# Thermal misalignment of Scalar Dark Matter

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(2211.xxxx hep-ph)



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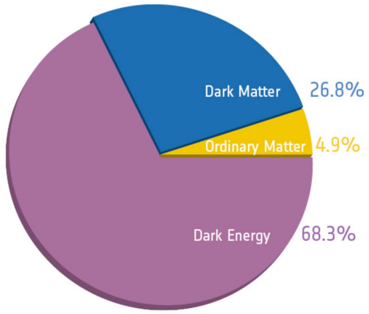
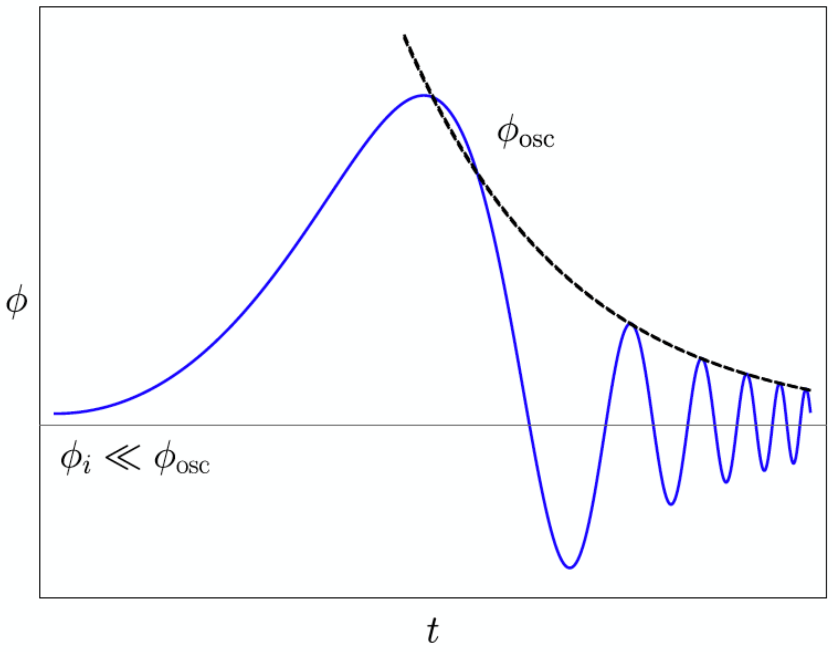
# Motivation

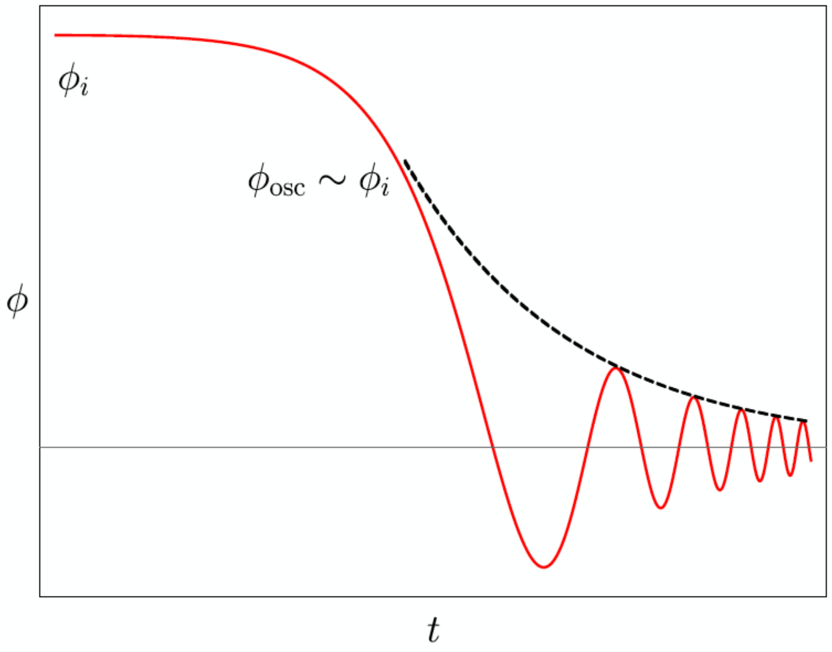



- A scalar field coupled through the Higgs portal provides a minimal and well motivated model of ultralight DM.[1]
- Several dynamical sources of scalar field misalignment during the radiation era exist in this model. We have computed the relic abundance over a broad range of masses and for different initial conditions.
- For larger scalar masses, thermal misalignment, due to the thermal potential, is the dominant misalignment mechanism, and provides a robust relic density target, which is largely independent of the initial conditions.
- For smaller masses, misalignment from the shift in scalar vev triggered by the EWPT dominates and the precise relic density prediction depends on the initial conditions.
- A variety of experimental and astrophysical constraints on the model exist, but new ideas are needed to further explore the cosmologically motivated parameter space.

[1] Piazza and Pospelov, Phys.Rev.D82:043533,2010
[2] Brian Batell and Akshay Ghalsasi, arXiv 2109.04476 [hep-ph]

# Thermal Misalignment vs Standard Misalignment



# Higgs portal model

- Light scalar $\phi$ with small coupling to Higgs(h) in thermal bath:

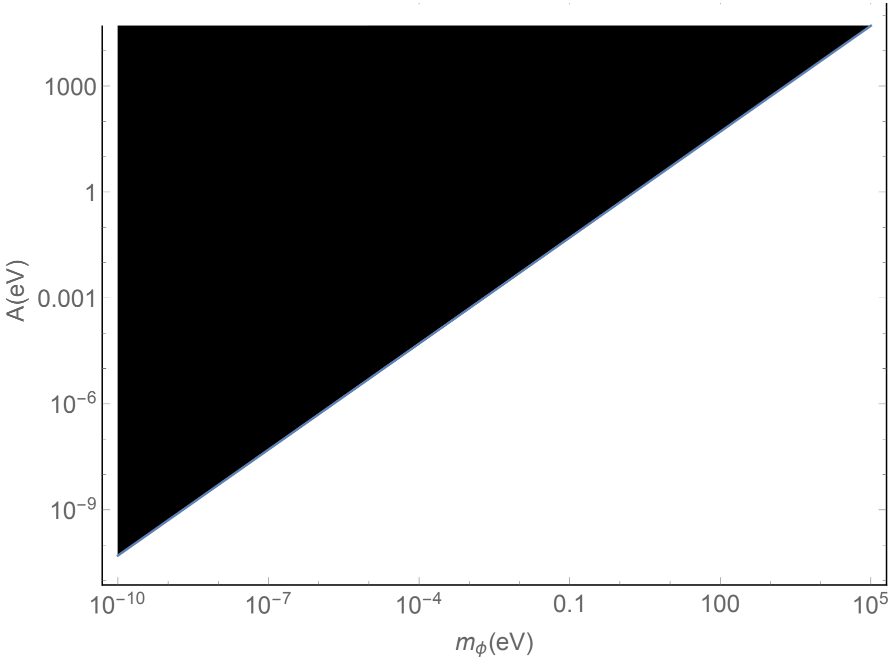
$$V = -\frac{1}{2}\mu^2 h^2 + \frac{1}{4}\lambda h^4 + \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}A\,\phi\,h^2$$

- Since we are always in regime where $A^2 \lesssim m_\phi^2 \ll \lambda v^2$,

$$\theta \sim \frac{A}{2\lambda v} \simeq \frac{Av}{M_h^2}, \qquad M_h^2 \simeq 2\lambda v^2 + \frac{A^2}{2\lambda}, \qquad M_\phi^2 \simeq m_\phi^2 - \frac{A^2}{2\lambda}.$$

$$\frac{A^2}{m_\phi^2} < 2\lambda$$

Black : No Higgs vev



# Effective potential

- There are three contributions to the effective potential:

$$V_{eff}(\phi, h, T) = V_0(\phi, h) + V_{CW}(\phi, h) + V_{th}(\phi, h, T)$$

- The first term is the usual zero temperature potential.
- In our study, the CW potential only effects the Higgs transition slightly and does not have a major impact on our final results, thus ignored.
- $\phi$ is not in thermal equilibrium, but experiences a thermal potential due to its coupling to SM via Higgs, all of which is in thermal equilibrium.

# 1-loop finite temperature effective potential

- For our model, the thermal potential is given as:

$$V_1^T(\phi,h,T) = \frac{1}{2\pi^2}T^4 J_B\left[\frac{m_h^2(\phi,h)}{T^2}\right] + \frac{3}{2\pi^2}T^4 J_B\left[\frac{m_\chi^2(\phi,h)}{T^2}\right] + \frac{6}{2\pi^2}T^4 J_B\left[\frac{m_W^2(h)}{T^2}\right]$$
$$+ \frac{3}{2\pi^2}T^4 J_B\left[\frac{m_Z^2(h)}{T^2}\right] - \frac{12}{2\pi^2}T^4 J_F\left[\frac{m_t^2(h)}{T^2}\right] - \frac{12}{2\pi^2}T^4 J_F\left[\frac{m_b^2(h)}{T^2}\right] + \dots$$

where

$$J_B(w^2) = \int_0^\infty dx\, x^2 \log[1 - e^{-\sqrt{x^2+w^2}}]$$
$$J_F(w^2) = \int_0^\infty dx\, x^2 \log[1 + e^{-\sqrt{x^2+w^2}}].$$

- We account for the hard thermal loops by using the Truncated dressing, where the masses are replaced by [1]

$$m^2 = m_{tree}^2 + \Pi(T),\ \Pi(T) \propto T^2$$

[1]David Curtin et al, https://arxiv.org/pdf/1612.00466.pdf

# Higgs field

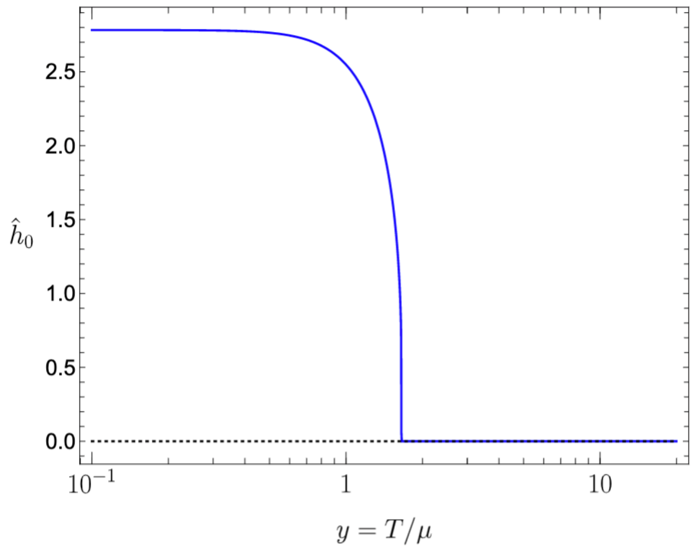
- Dimensionless variables: $y = \frac{T}{\mu}, \quad \hat{\phi} = \frac{\phi}{M_{\rm pl}}, \quad \hat{h} = \frac{h}{\mu}, \quad \kappa = \frac{m_\phi M_{\rm pl}}{\mu^2}, \quad \beta = \frac{AM_{pl}}{\mu^2}$

- Higgs field tracks its minima, which can be derived by minimizing the potential, $\frac{\partial V}{\partial h} = 0$ :

$$0 = \lambda \hat{h}^2 - (1 - \beta\hat{\phi}) + \frac{y^2}{2\pi^2}\left(6\lambda(J_B'[\eta_h] + J_B'[\eta_\chi]) + g^2\left(J_B'[\eta_{W_T}] + J_B'[\eta_{W_L}]\right) + (g^2 + g'^2)J_B'[\eta_{Z_T}]\right)$$
$$+ \frac{y^2}{2\pi^2}\left(\frac{\partial \eta_{Z_L}}{\partial z} J_B'[\eta_{Z_L}] + \frac{\partial \eta_{A_L}}{\partial z} J_B'[\eta_{A_L}]\right) - \frac{y^2}{2\pi^2}\left(12 y_t^2 J_F'[\eta_t]\right)$$

- Ansatz :

$$\hat{h}^2(\hat{\phi}, y) \approx \hat{h}_0^2(y) + \left(\frac{\partial \hat{h}^2}{\partial \hat{\phi}}\right)\hat{\phi}$$



# Evolution of Scalar Dark Matter

- EoM for $\phi$ :

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V_{eff}}{\partial \phi} = 0$$

- In terms of dimensionless quantities and temperature:

$$\hat{\phi}'' + \frac{1}{\gamma^2 y^6}\left[\kappa^2\hat{\phi} + \frac{\beta\hat{h}^2}{2} + \frac{\beta y^2}{2\pi^2}\left(J_B'[\eta_h] + 3J_B'[\eta_\chi]\right)\right] = 0.$$

- We solve it numerically by inserting the Higgs solution.

# Initial Conditions

- We consider two sets of initial conditions as our benchmark models :
- For a long enough period of inflation and a low enough Hubble, $H_I < \mathrm{v}$, the effective temperature experienced by the scalar field is T ~ $H_I$.
- Since $H_I \ll \mathrm{v}$, the Higgs is close to its vev and the true minima of $\phi$ is approximately given by it's 0 T value :

$$\phi[y_i] = \phi_0 = \frac{\beta M_{pl}}{\beta^2 - 2\lambda\kappa^2}$$

- $\phi_i = 0$, serves as a representative example of the general situation where $\phi_i$ is vastly different than $\phi_0$ , and Higgs VEV misalignment controls the final relic density for low masses.

# Onset of oscillations

- For the onset of oscillations, we require,

$$(3H)^2 \sim m_\phi^2(T)$$

- We will focus on 2 regions, where in both cases:

$$3H \sim m_\phi \Rightarrow y_{osc} \sim \sqrt{\frac{\kappa}{3\gamma}}$$

- Region 1 (small $\beta$, large $\kappa$, high T ):

$$\kappa > 3\gamma,\ y_{osc} \gg 1$$

- Region 2 (small $\kappa$ , low T ):

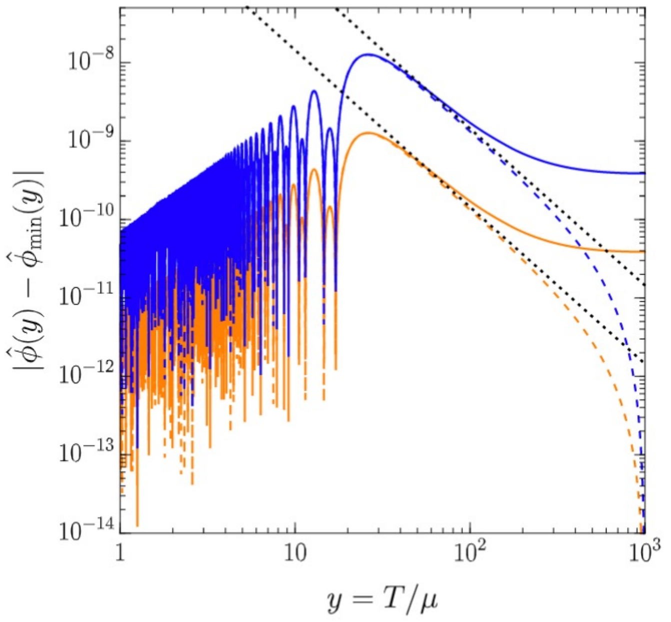
$$\kappa < 3\gamma, y_{osc} < 1$$

# Approximate DM density : Region I

- Region I is defined as : $(\kappa \gtrapprox 10^3,\ m_\phi \gtrapprox 3\times 10^{-3}\mathrm{eV})$

- In this region, the thermal misalignment dominates over the kick due to Higgs transition, hence we drop the Higgs dependent term to get an approximate form of the equation:

$$\hat{\phi}''(y) + \frac{\beta}{2\pi^2\gamma^2 y^4}\left(J_B'[\eta_h] + 3(J_B'[\eta_\chi]\right) = 0,$$




- This yields :

$$\hat{\phi}(y) = -\frac{\beta}{6\pi^2\gamma^2 y^2} + \phi_i \qquad \hat{\phi}(y_{osc}) = -\frac{\beta}{2\pi^2\gamma\kappa} + \phi_i$$

- The DM density can be given by a simple approx. form:

$$\Omega_{DM} = \frac{\rho(T_0)}{\rho_{tot}} = \frac{\rho(y_{osc})}{\rho_{tot}}\left(\frac{y_0}{y_{osc}}\right)^3\left(\frac{g_{*,0}}{g_{*,osc}}\right)$$
$$= 0.26\left(\frac{\beta}{0.05}\right)^2\left(\frac{1000}{\kappa}\right)^{3/2}$$

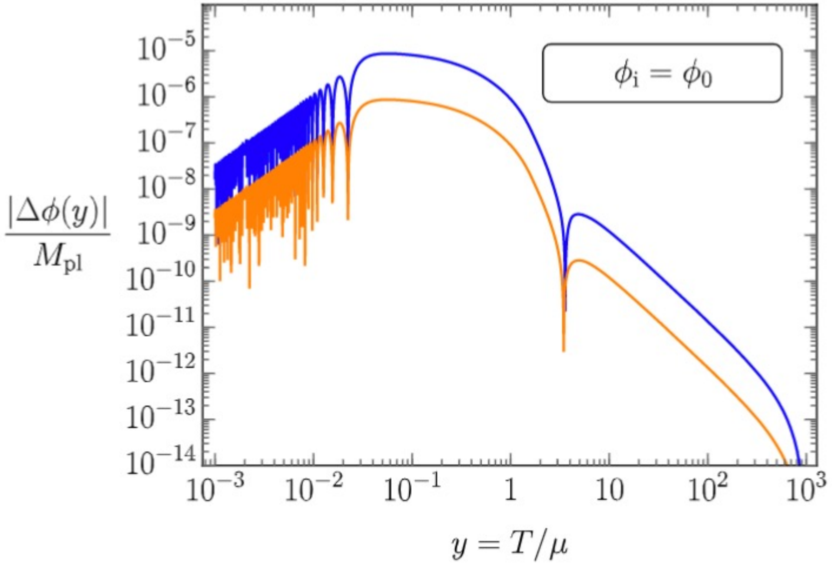
# Approximate DM density: Region 2

- Region 2 is defined as : $\kappa < 1, m_\phi < 10^{-5} eV$
- The thermal potential is not relevant in this region, thus we get :

$$\phi''(y) + \frac{1}{\gamma^2 y^6}\left(\kappa^2 \hat{\phi}\right) = 0,$$

- Solution:

$$\phi(y) = \frac{1}{y^4}\frac{\beta}{24\gamma^2\lambda} + \phi_0$$




- The DM density is given by :

$$\Omega_{DM} = \frac{\rho(T_0)}{\rho_{tot}} = \frac{\rho(y_{osc})}{\rho_{tot}}\left(\frac{y_0}{y_{osc}}\right)^3\left(\frac{g_{*,0}}{g_{*,osc}}\right)$$
$$= 0.26\left(\frac{\beta}{2\times 10^{-4}}\right)^2\left(\frac{\kappa}{10^{-3}}\right)^{1/4}$$

# Peaky behavior ( intermediate masses)

- $3 * 10^{-3} eV \lesssim m_\phi \lesssim 10^{-2} eV$, thermal and Higgs transition compete.
- If the scalar happens to be near its peak amplitude of oscillation when the Higgs transitions, then it leads to reduction in its amplitude, thus larger coupling A in required to get the right abundance, which leads to peaks.

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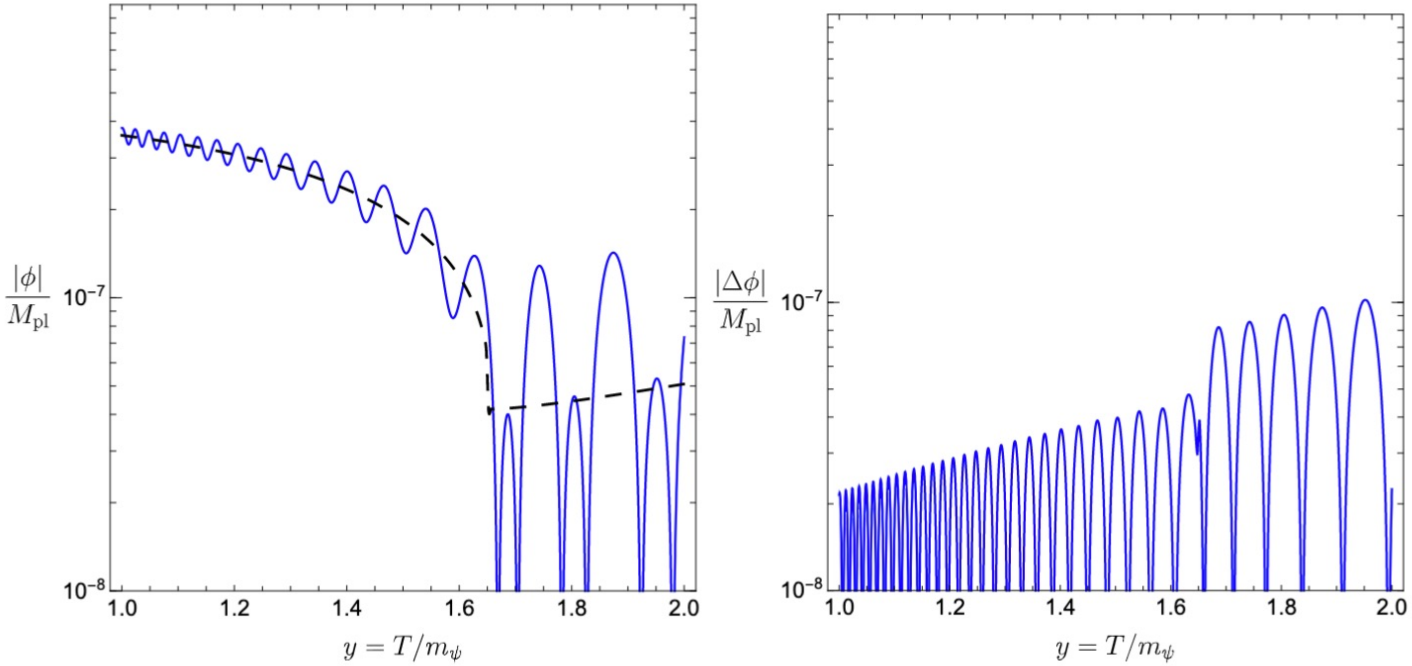
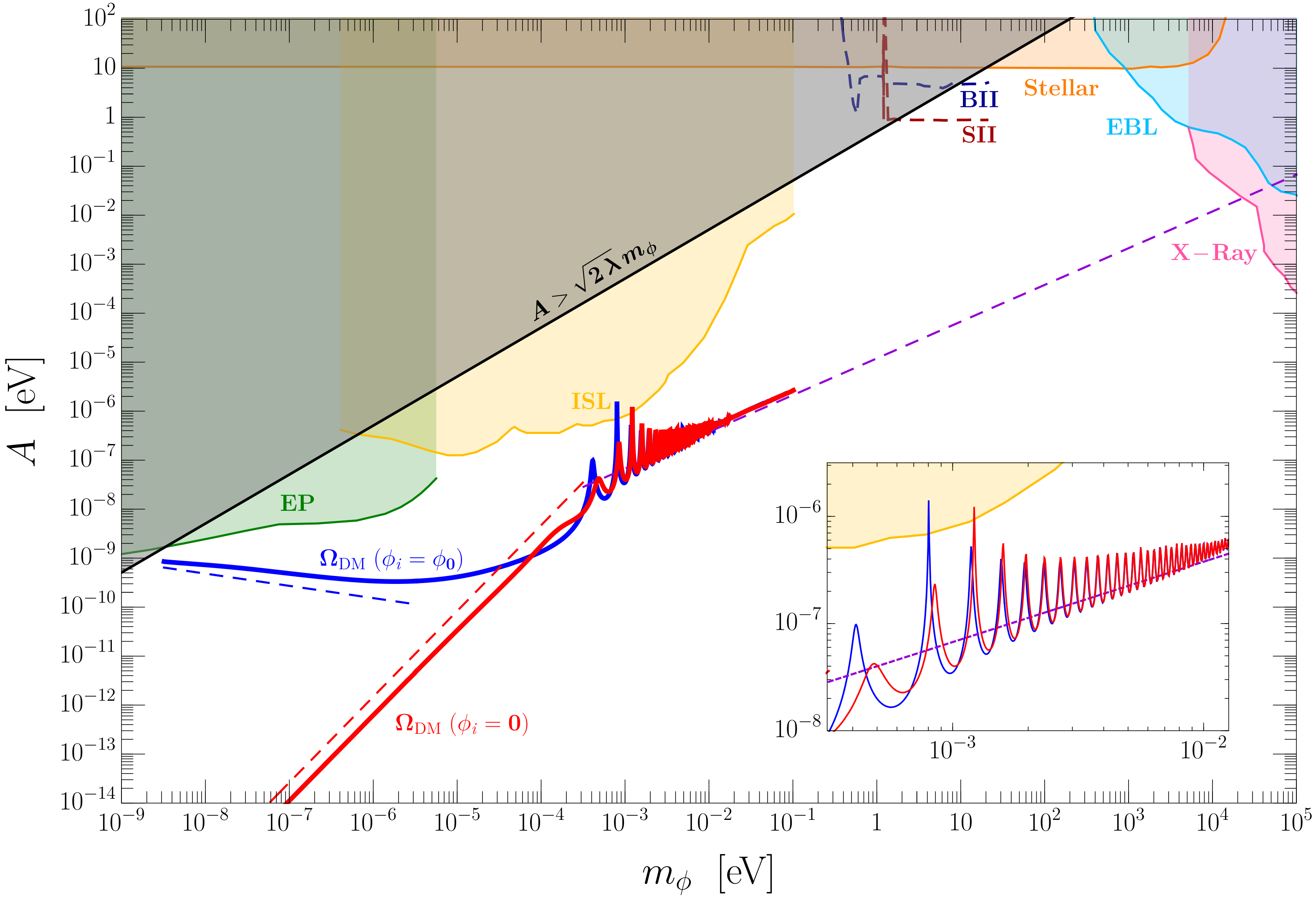


# Relic Density Plot



# Conclusions

- Ultralight scalars in DM models lead to a well-motivated and phenomenologically distinct viable scenarios.
- Particularly, we have focused on the phenomenology of a realistic scenario where the DM couples to the Higgs and the SM.
- Relic abundance is fairly insensitive to initial conditions and is dictated by the couplings and masses.
- This is one of the most minimal setup which is also experimentally viable.
- In future, more work is needed to conceive of ways to probe the model experimentally.

THANK YOU!

# BACKUP Slides

# Mass eigenstates

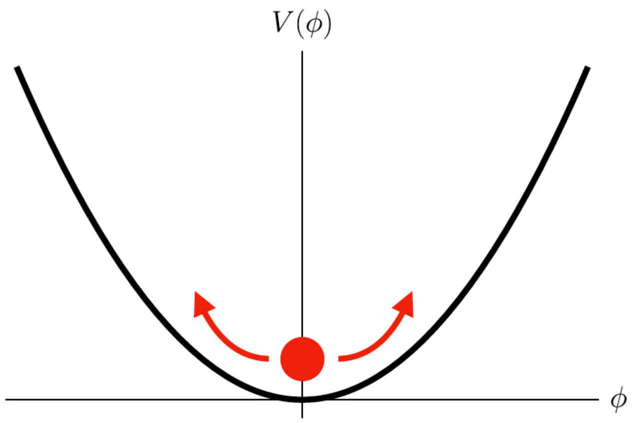
- Mass eigenvalues :

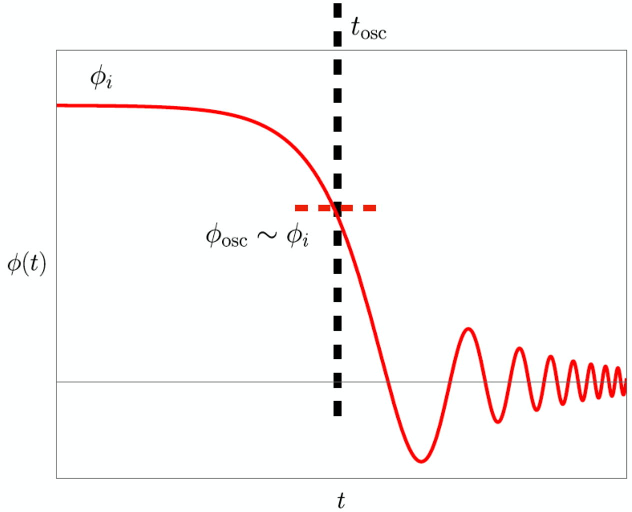
$$M^2_{h,\phi} = \frac{1}{2}\left[2\lambda v^2 + m^2_\phi \pm \sqrt{(2\lambda v^2 - m^2_\phi)^2 + 4A^2v^2}\right]$$

# Standard Misalignment mechanism

$$\ddot{\phi} + 3H\dot{\phi} + m_\phi^2\phi = 0$$

- During early times (high T) the scalar is held up by Hubble friction and remains fixed at its initial value.
- As the universe cools, $H < m$. This signals the onset of scalar oscillations.
- At late times, the scalar oscillates about its minimum and is diluted due to Hubble expansion.



# Standard Misalignment mechanism

- The energy density redshifts as matter

$$\rho_\phi = \frac{1}{2} m_\phi^2 \langle \phi^2(t) \rangle \sim a(t)^{-3} \sim t^{-3/2} \sim T^3$$

- The relic abundance at late times will depend on the initial value of field via the oscillation field value:

$$\Omega_\phi\big|_0 = \frac{\rho_{\phi,0}}{\rho_{c,0}} \simeq \frac{\frac{1}{2} m_\phi^2 \phi_{\rm osc}^2 (T_0/T_{\rm osc})^3 (g_{*S}^0/g_{*S}^{\rm osc})}{\rho_{c,0}}$$

# Potential in dimensionless terms

- We do calculations in dimensionless terms, by defining,

$$y = \frac{T}{\mu}, \quad \hat{\phi} = \frac{\phi}{M_{\rm pl}}, \quad \hat{h} = \frac{h}{\mu}, \quad \kappa = \frac{m_\phi M_{\rm pl}}{\mu^2}, \quad \beta = \frac{AM_{pl}}{\mu^2}$$

- The potential becomes :

$$\hat{V} = -\frac{1}{2}\hat{h}^2(1-\beta\hat{\phi}) + \frac{1}{4}\lambda\hat{h}^4 + \frac{1}{2}\kappa^2\hat{\phi}^2 \tag{21}$$
$$+\frac{y^4}{2\pi^2}\left(J_B[\eta_h] + 3J_B[\eta_\chi] + 4J_B[\eta_{W_T}] + 2J_B[\eta_{Z_T}] + 2J_B[\eta_{W_L}] + J_B[\eta_{Z_L}] + J_B[\eta_{A_L}] - 12J_F[\eta_t]\right)$$

- The potential leads to a set of coupled EoM for the two fields, and we solve them numerically, by first solving for Higgs.

# Thermal potential : Basics

- Thermal potentials can be understood from the phase space distributions.
- Consider a field $\psi$ with mass $m_\psi$ in thermal bath, then it's free energy density($\mu = 0$) gives the thermodynamic effective potential ( - : bosons, + : fermion)

$$V_{th}(\chi) = \mathcal{F} = -P$$

$$V_{th}(\chi) = \frac{(-1)^n g}{6\pi^2} T^4 \int_0^\infty dx \frac{x^4}{\sqrt{x^2 + m_\psi^2(\chi)/T^2}} \{exp[(\sqrt{x^2 + m_\psi^2(\chi)/T^2}] \pm 1\}^{-1}$$

$$= \frac{(-1)^n g}{2\pi^2} T^4 \int_0^\infty dx\, x^2 \log[1 \pm e^{-\sqrt{x^2 + m_\psi^2(\chi)/T^2}}]$$

$x = p/T$

- Where the Phase space and pressure is given as :

$$f(p) = \{exp[(\sqrt{p^2 + m_\psi^2(\chi)} - \mu)/T] \pm 1\}^{-1}$$

$$P = \frac{g_\psi}{2\pi^2} \int_0^\infty dp \frac{p^4}{3E(p)} f(p)$$

# Finite temperature J functions

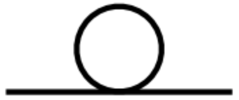
- At high temperature, one can expand them as :

$$J_B(y^2) \approx J_B^{\text{high}-T}(y^2) = -\frac{\pi^4}{45} + \frac{\pi^2}{12}y^2 - \frac{\pi}{6}y^3 - \frac{1}{32}y^4 \log\left(\frac{y^2}{a_b}\right)$$

$$J_F(y^2) \approx J_F^{\text{high}-T}(y^2) = \frac{7\pi^4}{360} - \frac{\pi^2}{24}y^2 - \frac{1}{32}y^4 \log\left(\frac{y^2}{a_f}\right) \qquad \text{for } |y^2| \ll 1$$

- At low temperature, they are Boltzmann suppressed, thus the analysis reverts to the Tree level potential.

# Hard Thermal loops basics



(a)

(b)

$$V = \frac{-\mu^2\phi^2}{2} + \frac{\lambda\phi^4}{4}$$

1-loop mass correction

$\lambda T^2$

higher-loop daisy correction

$\frac{\lambda^n T^{2n-1}}{\mu^{2n-3}}$

- Large ratios of T /μ have to be resumed ($\mu^2 \sim \lambda T^2$), which can be done by replacing the tree mass by

$$m^2(\phi) = m^2_{\text{tree}}(\phi) + \Pi(\phi, T),$$

- For scalars, Π gives the leading contribution in T to the one-loop thermal mass, and is obtained by differentiating $V_{th}$ with respect to field:

$$\Pi \sim \lambda T^2 + \dots$$

- This includes the hard thermal loops and daisy contributions to all orders.

# Potential including thermal effects

- Thus, by resuming the thermal mass in the arguments of the thermal potential, ("Truncated Full Dressing"), we get:

$$\hat{V} = -\frac{1}{2}\hat{h}^2(1-\beta\hat{\phi}) + \frac{1}{4}\lambda\hat{h}^4 + \frac{1}{2}\kappa^2\hat{\phi}^2 + \frac{y^4}{2\pi^2}\left(J_B[\eta_h] + 3J_B[\eta_\chi] + 4J_B[\eta_{W_T}] + 2J_B[\eta_{Z_T}] + 2J_B[\eta_{W_L}] + J_B[\eta_{Z_L}] + J_B[\eta_{A_L}] - 12J_F[\eta_t]\right)$$

- For Higgs and the Goldstones, the correction is given by

$$\eta_h = \frac{1}{y^2}\left(3\lambda\hat{h}^2 - (1-\beta\hat{\phi}) + \frac{y^2}{4}\left(2\lambda + y_t^2 + \frac{3}{4}g^2 + \frac{1}{4}g'^2\right)\right)$$
$$\eta_\chi = \frac{1}{y^2}\left(\lambda\hat{h}^2 - (1-\beta\hat{\phi}) + \frac{y^2}{4}\left(2\lambda + y_t^2 + \frac{3}{4}g^2 + \frac{1}{4}g'^2\right)\right),$$

$$y = \frac{T}{\mu}, \quad \hat{\phi} = \frac{\phi}{M_{\rm pl}}, \quad \hat{h} = \frac{h}{\mu}, \quad \kappa = \frac{m_\phi M_{\rm pl}}{\mu^2}, \quad \beta = \frac{AM_{pl}}{\mu^2}$$

- For Longitudinal vector boson modes, it is given as(gauge basis):

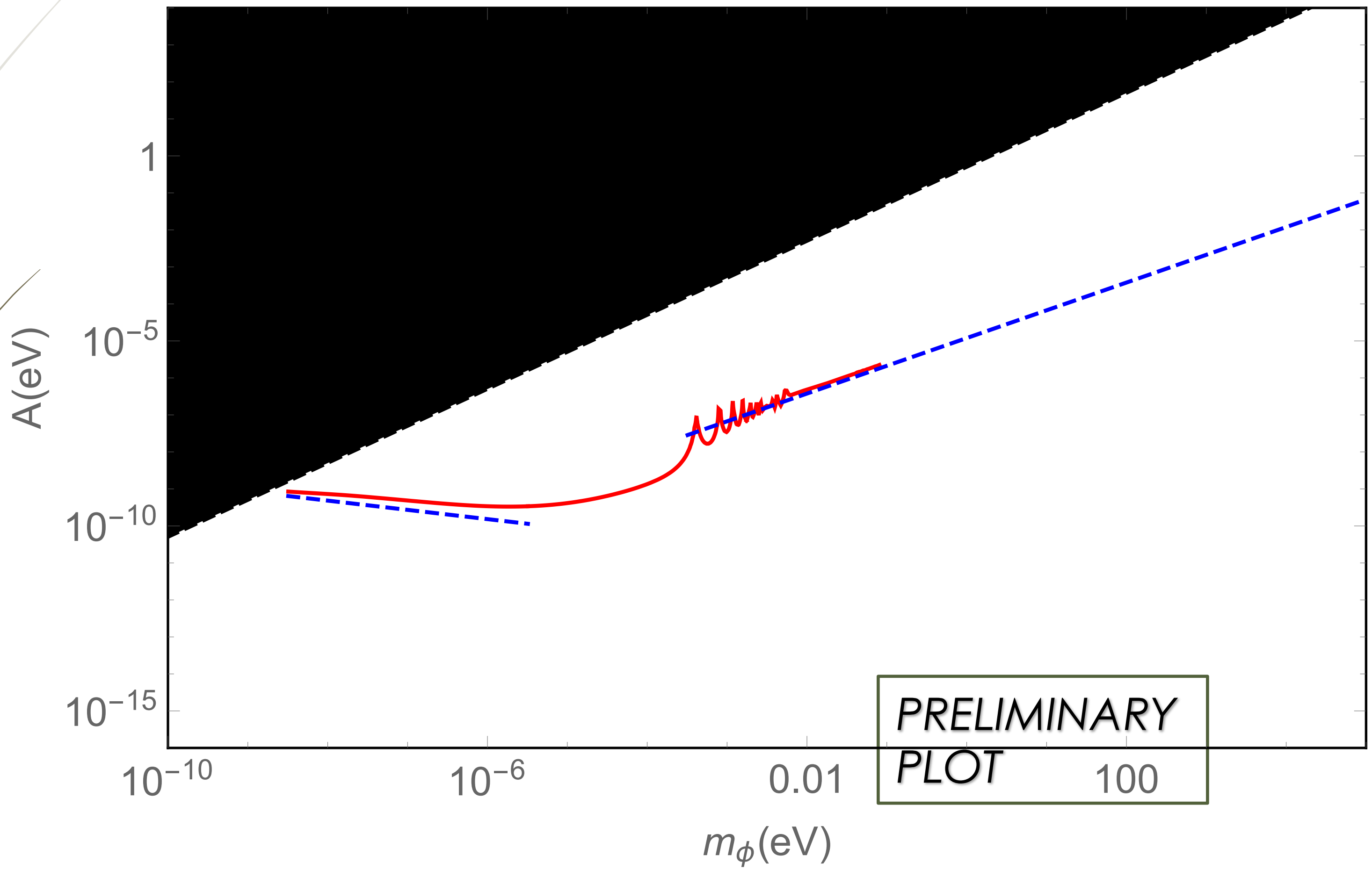
$$\Pi^L_{GB}(0) = \frac{11}{6}T^2\,\mathrm{diag}(g^2, g^2, g^2, g'^2)$$

- Contributions to Fermions(no zero modes, thus no IR divergence in propagators) and transverse vector boson modes(gauge symmetry) are suppressed.

# Initial Conditions

- The amplitude of the oscillations are controlled by two different sources (misalignment at end of inflation, $\phi_i$ and Thermal Misalignment, $\phi_T$)
- Total misalignment, and the energy density is dictated by $\phi_T$+ $\phi_i$- $\phi_{min}$.
- For a long enough period of inflation and a low enough Hubble, $H_I < \mathrm{v}$, the effective temperature experienced by the scalar field is T ~ $H_I$.
- Since $H_I \ll \mathrm{v}$, the Higgs is close to its vev and the true minima of $\phi$ is approximately at $\phi_0$.
- The long period helps the field to reach it's true minima.

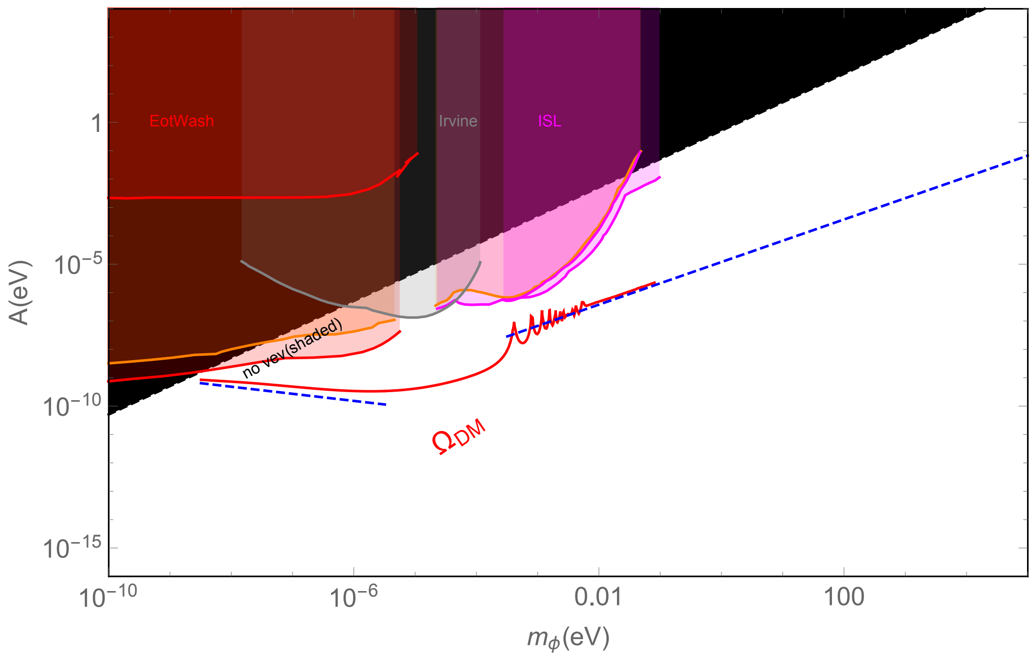
# Overall comparison with numerics

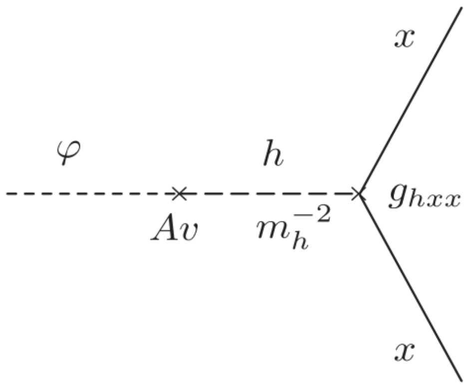



Blue Dashed: Approx Reg I and II
Black : no vev
Red : DM numerical

# Phenomenological constraints

# Fifth force experiments Constraints




- In the limit of a very long-range force of range ~ $m_\phi^{-1}$, bounds are derived from post-Newtonian tests of relativity.
- The universal coupling turns out to be :

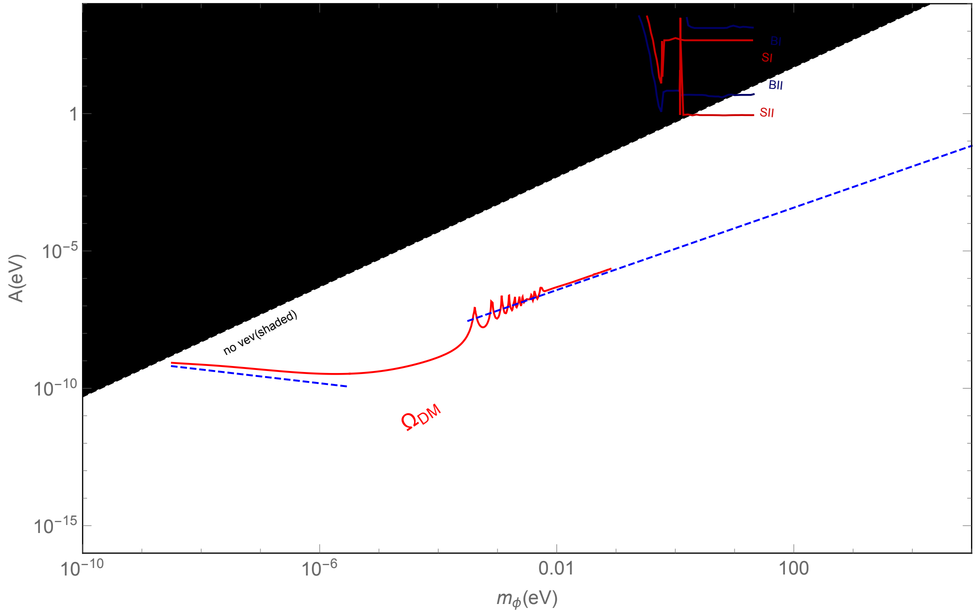
$$V(r) = -\frac{Gm^2}{r}(1 + \alpha^2 e^{-m_\phi r})$$

$$\alpha = g_{hNN} \frac{\sqrt{2} M_P}{m_{\rm nuc}} \frac{Av}{m_h^2} \simeq 10^{-3} \left(\frac{m_h}{115\,\rm GeV}\right)^{-2} \frac{A}{10^{-8}\rm eV}.$$

$$A = \frac{\beta \mu^2}{M_{pl}}$$

[1]Pospelov et al, Phys.Rev.D82:043533,2010

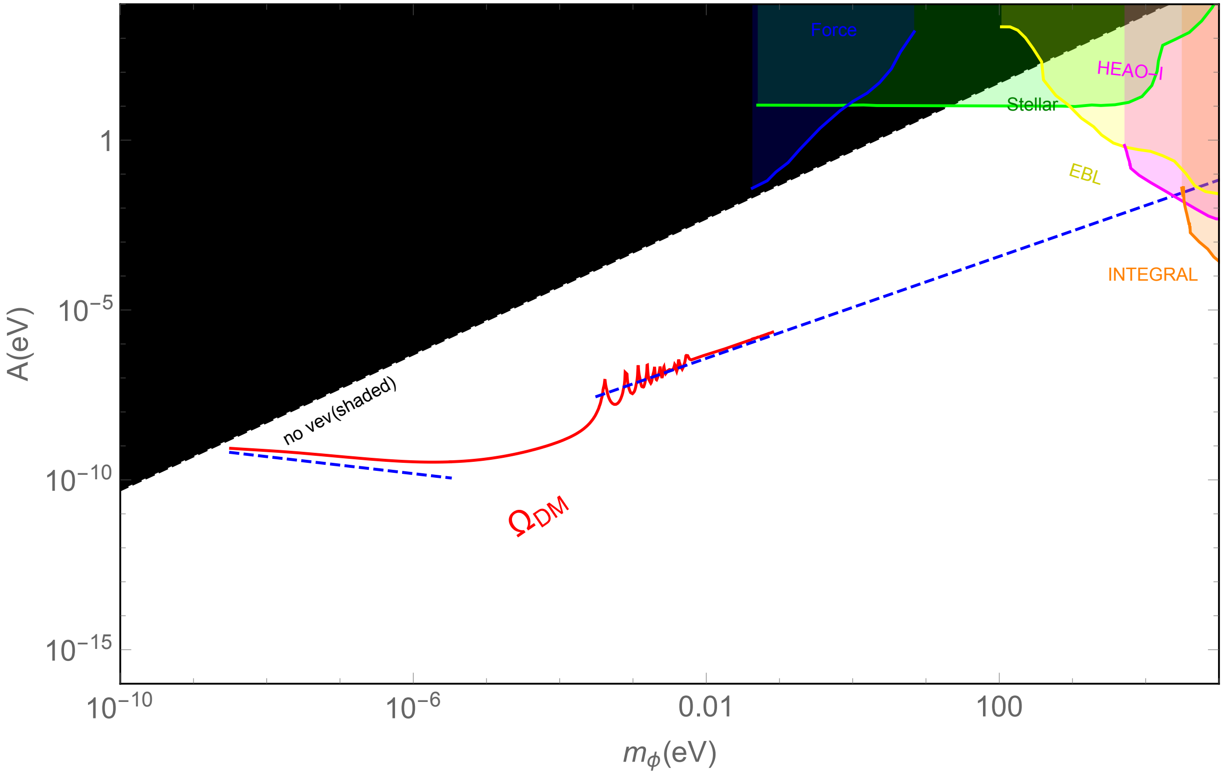
# Resonant absorption in gas chamber




- Bosonic dark matter (DM) detectors based on resonant absorption onto a gas of small polyatomic molecules.
- The excited molecules emit the absorbed energy into fluorescence photons that are picked up by sensitive photodetectors with low dark count rates.
- DM masses between 0.2 eV and 20 eV are targeted, with Bulk and Stack configurations being focused on.

Arvanitaki et al [https://arxiv.org/pdf/1709.05354.pdf]

# Stellar Cooling bounds




- Stellar cooling constraints relies upon the draining and cooldown of stars due to production of ultralight particles (like $\phi$) in stars.
- We consider the bounds coming from red giants (RG) and horizontal branch (HB) stars cooling.

Edward Hardy et al[arXiv:1611.05852 [hep-ph]]

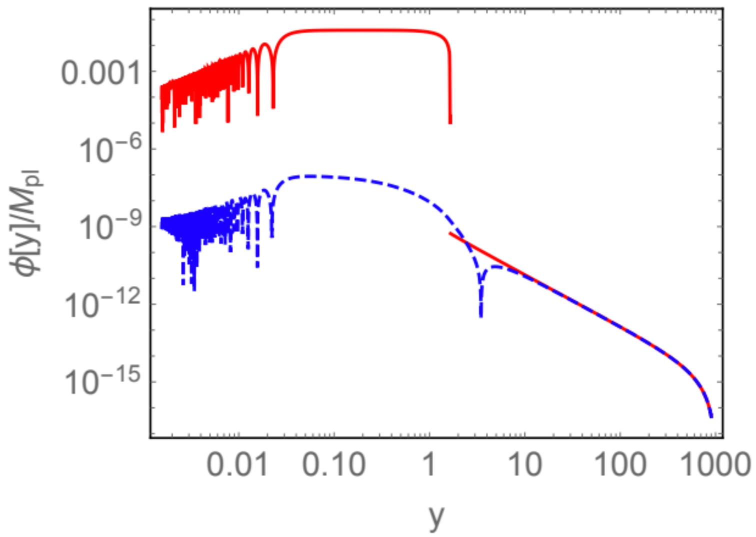
# 2 body photon decay

- Extragalactic bounds
  - Photons emitted from very late decays that do not lie in ultraviolet range, can be observed today as a distortion of the diffuse extragalactic background light (EBL).
  - Together these bounds cover the wavelength range between 0.1 and 1000 μm, that is roughly the mass range between 0.1 eV and 1 keV.
- Two body photon decays($\phi \rightarrow \gamma\gamma$)
  - HEAO-1 : Data is from observations of 3-50 keV photons made with the A2 High-Energy Detector on HEAO-1 . Other datasets from the experiment are significantly weaker than those from the INTEGRAL experiment.
  - INTEGRAL : Data is from observations of 20 keV to 2 MeV photons.

Thomas Flacke et al(https://arxiv.org/pdf/1610.02025.pdf)
Rouven Essig et al(https://arxiv.org/pdf/1309.4091.pdf)

# Initial Conditions: Comparison

We will compare the case where initially if the field starts from 0, which can be the case of high scale inflation. We observe that the High T behavior is same in both cases.




$\phi$ evolution(Higgs vev subtracted) comparison for different choices of initial conditions, with benchmark points being : $(A, m_\phi) = (3.21 \times 10^{-12}, 0.032 \times 10^{-8})\, eV$. Gray curve : $\phi_i = \phi_0$, Red Curve $\phi_i = 0$: Dashed curves corresponds to respective initial conditions.

# Conclusions

- Particularly, we have focused on the phenomenology of a realistic scenario where the DM couples to the Higgs and the SM.
- This is one of the most minimal setup which is also experimentally viable.
- A variety of opportunities for probing this scenario in the future exist.
- Our DM density line can be interpreted as a constraints coming from cosmology(thermal effects),
- The High T thermal effects are fairly insensitive to the initial conditions, and thus for heavier mass ranges, it would be hard to escape these bounds modulo fine tuned initial conditions.