Quarkonium plus jet at large rapidity separation

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Hybrid collinear/high-energy factorization

J/ψ production

J/ψ plus jet production at low-p_T

J/ψ production from single parton fragmentation

Summary and outlook

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Summary and outlook

Motivation

- Heavy flavor physics has long been considered as a perfect framework for testing perturbative QCD at colliders, due to the smallness of the running coupling
- However, at modern colliders, heavy-flavor production enters a two-scale regime, called semi-hard
- **Semi-hard** collision process $\to$ stringent *scale hierarchy*

$$s \gg Q^2 \gg \Lambda^2_{\rm QCD}, \qquad Q^2 \text{ a hard scale,}$$

$\nearrow$

Regge kinematic region

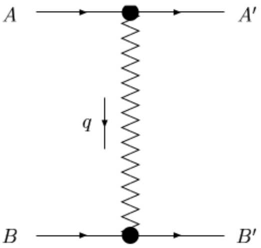
$$\alpha_s(Q^2)\,\ln\left(\frac{s}{Q^2}\right) \sim 1 \implies \text{all-order } \textbf{resummation} \text{ needed}$$

- The Balitsky-Fadin-Kuraev-Lipatov (**BFKL**) approach is the general framework for this *high-energy* resummation
  - Leading-Logarithmic-Approximation (**LLA**): $(\alpha_s \ln s)^n$
  - Next-to-Leading-Logarithmic-Approximation (**NLLA**): $\alpha_s(\alpha_s \ln s)^n$
- Progress on **NNLLA**

The Reggeized gluon

Scattering process $A + B \longrightarrow A' + B'$

- Gluon quantum numbers in the t-channel
- Regge limit: $s \simeq -u \to \infty$, t fixed (i.e not growing with s)
- All-order resummation:
  leading logarithmic approximation (LLA): $(\alpha_s \ln s)^n$
  next-to-leading logarithmic approximation (NLA): $\alpha_s(\alpha_s \ln s)^n$



$$(\mathcal{A})_{AB}^{A'B'} = \Gamma_{A'A}^{c} \left[\left(\frac{-s}{-t} \right)^{j(t)} - \left(\frac{s}{-t} \right)^{j(t)} \right] \Gamma_{B'B}^{c}$$

$$j(t) = 1 + \omega(t), \qquad j(0) = 1$$

$j(t)$-**Reggeized gluon trajectory**

$$\Gamma_{A'A}^{c} = g \, \langle A' | T^c | A \rangle \, \Gamma_{A'A}$$

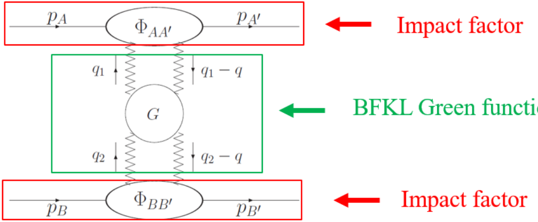
T^c- fundamental(quarks) or adjoint(gluons)

- LLA [Ya. Ya. Balitsky, V.S. Fadin, L.N. Lipatov (1979)]

$$\Gamma_{A'A}^{(0)} = \delta_{\lambda_{A'} \lambda_A}, \quad \omega^{(1)}(t) = \frac{g^2 t}{(2\pi)^{(D-1)}} \frac{N}{2} \int \frac{d^{D-2} k_\perp}{k_\perp^2 (q-k)_\perp^2} = -g^2 \frac{N\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)} (\vec{q}^{\,2})^\epsilon$$

BFKL resummation

- Diffusion $A + B \longrightarrow A' + B'$ in the Regge kinematical region
- Gluon Reggeization
- BFKL factorization for $\Im \mathcal{A}_{AB}^{A'B'}$: convolution of a Green function (process independent) with the Impact factors of the colliding particles (process dependent)



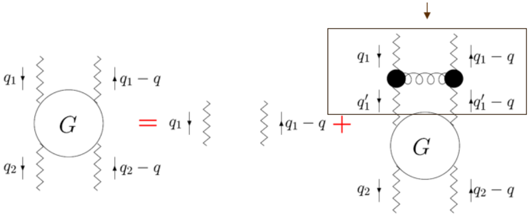
$$\Im_s \mathcal{A}_{AB}^{A'B'} = \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2} q_1}{\vec{q}_1^{\,2} (\vec{q}_1 - \vec{q}\,)^{\,2}} \frac{d^{D-2} q_2}{\vec{q}_2^{\,2} (\vec{q}_2 - \vec{q}\,)^{\,2}}$$

$$\times \sum_{\nu} \Phi_{A'A}^{(R,\nu)}(\vec{q}_1, \vec{q}, s_0) \int \frac{d\omega}{2\pi i} \left[\left(\frac{s}{s_0} \right)^{\omega} G_{\omega}^{(R)}(\vec{q}_1, \vec{q}_2; \vec{q}\,) \right] \Phi_{B'B}^{(R,\nu)}(-\vec{q}_2, \vec{q}, s_0)$$

BFKL resummation

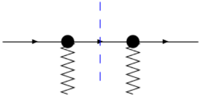
- $G_\omega^{(R)}(\vec{q}_1, \vec{q}_2; \vec{q}\,)$-Mellin transform of the Green function for the Reggeon-Reggeon scattering

$$\omega G_\omega^{(R)}(\vec{q}_1, \vec{q}_2; \vec{q}\,) = \vec{q}_1^{\,2}(\vec{q}_1 - \vec{q}\,)^2 \delta^{(D-2)}(\vec{q}_1 - \vec{q}_2) + \int \frac{d^{D-2}q_1'}{\vec{q}_1^{\,\prime 2}(\vec{q}_1^{\,\prime} - \vec{q}\,)^2} \mathcal{K}^{(R)}(\vec{q}_1, \vec{q}_1^{\,\prime}; \vec{q}\,) G_\omega^{(R)}(\vec{q}_1^{\,\prime}, \vec{q}_2; \vec{q}\,)$$



- $\Phi_{P'P}^{(R,\nu)}$- LO impact factor in the t-channel color state (R, ν)

$$\Phi_{PP'}^{(R,\nu)} = \langle cc' | \hat{\mathcal{P}} | \nu \rangle \sum_{\{f\}} \int \frac{ds_{PR}}{2\pi} d\rho_f \Gamma^c_{\{f\}P} (\Gamma^{c'}_{\{f\}P'})^*$$

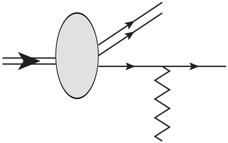


Hybrid collinear/high-energy factorization

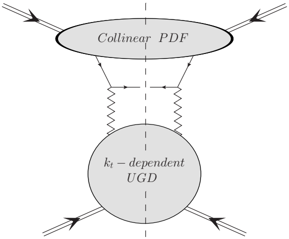
- Infrared safety of impact factor for colorless particles
  [V. S. Fadin, A. D. Martin (1999)]
- Impact factors of colored particles afflicted by *infrared singularities*

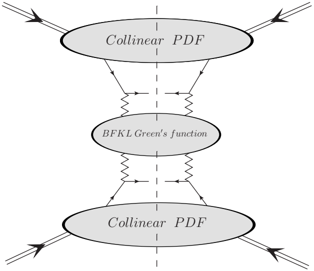
$$p_J = z_J p_1 + \frac{\vec{p}_J^{\,2}}{z_J s} p_2 + p_{J\perp}$$

$$\frac{d\Phi^J_{PP}}{dx_J d^2\vec{p}_J} = \int_{x_J}^1 \frac{dz_J}{z_J} \sum_{a=q,\bar{q},g} f_a\left(\frac{x_J}{z_J}\right) \frac{d\Phi^J_{aa}}{dz_J d^2\vec{p}_J}$$



- **Hybrid factorization(s)**





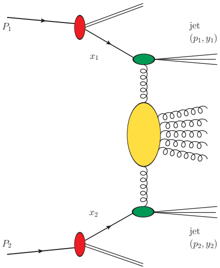
Mueller-Navelet jets

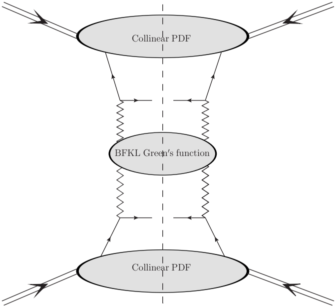
- Inclusive production of **rapidity-separated jets** in proton-proton collision
- Large energy logarithms → *BFKL resummed partonic cross section*
- Moderate values of parton x → *collinear PDFs*

[A.H. Mueller, H. Navelet (1987)]

- **Full next-to-leading analysis**

[B. Ducloué, L. Szymanowski, S. Wallon (2013,2014)]
[F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa (2014,2015)]





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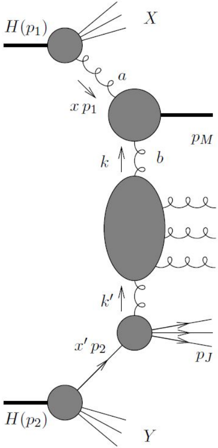
Treatment of heavy-quarks masses

- The mass of light quarks ($q = u, d, s$) is always set to zero. They are always present in the initial state
- The presence in the initial state and the way one must treat the mass of an heavy-quark ($Q = c, b$) depends on kinematical conditions
- **Zero-mass variable flavor number scheme (0M-VFNS)**
  - $m_Q = 0$
  - Heavy quark is present in the initial state above a fixed threshold.
  - Powers of $m_Q^2/p_{T,HQ}^2$ missed by the scheme
  - It is appropriate in region of high $p_{T,HQ}^2 \gg m_Q^2$
- **Fixed flavor number scheme (FFNS)**
  - $m_Q \neq 0$
  - Heavy quark is present only in the final state
  - Logarithms of $p_{T,Q}^2/m_Q^2$ missed by the scheme
  - It is appropriate in regions of moderate $p_{T,Q}^2$
- **General-mass variable flavor number schemes (GM-VFNS)**
  - It is a matching between the previous schemes

J/ψ plus jet production at low-p_T

- Process: proton(p_1) + proton(p_2) $\rightarrow J/\psi + X +$ jet

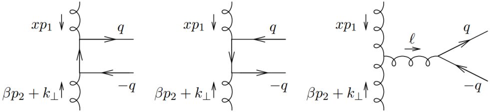
- **hybrid collinear/BFKL approach**
- high-energy hadroproduction of a J/Ψ meson and a jet, with a remnant X
- both the J/Ψ and the jet emitted with large transverse momenta and well separated in rapidity
- NLA BFKL + NLO jet + LO J/Ψ
  - LO J/Ψ IF calculated in **NRQCD** (Color-singlet and Color-octect)
  - LO J/Ψ IF calculated in **color evaporation model (CEM)**
- Realistic CMS and CASTOR rapidity ranges, fixed p_T final states



[R. Boussarie, B. Ducloué, L. Szymanowski, S. Wallon (2018)]

- Impact factor in **Color Evaporation Model**

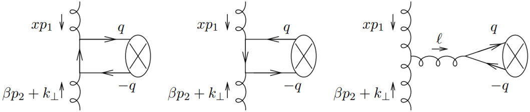
$$\mathcal{V}_{J/\psi} = F_{J/\psi} \int_{4m_c^2}^{4m_D^2} dM^2 \frac{d\mathcal{V}_{c\bar{c}}}{dM^2}$$



- **NRQCD expansion**

$$|J/\psi\rangle = O(1) \left| Q\bar{Q} \left[{}^3S_1^{(1)} \right] \right\rangle + O(v) \left| Q\bar{Q} \left[{}^3S_1^{(8)} \right] g \right\rangle + O\left(v^2\right)$$

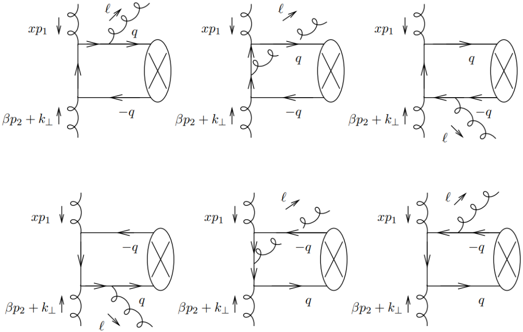
- Impact factor in *Color Octect* case



$$[v(q)\,\bar{u}(q)]_{\alpha\beta}^{ji\to a} \to t_{ji}^a d_8 \left(\frac{\langle \mathcal{O}_8 \rangle_{J/\psi}}{m} \right)^{\frac{1}{2}} \left[\hat{\varepsilon}_{J/\psi}^* \left(2\hat{q} + M \right) \right]_{\alpha\beta}$$

Impact factors in NRQCD

- Impact factor in *Color Singlet* case



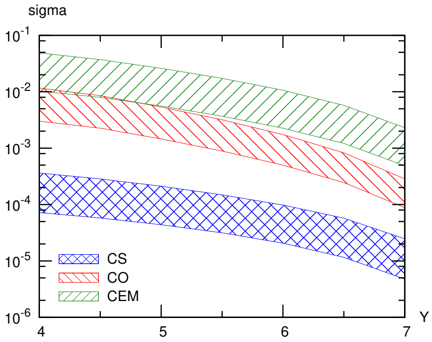
$$v\left[(q)\,\bar{u}\,(q)\right]^{ij}_{\alpha\beta} \to \frac{\delta^{ij}}{4N_c}\left(\frac{\langle \mathcal{O}_1\rangle_{J/\psi}}{m}\right)^{\frac{1}{2}}\left[\hat{\varepsilon}^*_{J/\psi}\,(2\hat{q}+M)\right]_{\alpha\beta}$$

J/ψ plus jet production at low-p_T

- Realistic CMS and CASTOR rapidity ranges, fixed p_T final states

$$\frac{d\sigma}{d|k_{J/\psi}|d|k_{\text{jet}}|dY}[\text{nb GeV}^{-2}]$$

$$|k_{J/\psi}| = |k_{\text{jet}}| = 10 \text{ GeV}$$



[R. Boussarie, B. Ducloué, L. Szymanowski, S. Wallon (2018)]

- Double prompt J/Ψ in Parton Reggeization Approach (PRA)

[Z. He, B.A. Kniehl, M.A. Nefedov, and V. A. Saleev (2019)]

- Diffractive photoproduction and inclusive hadroproduction of J/Ψ and Υ

[P. Kotko, L. Motyka, M. Sadzikowski, A. M. Stasto (2019)]

J/ψ production from single parton fragmentation

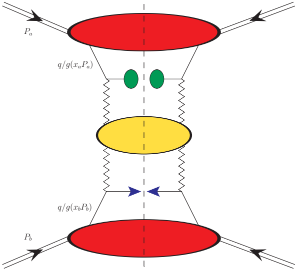
$$p(P_a) + p(P_b) \to \mathcal{Q}(p_\mathcal{Q}, y_\mathcal{Q}) + X + \mathrm{jet}(p_J, y_J)$$

- **Hybrid cross section**

$$\frac{d\sigma}{dy_\mathcal{Q}\, dy_J\, d^2\vec{p}_\mathcal{Q}\, d^2\vec{p}_J} = \frac{1}{(2\pi)^2}$$

$$\times \int \frac{d^2\vec{q}_1}{\vec{q}_1^{\,2}} V_\mathcal{Q}(\vec{q}_1, x_g, \vec{p}_\mathcal{Q}) \int \frac{d^2\vec{q}_2}{\vec{q}_2^{\,2}} V_J(\vec{q}_2, x_J, \vec{p}_J)$$

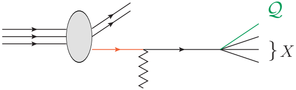
$$\times \int_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left(\frac{x_\mathcal{Q} x_J s}{s_0} \right)^\omega G_\omega(\vec{q}_1, \vec{q}_2)$$

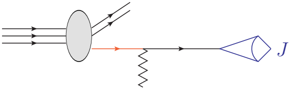


- Impact factors

$$V_\mathcal{Q}(\vec{q}_1, x_\mathcal{Q}, \vec{p}_\mathcal{Q}) = f_{q/g} \otimes H \otimes D^{\mathcal{Q}}_{q/g}$$

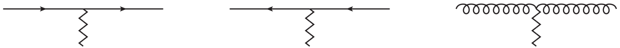
$$V_J(\vec{q}_2, x_J, \vec{p}_J) = f_{q/g} \otimes H \cdot \mathcal{J}$$





LO Impact factors

- Parton-Parton-Reggeon (PPR) vertices



- LO jet impact factor

$$c_J(n,\nu,|\vec{p}\,|,x) = 2\sqrt{\frac{C_F}{C_A}}(|\vec{p}\,|^2)^{i\nu-1/2}\left(\frac{C_A}{C_F}f_g(x) + \sum_{\beta=q,\bar{q}} f_\beta(x)\right) \;,$$

- LO light hadron impact factor

$$c_{\mathcal{Q}}(n,\nu,|\vec{p}\,|,x) = 2\sqrt{\frac{C_F}{C_A}}(|\vec{p}\,|^2)^{i\nu-1/2}\int_x^1 \frac{d\zeta}{\zeta}\left(\frac{\zeta}{x}\right)^{2i\nu-1}$$

$$\times\left[\frac{C_A}{C_F}f_g(\zeta)D_g^{\mathcal{Q}}\left(\frac{x}{\zeta}\right) + \sum_{\alpha=q,\bar{q}} f_\alpha(\zeta)D_\alpha^{\mathcal{Q}}\left(\frac{x}{\zeta}\right)\right]$$

- Both known at NLO

[J. Bartels, D. Colferai, G.P. Vacca (2003)]
[F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa (2014)]
[D.Yu. Ivanov, A. Papa (2012)]

J/Ψ fragmentation: heavy-quark channel

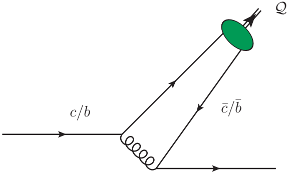
- NRQCD FFs

$$D_i^{\mathcal{Q}}(z,\mu_F) = \sum_{[n]} \mathcal{D}_i^{Q\bar{Q}}(z,\mu_F,[n])\langle \mathcal{O}^{\mathcal{Q}}([n])\rangle$$

- Spin-triplet (vector) and color-singlet quarkonium state, $^3S_1^{(1)}$
- Heavy-quark fragmentation function computed at $\mu_0 = 3m_Q$ in NRQCD

$$D_Q^{\mathcal{Q}}(z,\mu_F \equiv \mu_0) = D_Q^{\mathcal{Q},\mathrm{LO}}(z)$$

$$+\frac{\alpha_s^3(\mu_R)}{m_Q^3}\,|\mathcal{R}_{\mathcal{Q}}(0)|^2\,\Gamma^{\mathcal{Q},\mathrm{NLO}}(z)$$



- LO fragmentation function

[E. Braaten, K. Cheung, T.C. Yuan (1993)]

$$D_Q^{\mathcal{Q},\mathrm{LO}}(z) = \frac{\alpha_s^2(\mu_R)}{m_Q^3}\frac{8z(1-z)^2}{27\pi(2-z)^6}\,|\mathcal{R}_{\mathcal{Q}}(0)|^2\,(5z^4-32z^3+72z^2-32z+16)$$

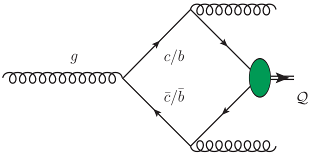
- The NLO correction is given by a polynomial function

[X.C. Zheng, C.H. Chang, and X.G. Wu (2019)]

J/Ψ fragmentation: gluon and light-quark channels

- Gluon fragmentation function computed at $\mu_0 = 2m_Q$ in NRQCD [E. Braaten, T. C. Yuan (1993)]

$$D_g^{\mathcal{Q}}(z, 2m_Q) = \frac{5}{36(2\pi)^2} \alpha_s^3(2m_Q) \frac{|\mathcal{R}_{\mathcal{Q}}(0)|^2}{m_{\mathcal{Q}}^3} \int_0^z d\xi \int_{(\xi+z^2)/2z}^{(1+\xi)/2} d\tau \frac{1}{(1-\tau)^2(\tau-\xi)^2(\tau^2-\xi)^2}$$
$$\sum_{i=1}^{2} z^i \left[f_i^{(g)}(\xi,\tau) + g_i^{(g)}(\xi,\tau) \frac{1+\xi-2\tau}{2(\tau-\xi)\sqrt{\tau^2-\xi}} \ln\left(\frac{\tau-\xi+\sqrt{\tau^2-\xi}}{\tau-\xi-\sqrt{\tau^2-\xi}} \right) \right]$$

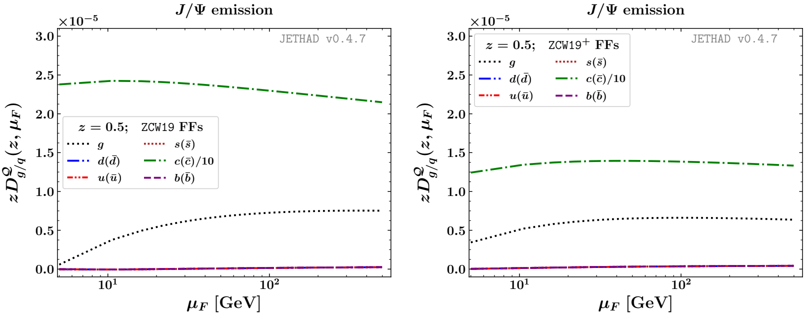


- FFs evolved from the initial scale through the DGLAP evolution equations
- Light-quarks FFs (LQFFs) are zero at the initial scale: $D_q^{\mathcal{Q}}(z, 2m_Q) = 0$
- At higher scales LQFFs dynamically generated by evolution

$$D_q^{\mathcal{Q}}(z, \mu > 2m_Q) = \underbrace{D_q^{\mathcal{Q}}(z, 2m_Q)}_{0} + \frac{\alpha_s}{2\pi} \ln\left(\frac{\mu^2}{(2m_Q)^2}\right) \int_z^1 \frac{dx}{x} P_{qg}(x) D_g^{\mathcal{Q}}\left(\frac{z}{x}, 2m_Q\right)$$

J/ψ production from single parton fragmentation

- J/Ψ collinear FFs



$$\left\{ D_Q^{\mathcal{Q}}(z, 3m_Q) \right\}_{\text{NRQCD}} \xrightarrow{\text{APFEL++}} \boxed{\textbf{ZCW19 Onium FFs}}$$

$$\left\{ D_Q^{\mathcal{Q}}(z, 3m_Q), D_g^{\mathcal{Q}}(z, 2m_Q) \right\}_{\text{NRQCD}} \xrightarrow{\text{APFEL++}} \boxed{\textbf{ZCW19}^{+}\textbf{ Onium FFs}}$$

[F. G. Celiberto, M.F. (2022)]

J/ψ production from single parton fragmentation

- Differential cross section in terms of azimuthal-angle coefficients

$$\frac{d\sigma}{dy_{\mathcal{Q}} dy_J d|p_{\mathcal{Q}}| d|p_J| d\phi_{\mathcal{Q}} d\phi_J} = \frac{1}{(2\pi)^2} \left[\mathcal{C}_0 + 2 \sum_{n=1}^{\infty} \cos(n\varphi)\, \mathcal{C}_n \right]$$

- Unintegrated azimuthal-angle coefficients at NLO

$$\mathcal{C}_n = \int_0^{2\pi} d\phi_{\mathcal{Q}} \int_0^{2\pi} d\phi_J \cos(n\varphi) \frac{d\sigma_{\rm NLA}}{dy_{\mathcal{Q}} dy_J d|\vec{p}_{\mathcal{Q}}|\, d|\vec{p}_J| d\phi_{\mathcal{Q}} d\phi_J} = \frac{e^{\Delta Y}}{s} \int_{-\infty}^{+\infty} d\nu$$

$$e^{\Delta Y \bar{\alpha}_s(\mu_R) \left\{ \chi(n,\nu) + \bar{\alpha}_s(\mu_R) \left[\bar{\chi}(n,\nu) + \frac{\beta_0}{8N_c} \chi(n,\nu) \left[-\chi(n,\nu) + \frac{10}{3} + 4 \ln \left(\frac{\mu_R}{\sqrt{|\vec{p}_{\mathcal{Q}}||\vec{p}_J|}} \right) \right] \right] \right\}}$$

$$\times\, \alpha_s^2(\mu_R) \left[c_{\mathcal{Q}}^{\rm NLO}(n,\nu,|\vec{p}_{\mathcal{Q}}|,x_1) [c_J^{\rm NLO}(n,\nu,|\vec{p}_J|,x_2)]^* + \bar{\alpha}_s^2(\mu_R) \Delta Y \frac{\beta_0}{4N_c} \chi(n,\nu) f(\nu) \right]$$

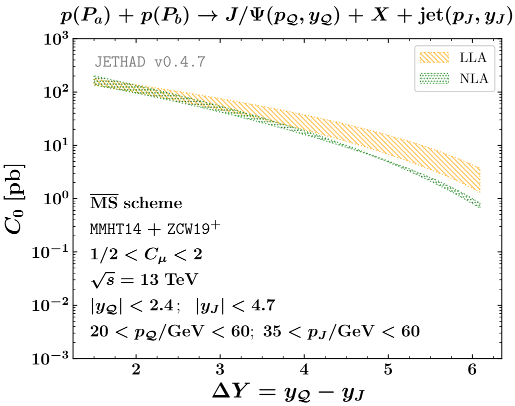
- Azimuthal-angle coefficients at NLO

$$C_n = \int_{y_{\mathcal{Q}}^{\rm min}}^{y_{\mathcal{Q}}^{\rm max}} dy_{\mathcal{Q}} \int_{y_J^{\rm min}}^{y_J^{\rm max}} dy_J \int_{p_{\mathcal{Q}}^{\rm min}}^{p_{\mathcal{Q}}^{\rm max}} d|\vec{p}_{\mathcal{Q}}| \int_{p_J^{\rm min}}^{p_J^{\rm max}} d|\vec{p}_J|\ \delta(\Delta Y - y_{\mathcal{Q}} - y_J)\ \mathcal{C}_n$$

J/ψ production from single parton fragmentation

- ΔY-behaviour of the cross section

$$C_0 = \int_{y_Q^{\min}}^{y_Q^{\max}} dy_Q \int_{y_J^{\min}}^{y_J^{\max}} dy_J \int_{p_Q^{\min}}^{p_Q^{\max}} d|\vec{p}_Q| \int_{p_J^{\min}}^{p_J^{\max}} d|\vec{p}_J| \; \delta(\Delta Y - y_Q - y_J) \; \mathcal{C}_0 \left(|\vec{p}_Q|, |\vec{p}_J|, y_Q, y_J\right)$$

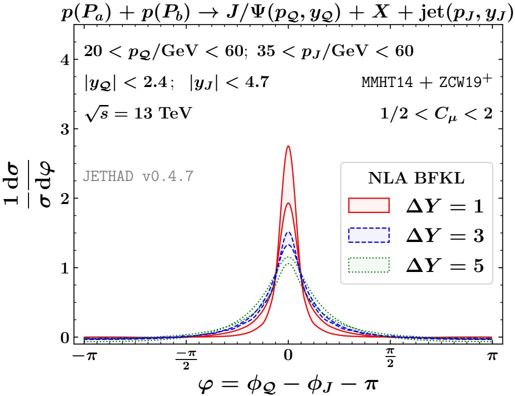


[F. G. Celiberto, M.F. (2022)]

J/ψ production from single parton fragmentation

- Azimuthal distribution

$$\frac{1}{\sigma}\frac{d\sigma}{d\varphi} = \frac{1}{2\pi}\left\{1 + 2\sum_{n=1}^{\infty}\cos(n\varphi)\langle\cos(n\varphi)\rangle\right\} = \frac{1}{2\pi}\left\{1 + 2\sum_{n=1}^{\infty}\cos(n\varphi)R_{n0}\right\} .$$

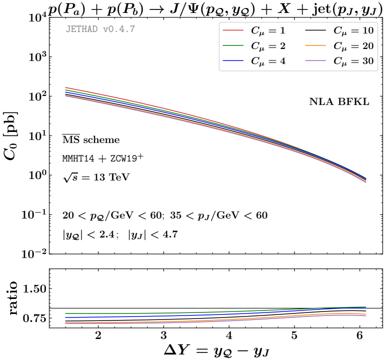


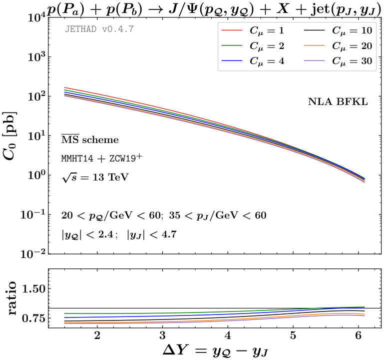
[F. G. Celiberto, M.F. (2022)]

J/ψ production from single parton fragmentation

- ΔY-behaviour of the cross section under scale variation

  $C_\mu = \mu_{R,F}/\mu_N$





[F. G. Celiberto, M.F. (2022)]

Outline

Introduction and motivations

- BFKL resummation
- Hybrid collinear/high-energy factorization

J/ψ production

- J/ψ plus jet production at low-p_T
- J/ψ production from single parton fragmentation

Summary and outlook

Summary and conclusions

Conclusions

- We considered **J/Ψ plus jet** production at large p_T and rapidity-separation, in the *fragmentation approximation* and within NLLA
- Description of the J/Ψ fragmentation

$$\left\{ D_Q^{\mathcal{Q}}(z, 3m_Q), D_g^{\mathcal{Q}}(z, 2m_Q) \right\}_{\text{NRQCD}} \xrightarrow{\textbf{APFEL++}} \boxed{\textbf{ZCW19}^{+}\ \textbf{Onium FFs}}$$

- **Stability** effects occur in the J/ψ-channel

Outlook

- Inclusion of subleading corrections from the heavy-quark pair impact factors, needed to produce full-NLA predictions
- Improvement of the description at large-p_T including the color octect mechanism
- Investigation of the single-forward J/Ψ photo/electro-production

Thank you for the attention

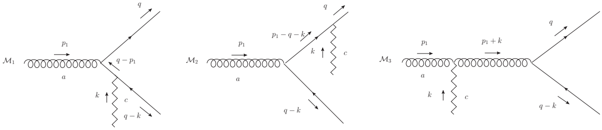
Backup

LO heavy-quark impact factors

- Gluon-initiated impact factor

[A.D. Bolognino, F.G. Celiberto, M. F., D.Yu. Ivanov, A. Papa (2019)]

- Feynman diagrams



- Impact factor

$$d\Phi_{gg}^{\{Q\bar{Q}\}}(\vec{k},\vec{q},z) = \frac{\alpha_s^2\sqrt{N_c^2-1}}{2\pi N_c}\left[\left(m^2\,(R+\bar{R})^2+\left(z^2+\bar{z}^2\right)\left(\vec{P}+\vec{\bar{P}}\right)^2\right)\right.$$
$$\left.-\frac{N_c^2}{N_c^2-1}\left(2m^2R\bar{R}+\left(z^2+\bar{z}^2\right)2\vec{P}\cdot\vec{\bar{P}}\right)\right]d^2\vec{q}\;dz\;,$$

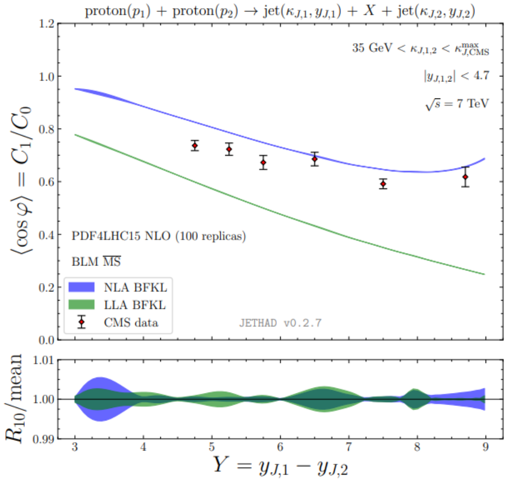
- Projection onto the LO BFKL eigenfunctions

$$\frac{d\Phi_{gg}^{\{Q\bar{Q}\}}(n,\nu,\vec{q},z)}{d^2\vec{q}\;dz} \equiv \int\frac{d^2\vec{k}}{\pi\sqrt{2}}(\vec{k}^{\,2})^{i\nu-\frac{3}{2}}e^{in\theta}\,\frac{d\Phi_{gg}^{\{Q\bar{Q}\}}(\vec{k},\vec{q},z)}{d^2\vec{q}\;dz} \equiv \alpha_s^2\;e^{in\varphi}c(n,\nu,\vec{q},z)$$

- Photon-initiated impact factor

[F.G. Celiberto, D.Yu. Ivanov, B. Murdaca, A. Papa (2017)]

Muller-Navelet: Theory vs Experiment



[B. Ducloué, L. Szymanowski, S. Wallon (2013)]

[F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa (2014)]

In this slide: [F.G. Celiberto (2021)]

Mueller-Navelet: Theory vs Experiment

- CMS **@**7Tev with symmetric p_T-ranges, only!

  [CMS collaboration (2016)]
- LHC kinematic **domain** in between the sectors described by BFKL and DGLAP approaches
- Clearer manifestation of high-energy signatures expected at increasing energies (higher hadronic center-of-mass energy or higher rapidity difference between tagged jets)
- Need for more exclusive final states as well as more sensitive observables

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- Strong manifestation of higher-order **instabilities** via scale variation

NLA BFKL corrections to cross section with opposite sign with respect to the leading order (LO) result and large in absolute value...

- ◇ ...call for some optimization procedure...
- ◇ ...choose scales to mimic the most relevant subleading terms

- **BLM** [S.J. Brodsky, G.P. Lepage, P.B. Mackenzie (1983)]
  - ✓ preserve the conformal invariance of an observable...
  - ✓ ...by making vanish its β_0-dependent part

* "Exact" BLM:

  suppress NLO IFs + NLO Kernel β_0-dependent factors

Partially inclusive processes in NLA

- Mueller-Navelet jet production

  [J. Bartels, D. Colferai, G.P. Vacca (2003)]
  [F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa, A. Perri (2011)]
  [D.Yu. Ivanov, A. Papa (2012)]
  [D. Colferai, A. Niccoli (2015)]
  [B. Ducloué, L. Szymanowski, S. Wallon (2013,2014)]
  [F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa (2014)]
  [F.G. Celiberto, D.Yu. Ivanov, B. Murdaca, A. Papa (2015)]

- Light hadron-light hadron production

  [D.Yu. Ivanov, A. Papa (2012)]
  [F.G. Celiberto, D.Yu. Ivanov, B. Murdaca, A. Papa (2016, 2017)]

- Light hadron-jet production

  [A.D. Bolognino, F.G. Celiberto, D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2018)]

- Heavy-light hadrons in VFNS

  [F.G. Celiberto, M.F, D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2021)]
  [F.G. Celiberto, M.F, D.Yu. Ivanov, A. Papa (2021)]

Other partially-inclusive reactions

- Three / four jet production (partial NLA)

  [F. Caporale, G. Chachamis, B. Murdaca, A. Sabio Vera (2016)]

  [F. Caporale, F.G. Celiberto, G. Chachamis, A. Sabio Vera (2016)]

  [F. Caporale, F.G. Celiberto, G. Chachamis, D.G. Gomez, A. Sabio Vera (2016, 2017)]

- Drell-Yan pair - jet (partial NLA)

  [K. Golec-Biernat, L. Motyka, T. Stebel (2018)]

- Higgs - jet

  - Partial NLA

  [F.G. Celiberto, D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2020)]

  - Full NLA, $m_t \to \infty$ limit

  [F.G. Celiberto, D.Yu. Ivanov, M.F., M.M.A. Mohammed, A. Papa (2022, in preparation)]

- Heavy-quark pair photo/hadro-production (partial NLA)

  [F.G. Celiberto, D.Yu. Ivanov, B. Murdaca, A. Papa (2017)]

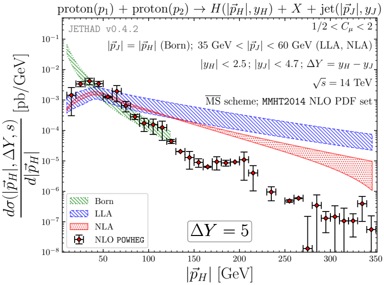
  [A.D. Bolognino, F.G. Celiberto, M.F., D.Yu. Ivanov, A. Papa (2019)]

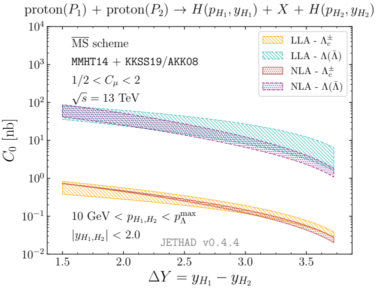
- J/Ψ - jet production (partial NLA)

  [R. Boussarie, B. Ducloué, L. Szymanowski, S. Wallon (2018)]

Stabilization effects

- Stabilization effects in Higgs and heavy flavor production
- Λ-baryon FFs
  - heavy species $\longrightarrow \Lambda_c$
    KKSS19 [B.A. Kniehl, G. Kramer, I. Schienbein, H. Spiesberger (2020)]
  - light species $\longrightarrow \Lambda$
    AKK08 [S.Albino, B.A. Kniehl, and G. Kramer (2008)]



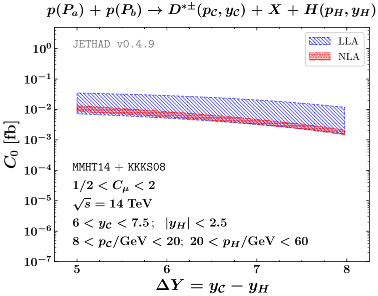


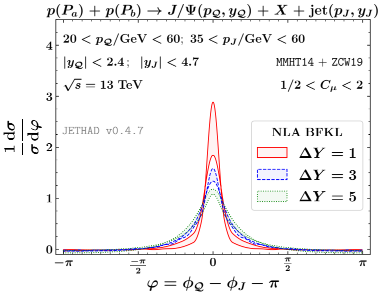
[F.G. Celiberto, D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2020)]

[F.G. Celiberto, D.Yu. Ivanov, M. F., A. Papa (2021)]

D^* plus Higgs production at the FPF

- ATLAS + FPF ultra-forward regime: Strong high-energy enhancement
- Final state with two stabilizers: Heavy-flavor and Higgs
- FPF studies on high-energy QCD: see yesterday's WG6 talk by **J. Rojo**





[FPF Collaboration (2022)]

[F. G. Celiberto, M.F., M. M. A. Mohammed (in preparation)]

Λ-baryon production

- Process: proton(p_1) + proton(p_2) $\to \Lambda + X + \Lambda$
  [F.G. Celiberto, M. F., Dmitry Yu. Ivanov, Alessandro Papa (2021)]
- Zero-mass variable flavor number scheme (ZM-VFNS)
- Light parton NLO impact factors $\to$ Heavy baryon NLO impact factor
  [M. Ciafaloni and G. Rodrigo (2000)] [V.S. Fadin et al. (2000)]
  [D.Yu. Ivanov, A. Papa (2012)]

- Lambda FFs
  - heavy species $\longrightarrow \Lambda_c$
    KKSS19 [B.A. Kniehl, G. Kramer, I. Schienbein, H. Spiesberger (2020)]
  - light species $\longrightarrow \Lambda^0$
    AKK08 [S.Albino, B.A. Kniehl, and G. Kramer (2008)]