

DE LA RECHERCHE À L'INDUSTRIE



Benjamin Audurier - Aussois classes - Jan. 2023

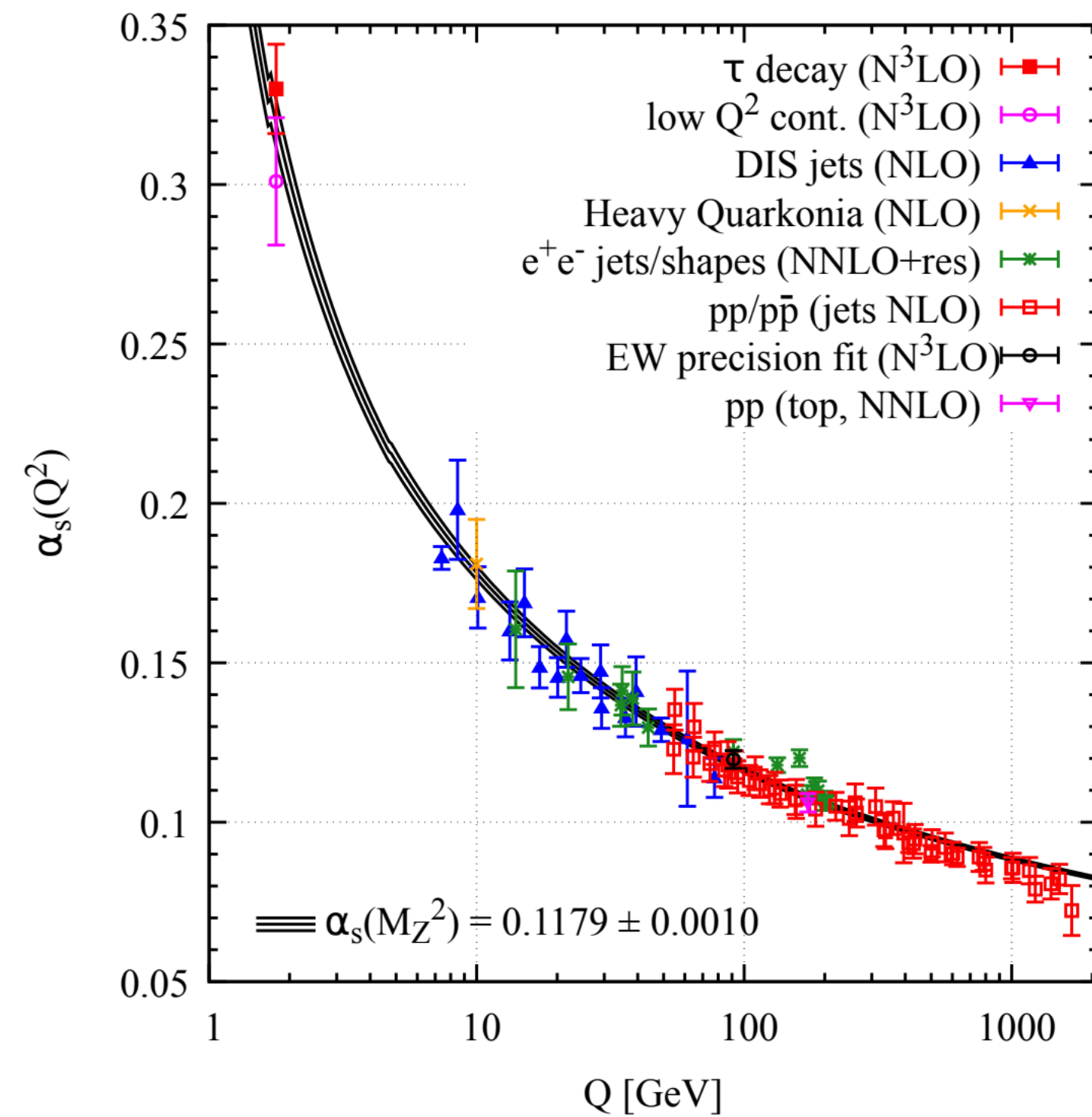
GENERAL INTRODUCTION TO HEAVY-ION PHYSICS

« Heavy-ion talks always start with someone doing a weird clap with flat hands »

-A physicist from Strasbourg that I know.

The best of two worlds

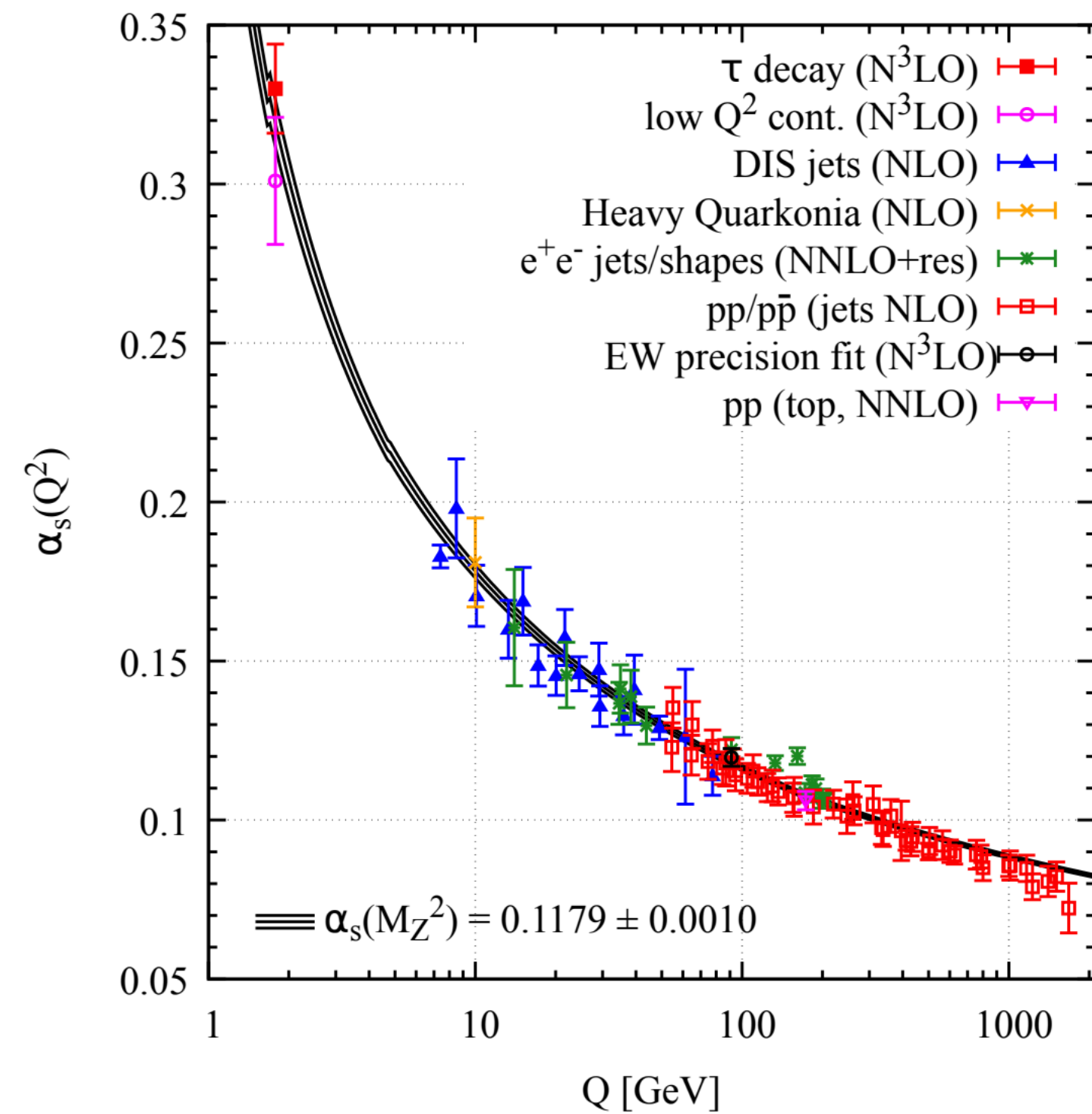
Quantum Chromodynamics



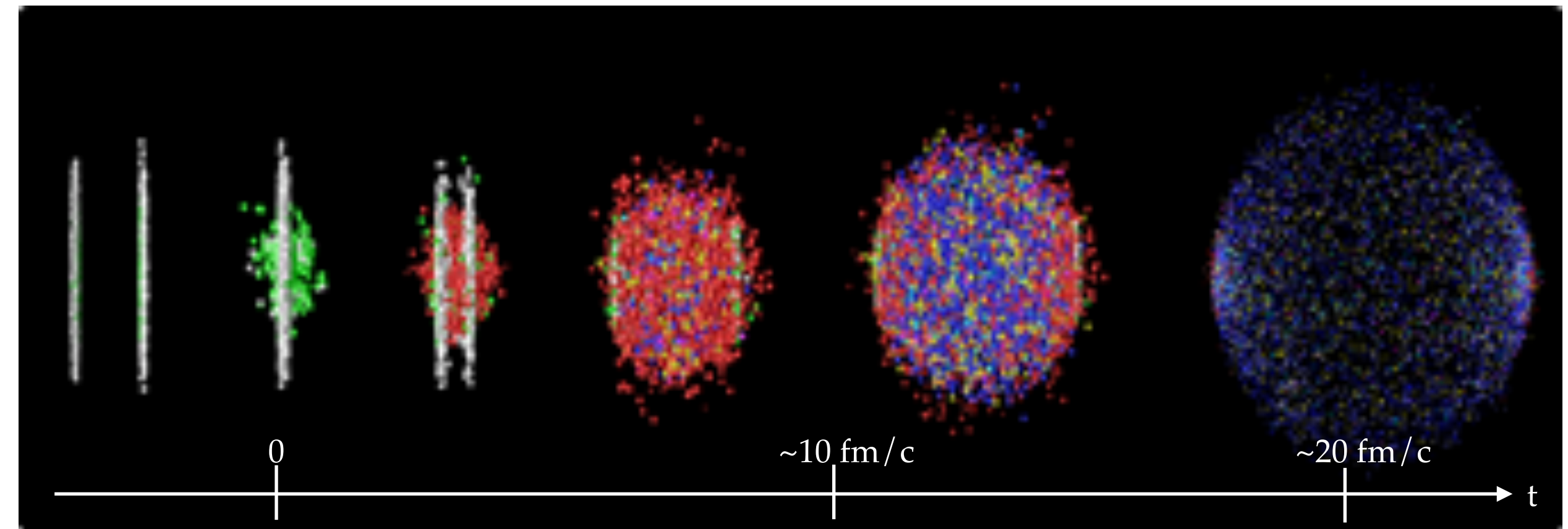
- Non-renormalizable.
- Headache to solve numerically.

The best of two worlds

Quantum Chromodynamics



Dynamic colliding nuclear medium

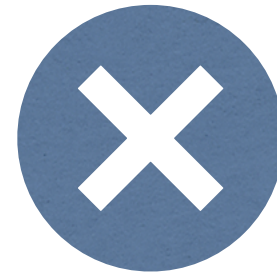
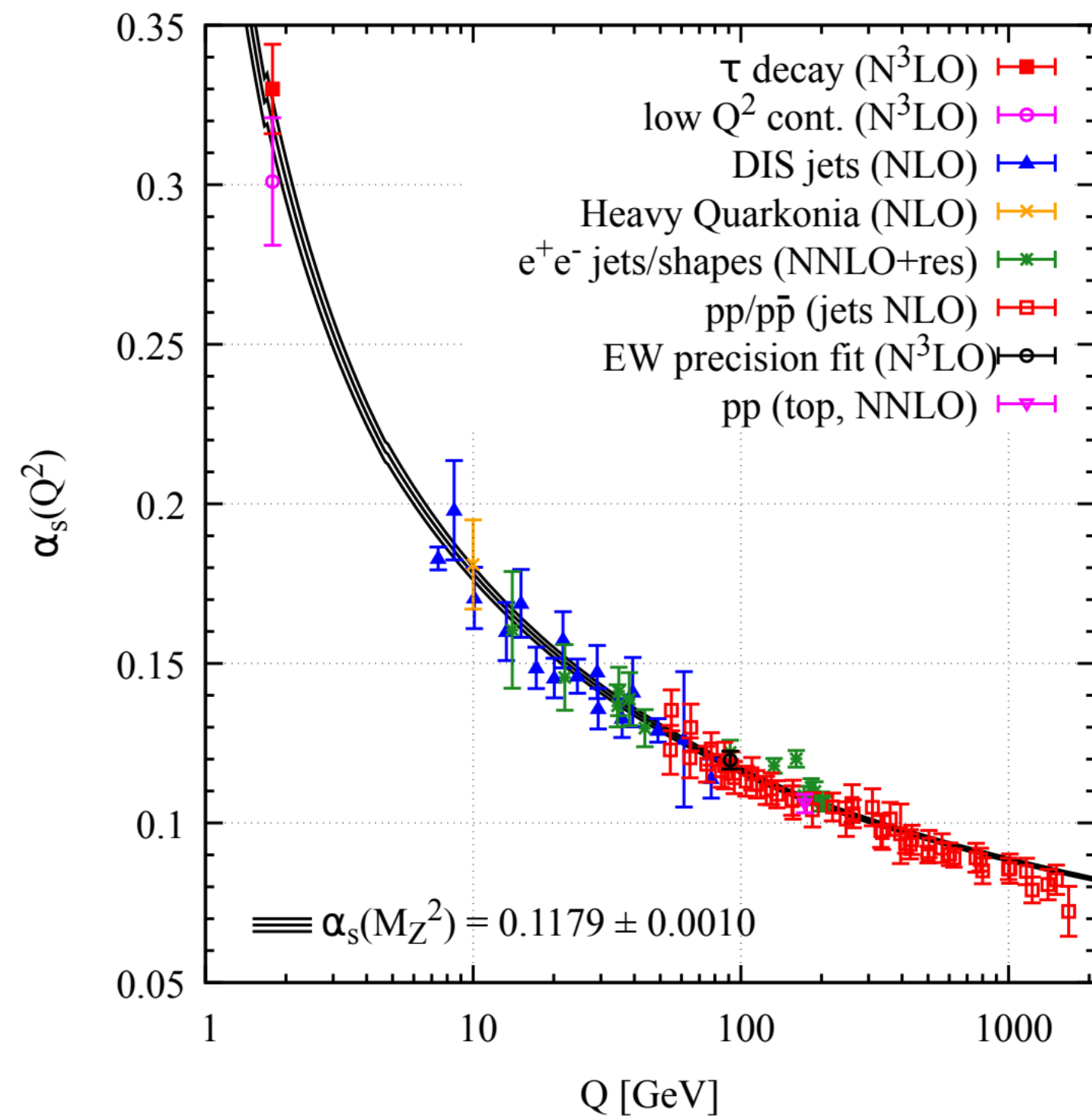


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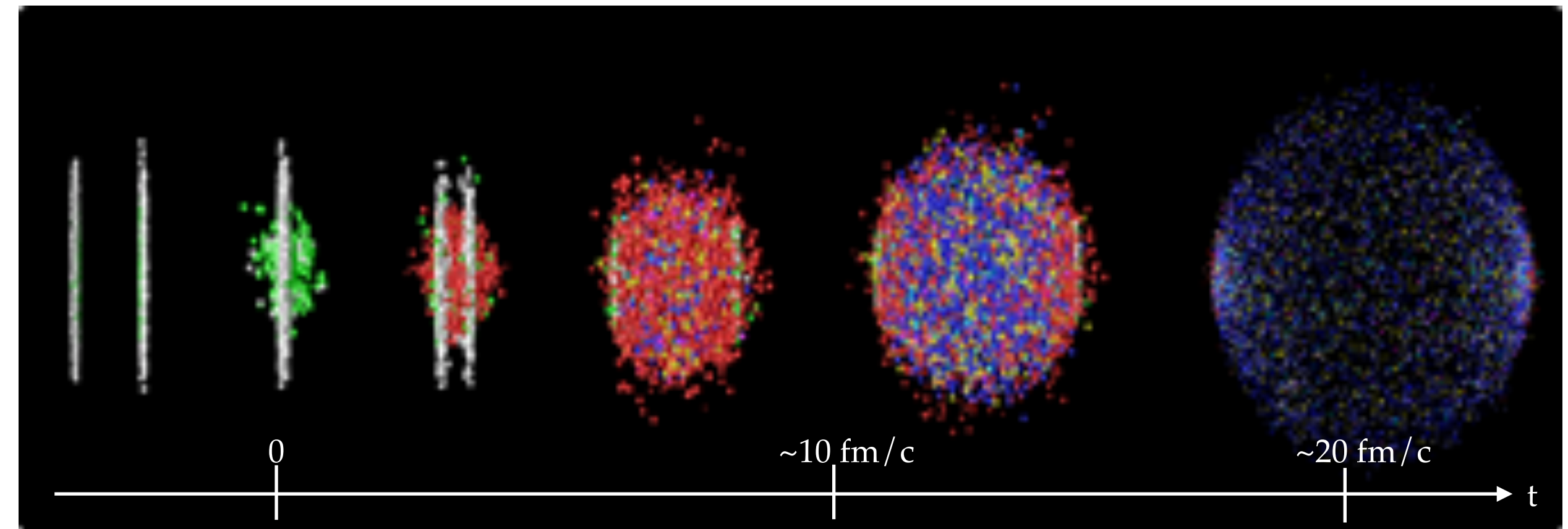
- Initial conditions are different from proton-proton collisions.
- Expanding medium in the collision.

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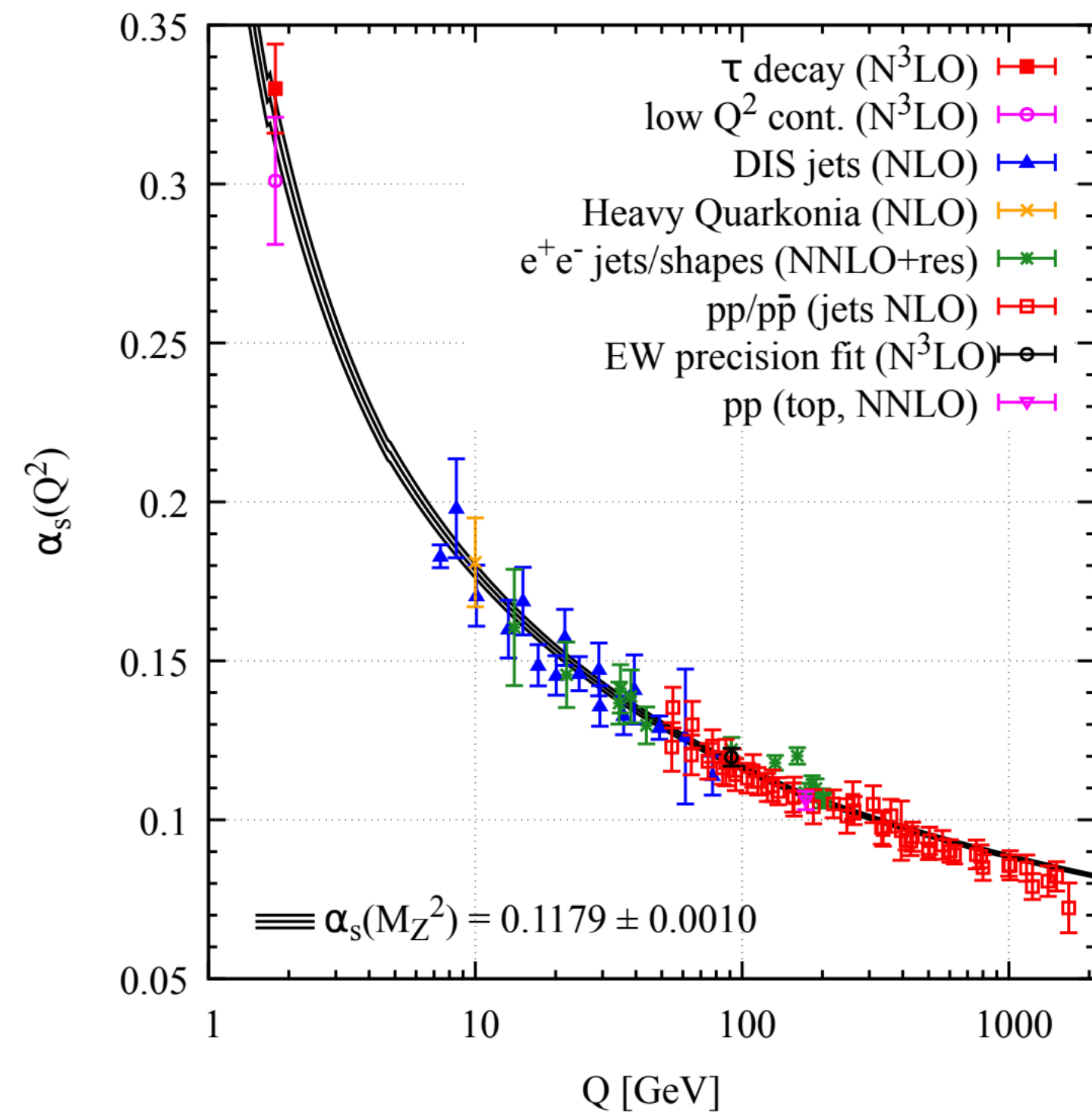
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Heavy-ion physics: interface between effective theory, modeling and phenomenology

One medium to rule them all

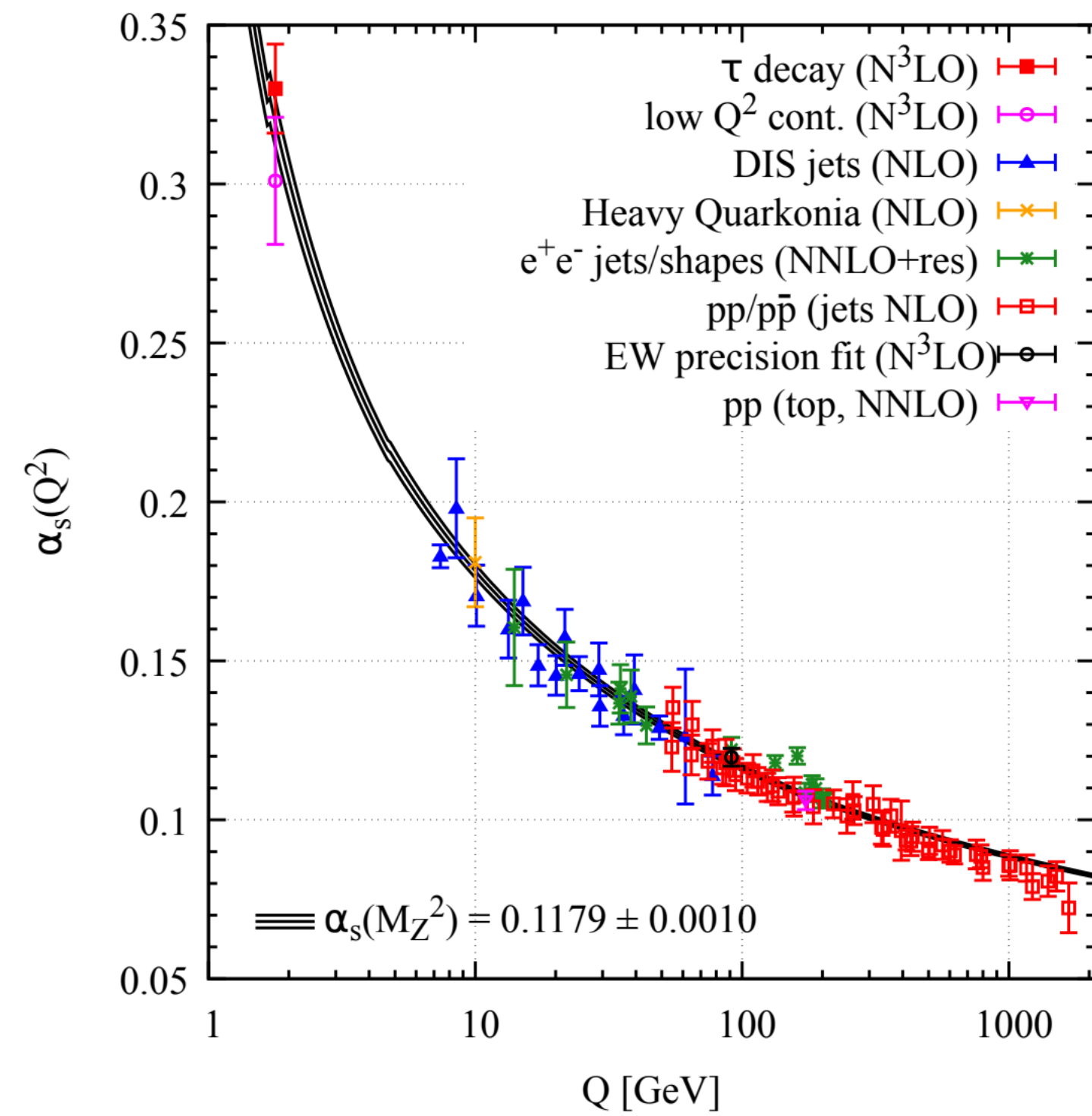
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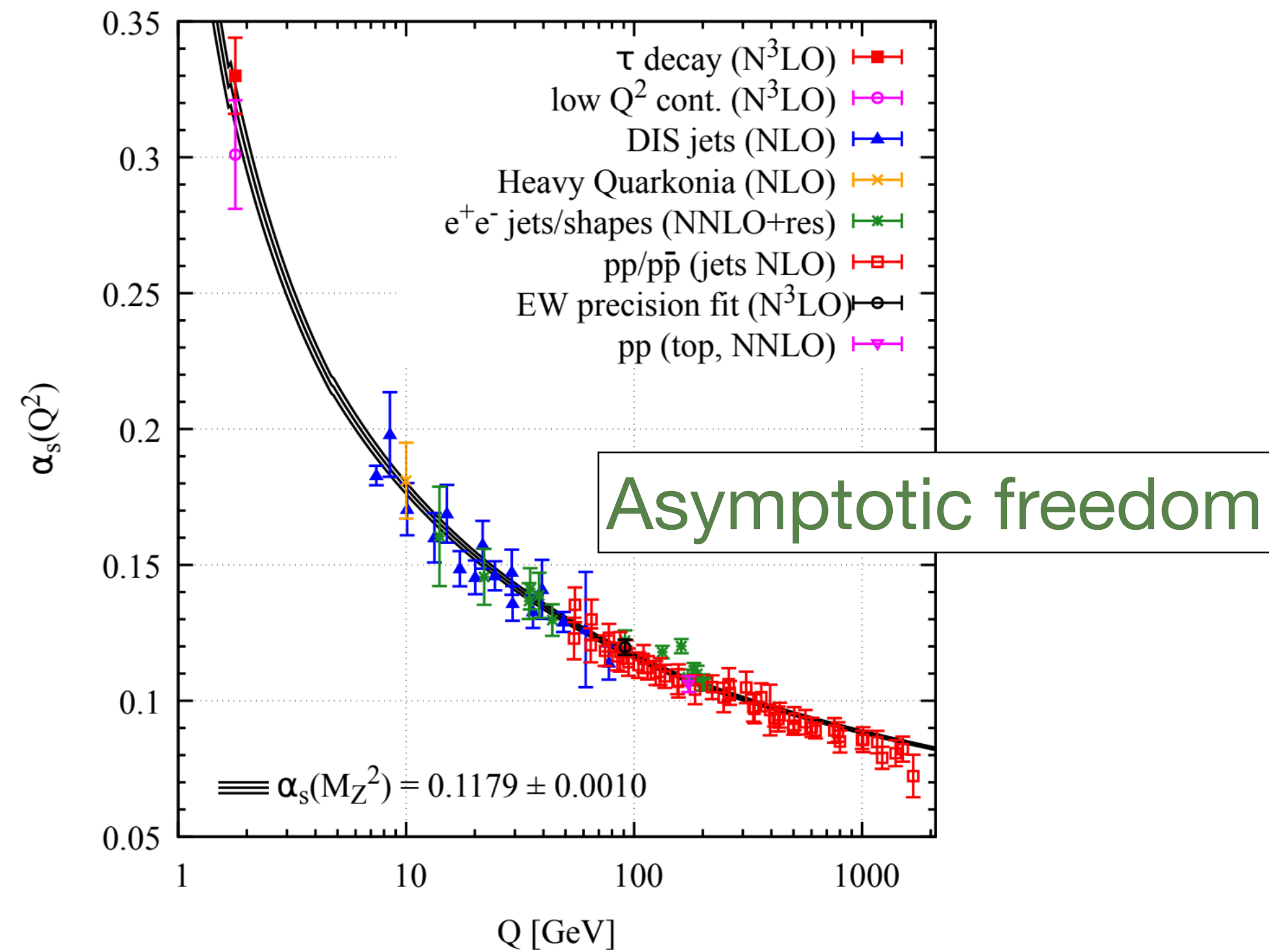
Confinement



One medium to rule them all

Quantum Chromodynamics

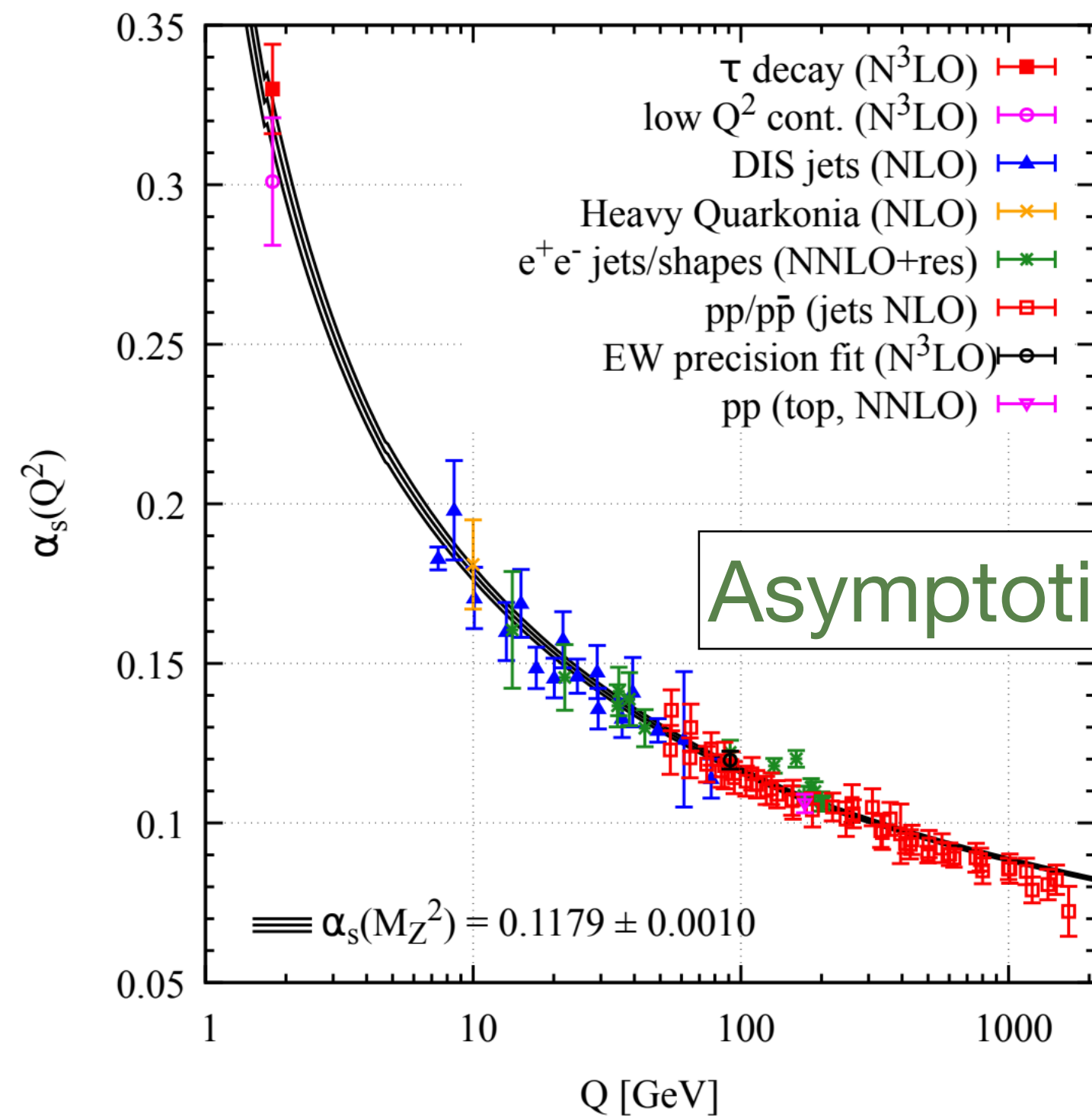
Confinement



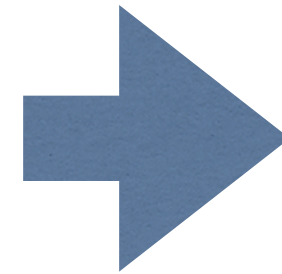
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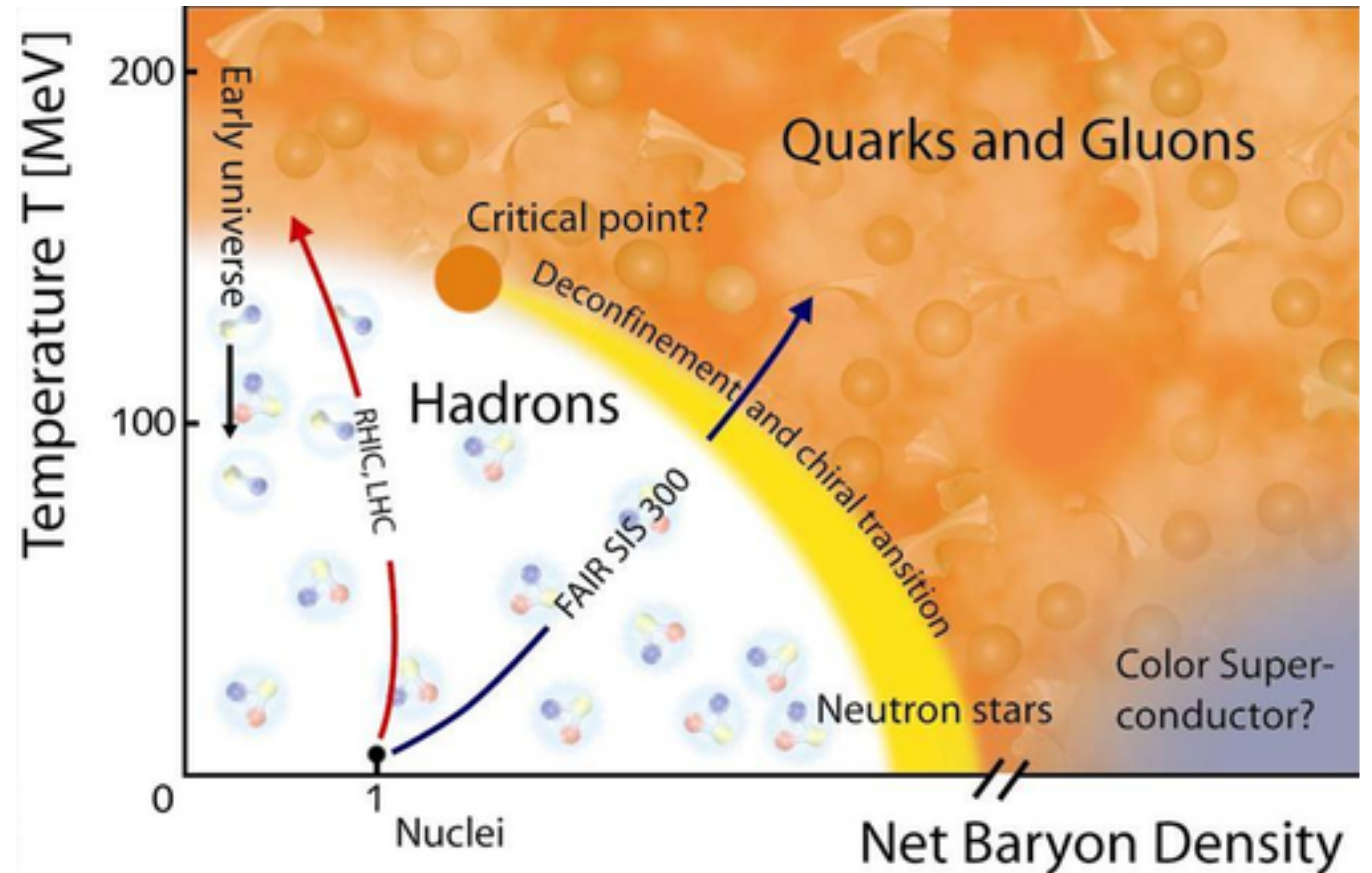
Confinement



Asymptotic freedom



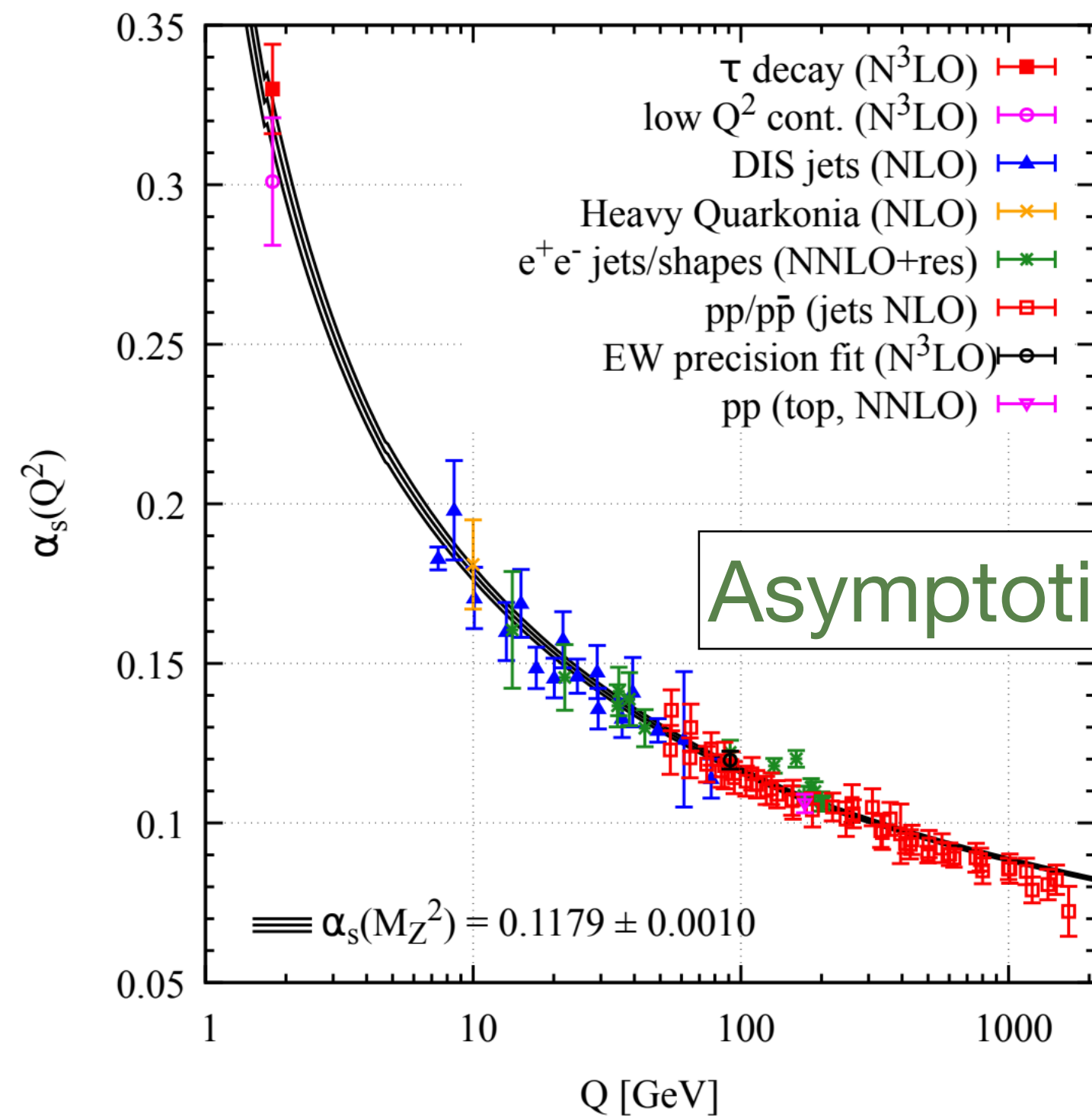
Phase diagram of hadronic matter



One medium to rule them all

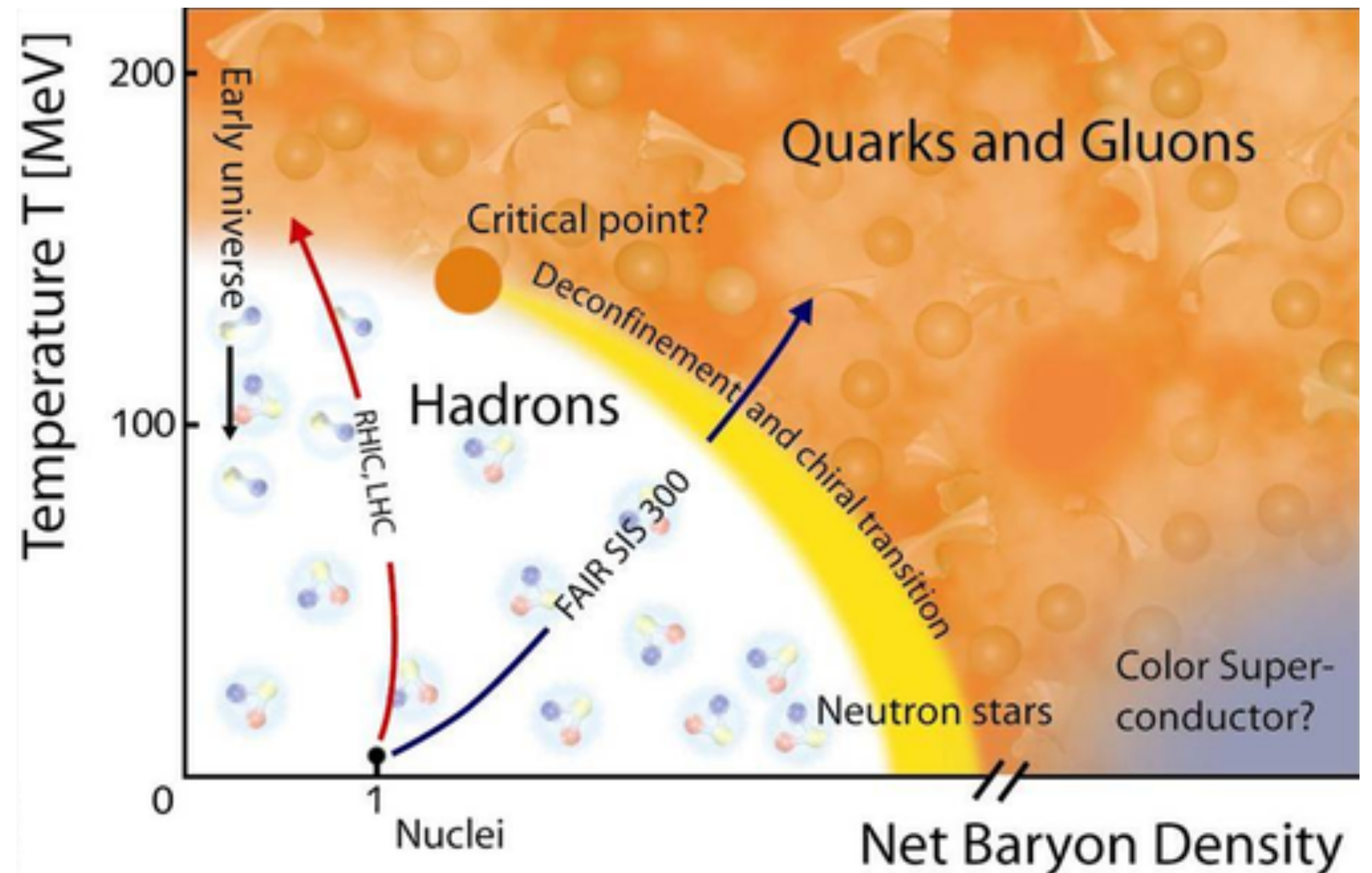
Quantum Chromodynamics

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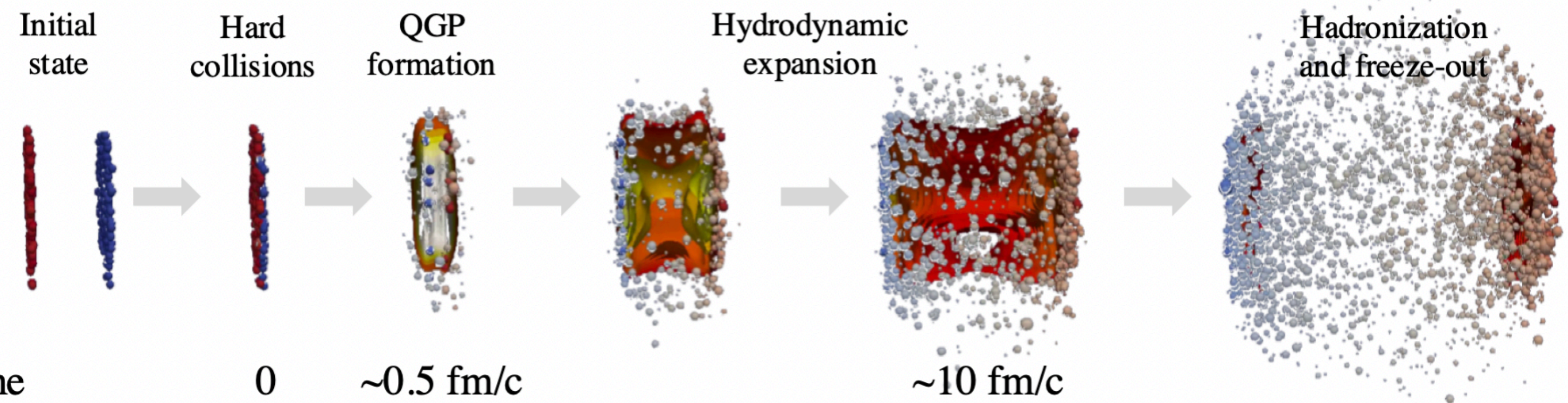
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Phase diagram of hadronic matter



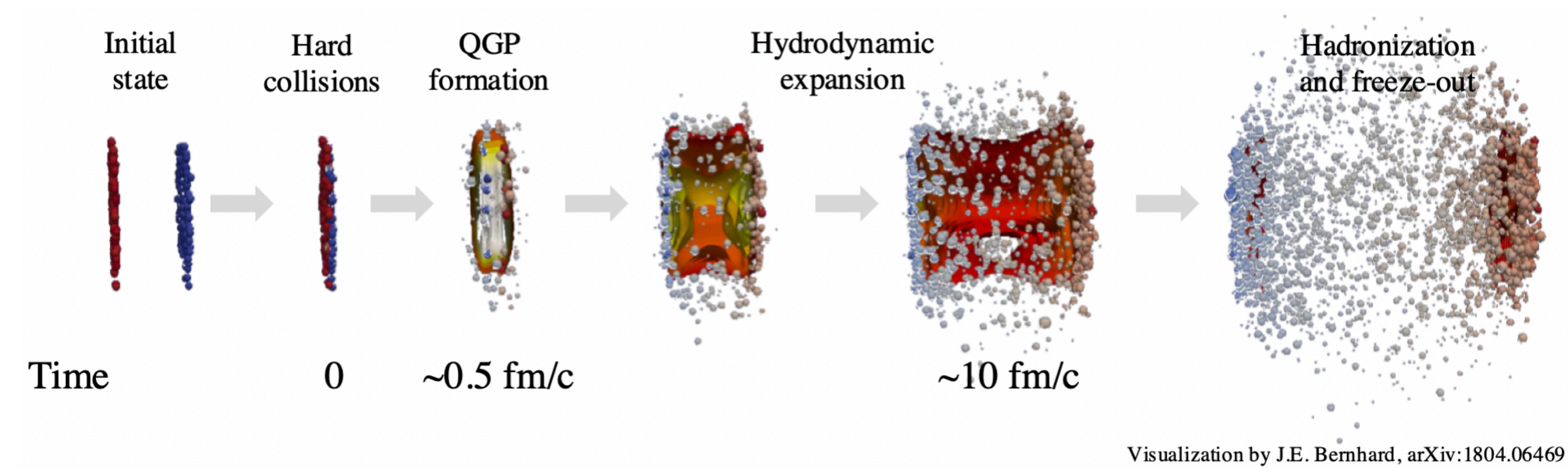
High temperature/density: formation of the Quark-Gluon Plasma

Why study the QGP?



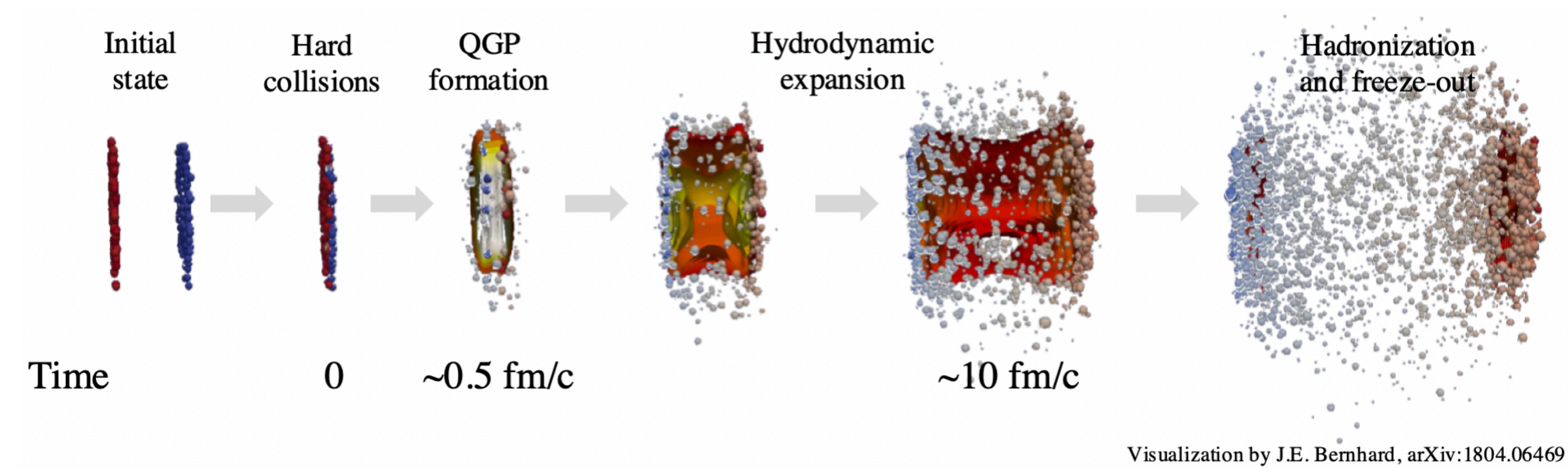
Visualization by J.E. Bernhard, arXiv:1804.06469

Why study the QGP?



* QGP studies at CERN:

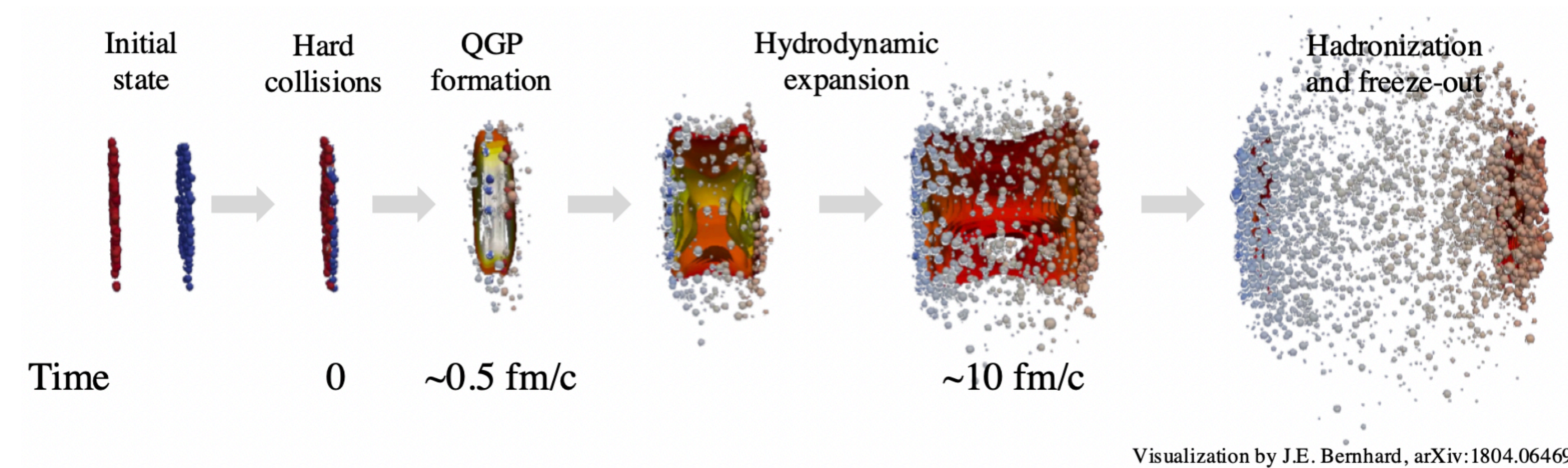
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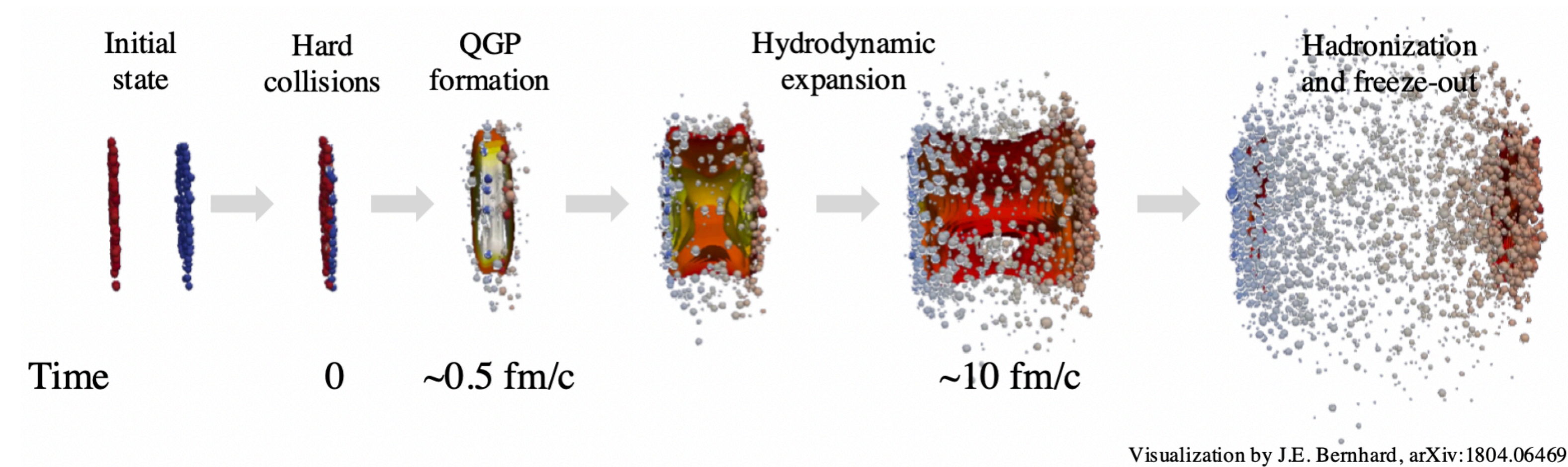
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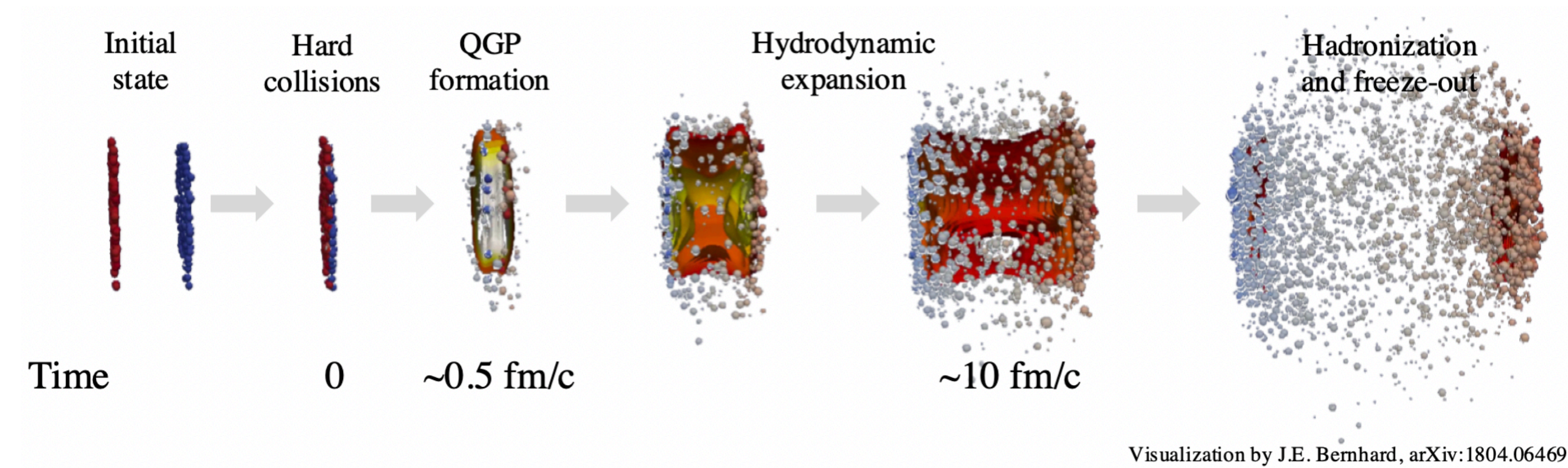
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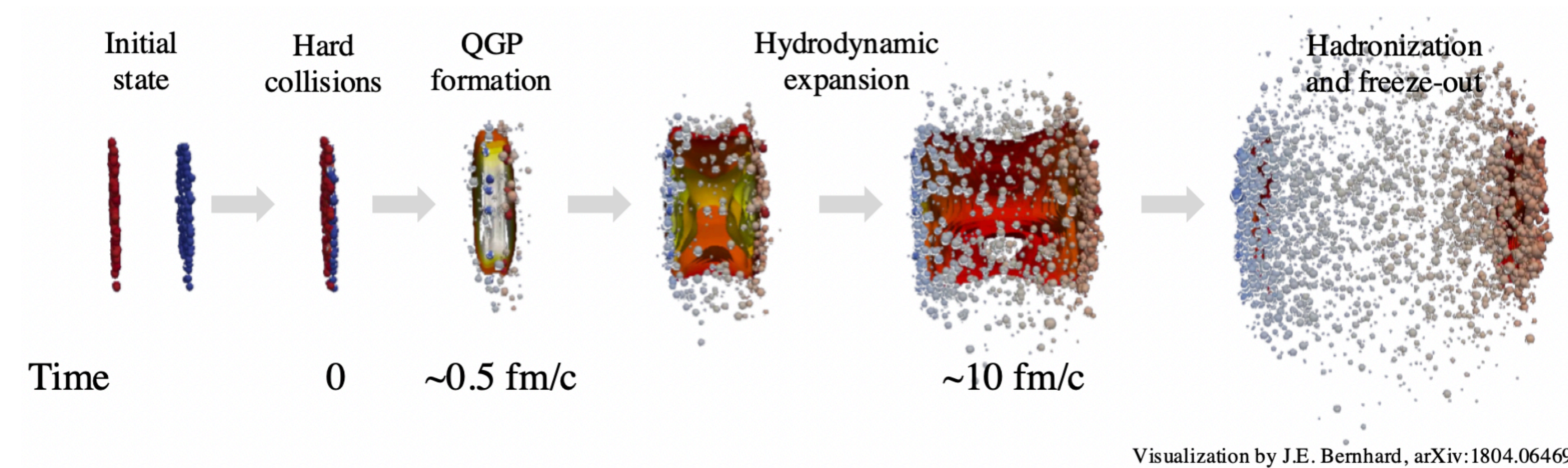
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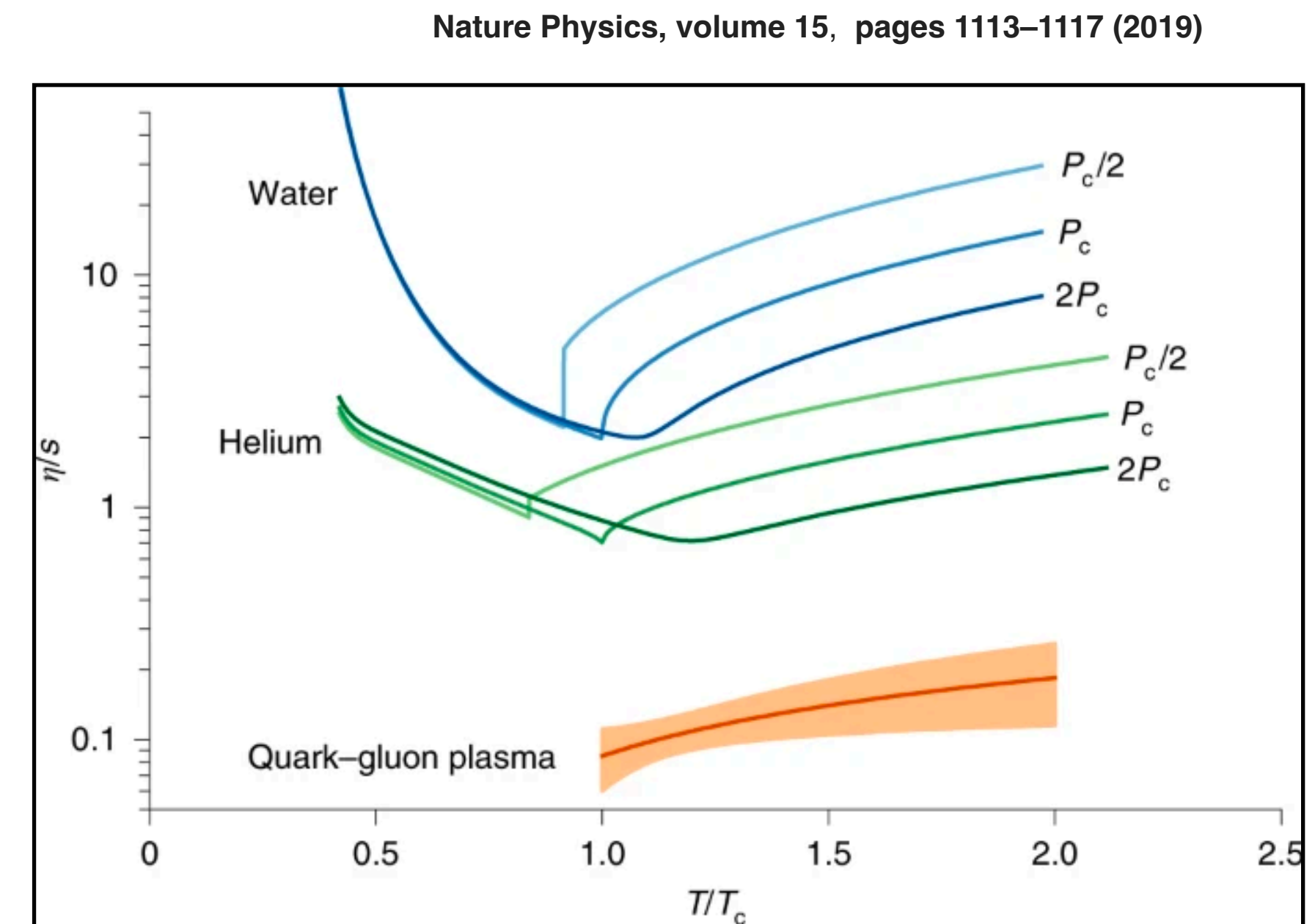
- Study of nuclear matter under extreme temperature.
- Study the phase transition and confinement.
- Study of N-body problems: hydrodynamics.
- **Study baby Universe!**

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QGP = perfect fluid

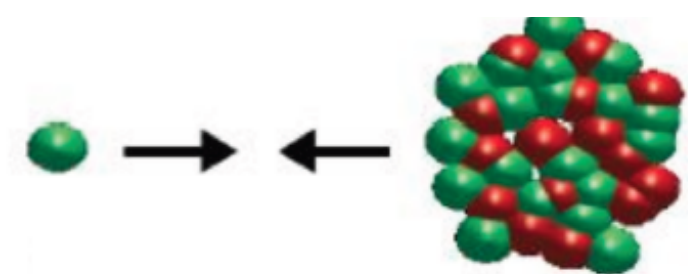
Where it gets 'messy'

Where it gets 'messy'

The orthodox approach



- The QCD 'vacuum'.



p Pb
b bP

- The confined matter.



Pb Pb
bP bP

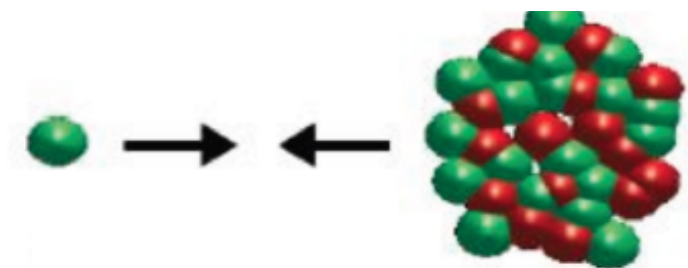
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The probes

Hard probes

- Heavy-quark mesons, quarkonia, jets...

Soft probes



- Charged particles, light hadrons, low-mass hadrons ...

Electromagnetic probes

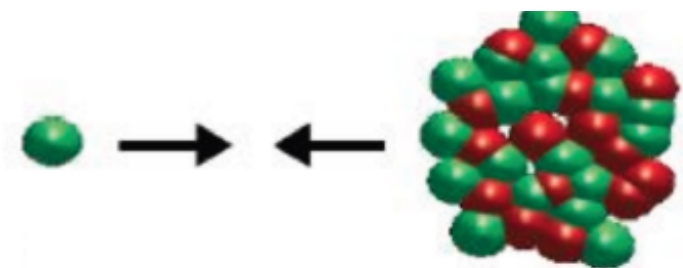
- Drell-Yan, photons, weak bosons ...

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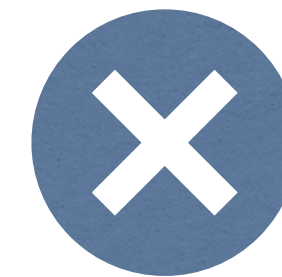
The observables

Production

- Cross-sections, Nuclear modification factor, Relative ratios ...

Correlations

- Multiplicity dependence, flow measurements...

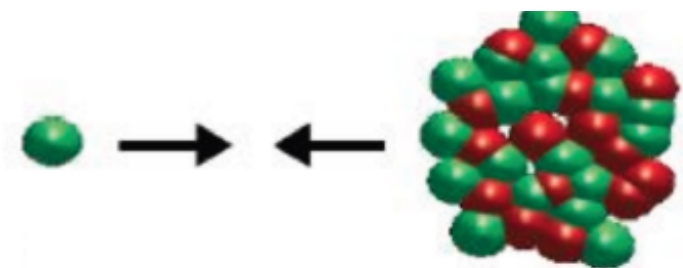


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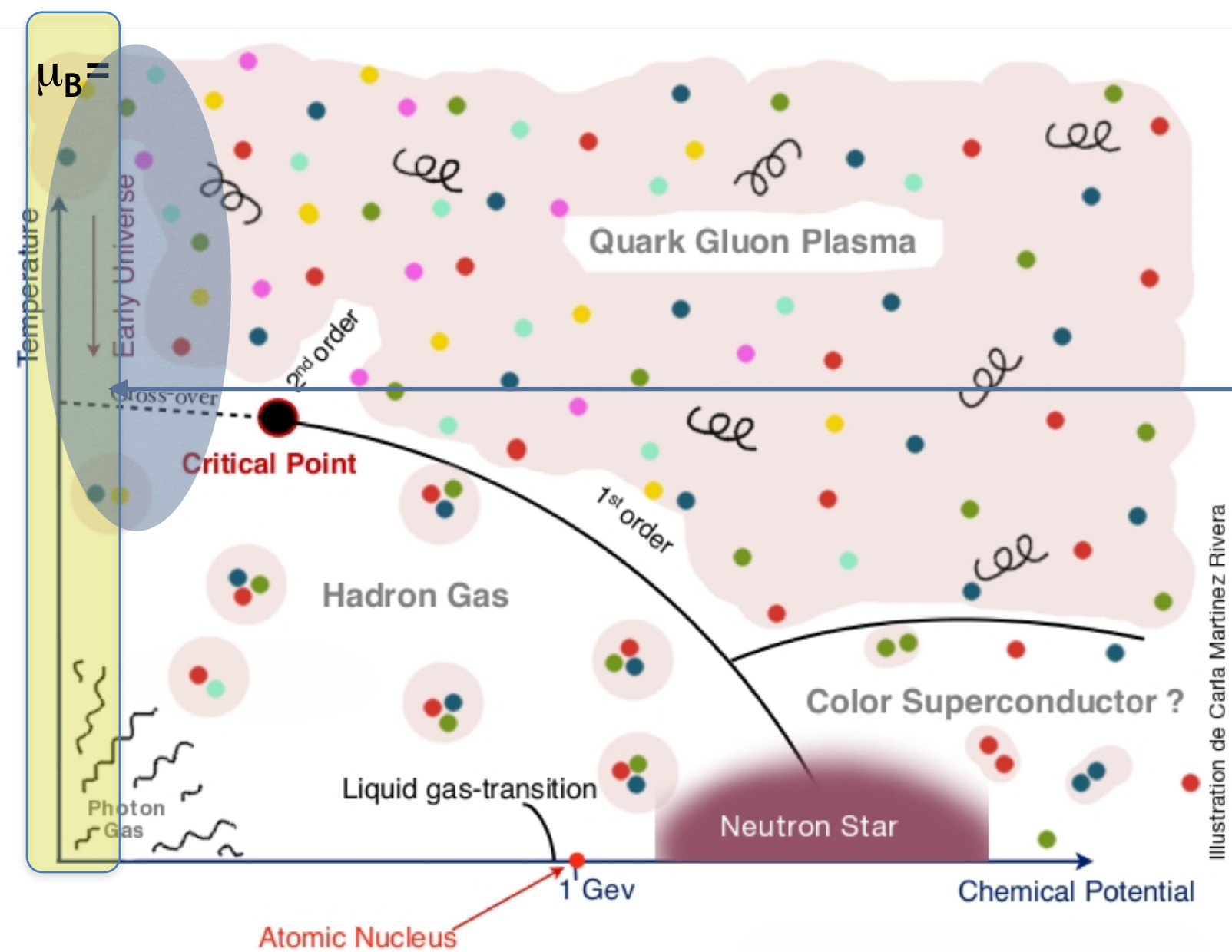
- Multiplicity dependence, flow measurements...

A QGP physicist should know everything about his/her favorite probs !

MODELING A HIC : HYDRODYNAMICS

- [Eur. J. Phys. 29 \(2008\) 275-302](#)

From the phase diagram to the collision



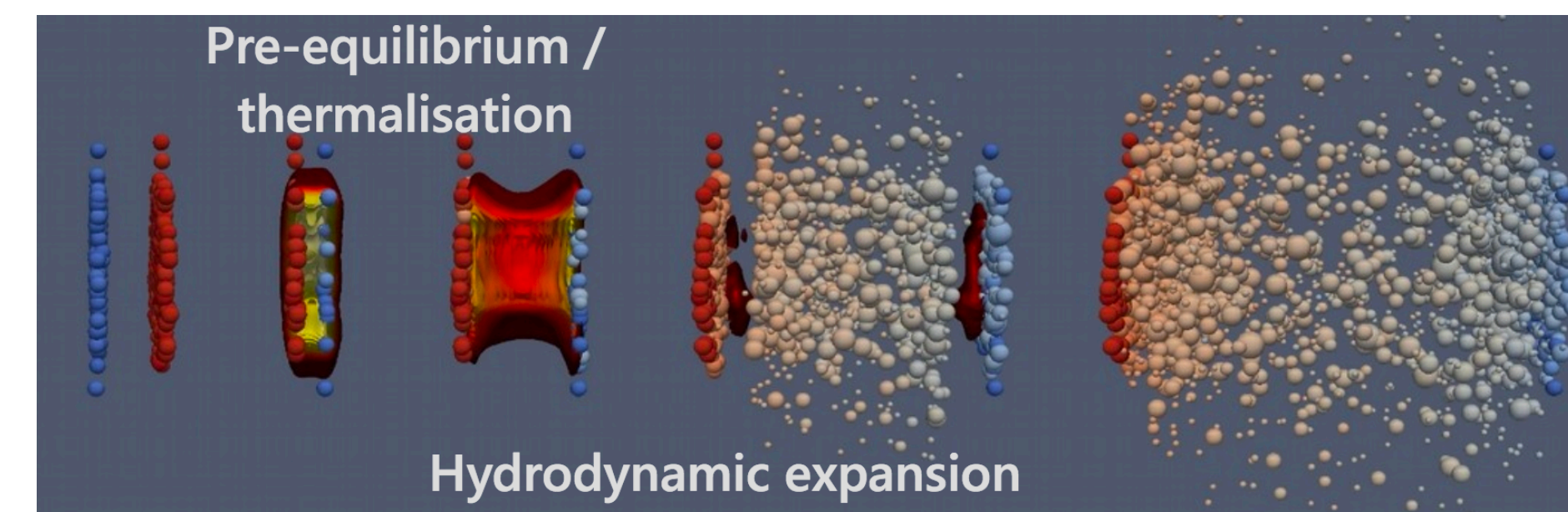
We now want to study this area

- * The basic approach strategy :

- « Hey, let's take two big nuclei and smash them as hard as we can ! »

- * The immediate answer :

- « Oh cool ! ... but how does it look like, a heavy-ion collisions ? »



Goal for today : get to this representation !

Hydro : a good approach

* Why Hydrodynamics :

- Heavy-ion collisions (HIC) : a bunch of quarks and gluons happily colliding together.
- Ideal tool to study **bulk properties** and **dynamical evolution**.

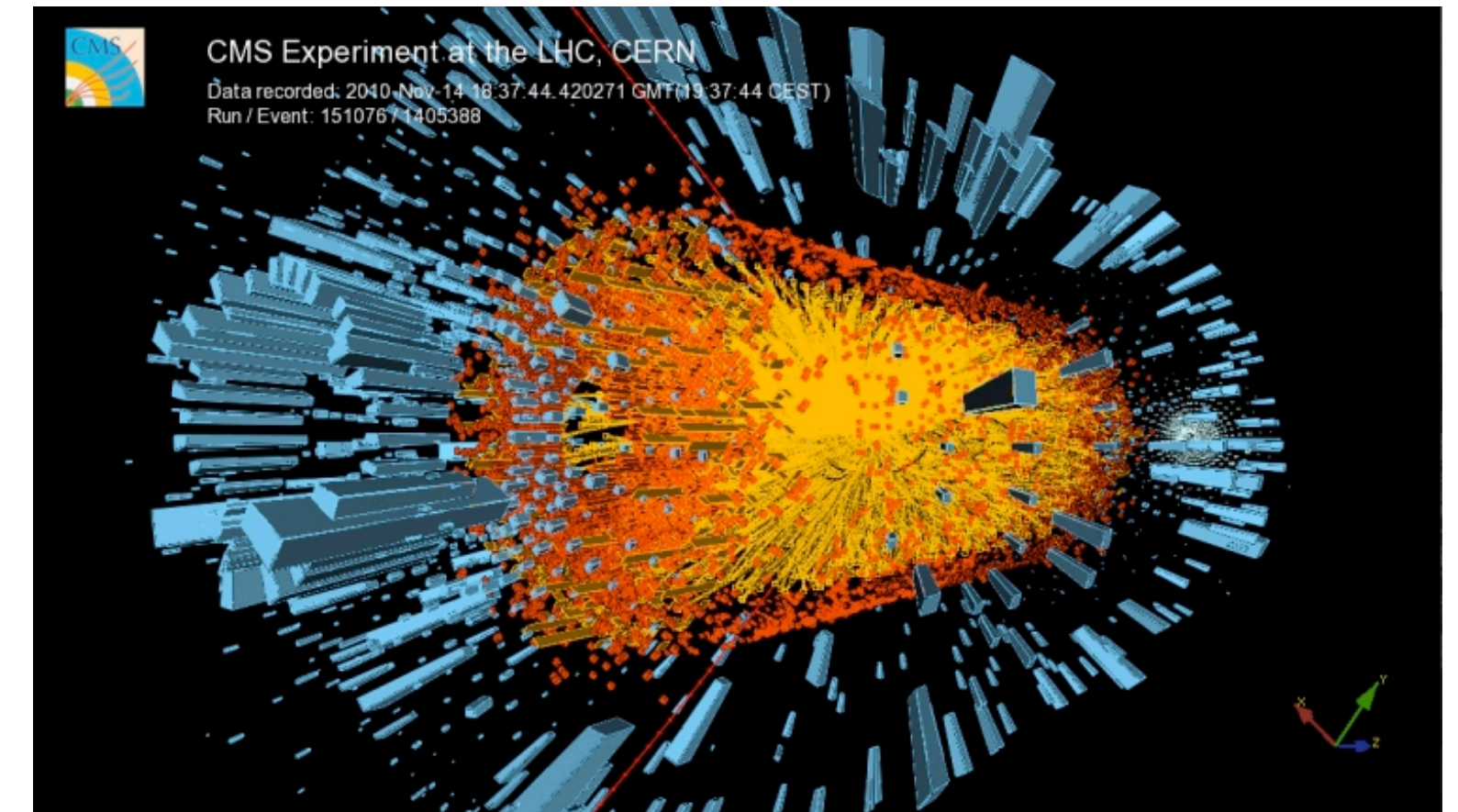
* Assumptions : $\langle \lambda \rangle \ll L$ where :

- $\langle \lambda \rangle$: the mean free path of particles between two interactions.
- L : the size of the system.

* Hydro's pros :

- Simple : all the informations encoded in thermo. properties.
- General : only assume **thermal equilibrium**.

* **This talk : we go through basic hydro. model and derive basic hydro. features !**



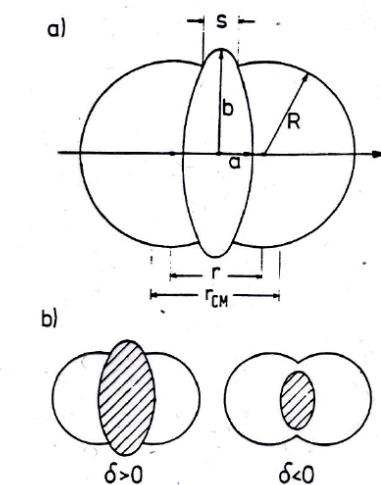
32, NUMBER 13 PHYSICAL REVIEW LETTERS 1 APRIL 1974

Nuclear Shock Waves in Heavy-Ion Collisions

Werner Scheid, Hans Müller, and Walter Greiner

Institut für Theoretische Physik der Universität Frankfurt, Frankfurt am Main, Germany
(Received 19 November 1973)

It is shown that nuclear matter is compressed during the encounter of heavy ions. If the relative velocity of the nuclei is larger than the velocity of first sound in nuclear matter (compression sound for isospin $T=0$), nuclear shock waves occur. They lead to densities which are 3–5 times higher than the nuclear equilibrium density ρ_0 , depending on the energy of the nuclei. The implications of this phenomenon are discussed.



The first paper

FIG. 1. (a) Geometric parameters of the model. (b) Two cases $\delta > 0$ and $\delta < 0$. The unphysical situation $\delta < 0$ is excluded by forces of constraints.

Relativistic thermodynamics

- * Standard thermodynamics : assume a global equilibrium of the system.
- * This is clearly **not the case** in HIC collisions !
- * But at first we can assume **local** thermodynamic equilibrium :
 - For any point, P and T vary slowly in some neighborhoods around it (fluid element)
- * In the ideal (inviscid) case, one must derive the following equations :

**Local conservation
of energy-momentum**

$$\partial_\mu T_{\text{id}}^{\mu\nu} = 0, \quad \partial_\mu J_B^\mu = 0$$

↑
Energy-momentum
tensor

↑
Net Baryon
current

**Conservation
of baryon flux**

Let's derive those equations
in the fluid element rest
frame!

The Energy-Momentum tensor

Tensor in the fluid rest frame

$$T_{(0)} = \begin{pmatrix} \epsilon & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{pmatrix}$$

Lorentz transformation for any moving fluid cells

$T_{\text{id}}^{\mu\nu}$

$T =$

Tensor in the first order in velocity

Density of the j^{th} component of momentum

$$T = \begin{pmatrix} \epsilon & (\epsilon + P)v_x & (\epsilon + P)v_y & (\epsilon + P)v_z \\ (\epsilon + P)v_x & P & 0 & 0 \\ (\epsilon + P)v_y & 0 & P & 0 \\ (\epsilon + P)v_z & 0 & 0 & P \end{pmatrix}$$

Energy flux along axis i

Reduce formula

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu - P g^{\mu\nu}$$

$$g^{\mu\nu} \equiv \text{diag}(1, -1, -1, -1)$$

$$u^\mu = \left(\frac{1}{\sqrt{1 - |\mathbf{v}|^2}}, \frac{\vec{\mathbf{v}}}{\sqrt{1 - |\mathbf{v}|^2}} \right) \equiv \text{4velocity of the fluid cell}$$

- By definition $v_x = v_y = v_z = 0$ in the fluid rest frame
- **Thanks captain obvious**, but not really useful...

* To go to non-relativistic limits :

- $(\epsilon + P)\cdot\mathbf{v} \sim \epsilon\cdot\mathbf{v} \approx \rho\mathbf{v}$

* **The tensor is symmetric** in all rest frame.

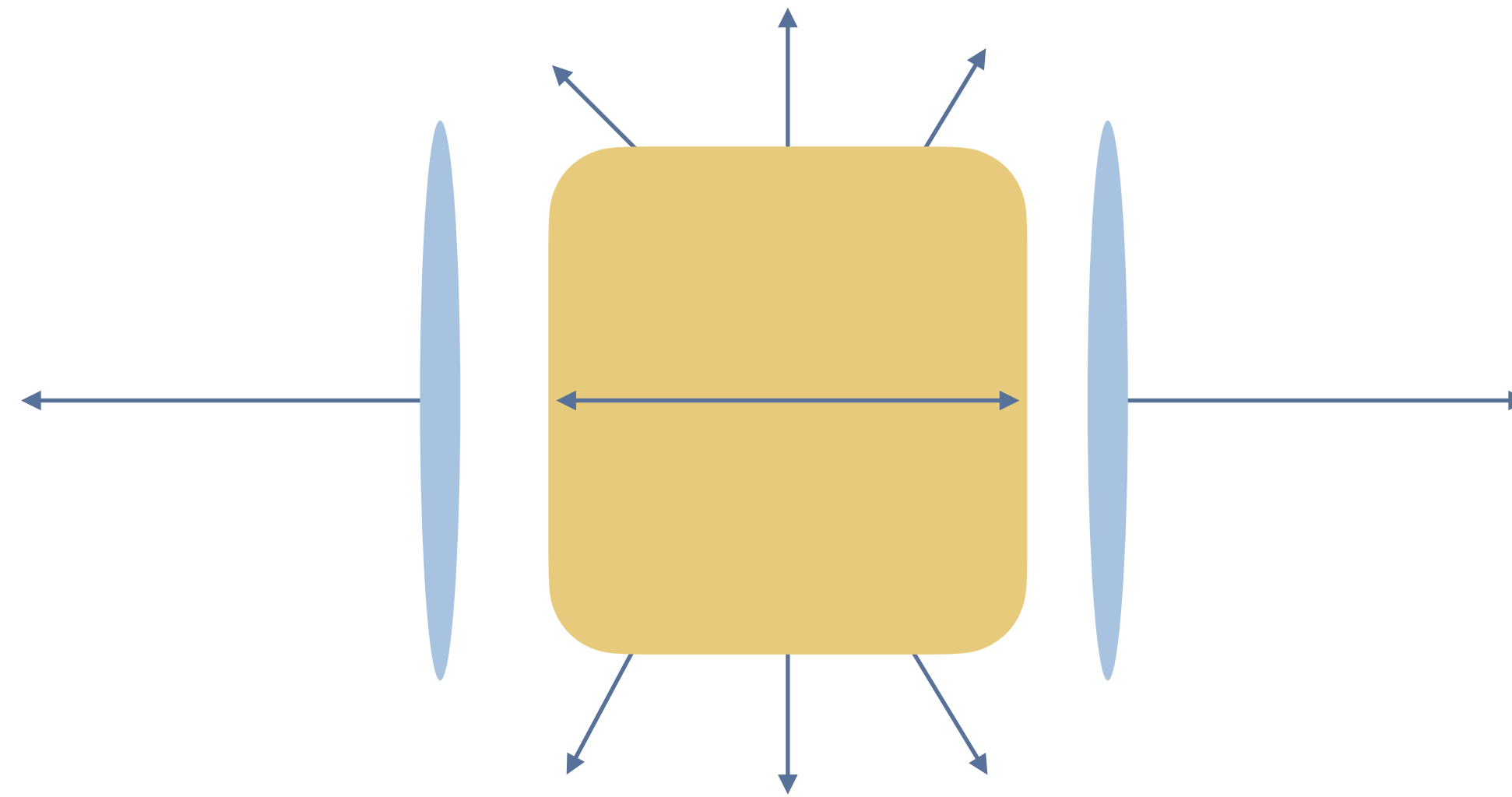
- **momentum density = energy flux**

Baryon number conservation

- * The equation $\partial_\mu J_B^\mu = 0$ can also be written as $\partial_\mu (nu^\mu) = 0$
- * There are similar equations for any conserved charges.
- * To close the equations system, one must add the *equation of state* :
$$P = P(\epsilon, n_i)$$
- * For the sake of arguments :
 - 3 comp. of $u + \epsilon + P + n_i(i=1,\dots,M) = 4 + M + 1$ equations
- * **Let's now try to get a model out of all these eq. !**

THE BJORKEN SCENARIO

Simple picture : the Bjorken scenario



* Initial conditions :

- Highly boosted « pancakes » in the centre-of-mass system, that cross within $\tau_{\text{cross}} \sim 2R/\gamma$.
 - Note that $\gamma_{\text{SPS}} \sim 10 < \gamma_{\text{RHIC}} \sim 100 < \gamma_{\text{LHC}} \sim 2500-7000$, thus $\tau_{\text{cross}} < \tau_{\text{QCD}} \sim 1 / \Lambda_{\text{QCD}} \sim 1 \text{ fm}/c$.
 - We assume fast thermalization.
 - We assume $\langle p_x \rangle = \langle p_y \rangle = 0 \rightarrow$ **Flow (see later) only comes from Hydro. phase.**
- * Bjorken's prescriptions : the fluid rapidity = the space-time rapidity

New variables

Bjorken's prescriptions

$$t = \tau \cosh \eta_s$$

$$\tau = \sqrt{t^2 - z^2}$$

Proper time

$$z = \tau \sinh \eta_s$$

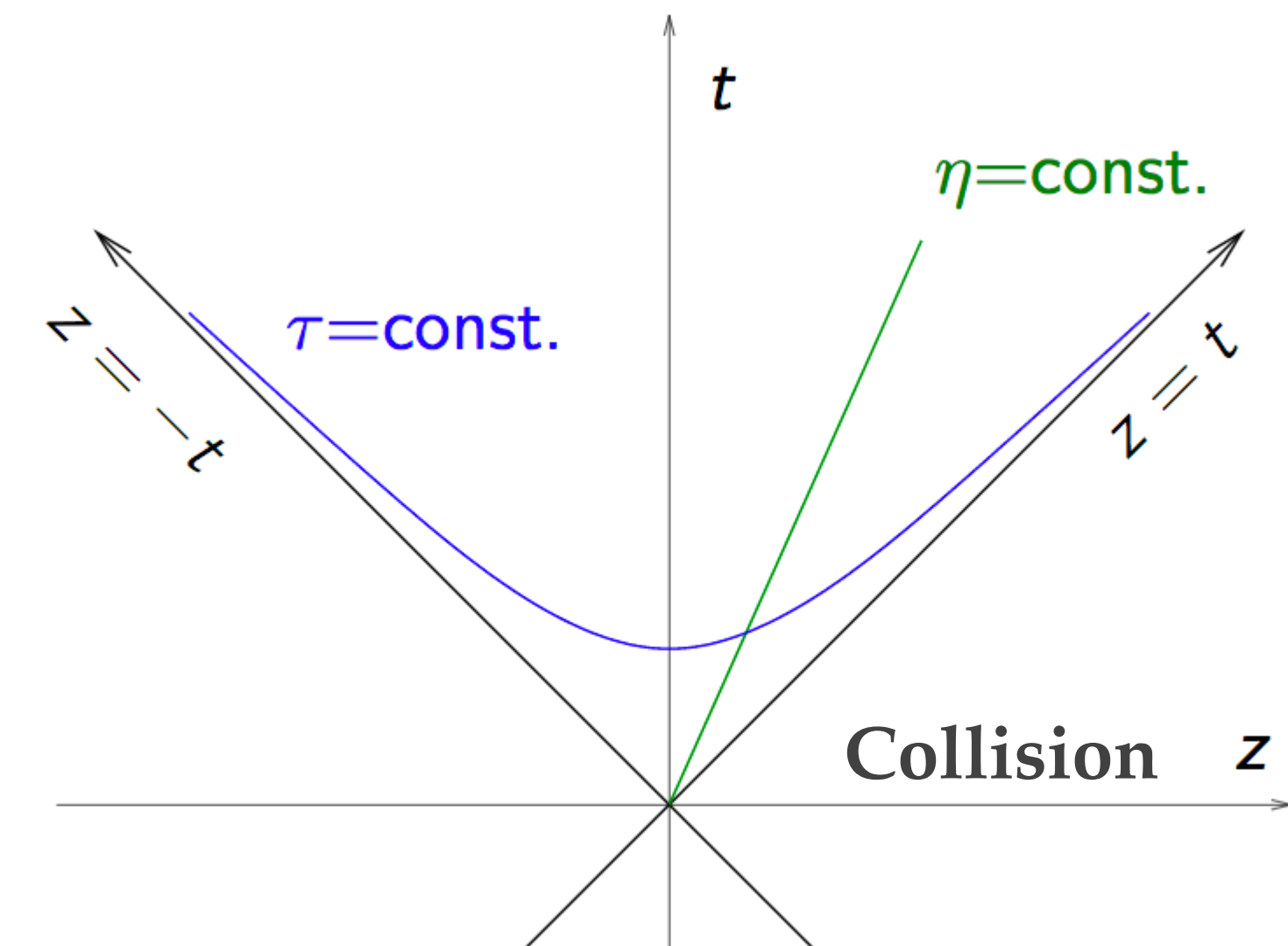
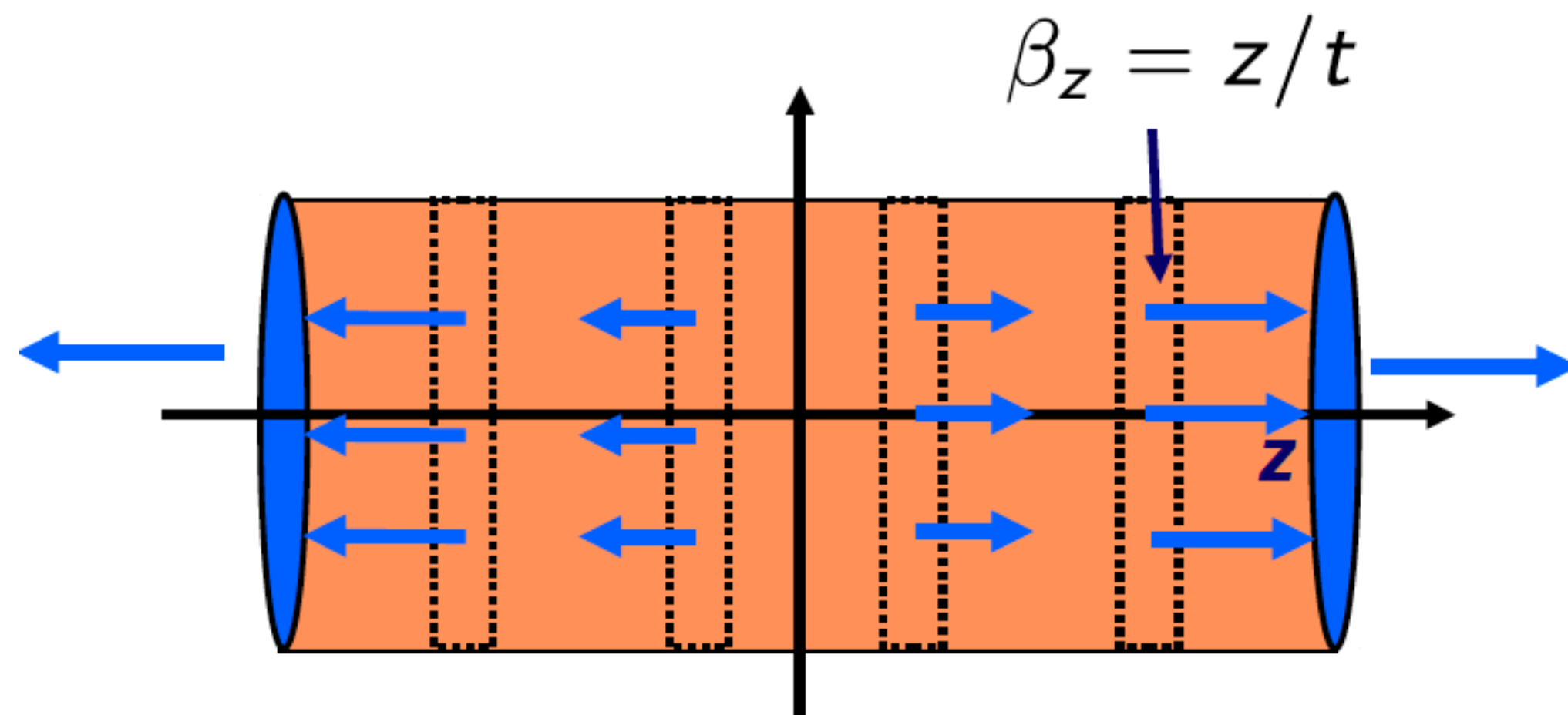
$$\eta_s = \frac{1}{2} \log \frac{t+z}{t-z}$$

Space-time rapidity

$$v_z = \tanh Y$$

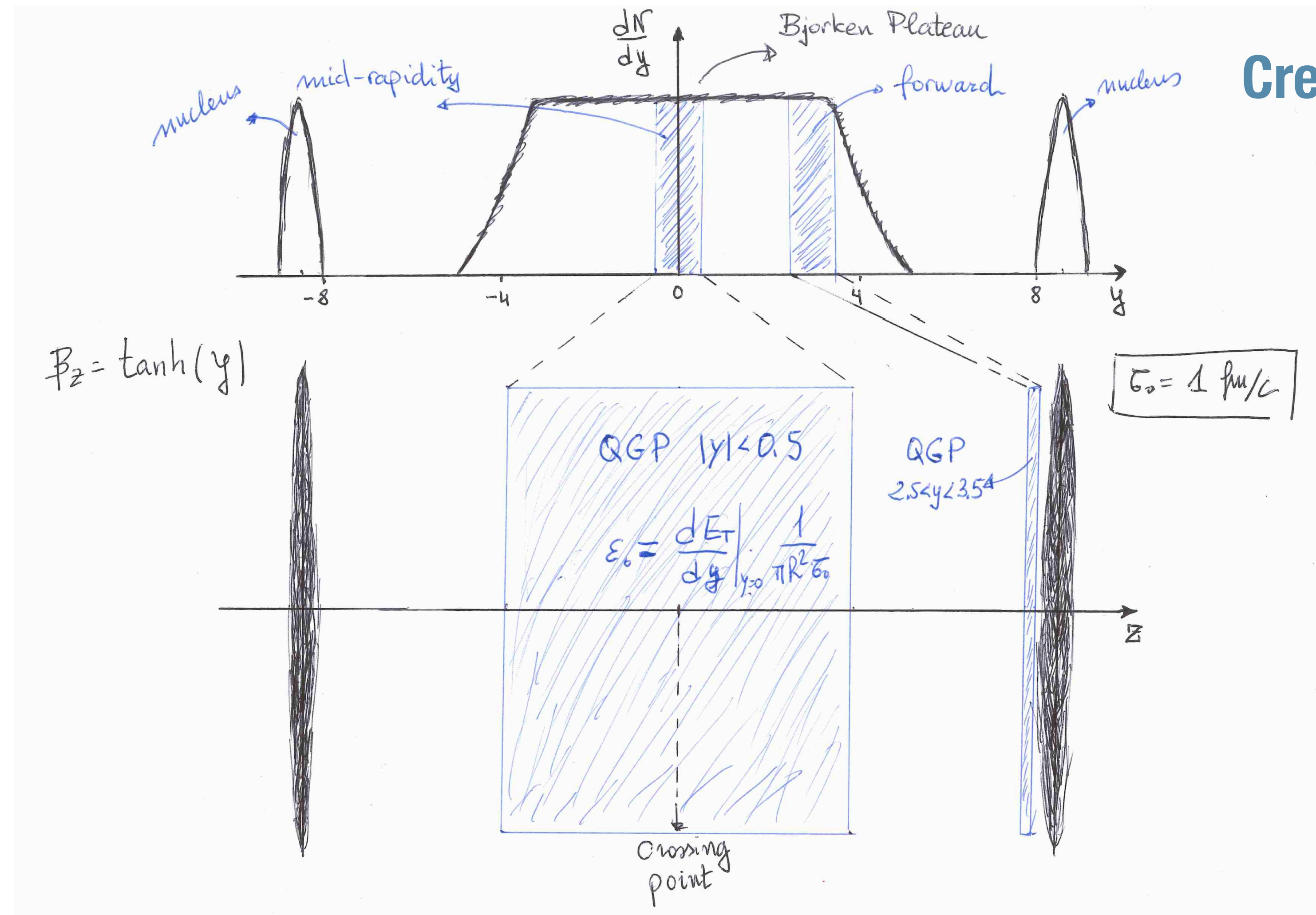
$$Y = \frac{1}{2} \log \frac{1+v_z}{1-v_z}$$

Fluid rapidity



Space -time representation of a collision

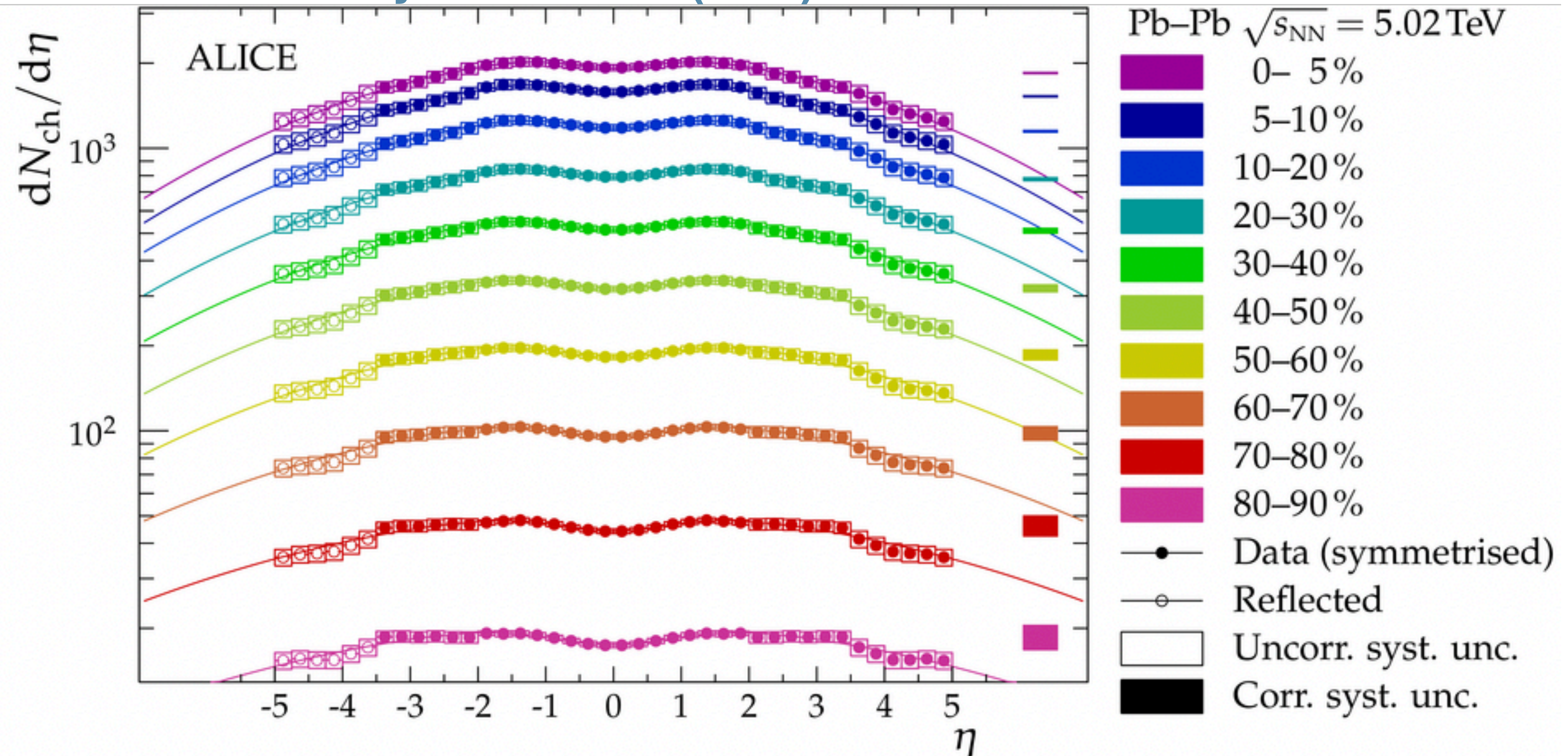
The famous Bjorken's plateau



Credit : Ginés Martínez

The famous Bjorken's plateau?

Phys. Lett. B 772 (2017)



Back to hydro with Bjorken's prescription

$$\partial_\mu T_{\text{id}}^{\mu\nu} = 0$$

$$\begin{aligned} \frac{\partial \epsilon}{\partial t} + \vec{\nabla} \cdot ((\epsilon + P)\vec{v}) &= 0 \\ \frac{\partial}{\partial t}((\epsilon + P)\vec{v}) + \vec{\nabla} P &= \vec{0}. \end{aligned}$$

Versus z

$$v_z = 0$$

$$\frac{d\epsilon}{\epsilon + P} = \frac{ds}{s}$$

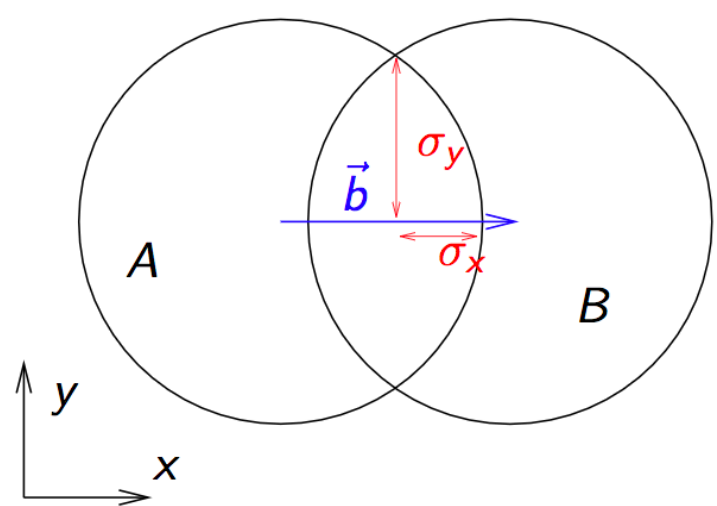
$$c_s = \left(\frac{\partial P}{\partial \epsilon} \right)^{1/2}$$

$$\frac{\partial}{\partial t}((\epsilon + P)v_z) + \frac{\partial}{\partial z} P = 0$$

$$\frac{\partial v_z}{\partial t} = -\frac{1}{\epsilon + P} \frac{\partial P}{\partial z} = -c_s^2 \frac{\partial \ln s}{\partial z}$$

Near $z = 0 \dots$

Toy model for initial density (entropy)



$$s(x, y, \eta_s) \propto \exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} - \frac{\eta_s^2}{2\sigma_\eta^2}\right)$$

$$\frac{\partial Y}{\partial \tau} = \frac{c_s^2}{\tau} \frac{\eta_s}{\sigma_\eta^2}$$

fluid cell rapidity \longrightarrow

$$Y(\tau) = \left(1 + \frac{c_s^2 \ln(\tau/\tau_0)}{\sigma_\eta^2} \right) \eta_s$$

Space-time rapidity

To which extend Bjorken is valid ?

- * c_s = speed of sound
- * Y **proportional** to η_s

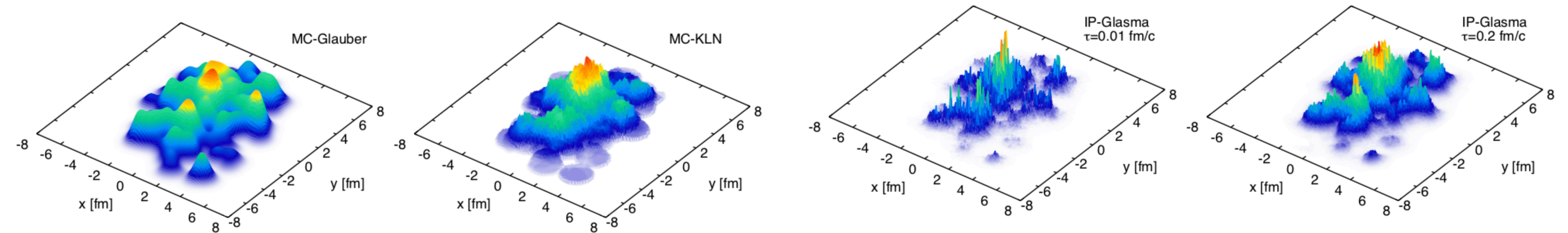
$$Y(\tau) = \left(1 + \frac{c_s^2 \ln(\tau/\tau_0)}{\sigma_\eta^2} \right) \eta_s$$

- * **Bjorken** : $Y = \eta_s$
- * **Good approximation** as long as $t \ll \sigma_x/c_s, \sigma_y/c_s \rightarrow$ **before longitudinal expansion !**
- * We will see later that transverse expansion acts like a cut-off for the longitudinal expansion.

Initial conditions

- * For our toy model, we assume
- * In reality, many prescriptions can be used :

$$s(x, y, \eta_s) \propto \exp \left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} - \frac{\eta_s^2}{2\sigma_\eta^2} \right)$$



Longitudinal expansion

- * Similarly with the baryon density around $z=0$:

$$\frac{\partial n}{\partial t} + \frac{n}{t} = 0 \quad \implies \quad nt = \text{const.}$$

Conserved

- * The energy density :

$$\begin{cases} \frac{\partial \epsilon}{\partial t} + \vec{\nabla} \cdot ((\epsilon + P)\vec{v}) = 0 \\ \frac{\partial}{\partial t}((\epsilon + P)\vec{v}) + \vec{\nabla} P = \vec{0}. \end{cases}$$



$$\frac{\partial \epsilon}{\partial t} + \frac{\epsilon + P}{t} = 0$$



A bit of shuffling that you can do yourself...



$$\implies d(\epsilon t) = -P dt$$

Not Conserved

$$\frac{\partial V_z}{\partial z} = \frac{1}{t}$$

- * Comments

- Longitudinal cooling $\rightarrow \epsilon_{\text{init}} > \epsilon_{\text{fin}}$
- No experimental evidence of longitudinal cooling (we measure final state particles), but we can still probe the initial stage with electromagnetic probes (thermal photons ...).
- Comoving energy decreases due to negative work of pressure forces \rightarrow only appear as a result of a thermalization process.

- * However, entropy density is conserved :

$$d\epsilon = T ds + \mu dn$$

From thermodynamics



$$\frac{\partial s}{\partial t} + \frac{s}{t} = 0$$

$$\implies st = \text{const.}$$

= No heat diffusion between fluid cells

Transverse momentum expansion

- * The initial transverse velocity is usually zero, but acceleration is not :

$$\frac{\partial v_x}{\partial t} = -\frac{1}{\epsilon + P} \frac{\partial P}{\partial x} = -c_s^2 \frac{\partial \ln s}{\partial x}$$

- * Assuming c_s constant :

$$v_x = \frac{c_s^2 x}{\sigma_x^2} t, \quad v_y = \frac{c_s^2 y}{\sigma_y^2} t.$$

- * Comments :

- Very smooth process.
- Typical time-scale σ_x/c_s . For $t \ll \sigma_x/c_s \sim R/c_s$, longitudinal expansion dominates.

Predictions from Hydro - angular correlation

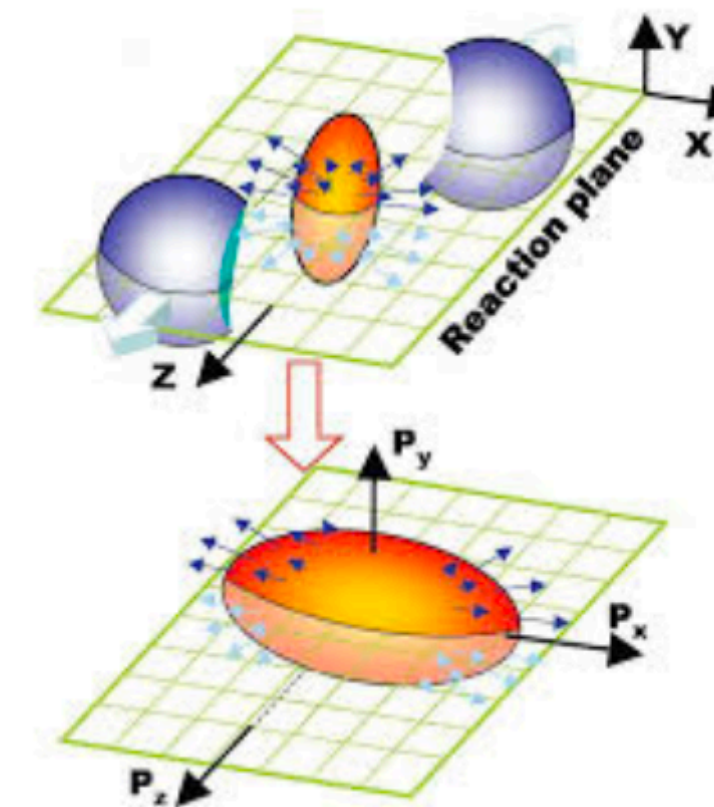
* In non-central collisions : $\sigma_x < \sigma_y$

* This leads to a *elliptical flow* v_2 : $\langle v_x^2 \rangle > \langle v_y^2 \rangle$

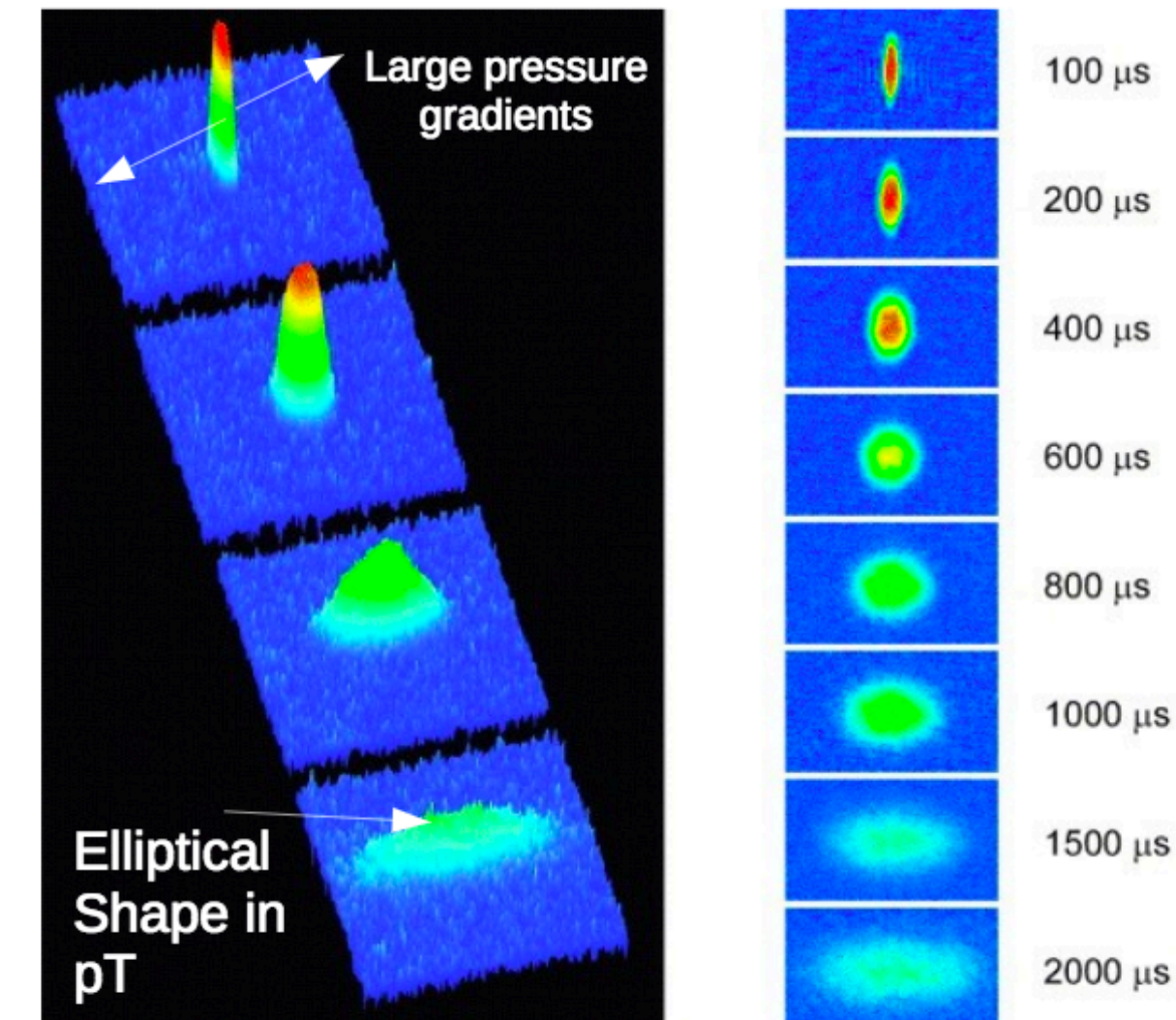
* Typical time scale:

Transverse expansion starts

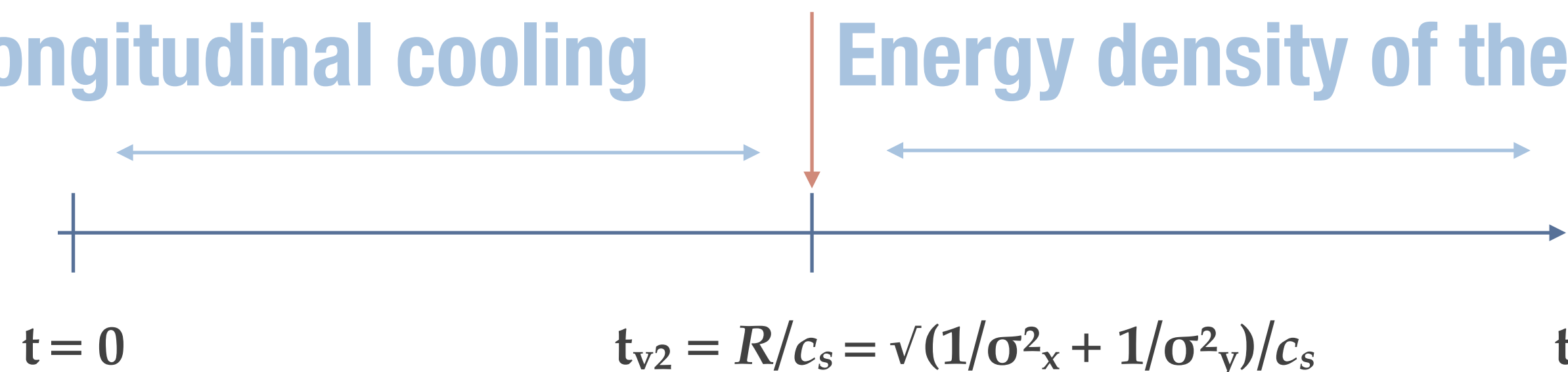
Assuming Gold ions are spheres...



Cold Atoms



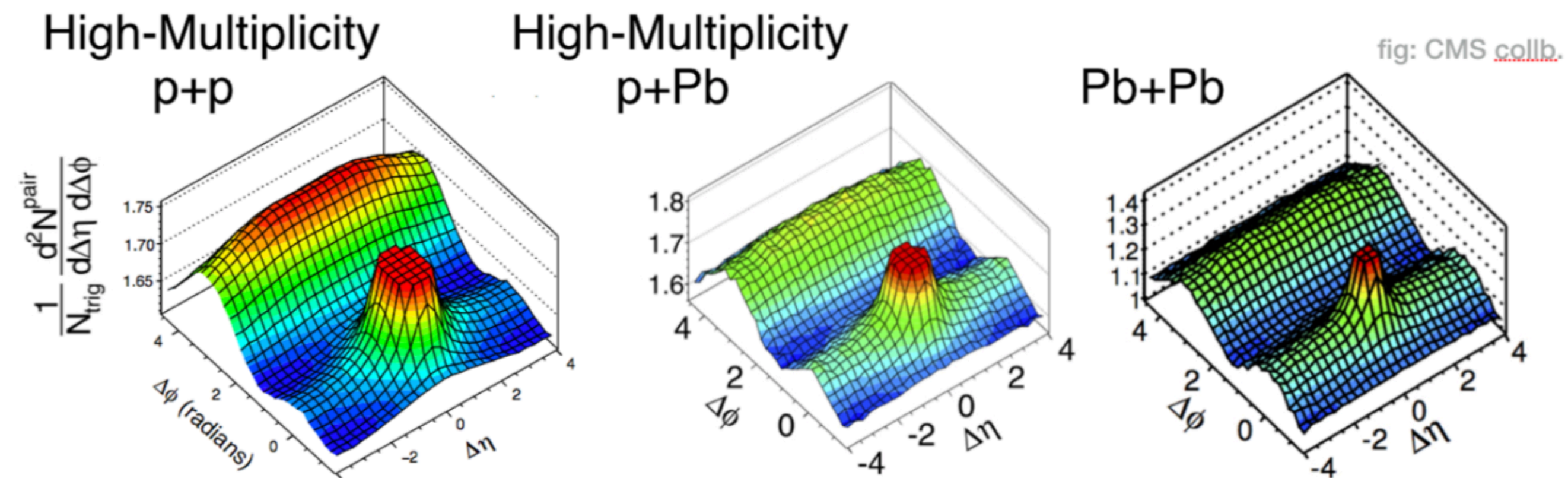
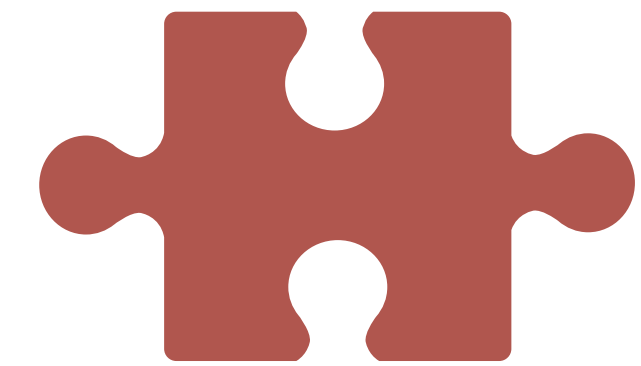
Longitudinal cooling | Energy density of the medium is constant



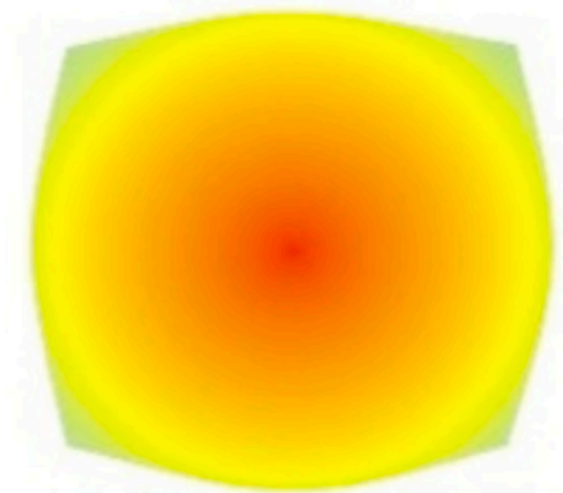
$$\frac{dN}{d\phi} \propto 1 + 2v_2 \cos 2\phi$$

A puzzle : high-multiplicity events in small system

- * Striking similarities between all the colliding system.
- * Many questions and possible origin :
 - * Momentum space correlation
 - * Position space correlation
 - * Other ?



Last step : particle spectra



Hydro

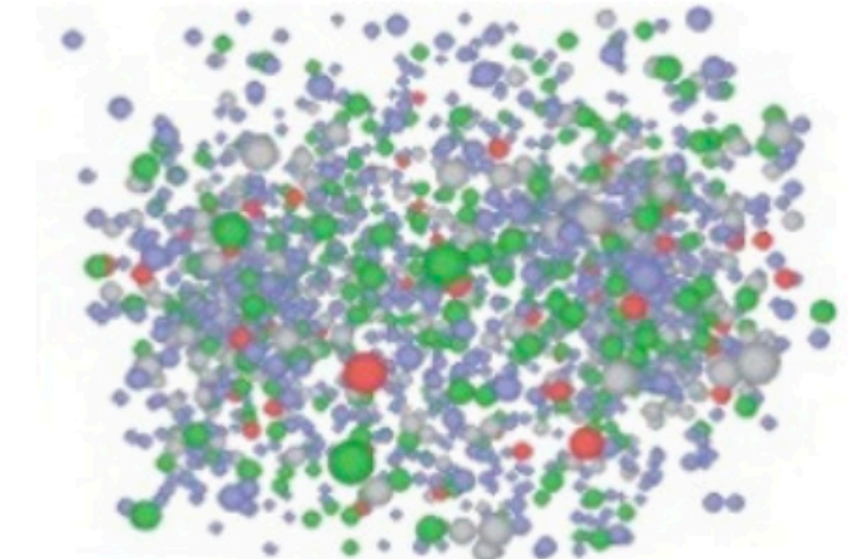
The Freeze-out surface

Cooper-Frye formula

$$E \frac{dN}{d^3p} = \int_{\Sigma} d\Sigma_{\mu} p^{\mu} f(T, p_{\mu} u^{\mu}, \pi^{\mu\nu})$$

Particle distribution function

Forget this beast for the moment



Particles

* Basic assumptions :

- $p^{\mu}_{\text{fluid}} = p^{\mu}_{\text{freeze-out}}$ for all the particles.
- The fluid = ideal gaz.

* If we ask momentum distributions to follow Boltzmann statistics :

Spin degrees

Energy of the particle in the fluid rest frame = $p_{\mu} u^{\mu}$

$$\frac{dN}{d^3x d^3p} = \frac{2S + 1}{(2\pi\hbar)^3} \exp\left(-\frac{E^*}{T}\right)$$

Freeze-out temperature

Last step : particle spectra

* What does this formula tell us :

$$\frac{dN}{d^3x d^3p} = \frac{2S+1}{(2\pi\hbar)^3} \exp\left(-\frac{E^*}{T}\right)$$

- More particles when E^* is minimum \rightarrow particles **move** with the fluid.
- For fast particles ($E^* > m$), E^* is minimum when particle momentum **is parallel** to the fluid momentum.
- For simplicity, we assume $p_z=0$ and that the fluid velocity is parallel to the particle velocity to write down :

$$E^* = p^\mu u_\mu = m_t u^0 - p_t u,$$

$$m_t = \sqrt{p_T^2 + m^2}$$

Particle spectra

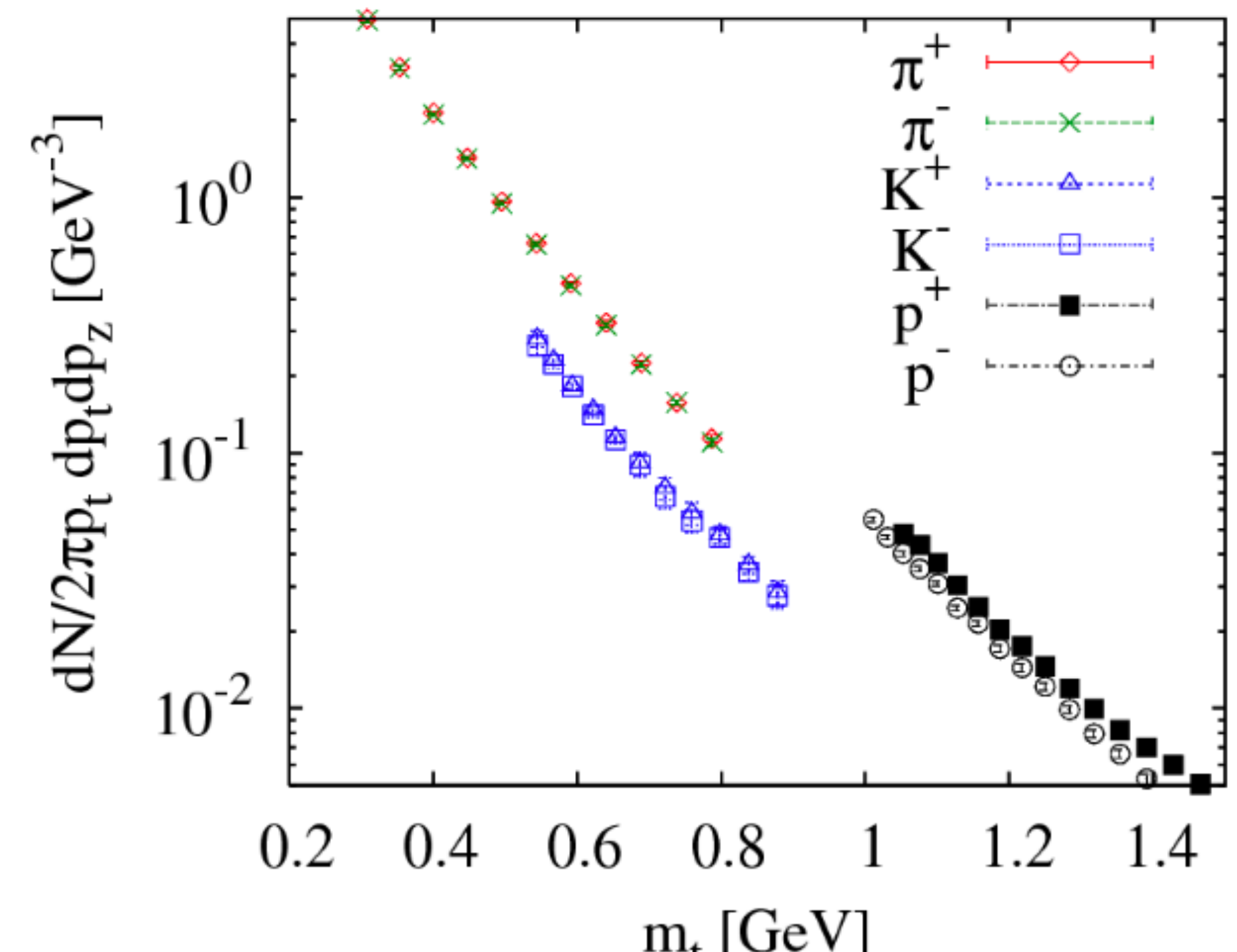
$$\frac{dN}{d^3x d^3p} = \frac{2S+1}{(2\pi\hbar)^3} \exp\left(-\frac{E^*}{T}\right)$$

$$E^* = p^\mu u_\mu = m_t u^0 - p_t u,$$

$$\frac{dN}{2\pi p_t dp_t dp_z} \propto \exp\left(\frac{-m_t u^0 + p_t u}{T}\right)$$

Adams J et al. [STAR Collaboration] 2004 Phys. Rev. Lett. 92 112301

- * pp collisions at $\sqrt{s} = 200$ GeV near $p_z = 0$.
- * Clear scaling with transverse mass.
- * What about Au-Au ?



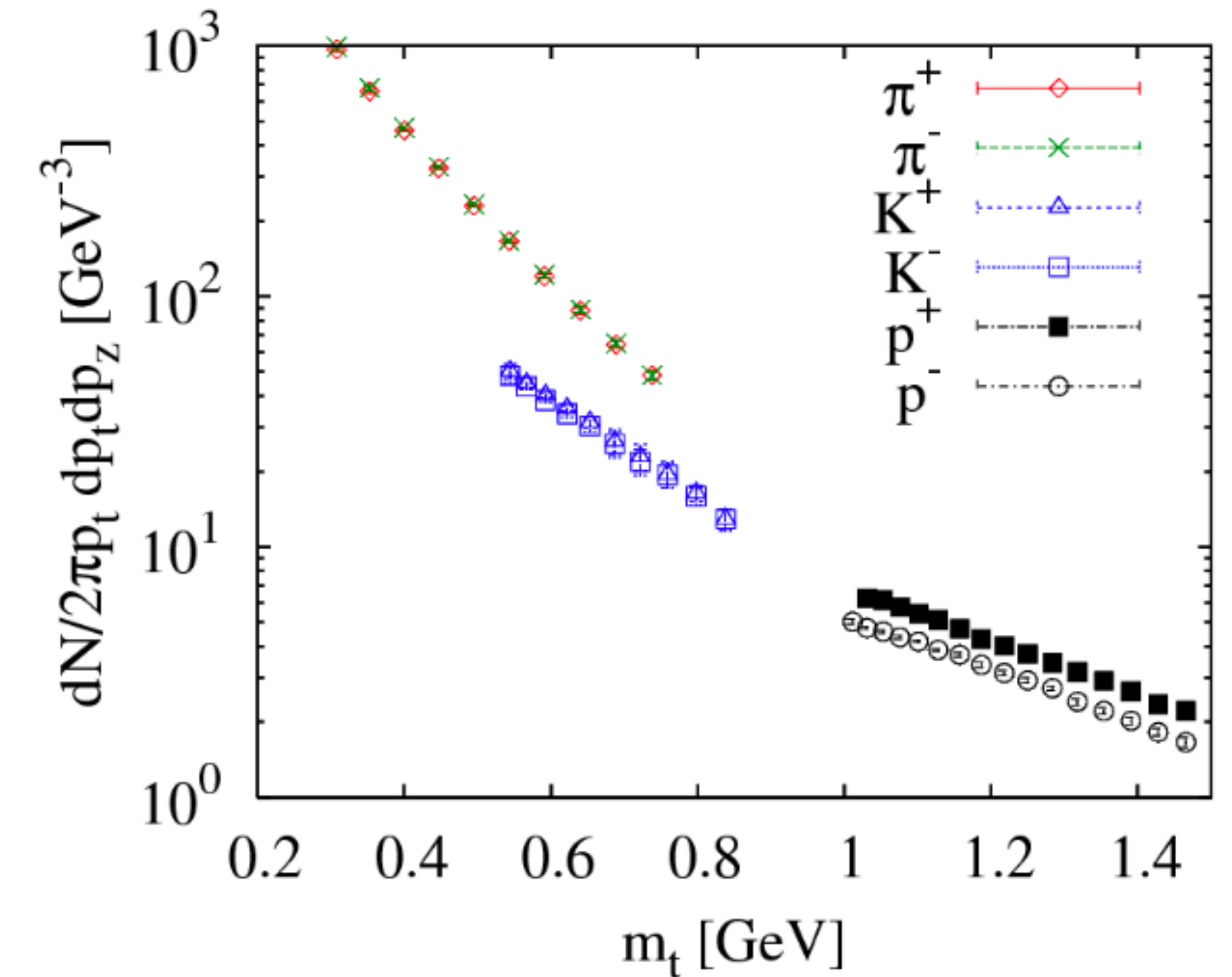
Particle spectra

- * Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV near $p_z = 0$
- * m_t scaling is broken \rightarrow **collective velocity**.
- * To convince you :

$$\frac{d}{dm_t} \log \left(\frac{dN}{2\pi p_t dp_t dp_z} \right) = \frac{-u_0 + um_t/p_t}{T}$$

- * Those results show evidence of **transverse flow**.

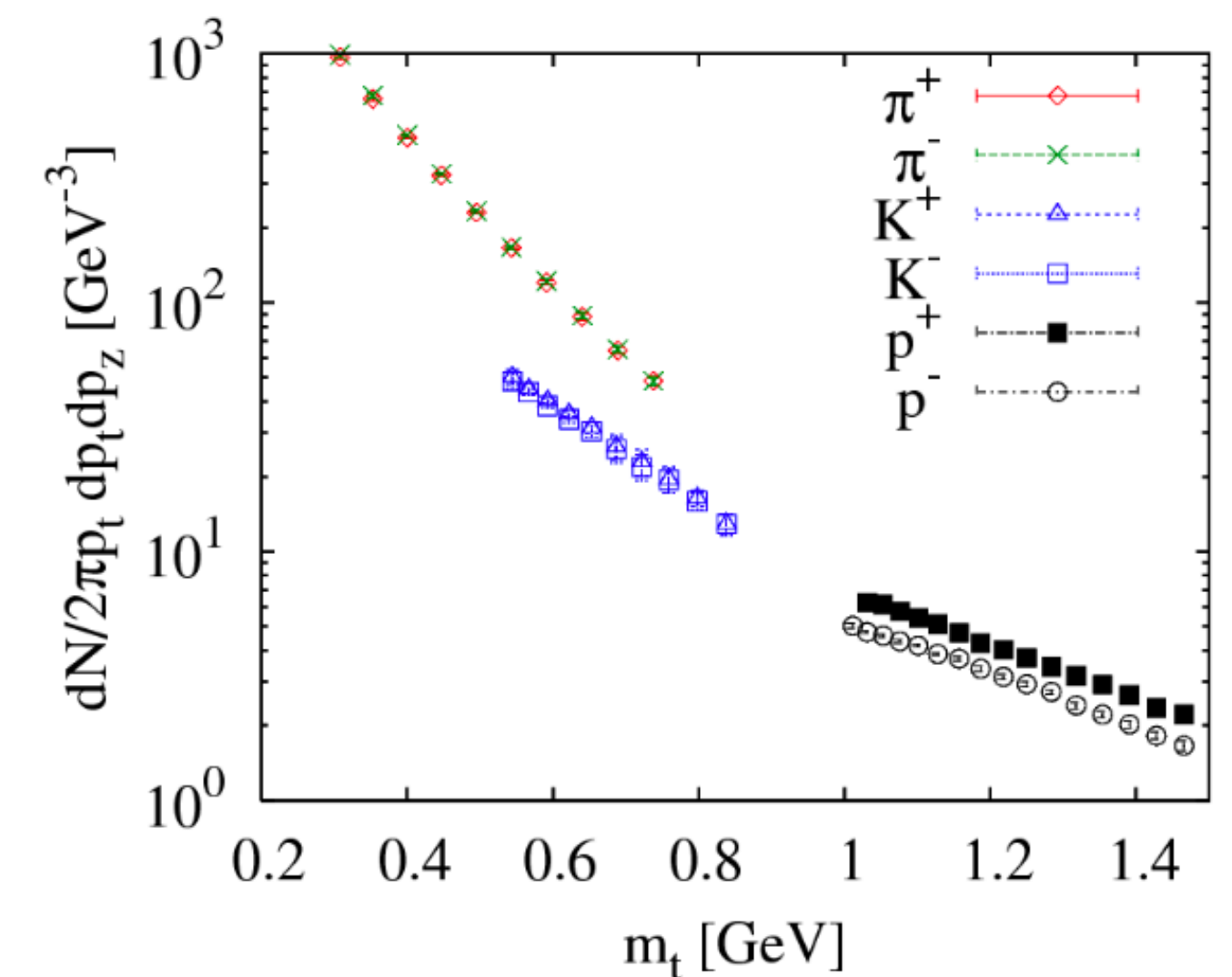
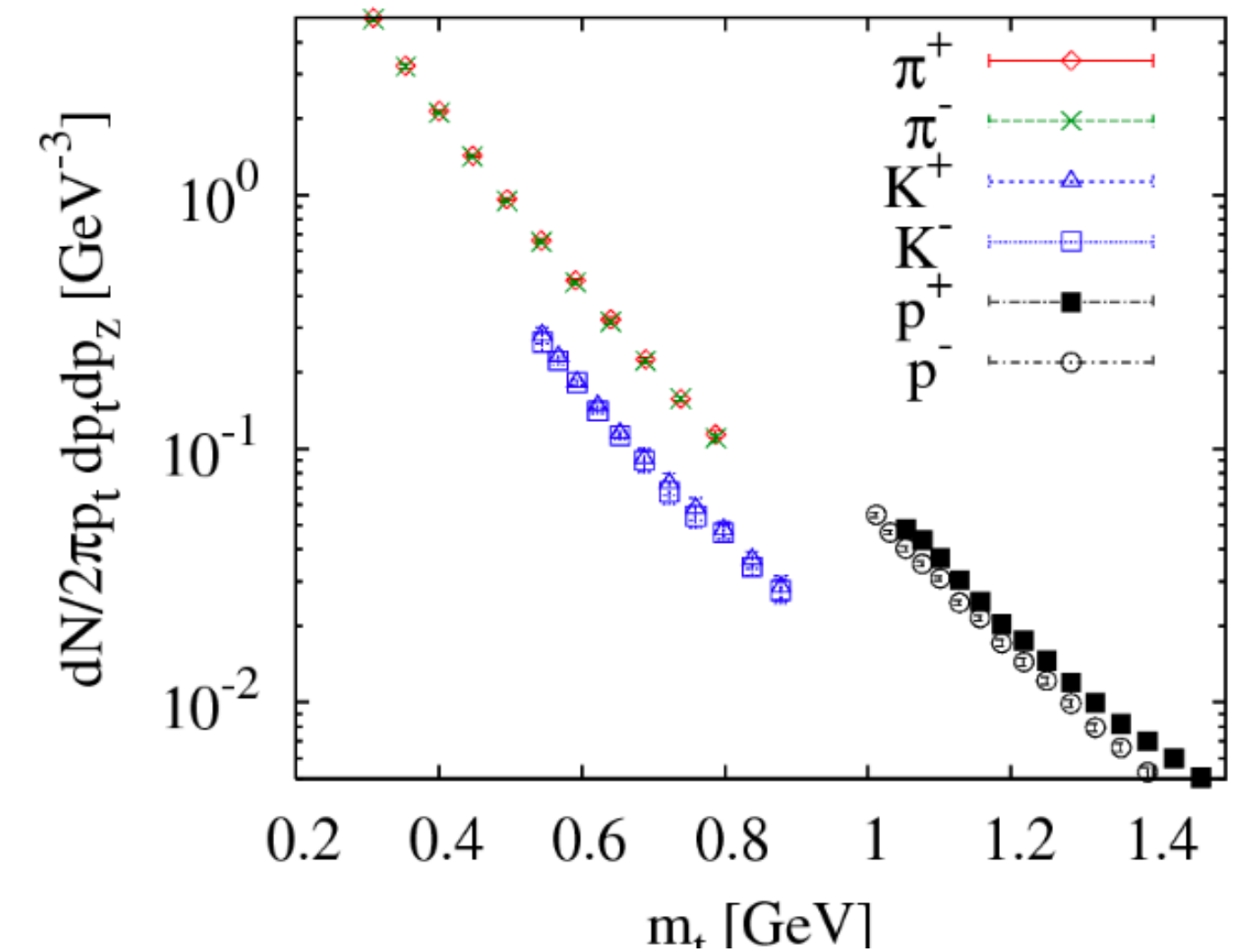
Adams J et al. [STAR Collaboration] 2004 Phys. Rev. Lett. 92 112301



On the freeze-out temperature

$$\frac{dN}{2\pi p_t dp_t dp_z} \propto \exp\left(\frac{-m_t u_0 + p_t u}{T}\right)$$

- * Same relative abundances of pions, kaons, and (anti)protons in pp and Au-Au.
- * Particle ratios only depends on T :
 - « **Chemical Freeze-out** » $T_c \approx 170$ MeV
- * But in Au-Au due to radial flow, T **must be lower**:
 - « **Kinetic Freeze-out** » $T_f \approx 100$ MeV



Consequences on V_2

- * We can rewrite :

$$\boxed{\frac{dN}{2\pi p_t dp_t dp_z} \propto \exp\left(\frac{-m_t u_0 + p_t u}{T}\right)} \longrightarrow \boxed{\frac{dN}{p_t dp_t dp_z d\phi} \propto \exp\left(\frac{-m_t u_0(\phi) + p_t u(\phi)}{T}\right)}$$

- * Since the fluid velocity is larger on the x -axis than the y -axis, $u(\phi)$ can be parameterized as :

$$\boxed{u(\phi) = u + 2\alpha \cos 2\phi,}$$

- * Finally, experiment suggests $\alpha \sim 4\%$. Using $u^0 = \sqrt{u^2 + 1}$ and expanding to first order in α :

$$\boxed{u^0(\phi) = u^0 + 2v\alpha \cos 2\phi}$$

$$\boxed{v = u/u_0}$$

Consequences on v_2

$$\frac{dN}{p_t dp_t dp_z d\phi} \propto \exp\left(\frac{-m_t u_0(\phi) + p_t u(\phi)}{T}\right)$$

$$u(\phi) = u + 2\alpha \cos 2\phi,$$

$$\frac{dN}{d\phi} \propto 1 + 2v_2 \cos 2\phi$$

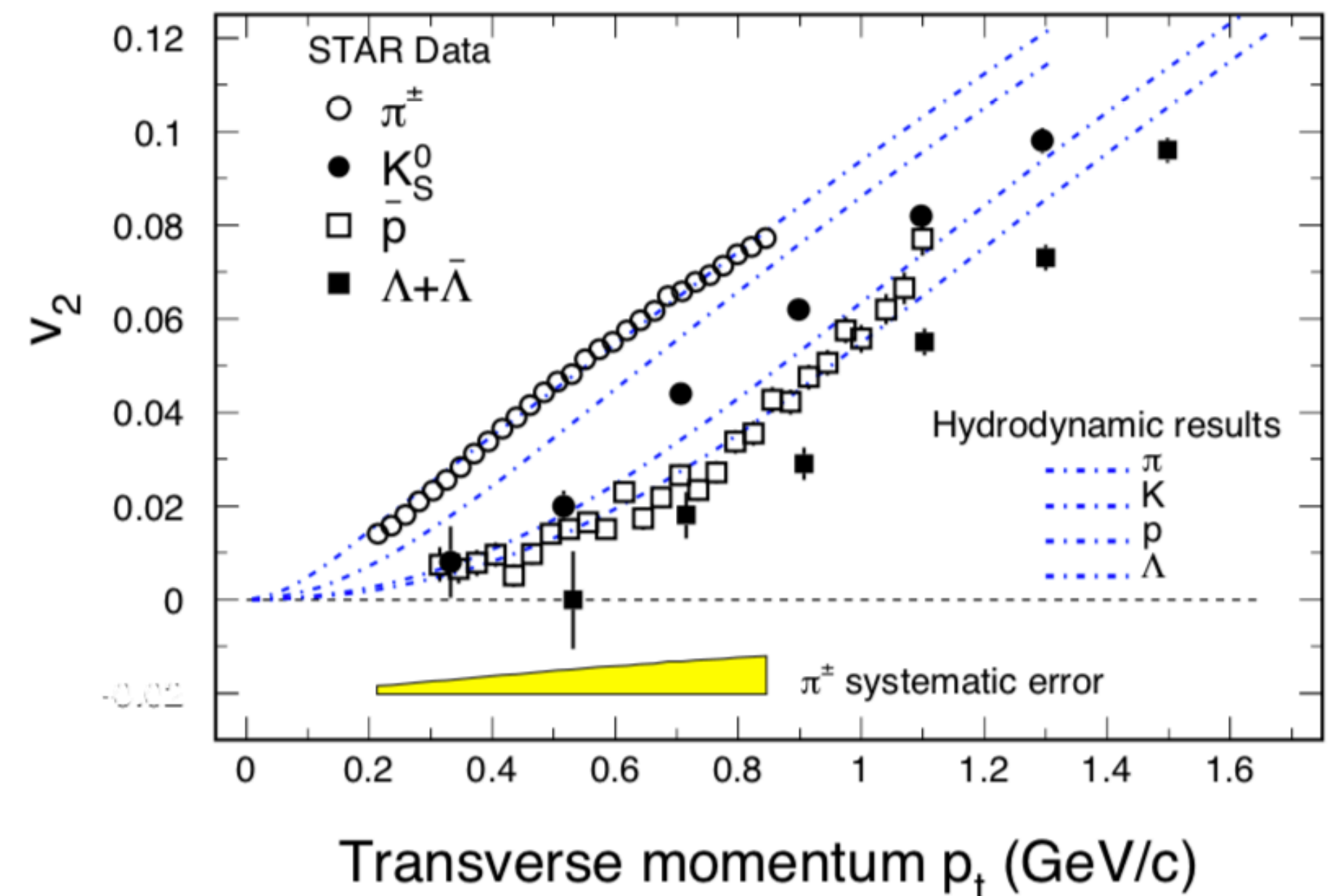
Take all the formula at first order in α ...



$$v_2 = \frac{\alpha}{T} (p_t - v m_t)$$

$$u^0(\phi) = u^0 + 2v\alpha \cos 2\phi$$

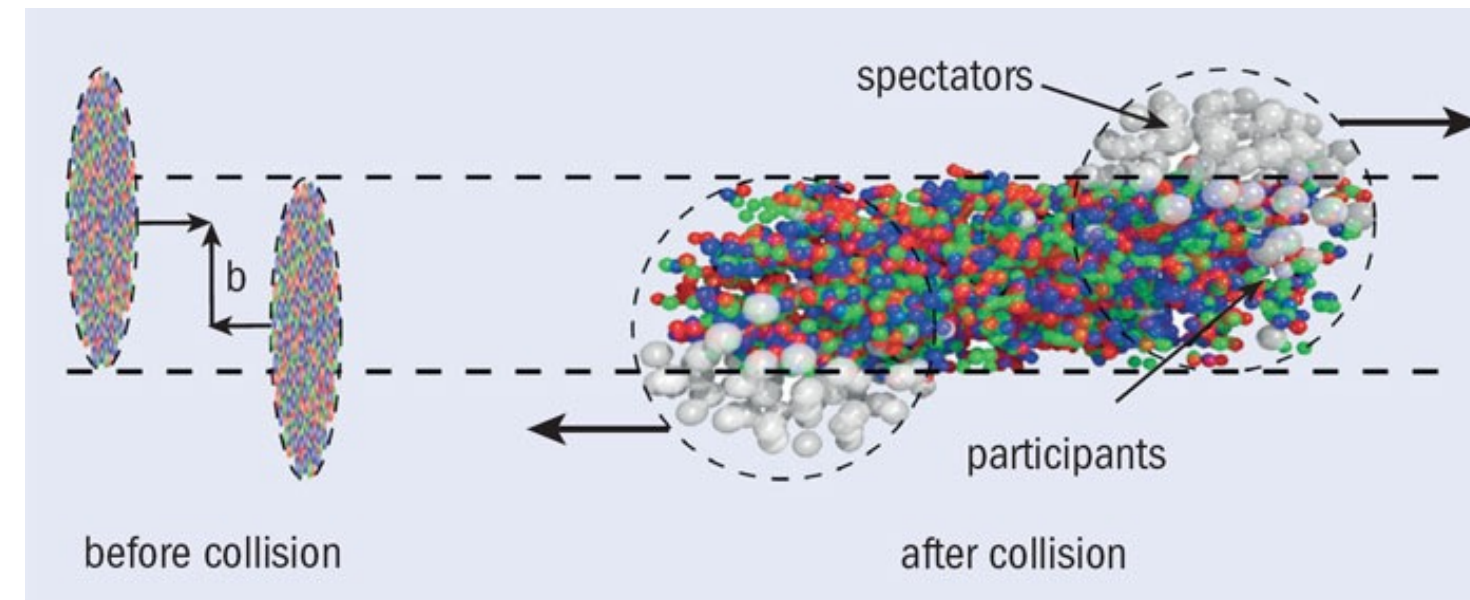
- * Mass ordering for the v_2 .
- * Good agreement data/theory up to $p_t \sim 2$ GeV
- * At higher p_t : v_2 saturates (viscous effect ?).



THE GLAUBER MODEL

Centrality and Glauber Model

- * The quantity that relate if a (A-A) collision is head-on or more peripheral is called *centrality*.



- * **Question** : How do we determine this quantity ?

$$\frac{dN_{ch}}{d\eta} \propto \underbrace{(N_{part} \leftrightarrow N_{coll})}_{\substack{\uparrow \\ \uparrow \\ \uparrow}} \propto b \rightarrow \text{Centrality}$$

Experimental
Observable

Glauber Model

What we want

History of Glauber Model



* Roy Glauber :

- 1950's: Used quantum mechanical scattering techniques to analytically describe multi-body scattering of composite systems.
- 1970's: Beams of protons and ions are scattered off nuclear targets and Glauber's work was found useful for computing total cross-sections.
- 2005 : Nobel prize in physics for his contributions to quantum optics
- Present: Glauber Monte Carlo models are used to determine centrality in HIC (among other things).

* Glauber model assumptions :

- Nucleons travel on straight trajectories
- Nucleon-nucleon cross-section is independent of the number of collisions a nucleon underwent before
- Assume « optical limit » : particles have momenta such that they are deflected very little as they pass through each other

* Complete review : <https://arxiv.org/pdf/nucl-ex/0701025.pdf>

Analytical model

- * Thickness function :

$$\hat{T}_A(\mathbf{s}) = \int \hat{\rho}_A(\mathbf{s}, z_A) dz_A \quad \int T_A(\vec{s}) d^2s = A$$

- * Overlap Function:

$$\hat{T}_{AB}(\mathbf{b}) = \int \hat{T}_A(\mathbf{s}) \hat{T}_B(\mathbf{s} - \mathbf{b}) d^2s.$$

- * Probability of having n such NN Collisions:

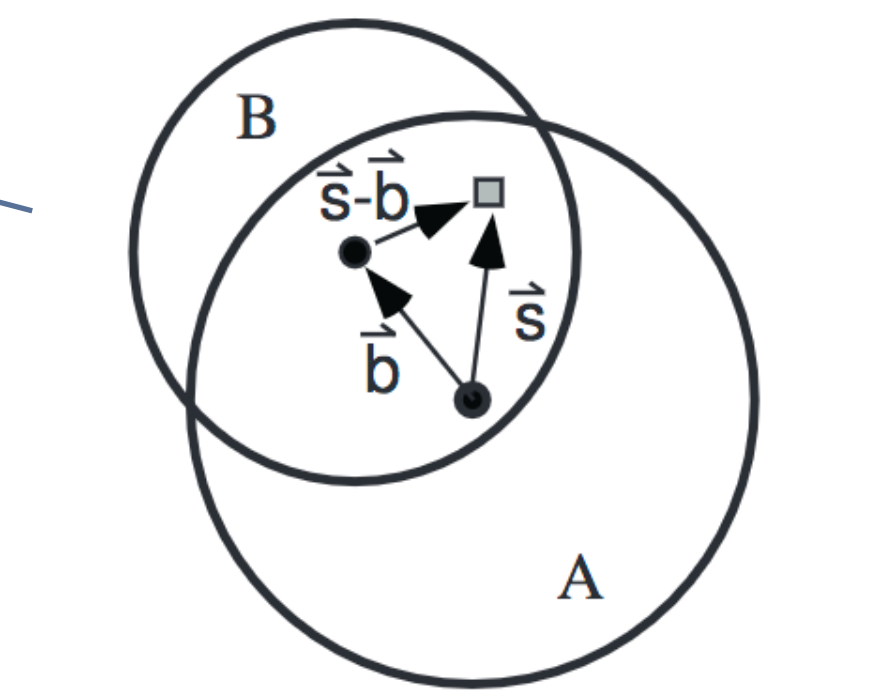
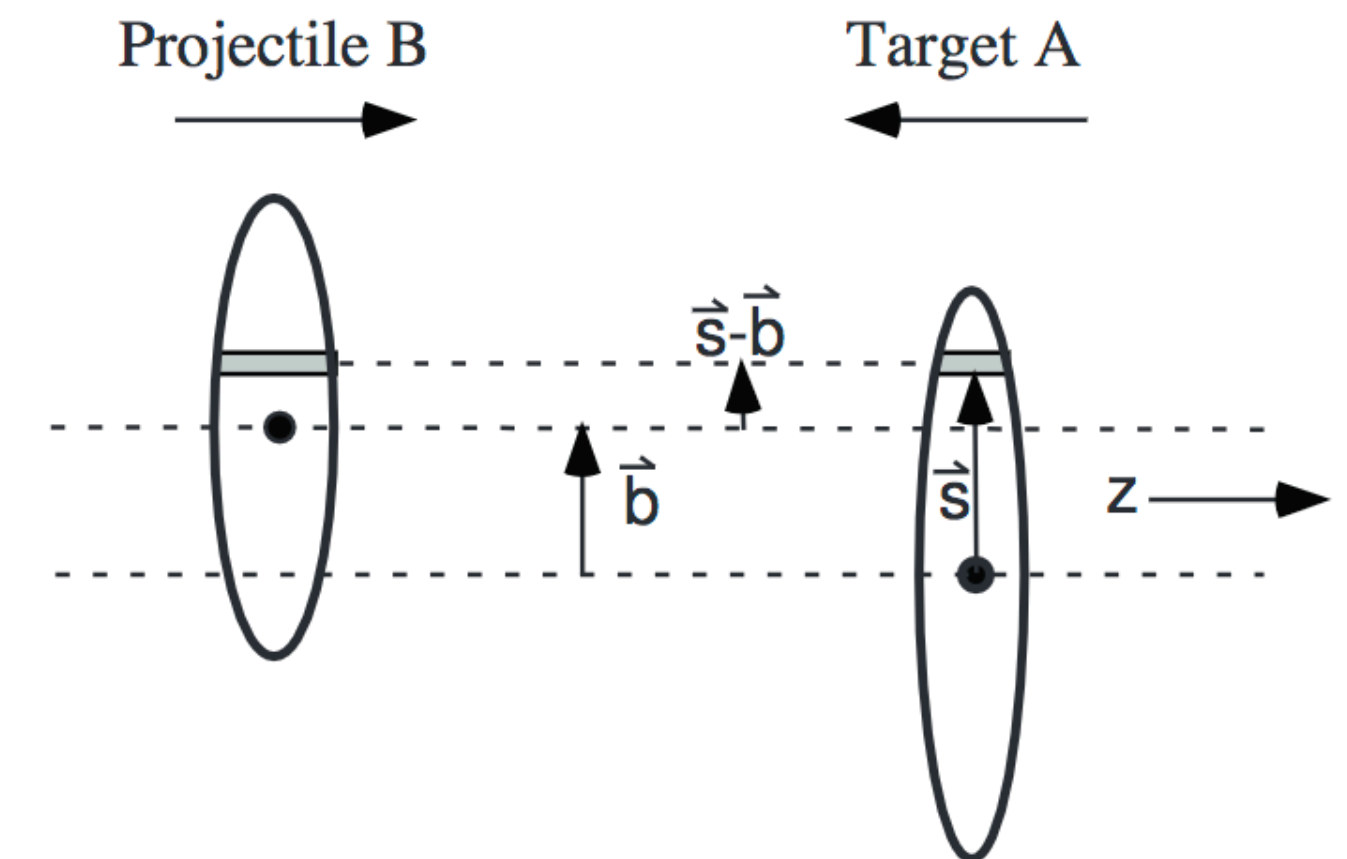
$$P(n, \mathbf{b}) = \binom{AB}{n} [\hat{T}_{AB}(\mathbf{b}) \sigma_{\text{inel}}^{\text{NN}}]^n [1 - \hat{T}_{AB}(\mathbf{b}) \sigma_{\text{inel}}^{\text{NN}}]^{AB-n}$$

$$\binom{AB}{n} = \frac{(AB)!}{n!(AB-n)!}$$

Probability of n hits

Probability of AB-n miss

Side view



Beam-line view

Small definition

$$\hat{T}_B(\vec{x}) := T_B(\vec{x})/B$$

Analytical model

- * Total inelastic cross-section :

$$\sigma_{\text{inel}}^{A+B} = \int_0^{\infty} 2\pi b db \left\{ 1 - \left[1 - \hat{T}_{AB}(b) \sigma_{\text{inel}}^{\text{NN}} \right]^{AB} \right\}$$

- * Number of Binary collisions :

$$N_{\text{coll}}(b) = \sum_{n=1}^{AB} n P(n, b) = AB \hat{T}_{AB}(b) \sigma_{\text{inel}}^{\text{NN}}$$

- * Number of participant (wounded) nucleons :

$$N_{\text{part}}(\mathbf{b}) = A \int \hat{T}_A(\mathbf{s}) \left\{ 1 - \left[1 - \hat{T}_B(\mathbf{s} - \mathbf{b}) \sigma_{\text{inel}}^{\text{NN}} \right]^B \right\} d^2s + \\ B \int \hat{T}_B(\mathbf{s} - \mathbf{b}) \left\{ 1 - \left[1 - \hat{T}_A(\mathbf{s}) \sigma_{\text{inel}}^{\text{NN}} \right]^A \right\} d^2s,$$

MC Glauber

- * In practice, solving analytically Glauber requires a $2x(A+B+1)$ dimensional integral ... No thanks !

* Receipt for a MC Glauber model :

* Step 1 : Create nuclei

- * For each nucleon specify a position vector by drawing random $p = (x,y,z)$ location from Woods-Saxon

* Step 2 : Define Orientations of Nuclei

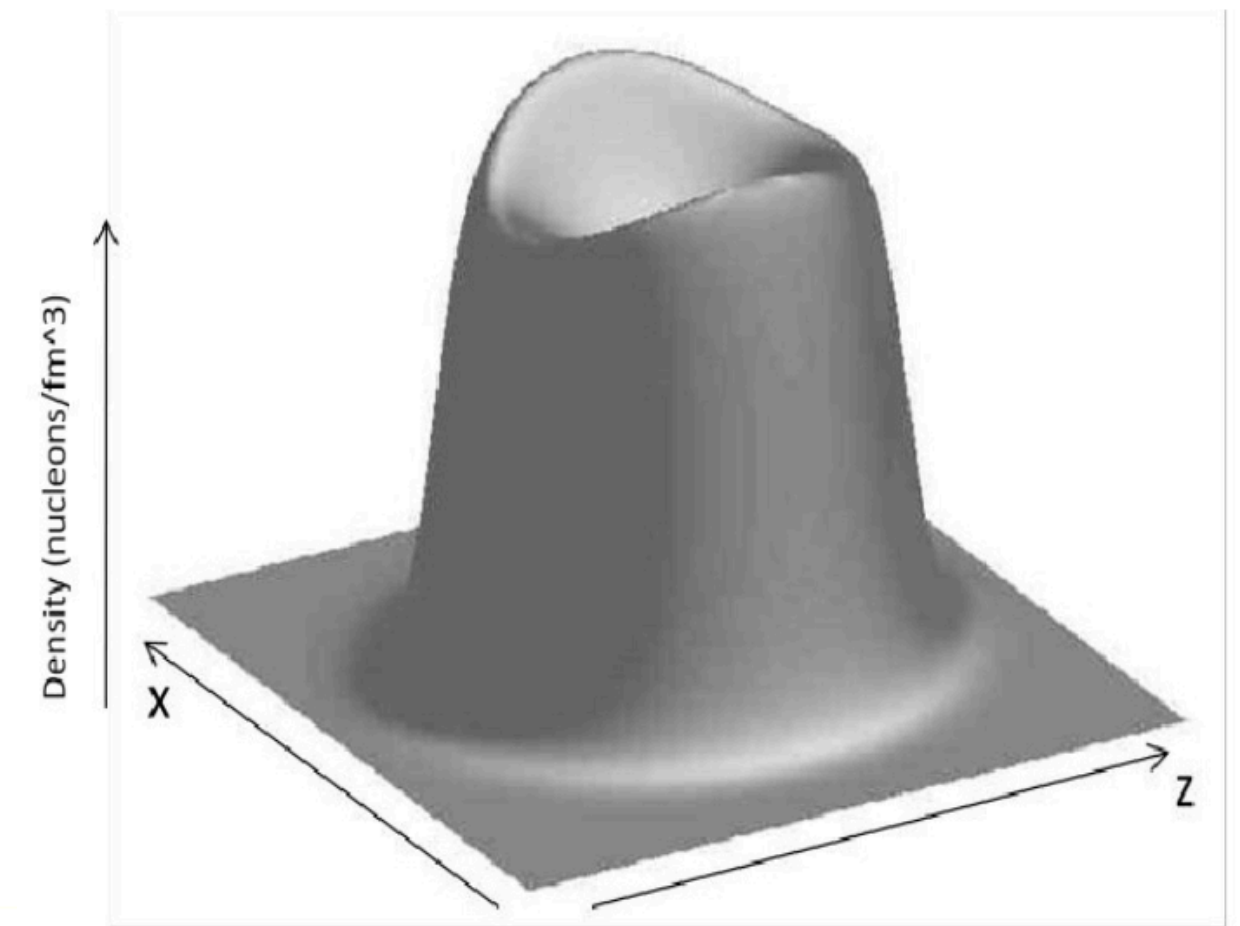
- * Generate 3 random numbers in range $[0,1]$ to uniformly sample phase space transform thusly...
- * Draw Random Impact Parameter from $d\sigma/db = 2\pi b$
- * Rotate Nucleon position vectors and translate by b

* Step 3 : Compute Ncoll and Npart

Step 1 – Creating Nuclei

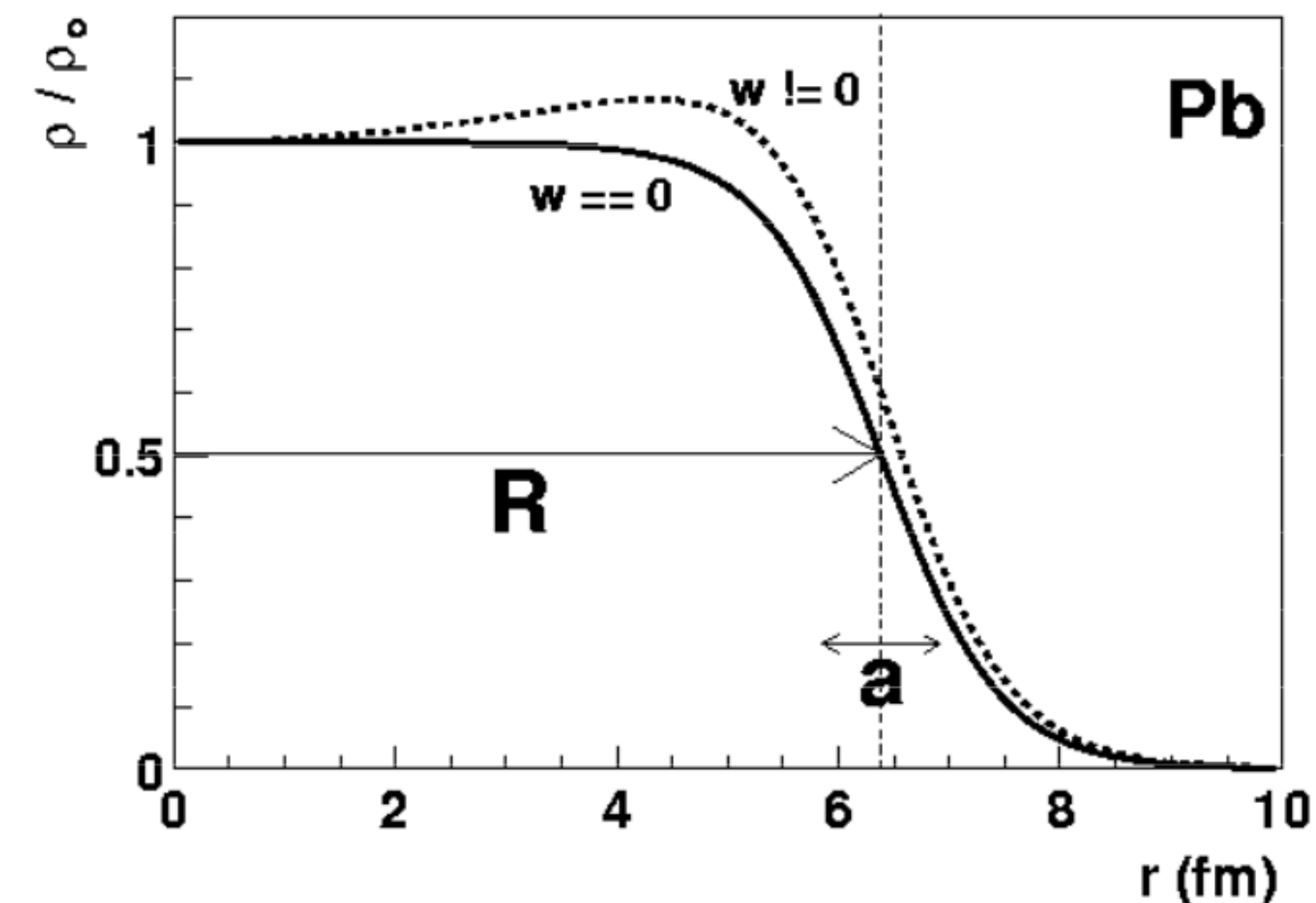
- * Woods-Saxon nuclear density profile :
- * Woods-Saxon parameters typically determined by electron scattering.
- * Differences between neutron and proton distributions are small and typically neglected.

$$\rho(r) = \frac{\rho_0 (1 + wr^2/R^2)}{1 + \exp((r - R)/a)}$$

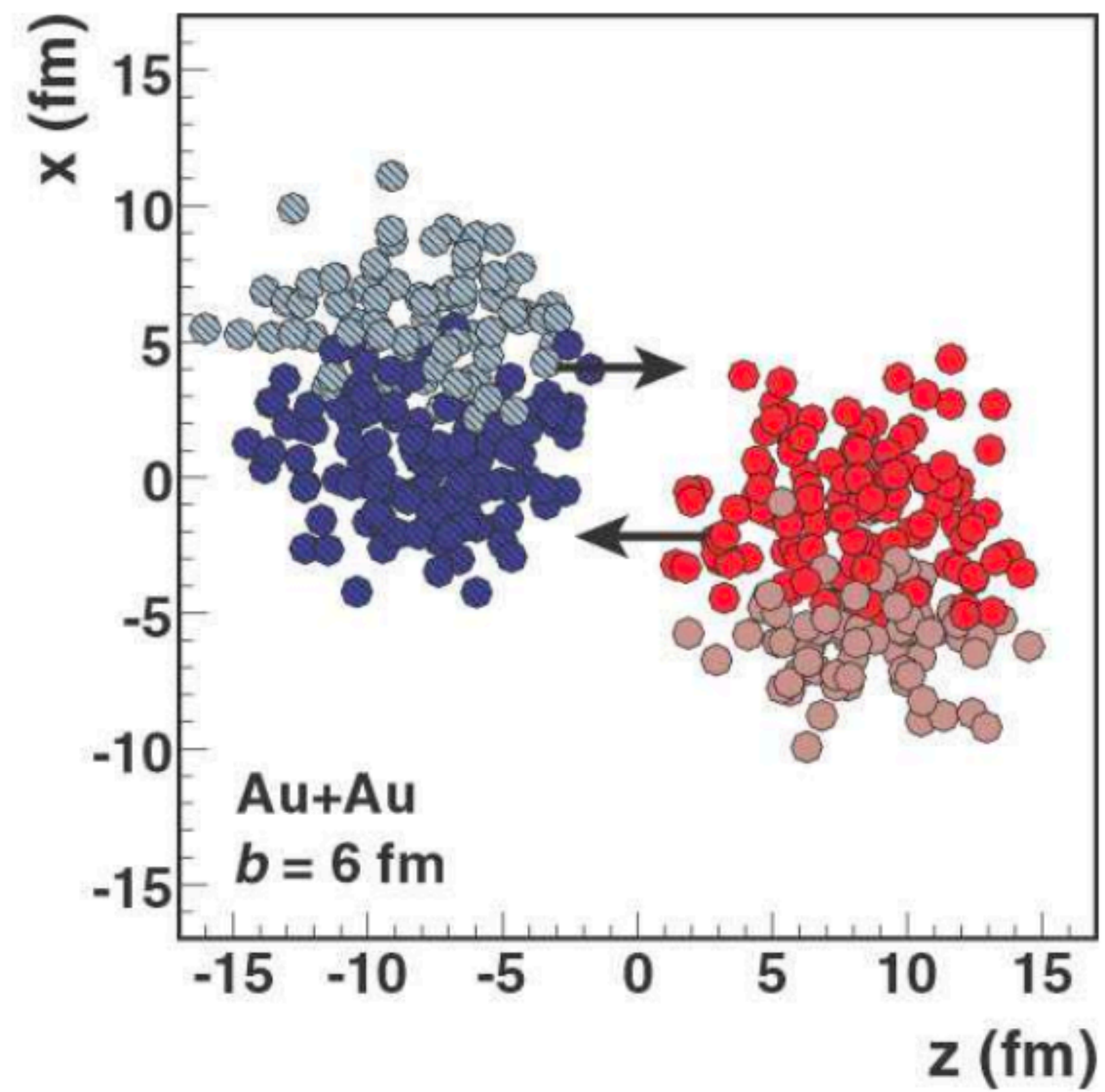
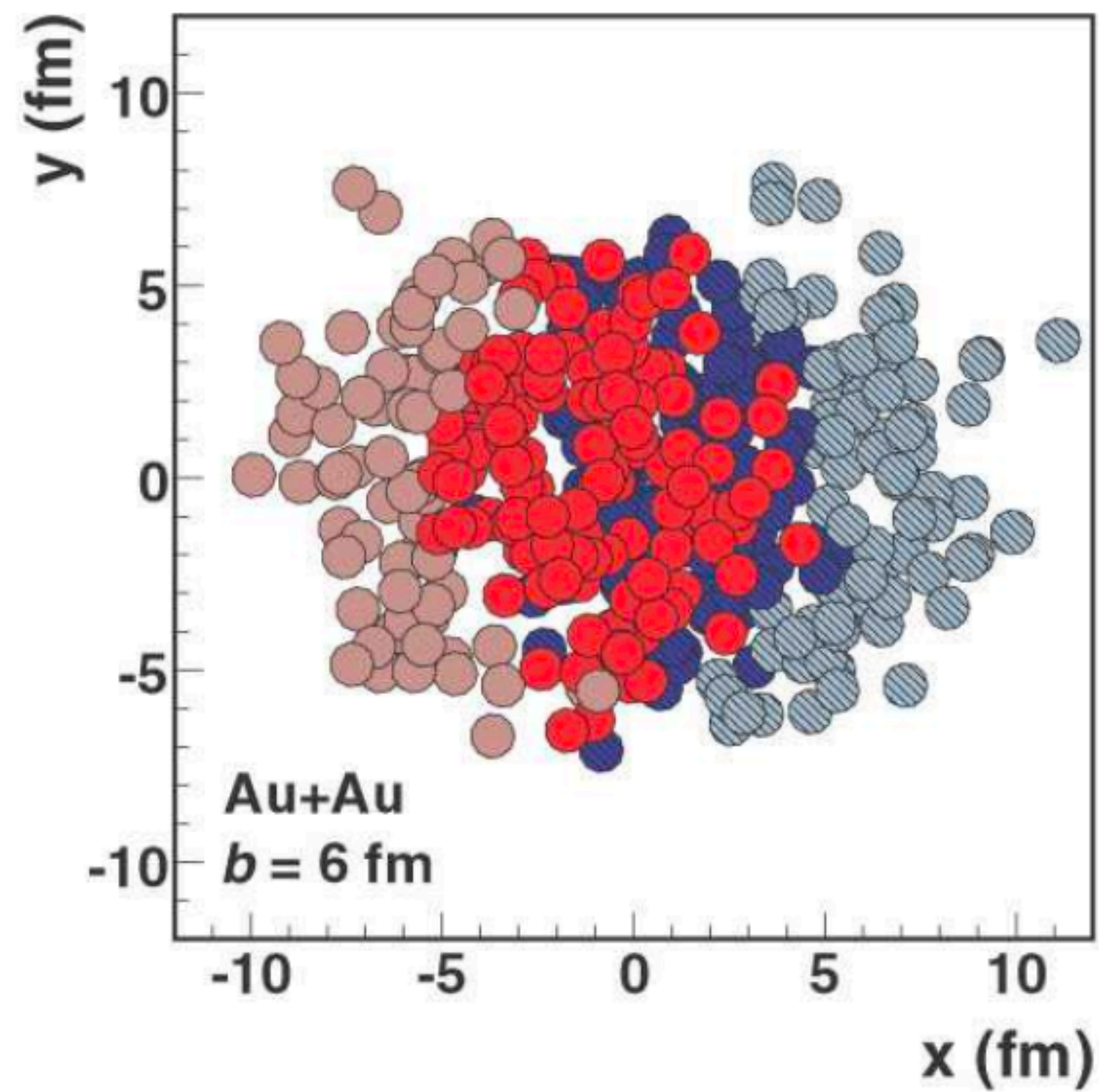


Nucleus	A	R (fm)	a (fm)	w
C	12	2.47	0	0
O	16	2.608	0.513	-0.051
Al	27	3.07	0.519	0
S	32	3.458	0.61	0
Ca	40	3.76	0.586	-0.161
Ni	58	4.309	0.516	-0.1308
Cu	63	4.2	0.596	0
W	186	6.51	0.535	0
Au	197	6.38	0.535	0
Pb	208	6.68	0.546	0
U	238	6.68	0.6	0

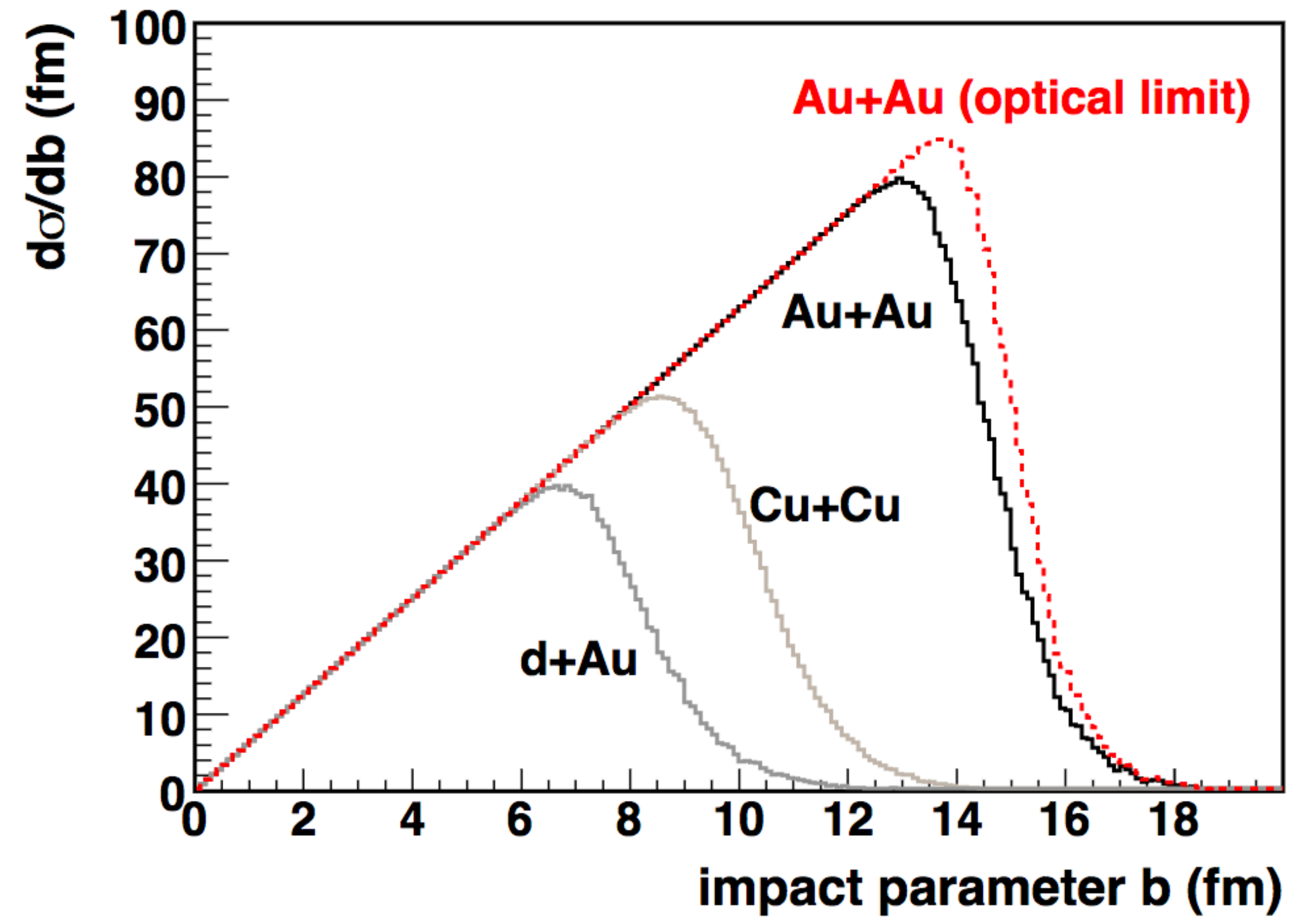
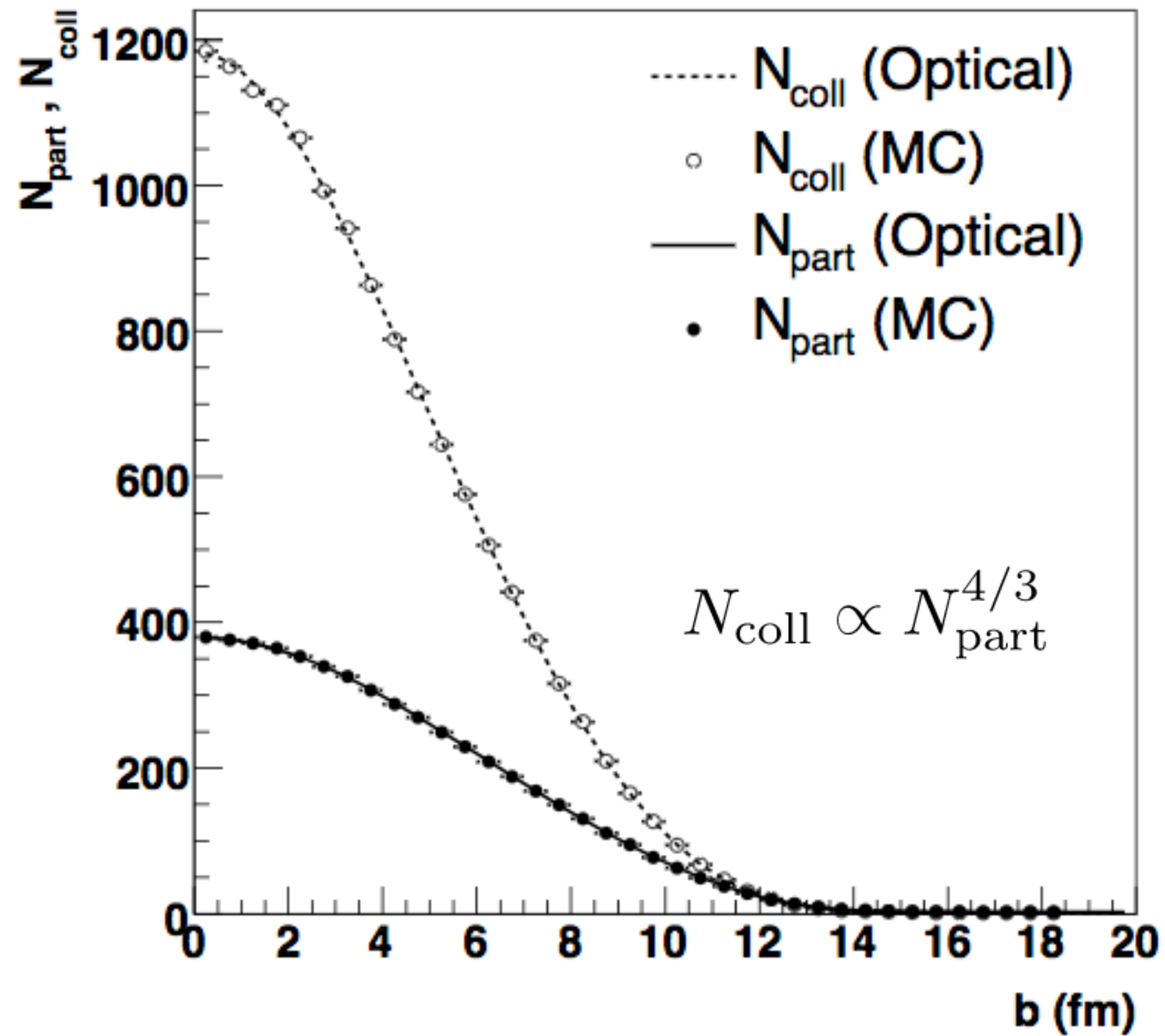
H. DeVries, C.W. De Jager, C. DeVries, 1987



MC Glauber illustrated



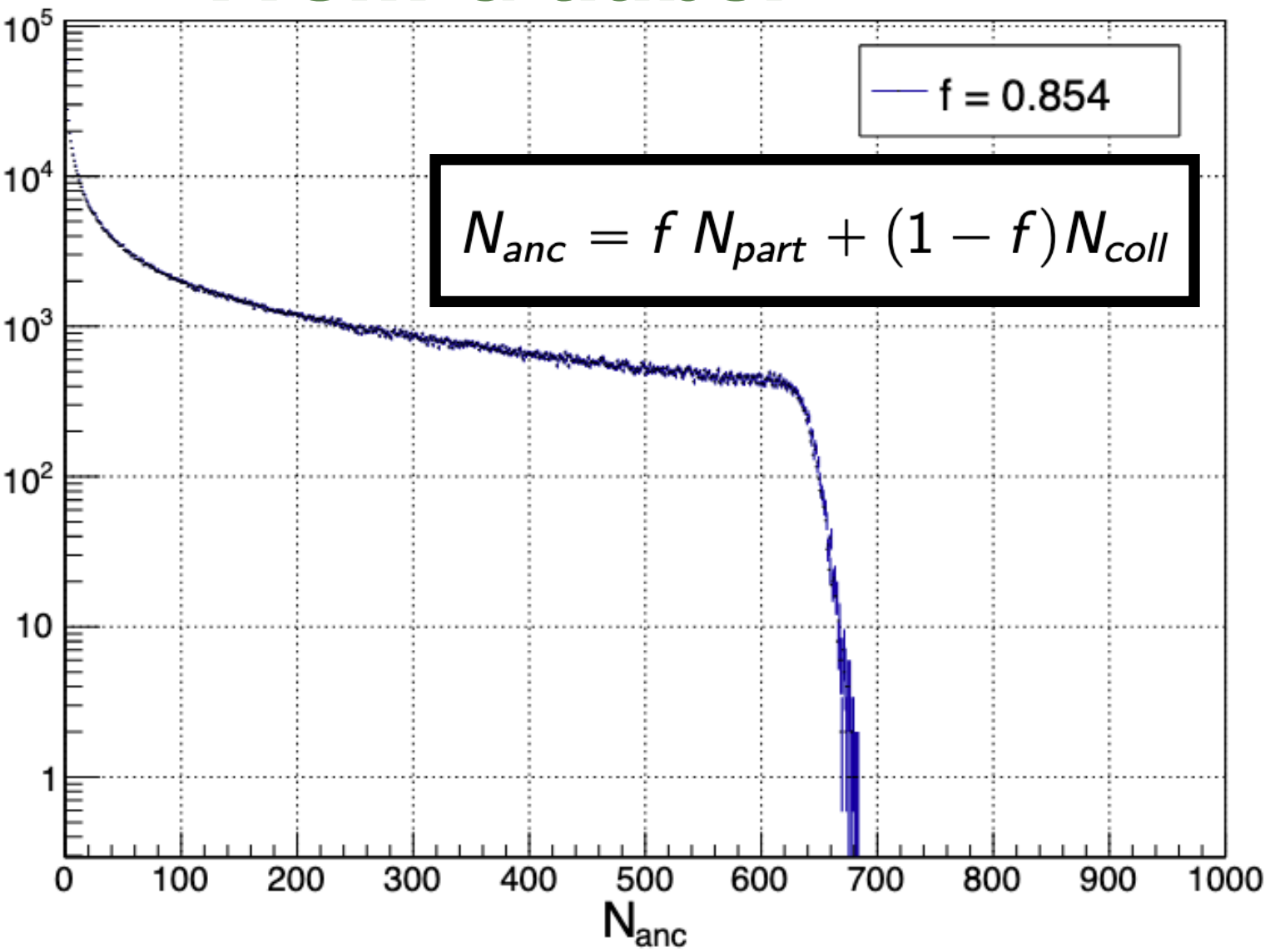
MC Glauber illustrated



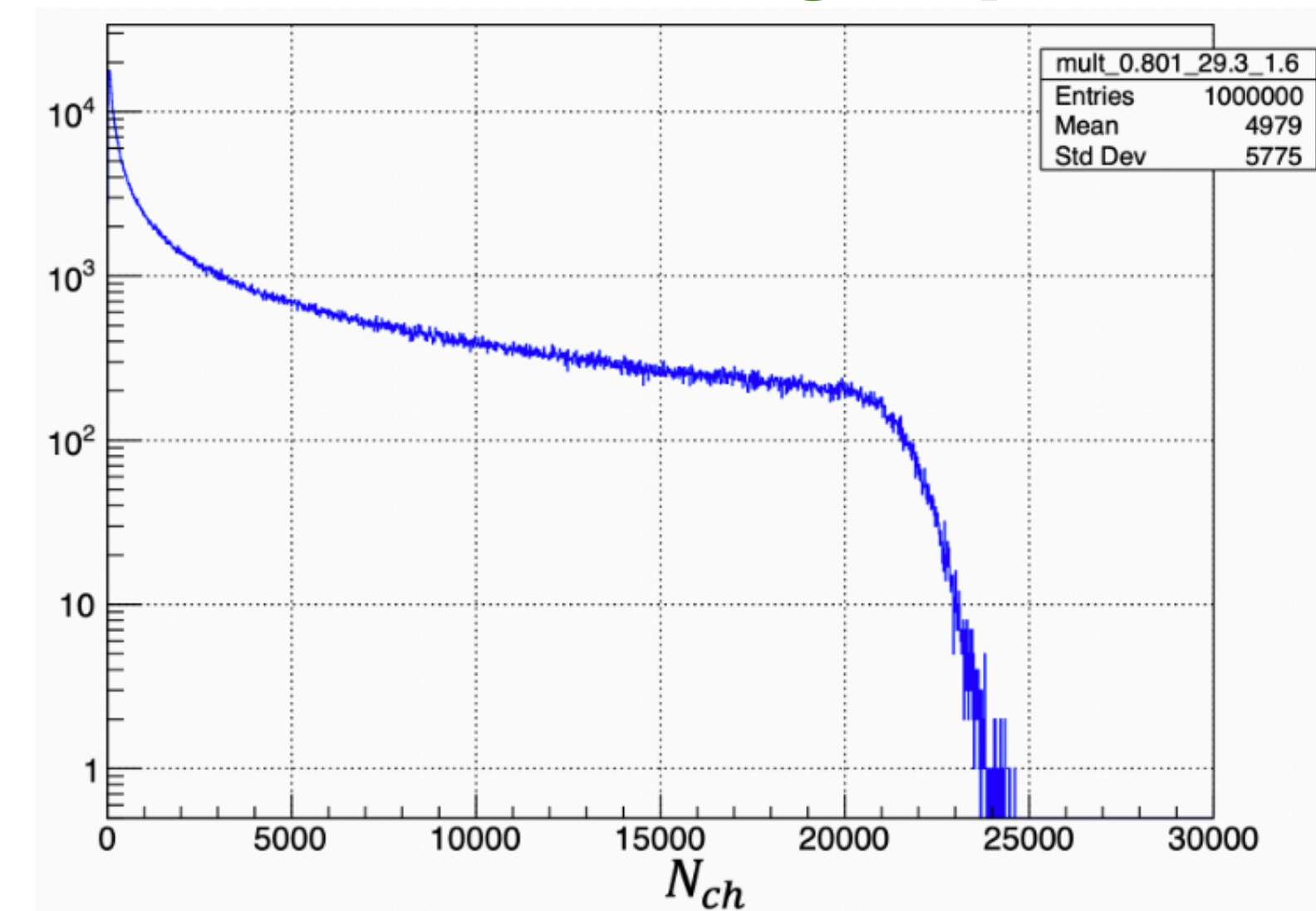
Connecting MC Glauber to Centrality : LHCb case

Step one : translate $\langle N_{coll} \rangle$ into an equivalent number of charged particles.

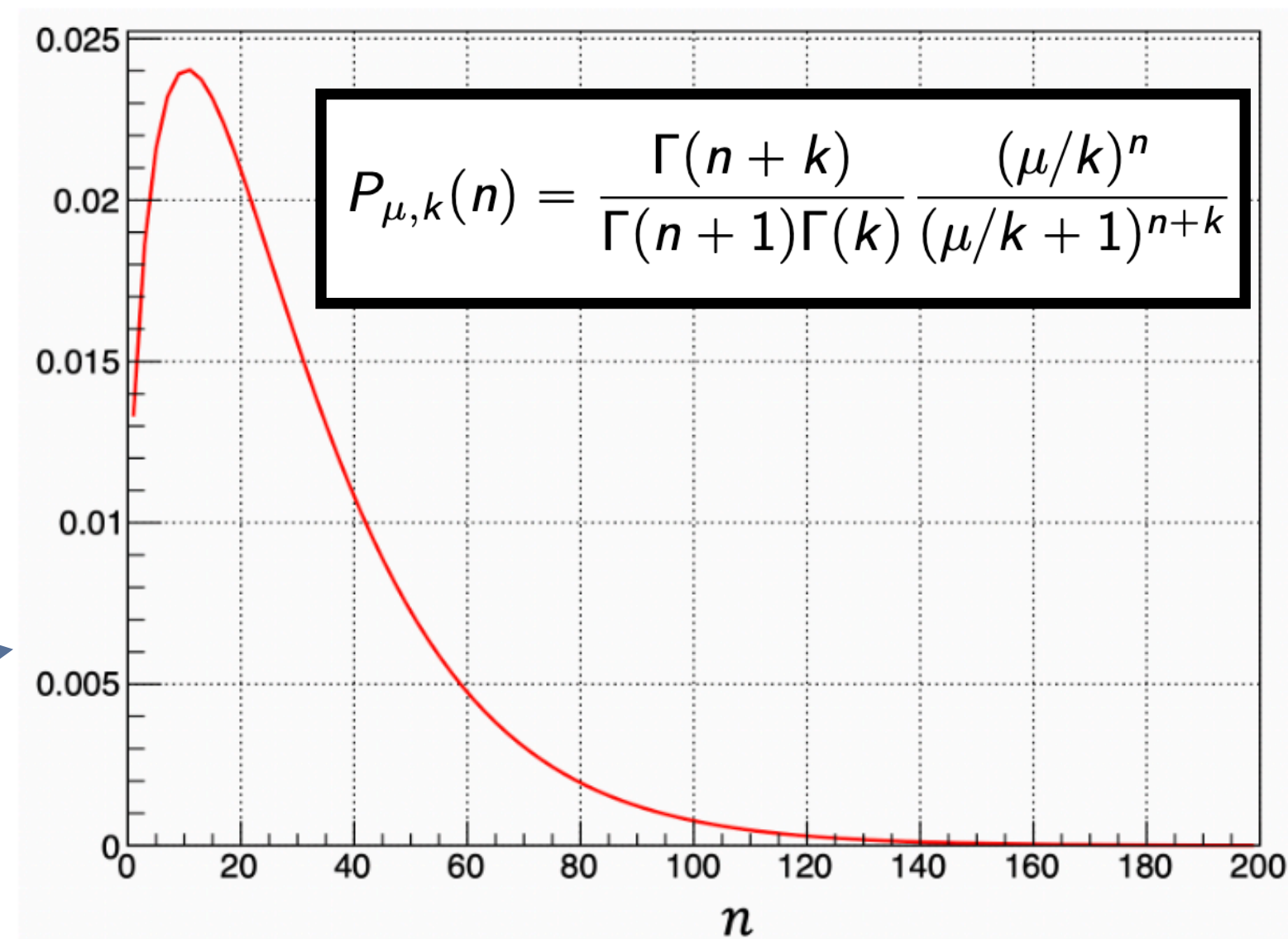
From Glauber



Equivalent number of charged particles



**P := negative binomial distributions
:= charged particles distribution per ancestor**



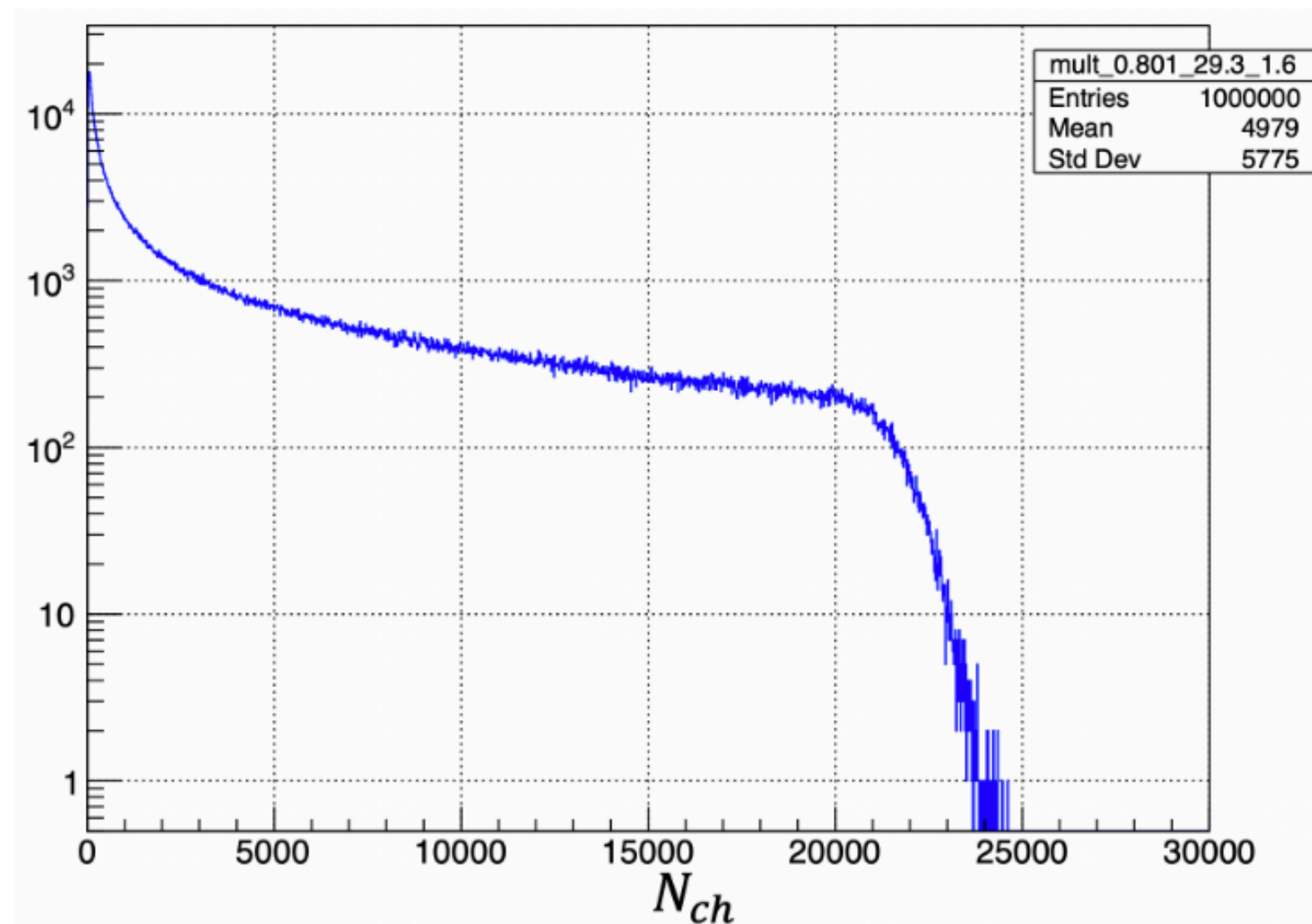
Source to sample ...

Sampled for every event N_{anc} times...

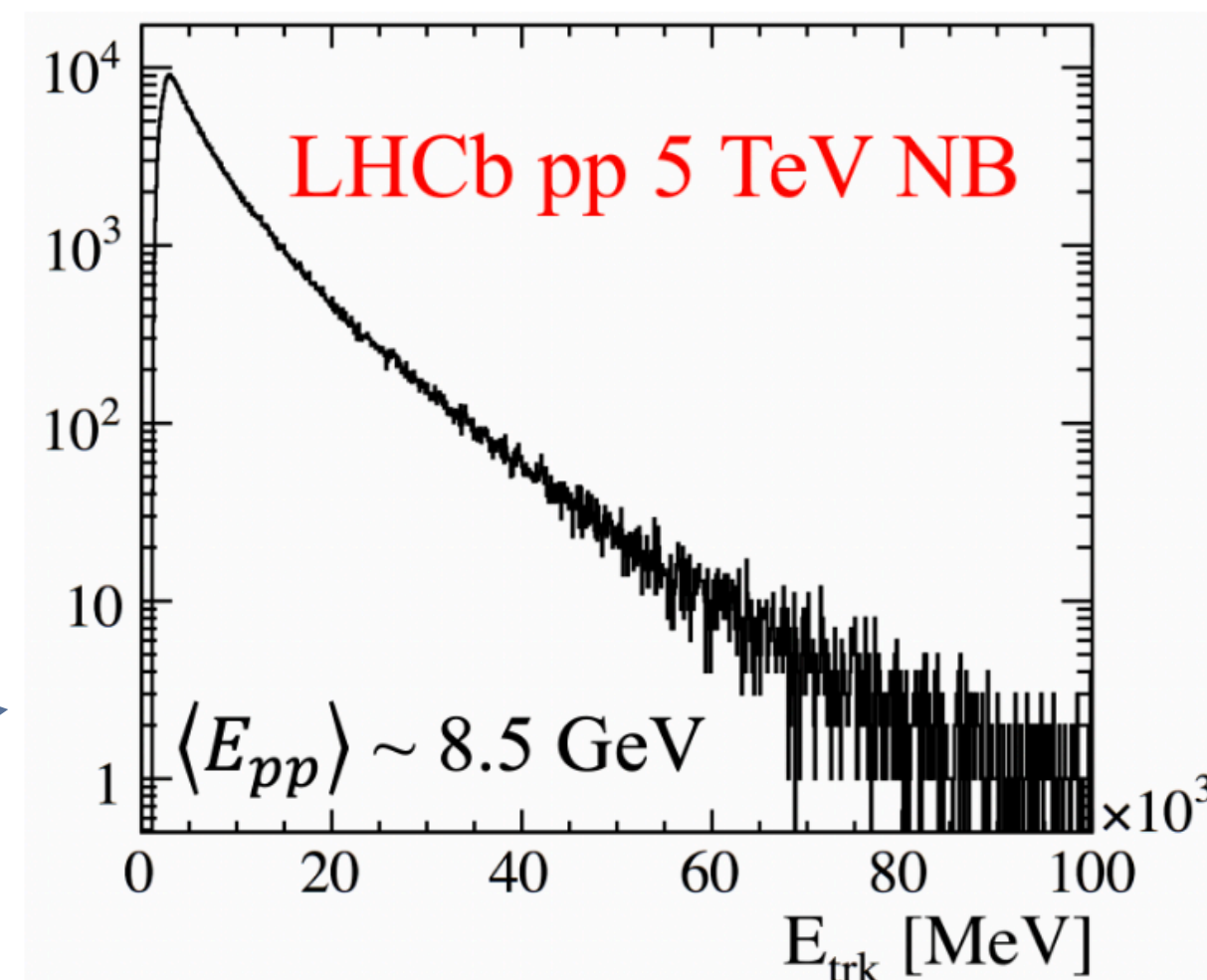
Connecting MC Glauber to Centrality : LHCb case

Step two : translating to energy deposit.

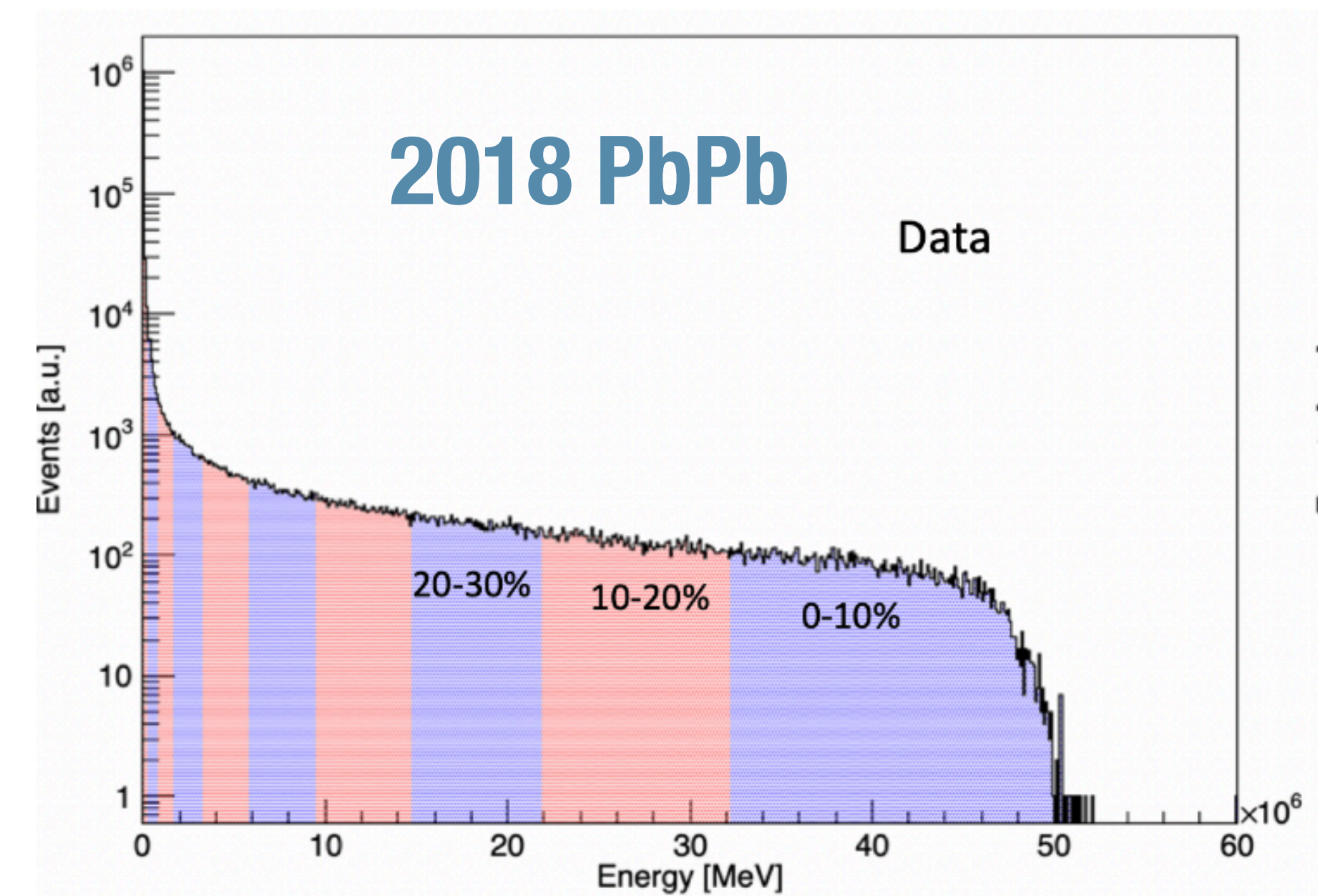
Equivalent number of charged particles



Source to sample ...

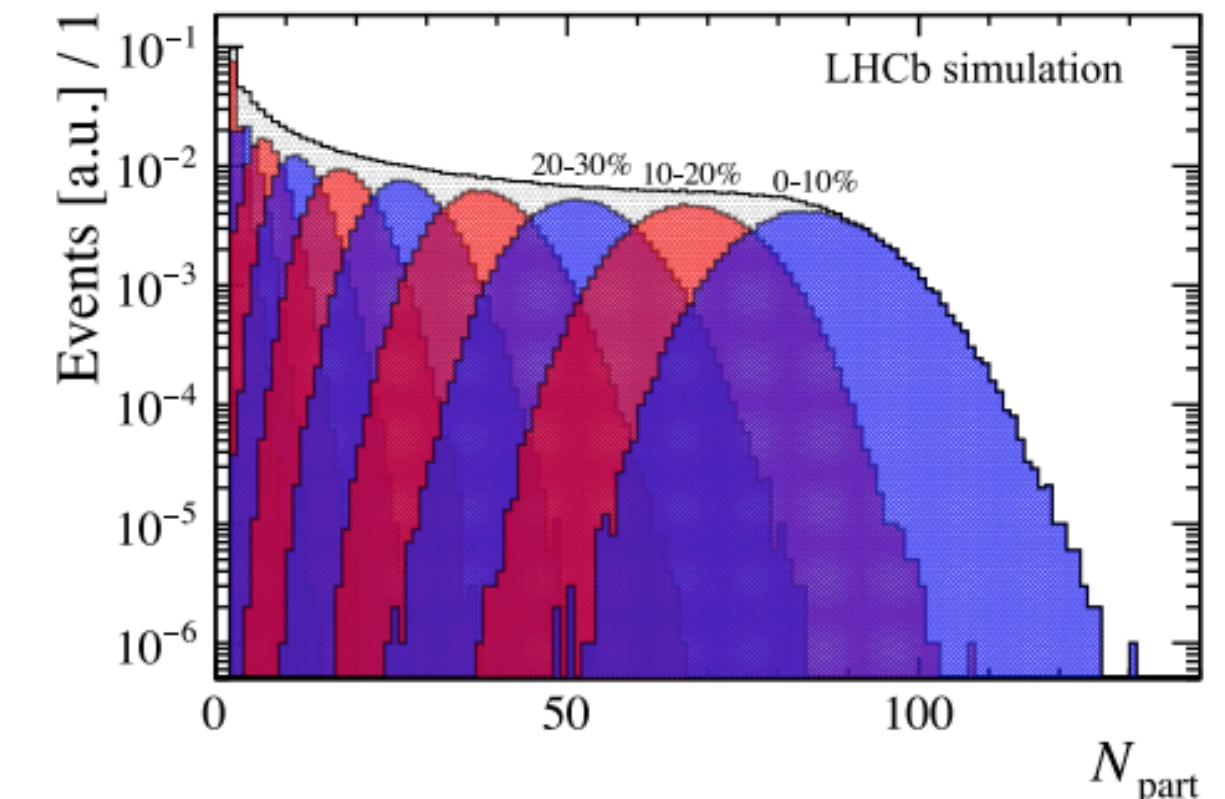
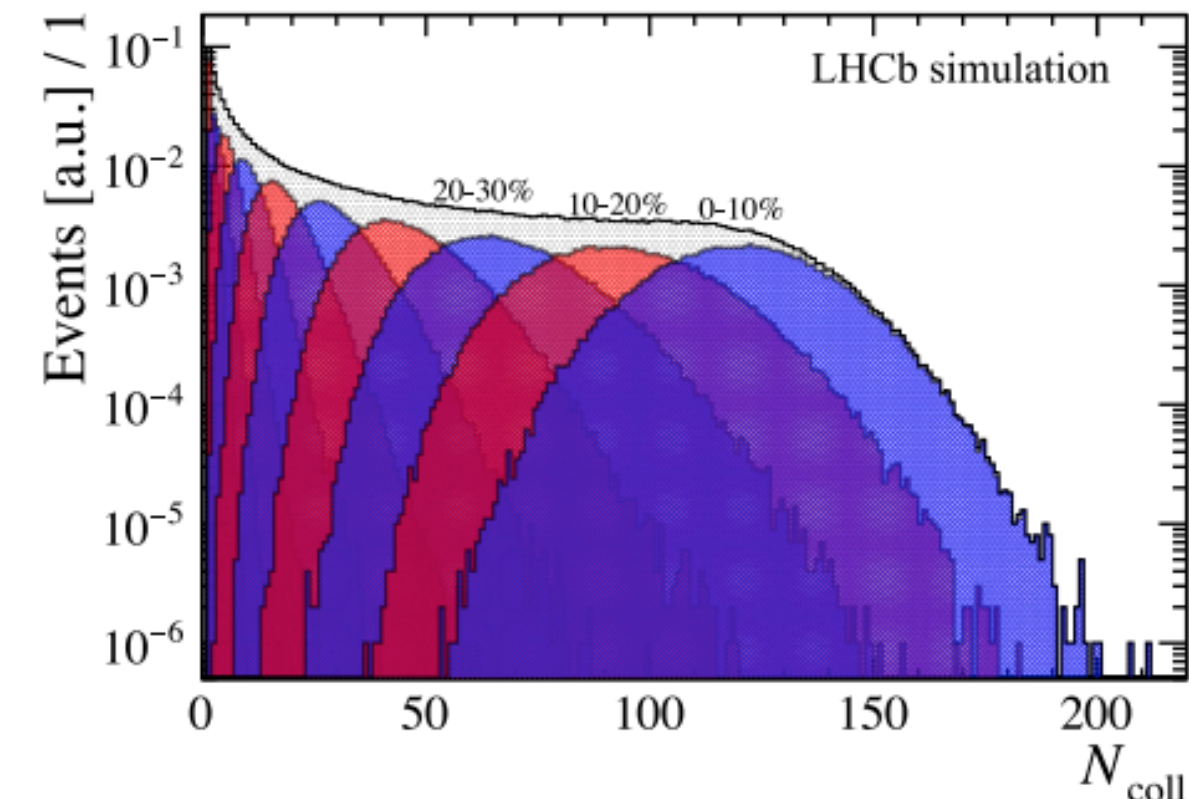
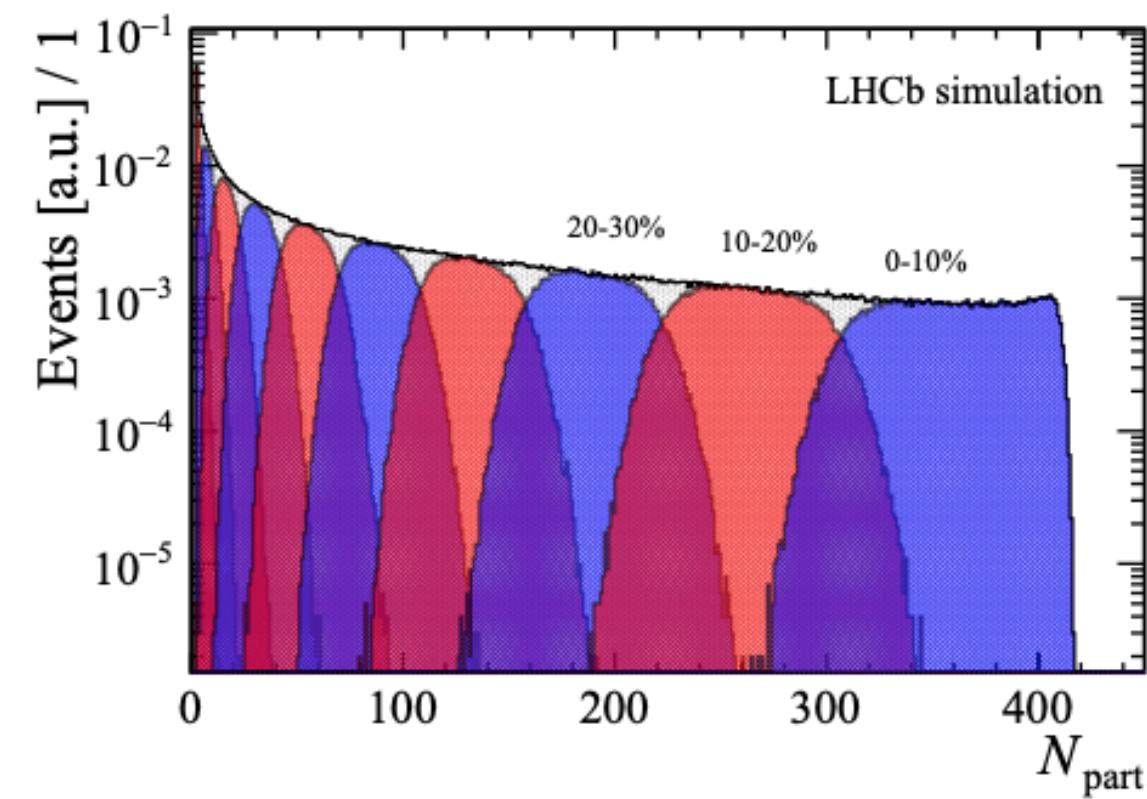
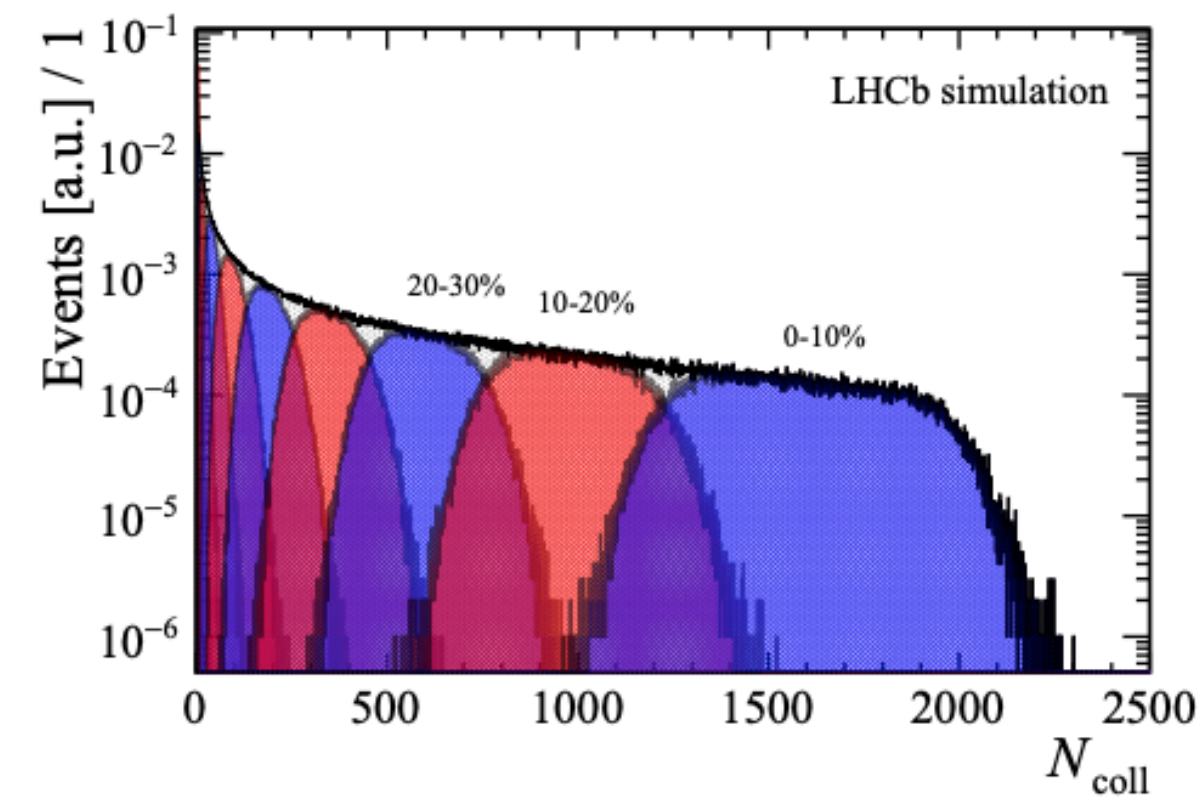
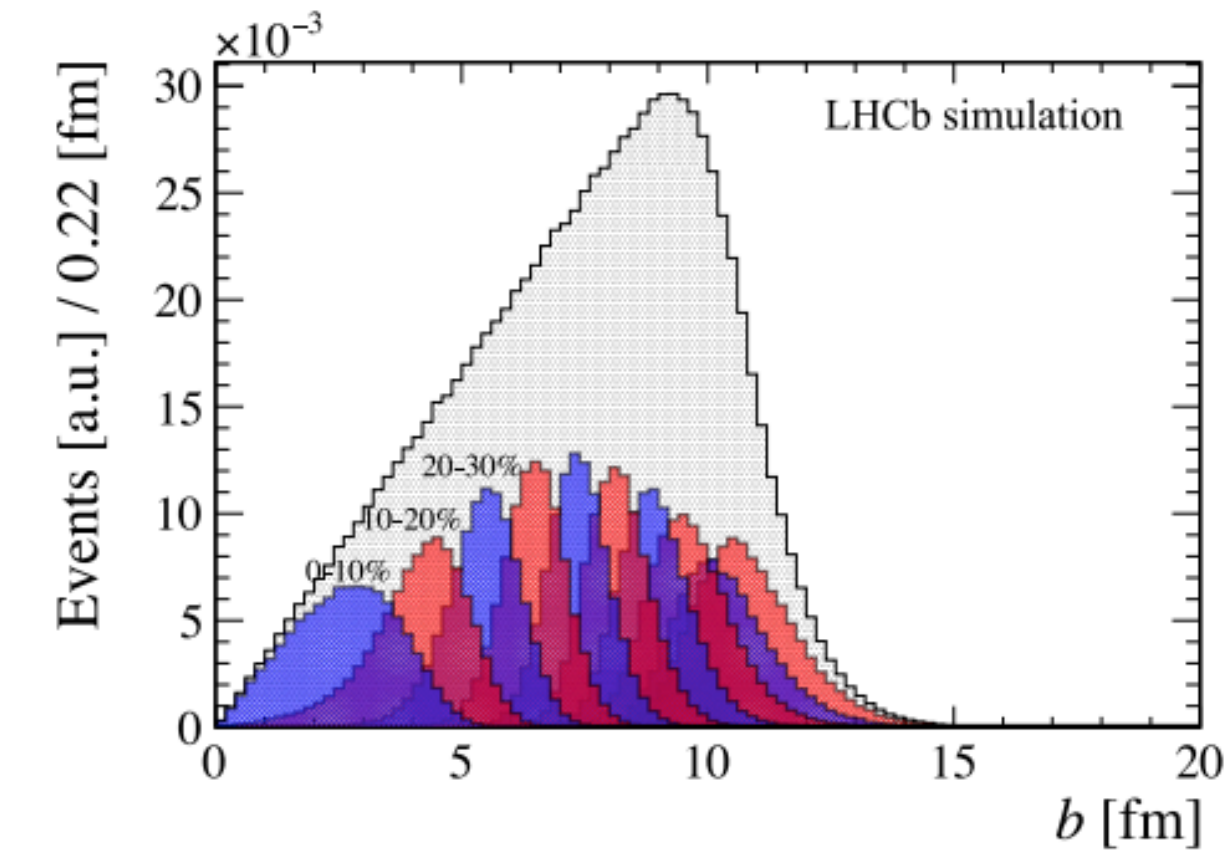
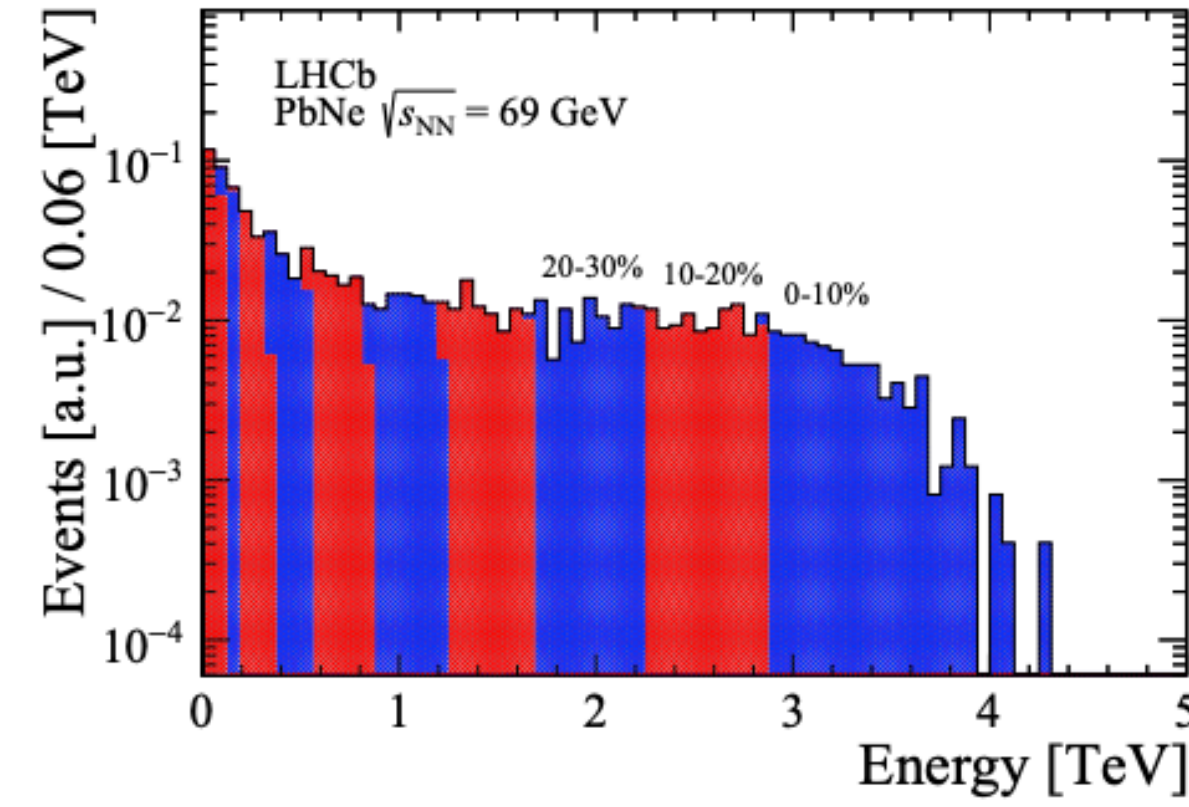
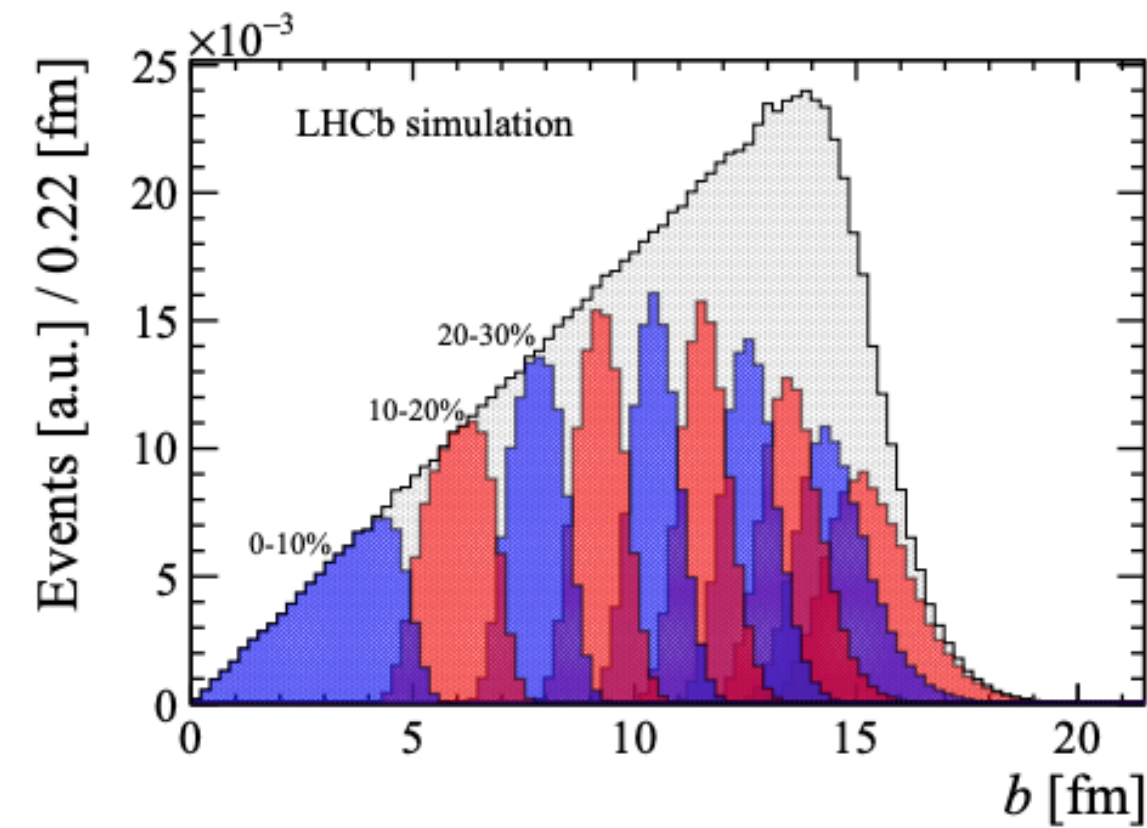
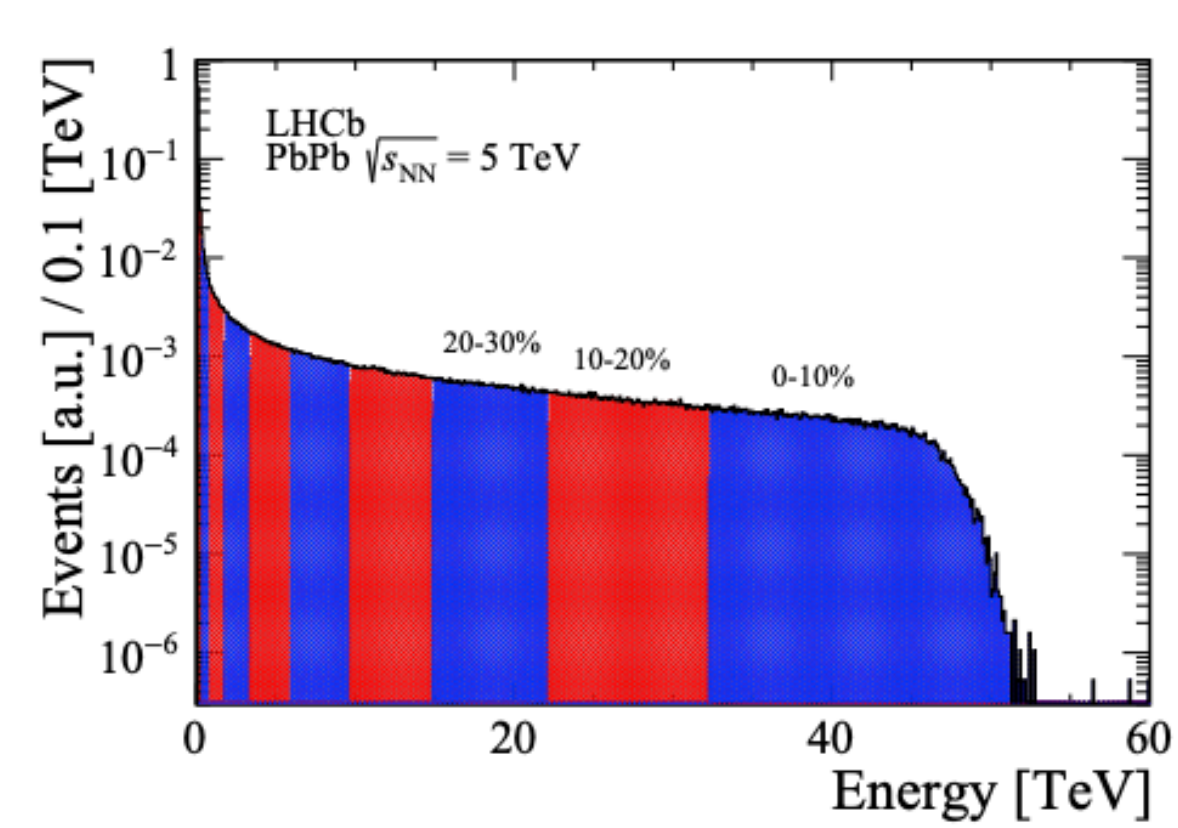


Centrality class



Use $\langle E_{pp} \rangle$ and fit eCal distribution...

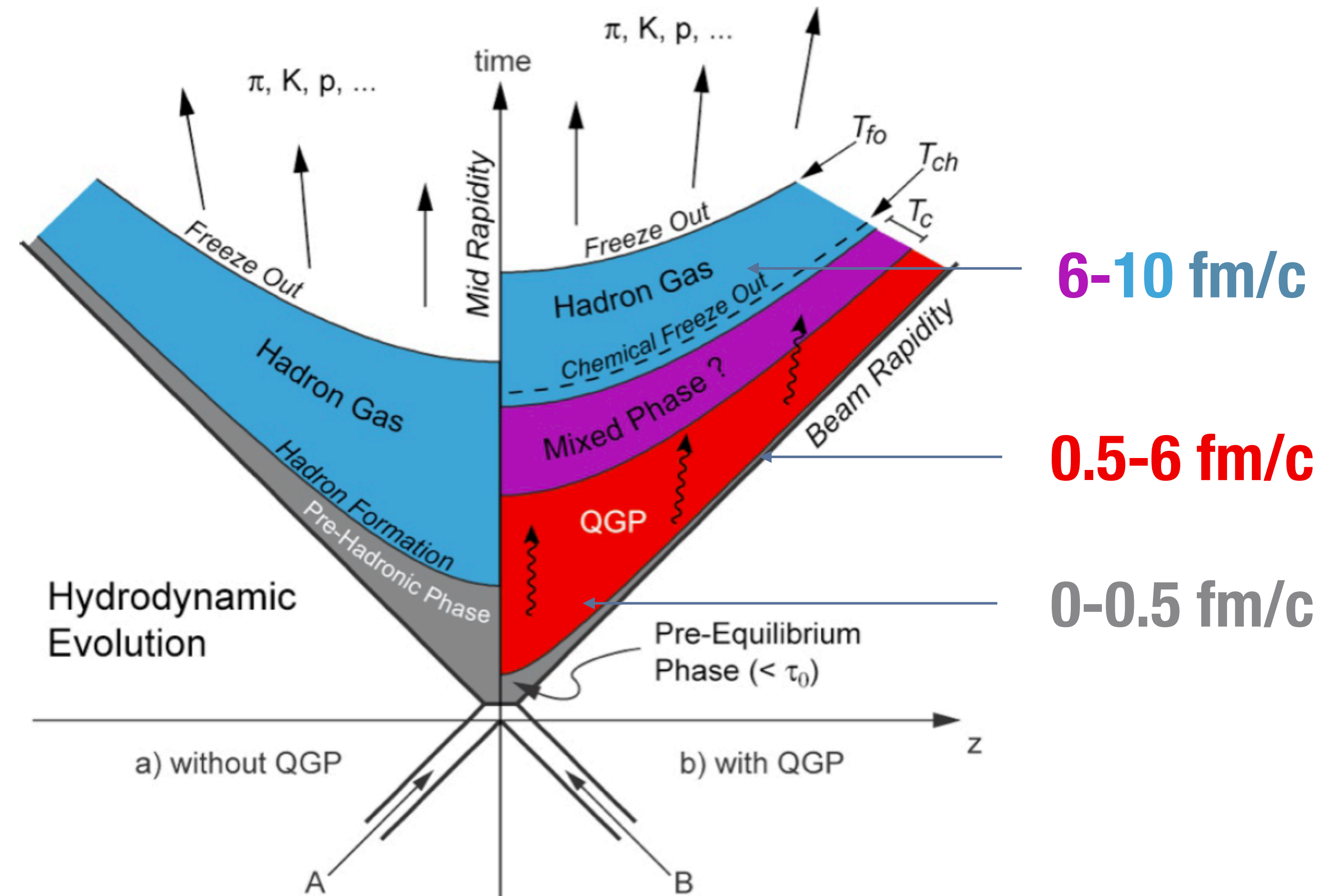
Small detour: centrality



$E_{tot}^{cal} \otimes$ Glauber fit = centrality

Points to take home: Evolution of a nucleus-nucleus collisions

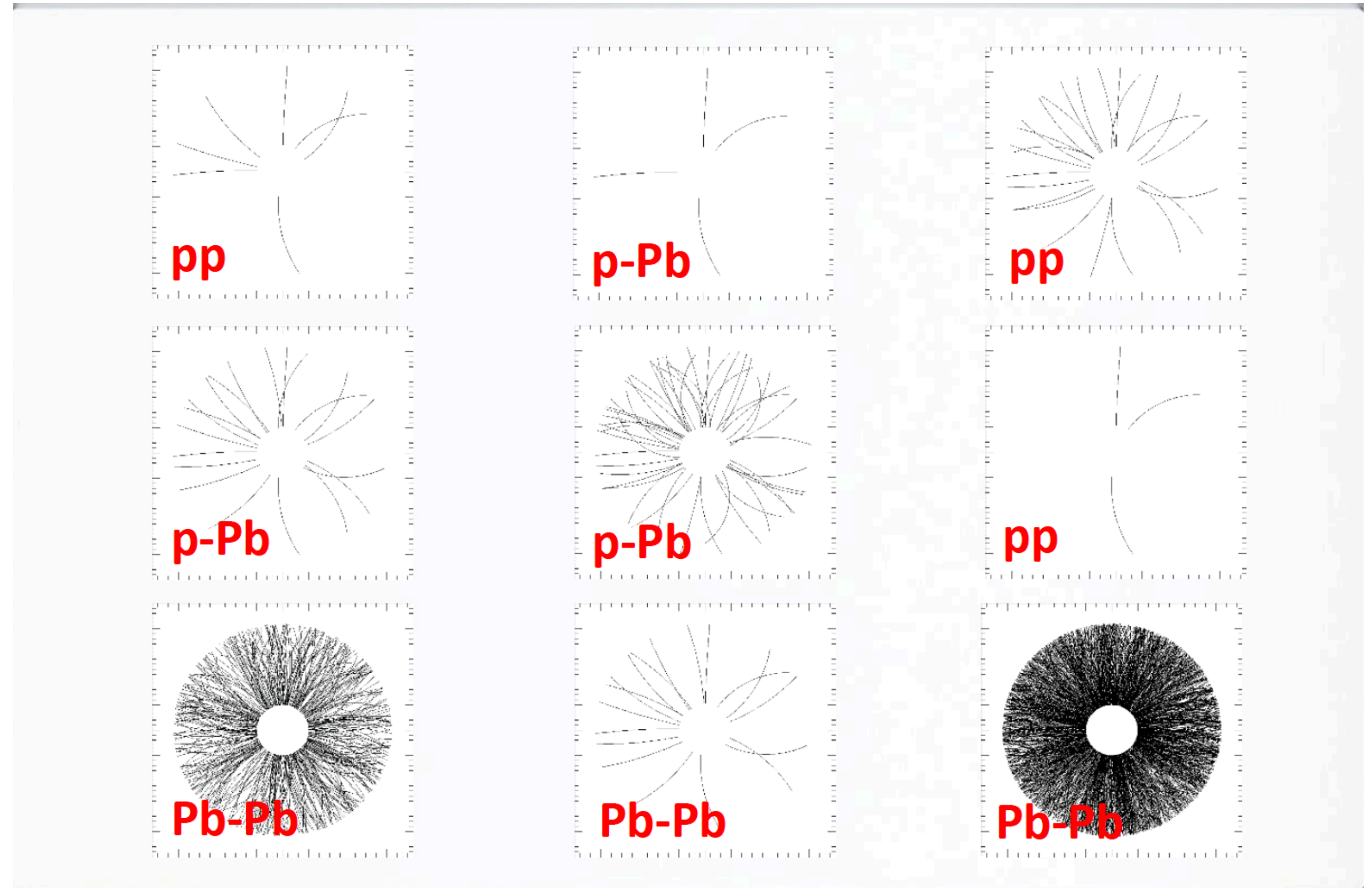
- * Hydro : very nice approach to describe the evolution of the bulk :
 - Predicts particle correlations and mass ordering of the v_2 .
- * From fitting the current data with hydro models from LHC@2.76TeV:
 - Initial energy density : 12–14 GeV/fm³
 - Effective temperature of the early QGP ≈ 297 MeV
 - Chemical freeze-out : $T_{ch} \approx 156$ MeV
 - Kinetic freeze-out : $T_{kin} \approx 96$ MeV
 - Size of the QGP in 0-5% centrality ~ 5000 fm³



OUTRO: HYDRO IN SMALL SYSTEMS ??

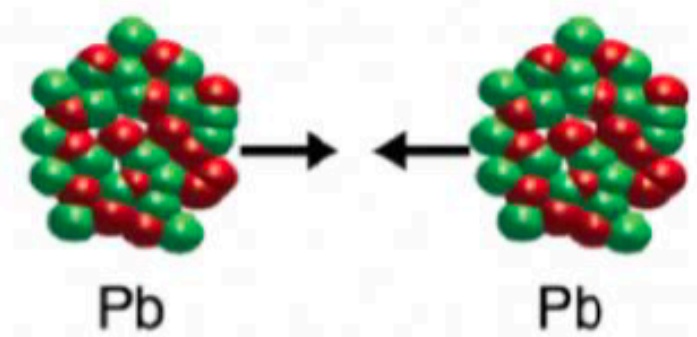
What are « small systems » ?

- * Small refers to :
 - Size of **colliding objects**.
 - * $ee < pp < pA < AA$
 - Size of **created medium**.
 - * N_{coll}, N_{part}
 - * Multiplicity

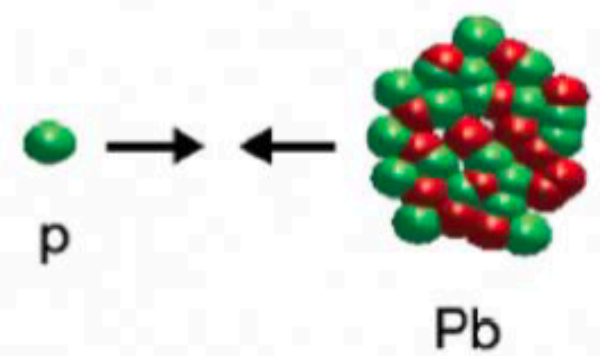


What do we search for in small system ?

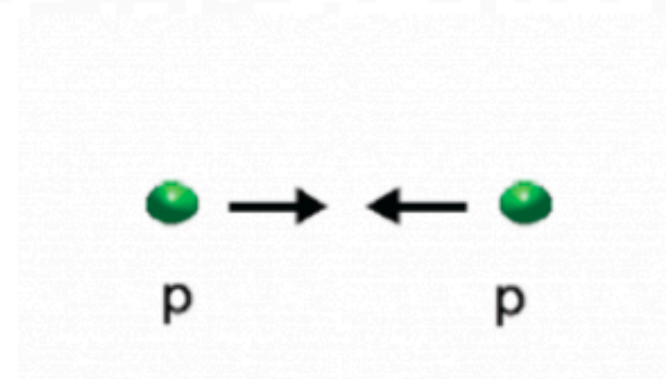
- * Small systems are traditionally considered « control systems » for HIC.



QGP

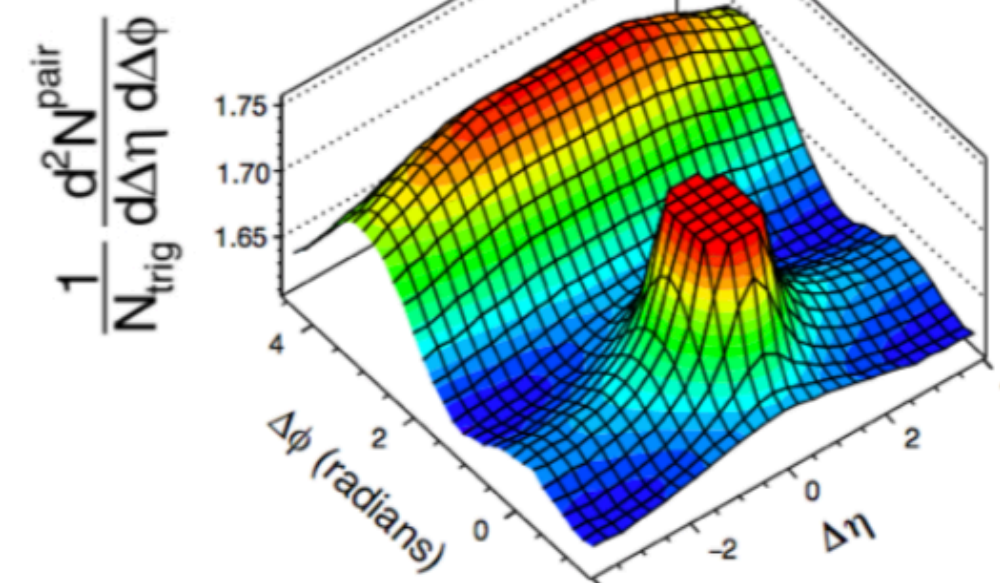


Cold nuclear matter effect

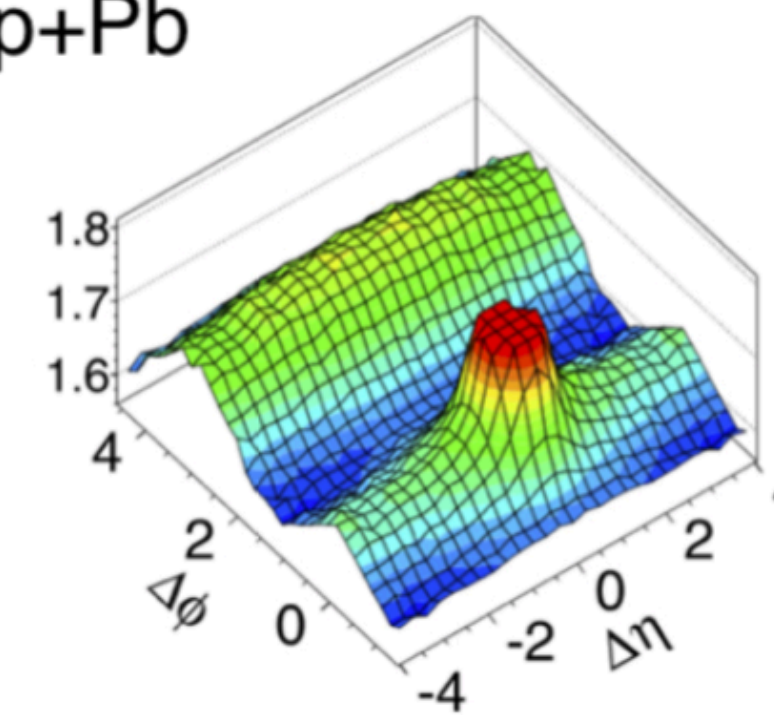


Baseline

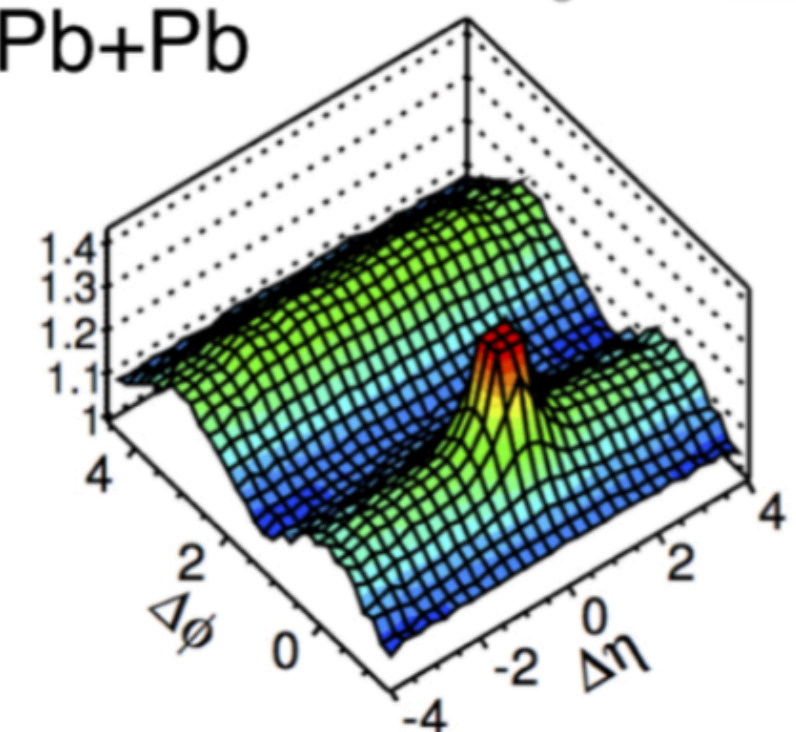
High-Multiplicity p+p



High-Multiplicity p+Pb



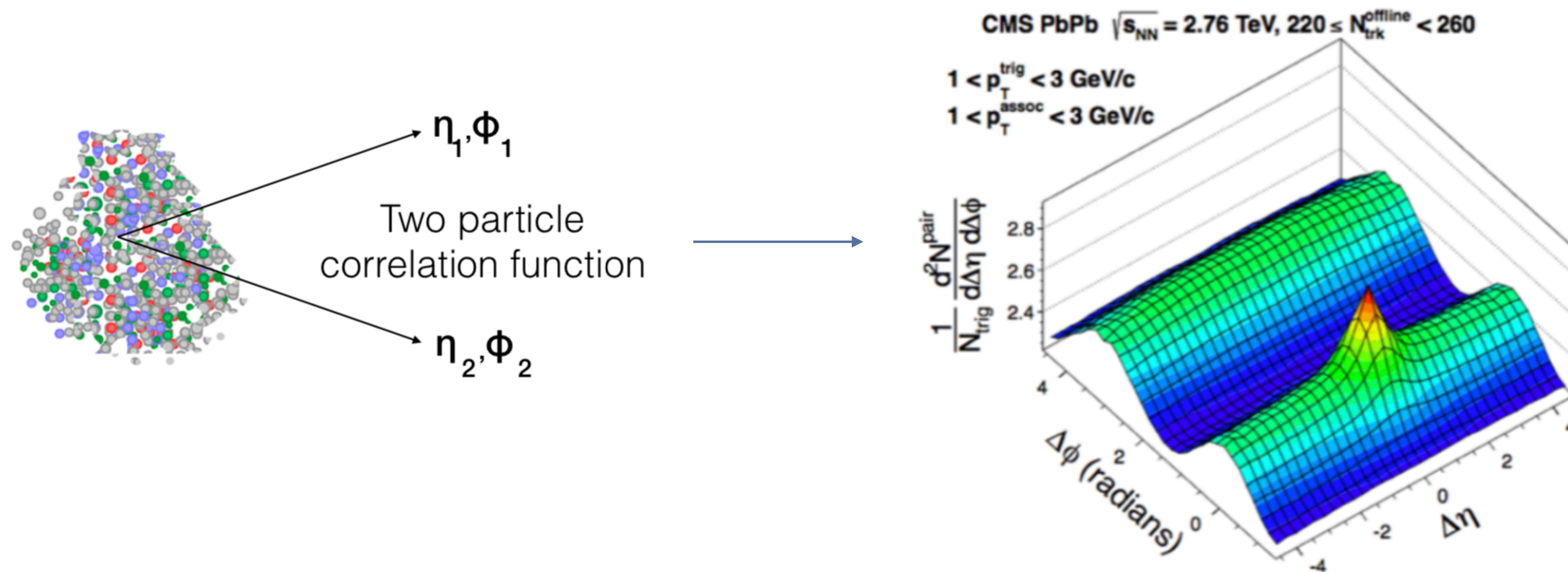
Pb+Pb



QGP in small system ?

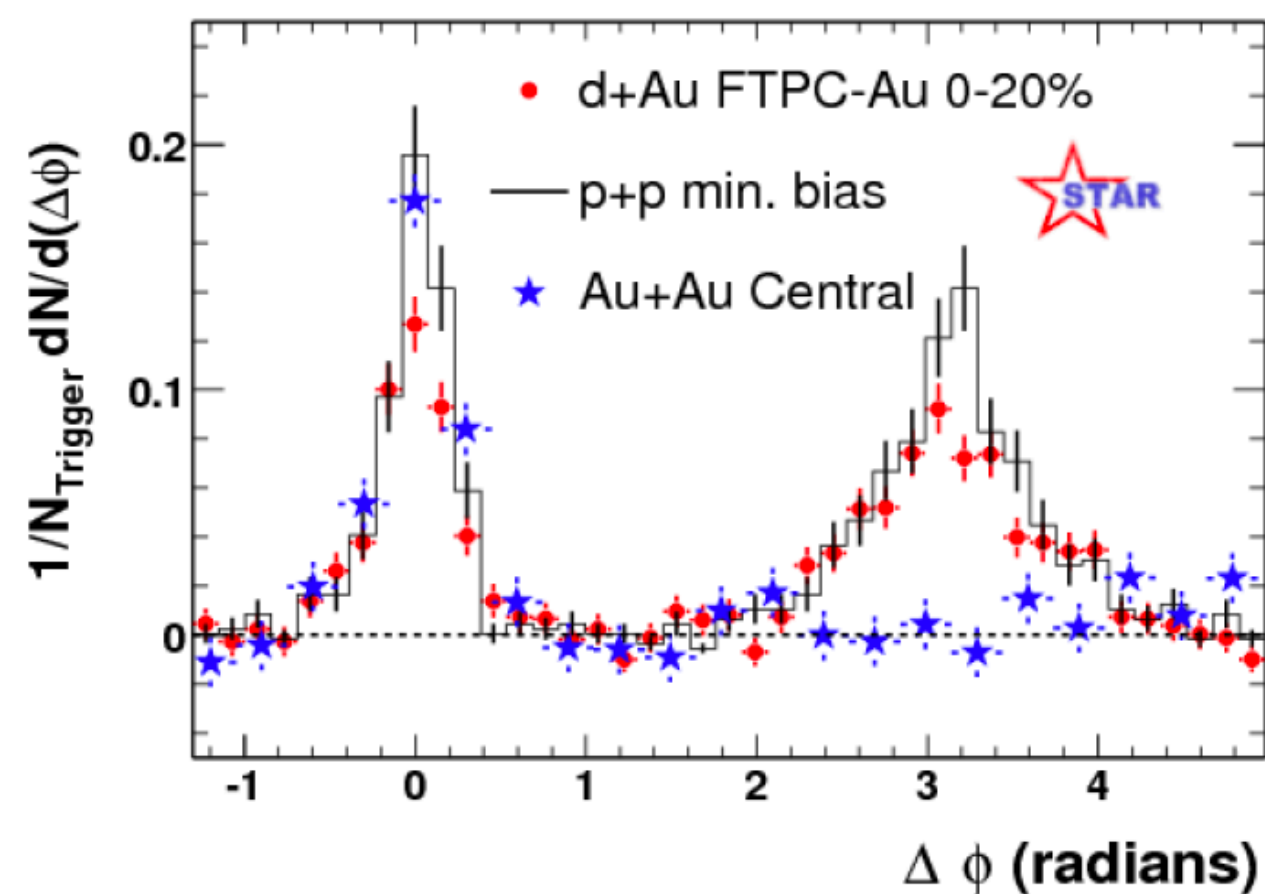
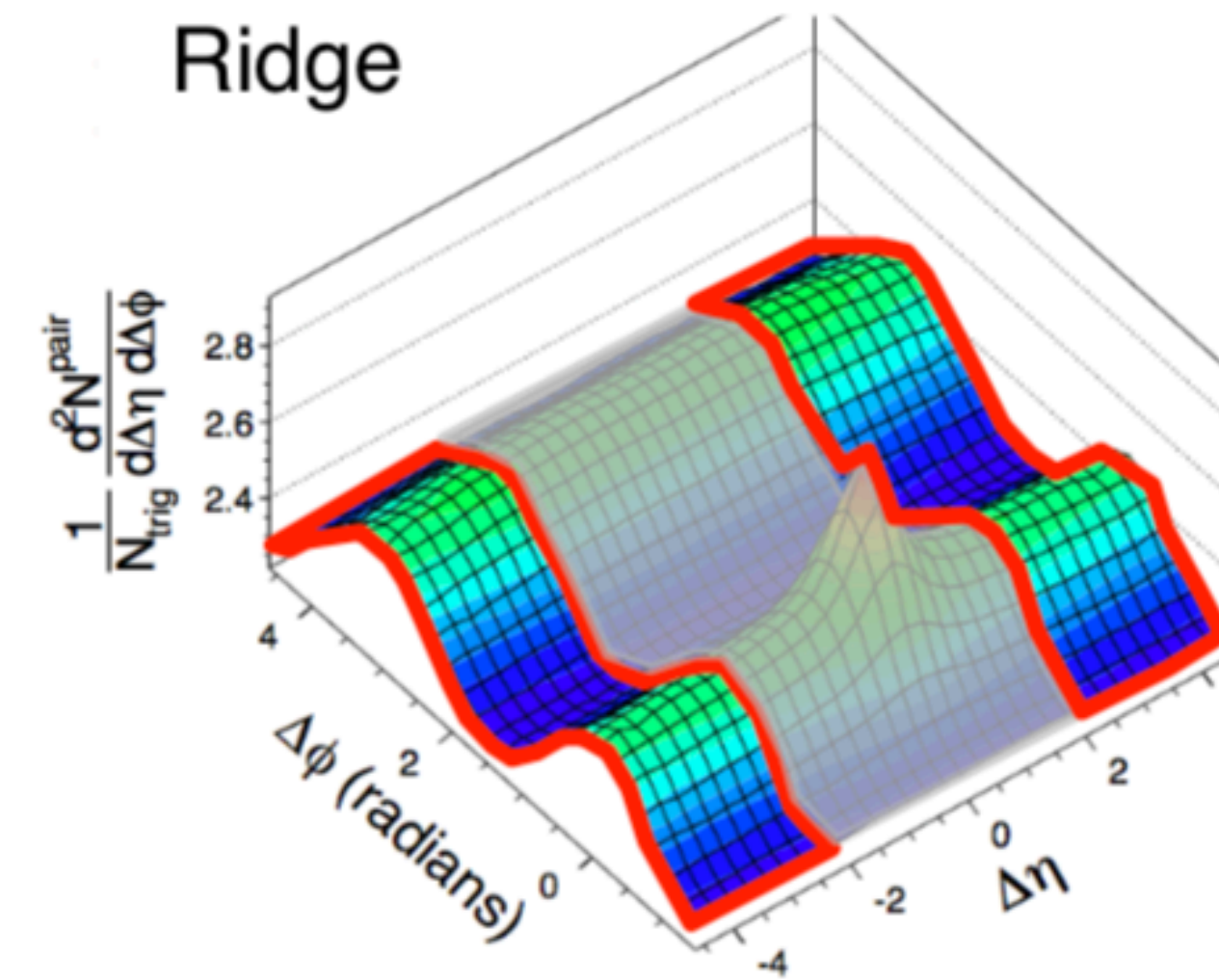
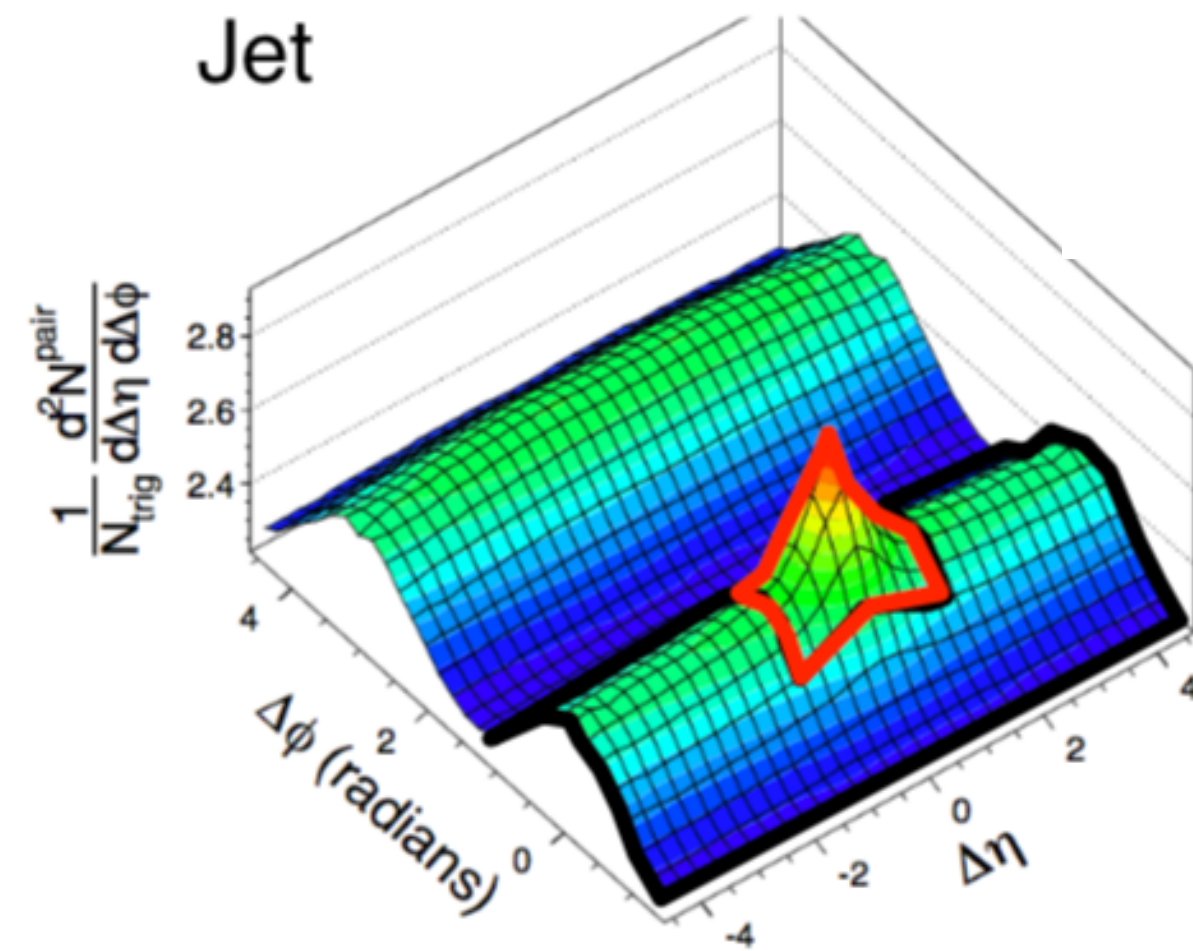
We need to change our perspectives and look closely at small systems

Angular correlation : two-particles correlations

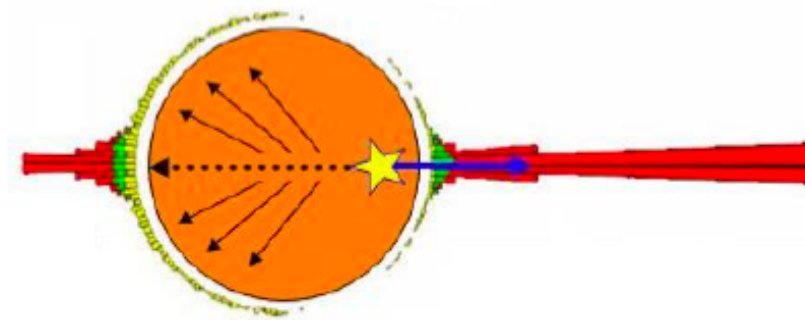


- * What are those structures on the correlation matrix ?

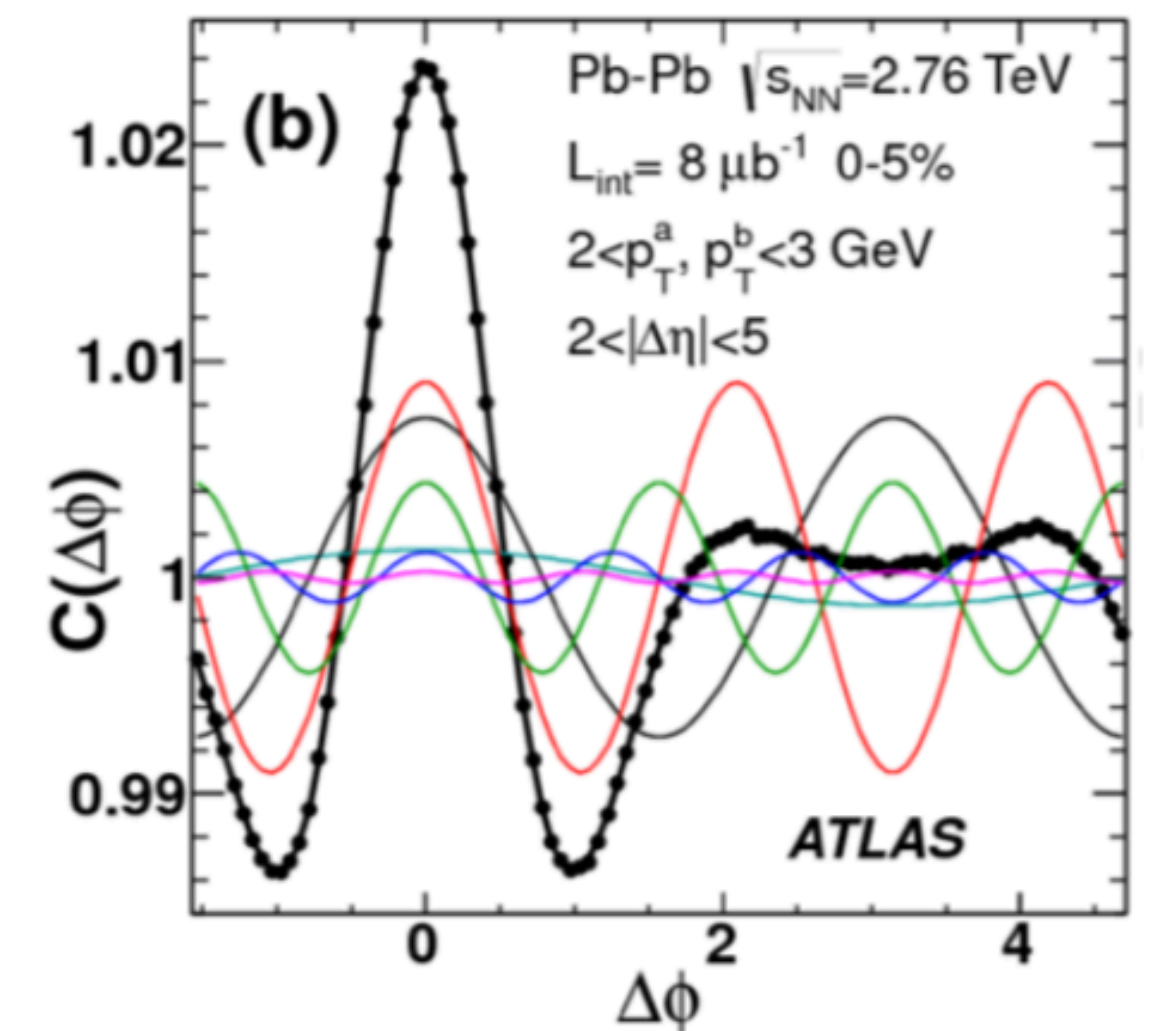
Angular correlation : two-particles correlations



Jet Quenching

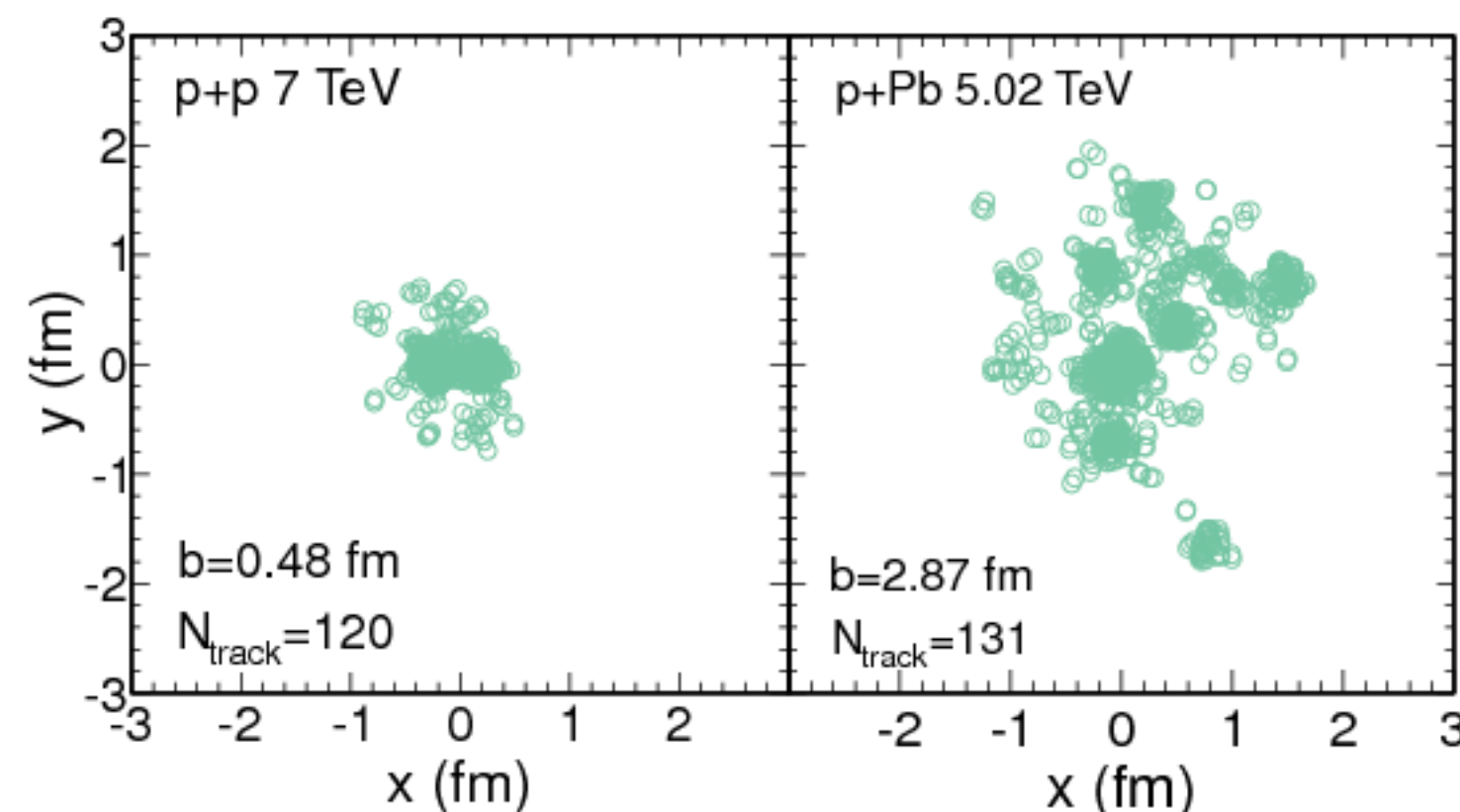
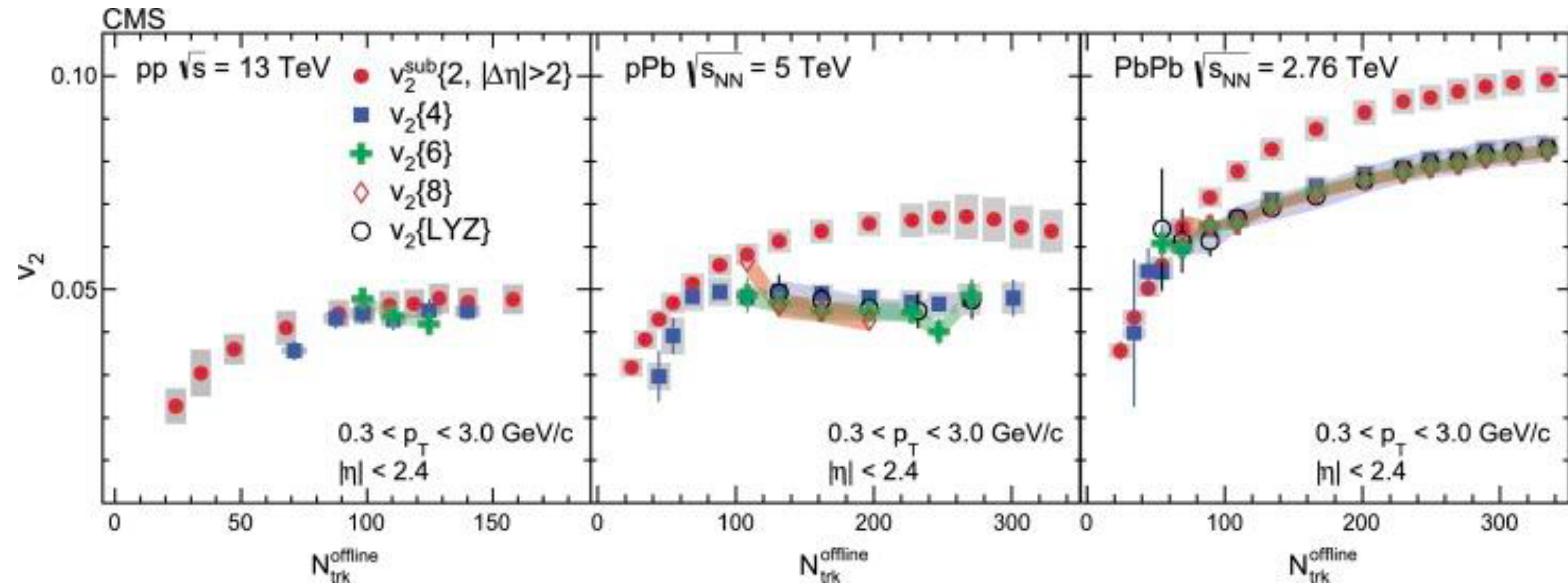


Anisotropic flow

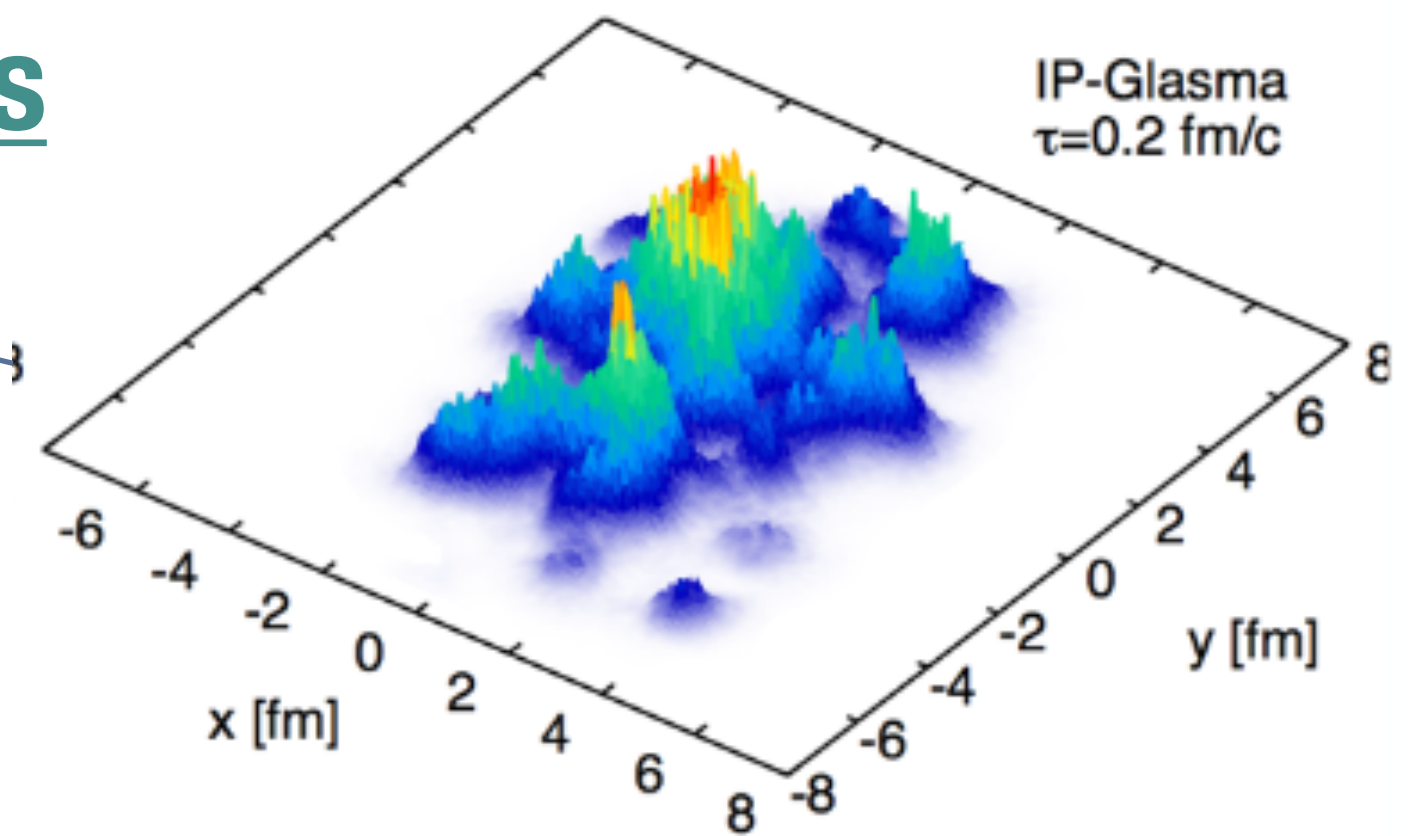


Elliptic flow in small systems

- * $v_2 > 0$ for all systems !
- * Is it really hydro ?
 - If hydro \rightarrow flow
 - If flow \Rightarrow hydro
- * **Flow = correlation!**



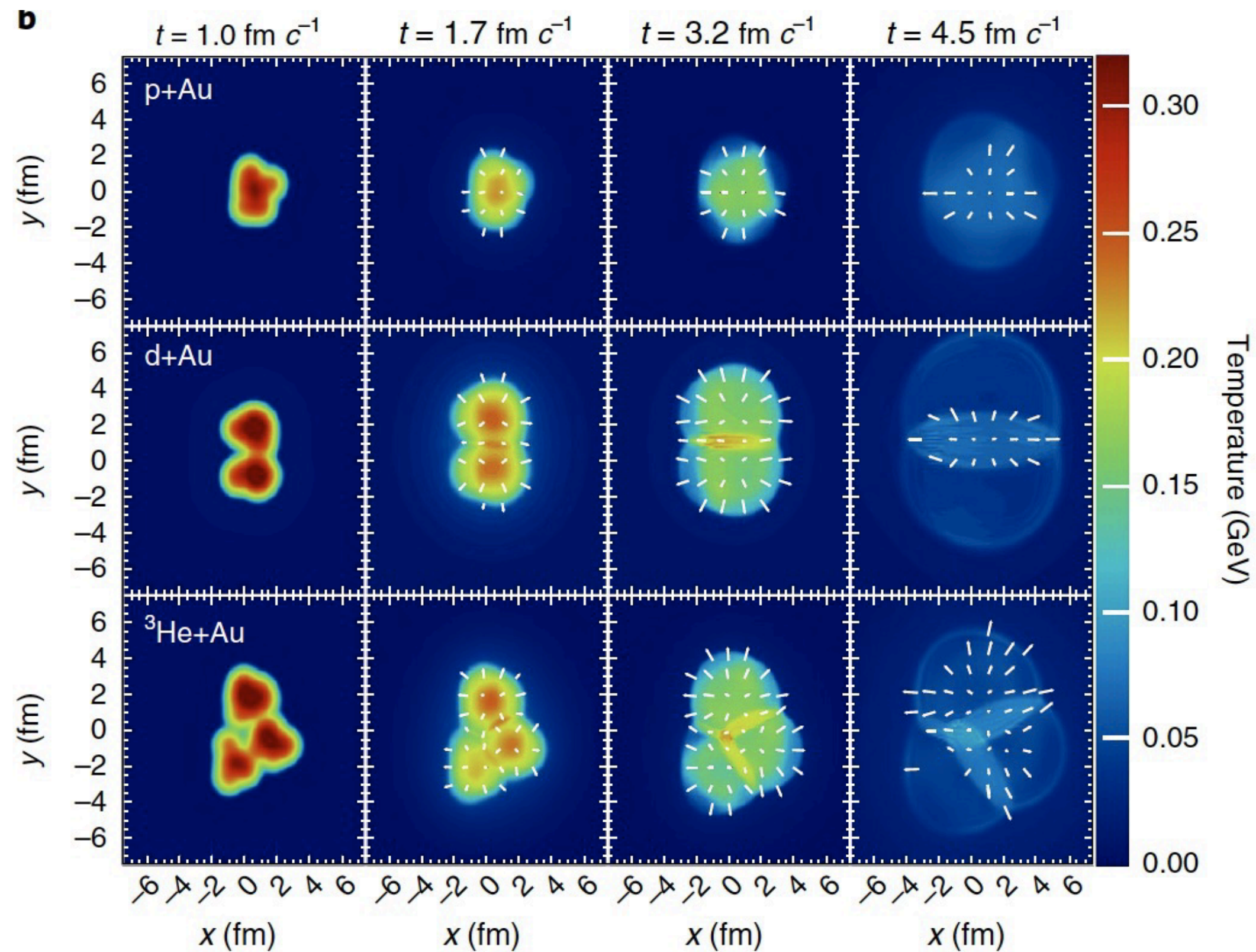
Initial density distributions



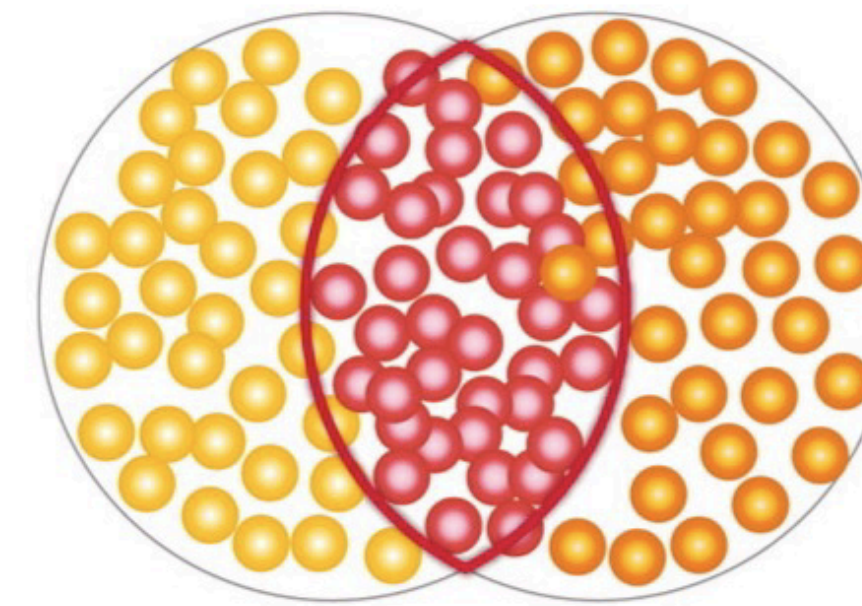
Angular flow: latest result from PHENIX

PHENIX, *Nature Physics* 15 214–220 (2019)

Creation of quark–gluon plasma droplets with three distinct geometries

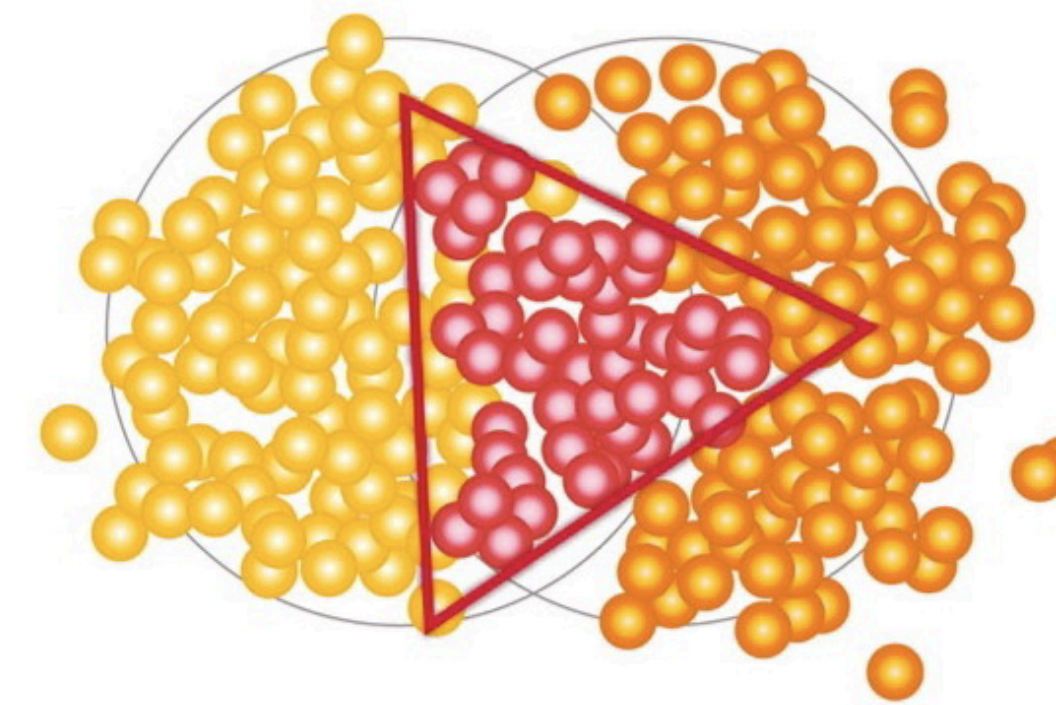


Hydro simulation



Elliptic flow

d+Au collisions have intrinsic ellipticity: enhanced v_2



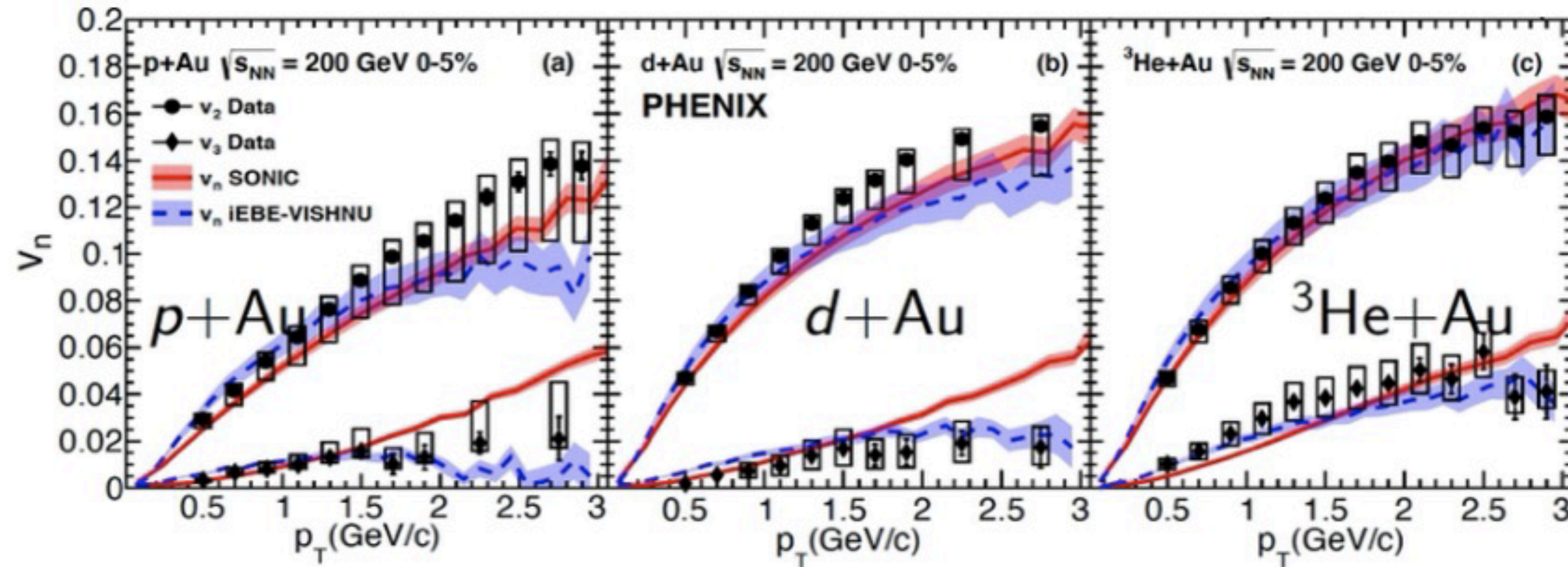
Triangular flow

$^3\text{He}+\text{A}$ collisions have intrinsic triangularity: enhanced v_3

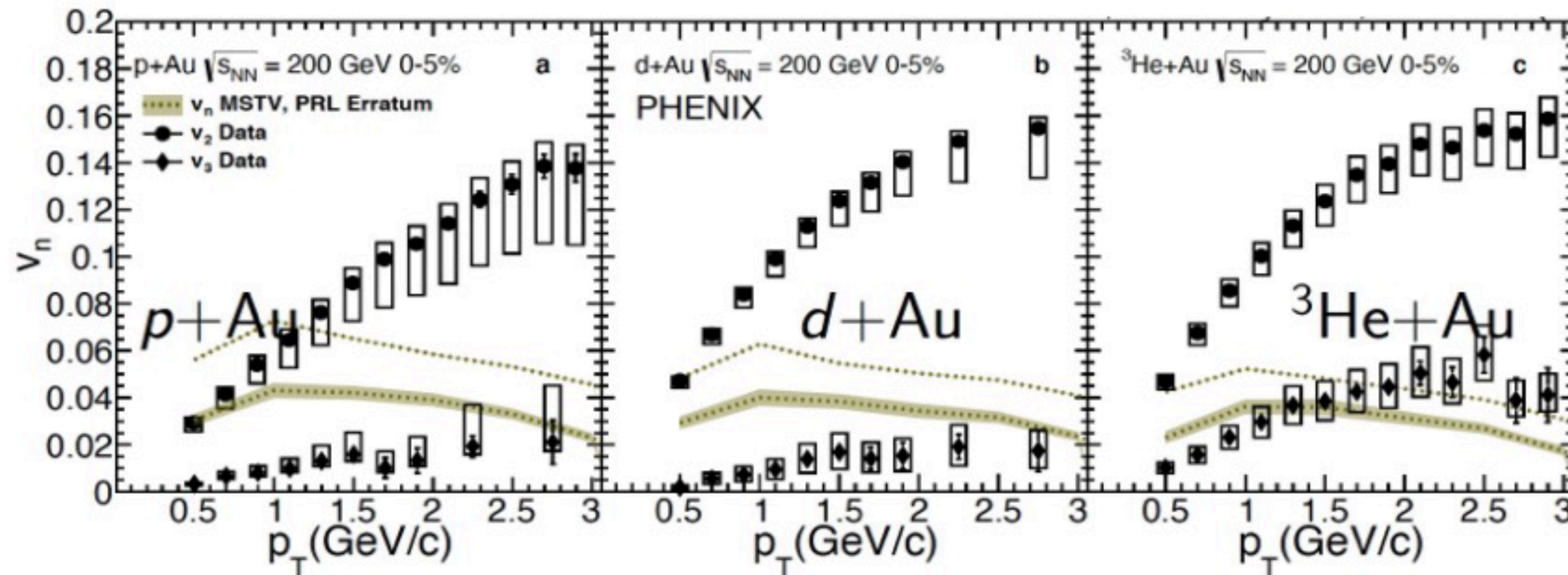
$$v_2^{\text{p+Au}} < v_2^{\text{d+Au}} \approx v_2^{\text{3He+Au}}$$

$$v_3^{\text{p+Au}} \approx v_3^{\text{d+Au}} < v_3^{\text{3He+Au}}$$

The results



SONIC and **iEBE-VISHNU**:
hydrodynamics predictions



MSTV: initial state calculation,
does not match data

Small system flow depends on
initial geometry, just like large
systems. Points to similar origin.