Top-down restrictions on scale-separation

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Based on [2005.11421] and [230x.xxxxx]

Motivation

The Universe appears four-dimensional; non-supersymmetric and has a small positive curvature (cosmological constant?)

String/M-theory : 10 or 11 dimensional; supersymmetric; no c.c.

Compactifications should produce positive external curvature, susy breaking and scale separation.

No-go results in 2-derivative supergravity against positive curvature and/or scale-separation, but string theory is not just supergravity

Are scale-separated solutions possible in string theory? If so, what are the necessary ingredients? If not, what principles forbid it?

Perturbative expansions in string theory

String/M-theory has no free parameters

Equations of motion are arranged in a derivative expansion from powers of curvatures and field strengths

Asymptotic regimes defined by small/large expectation values

Once a background is chosen, can expand in powers of the background fields

Need to treat possible coordinate dependence of the fields especially if they exit the asymptotic regime

Asymptotic expansions and transseries

Perturbative expansions are typically divergent, asymptotic series

Can be Borel summed, but induces "non-perturbatively" small ambiguities.

Can extend the asymptotic series to a transseries

$$\sum_k A_k(\epsilon) e^{-s_k/\epsilon^{p_k}} \left(1 + \sum_n a_{k,n} \epsilon^n\right) \tag{1}$$

such that the ambiguities cancel.

Can in principle be reconstructed from the perturbative series alone via resurgence relations.

Well defined at any value of the expansion parameter, even far from the original asymptotic regime.

Types of corrections

- Perturbative bulk corrections, e.g.

$$\int d^D x \sqrt{g} R^{l_1} \nabla^k R^{l_2} F^{n_1} \nabla^m F^{n_2} \tag{2}$$

- Localized terms from extended objects

$$\int d^D x \sqrt{g} \frac{1}{\sqrt{g_\perp}} \delta(x_\perp)(\mu_p + \ldots) \tag{3}$$

- Non-local corrections

$$\int d^D x d^D x' \sqrt{g(x) g(x')} O(x) G(x - x') O'(x') \tag{4}$$

Non-local corrections and the transseries structure

The non-local corrections will dominate at low energy unless exponentiated

$$\begin{aligned} &\int d^D x \sqrt{g(x)} O(x) \left(a_1 e^{-\mathcal{I}(x)} + a_2 e^{-2\mathcal{I}(x)} + \ldots \right) \\ &\mathcal{I}(x) = \int d^D x' \sqrt{g(x')} G(x - x') O'(x') \end{aligned} \tag{5}$$

This promotes the derivative expansion to a (resurgent?) transseries expression, which we will assume can be consistently resummed and therefore valid for all field values!

Gives non-local corrections a natural interpretation in terms of brane instantons or similar effects

Background ansatz

We make the (string frame) metric ansatz

$$ds^2 = w^{2A} \frac{1}{\Lambda x_3^2} \eta_{ij} dx^i dx^j + v^{2B} g_{mn} dy^m dy^n \tag{6}$$

with $\tau = e^{-\phi}$ as well as v, w functions of y

Assume $v > 1$ on most of the internal manifold and $B > 0$ such that $\int \sqrt{g} d^6 y = 1$. Weak string coupling and large volume expansions in 4d require $v \gg 1$, $\tau \gg 1$ on most of the manifold

Scale separation when $\Lambda \langle v^{2B-2A} \rangle \ll 1$ for slowly varying v, w.

Expanding the Fluxes and EOM

Expand all other fields as

$$\Phi = v^a \left(\Phi^{(0)} + \sum_n v^{-\beta n} \Phi^{(n)} \right) + \text{N.P. terms} \qquad (7)$$

where $\Phi^{(n)}$ can still have y dependence that isn't proportional to any powers of v.

Choose A such that $w \sim v + \dots$

Treat derivatives hitting v as a further expansion in $\frac{\partial v}{v}$, $\frac{\Box v}{v}$ etc.

Equations of motion will now split in powers (and nonperturbative exponentials) of v

This is an expansion in a basis of linearly independent functions, so cancellations must happen order by order, even in regions where v is not large!

Scalings in Einstein Equation

We consider Einstein's equation with one index raised so that the v scaling of each term is the same as that of a fully contracted scalar.

$$g^{MP}\frac{\delta S}{\delta g^{PN}} \sim S \tag{8}$$

${R^M}_N$ has the schematic form $g^{-2}\partial^2 g + g^{-3}\partial g \partial g$

For block-diagonal metric, curvature components scale according to the inverse metric that contracts the derivatives, giving two types of leading order scaling

$$R_{(ext.)} \sim v^{-2A} \;, \qquad R_{(int.)} \sim v^{-2B} \tag{9}$$

The mismatch

The v^A dependence of the flux EOM and stress tensor cancels out!

For generic $A \neq B$, $R_{(ext)}$ and $R_{(int)}$ will not have the same scaling, so will need several types of ingredients.

Not a problem for compactifications to flat space.

Invisible when averaging over the internal manifold.

Localized sources can't help even if they have the right scaling.

KKLT-type scenario

Kachru et al. '03

Type IIB with τ constant and $v^B = w^{-A} = h(y)$

$$ds^2 = \frac{1}{h^2}\frac{1}{\Lambda x_3^2}\eta_{ij}dx^i dx^j + h^2 g_{mn}dy^m dy^n \tag{10}$$

then

$$R_{(ext.)} \sim h^2 \ , \qquad R_{(int.)} \sim h^{-2} \tag{11}$$

Fluxes must scale as

$$C_4 \sim h^{-4} \ , \qquad F_3 \sim H_3 \sim h^2 \implies T^M_N \sim h^{-2} \tag{12}$$

for all fluxes. Need a relative factor of h^4 to cancel external curvature.

Nonlocal terms and D3-instantons

The stress tensor contribution from a nonlocal correction is

$$T^M{}_N(x) = g^{MP}(x) \int d^D x' \sqrt{g(x')} O(x') \frac{\delta \mathcal{I}(x')}{\delta g^{PN}(x)} e^{-\mathcal{I}(x')} + \ldots \quad (13)$$

Exponentially suppressed and carries positive powers of v.

Taking $G(X - X') = \delta^{(4)}(x - x')\delta^{(2)}(y_\perp - y'_\perp)$ gives

$$g^{MP} \frac{\delta \mathcal{I}(x')}{\delta g^{PN}(x)} e^{-\mathcal{I}(x')} \sim h^4 e^{-\langle h^4 \rangle} \implies \Lambda \sim e^{-\langle h^4 \rangle} \quad (14)$$

and so $\Lambda \langle h^4 \rangle \ll 1$ despite $h \gg 1$

This is precisely the D3-instantons used in KKLT!

Smeared DGKT

De Wolfe et al. '05

The smeared DGKT metric has $A = 3B$. Fluxes scale as

$$F_2 \sim v^{2B} \ , \qquad F_4 \sim v^{4B} \ , \qquad H_3 \sim v^0 \ , \qquad \tau \sim v^{3B} \tag{15}$$

with $F_6 = 0$, leading to all field stress tensors $T^M{}_N \sim v^0$. Meanwhile

$$\tau^2 R_{(ext.)} \sim v^0 \ , \qquad \tau^2 R_{(int.)} \sim v^{4B} \tag{16}$$

so $R_{(ext.)}$ is formally subleading to $R_{(int.)}$ in the v^{-1} expansion, however $R_{(int.)}$ actually vanishes in the smeared solution, so no mismatch!

Unsmeared DGKT

Junghans '20, see also Marchesano et al. '20

The unsmeared solution is warped and $R_{(int.)}$ no longer vanishes.

Need a stress tensor that scales as v^{4B}. D3-instantons unavailable!

Could consider for example $F_6 \sim v^{8B}$ but this is not the correct scaling of the leading order backreaction.

Leading order unsmeared solution treats corrections to metric and warp factor as fluctuations so they do not yet affect scalings.

Need to check next to leading order!

Conclusions

Transseries expansion of equations of motion organizes field and derivative expansions as well as non-perturbative effects.

Expansion in regime-defining local field values provides restrictions on possible solutions or guidance towards necessary ingredients.

Warped compactifications with scale-separation typically feature a "mismatch" of scalings requiring several types of ingredients.

KKLT-type scale-separation allowed. Scaling mismatch requires D3-instantons as expected.

Smeared DGKT is special due to trivial flux EOM, and vanishing internal curvature.

Puzzles for unsmeared DGKT. Invisible at leading order unsmearing. Need quantum effects?

Thank You!