Revised TMD shape function in SIDIS

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Onia as Tools 2023

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Outline

- TMD factorization involving shape function
- Matching procedure
  - Hard amplitude pole structure
- TMD shape function and J/ψ polarization
- Conclusions and outcome

Gluon TMDs

Processes involving J/ψ are great to access gluon TMDs

No true extractions of gluon TMDs (yet)

*leading twist

Gluon polar. / Nucleon polar.	Unpolarized	Circular	Linear
Unpolarized	f_1		$h_1^{\perp}$
Longitudinal		g_{1L}	$h_{1L}^{\perp}$
Transverse	$f_{1T}^{\perp}$	g_{1T}	$h_1, h_{1T}^{\perp}$

Mulders, Rodriguez, *PRD 63* (2001)

Our knowledge will deepen by extracting gluon TMDs from few processes

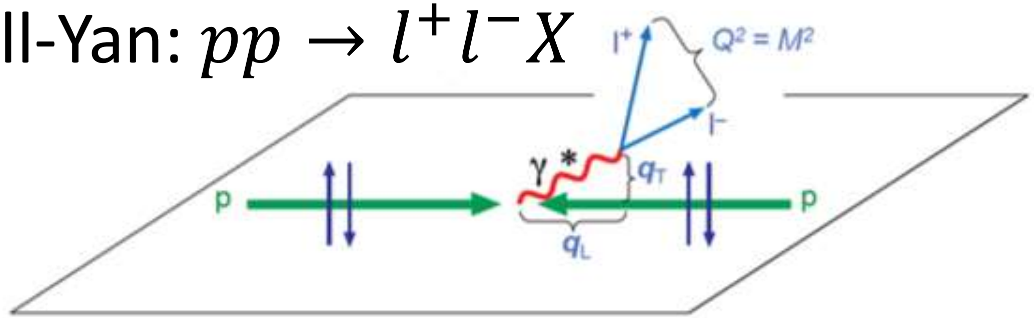
Theoretical TMD established processes

TMD factorization is formally proven only for few processes

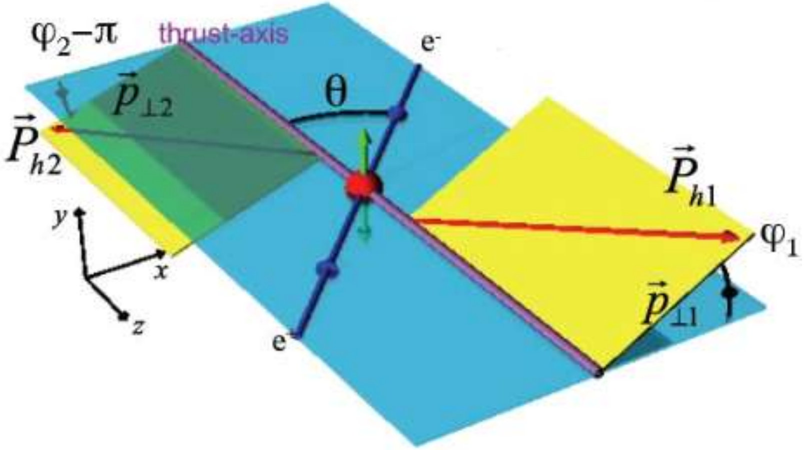
Collins, *Cambridge University Press* (2011)

Echevarría Idibi Scimemi, *JHEP 07* (2012)

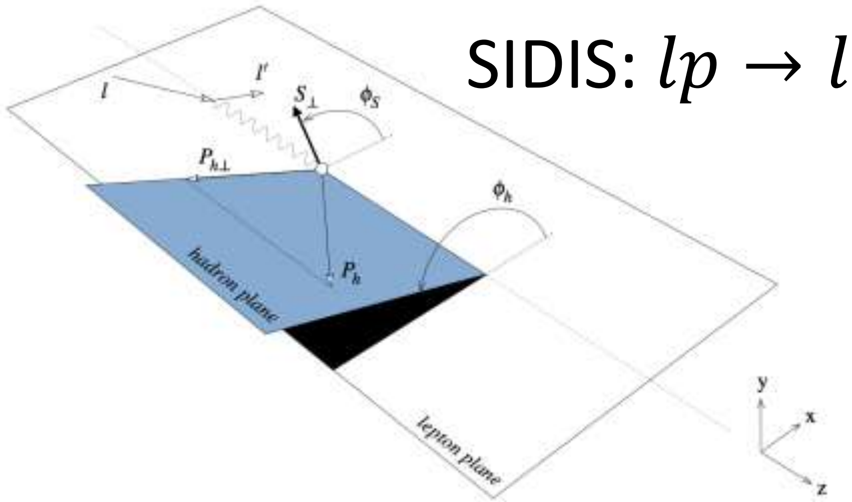
Drell-Yan: $pp \rightarrow l^+ l^- X$



lepton annihilation: $e^+ e^- \rightarrow \pi\pi X$



SIDIS: $lp \rightarrow l' hX$



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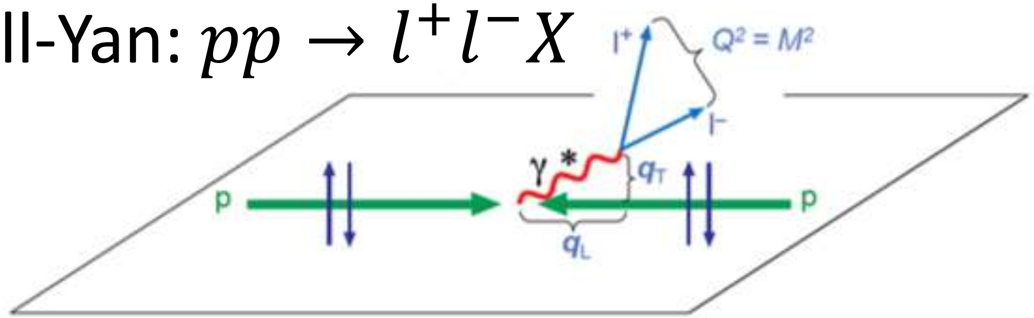

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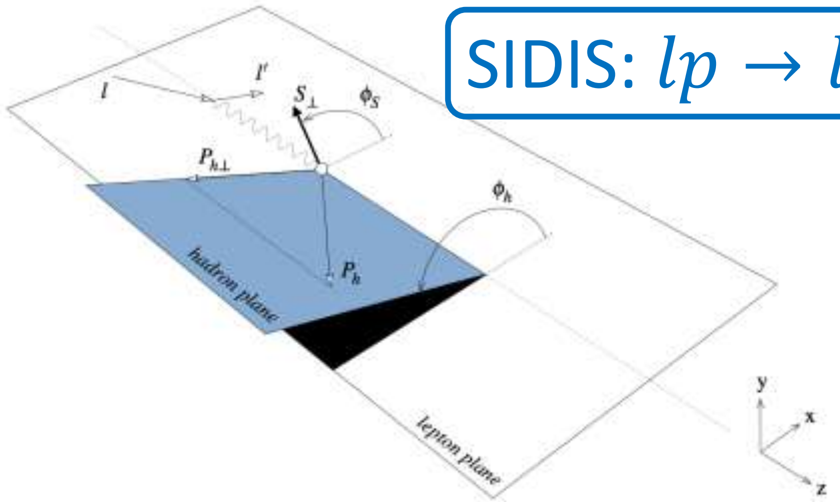

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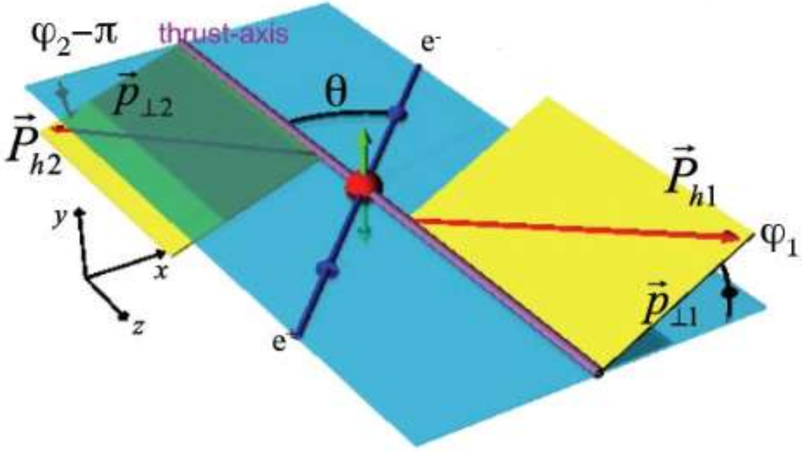
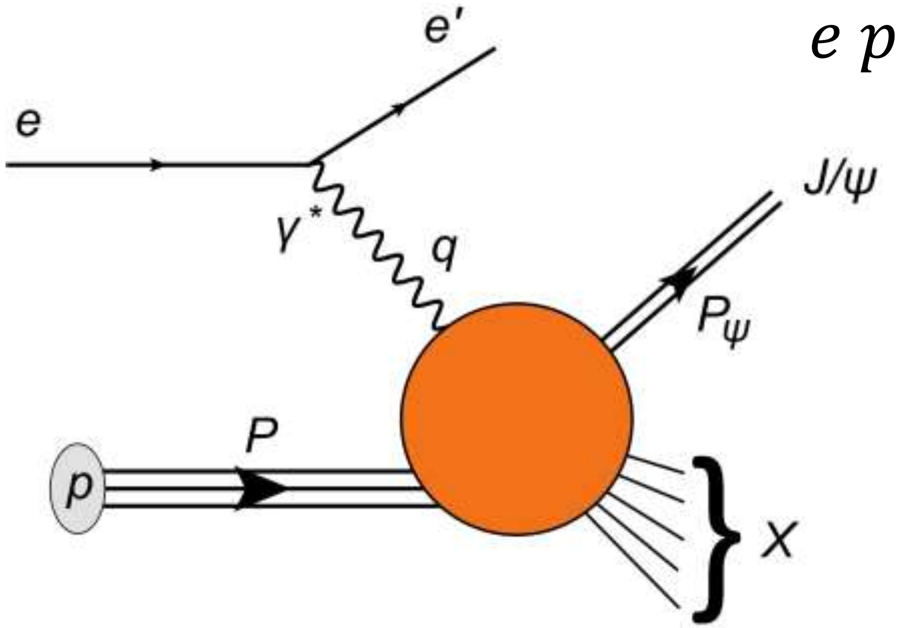


Bacchetta, Boer, Pisano, Taels, *Eur. Phys. J. C 80* (2020)

Quarkonium production has same color flow

➡ No factorization breaking expected

Semi-Inclusive DIS and structure functions



$$e\,p(P) \to e'\gamma^*(q)\,p(P) \to e'\,J/\psi\big(P_\psi\big) + X$$

SIDIS variables

$$Q^2 = -q^2,\, S \approx 2P\cdot l$$

$$x_B = \frac{Q^2}{2P\cdot q},\, y = \frac{P\cdot q}{P\cdot l},\, z = \frac{P\cdot P_\psi}{P\cdot q}$$

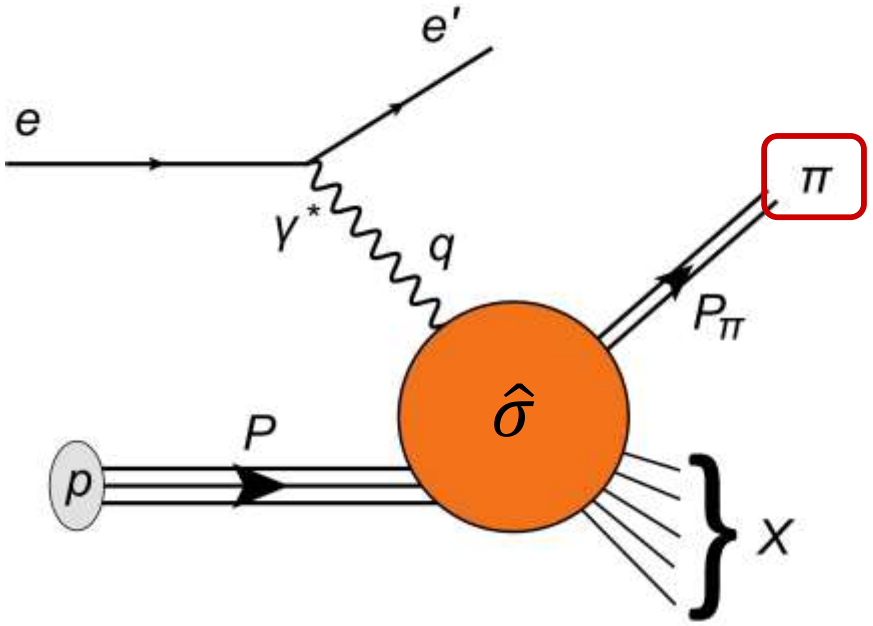
phase spaces

$$\frac{\mathrm{d}^3 l'}{2E'} = y\,S\,\mathrm{d}x_B\mathrm{d}y\mathrm{d}\phi'$$

$$\frac{\mathrm{d}^3 P_\psi}{2E_\psi} = \frac{\mathrm{d}z}{z}\mathrm{d}P_\perp^2\,\mathrm{d}\phi_\psi$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_B\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}\boldsymbol{P}_\perp^2\,\mathrm{d}\phi_\psi} = \frac{\alpha}{y\,Q^2}\Big\{[1-(1-y)^2]F_{UU,T} + (1-y)F_{UU,L}$$

Collinear factorized formula



(standard) SIDIS

Hard scale

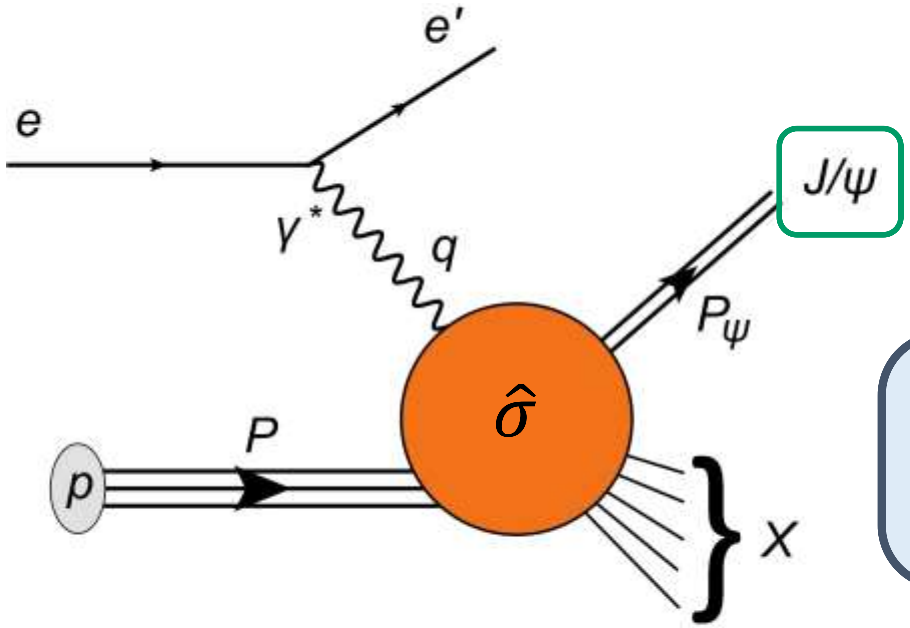
$\mu^2 \gg \Lambda^2_{\mathrm{QCD}} \quad \Rightarrow \quad \alpha_s(\mu^2) \ll 1$

Outgoing hadron T.M.

$P_\perp^2 \gg \Lambda^2_{\mathrm{QCD}}$

$$\sigma^{ep\to h+X} = \underbrace{\hat{\sigma}(\mu^2)}_{\text{partonic cross section}} \otimes \underbrace{f_p(\xi;\mu^2)}_{\text{PDF}} \otimes \underbrace{D_h(\hat{z};\mu^2)}_{\text{FF}}$$

Collinear factorized formula



SIDIS involving J/ψ
(NRQCD to describe the hadronization)

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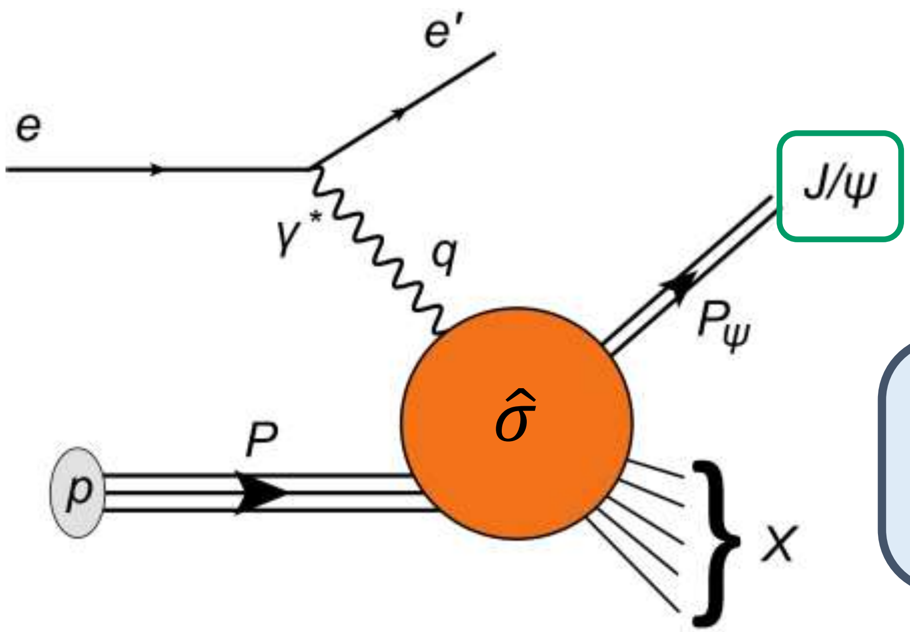
TMD factorized formula



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$$\sigma^{ep\to J/\psi+X} = \underbrace{\hat{\sigma}(\mu^2)}_{\text{partonic cross section}} \otimes \underbrace{f_p(\xi, \boldsymbol{p}_\perp^2; \mu^2)}_{\text{TMD-PDF}} \otimes \underbrace{\Delta^{[n]}(\mathbf{k}_\perp^2; \mu^2)}_{\text{TMD-shape function}}$$

partonic cross section

TMD-PDF

TMD-shape function

Echevarria, *JHEP 10* (2019)

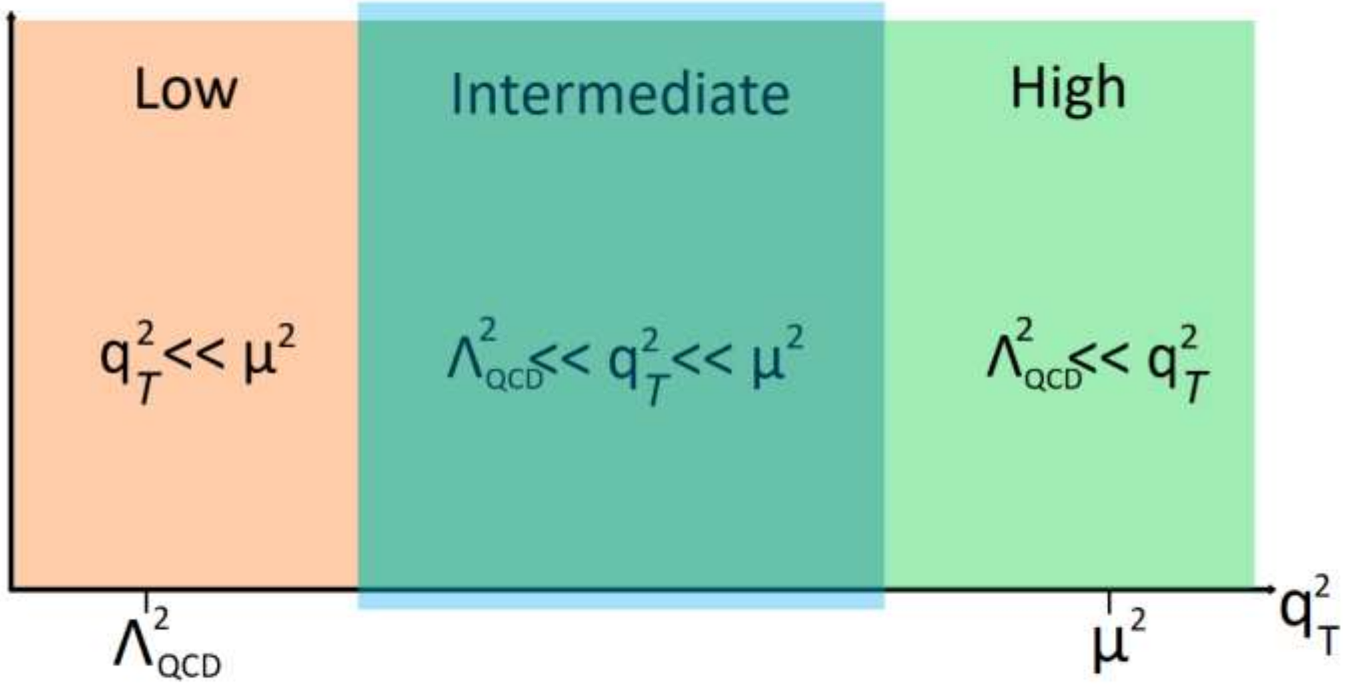
Fleming, Makris, Mehen, *JHEP 04* (2020)

Matching factorization schemes

$P_\perp$: J/ψ transverse momentum if γ^*-p along z-axis

q_T: γ^* transverse momentum if J/ψ-p along z-axis

$$|\boldsymbol{q}_T| = \frac{|\boldsymbol{P}_\perp|}{z}$$



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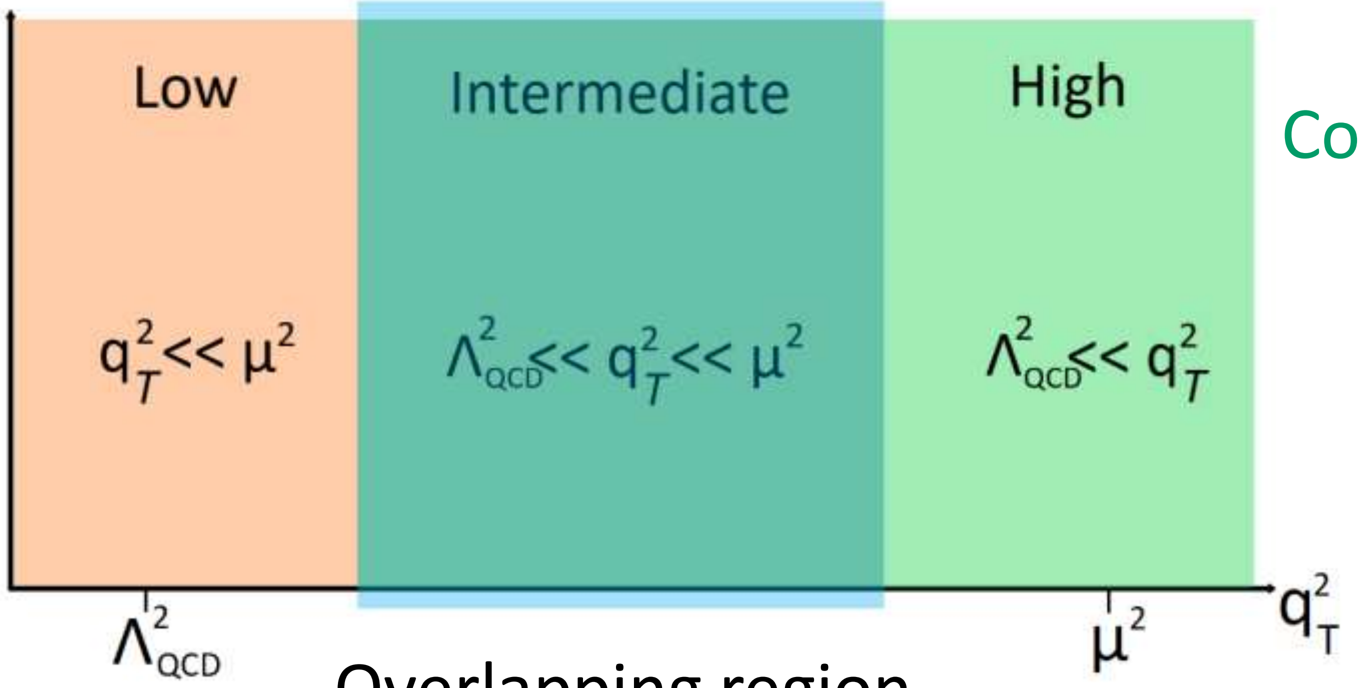
TMD factorization

↓

small transverse momentum

$q_T^2 \ll \mu^2$



Collinear factorization

↓

high transverse momentum

$q_T^2 \gg \Lambda_{QCD}^2$

Overlapping region

$\Lambda_{QCD}^2 \ll q_T^2 \ll \mu^2$

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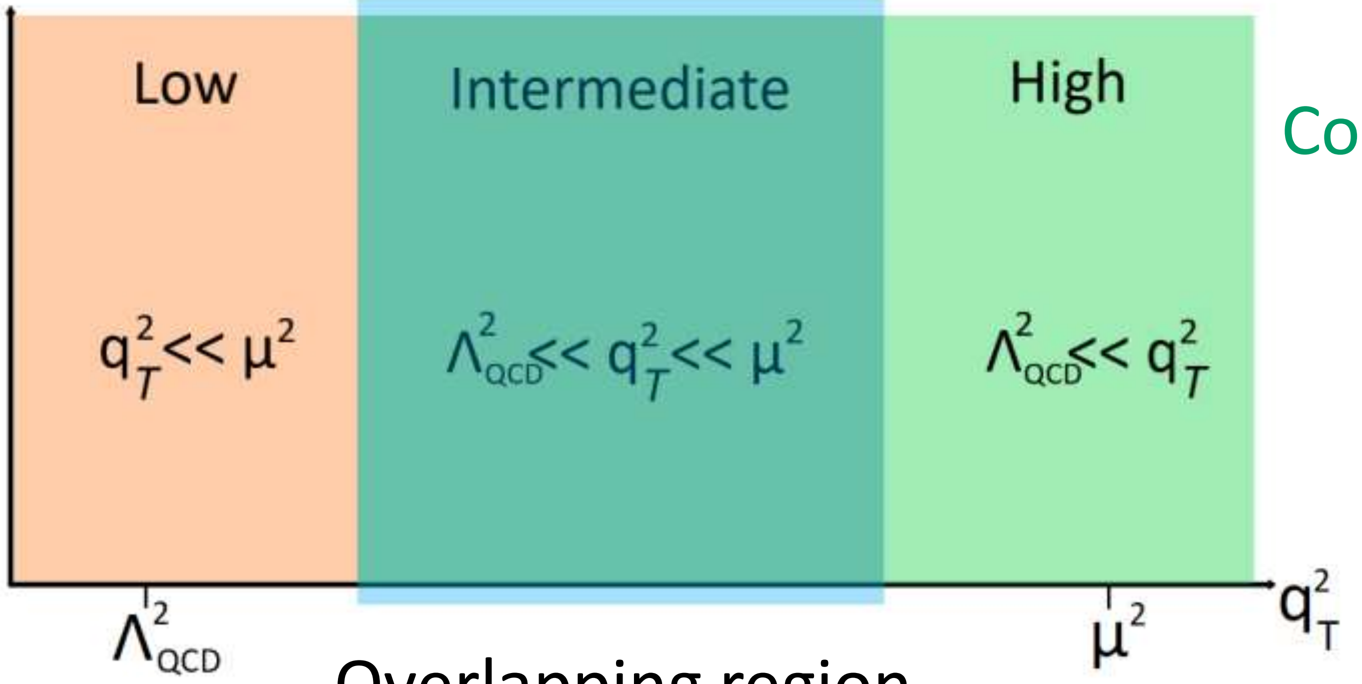


Collinear factorization

high transverse momentum

$q_T^2 \gg \Lambda^2_{QCD}$

Overlapping region

$\Lambda^2_{QCD} \ll q_T^2 \ll \mu^2$

Description of same dynamics?
Need to be matched!

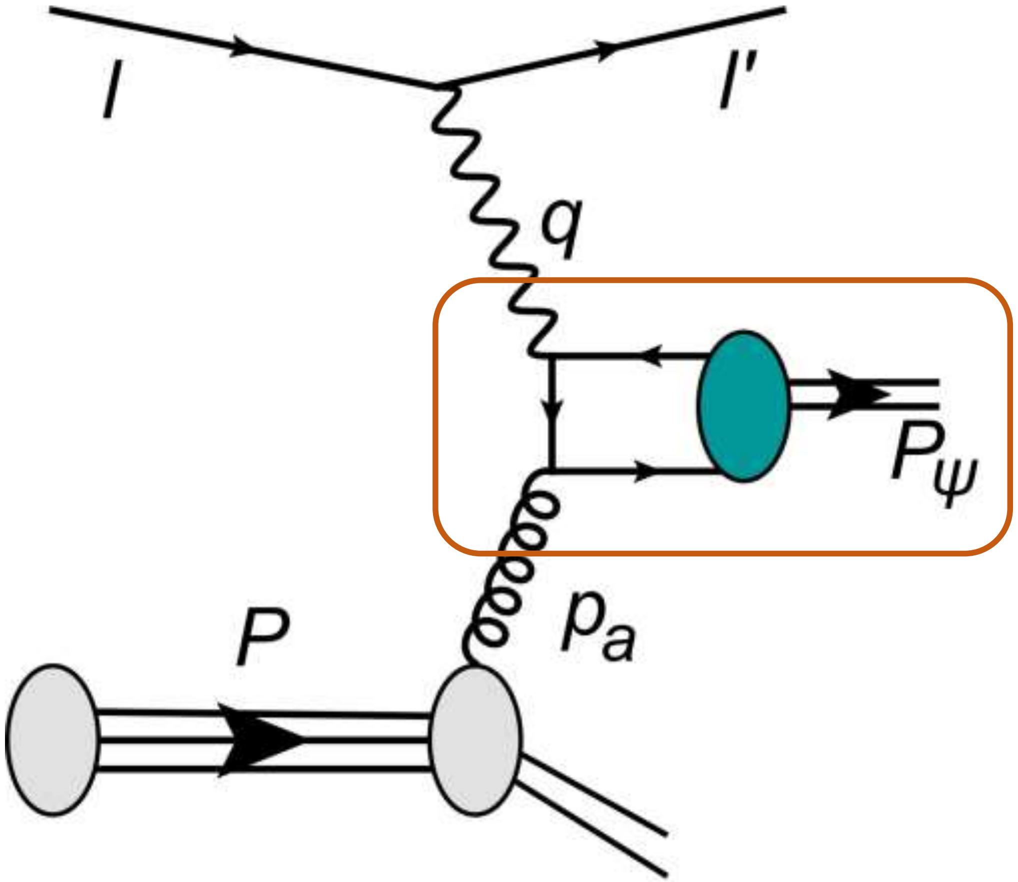
Bacchetta, Boer, Diehl, Mulders, *JHEP 08* (2008)

Boer, D'Alesio, Murgia, Pisano, Taels, *JHEP 09* (2020)

TMD calculation

Bacchetta, Boer, Pisano, Taels, *Eur. Phys. J. C 80* (2020)

$$\mathrm{d}\sigma \propto \int \mathrm{d}\xi\, \mathrm{d}^2\boldsymbol{p}_T\, \frac{L^{\mu\nu}\, \mathcal{H}^{[n]}_{\mu\alpha}\, \mathcal{H}^{[n]*}_{\nu\beta}\, \Gamma_U^{\alpha\beta}}{\mathrm{Q}^4}\, \delta^4\!\left(q + P - P_\psi\right)$$



Hard amplitude (order $\alpha\alpha_s$)

from $\gamma^*(q) + g(p_a) \rightarrow c\bar{c}[n](P_\psi)$

TMD calculation

$$d\sigma \propto \int d\xi \, d^2\boldsymbol{p}_T \, \frac{L^{\mu\nu} \mathcal{H}^{[n]}_{\mu\alpha} \mathcal{H}^{[n]*}_{\nu\beta} \Gamma^{\alpha\beta}_U}{Q^4} \delta^4(q + P - P_\psi)$$

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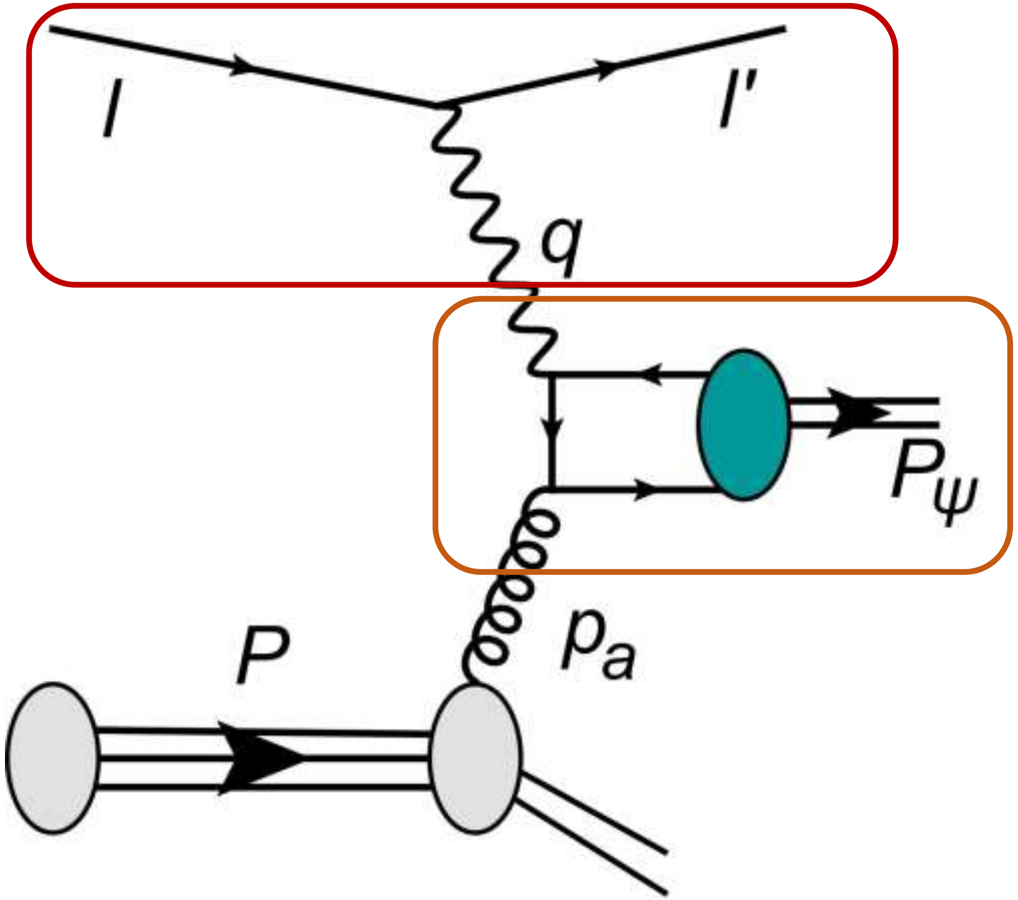

Bacchetta et al., *JHEP 02* (2007)

Lepton tensor

$$L^{\mu\nu} = (4\pi\alpha)Q^2 \left[-g^{\mu\nu} + 2\frac{l^{\{\mu} l'^{\nu\}}}{Q^2} \right]$$

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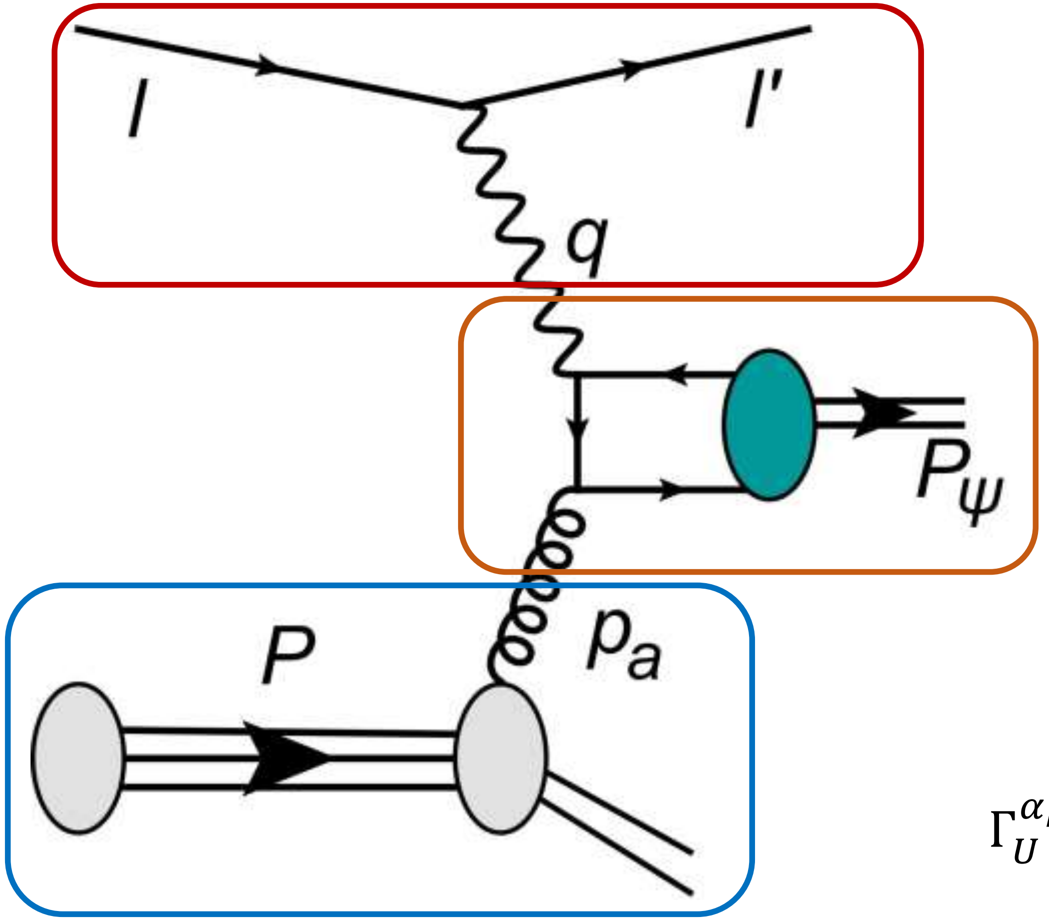

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Gluon correlator

Mulders, Rodriguez, *PRD 63* (2001)

$$\Gamma_U^{\alpha\beta} = \frac{1}{2x}\left[-g_T^{\alpha\beta} f_1^g(x, \boldsymbol{p}_T^2) + \frac{1}{2M_p^2}\left(2p_T^\alpha p_T^\beta + g_T^{\alpha\beta}\boldsymbol{p}_T^2\right) h_1^{\perp g}(x, \boldsymbol{p}_T^2)\right]$$

Small-q_T result (TMD)

valid at $z = 1$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_B\,\mathrm{d}y\,\mathrm{d}^2\boldsymbol{P}_\perp} \propto [1-(1-y)^2]\mathcal{F}_{UU,T} + (1-y)\mathcal{F}_{UU,L} + (1-y)\cos 2\phi_\psi \mathcal{F}_{UU}^{\cos 2\phi_\psi}$$

Involving the convolutions

$$\mathcal{C}\left[f_1^g \Delta^{[n]}\right](x, \boldsymbol{q}_T^2)$$

$$\mathcal{C}\left[w h_1^{\perp g} \Delta_h^{[n]}\right](x, \boldsymbol{q}_T^2)$$

➡ questions on this term ask Jelle!

Boer, Bor, *PRD 106* (2022)

Evolving the TMD result at $\Lambda_{QCD}^2 \ll q_T^2 \ll \mu^2$ we can compare with collinear one

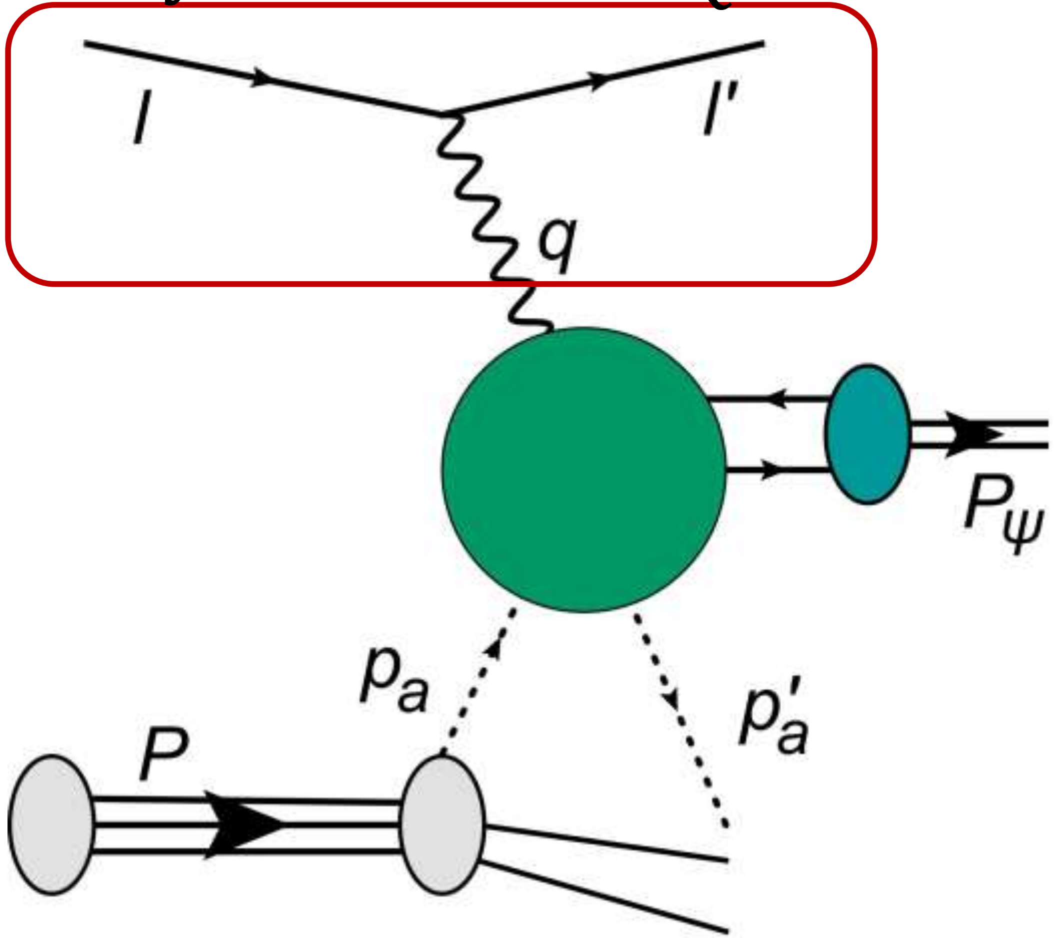
➡ extracting the TMDShF perturbative tail

Collinear calculation

$$d\sigma \propto \int \frac{d\hat{x}}{\hat{x}} \frac{d\hat{z}}{\hat{z}} \frac{L^{\mu\nu} \mathcal{H}_{\mu}^{a[n]} \mathcal{H}_{\nu}^{a[n]*}}{Q^6} f_1^a\left(\frac{x_B}{\hat{x}}; \mu^2\right) \delta\big(F(\hat{x},\hat{z})\big)\, \delta(\hat{z} - z)$$

$$\hat{x} = \frac{Q^2}{2p_a \cdot q}$$

$$\hat{z} = \frac{p_a \cdot P_\psi}{p_a \cdot q}$$



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Collinear calculation

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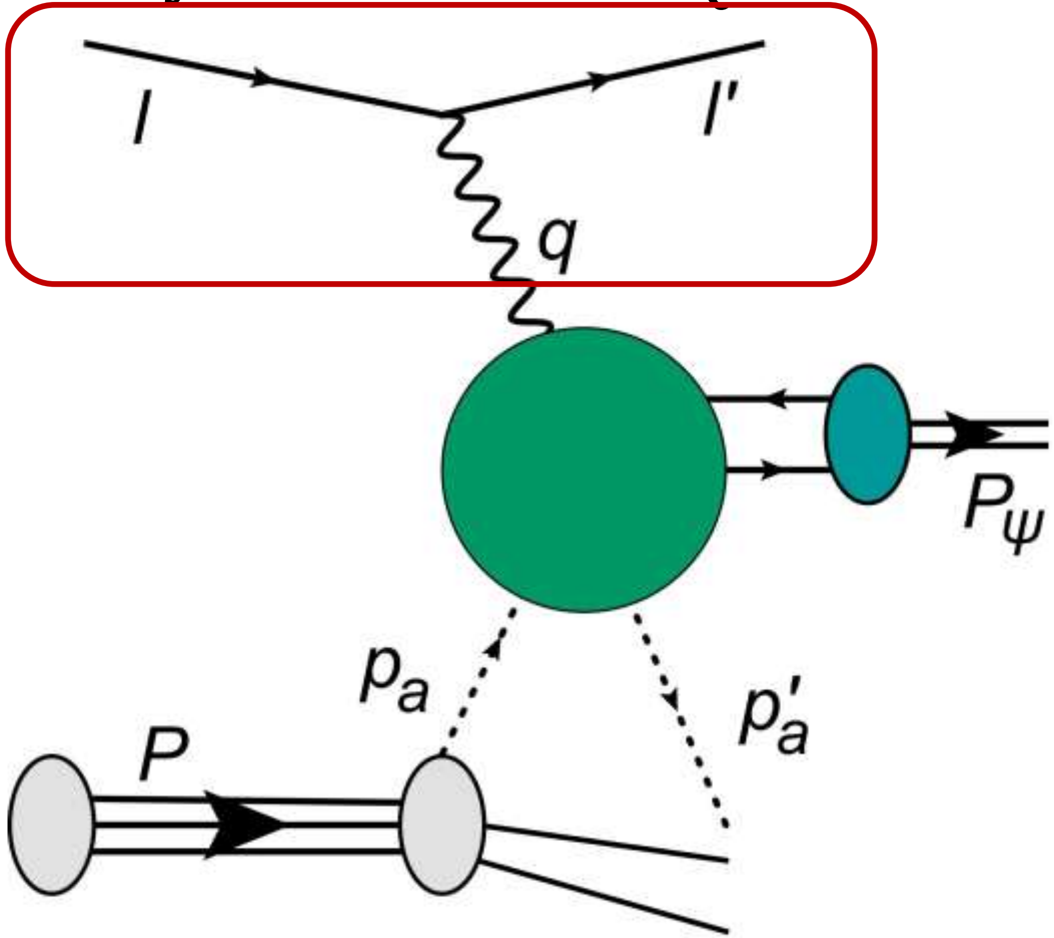

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Dirac-delta

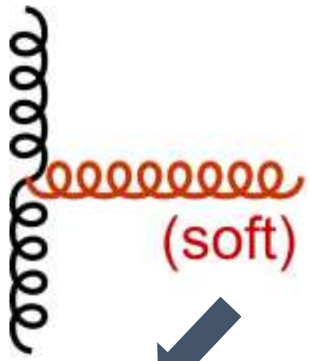
$$F(\hat{x},\hat{z}) = \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\hat{z}} - \frac{1-\hat{z}}{\hat{z}^2}\frac{M_\psi^2}{Q^2} - \frac{\boldsymbol{q}_T^2}{Q^2}$$

From high to intermediate q_T

Transverse momentum regions

$\Lambda^2_{QCD} \ll q_T^2 \ll \mu^2$

$q_T^2 \gg \Lambda^2_{QCD}$



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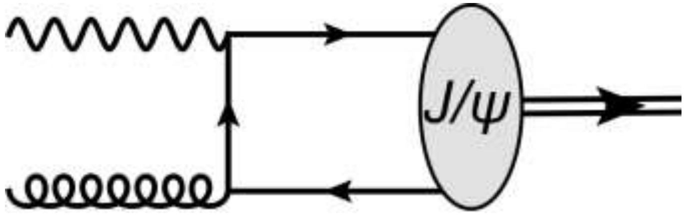
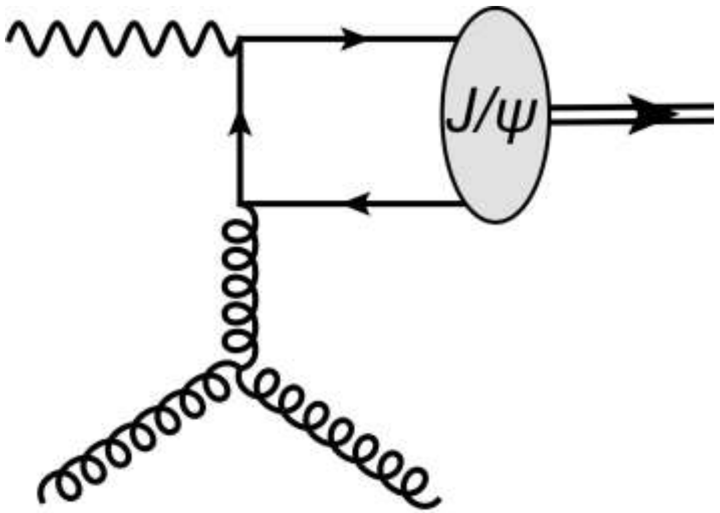




q_T divergent behavior
(resumed into Sudakov factor)

partonic cross sections $\hat{\sigma}$
appearing in TMD

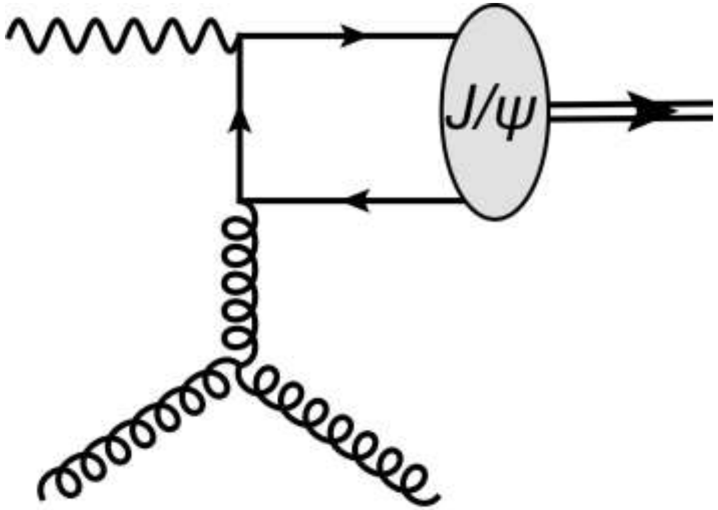
Obtained via the delta expansion

$$\delta\big(F(\hat{x},\hat{z})\big) \propto \frac{\hat{x}'}{(1-\hat{x}')_+}\delta(1-\hat{z}) + \log\frac{M_\psi^2+Q^2}{\boldsymbol{q}_T^2}\delta(1-\hat{x}')\delta(1-\hat{z})$$

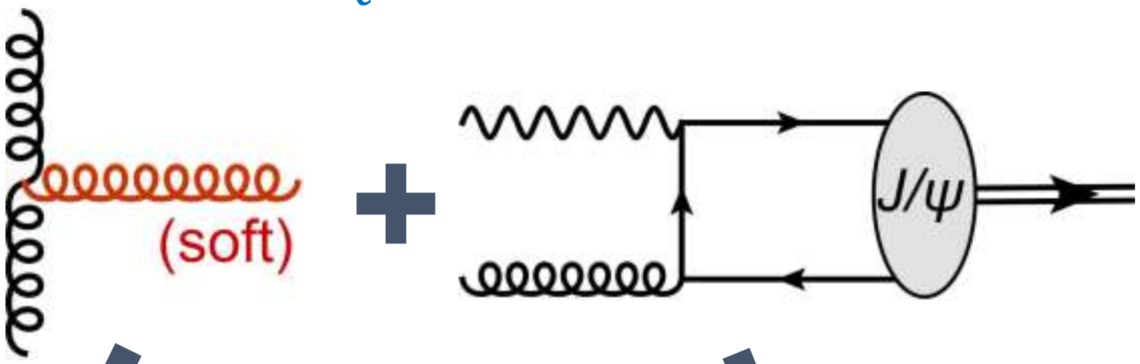
From high to intermediate q_T

Transverse momentum regions

$q_T^2 \gg \Lambda_{QCD}^2$



$\Lambda_{QCD}^2 \ll q_T^2 \ll \mu^2$



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From high to intermediate q_T (part II)

Dirac-delta expansion is applied only to continuous functions

$$F_{UU,\mathcal{P}} = F^{(0)}_{UU,\mathcal{P}}(\hat{x}',\hat{z}) + \Sigma_k \left(\frac{1-\hat{z}}{1-\hat{x}'}\right)^k F^{(k)}_{UU,\mathcal{P}}(\hat{z})$$

$$\hat{x}' = \frac{M_\psi^2 + Q^2}{2p_a \cdot q}$$

$$\hat{z} = \frac{p_a \cdot P_\psi}{p_a \cdot q}$$

Boer, Bor, Maxia, Pisano, *in progress*

Subleading, non-negligible term in the double delta*

*$\delta(1-\hat{x}')\delta(1-\hat{z})$

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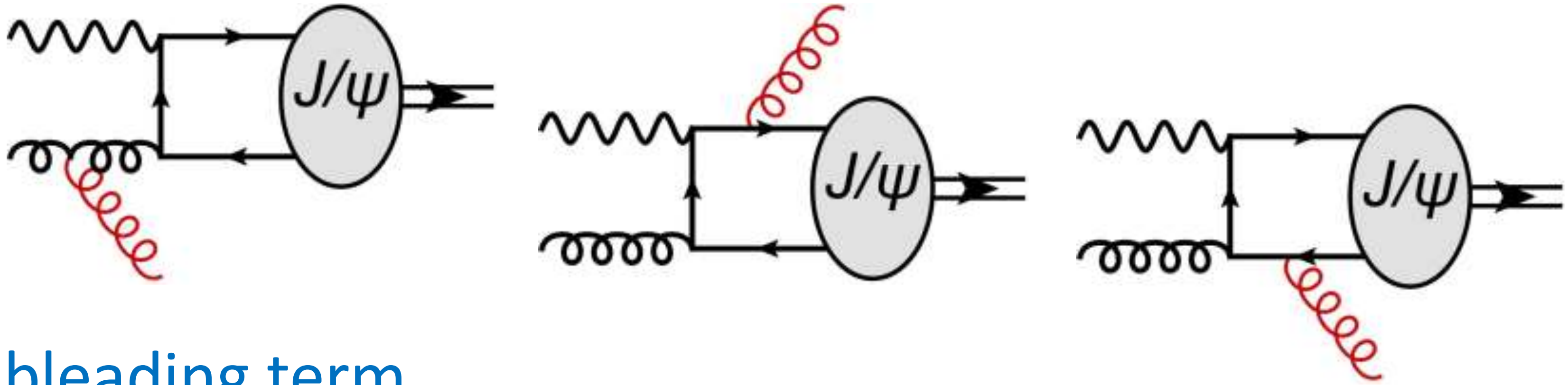
Subleading, non-negligible term in the double delta*

$*\delta(1-\hat{x}')\delta(1-\hat{z})$

Soft emission in diagrams



also indicate the need of subleading term

Comparison in the overlapping region

Transverse momentum region: $\Lambda_{QCD} \ll q_T \ll Q$

$$\mathcal{F}_{UU,\mathcal{P}}\Big|_{TMD} \neq F_{UU,\mathcal{P}}\Big|_{coll}$$

$$\Delta^{[n]}(\boldsymbol{k}_T^2) = \frac{\alpha_s}{2\pi^2 \boldsymbol{k}_T^2} C_A \left(\ln \frac{\mu^2}{\boldsymbol{k}_T^2} \right) \langle O[n] \rangle$$

$$+\mathcal{O}(\alpha_s)$$

Boer, D'Alesio, Murgia, Pisano, Taels, *JHEP 09* (2020)

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$$+\mathcal{O}(\alpha_s)$$

$$\mathcal{F}_{UU}^{\cos 2\phi_\psi}\Big|_{TMD} = F_{UU}^{\cos 2\phi_\psi}\Big|_{coll}$$

$$\Delta_h^{[n]}(\boldsymbol{k}_T^2) = \delta^{(2)}(\boldsymbol{k}_T^2)\langle O[n]\rangle$$

$$+\mathcal{O}(\alpha_s)$$

Boer, Bor, Maxia, Pisano, *in progress*

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Boer, Bor, Maxia, Pisano, *in progress*

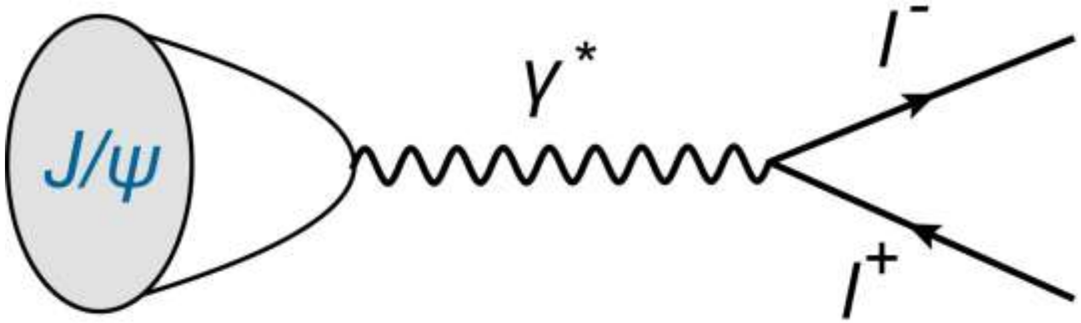
Precision up to the order

$$\mathcal{O}\left(\frac{\Lambda_{QCD}}{|\boldsymbol{q}_T|}\right), \quad \mathcal{O}\left(\frac{|\boldsymbol{q}_T|}{\mu}\right), \quad \mathcal{O}(\alpha_s^2), \quad \mathcal{O}(v^5)$$

→ dominated by CO waves: ${}^1S_0^{[8]}$, ${}^3P_J^{[8]}$

J/ψ polarization parenthesis

We can study the J/ψ polarization states by including the decay into $l^+ l^-$



$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_B\,\mathrm{d}y\,\mathrm{d}^4P_\psi\,\mathrm{d}\Omega} \propto \mathcal{W}_T(1+\cos^2\theta) + \mathcal{W}_L(1-\cos^2\theta) + \mathcal{W}_\Delta \sin 2\theta\cos\phi + \mathcal{W}_{\Delta\Delta}\sin^2\theta\cos 2\phi$$

J/ψ polarization parenthesis

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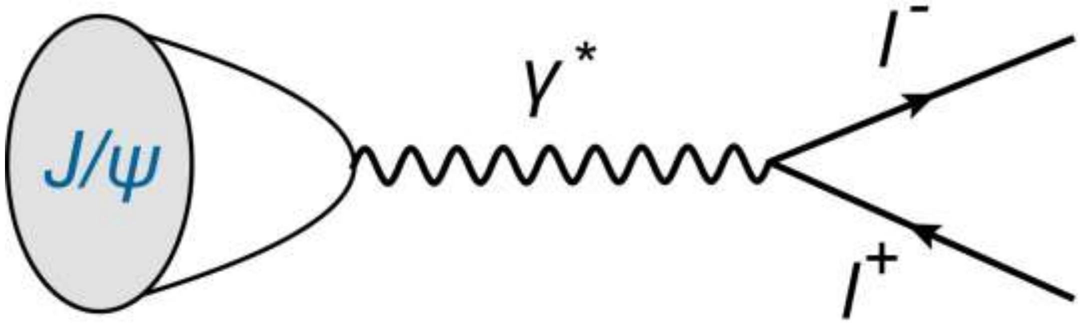

$$\frac{d\sigma}{dx_B\, dy\, d^4P_\psi\, d\Omega} \propto \underbrace{\mathcal{W}_T(1+\cos^2\theta) + \mathcal{W}_L(1-\cos^2\theta)} + \mathcal{W}_\Delta \sin 2\theta \cos\phi + \underbrace{\mathcal{W}_{\Delta\Delta}} \sin^2\theta \cos 2\phi$$

TMD convolutions: $\mathcal{C}\left[f_1^g \Delta_{\Lambda_\psi}^{[n]}\right]$ $\mathcal{C}\left[w h_1^{\perp g} \Delta_{\Lambda_\psi}^{[n]}\right]$

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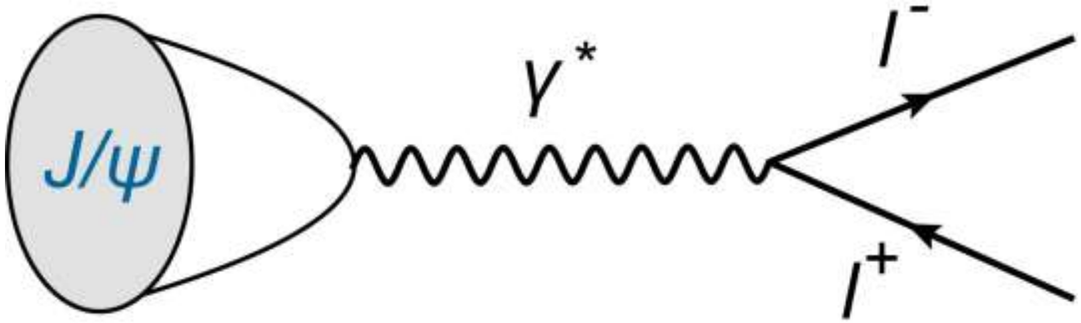

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Matching works in the same way, with: $\mathcal{W}_{\Delta\Delta}\Big|_{TMD} = \mathcal{W}_{\Delta\Delta}\Big|_{coll}$ $\mathcal{W}_{T,L}\Big|_{TMD} \neq \mathcal{W}_{T,L}\Big|_{coll}$

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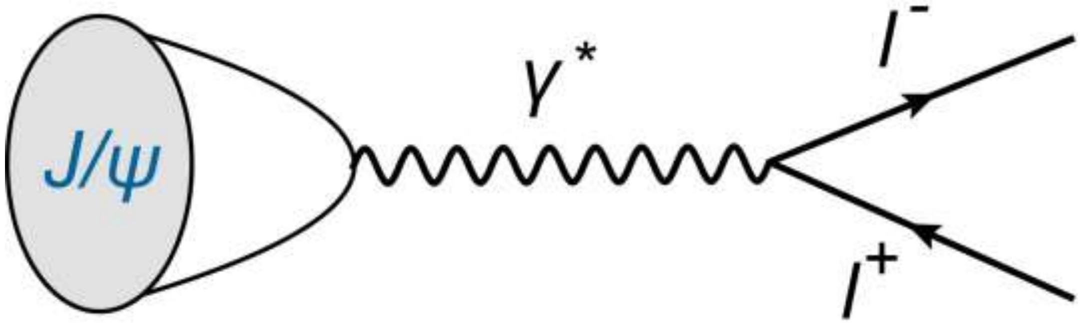

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TMD convolutions: $\mathcal{C}\left[f_1^g \Delta_{\Lambda_\psi}^{[n]}\right]$ (for $\mathcal{W}_T$, $\mathcal{W}_L$) $\qquad \mathcal{C}\left[w h_1^{\perp g} \Delta_{\Lambda_\psi}^{[n]}\right]$ (for $\mathcal{W}_{\Delta\Delta}$)

Matching works in the same way, with: $\mathcal{W}_{\Delta\Delta}\Big|_{TMD} = \mathcal{W}_{\Delta\Delta}\Big|_{coll}$ $\quad \mathcal{W}_{T,L}\Big|_{TMD} \neq \mathcal{W}_{T,L}\Big|_{coll}$

What are the TMD shape functions?

Still encodes the $c\bar{c}$ hadronization into J/ψ ➡ LDMEs $\langle O[n] \rangle$

Exchange of transverse momentum via soft gluons

- Deduced properties
  - Independence on J/ψ polarization
  - Same k_T dependence for different states n

$$\Delta^{[n]}(\boldsymbol{k}_T^2) = \frac{\alpha_s}{2\pi^2 \boldsymbol{k}_T^2} C_A \left(1 + \ln \frac{M^2}{M^2 + Q^2}\right) \langle O[n] \rangle$$

- Work in progress
  - Universality

Summary and outlook

- Introducing a new TMD factorized form involving TMD shape functions
- Matching procedure to extract the TMD shape function perturbative tail
  - We highlighted the importance of the hard amplitude pole structure
- Upon testing universality we can extract TMD shape functions from other processes
- Perturbative tail at higher order allow to deduce the TMDShF related to $h_1^{\perp g}$
- Non-perturbative dependence needed (also for Colour-Singlet states)

Echevarria, *JHEP 10* (2019) Fleming, Makris, Mehen, *JHEP 04* (2020)

- The advent of EIC can shed light on the role of TMD shape function and its properties

Thanks for the attention

BACK-UP

Operator definition

$$\mathrm{d}\sigma \propto \boxed{L} \otimes \boxed{|H|} \otimes \boxed{\Phi}$$

$$i\mathcal{M} = \boxed{\langle e'|(-iee_q)j^{\beta}(0)|e\rangle\left(i\frac{g_{\alpha\beta}}{Q^2}\right)}\boxed{\left(-i\mathcal{A}^{\alpha\mu}(q,p)\right)}\boxed{\left(-i\frac{g_{\mu\nu}}{m^2}\right)\langle P_X|igt^a j^{\nu}(0)|P\rangle}$$

$$\int \mathrm{d}p^- \, \Phi^{\mu\nu}(p) \equiv \Gamma^{\mu\nu}$$

$$\Gamma_U^{\mu\nu} = \frac{1}{2p^+}\left[-g_T^{\alpha\beta} f_1^g(x,\boldsymbol{p}_T^2) + \frac{1}{2M_p^2}\left(2p_T^{\alpha}p_T^{\beta} + g_T^{\alpha\beta}\boldsymbol{p}_T^2\right) h_1^{\perp g}(x,\boldsymbol{p}_T^2)\right]$$

unpolarized gluonTMD

linearly polarized gluonTMD

More details on TMD calculations

$$\mathrm{d}\sigma \propto \int \mathrm{d}\xi\, \mathrm{d}^2\boldsymbol{p}_T \frac{L^{\mu\nu}\; \mathcal{H}_{\mu\alpha}^{[n]}\, \mathcal{H}_{\nu\beta}^{[n]*}\; \Gamma_U^{\alpha\beta}}{\mathrm{Q}^4} \delta^4(q + P - P_\psi)$$

Lepton tensor

Bacchetta et al., *JHEP 02* (2007)

$$L^{\mu\nu} = (4\pi\alpha)\frac{\mathrm{Q}^2}{y}\left\{-[1 + (1 - y)^2]g_\perp^{\mu\nu} + 4(1 - y)\epsilon_L^\mu \epsilon_L^\nu\right.$$

$$g_\perp^{\mu\nu} = g^{\mu\nu} - \frac{1}{P \cdot q}\left(P^\mu q^\nu + P^\nu q^\mu - \frac{Q^2}{(P \cdot q)^2} P^\mu P^\nu\right)$$

$$\epsilon_L^\mu = \frac{1}{Q}\left(q^\mu - \frac{Q^2}{P \cdot q} P^\mu\right)$$

More details on TMD calculations

$$\mathrm{d}\sigma \propto \int \mathrm{d}\xi\, \mathrm{d}^2\boldsymbol{p}_T \frac{L^{\mu\nu}\ \ \mathcal{H}^{[n]}_{\mu\alpha}\,\mathcal{H}^{[n]*}_{\nu\beta}\ \ \Gamma^{\alpha\beta}_U}{\mathrm{Q}^4} \boxed{\delta^4(q+P-P_\psi)}$$

Dirac-delta fixes our variables

$$\delta^4(q+P-P_\psi) = \frac{2x_B}{Q^2}\delta(\xi - x)\delta^{(2)}(\boldsymbol{q}_T - \boldsymbol{p}_T)\delta(1-z)$$

$$x = x_B \frac{M^2+Q^2}{Q^2}$$

$$z = \frac{P\cdot P_\psi}{P\cdot q}$$

TMD structure functions

$$\mathcal{F}_{UU,T} = \frac{2\pi^2 e_c^2 \alpha \alpha_s}{M_\psi (M_\psi^2 + Q^2)} \left(\mathcal{C}\left[f_1^g \Delta\left[{}^1S_0^{(8)} \right] \right] + 4 \frac{7M_\psi^4 + 2M_\psi^2 Q^2 + 3Q^4}{M_\psi^2 (M_\psi^2 + Q^2)^2} \mathcal{C}\left[f_1^g \Delta\left[{}^3P_0^{(8)} \right] \right] \right)$$

$$\mathcal{F}_{UU,L} = \frac{2\pi^2 e_c^2 \alpha \alpha_s}{M_\psi (M_\psi^2 + Q^2)} \left(16 \frac{Q^2}{(M_\psi^2 + Q^2)^2} \mathcal{C}\left[f_1^g \Delta\left[{}^3P_0^{(8)} \right] \right] \right)$$

$$\mathcal{F}_{UU}^{\cos 2\phi_\psi} = \frac{2\pi^2 e_c^2 \alpha \alpha_s}{M_\psi (M_\psi^2 + Q^2)} \left(-\mathcal{C}\left[w h_1^{\perp g} \Delta\left[{}^1S_0^{(8)} \right] \right] + 4 \frac{3M_\psi^2 - Q^2}{M_\psi^2 (M_\psi^2 + Q^2)} \mathcal{C}\left[w h_1^{\perp g} \Delta\left[{}^3P_0^{(8)} \right] \right] \right)$$

TMD evolutions

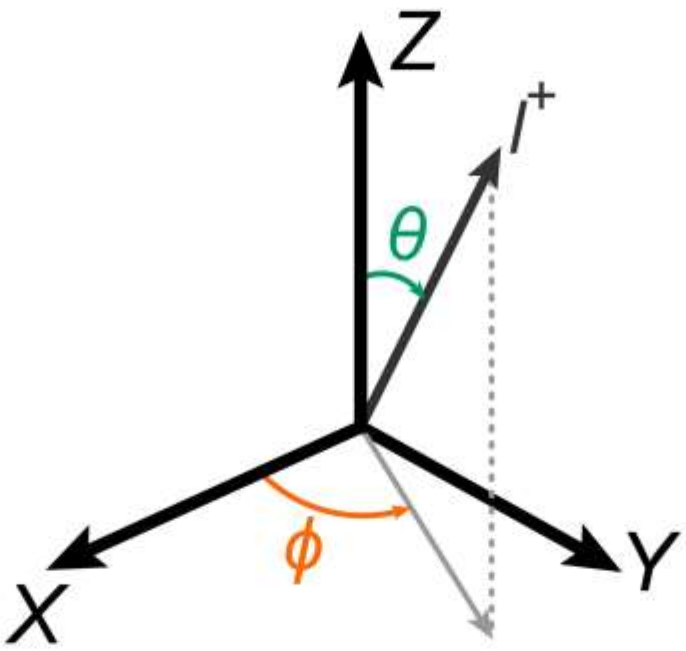
$$f_1^g(x, \boldsymbol{p}_T^2; \mu^2) = \frac{1}{2\pi} \int \mathrm{d}\boldsymbol{b}_T \mathrm{e}_T^{-i\boldsymbol{b}_T \cdot \boldsymbol{p}_T} \hat{f}_1^g(x, \boldsymbol{b}_T^2; \mu^2)$$

$$= \frac{\alpha_s}{2\pi^2 \boldsymbol{p}_T^2} \left[\left(C_A \log \frac{\mu}{p_T^2} - \frac{11C_A - 2n_f}{6} \right) f_1^g(x; \mu^2) \; + \left(P_{gg} \otimes f_1^g + P_{gi} \otimes f_1^i \right)(x; \mu^2) \right]$$

$$\frac{\boldsymbol{p}_T^2}{2m_p^2} h_1^{\perp g}(x, \boldsymbol{p}_T^2; \mu^2) = \frac{\alpha_s}{\pi} \frac{1}{\boldsymbol{p}_T^2} \left[\left(\delta P_{gg} \otimes f_1^g + \delta P_{gi} \otimes f_1^i \right)(x; \mu^2) \right]$$

$$\delta P_{gg}(z) = C_A \frac{1-z}{z} \qquad \delta P_{gi}(z) = C_F \frac{1-z}{z}$$

Frame choices in polarization

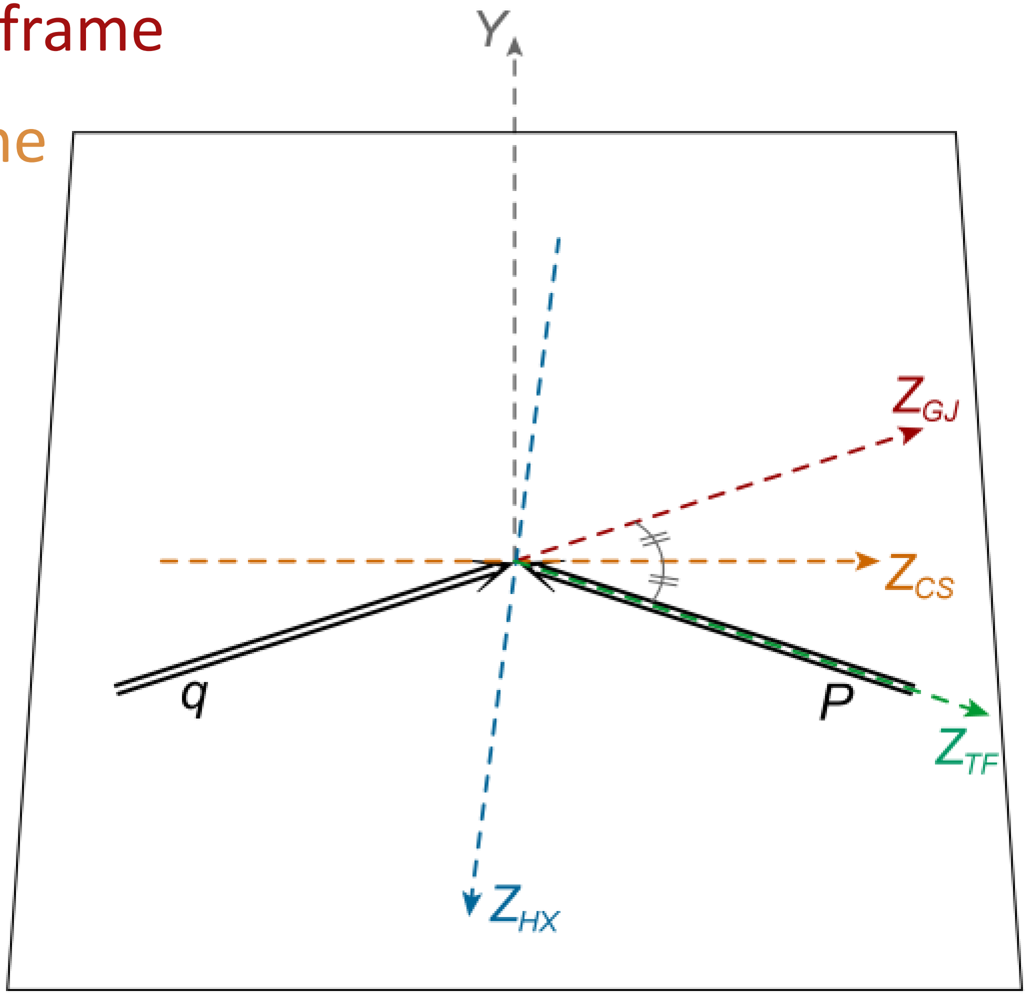


GJ *Gottfried-Jackson* frame

CS *Collins-Soper* frame

HX *Helicity* frame

TF *Target* frame



Solid angle $\Omega(\theta, \phi)$ of l^+ measured w.r.t. a specific frame

From polarization to unpolarized

Tracing over J/ψ helicity states connects J/ψ polarization cross section to unpolarized J/ψ

$$2\hat{\sigma}_T^{\mathcal{P}}\ \mathcal{C}\left[f_1^g \Delta_T^{[n]}\right] + \hat{\sigma}_L^{\mathcal{P}}\ \mathcal{C}\left[f_1^g \Delta_L^{[n]}\right] \propto \hat{\sigma}_{UU,\mathcal{P}}\ \mathcal{C}[f_1^g \Delta^{[n]}]$$

$\mathcal{P}$ is the virtual photon polarization

where $2\hat{\sigma}_T^{\mathcal{P}} + \hat{\sigma}_L^{\mathcal{P}} \propto \hat{\sigma}_{UU,\mathcal{P}}$

deducing $\Delta_T^{[n]} = \Delta_L^{[n]} = \Delta^{[n]}$