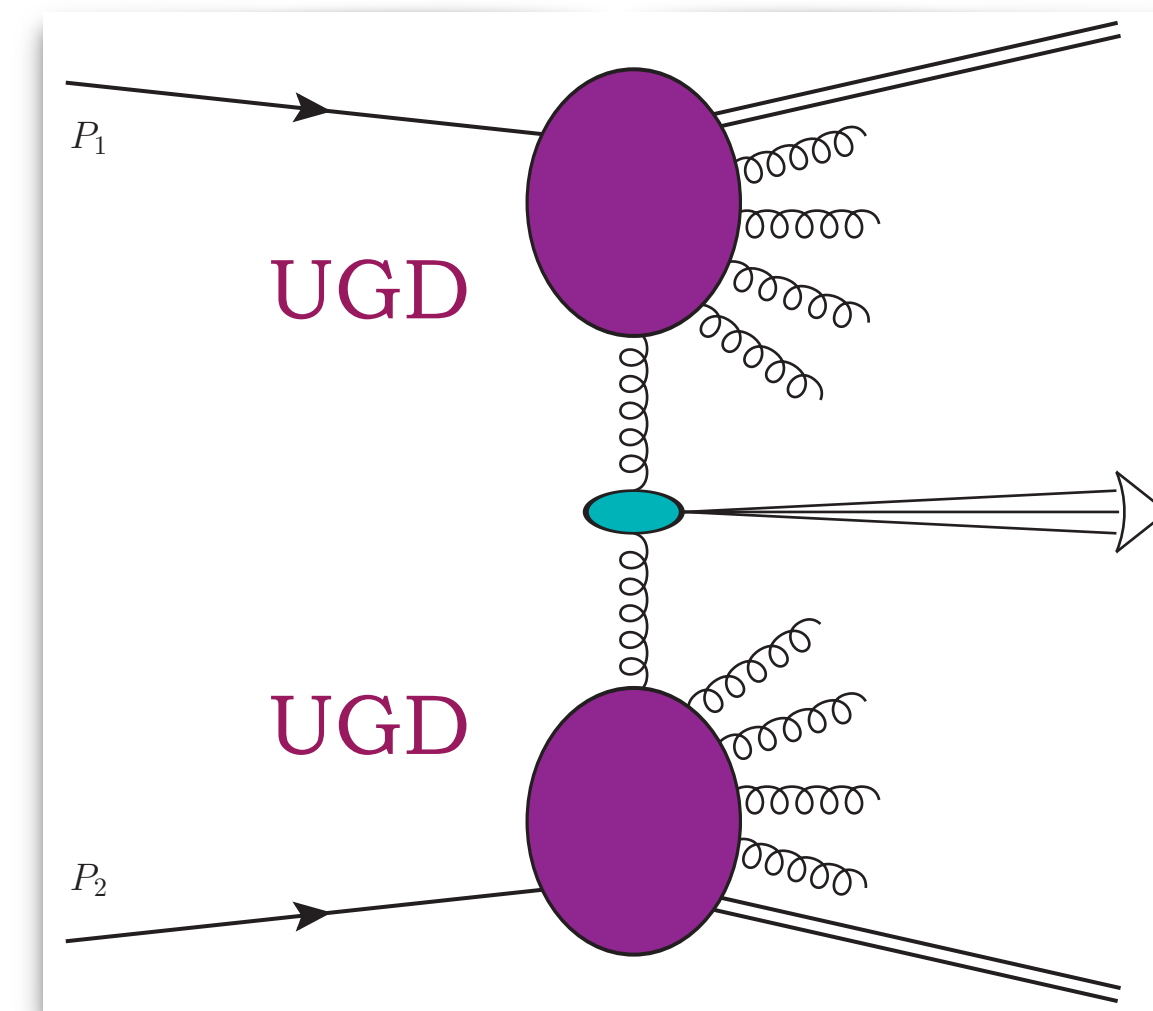
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- * *Distinctive signals* of small- x dynamics **expected**
- * Phenomenology:
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Central emissions

- * *Gluon induced* \leftrightarrow small- x gluons
- * Pure **high-energy** factorization

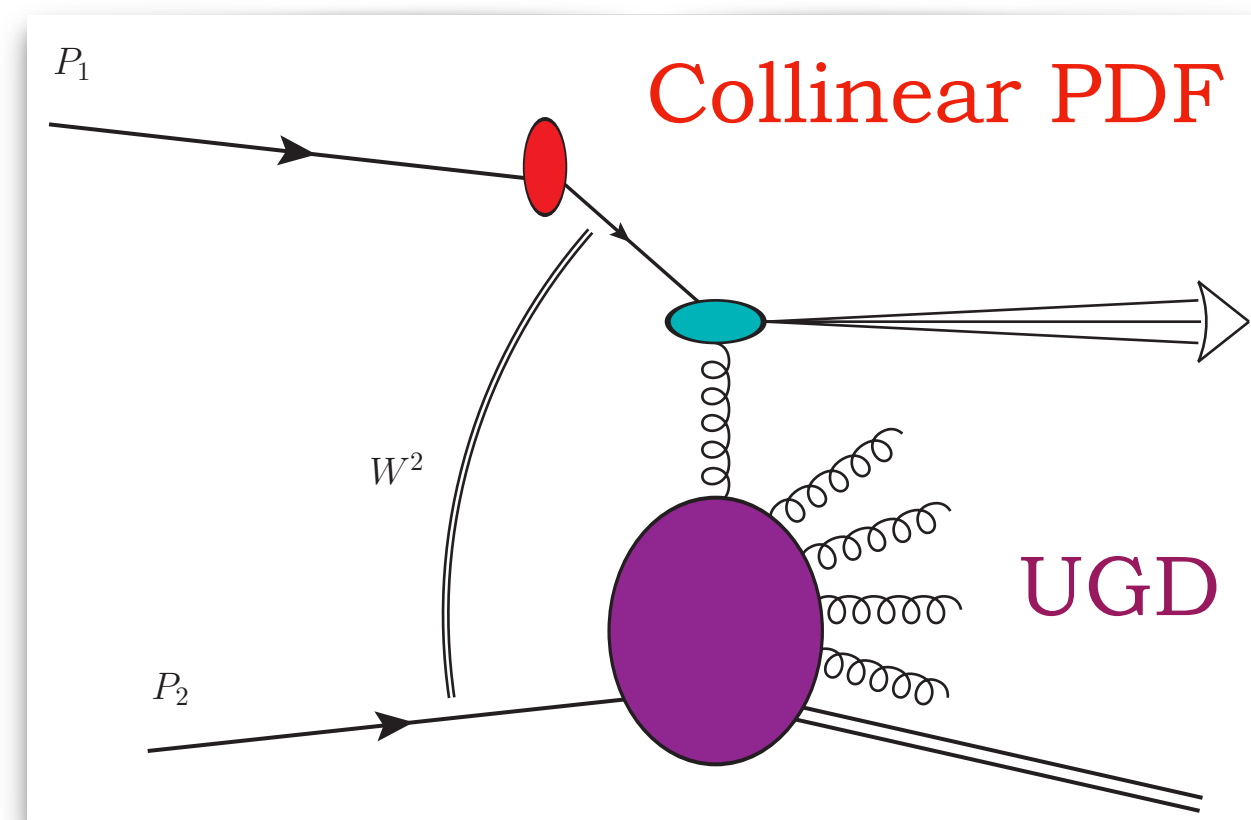


- * Small- x dynamics to **enhance** f.o. description
- * Phenomenology:
central jet, Higgs or vector meson

Hybrid or pure factorization?

Forward emissions

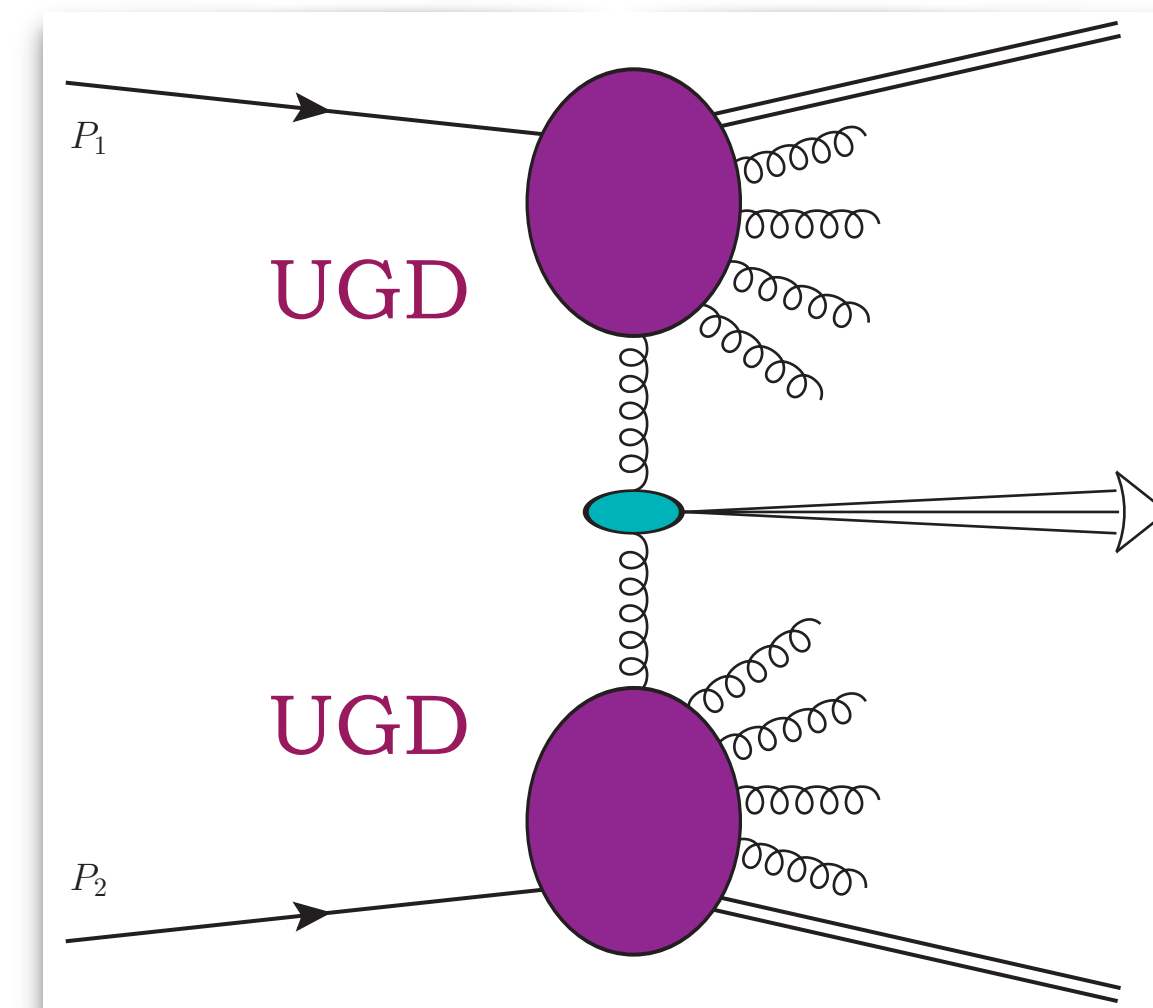
- * *Asymmetric* config. \leftrightarrow fast parton + small- x gluon
- * Hybrid **high-energy** / **collinear** factorization



- * *Distinctive signals* of small- x dynamics **expected**
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Central emissions

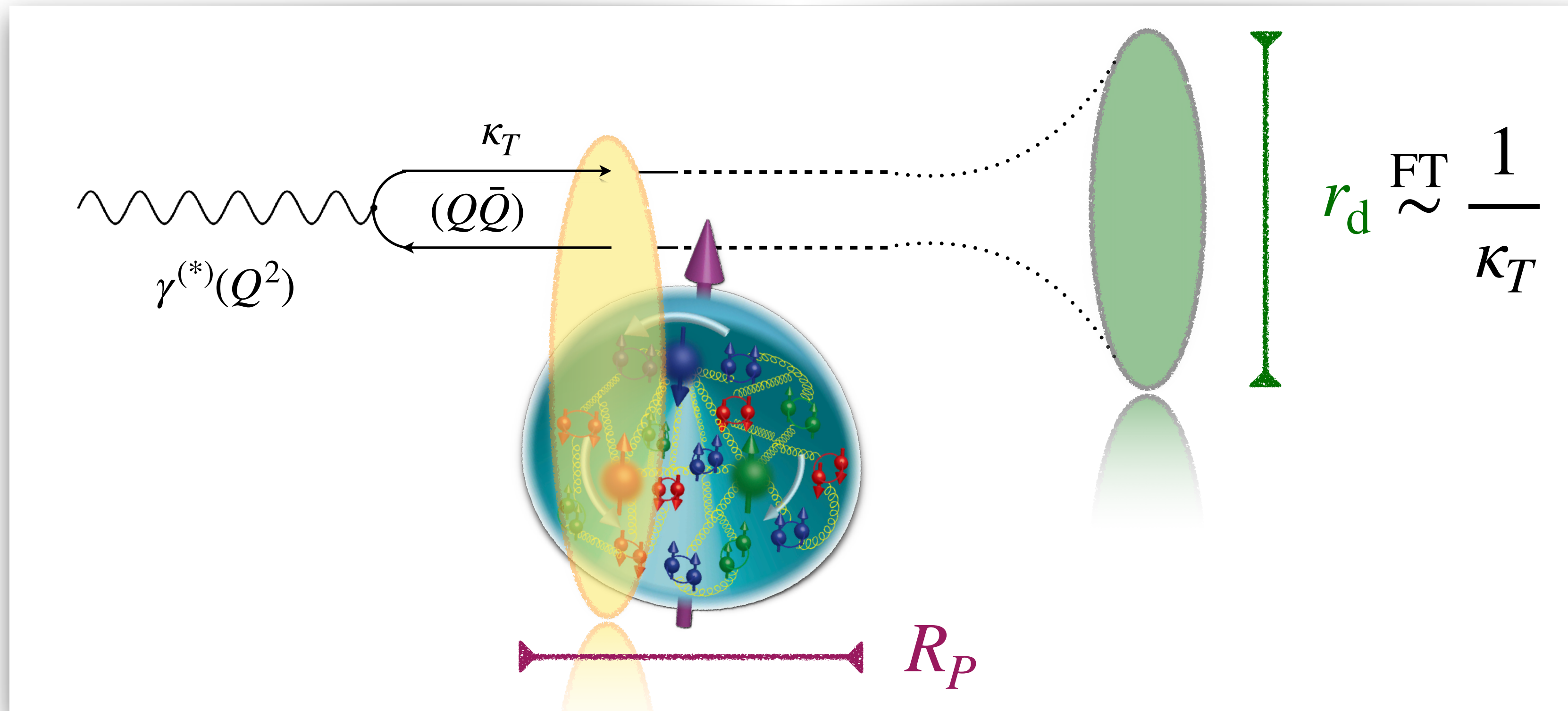
- * *Gluon induced* \leftrightarrow small- x gluons
- * Pure **high-energy** factorization



- * Small- x dynamics to **enhance** f.o. description
- * Phenomenology:
central jet, Higgs or vector meson

Table complemented by *exclusive* counterparts and *lepto-hadronic* channels

Diffractive γ^*P scatterings and color dipoles



$$W_{\mu\nu} \propto \text{Im} \left\{ i \int d^4x e^{iq \cdot x} \langle P | T [J_\mu(x) J_\nu(0)] | P \rangle \right\}$$

- * Small- $x \Rightarrow$ Ioffe time $\gg R_P$
- * At least one J_μ outside proton...
- * ...color dipole picture!

The background of the slide features a repeating pattern of Feynman diagrams. Each diagram is contained within a semi-transparent, light-colored circular frame. The diagrams themselves are composed of various colored lines and vertices: yellow wavy lines representing photons or gluons, blue and red straight lines representing fermions, and green and red vertices representing interactions. Some diagrams include arrows indicating the direction of particle flow. The overall aesthetic is scientific and technical, with a light blue and green color palette.

Exclusive forward ρ meson leptonproduction

Exclusive light VM: ρ^0, ω, ϕ

* *Small-size* dipoles \Rightarrow large κ_T

* **Collinear** description: twist-2/-3 LVM NP **DAs**

$$\Phi^{\gamma^* \rightarrow \rho} \propto \int_0^1 dz T_H^{\gamma^* \rightarrow \rho}(z, \kappa_T, Q, \mu_R, \mu_F) \phi^{\lambda_\rho}(z, \mu_F)$$

* Significance of small κ_T under investigation...

* HERA indication: no large- r_d dynamics

* **LVMs as tools**: discrimination among UGD models

* **LVMs as tools**: UGD extraction \Leftarrow HERA + EIC fits

Single forward emissions

Exclusive light VM: ρ^0, ω, ϕ

- * *Small-size* dipoles \Rightarrow large κ_T
- * **Collinear** description: twist-2/-3 LVM NP **DAs**
- $$\Phi^{\gamma^* \rightarrow \rho} \propto \int_0^1 dz T_H^{\gamma^* \rightarrow \rho}(z, \kappa_T, Q, \mu_R, \mu_F) \phi^{\lambda_\rho}(z, \mu_F)$$
- * Significance of small κ_T under investigation...
- * HERA indication: no large- r_d dynamics
- * **LVMs as tools**: discrimination among UGD models
- * **LVMs as tools**: UGD extraction \Leftarrow HERA + EIC fits

Quarkonia

- * Size of dipoles \Rightarrow wide range of κ_T
- * Description: **NRQCD** (combined with LFWFs)
- $$[\text{LFWF} \otimes \mathcal{A}_{\text{dip.}}] \xleftrightarrow{\text{dilute}} [\Phi^{\gamma^* \rightarrow J/\Psi} \otimes \text{UGD}]$$
- * Validity of *small-size* dipoles questionable...
- * NRQCD: large- r_d dynamics for $\Psi(2s)$ ($\Upsilon(2s)$?)
- [\[K. Suzuki et al. \(2000\)\]](#); [\[J. Cepila et al. \(2019\)\]](#); [\[M. Hentschinski et al. \(2020\)\]](#)
- * **Onia as tools**: scan of TMD/HEF intersection range