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NANJING NORMAL UNIVERSITY

# Gravitational waves from first order phase transitions

Peter Athron  
(Nanjing Normal University)

This talk is based on:

- PA, C. Balázs, L. Morris, JCAP 03 (2023), 006 , 57 pages
- PA, C. Balázs, A. Fowlie, L. Morris, L. Wu, arxiv:2305.02357, (Invited review for Progress in Particle and Nuclear Physics), 155 pages
- PA, A. Fowlie, Chih-Ting Lu, L. Morris, L. Wu, Yongcheng Wu, Zhongxiu Xu, arXiv:2306.17239
- PA, C. Balázs, T. Gonzalo, M. Pearce, arXiv:2307.02544
- PA, C. Balázs, A. Fowlie, L. Morris, G. White, Y. Zhang, JHEP 01 (2023) 050, 45 pages
- PA, L. Morris, Z. Xu , In preparation

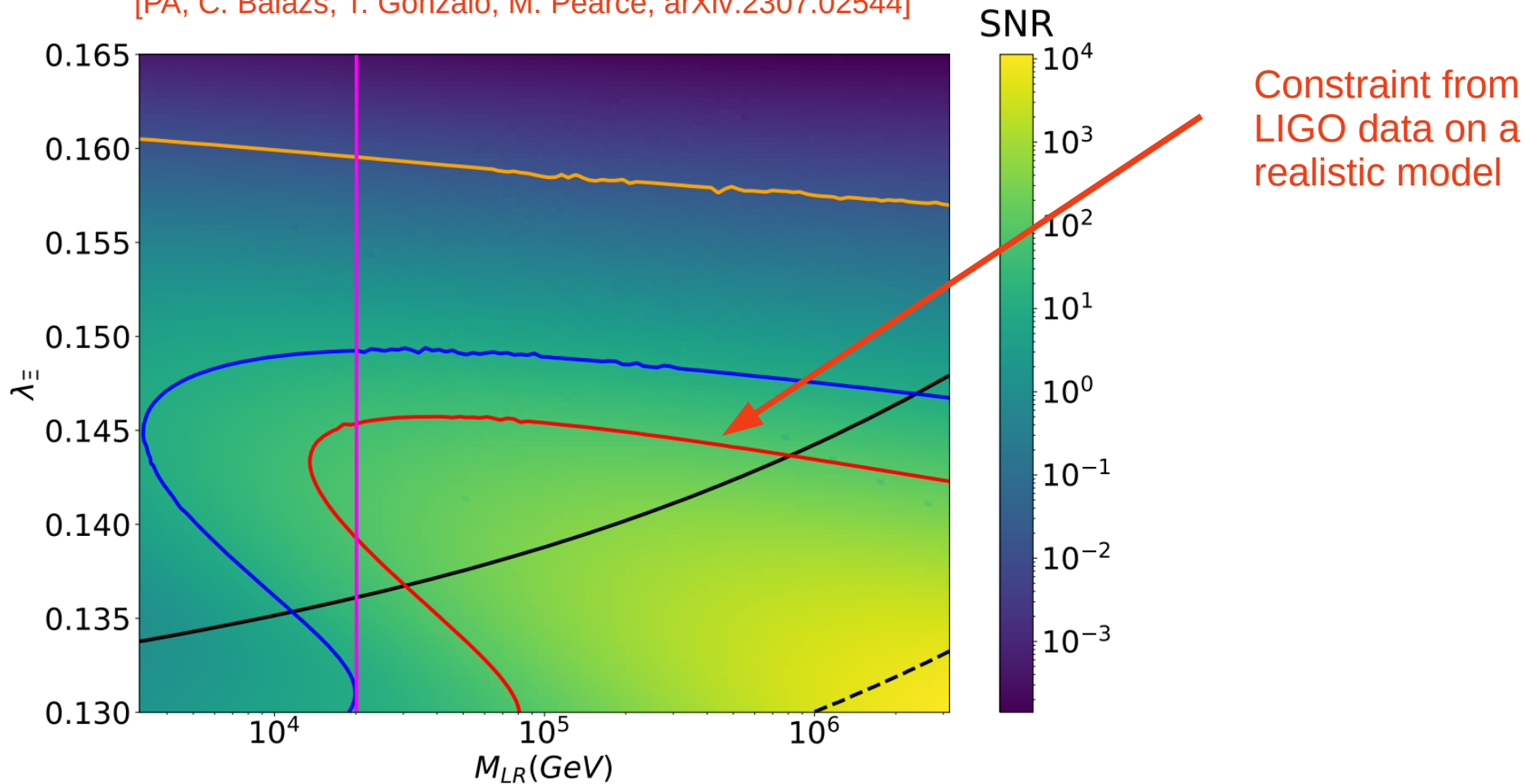
So I have to be fast and skip a few details to stay within the time limit...

We are entering an era  
where  
precise GWs predictions matter

# Precise GWs predictions matter

LIGO data already constrains well motivated Pati-Salam GUT models

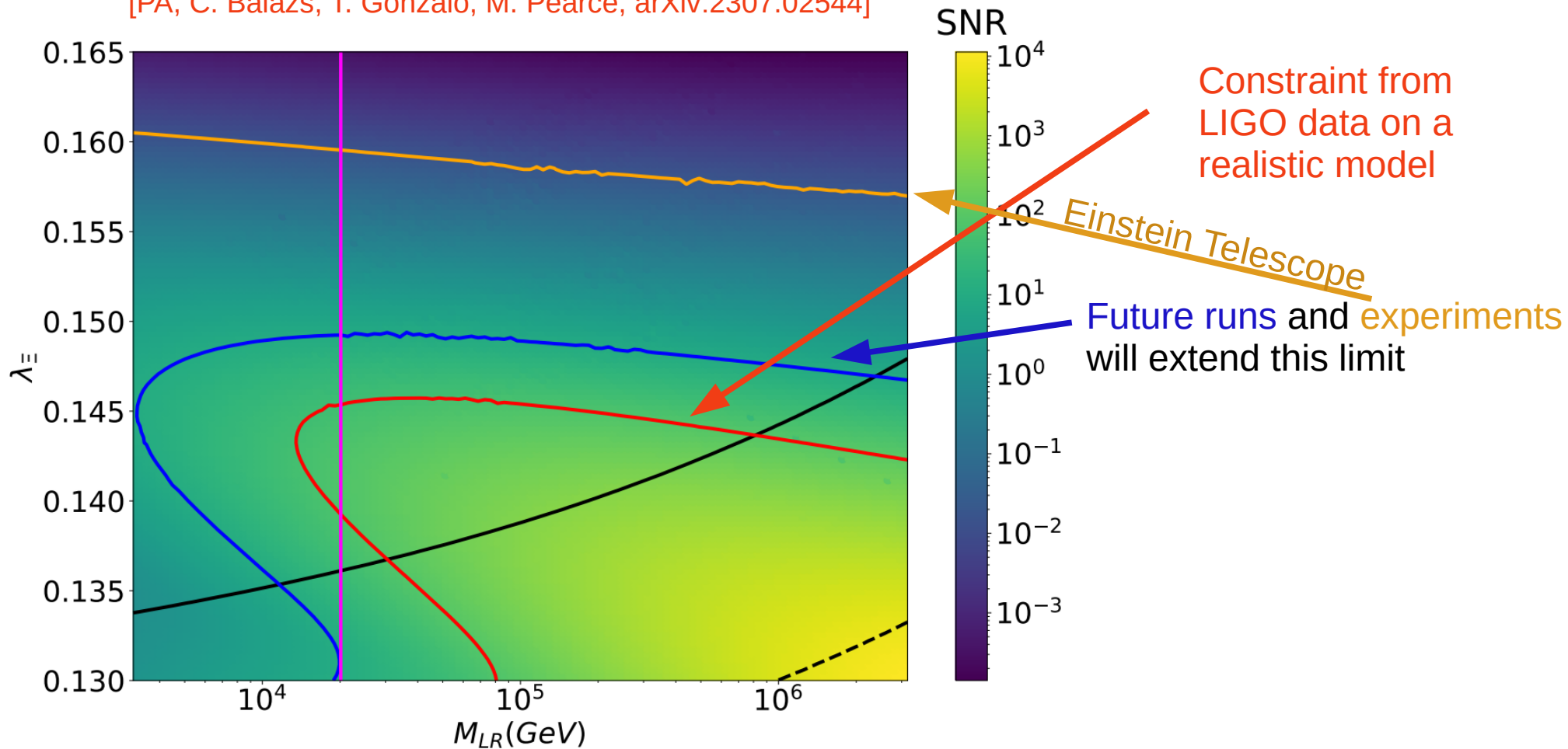
[PA, C. Balázs, T. Gonzalo, M. Pearce, arXiv:2307.02544]



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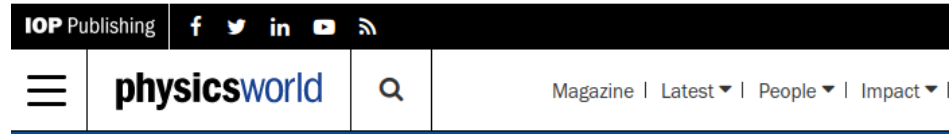
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# Big news last month:

## A stochastic gravitational wave background has been observed by multiple Pulsar Timing Arrays experiments



ASTRONOMY AND SPACE | RESEARCH UPDATE

### Pulsar timing irregularities reveals hidden gravitational-wave background

29 Jun 2023



Cosmic claims: researchers have used radiotelescopes around the world to hunt for gravitational waves using the subtle variations in the timing of pulsars. (Courtesy: Aurore Simonnet for the NANOGrav Collaboration)

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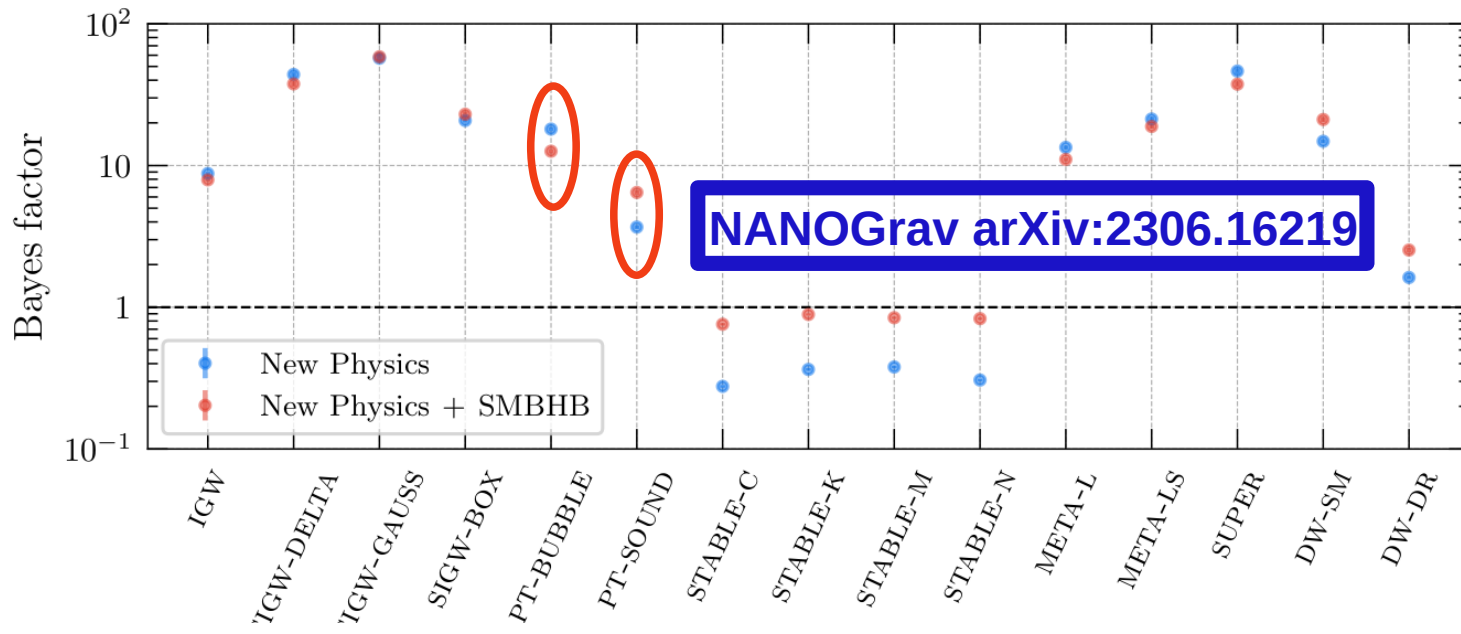
A stochastic gravitational wave background has been observed  
by multiple Pulsar Timing Arrays experiments

More excitement:

first order phase transitions

fit the data better (slightly)

than super massive black hole binaries



## WARNING

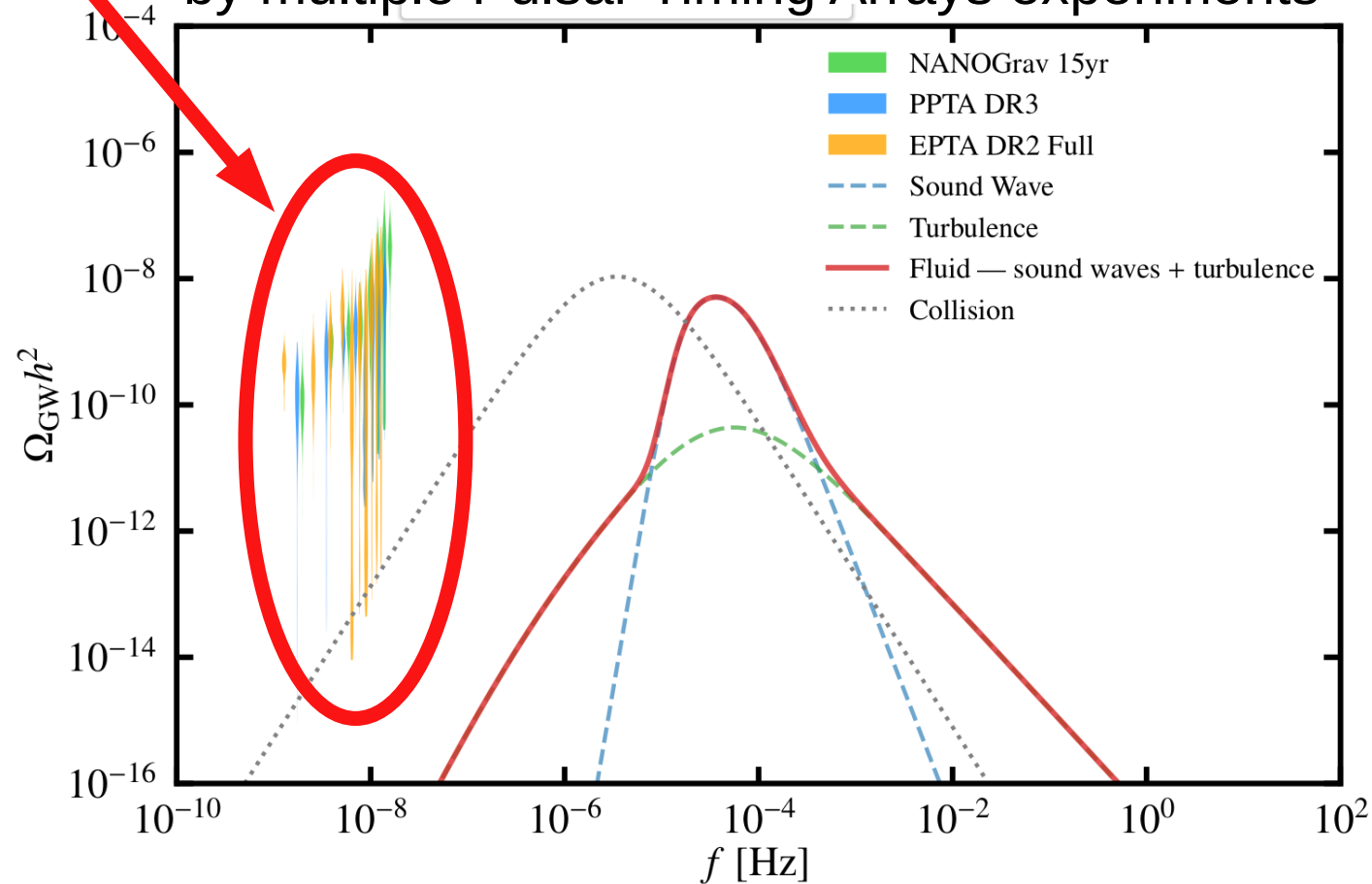
However for specific models these predictions require great care!

We looked at one model prominently cited as able to fit nHz signals from PTAs...



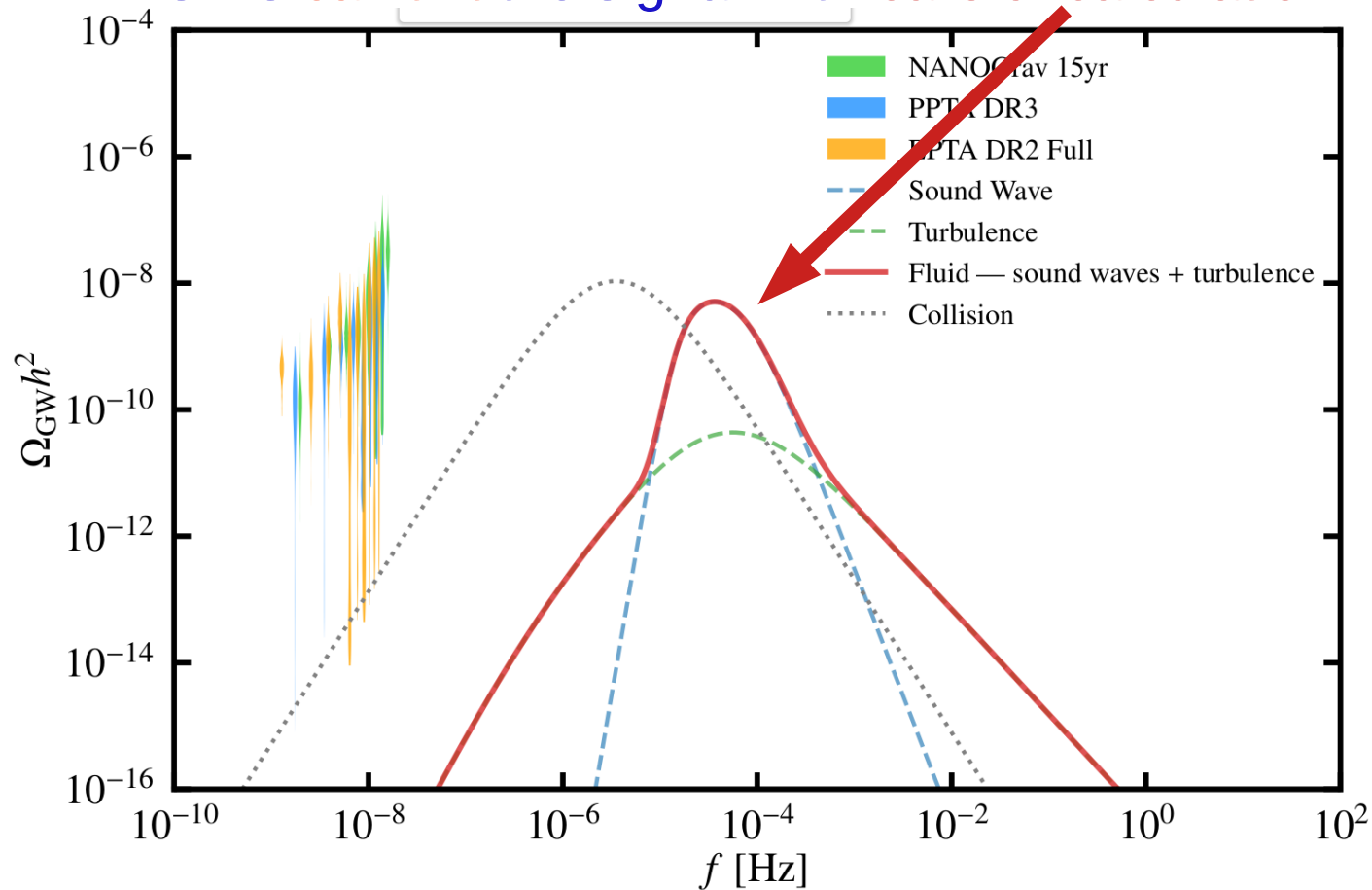
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But for the prototypical model of supercooled PTs  
cited by NANOgrav as a possible explanation:

**GWs can't fit the signal with careful calculation**



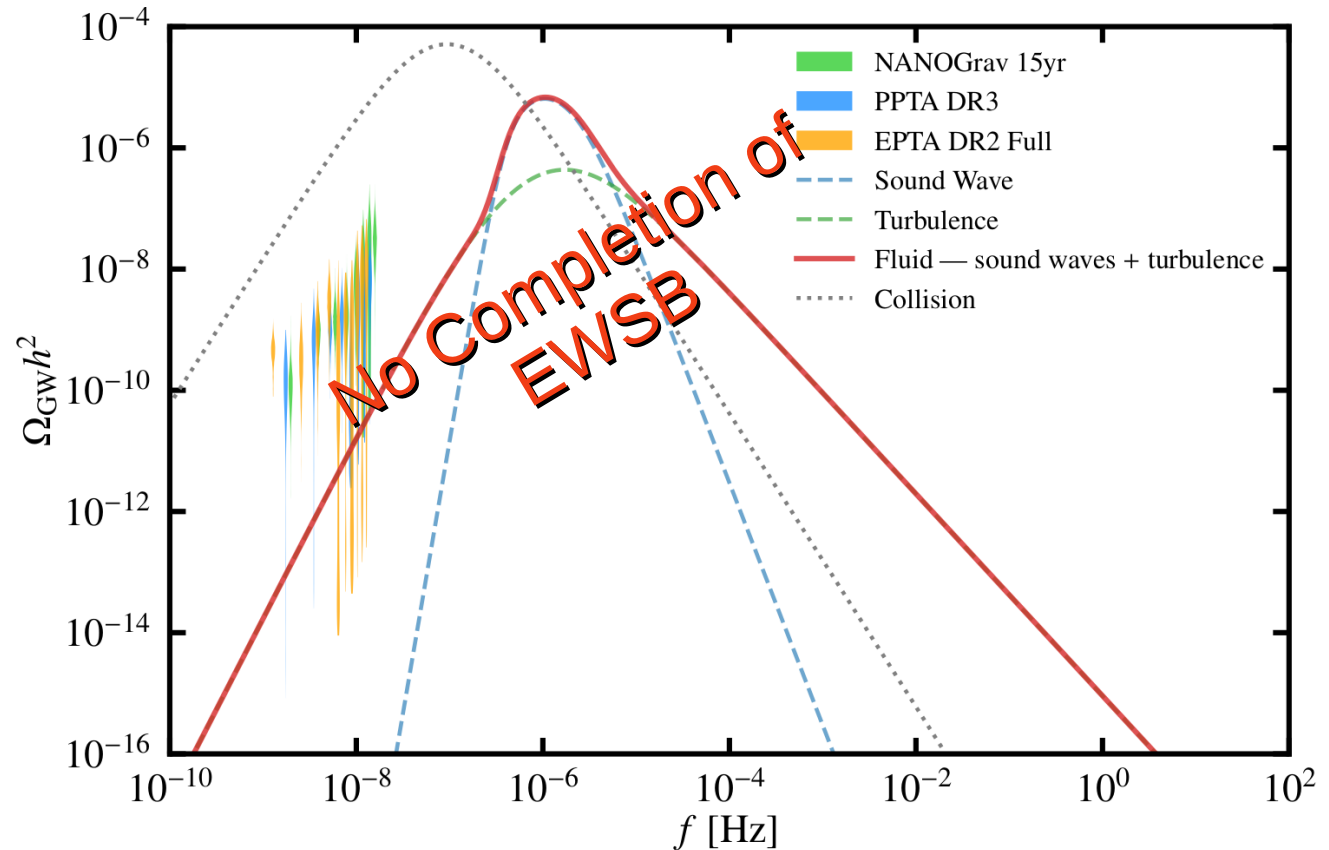
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Big news last month:

A stochastic gravitational wave background has been observed  
by multiple Pulsar Timing Arrays experiments

Larger signals are ruled  
out in this model  
because the PT does not  
complete

This is the first of the  
subtle effects I will  
discuss today!



## Check the phase transition completes

Especially for EWBG studies a common procedure was just checking if a FOPT had  $\gamma = v/T_c$

More careful studies checked that bubbles nucleate (one per Hubble volume )

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Much better to calculate the false vacuum fraction

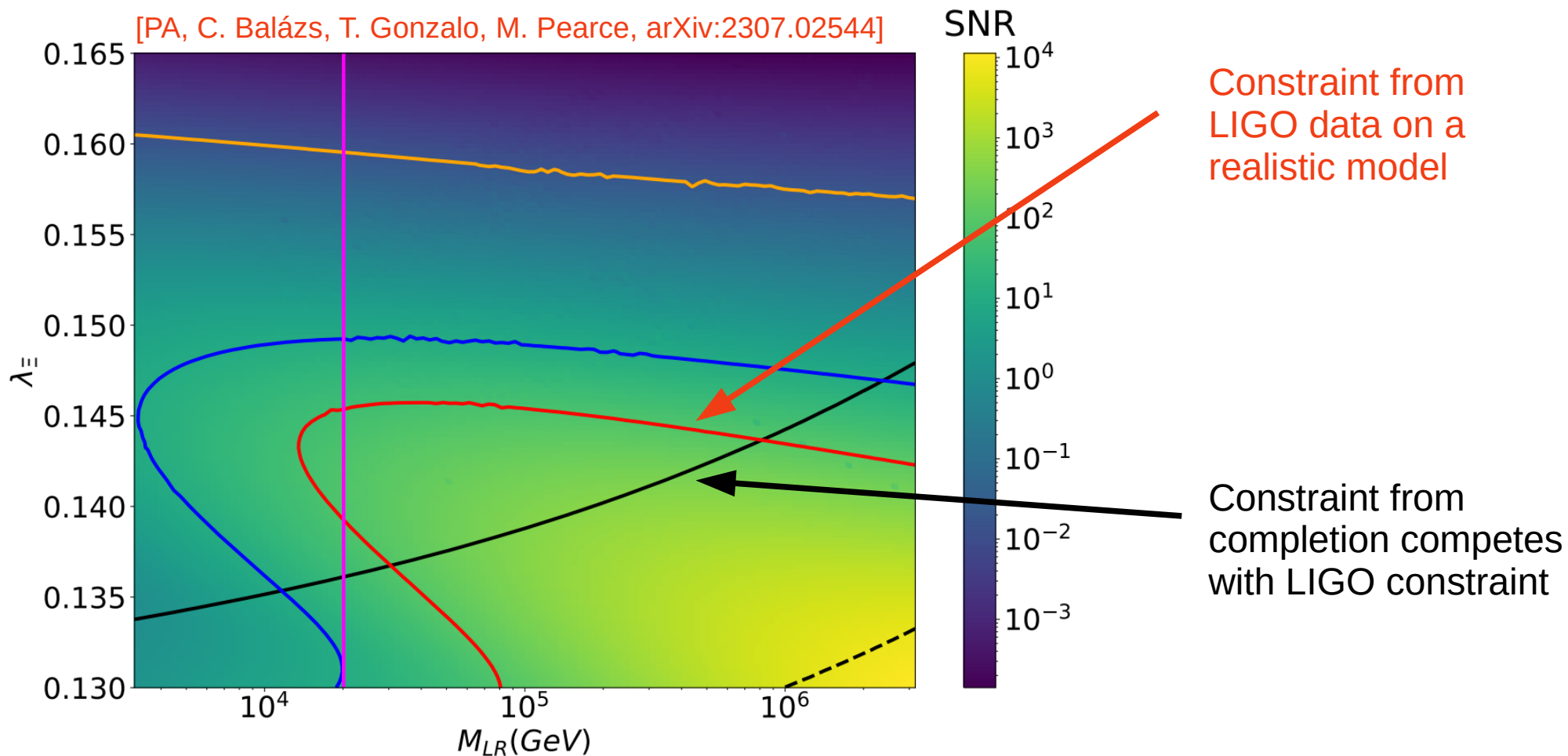
$$P_f(T) = \exp \left[ -\frac{4\pi}{3} v_w^3 \int_T^{T_c} \frac{\Gamma(T') dT'}{T'^4 H(T')} \left( \int_T^{T'} \frac{dT''}{H(T'')} \right)^3 \right]$$

Check this can be reduced to a sufficiently small value, e.g.  $P_f(T_f) < 0.01$

# LIGO data already constrains well motivated Pati-Salam GUT models

But checking completion is essential here too!

[PA, C. Balázs, T. Gonzalo, M. Pearce, arXiv:2307.02544]





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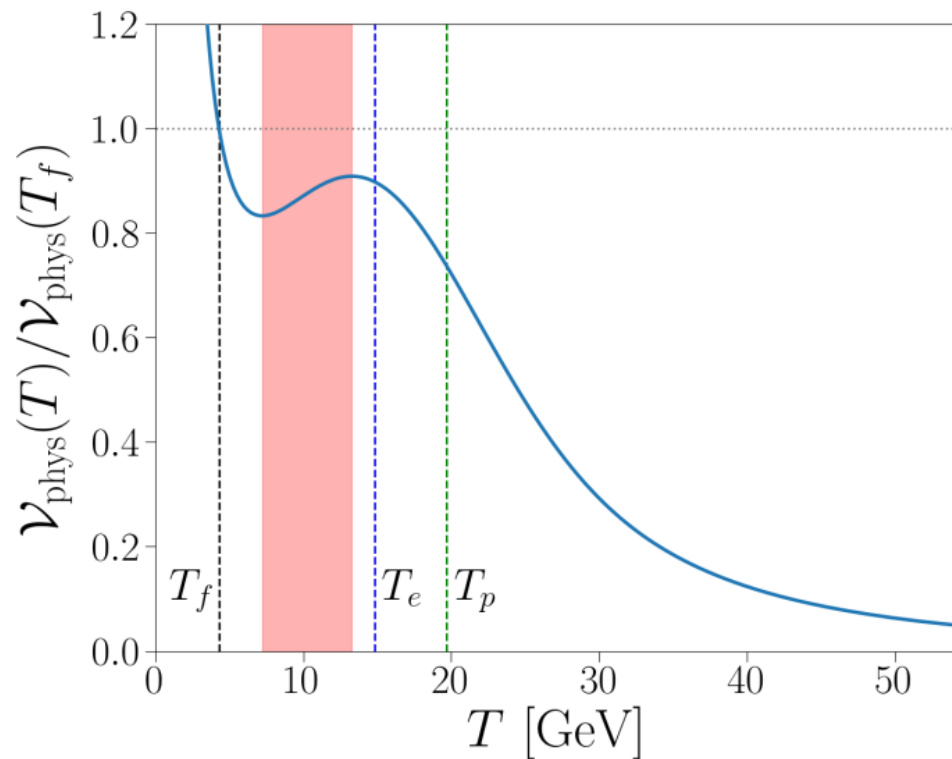
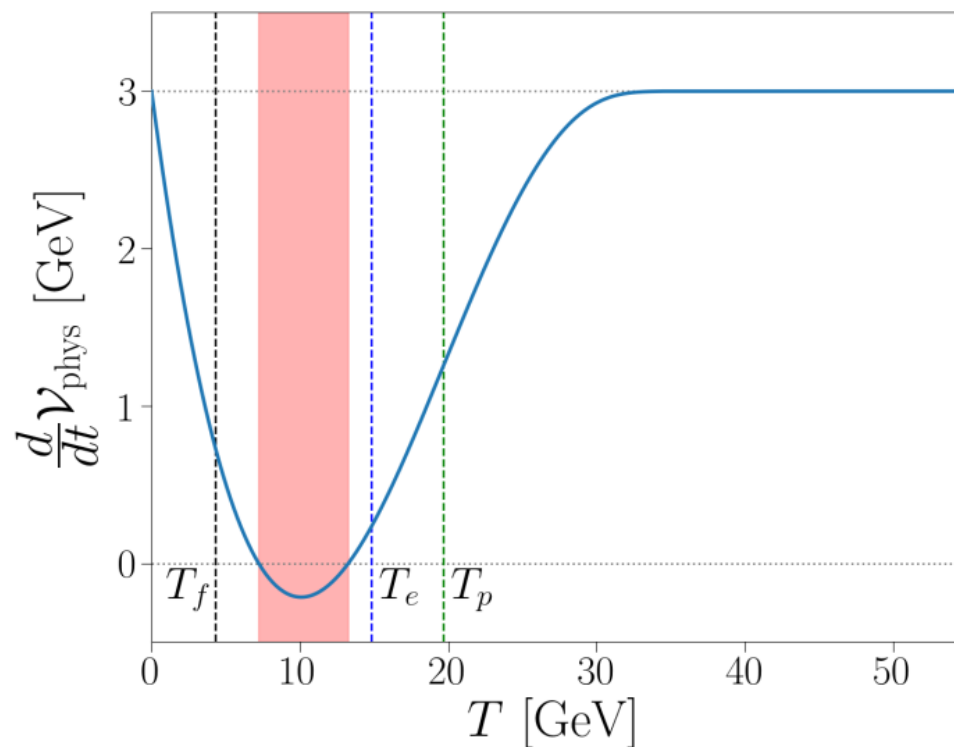
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Check the this can be reduced to a sufficiently small value, e.g.  $P_f(T_f) < 0.01$

Warning: even this may not be enough to guarantee completion since space between the bubbles is also growing.

## Tricky Effects from expansion of space



[PA, C. Balázs, L. Morris, JCAP 03 (2023), 006]

To ensure it really completes also require:

$$3 + T_p \left. \frac{d\mathcal{V}_t^{\text{ext}}}{dT} \right|_{T_p} < 0,$$

## Temperature dependence

Many studies have nucleation temperature as the reference temperature

But the nucleation temperature is not really connected to bubble collisions

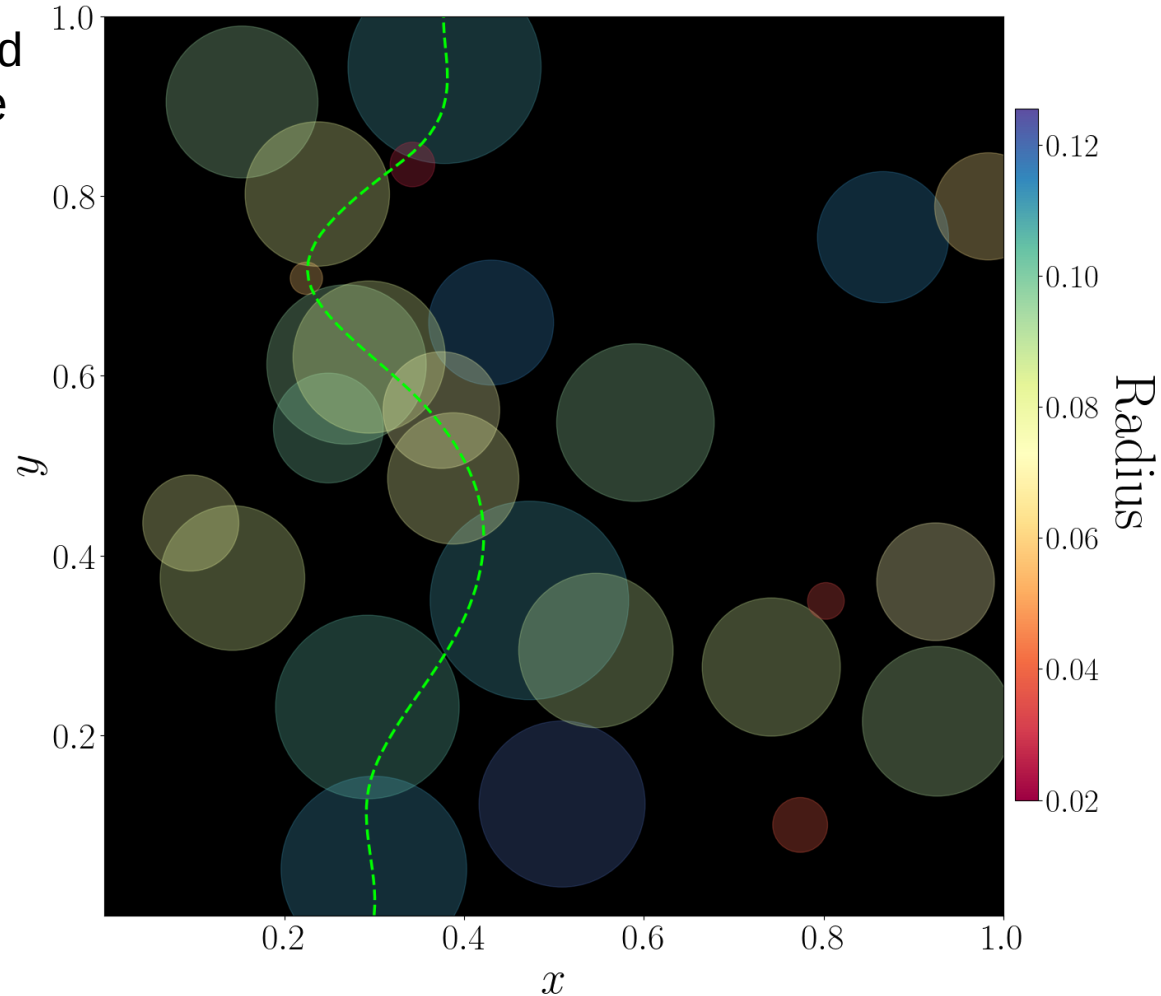
Percolation is directly defined in terms of contact between bubbles

# Percolation tempearture

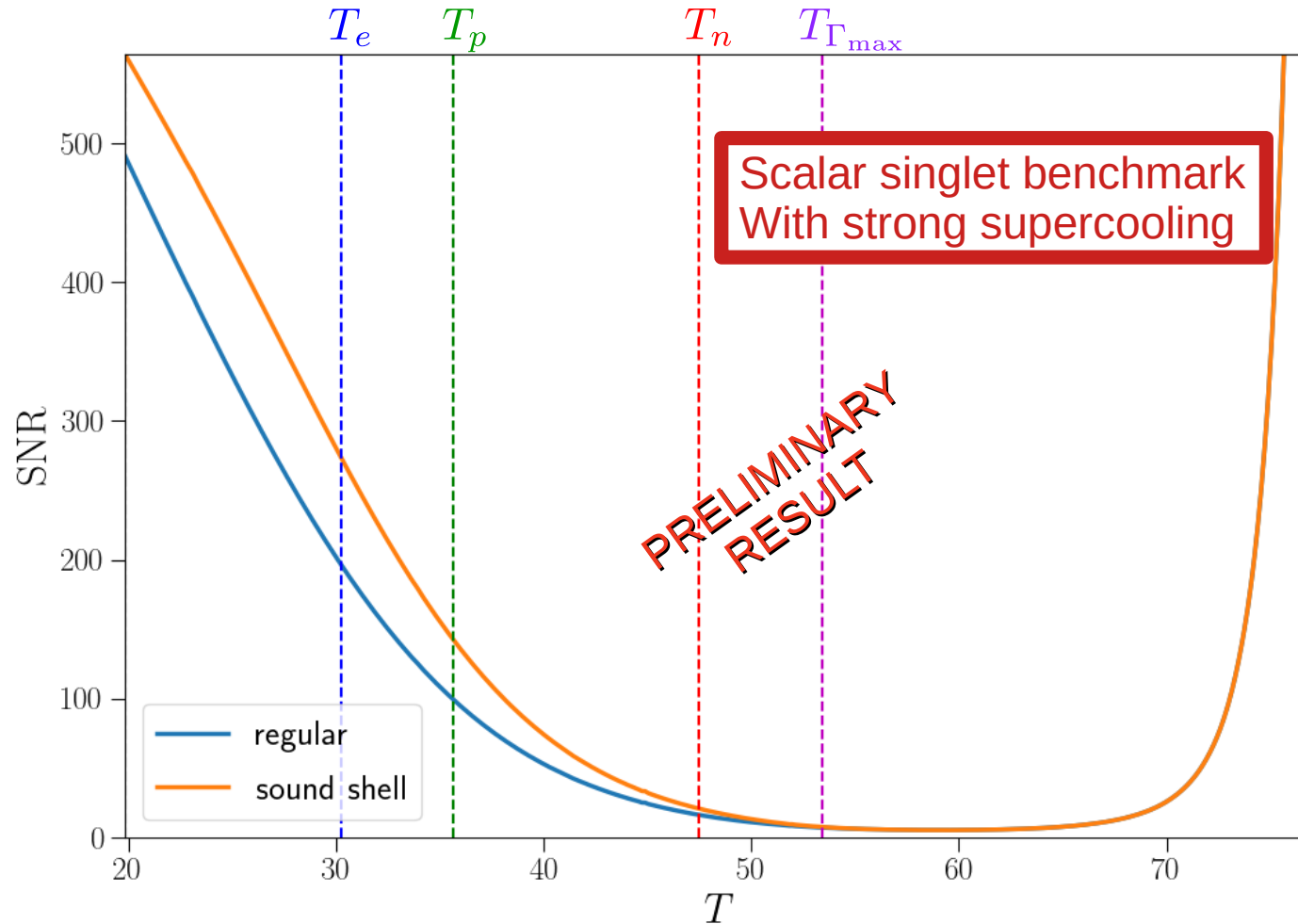
Percolation is when there is a connected path between bubbles across the space

Strongly linked to bubble collisions

Good choice for a temperature at which to evaluate thermal parameters determining the GWs spectrum



# Temperature dependence



PRELIMINARY  
RESULT

Very significant difference  
between SNR at  
percolation vs nucleation!

[PA, Lachlan Morris, Zhongxiu Xu, Preliminary findings]

# Temperature dependence

Many studies have nucleation temperature as the reference temperature

But the nucleation temperature is not really connected to bubble collisions

Percolation is directly defined in terms of contact between bubbles

We recommend use of percolation temperature based on this reasonable argument

But its still not established what the “right” reference temperature is or what the uncertainty using any single reference temperature is.

These issues could be probed with a hydrodynamic simulation performed alongside an analysis of the false vacuum fraction and thermal parameters.

This could help bridge the gap between lattice and non-lattice studies.

# Thermal parameters and gravitational waves

Many of the commonly used approximations in thermal parameters and GW spectra calculations/simulations/fits may not apply to scenarios considered.

Here I can only list a few examples:

**Bag model** frequently assumed in e.g. approximations for “strength” of transition  $\alpha$

**Radiation domination** assumed for Hubble parameter in e.g. false vacuum fraction, or maybe even in redshift factors

**Simultaneous nucleation** often used in simulations, but in realistic models the rate is often exponential or gaussian and it can vary with parameter space requiring extrapolation.

**Further extrapolation** beyond the range of validity for in terms of transition strength, bubble wall velocity and expansion modes, as well as with assumptions about the bag model

# Transition Solver

The good news is many of these issues can be avoided with careful numerical implementations

[TransitionSolver](#) is designed to treat these issues as well as can feasibly be done in BSM studies

[TransitionSolver](#) finds possible FOPTs, checks they complete, computes thermal parameters and gravitational wave spectra as well as we are able.

→ v1 Release is imminent, ETA by end of summer 2023

→ Future releases (v2) will automate effective potential, link to [DRalgo](#) for best feasible handling of effective potential as well!



# Conclusions

Very exciting recent results indicate we have entered an era where GW experiments have sensitivity to SGBG from BSM physics

Now things are real and we really need to understand uncertainties and make reliable predictions of GW spectra from BSM physics scenarios.

There are many subtle issues and hidden assumptions on predictions applied in the literature.

These can have a big impact on the predictions, substantially affecting whether exclusion bounds or whether a signal is fitted

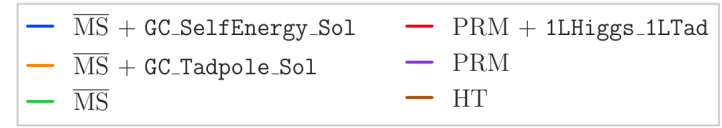
It's very important that the theory community takes this seriously and BSM predictions are done as well as possible

[TransitionSolver](#) is here to help!

The END

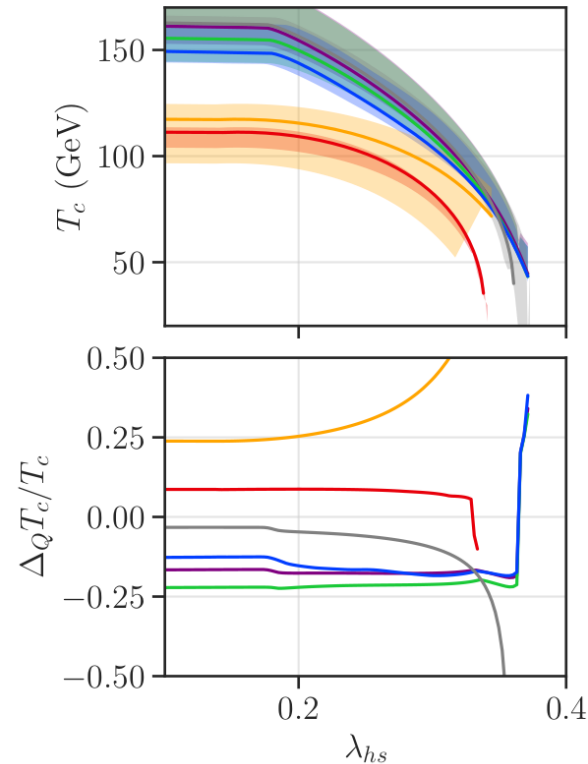
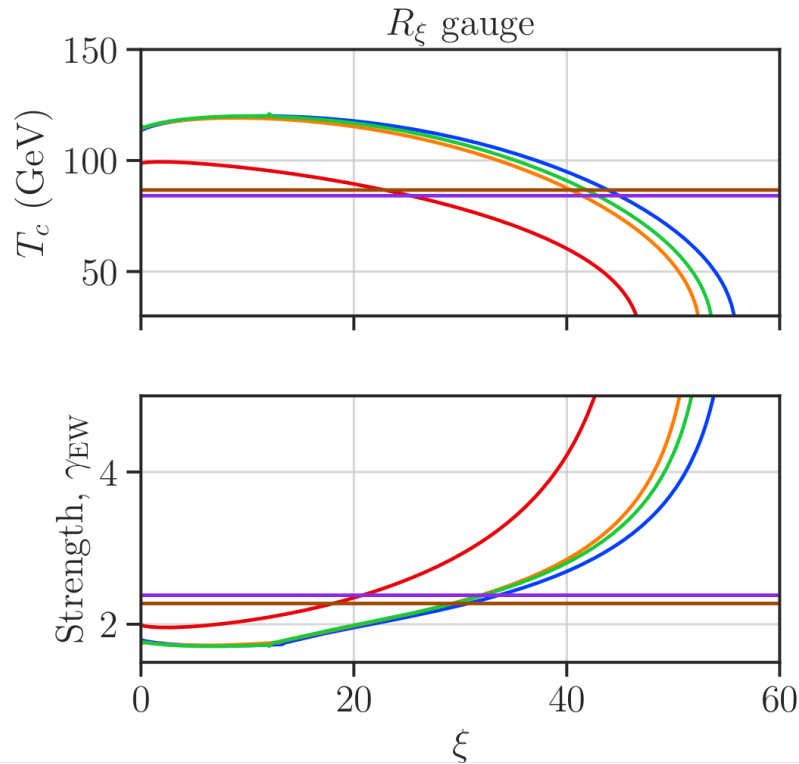
Thanks for listening!

# Effective Potential



Perturbative estimates of the effective potential can be tricky

Significant variance from gauge and renormalisation scale



# Effective Potential

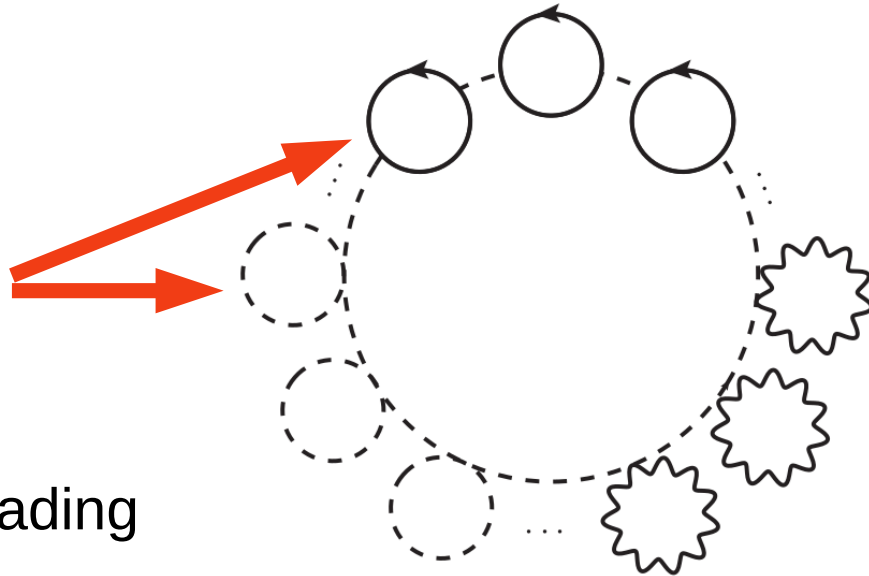
Perturbative estimates of the effective potential can be tricky

Significant variance from gauge and renormalisation scale

Resummation needed to deal with high temperatures spoiling perturbativity

Daisy diagram with N-loops

Individual petals are inserted  
one-loop corrections

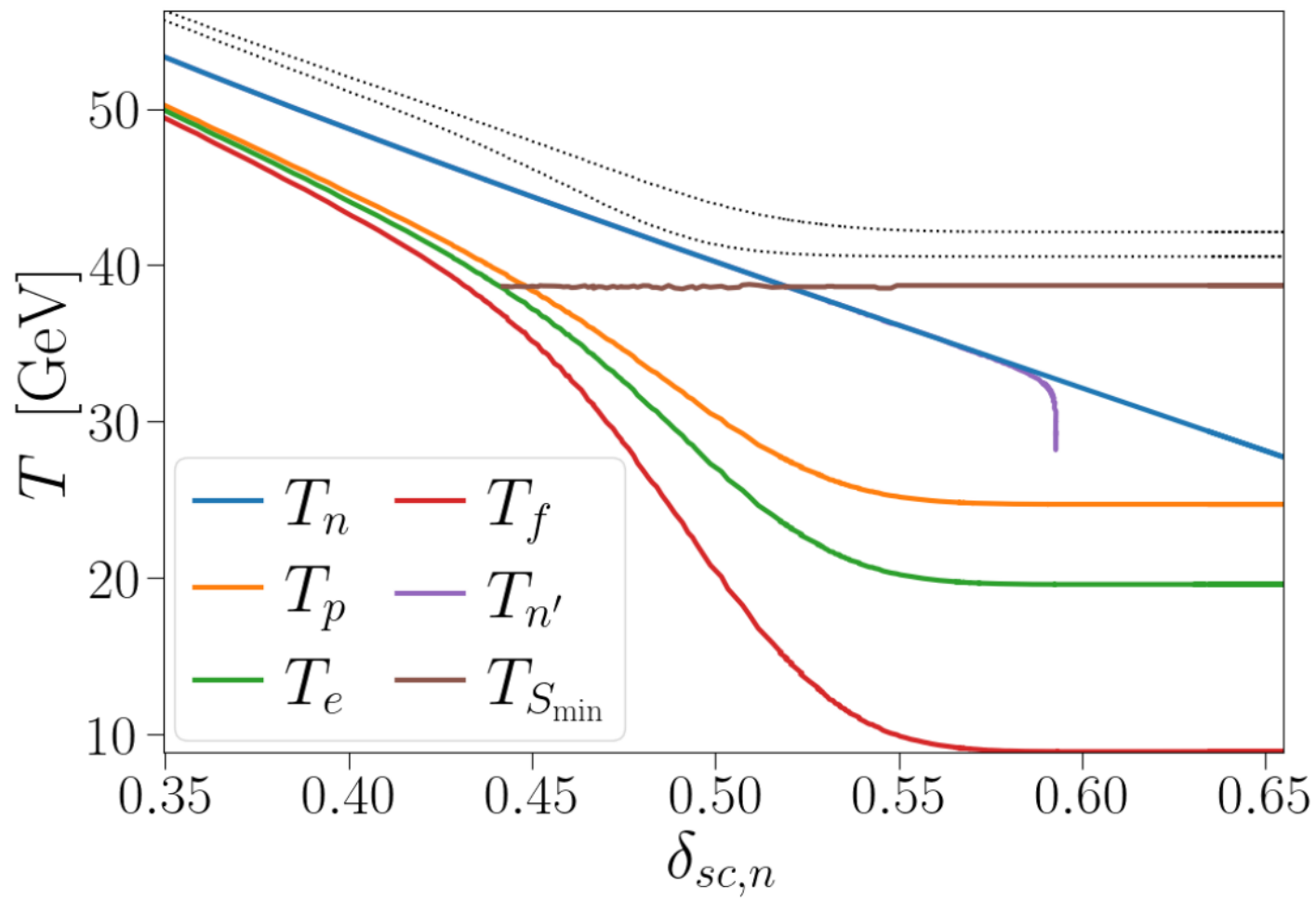


Resum daisy diagrams for leading  
order  $\frac{T^2}{m^2}$

## Effective Potential

- Better resummation by constructing a 3DEFT often called Dimensional Reductions (see e.g. D.Croon, O.Gould, P.Schicho, T.Tenkanen and G.White [JHEP 04 \(2021\) 055](#) )
- This can be done via automation of [DRalgo](#) for best feasible handling of effective potential as well!
- Gold standard is really to do things non-peturbatively on the lattice
- Not really feasible for scans in BSM models with many parameters
- But can be done for most exciting cases.

## Temperature dependence



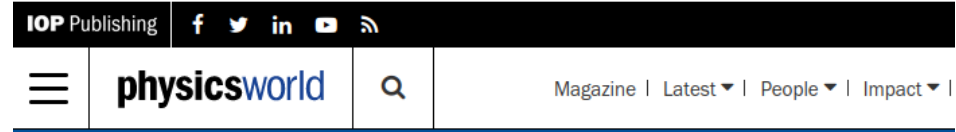
Big news last month:

A stochastic gravitational wave background has been observed  
by multiple Pulsar Timing Arrays experiments

Conservative interpretation would  
be supermassive black holes

Nonetheless SGWB is now a real  
thing to be used as data!

Now we really need to think about  
how precise our calculations are!



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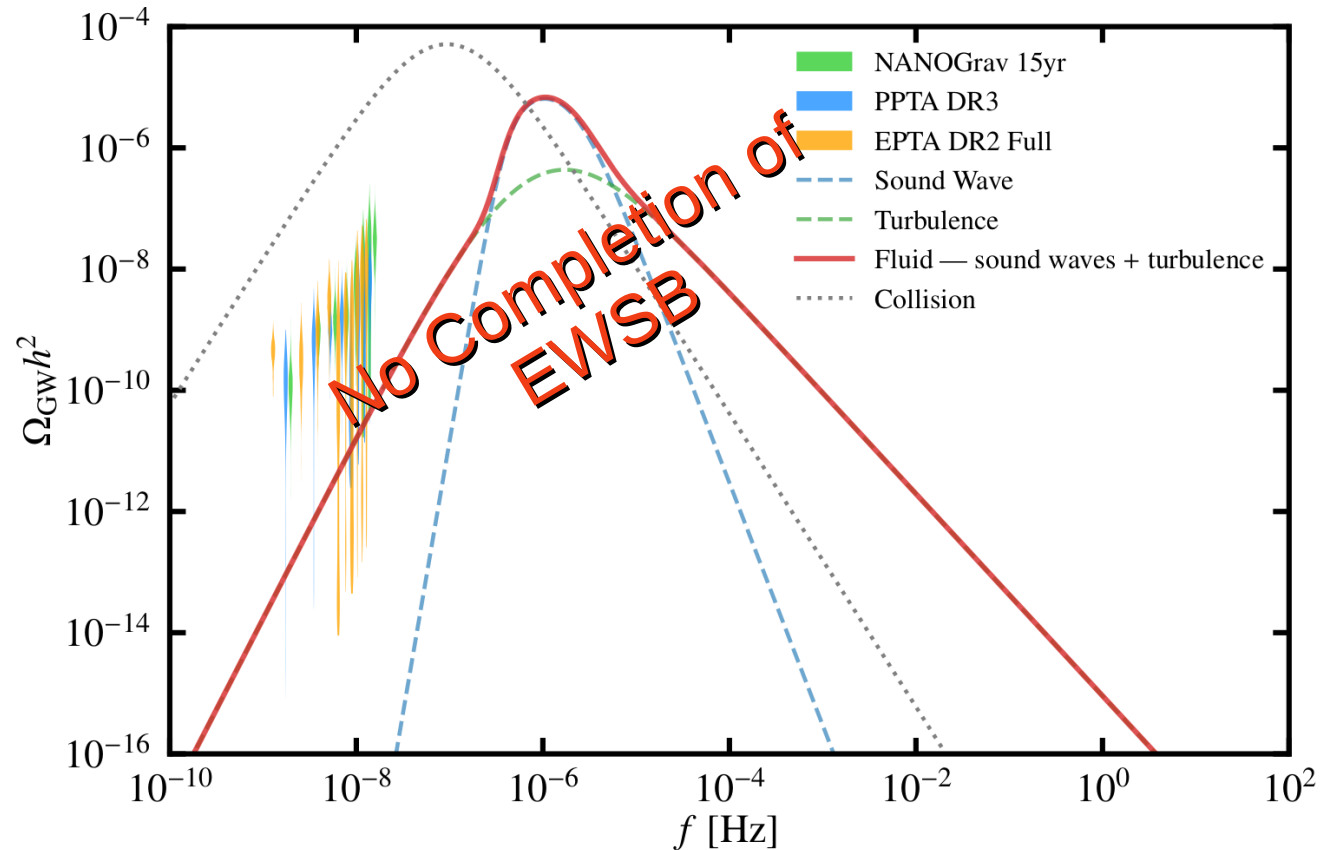


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precisely because of one  
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# GWs from First Order Phase Transitions

There are many subtleties and challenges in calculating GW spectra from cosmological PTs

For example this makes it easy to mistakenly predict a given model explains the data, get the wrong projections for future experiments or miss correlated features/constraints

I will discuss some subtle issues from JCAP 03 (2023), 006 and our review arxiv:2305.02357

- Nucleation is not enough – check PT completes
- Temperature dependence is very important, using most relevant temperatures really matters
- Hidden assumptions and approximations in thermal parameters and fits to calculations or simulations of gravitational wave spectra
- Resummation and gauge invariance in the effective potential treatment

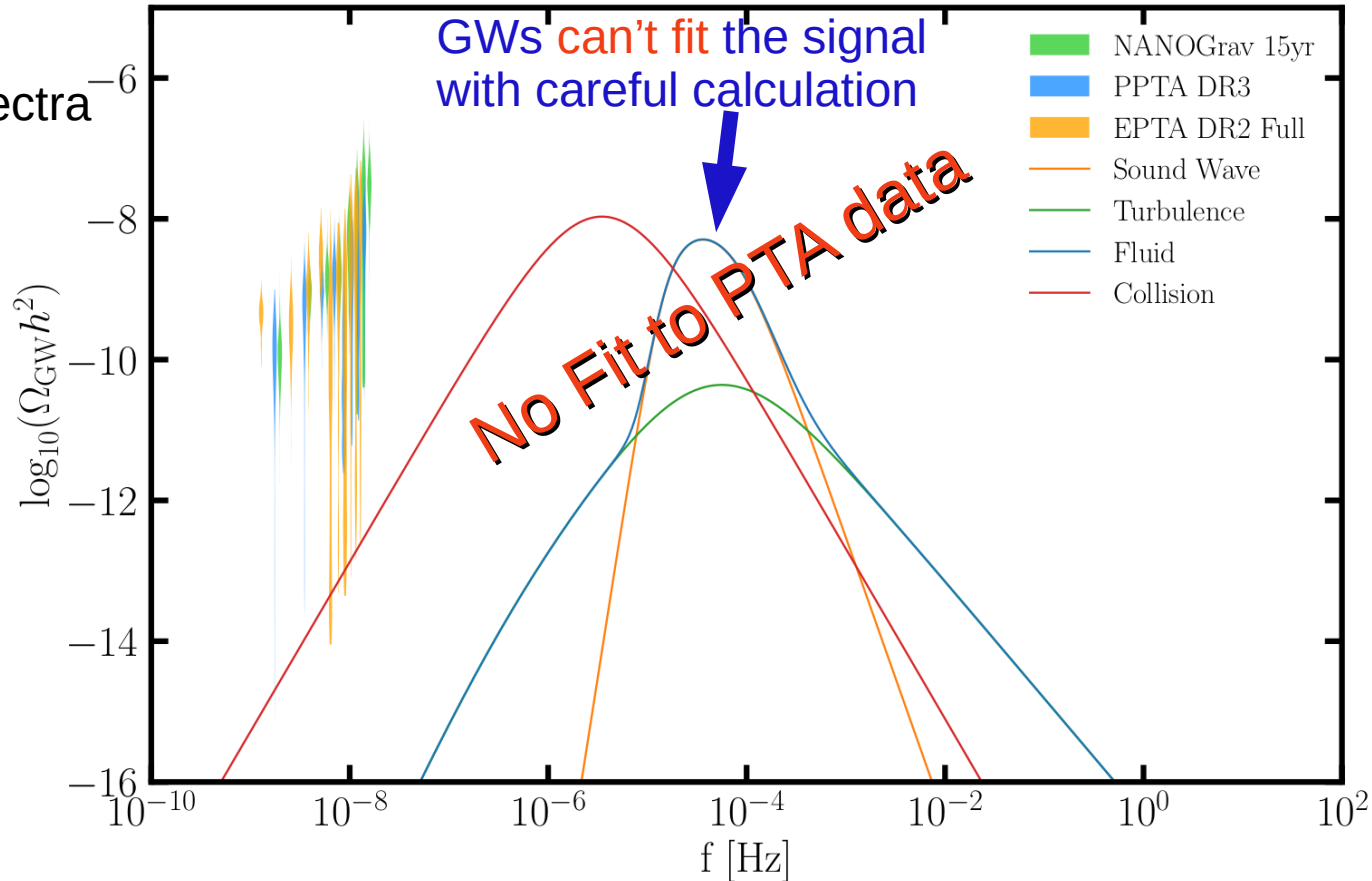
There are many other details I can't cover, see original papers for details

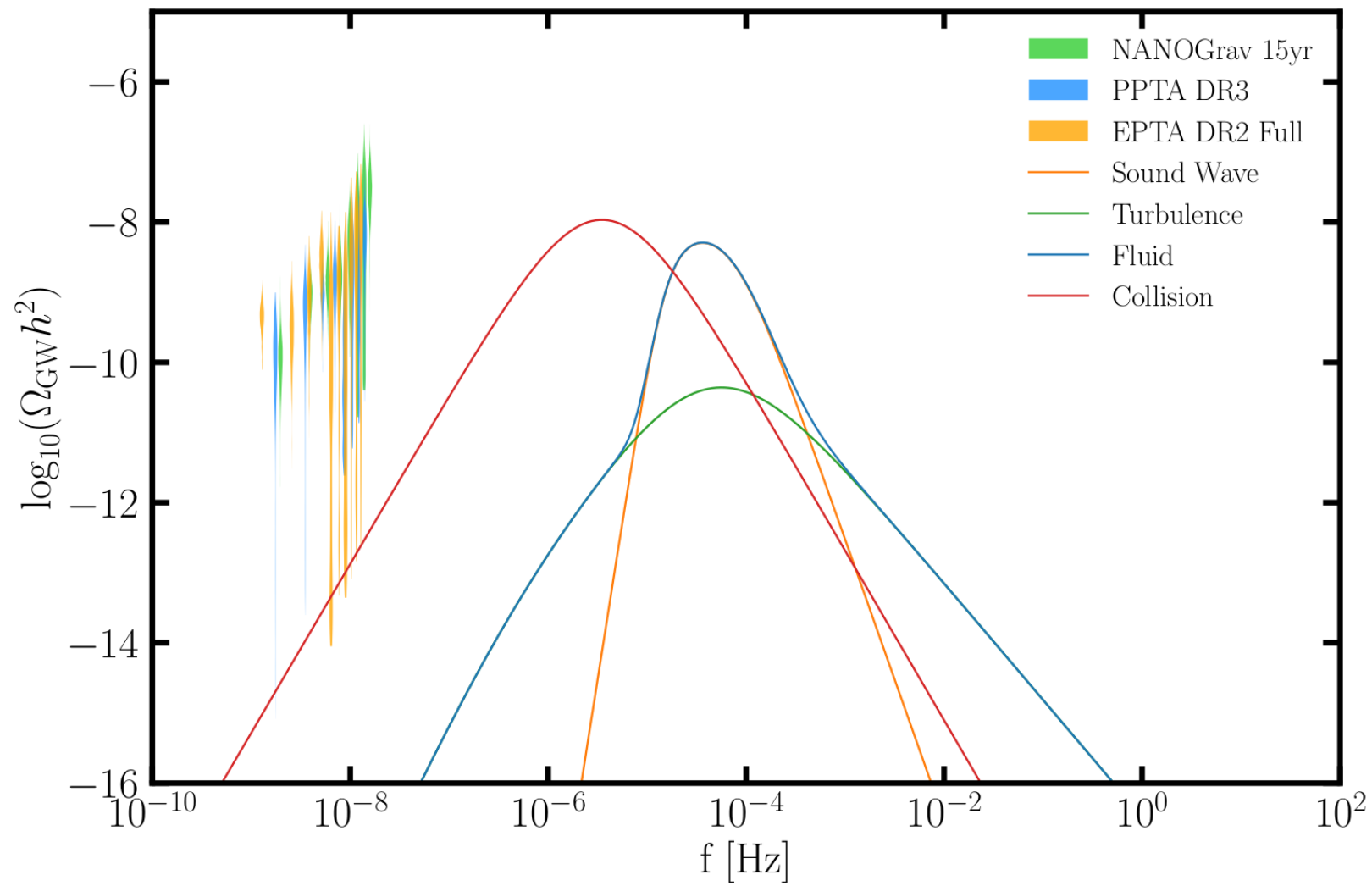
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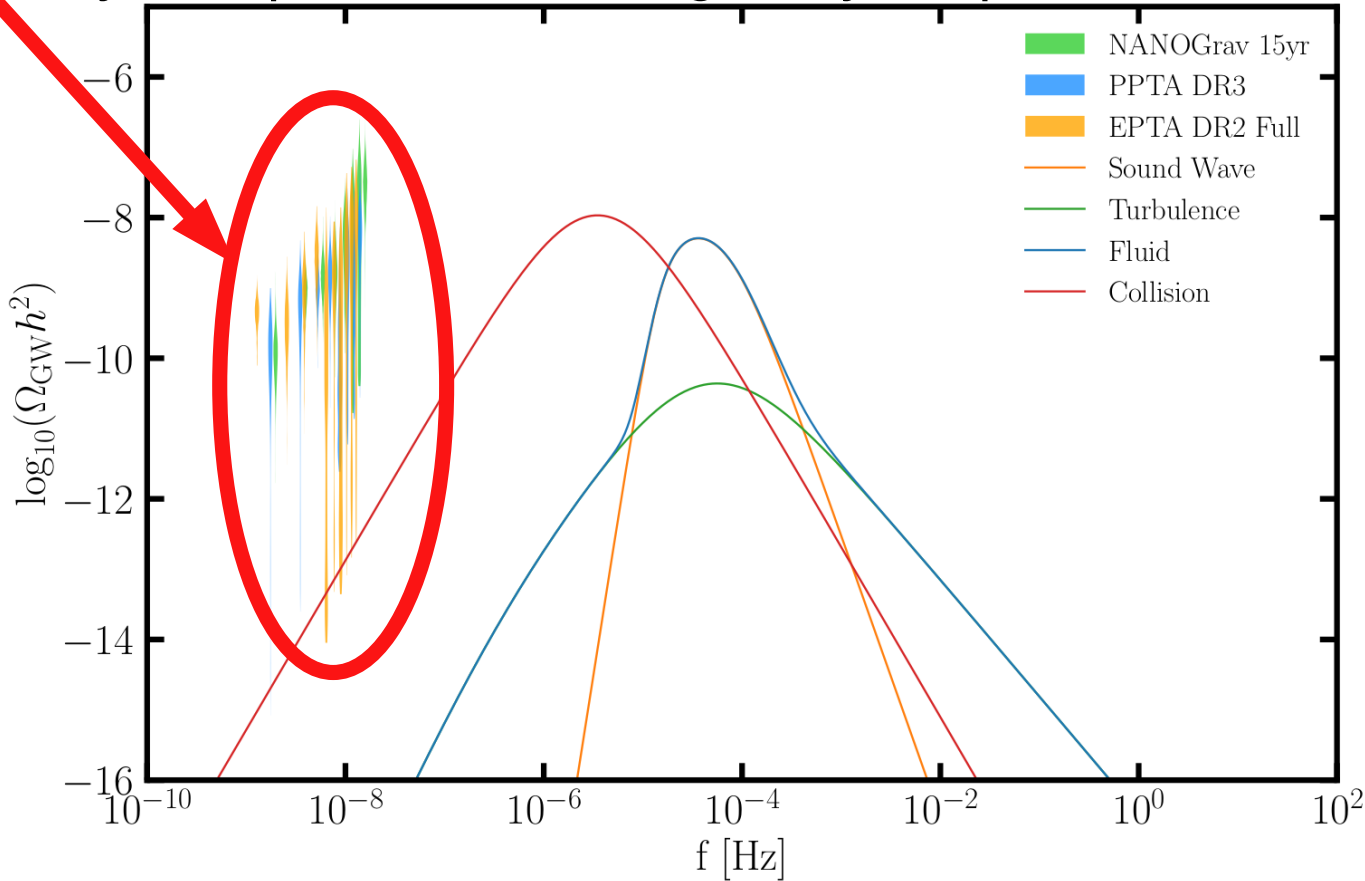
Easy to mistakenly predict a  
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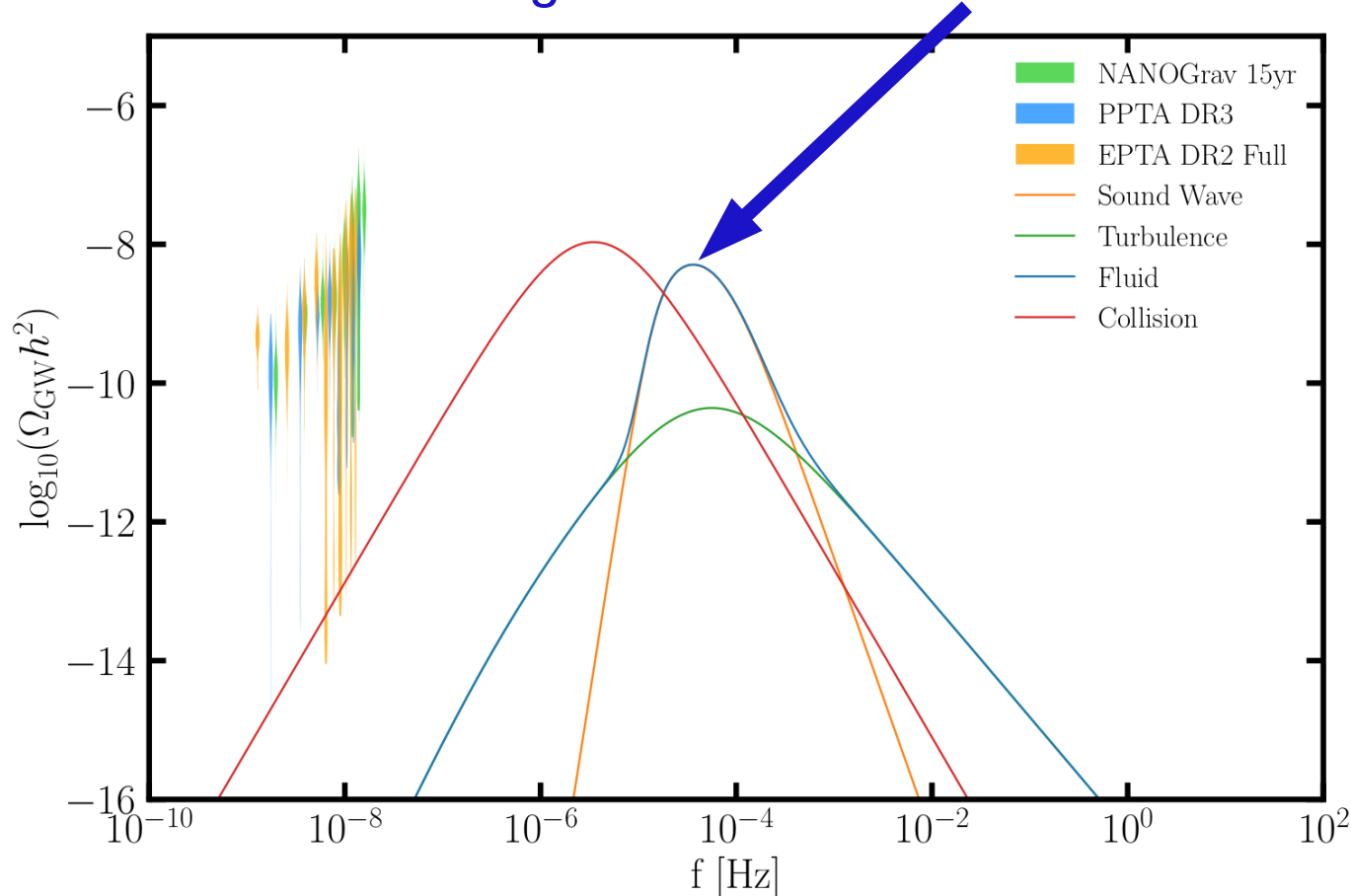


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