Unified *ab initio* treatment of ΔS=0 parity violation in low-energy nuclear processes

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based on work with Prof. Susan Gardner

1)S. Gardner, **GM**, QCD analysis of S = 0 hadronic parity violation, Phys. Lett. B 833 (2022) 137372. arXiv:2203.00033v2, doi:10.1016/j.physletb.2022.137372.

2)Towards a unified treatment of ΔS=0 parity violation in low-energy nuclear processes, arXiv:2210.03567

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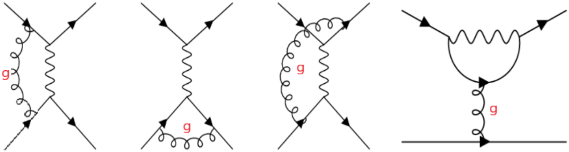
PV In Hadronic Processes

- Since its first experimental realization[1], research efforts have tried to capture and quantify parity violation (PV), including in hadronic processes.
- Hadronic PV is complex since weak-interactions between quarks, receive significant corrections due to QCD gluon loops. For example, at leading order in α_s,



[1] C.S.Wu et al., 1957

Theoretical Developments

- At low energies, modeled using *meson* exchange *nucleon-nucleon* interactions[1], DDH introduced strong interaction modifications of the weak interactions via phenomenological factors.

- In later works[2], RG methods were used to analyse hadronic PV in the isovector sector.

- More recently[3], and in the light of new experimental results, it has been argued that better theoretical estimations and comparison of different isosectors are required.

- This calls for a concrete QCD analysis and estimates comparing the weak meson-nucleon couplings for all three isosectors.

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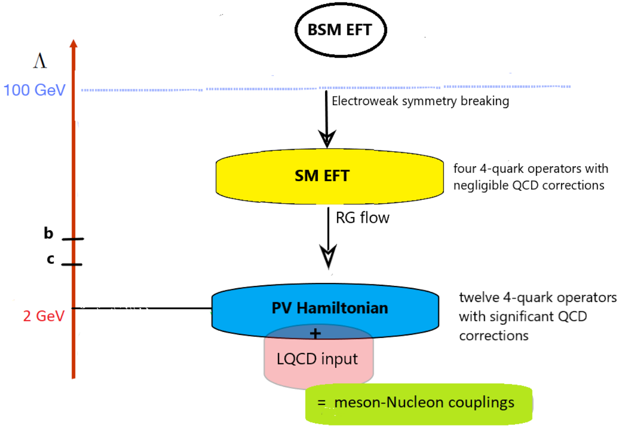
[1] Desplanques, Donoghue, and Holstein (DDH), 1980

[2] Dai et al., 1991, Tiburzi, 2012

[3] Gardner et al., 2017, Schindler et al., 2016

QCD Evolution Across Energy Scales: Our Work



PV Effective Hamiltonian at M_W

Summing all $\Delta S = 0$ tree level amplitudes, (we keep the $W^{\pm}$ and Z^0 contributions separate for clarity):

$$\mathscr{H}^{PV} \equiv \mathscr{H} = \mathscr{H}_Z + \mathscr{H}_W$$

$$\mathscr{H}_Z(M_W) = \frac{G_F s_w^2}{3\sqrt{2}} \left(\Theta_1 - 3(\frac{1}{2s_w^2} - 1)\Theta_5 \right)$$

$$\Theta_1 = [(\bar{u}u)_V + (\bar{d}d)_V + (\bar{s}s)_V]^{\alpha\alpha}[(\bar{u}u)_A - (\bar{d}d)_A - (\bar{s}s)_A]^{\beta\beta}$$

$$\Theta_5 = [(\bar{u}u)_V - (\bar{d}d)_V - (\bar{s}s)_V]^{\alpha\alpha}[(\bar{u}u)_A - (\bar{d}d)_A - (\bar{s}s)_A]^{\beta\beta}$$

$$\mathscr{H}_W(M_W) = -\frac{G_F}{\sqrt{2}} \left(cos^2(\theta_c)\Theta_9 + sin^2(\theta_c)\Theta_{11} \right)$$

$$\Theta_9 = (\bar{u}d)_V^{\alpha\alpha}(\bar{d}u)_A^{\beta\beta} + (\bar{d}u)_V^{\alpha\alpha}(\bar{u}d)_A^{\beta\beta}$$

$$\Theta_{11} = (\bar{u}s)_V^{\alpha\alpha}(\bar{s}u)_A^{\beta\beta} + (\bar{s}u)_V^{\alpha\alpha}(\bar{u}s)_A^{\beta\beta}$$

Operator group for $Z^0 - channel$

Under QCD corrections, operators rescale and mix with other operators. For example, Θ_1,

$$\Theta_1 \rightarrow \Theta_1 + \frac{g^2\Gamma(2-\frac{d}{2})}{(4\pi)^2(\mu^2)^{2-\frac{d}{2}}}\left(\frac{2}{9}\Theta_1 - \frac{2}{3}\Theta_2 + 1\Theta_3 - 3\Theta_4\right)$$

Continuing the analysis for every operator (existing and newly generated) we get the following operator set that is closed under mixing,

$$\Theta_1 = [(\bar{u}u)_V + (\bar{d}d)_V + (\bar{s}s)_V]^{\alpha\alpha}[(\bar{u}u)_A - (\bar{d}d)_A - (\bar{s}s)_A]^{\beta\beta}$$
$$\Theta_2 = [(\bar{u}u)_V + (\bar{d}d)_V + (\bar{s}s)_V]^{\alpha\beta}[(\bar{u}u)_A - (\bar{d}d)_A - (\bar{s}s)_A]^{\beta\alpha}$$
$$\Theta_3 = [(\bar{u}u)_A + (\bar{d}d)_A + (\bar{s}s)_A]^{\alpha\alpha}[(\bar{u}u)_V - (\bar{d}d)_V - (\bar{s}s)_V]^{\beta\beta}$$
$$\Theta_4 = [(\bar{u}u)_A + (\bar{d}d)_A + (\bar{s}s)_A]^{\alpha\beta}[(\bar{u}u)_V - (\bar{d}d)_V - (\bar{s}s)_V]^{\beta\alpha}$$
$$\Theta_5 = [(\bar{u}u)_V - (\bar{d}d)_V - (\bar{s}s)_V]^{\alpha\alpha}[(\bar{u}u)_A - (\bar{d}d)_A - (\bar{s}s)_A]^{\beta\beta}$$
$$\Theta_6 = [(\bar{u}u)_V - (\bar{d}d)_V - (\bar{s}s)_V]^{\alpha\beta}[(\bar{u}u)_A - (\bar{d}d)_A - (\bar{s}s)_A]^{\beta\alpha}$$
$$\Theta_7 = [(\bar{u}u)_A + (\bar{d}d)_A + (\bar{s}s)_A]^{\alpha\alpha}[(\bar{u}u)_V + (\bar{d}d)_V + (\bar{s}s)_V]^{\beta\beta}$$
$$\Theta_8 = [(\bar{u}u)_A + (\bar{d}d)_A + (\bar{s}s)_A]^{\alpha\beta}[(\bar{u}u)_V + (\bar{d}d)_V + (\bar{s}s)_V]^{\beta\alpha}$$

Anomalous Dimension Matrix γ_Z

The final γ matrix for the Z-sector that takes the open quark flavors into account is

$$\gamma_Z(\mu) = -\frac{g_s}{8\pi^2}\times$$

$$\begin{pmatrix}
\frac{2}{9} & \frac{-2}{3} & 1 & -3 & 0 & 0 & 0 & 0 \\
-\frac{3}{2}+\frac{2}{9}n_f & \frac{9}{2}-\frac{2}{3}n_f & \frac{-3}{2} & \frac{-7}{2} & 0 & 0 & 0 & 0 \\
\frac{11}{9} & \frac{-11}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{-3}{2} & \frac{-7}{2} & \frac{-3}{2} & \frac{9}{2} & 0 & 0 & -\frac{2}{9}n_Q & \frac{2}{3}n_Q \\
0 & 0 & 0 & 0 & 1 & -3 & \frac{2}{9} & -\frac{2}{3} \\
-\frac{2}{9}n_Q & \frac{2}{3}n_Q & 0 & 0 & -3 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{11}{9} & -\frac{11}{3} \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{2n_f}{9}-3 & 1-\frac{2n_f}{3}
\end{pmatrix}$$

Here n_f =number of dynamical quarks at the considered energy scale.
$n_Q = \#d - \#u$.

Operator group for $W^{\pm}-channel$

The operators from the W-channel form a separate closed group,

$$\boxed{\begin{aligned}
\Theta_9 &= (\bar{u}d)_V^{\alpha\alpha}(\bar{d}u)_A^{\beta\beta} + (\bar{d}u)_V^{\alpha\alpha}(\bar{u}d)_A^{\beta\beta}\\
\Theta_{10} &= (\bar{u}d)_V^{\alpha\beta}(\bar{d}u)_A^{\beta\alpha} + (\bar{d}u)_V^{\alpha\beta}(\bar{u}d)_A^{\beta\alpha}\\
\Theta_{11} &= (\bar{u}s)_V^{\alpha\alpha}(\bar{s}u)_A^{\beta\beta} + (\bar{s}u)_V^{\alpha\alpha}(\bar{u}s)_A^{\beta\beta}\\
\Theta_{12} &= (\bar{u}s)_V^{\alpha\beta}(\bar{s}u)_A^{\beta\alpha} + (\bar{s}u)_V^{\alpha\beta}(\bar{u}s)_A^{\beta\alpha}
\end{aligned}}$$

$$\gamma_W(\mu) = -\frac{g_s}{8\pi^2}\begin{pmatrix} 1 & -3 & 0 & 0\\ -3 & 1 & 0 & 0\\ 0 & 0 & 1 & -3\\ 0 & 0 & -3 & 1 \end{pmatrix}$$

Using the γ matrices, we perform the RG flow:

$$\vec{C}(\mu) = \exp\left[\int_{g_s(M_w)}^{g_s(\mu)} dg\,\frac{\gamma^T(\mu)}{\beta(g_s)}\right]\vec{C}(M_w)$$

Hadronic Wilson Coefficients

$$\vec{C}(M_W) = (1,0,0,0,-3.49,0,0,0,-13.0\cos^2\theta_c,0,-13.0\sin^2\theta_c,0),$$

$$\vec{C}(2\,\mathrm{GeV}) = \begin{pmatrix} 1.09 & [1.17\ldots 1.06][1.08\ldots 1.04] & [1.07][1.06] \\ 0.018 & [0.014\ldots 0.021][0.033\ldots 0.006] & [-0.006][-0.006] \\ 0.199 & [0.321\ldots 0.133][0.193\ldots 0.127] & [0.158][0.153] \\ -0.583 & [-0.990\cdots -0.385][-0.571\cdots -0.374] & [-0.460][-0.456] \\ -4.36 & [-4.99\cdots -4.05][-4.34\cdots -4.03] & [-4.16][-4.14] \\ 1.72 & [2.63\ldots 1.19][1.67\ldots 1.16] & [1.40][1.36] \\ -0.170 & [-0.288\cdots -0.110][-0.165\cdots -0.105] & [-0.134][-0.129] \\ 0.332 & [0.496\ldots 0.235][0.322\ldots 0.225] & [0.275][0.268] \\ -16.2 & [-18.6\cdots -15.0][-16.1\cdots -15.0] & [-15.48][-15.4] \\ 6.38 & [9.76\ldots 4.44][6.22\ldots 4.30] & [5.19][5.05] \\ -16.2 & [-18.6\cdots -15.0][-16.1\cdots -15.0] & [-15.48][-15.4] \\ 6.38 & [9.76\ldots 4.44][6.22\ldots 4.30] & [5.19][5.05] \end{pmatrix}$$

where the last four entries should be multiplied by factors of $\cos^2\theta_c, \cos^2\theta_c, \sin^2\theta_c$, and $\sin^2\theta_c$, respectively. The primary result is given by the leftmost column of numbers. The other columns illustrate the uncertainties in the computation. In the central column, the left set shows the ranges of WC that result in the $N_f = 2+1$ theory for renormalization scales of $\mu = 1-4\,\mathrm{GeV}$ and the right set shows them in the $N_f = 2+1+1$ theory with $\mu = 2-4\,\mathrm{GeV}$. The rightmost column gives WC if the α_s running and matching is computed at NLO (left) and NNLO (right).

Iso-sector Separation

In terms of Hamiltonian :

$$\mathscr{H}_{eff}(\mu) = \frac{G_F s_w^2}{3\sqrt{2}} \sum_i C_i(\mu)\Theta_i \longrightarrow \mathscr{H}_{eff}^{I=1} = \frac{G_F s_w^2}{3\sqrt{2}} \sum_i C_i^{I=1}\Theta_i^{I=1}$$

$$\mathscr{H}_{eff}(\mu) = \frac{G_F s_w^2}{3\sqrt{2}} \sum_i C_i(\mu)\Theta_i \longrightarrow \mathscr{H}_{eff}^{I=0\oplus 2} = \frac{G_F s_w^2}{3\sqrt{2}} \sum_i C_i^{I=0\oplus 2}\Theta_i^{I=0\oplus 2}$$

$$C^{I=1}(2\,\mathrm{GeV}) = \begin{pmatrix} 1.091 \\ 0.018 \\ 0.199 \\ -0.583 \\ 4.363 \\ -1.715 \\ 4.363 \\ -1.715 \\ -16.221\sin^2\theta_c \\ 6.377\sin^2\theta_c \end{pmatrix} \qquad \mathrm{C}^{\mathrm{I}=0\oplus 2}(2\,\mathrm{GeV}) = \begin{pmatrix} -1.091 \\ -0.018 \\ -0.199 \\ 0.583 \\ -4.363 \\ 1.715 \\ -0.170 \\ 0.332 \\ -16.221\cos^2\theta_c \\ 6.377\cos^2\theta_c \end{pmatrix}$$

The uncertainties in these WCs (not shown here) are also obtained alongside the extraction.

Meson-Nucleon couplings: h_M^I

- The DDH's meson-exchange phenomenological HPV Hamiltonian is dictated by couplings h_M^I for meson M and isosector I:
  $\mathrm{h}_\pi^1, \quad \mathrm{h}_{\rho^0}^1, \quad \mathrm{h}_\rho^0, \quad \mathrm{h}_\rho^2, \quad \mathrm{h}_\omega^0 \quad \text{and} \quad \mathrm{h}_\omega^1$
- To obtain them from our RG Hamiltonian, we make the following matching from the quark to hadron level: $\langle MN'|\mathscr{H}_{\mathrm{eff}}^I|N\rangle = \langle MN'|\mathscr{H}_{\mathrm{DDH}}|N\rangle$
- For example, the pion contribution to hadronic PV:

$$\mathscr{H}_{\mathrm{DDH}}^{\pi} = ih_\pi^1(\pi^+\bar{p}n - \pi^-\bar{n}p) \quad \Longrightarrow \; -ih_\pi^1\bar{u}_n u_p = \left\langle n\pi^+ \right| \mathscr{H}_{\mathrm{eff}}^{I=1} \left| p \right\rangle$$

  u_N is a Dirac spinor.
- Next, we make use of factorization approximation to evaluate these matrix elements. If we consider vector meson (V) emission, the factorization approximation for long-distance hadronic interaction matrix elements in terms of four-quark operators separate as

$$\left\langle VN' \right| (\bar{q}_1 q_2)_v \bar{q}_3 q_4)_A \left| N \right\rangle = \langle V| (\bar{q}_1 q_2)_v |0\rangle \langle N'| (\bar{q}_3 q_4)_A |N\rangle$$

h_π^1

As a pseudoscalar meson

$$\langle \pi^+ n | \mathscr{H}_{I=1} | p \rangle = -i h_\pi^1 \bar{u}_n u_p =$$
$$= \frac{G_F s_w^2}{3\sqrt{2}} \langle \pi^+ | (\bar{u}\gamma_5 d) | 0 \rangle \left(\frac{2c_1^{I=1}}{3} + 2c_2^{I=1} - \frac{2c_3^{I=1}}{3} + 2c_4^{I=1} \right) \langle n | \bar{d} u | p \rangle$$

With f_π the charged pion decay constant

$$\langle \pi^+ | (\bar{u}\gamma_5 d) | 0 \rangle = \frac{m_\pi^2 f_\pi}{i(m_u + m_d)}$$

$$m_\pi = 135\,\mathrm{MeV}\,;\, f_\pi = 130\,;\, (m_u + m_d)[\mathrm{RGI}] = 2(4.736(60)_m (1.5)_\Lambda)\,\mathrm{MeV}$$

and isovector quark scalar charge of the nucleon[1]

$$\langle n | \bar{d} u | p \rangle = g_s^{u-d} \bar{u}_n u_p; \quad g_s^{u-d} = 1.06(10)(06)_{sys}$$

$$h_\pi^1 = (3.06 \pm 0.34 + \left({}^{+1.29}_{-0.64} \right) + 0.42 + (1.00)) \times 10^{-7} (\mathrm{npdGamma}^2 : 2.6(1.2)(0.2) \times 10^{-7}$$

[1] m_{ud}: FLAG Review 2021, 2111.09849; g_s: $(N_f = 2+1)$ [S. Parke et al., 2103.05599]

[2] D. Blyth, et al., Phys. Rev. Lett. 121 (24) (2018) 242002.

h^0_ω, h^1_ω, h^1_ρ, h^0_ρ and h^2_ρ

Rest of the couplings defined in DDH:

$$h^1_\omega = +1.825 \pm 0.111 + \begin{pmatrix} -0.047 \\ 0.125 \end{pmatrix} - 0.040 + (-0.020)) \times 10^{-7} \ ;$$

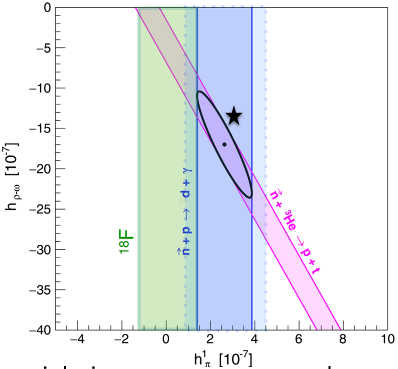
$$h^0_\omega = +0.270 \pm 0.015 + \begin{pmatrix} -0.32 \\ 0.55 \end{pmatrix} - 0.202 + (1.148)) \times 10^{-7}$$

$$h^1_\rho = -0.294 \pm 0.045 + \begin{pmatrix} 0.014 \\ -0.036 \end{pmatrix} + 0.009 + (0.026)) \times 10^{-7}$$

$$h^0_\rho = -11.05 \pm 0.672 + \begin{pmatrix} 1.079 \\ -2.051 \end{pmatrix} + 0.673 + (-4.039)) \times 10^{-7};$$

$$h^2_\rho = +8.57 \pm 0.519 + \begin{pmatrix} 1.129 \\ -1.736 \end{pmatrix} + 0.802 + (-3.749)) \times 10^{-7}$$

Implications



Constraints on the parity-violating vector-meson-nucleon coupling constants:

n3He: $h_{\rho-\omega} \equiv h^0_\rho + 0.605 h^0_\omega - 0.605 h^1_\rho - 1.316 h^1_\omega + 0.026 h^2_\rho = (-17.0 \pm 6.56) \times 10^{-7}$

LOQCD+LQCD: $h_{\rho-\omega} = -12.9 \pm 0.52 + \binom{0.97}{-1.9} + 0.62 + (-3.4)) \times 10^{-7}$;

Along with the pion coupling, these predictions are within $\pm 1\sigma$ of experiment.

Perspective

The analysis of ^{18}F radiative decay[3] from its excited state yields the bound $| h^1_\pi | < 1.3 \times 10^{-7}$. This is the the only very precise determination in a complex system that challenges the few-body estimations of pion-coupling, as shown in the figure.

As an example of a complex system the above tension may be reflective of an extraction in a different physical setting.

The couplings are not direct physical observables and thus can be sensitive to the energy scale of the system under consideration, intrinsically depending on the physical momentum scale of the studies in which they are extracted.

[3] W. Haxton et al., Prog. Part. Nucl. Phys. 71 (2013) 185–203

Summary

- We have presented a LO QCD + LQCD analysis of the parity-violating meson-nucleon coupling constants.
- Our results compare favorably to the couplings determined in the NPDGamma and n3He experiments

$$\text{LO QCD+LQCD:}\, h_{\rho-\omega} = (-12.9 \pm \ldots \times 10^{-7}\, [h_{\rho-\omega} = (-17.0 \pm 6.56) \times 10^{-7}]$$
$$\text{LO QCD+LQCD:}\, h_{\pi}^{1} = (3.03 \pm \ldots) \times 10^{-7} \quad [h_{\pi}^{1} = (2.6 \pm 1.2) \times 10^{-7}]$$

  suggesting that nonfactorizable corrections are small.
- Our study suggests that extraction of h_{π}^{1} could vary with the cutoff scale of the physical description. We hope that further studies of hadronic parity violation in complex systems could be made and be of sufficient precision to reveal this effect in other isosectors as well.
- A study of NLO effects in the current study could also help in cementing these observations. This is underway.

Thank You!

Isospin Structure of (u,d) 4-quark Operators

Upto a normalization, we get,

$$|0,0\rangle = |1,-1\rangle \otimes |1,1\rangle + |1,1\rangle \otimes |1,-1\rangle - |1,0\rangle \otimes |1,0\rangle$$

$$\boxed{\Theta_{I=0} = (\bar{u}d)_V(-\bar{d}u)_A + (-\bar{d}u)_V(\bar{u}d)_A - \left(\frac{(\bar{u}u - \bar{d}d)_V}{\sqrt{2}}\frac{(\bar{u}u - \bar{d}d)_A}{\sqrt{2}}\right)}$$

$$|1,0\rangle = |0,0\rangle \otimes |1,0\rangle$$

$$\boxed{\Theta_{I=1} = \left(\frac{(\bar{u}u + \bar{d}d)_V}{\sqrt{2}}\frac{(\bar{u}u - \bar{d}d)_A}{\sqrt{2}}\right)}$$

$$|2,0\rangle = |1,-1\rangle \otimes |1,1\rangle + |1,1\rangle \otimes |1,-1\rangle + 2|1,0\rangle \otimes |1,0\rangle$$

$$\boxed{\Theta_{I=2} = (\bar{u}d)_V(-\bar{d}u)_A + (-\bar{d}u)_V(\bar{u}d)_A + 2\left(\frac{(\bar{u}u - \bar{d}d)_V}{\sqrt{2}}\frac{(\bar{u}u - \bar{d}d)_A}{\sqrt{2}}\right)}$$