

EFT interpretation of $b \rightarrow c \tau \nu$ and connections with high- p_T tails

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Based on:

J. Aebischer, G. Isidori, M. Pesut, B.A. Stefanek, FW [2210.13422] L. Allwicher, D.A. Faroughy, F. Jaffredo, O. Sumensari, FW [2207.10714] [2207.10756]

LHC EFT WG 6 Meeting – CERN

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Universität Zürich^{uz}

- Complementarity of:
 - Low-energy data (precision frontier)
 - High- p_T data (energy frontier)
- EFT analysis for $b \rightarrow c \tau \nu$ transition at low energies
- Construction of full flavor likelihood for NP in Drell-Yan
 - Implemented in Mathematica code: HighPT
 - Constraints on SMEFT and leptoquark models
- High- p_T constraints on $b \rightarrow c \tau \nu$ transition



https://highpt.github.io/

The flavor pattern of NP



- Model independent NP analysis using EFTs \rightarrow in particular the SMEFT
- The SMEFT has a very rich flavor structure
 - d = 6: 59 electroweak structures \leftrightarrow 2499 parameters
 - How to constrain all these parameters?
- Focus only on subset of operators

The flavor pattern of NP

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 - How to constrain all these parameters?
- Focus only on subset of operators
- Hints for NP: indication of LFUV in semileptonic B decays



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Probing semileptonic operators at different scales:







Probing semileptonic operators at different scales:







Flavor in Drell-Yan



- \rightarrow see also the talks by Aleks and Arne
- Hadronic cross-section:

$$\sigma_{\text{had}}(pp \to \ell_{\alpha} \ell_{\beta}) = L_{ij} \otimes [\hat{\sigma}]_{ij}^{\alpha\beta}$$

- L_{ij} parton luminosities / PDFs \rightarrow all quark flavors contribute (except for top)

$$\mathcal{L}_{ij}(\hat{s}) = \int_{\frac{\hat{s}}{s}}^{1} \frac{\mathrm{d}x}{x} \left[f_{\bar{q}_i}(x,\mu) f_{q_j}\left(\frac{\hat{s}}{sx},\mu\right) + (\bar{q}_i \leftrightarrow q_j) \right]$$

- $\left[\hat{\sigma}\right]_{ij}^{\alpha\beta}$ partonic cross section \rightarrow energy enhanced in EFT $\left[\hat{\sigma}\right]_{ij}^{\alpha\beta} \propto \frac{\hat{s}}{\Lambda^4} \left|C\right|^2$
- τ -tails particularly relevant for models with large 3rd generation couplings Faroughy, Greljo, Kamenik [1609.07138]



Low-energy constraints on $b \rightarrow c \, \tau \, \nu$ transitions

An EFT analysis under the U_1 hypothesis

EFT for the U_1 leptoquark

• Working hypothesis: vector leptoquark field $U_1 \sim (3,1)_{2/3}$ with current:

$$J_U^{\mu} = \frac{g_U}{\sqrt{2}} \left[\overline{q}_L^3 \gamma^{\mu} \ell_L^3 + \beta_R \overline{d}_R^3 \gamma^{\mu} e_R^3 + \sum_{k=1,2} \epsilon_{q_k} \overline{q}_L^k \gamma^{\mu} \ell_L^3 \right]$$

- Coupled only to 3rd generation leptons
- Variable coupling β_R to right-handed fields
- Suppressed coupling ϵ_{q_k} to light quarks
- Corresponding EFT Lagrangian:

$$\mathscr{L}_{\rm EFT}^{\rm LQ} = \frac{2}{v^2} \Big[C_{LL}^{ij\alpha\beta} O_{LL}^{ij\alpha\beta} + C_{RR}^{ij\alpha\beta} O_{RR}^{ij\alpha\beta} + \left(C_{LR}^{ij\alpha\beta} O_{LR}^{ij\alpha\beta} + {\rm h.c.} \right) \Big]$$

• Introduce effective scale $\Lambda_U = \sqrt{2}M_U/g_U \Rightarrow C_{LL}^{33\tau\tau} = \frac{v^2}{2\Lambda_U^2}$



 $O_{LL}^{ij\alpha\beta} = (\overline{q}_L^i \gamma_\mu \mathcal{C}_L^\alpha) (\overline{\mathcal{C}}_L^\beta \gamma^\mu q_L^j)$ $O_{LR}^{ij\alpha\beta} = (\overline{q}_L^i \gamma_\mu \mathcal{E}_L^\alpha) (\overline{e}_R^\beta \gamma^\mu d_L^j)$ $O_{RR}^{ij\alpha\beta} = (\overline{d}_R^i \gamma_\mu e_R^\alpha) (\overline{e}_e^\beta \gamma^\mu d_R^j)$



Quark flavor structure



- Approximate flavor symmetry: $U(2)^3 = U(2)_Q \times U(2)_U \times U(2)_D$ for light generations
- Symmetry breaking spurions: $\mathbf{e}_q = (\epsilon_{q_1}, \epsilon_{q_2})$

 $\begin{array}{ll} \mathbf{e}_q\,,\,\mathbf{V}_u\,,\,\mathbf{V}_d &\sim & \mathbf{2}_{\mathbf{Q}} & \textit{heavy} \rightarrow \textit{light mixing} \\ \mathbf{\Delta}_u\,,\,\mathbf{\Delta}_d &\sim & \overline{\mathbf{2}}_{\mathbf{U}(\mathbf{D})} \times \mathbf{2}_{\mathbf{Q}} & \textit{light Yukawas} \end{array}$



Baker, Fuentes-Martín, Isidori, König [1901.10480]

• Diagonalization of Y_f by rotation L_f :

$$L_f Y_f Y_f^{\dagger} L_f^{\dagger} = \operatorname{diag}(y_{f_1}, y_{f_2}, y_{f_3})$$

$$L_{f} \simeq \begin{pmatrix} O_{f}^{\mathsf{T}} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -\mathbf{V}_{f} \\ \mathbf{V}_{f}^{\dagger} & 1 \end{pmatrix} \quad \text{where } O_{f} = \begin{pmatrix} c_{f} & s_{f} \\ -s_{f} & c_{f} \end{pmatrix} \text{ diagonalizes } \mathbf{\Delta}_{f}$$

- Down-alignment of heavy \rightarrow light mixing
 - Closure of the algebra requires an operator $(\mathcal{O}(1)/\Lambda_U^2)(\overline{q}_L^3\gamma_\mu q_L^3)^2$
 - $B_{s(d)} \overline{B}_{s(d)}$ mixing requires setting $V_d = 0$

A Minimal breaking scenario: \mathbf{e}_q and \mathbf{V}_u aligned in the $U(2)_Q$ space

• Up alignment for light quarks: $s_u \simeq 0$ required by $K - \overline{K}$ and $D - \overline{D}$ mixing

Low-energy constraints



• EFT Lagrangian for $b \to c \tau \nu$

$$\mathscr{L}_{b\to c} = -\frac{G_F}{\sqrt{2}} V_{cb} \Big[\Big(1 + \mathscr{C}_{LL}^c \Big) (\overline{c}_L \gamma_\mu b_L) (\overline{\tau}_L \gamma^\mu \nu_L) - 2 \mathscr{C}_{LR}^c (\overline{c}_L b_R) (\overline{\tau}_R \nu_L) \Big]$$

where $\mathscr{C}_{LL(LR)}^{c} = C_{LL(LR)}^{cb\tau\tau} / V_{cb}$, $\mathscr{C}_{LR}^{c} = \beta_{R}^{*} \mathscr{C}_{LL}^{c}$

- Left-handed couplings only: $\mathscr{C}_{LR} = 0$
- Equal magnitude: $\mathscr{C}_{LR}^c = \mathscr{C}_{LL}^c$
- Observables relevant to low-energy fit:

-
$$R_D$$
, R_{D^*} , R_{Λ_b} , $\mathscr{B}(B_u^- \to \tau \overline{\nu})$

- Combined fit shows 3σ discrepancy with SM
- Compatible with both $\beta_R = 0$ and $\beta_R = -1$



High- p_T constraints on $b \rightarrow c \tau \nu$ transitions Drell-Yan tails $b\overline{b} \rightarrow \tau^+ \tau^-$

HighPT



A Mathematica package for high- p_T Drell-Yan Tails Beyond the Standard Model L. Allwicher, D.A. Faroughy, F. Jaffredo, O. Sumensari, FW [2207.10756] (and more to come)

Computation of:

- Drell-Yan cross sections
- Experimental observables
- Likelihoods

Implemented BSM models:

- SMEFT (d = 6 and d = 8)
- BSM mediators (leptoquarks)

Recasted searches available:

Full LHC run-II datasets \bullet



https://highpt.github.io/

Process	Experiment	Luminosity	,
$pp \to \tau\tau$	ATLAS	$139\mathrm{fb}^{-1}$	
$pp ightarrow \mu \mu$	CMS	$140{\rm fb}^{-1}$	
$pp \rightarrow ee$	CMS	$137{\rm fb}^{-1}$	
$pp \rightarrow \tau \nu$	ATLAS	$139{\rm fb}^{-1}$	[ATL
$pp ightarrow \mu u$	ATLAS	$139{\rm fb}^{-1}$	
$pp \rightarrow e\nu$	ATLAS	$139{\rm fb}^{-1}$	
$pp \rightarrow \tau \mu$	CMS	$138{\rm fb}^{-1}$	
$pp \to \tau e$	CMS	$138{\rm fb}^{-1}$	
$pp \rightarrow \mu e$	CMS	$138{\rm fb}^{-1}$	





[2	2002.1	2223]	
[2	2103.0	2708]	
[2	2103.0	2708]	
LAS	-CONF	-2021-	025
[1906.0	5609]	
[1906.0	5609]	
[2	2205.0	6709]	
[2	2205.0	6709]	
Ľ	2205.0	67091	

Observables and likelihoods



• High- p_T tail distributions:

- Particle-level distribution $\frac{d\sigma}{dx}$ computed from final state particles e, μ, τ, ν
- Detector-level distribution $\frac{d\sigma}{dx_{obs}}$ measured by experiments from reconstructed objects (isolated leptons, tagged jets, missing energy, ...)

• Relate
$$\frac{d\sigma}{dx}$$
 to $\frac{d\sigma}{dx_{obs}}$ using MC simulations (MadGraph+Pythia+Delphes)

$$\begin{array}{c} \textbf{measured} & & \\ \hline \sigma_q(x_{\text{obs}}) = \sum_{p=1}^M K_{pq} \sigma_p(x) \\ \hline & \\ \hline \end{array} \\ \begin{array}{c} \textbf{computed} \end{array} \\ \end{array}$$

object reconstruction efficiencies, detector response, phase-space mismatch

• Recasts of available experimental searches:

$$\chi^2 \sim \frac{(N_{\rm NP} + N_{\rm SM} - N_{\rm data})^2}{\sigma^2}$$

Observables and likelihoods



• High-*p_T* tail distributions:

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object reconstruction efficiencies, detector response, phase-space mismatch

• Recasts of available experimental searches:

HighPT
$$\chi^2 \sim \frac{(N_{\rm NP} + N_{\rm SM} - N_{\rm data})^2}{\sigma^2}$$
 provided by experiment

Observables and likelihoods



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object reconstruction efficiencies, detector response, phase-space mismatch

• Recasts of available experimental searches:

can be exported to python

HighPT
$$\frac{(N_{NP} + N_{SM} - N_{data})^2}{\sigma^2}$$
 provided by experiment

U_1 Leptoquark model



$$\mathcal{L}_{U_1} = [x_1^L]^{i\alpha} \,\bar{q}_i \psi_1 l_\alpha + [x_1^R]^{i\alpha} \,\bar{d}_i \psi_1 e_\alpha + [\bar{x}_1^R]^{i\alpha} \,\bar{u}_i \psi_1 \nu_\alpha + \text{h.c.} \xrightarrow{\mathsf{SMEFT}} [C_{lq}^{(1)}]_{\alpha\beta ij} = [C_{lq}^{(3)}]_{\alpha\beta ij} = -\frac{1}{2} [x_1^L]_{i\beta} [x_1^L]_{j\alpha}^*$$

- Consider couplings to left-handed fields only $q_{3,2}^L$ and \mathcal{C}_3^L
- Relevant processes: $b\bar{b} \rightarrow \tau^+ \tau^-$, $b\bar{s} \rightarrow \tau^+ \tau^-$, $b\bar{c} \rightarrow \tau^- \bar{\nu}$... (+ c.c.)

U_1 Leptoquark model



$$\mathcal{L}_{U_1} = [x_1^L]^{i\alpha} \,\bar{q}_i \psi_1 l_\alpha + [x_1^R]^{i\alpha} \,\bar{d}_i \psi_1 e_\alpha + [\bar{x}_1^R]^{i\alpha} \,\bar{u}_i \psi_1 \nu_\alpha + \text{h.c.} \xrightarrow{\mathsf{SMEFT}} [C_{lq}^{(1)}]_{\alpha\beta ij} = [C_{lq}^{(3)}]_{\alpha\beta ij} = -\frac{1}{2} [x_1^L]_{i\beta} [x_1^L]_{j\alpha}^*$$

- Consider couplings to left-handed fields only $q_{3,2}^L$ and $\mathcal{\ell}_3^L$
- Relevant processes: $b\bar{b} \rightarrow \tau^+ \tau^-$, $b\bar{s} \rightarrow \tau^+ \tau^-$, $b\bar{c} \rightarrow \tau^- \bar{\nu} \dots$ (+ c.c.)



SMEFT fit

U_1 Leptoquark model



 $\mathcal{L}_{U_1} = [x_1^L]^{i\alpha} \,\bar{q}_i \psi_1 l_\alpha + [x_1^R]^{i\alpha} \,\bar{d}_i \psi_1 e_\alpha + [\bar{x}_1^R]^{i\alpha} \,\bar{u}_i \psi_1 \nu_\alpha + \text{h.c.} \xrightarrow{\mathsf{SMEFT}} [C_{lq}^{(1)}]_{\alpha\beta ij} = [C_{lq}^{(3)}]_{\alpha\beta ij} = -\frac{1}{2} [x_1^L]_{i\beta} [x_1^L]_{j\alpha}^*$

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- Relevant processes: $b\bar{b} \rightarrow \tau^+ \tau^-, \ b\bar{s} \rightarrow \tau^+ \tau^-, \ b\bar{c} \rightarrow \tau^- \bar{\nu} \dots$ (+ c.c.)



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Di-tau tails



- Searches for $pp \rightarrow \tau \tau$
 - ATLAS (no excess) [2002.12223] [implemented in HighPT]
 - CMS (~ 3σ excess) [2208.02717] [not yet implemented in HighPT]
- Exploit *b*-tagging:
- Models with large 3rd generation couplings
- Particularly relevant for $b\bar{b} \rightarrow \tau^- \tau^+$
- Gluon splitting $g \rightarrow b\bar{b}$



High- p_T constraints on the U_1



$$J_U^{\mu} = \frac{g_U}{\sqrt{2}} \left[\overline{q}_L^3 \gamma^{\mu} \mathcal{C}_L^3 + \beta_R \overline{d}_R^3 \gamma^{\mu} e_R^3 + \sum_{k=1,2} \epsilon_{q_k} \overline{q}_L^k \gamma^{\mu} \mathcal{C}_L^3 \right]$$

- Relevant processes at high- p_T : $pp \rightarrow \tau \tau$ in particular $b\bar{b} \rightarrow \tau^+ \tau^-$
 - Effective scale: $\Lambda_U = \sqrt{2}M_U/g_U$
- Searches for $pp \rightarrow \tau \tau$
 - ATLAS (no excess) [2002.12223] [implemented in HighPT]
 - CMS ($\sim 3\sigma$ excess) [2208.02717]
- Exploit *b*-tagging for $b\bar{b} \rightarrow \tau^- \tau^+$
- Rescaled using NLO corrections computed in U. Haisch, L. Schnell, S. Schulte, [2209.12780]



Constraints on right-handed coupling scenarios

J. Aebischer, G. Isidori, M. Pesut, B.A. Stefanek, FW [2210.13422]

• A specific NP model would have many more collider signatures see e.g. Baker, Fuentes-Martin, Isidori, König [1901.10480]

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High- p_T vs. R_D and R_{D^*}



• Effective Lagrangian for $b \rightarrow c$ transitions:

$$\mathscr{L}_{b\to c} = -\frac{4G_F}{\sqrt{2}} V_{cb} \left[\left(1 + \mathscr{C}_{LL}^c \right) (\bar{c}_L \gamma_\mu b_L) (\bar{\tau}_L \gamma^\mu \nu_L) - 2\mathscr{C}_{LR}^c (\bar{c}_L b_R) (\bar{\tau}_R \nu_L) \right] \right]$$

- Match $\mathscr{C}_{\textit{LL}(\textit{LR})}^{c}$ to the U_1 model



J. Aebischer, G. Isidori, M. Pesut, B.A. Stefanek, FW [2210.13422]

High- p_T vs. R_D and R_{D^*}



- Effective Lagrangian for $b \to c$ transitions: $4G_F = \int (1 - c) (1$
 - $\mathscr{L}_{b\to c} = -\frac{4G_F}{\sqrt{2}} V_{cb} \left[\left(1 + \mathscr{C}_{LL}^c \right) (\bar{c}_L \gamma_\mu b_L) (\bar{\tau}_L \gamma^\mu \nu_L) 2\mathscr{C}_{LR}^c (\bar{c}_L b_R) (\bar{\tau}_R \nu_L) \right]$
- Match $\mathscr{C}_{LL(LR)}^c$ to the U_1 model
- Details of the fit:
 - $\mathscr{C}_{LL}^c \to 0$ corresponds to $|\beta_R| \to \infty$
 - More model dependence
 - Depends on 2nd gen. coupling ϵ_q
 - Small ϵ_q requires lower scale Λ_U
- Currently good compatibility of constraints
- Improvements expected by HL-LHC
- CMS excess would indicate scenario with large β_R



J. Aebischer, G. Isidori,

FW [2210.13422]

M. Pesut, B.A. Stefanek,

Conclusions

- High- p_T provides information complementary to low-energy experiments
 - Improvements expected with upcoming Run-3 and HL-LHC
 - Will help to scrutinize the origin of the B-anomalies
- Construction of full flavor likelihood for high- p_T Drell-Yan processes at LHC
 - For the SMEFT explicit heavy BSM mediators
- Future features for the **HighPT** code:
 - Addition of further observables <u>https://highpt.github.io/</u> (*b*-tagging, FB-asymmetries, other collider processes, low-energy, ...)
 - Assessment of PDF uncertainties & NLO corrections

Thank you for your attention !!!







Bounds on NP scenarios



Example:

LQ models for $R_{D^{(*)}}$

- Consider flavor indices: $\alpha\beta ij \in \{3333, 3323\}$
- Relevant experimental sear
 - $pp \rightarrow \tau \tau$
 - $pp \rightarrow \tau \nu$
- Perform fits for:
 - Wilson coefficients
 - NP couplings

 $\begin{aligned} \mathcal{L}_{S_{1}} &= [y_{1}^{L}]^{i\alpha} \, S_{1} \bar{q}_{i}^{c} \epsilon l_{\alpha} + [y_{1}^{R}]^{i\alpha} \, S_{1} \bar{u}_{i}^{c} e_{\alpha} + [\bar{y}_{1}^{R}]^{i\alpha} \, S_{1} \bar{d}_{i}^{c} \nu_{\alpha} + \text{h.c.} \\ \mathcal{L}_{R_{2}} &= -[y_{2}^{L}]^{i\alpha} \, \bar{u}_{i} R_{2} \epsilon l_{\alpha} + [y_{2}^{R}]^{i\alpha} \, \bar{q}_{i} e_{\alpha} R_{2} + \text{h.c.} \\ \mathcal{L}_{U_{1}} &= [x_{1}^{L}]^{i\alpha} \, \bar{q}_{i} \psi_{1} l_{\alpha} + [x_{1}^{R}]^{i\alpha} \, \bar{d}_{i} \psi_{1} e_{\alpha} + [\bar{x}_{1}^{R}]^{i\alpha} \, \bar{u}_{i} \psi_{1} \nu_{\alpha} + \text{h.c.} \end{aligned}$

SMEFT matching @ tree-level

Field	S_1	R_2	U_1	
Quantum Numbers	$(ar{3},1,1/3)$	$({f 3},{f 2},7/6)$	$({f 3},{f 1},2/3)$	
$\left[\mathcal{C}_{ledq} ight]_{lphaeta ij}$	—	_	$2[x_1^L]^{ilpha^*}[x_1^R]^{jeta}$	
$\left[{{\cal C}}_{lequ}^{(1)} ight]_{lphaeta ij}$	$rac{1}{2}[y_1^L]^{ilpha^*}[y_1^R]^{jeta}$	$-\tfrac{1}{2}[y_2^R]^{i\beta}[y_2^L]^{j\alpha^*}$	_	
$\left[{{\cal C}}_{lequ}^{(3)} ight]_{lphaeta ij}$	$-\tfrac{1}{8}[y_1^L]^{i\alpha^*}[y_1^R]^{j\beta}$	$-\tfrac{1}{8}[y_2^R]^{i\beta}[y_2^L]^{j\alpha^*}$	_	
$\left[\mathcal{C}_{eu} ight] _{lphaeta ij}$	$rac{1}{2}[y_1^R]^{jeta}[y_1^R]^{ilpha^*}$	_	_	
$[\mathcal{C}_{ed}]_{lphaeta ij}$	_	_	$-[x_1^R]^{i\beta}[x_1^R]^{j\alpha^*}$	
$[\mathcal{C}_{\ell u}]_{lphaeta ij}$	_	$-rac{1}{2}[y_2^L]^{ieta}[y_2^L]^{jlpha^*}$	_	
$\left[{{\cal C}_{qe}} ight]_{ijlphaeta}$	—	$-\tfrac{1}{2}[y_2^R]^{i\beta}[y_2^R]^{j\alpha^*}$	_	
$\left[\mathcal{C}_{lq}^{(1)} ight]_{lphaeta ij}$	$rac{1}{4}[y_1^L]^{ilpha^*}[y_1^L]^{jeta}$	_	$-\tfrac{1}{2}[x_1^L]^{i\beta}[x_1^L]^{j\alpha^*}$	
$\left[\mathcal{C}_{lq}^{(3)} ight] _{lphaeta ij}$	$-\tfrac{1}{4}[y_1^L]^{i\alpha^*}[y_1^L]^{j\beta}$	_	$-\tfrac{1}{2}[x_1^L]^{i\beta}[x_1^L]^{j\alpha^*}$	

R_2 Leptoquark (3, 2, 7/6)



$$\mathcal{L}_{R_2} = -[y_2^L]_{i\alpha} \,\bar{u}_i R_2 \epsilon l_\alpha + [y_2^R]_{i\alpha} \,\bar{q}_i e_\alpha R_2 + \text{h.c.}$$

$$\rightarrow \quad [C_{lequ}^{(1)}]_{\alpha\beta ij} = 4[C_{lequ}^{(3)}]_{\alpha\beta ij} = -\frac{1}{2}[y_2^R]_{i\beta}[y_2^L]_{j\alpha}^*$$

SMEFT fit

LQ mediator fit



L. Allwicher, D.A. Faroughy, F. Jaffredo, O. Sumensari, FW [2207.10714]

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S_1 Leptoquark ($\bar{3}, 1, 1/3$)



 $\mathcal{L}_{S_1} = [y_1^L]_{i\alpha} S_1 \bar{q}_i^c \epsilon l_{\alpha} + [y_1^R]_{i\alpha} S_1 \bar{u}_i^c e_{\alpha} + [\bar{y}_1^R]_{i\alpha} S_1 \bar{d}_i^c N_{\alpha} + \text{h.c.} \rightarrow [C_{lequ}^{(1)}]_{\alpha\beta ij} = -4[C_{lequ}^{(3)}]_{\alpha\beta ij} = \frac{1}{2} [y_1^L]_{i\alpha}^* [y_1^R]_{j\beta}$

SMEFT fit

LQ mediator fit



L. Allwicher, D.A. Faroughy, F. Jaffredo, O. Sumensari, FW [2207.10714]

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LFV in the U_1 model

- $U_1 \sim (3, 1, 2/3)$ leptoquark model:
- LFV requires 2 couplings turned on
 - LFV can be constrained by $pp \to \ell \, \overline{\ell}\,$ and $\, pp \to \ell \, \overline{\ell'}\,$
- Example: consider only 3rd generation quarks





CKM rotations



• Effects of up- / down-alignment assumption for NP constraints



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χ^2 likelihood vs CL_s



- χ^2 likelihood: combine experimental bins with low event count in the tails to validate the Gaussian approximation (1 σ , 2 σ , 3 σ contours)
- Compare to $CL_s = \frac{p_s}{1 p_0}$ method (1 σ , 2 σ , 3 σ dashed contours)
- CL_s tends to be more conservative, but overall good agreement with χ^2



EFT validity



- High- p_T tails: events with highest invariant mass are around $\sqrt{\hat{s}} \lesssim 4 \,\mathrm{TeV}$
- Validity of EFT approach for relatively light NP mediators (~few TeV) ???
 - Option 1: drop highest bins of all searches
 - Option 2: include higher dimensional operators
 - How sizable is the effect of d = 8 operators compared to d = 6?
 - Option 3: simulate with explicit NP mediator rather than EFT
 - How does the explicit model compare to d = 6, 8 EFT operators?
- Analyse these effects with **HighPT** for some specific models [w.i.p.]

see e.g.:

Dawson, Fontes, Homiller, Sullivan [2205.01561] Boughezal, Mereghetti, Petriello [2106.05337] Alioli, Boughezal, Mereghetti, Petriello [2003.11615] Kim, Martin [2203.11976]

Jack-knife plots





Clipped limits



- Constraints obtained with sliding upper cut $M_{\rm cut}$ for experimental observables
- Allows assessment of EFT validity range



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EFT validity



 $\Lambda = 4 \text{ TeV}$ $\Lambda = 6 \text{ TeV}$ $\Lambda = 2 \text{ TeV}$ $(\times 5)$ $(\times 5)$ $(\times 5)$ uuuuuu $> 4\pi v^2 / \Lambda^2$ ud $(\times 5)$ ud $(\times 5)$ ud $(\times 5)$ Constraints on form dd $(\times 5)$ $(\times 5)$ $(\times 5)$ ddddfactors ~ $C_{la}^{(1,3)}$: SSSSSScsCScsccccccSingle parameter 65 bbbbbblimits $\sim d = 6$ -0.06 - 0.04 - 0.02-0.06 - 0.04 - 0.020. 0.02 0.04 0.060. 0.02 0.04 $0.06 \quad -0.06 \quad -0.04 \quad -0.02$ 0. 0.020.040.06 $\left[\mathcal{F}_{V(0,0)}^{LL,\,qq\prime}\right]_{22ij}$ $\left[\mathcal{F}_{V(0,0)}^{LL,\,qq'}\right]_{22ij}$ $\left[\mathcal{F}_{V\left(0,0\right)}^{LL,\,qq\prime}\right]_{22ij}$ Marginalizing over d = 8 operators $\Lambda = 6 \text{ TeV}$ $\Lambda = 4 \text{ TeV}$ $\Lambda = 2 \text{ TeV}$ uuuuuu $\sim C^{(k)}_{l^2q^2D^2}$ udududddddddOperators of d = 6SSSSSSand d = 8 assuming CSCSCSccccCCZ' scenario bbbbbb0.06 -0.06 - 0.04 - 0.020. 0.02 0.04 -0.06 - 0.04 - 0.020. 0.020.04 $0.06 \quad -0.06 \quad -0.04 \quad -0.02$ 0. 0.020.040.06 $\left[\mathcal{F}_{V\,(0,0)}^{LL,\,qq\prime}\right]_{33ij}$ $\left[\mathcal{F}_{V\,(0,0)}^{\,LL,\,qq\prime}\right]_{33ij}$ $\left[\mathcal{F}_{V\,(0,0)}^{LL,\,qq\prime}\right]_{33ij}$

L. Allwicher, D.A. Faroughy, F. Jaffredo, O. Sumensari, FW [2207.10714]

LHC EFT WG Meeting: EFT interpretation of $b \rightarrow c \tau \nu$ and connections with high- p_T tails

Drell-Yan form-factors



 ℓ_{α}

• Drell-Yan processes:

 $\bar{u}_i u_j \to \ell_\alpha^- \ell_\beta^+, \quad \bar{d}_i d_j \to \ell_\alpha^- \ell_\beta^+, \quad \bar{u}_i d_j \to \ell_\alpha^- \bar{\nu}_\beta, \quad \bar{d}_i u_j \to \ell_\alpha^+ \nu_\beta$

• Amplitude form-factor decomposition:

$$\begin{split} \left[\mathcal{A}\right]_{ij}^{\alpha\beta} &\equiv \mathcal{A}\left(\bar{q}_{i}q'_{j} \rightarrow \bar{\ell}_{\alpha}\ell'_{\beta}\right) \\ &= \frac{1}{v^{2}}\sum_{X,Y} \left\{ \left(\bar{\ell}_{\alpha}\mathbb{P}_{X}\ell'_{\beta}\right) \left(\bar{q}_{i}\mathbb{P}_{Y}q'_{j}\right) \left[\mathcal{F}_{S}^{XY,qq'}(\hat{s},\hat{t})\right]_{ij}^{\alpha\beta} \quad \text{Scalar} \\ &+ \left(\bar{\ell}_{\alpha}\gamma_{\mu}\mathbb{P}_{X}\ell'_{\beta}\right) \left(\bar{q}_{i}\gamma^{\mu}\mathbb{P}_{Y}q'_{j}\right) \left[\mathcal{F}_{V}^{XY,qq'}(\hat{s},\hat{t})\right]_{ij}^{\alpha\beta} \quad \text{Vector} \\ &+ \left(\bar{\ell}_{\alpha}\sigma_{\mu\nu}\mathbb{P}_{X}\ell'_{\beta}\right) \left(\bar{q}_{i}\sigma^{\mu\nu}\mathbb{P}_{Y}q'_{j}\right) \delta^{XY} \left[\mathcal{F}_{D_{q}}^{XY,qq'}(\hat{s},\hat{t})\right]_{ij}^{\alpha\beta} \quad \text{Tensor} \\ &+ \left(\bar{\ell}_{\alpha}\sigma_{\mu\nu}\mathbb{P}_{X}\ell'_{\beta}\right) \left(\bar{q}_{i}\sigma^{\mu\nu}\mathbb{P}_{Y}q'_{j}\right) \frac{ik_{\nu}}{v} \left[\mathcal{F}_{D_{q}}^{XY,qq'}(\hat{s},\hat{t})\right]_{ij}^{\alpha\beta} \quad \text{Dipole} \\ &+ \left(\bar{\ell}_{\alpha}\sigma_{\mu\nu}\mathbb{P}_{X}\ell'_{\beta}\right) \left(\bar{q}_{i}\gamma^{\mu}\mathbb{P}_{Y}q'_{j}\right) \frac{ik^{\nu}}{v} \left[\mathcal{F}_{D_{\ell}}^{XY,qq'}(\hat{s},\hat{t})\right]_{ij}^{\alpha\beta} \quad \text{Dipole} \end{split}$$

- General parametrization of tree-level effects invariant under $SU(3)_c \times U(1)_e$
- Captures local and non-local effects

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Local and non-local contributions



Split form-factors into a regular and a singular piece

$$\mathcal{F}_{I}(\hat{s},\hat{t}) = \mathcal{F}_{I,\operatorname{Reg}}(\hat{s},\hat{t}) + \mathcal{F}_{I,\operatorname{Poles}}(\hat{s},\hat{t})$$

Form-factor framework can incorporate both EFT and explicit NP models

Felix Wilsch

Local and non-local contributions



Split form-factors into a regular and a singular piece

$$\mathcal{F}_{I}(\hat{s},\hat{t}) = \mathcal{F}_{I,\operatorname{Reg}}(\hat{s},\hat{t}) + \mathcal{F}_{I,\operatorname{Poles}}(\hat{s},\hat{t})$$

- Analytic function of \hat{s} , \hat{t}
- Describes EFT contact interactions
 - Can be matched to the SMEFT
- Formal expansion in validity range of the EFT: v^2 , $|\hat{s}|$, $|\hat{t}| < \Lambda^2$

$$F_{I,Reg}(\hat{s},\hat{t}) = \sum_{n,m=0}^{\infty} F_{I,(n,m)} \left(\frac{\hat{s}}{v^2}\right)^n \left(\frac{\hat{t}}{v^2}\right)^m$$

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Local and non-local contributions



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- Isolated simple poles in \hat{s} , \hat{t} (no branch-cuts at tree-level)

- Describes non-local effects due to exchange of mediators (SM & NP)

$$F_{I,\text{Poles}}(\hat{s},\hat{t}) = \sum_{a} \frac{v^2 \mathscr{S}_{I(a)}}{\hat{s} - \Omega_a} + \sum_{b} \frac{v^2 \mathscr{T}_{I(b)}}{\hat{t} - \Omega_b} - \sum_{c} \frac{v^2 \mathscr{U}_{I(c)}}{\hat{s} + \hat{t} + \Omega_c}$$

$$M = V \qquad \text{NP}$$

$$\Omega_n = m_n^2 - im_n \Gamma_n \qquad \hat{u} = -\hat{s} - \hat{t}$$

Form-factor framework can incorporate both EFT and explicit NP models

Regular form-factors $F_{I, \text{Reg}}(\hat{s}, \hat{t})$



- **Regular form-factors:** analytic functions of \hat{s} , \hat{t}
- Describe unresolved d.o.f. \rightarrow EFT
- Formal expansion in validity range of the EFT $|\hat{s}|, |\hat{t}| < \Lambda^2$:

- Derivative expansion:
$$F_{I,Reg}(\hat{s},\hat{t}) = \sum_{n,m=0}^{\infty} F_{I,(n,m)} \left(\frac{\hat{s}}{v^2}\right)^n \left(\frac{\hat{t}}{v^2}\right)^m$$

- EFT expansion: $F_{I,(n,m)} = \sum_{k=n+m+1} \mathcal{O}\left((v^2/\Lambda^2)^k\right)$

• Terms to consider at mass dimension *d*

-
$$d = 6$$
: $(n, m) = (0, 0)$

-
$$d = 8$$
: $(n, m) = (0, 0), (1, 0), (0, 1)$

Singular form-factors $F_{I, \text{Poles}}(\hat{s}, \hat{t})$

• **Pole form-factors:** non-analytic functions with finite number of simple poles

$$F_{I,\text{Poles}}(\hat{s},\hat{t}) = \sum_{a} \frac{v^2 \mathscr{S}_{I(a)}}{\hat{s} - \Omega_a} + \sum_{b} \frac{v^2 \mathscr{T}_{I(b)}}{\hat{t} - \Omega_b} - \sum_{c} \frac{v^2 \mathscr{U}_{I(c)}}{\hat{s} + \hat{t} + \Omega_c}$$

- ► *a* : sum over all *s*-channel (colorless) mediators
- ► *b* : sum over all *t*-channel (colorful) mediators
- c : sum over all u-channel (colorful) mediators
- SM contribution $\rightarrow \mathcal{S}_{V(a)} \ (a \in \{\gamma, Z, W\})$
- NP contribution $\rightarrow \mathcal{S}_{I(a)}, \mathcal{T}_{I(b)}, \mathcal{U}_{I(c)}$
- Residues can be made independent of \hat{s} , \hat{t} by partial fraction decomposition:

Universität

 $\hat{u} = -\hat{s} - \hat{t}$

 $\Omega_n = m_n^2 - i m_n \Gamma_n$

SMEFT



$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{\text{SM}} + \sum_{i} \frac{C_i^{(6)}}{\Lambda^2} Q_i^{(6)} + \sum_{i} \frac{C_i^{(8)}}{\Lambda^4} Q_i^{(8)} + \mathcal{O}(\Lambda^{-6})$$

• Cross-section in the SMEFT to $\mathcal{O}(\Lambda^{-4})$

$$\sigma \sim \left| A_{\rm SM} \right|^2 + \frac{1}{\Lambda^2} 2 \operatorname{Re} \left(A^{(6)} A_{\rm SM}^* \right) + \frac{1}{\Lambda^4} \left(\left| A^{(6)} \right|^2 + 2 \operatorname{Re} \left(A^{(8)} A_{\rm SM}^* \right) \right) + \mathcal{O}(\Lambda^{-6})$$

- Consistent description up to $\mathcal{O}(\Lambda^{-4})$
 - $|A^{(6)}|^2$ contribution can be energy enhanced
 - LFV only through $|A^{(6)}|^2$ (no SM interference)
- Requires inclusion of d = 8 operators Boughezal, Mereghetti, Petriello [2106.05337]
 - Only d = 8 interference with SM relevant

SMEFT



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• d = 6 Warsaw basis $\psi^4, \psi^2 H^2 D, \psi^2 X H$

Grzadkowski, Iskrzynski, Misiak, Rosiek [1008.4884]

• d = 8 basis (C. Murphy) $\psi^4 D^2, \psi^4 H^2, \psi^2 H^2 D^3, \psi^2 H^4 D$

 ψ^4 contact interactions non-local contributions Murphy [2005.00059]

see also: Li et al [2005.00008]

EFT contributions



• Feynman diagrams for Drell-Yan in the SMEFT to $\mathcal{O}(\Lambda^{-4})$



EFT operator counting and energy scaling

Dimension	d=6				d=8		
Operator classes	ψ^4	$\psi^2 H^2 D$	$\psi^2 X H$	$\psi^4 D^2$	$\psi^4 H^2$	$\psi^2 H^4 D$	$\psi^2 H^2 D^3$
Amplitude scaling	E^2/Λ^2	v^2/Λ^2	vE/Λ^2	E^4/Λ^4	$v^2 E^2 / \Lambda^4$	v^4/Λ^4	$v^2 E^2/\Lambda^4$

Only contributions interfering with the SM

EFT contributions



• Feynman diagrams for Drell-Yan in the SMEFT to $\mathcal{O}(\Lambda^{-4})$



EFT operator counting and energy scaling

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Operator classes	ψ^4	$\psi^2 H^2 D$	$\psi^2 X H$	$\psi^4 D^2$	$\psi^4 H^2$	$\psi^2 H^4 D$	$\psi^2 H^2 D^3$	
Amplitude scaling	E^2/Λ^2	v^2/Λ^2	vE/Λ^2	E^4/Λ^4	$v^2 E^2 / \Lambda^4$	v^4/Λ^4	$v^2 E^2/\Lambda^4$	
Only contributions interfering with the SM Most enhanced contributions								



• Example: vector form-factors
$$\begin{array}{l} \text{NC: } a \in \{\gamma, Z\} \\ \text{CC: } a \in \{W\} \end{array} \\ F_V = F_{V(0,0)} + F_{V(1,0)} \frac{\hat{s}}{v^2} + F_{V(0,1)} \frac{\hat{t}}{v^2} + \sum_a \frac{v^2}{\hat{s} - M_a^2 + iM_a\Gamma_A} \left(\mathscr{S}_{(a,\text{SM})} + \delta \mathscr{S}_{(a)} \right) \end{array}$$

• Schematic form-factor matching to $\mathcal{O}(\Lambda^{-4})$:

$$F_{V(0,0)} = \frac{v^2}{\Lambda^2} C_{\psi^4}^{(6)} + \frac{v^4}{\Lambda^4} C_{\psi^4 H^2}^{(8)} + \frac{v^2 m_a^2}{\Lambda^4} C_{\psi^2 H^2 D^3}^{(8)} + \cdots$$

$$F_{V(1,0)} = \frac{v^4}{\Lambda^4} C_{\psi^4 D^2}^{(8)} + \cdots$$

$$F_{V(0,1)} = \frac{v^4}{\Lambda^4} C_{\psi^4 D^2}^{(8)} + \cdots$$

$$\delta \mathcal{S}_{(a)} = \frac{m_a^2}{\Lambda^2} C_{\psi^2 H^2 D}^{(6)} + \frac{\nu^2 m_a^2}{\Lambda^4} \left(\left[C_{\psi^2 H^2 D}^{(6)} \right]^2 + C_{\psi^2 H^4 D}^{(8)} \right) + \frac{m_a^4}{\Lambda^4} C_{\psi^2 H^2 D^3}^{(8)} + \cdots$$



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$$\mathcal{S}_{(\gamma,\text{SM})} = 4\pi\alpha_{\text{em}}Q_lQ_q$$
$$\mathcal{S}_{(Z,\text{SM})} = \frac{4\pi\alpha_{\text{em}}}{c_W^2 s_W^2} g_l^X g_q^Y$$
$$\mathcal{S}_{(W,\text{SM})} = \frac{1}{2}g_2^2$$

$$F_{V(1,0)} = \frac{v^4}{\Lambda^4} C_{\psi^4 D^2}^{(8)} + \cdots$$

$$F_{V(0,1)} = \frac{v^4}{\Lambda^4} C_{\psi^4 D^2}^{(8)} + \cdots$$

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$$\begin{array}{ll} & \text{Example: vector form-factors} & \overset{\text{NC: } a \in \{\gamma, Z\}}{\text{C: } a \in \{W\}} & \text{Include BSM mediators similarly} \\ F_{V} = F_{V(0,0)} + F_{V(1,0)} \frac{\hat{s}}{v^{2}} + F_{V(0,1)} \frac{\hat{t}}{v^{2}} + \sum_{a} \frac{v^{2}}{\hat{s} - M_{a}^{2} + iM_{a}\Gamma_{A}} \left(\mathcal{S}_{(a, \text{SM})} + \delta \mathcal{S}_{(a)} \right) \\ & \text{Schematic form-factor matching to } \mathcal{O}(\Lambda^{-4}): \\ F_{V(0,0)} = \frac{v^{2}}{\Lambda^{2}} C_{\psi^{4}}^{(6)} + \frac{v^{4}}{\Lambda^{4}} C_{\psi^{4}H^{2}}^{(8)} + \frac{v^{2}m_{a}^{2}}{\Lambda^{4}} C_{\psi^{2}H^{2}D^{3}}^{(8)} + \cdots \\ F_{V(1,0)} = \frac{v^{4}}{\Lambda^{4}} C_{\psi^{4}D^{2}}^{(8)} + \cdots \\ F_{V(0,1)} = \frac{v^{4}}{\Lambda^{4}} C_{\psi^{4}D^{2}}^{(8)} + \cdots \\ & \delta \mathcal{S}_{(a)} = \frac{m_{a}^{2}}{\Lambda^{2}} C_{\psi^{2}H^{2}D}^{(6)} + \frac{v^{2}m_{a}^{2}}{\Lambda^{4}} \left(\left[C_{\psi^{2}H^{2}D}^{(6)} \right]^{2} + C_{\psi^{2}H^{4}D}^{(8)} \right) + \frac{m_{a}^{4}}{\Lambda^{4}} C_{\psi^{2}H^{2}D^{3}}^{(8)} + \cdots \\ \end{array}$$



• Example: vector form-factors
$$\overset{NC: a \in \{\gamma, Z\}}{CC: a \in \{W\}}$$
 include BSM mediators similarly
 $F_V = F_{V(0,0)} + F_{V(1,0)} \frac{\hat{s}}{v^2} + F_{V(0,1)} \frac{\hat{t}}{v^2} + \sum_a \frac{v^2}{\hat{s} - M_a^2 + iM_a \Gamma_A} \left(\mathscr{S}_{(a,SM)} + \delta \mathscr{S}_{(a)} \right)$
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