

Large lepton asymmetry from Affleck-Dine leptogenesis and Sterile neutrinos as dark matter

Kentaro Kasai

ICRR, The University of Tokyo

Collaboration with Masahiro Kawasaki and Kai Murai

Ongoing work

Motivation

“Sterile” neutrino

Neutrino mass

Dark matter

This presentation

Shi-Fuller mechanism :

Can explain the origin of all dark matter by sterile neutrinos with large initial lepton asymmetry

$$|\eta_L| (\gg |\eta_b^{\text{obs}}| \sim 10^{-10})$$

Affleck-Dine leptogenesis + Q-balls :

Can explain the large hierarchy between lepton/baryon asymmetry in SUSY setup

Outline of the scenario

Affleck-Dine leptogenesis



Large initial lepton asymmetry $\eta_L \gg$



Sphaleron process

Observed
baryon asymmetry
 $\eta_b \sim 10^{-10}$

Sterile neutrino dark matter
(by Shi-Fuller mechanism)

Question

- Q1. In Shi-Fuller mechanism,
what is the favorable initial value of lepton asymmetry
to explain all dark matter?
- Q2. Is there any favorable model parameter
to explain both the required lepton asymmetry
and observed baryon asymmetry
in Affleck-Dine leptogenesis?

Question

Q1. In Shi-Fuller mechanism,
what is the favorable initial value of lepton asymmetry
to explain all dark matter?

Q2. Is there any favorable model parameter
to explain both the required lepton asymmetry
and observed baryon asymmetry
in Affleck-Dine leptogenesis?

1. Review of Shi-Fuller mechanism
2. Revisiting the calculation of Shi-Fuller mechanism
3. Large lepton asymmetry from Affleck-Dine leptogenesis

1. Review of Shi-Fuller mechanism

2. Revisiting the calculation of Shi-Fuller mechanism

3. Large lepton asymmetry from Affleck-Dine leptogenesis

Sterile neutrino dark matter: Dodelson-Widrow mechanism

S. Dodelson, L. M. Widrow, (1993)

Mixing between active neutrino and sterile neutrino

$$|\nu_a\rangle = \cos\theta_a |\nu_1\rangle + \sin\theta_a |\nu_2\rangle, \quad |\nu_s\rangle = -\sin\theta_a |\nu_1\rangle + \cos\theta_a |\nu_2\rangle$$

$|\nu_a\rangle, |\nu_s\rangle$: Flavor eigenstate $|\nu_1\rangle, |\nu_2\rangle$: Mass eigenstate

We assume that active neutrinos are in thermal equilibrium in the early universe.

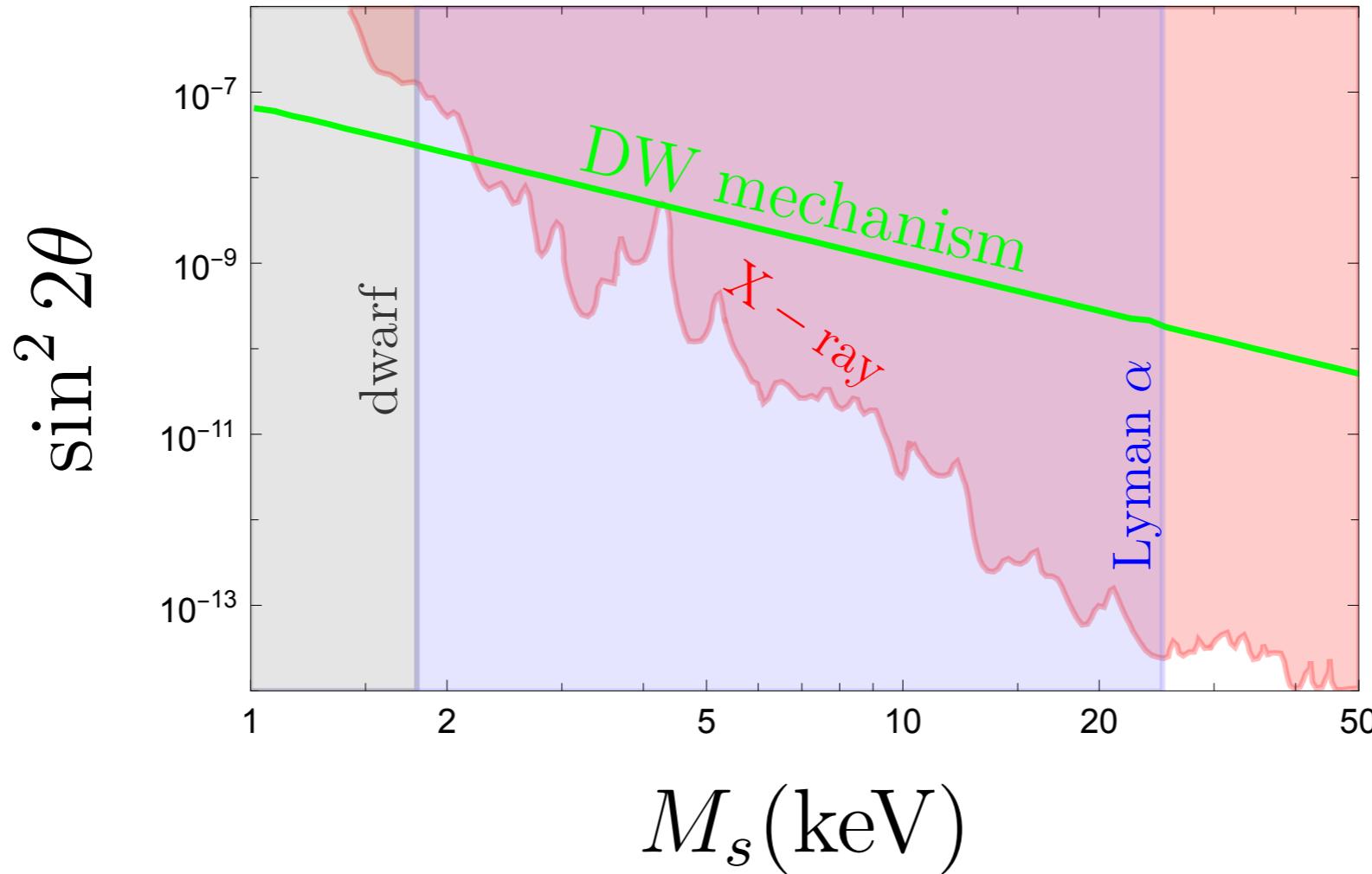
By neutrino oscillation effect,
some of them are finally observed as **“sterile” states**
= dark matter

Effective mixing angle between active and sterile states
(Thermal correction)

$$\sin^2 2\theta_{\alpha}^{\text{mat}} \equiv \frac{\sin^2 2\theta_{\alpha}}{\sin^2 2\theta_{\alpha} + (\cos 2\theta_{\alpha} + \frac{2r_{\alpha}G_F^2 E_{\nu}^2 T^4 / m_s^2}{})^2}$$

“Matter effect”

Constraints on sterile neutrino dark matter : X-ray observations, phase space constraint



Simple Dodelson-Widrow mechanism is already excluded by

1. X-rays from $\nu_s \rightarrow \nu_\alpha + \gamma$ (through active-sterile mixing) K. c. Y. Ng et al., (2019)
2. Upper bound of phase-space density due to Pauli blocking effect A. Boyarsky et al., (2009)
3. Effect on structure formation (Lyman α) C. Yache et.al., (2017)

Sterile neutrino dark matter : Shi-Fuller mechanism

X. D. Shi, G. M. Fuller, (1998)

We consider the effect of lepton asymmetry in SM neutrino sector

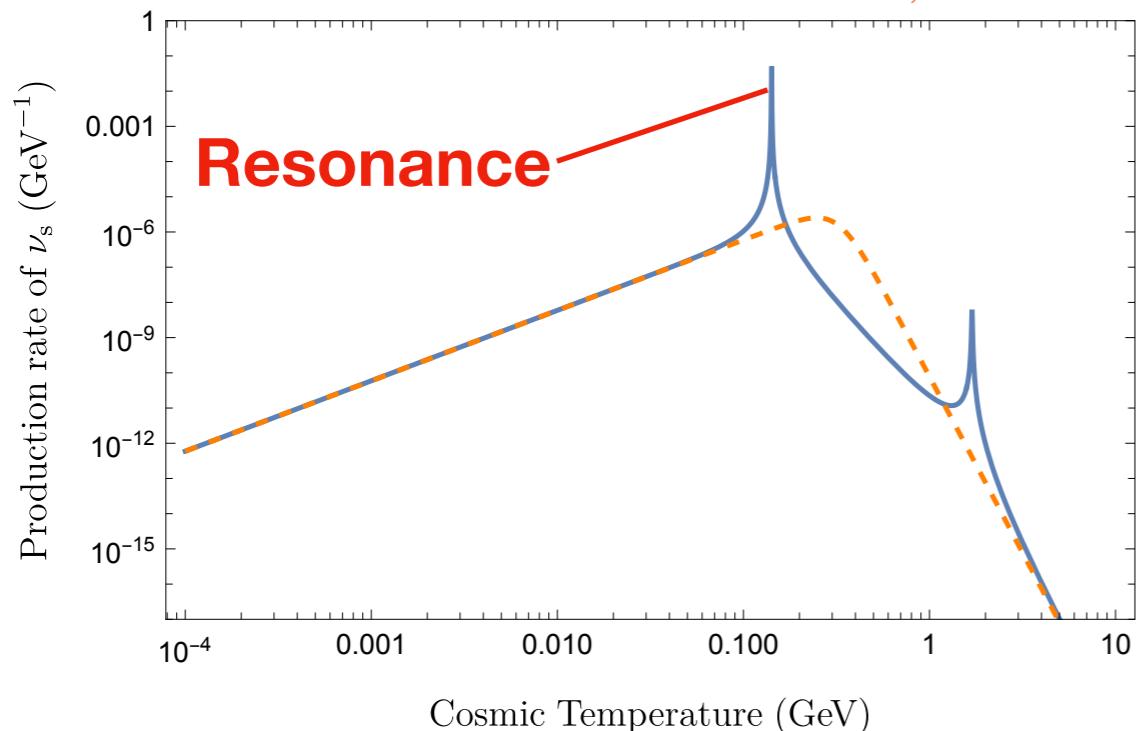
$$L_\alpha \equiv (n_{\nu_\alpha} - n_{\bar{\nu}_\alpha})/s_{\text{tot}} . \quad s_{\text{tot}} : \text{total entropy density}$$

Effective mixing angle :

$$\sin^2 2\theta_\alpha^{\text{mat}} \equiv \frac{\sin^2 2\theta_\alpha}{\sin^2 2\theta_\alpha + (\cos 2\theta_\alpha + 2E_\nu(-8\sqrt{2}G_F s_{\text{tot}} L_\alpha + r_\alpha G_F^2 E_\nu T^4)/m_s^2)^2} = 0 : \text{Resonance}$$

Production rate of ν_s (of mode with $\epsilon(\equiv E_\nu/T) = 1)$

$$m_s = 5\text{keV}, \sin^2 2\theta_0 = 10^{-11.65}, L_{e,0} = 10^{-4}$$



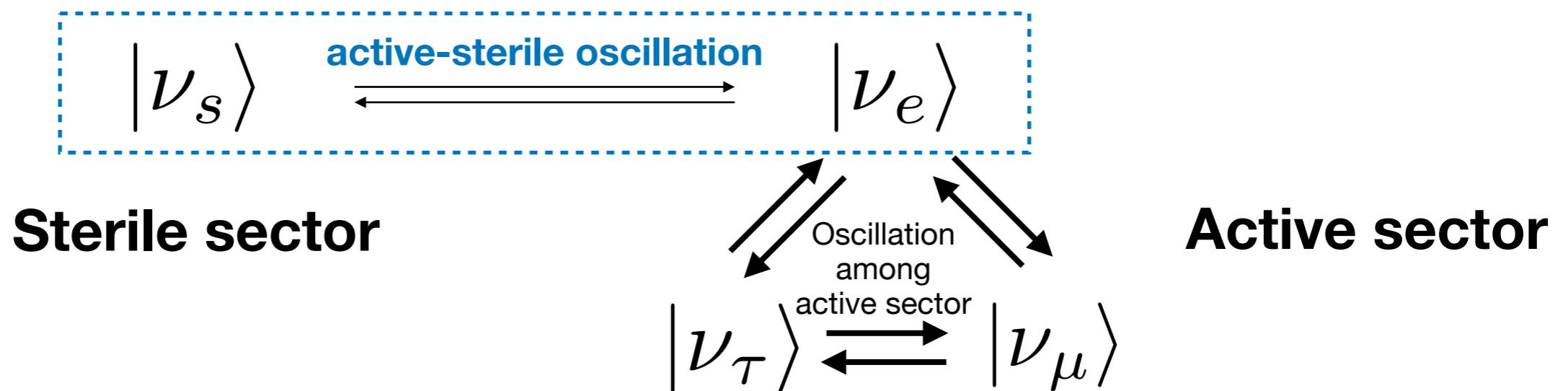
Dashed orange line: Dodelson-Widrow
Solid blue line: Shi-Fuller

By resonance effect,
we can explain all dark matter
with small mixing angle.

1. Review of Shi-Fuller mechanism
2. Revisiting the calculation of Shi-Fuller mechanism
3. Large lepton asymmetry from Affleck-Dine leptogenesis

Shi-Fuller mechanism : Calculation setup

For simplicity, we assume (electron neutrino)-(sterile neutrino) mixing.

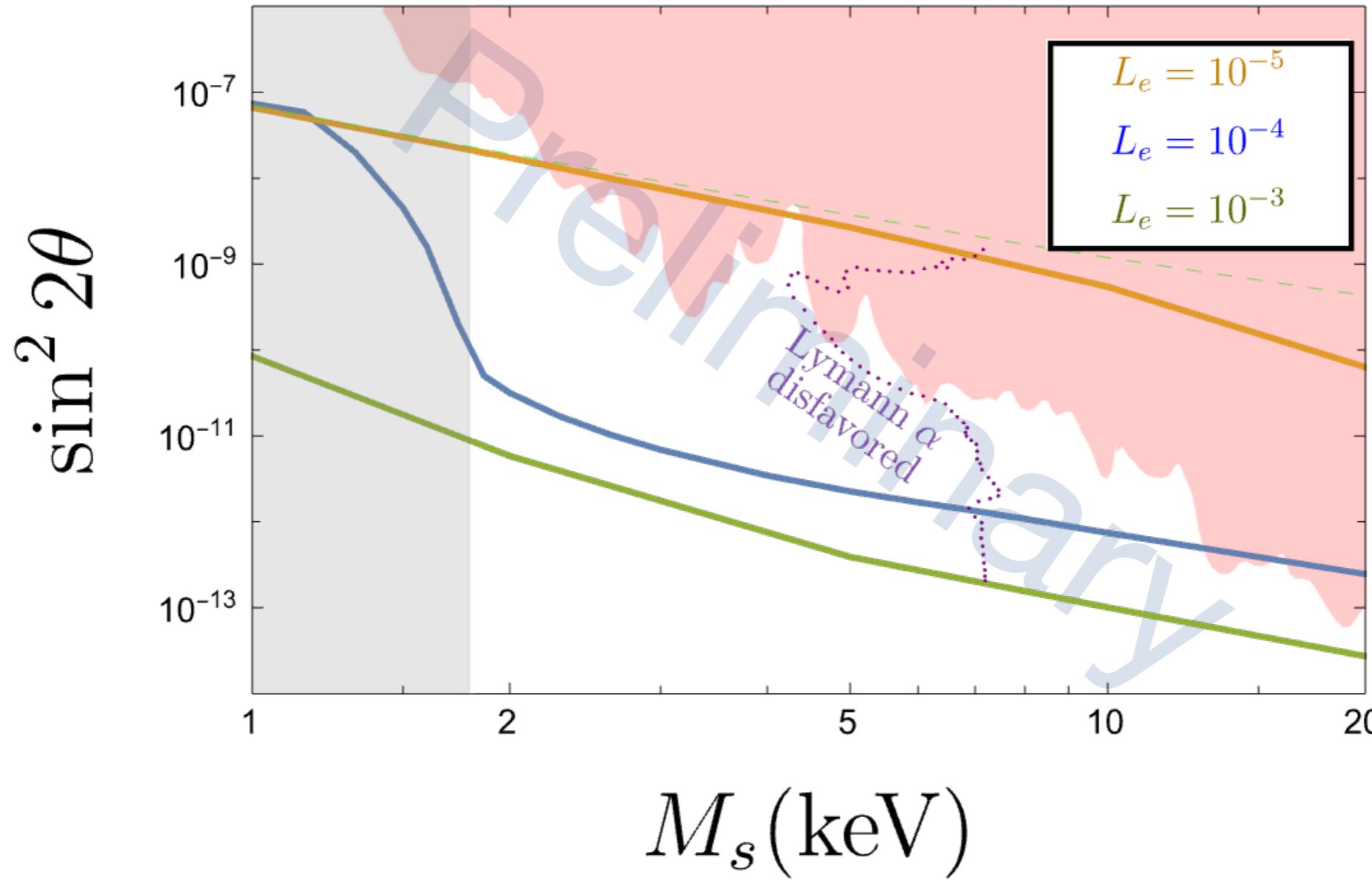


Assuming that neutrino oscillation in active sector proceeds efficiently,

$L_e \simeq L_\mu \simeq L_\tau$ is always satisfied.

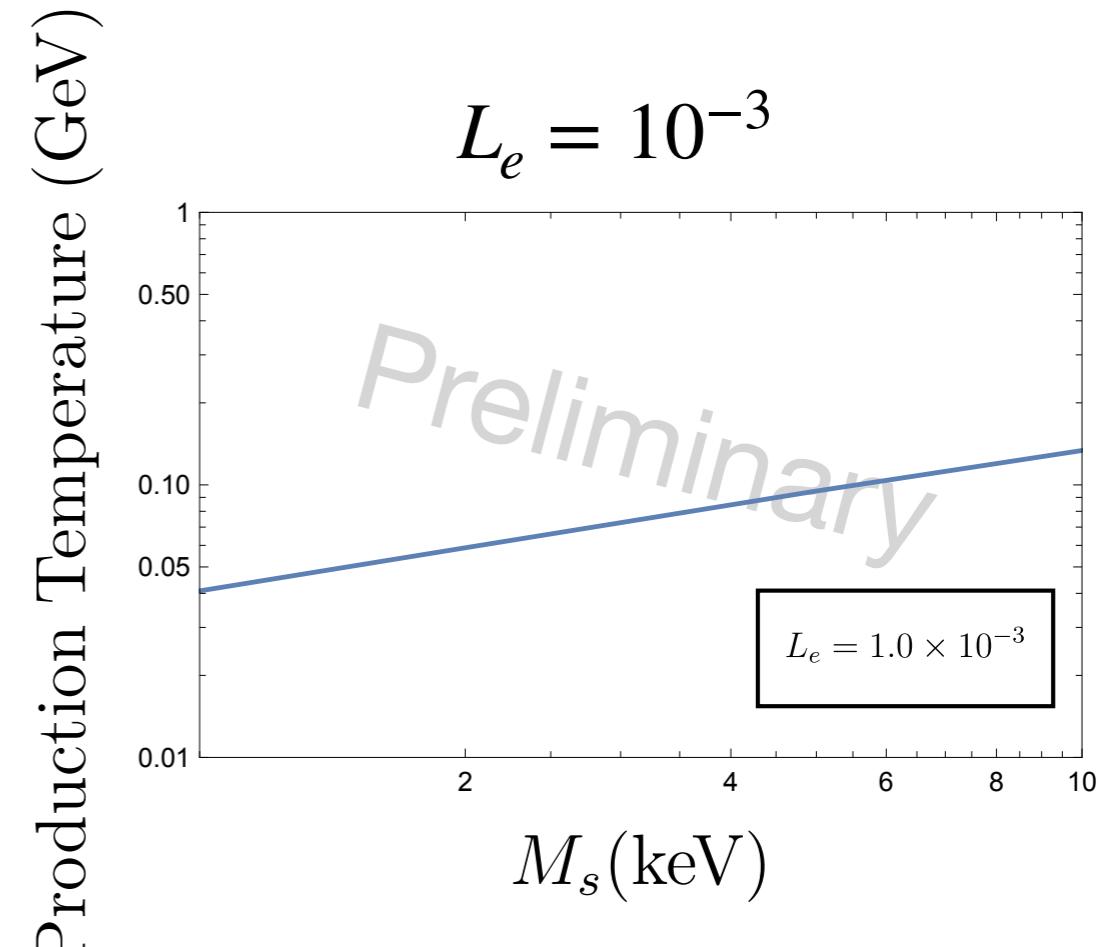
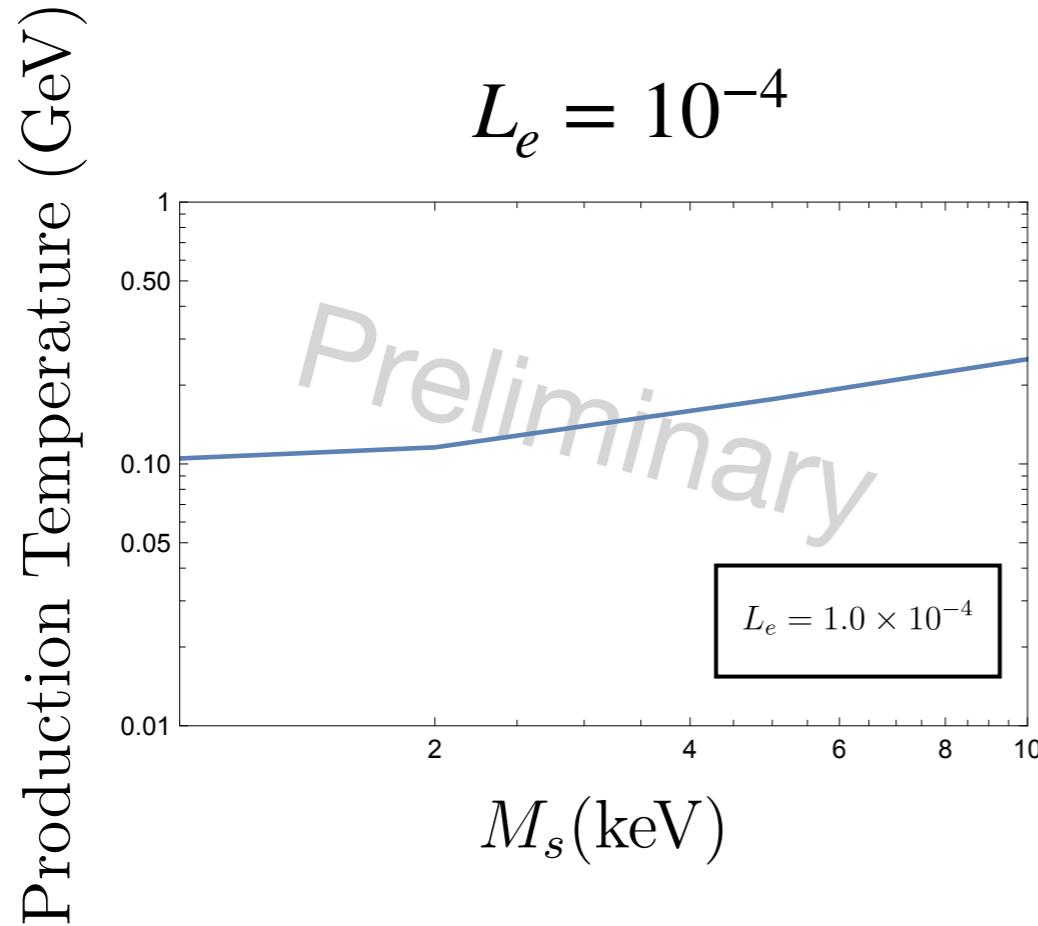
$L_s + \sum_{\alpha=e,\mu,\tau} L_\alpha$ is conserved during sterile neutrino production.

Shi-Fuller mechanism : Contour Plot



We can evade X-ray constraint
assuming larger initial lepton asymmetry in ν_e sector.
 $L_{e,\text{init}} = 10^{-4} - 10^{-3}$ is favored.

Shi-Fuller mechanism: The production temperature of sterile neutrinos



Production temperature T_p : Temperature at which $L_e(T_p) = 0.99L_{e,\text{init}} + 0.01L_{e,0}$

Sterile neutrino production occurs at $T = T_p \sim \mathcal{O}(0.1)\text{GeV}$.

The condition for leptogenesis in Shi-Fuller mechanism

To evade the X-ray constraints, the lepton asymmetry must be $L_e \gtrsim \mathcal{O}(10^{-4})$

Generation of lepton asymmetry must occur at

$$T \gtrsim 0.12\text{GeV} \text{ (with } L_{e,\text{init}} = 10^{-3} \text{)} \quad \equiv T_p$$
$$T \gtrsim 0.23\text{GeV} \text{ (with } L_{e,\text{init}} = 10^{-4} \text{)}$$

(If we focus on $\lesssim \mathcal{O}(10)$ keV sterile neutrinos)

Question

Q1. In this scenario,
what is the favorable initial value of lepton asymmetry
to explain all dark matter ? 

Q2. Is there any favorable model parameter
to explain both the required lepton asymmetry
and observed baryon asymmetry
in Affleck-Dine leptogenesis?

1. Review of Shi-Fuller mechanism
2. Revisiting the calculation of Shi-Fuller mechanism
3. Large lepton asymmetry from Affleck-Dine leptogenesis

Affleck-Dine (AD) mechanism

I. Affleck, M. Dine (1985), M. Dine, L. Randall, S. D. Thomas (1996)

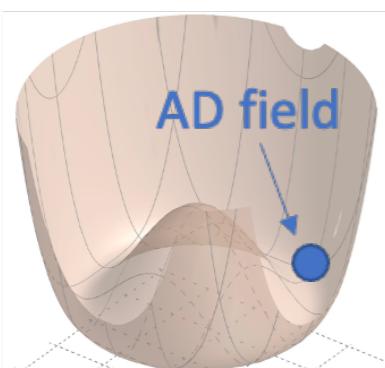
$$V(\phi) \simeq m_{3/2}^2 |\phi|^2 - cH^2 |\phi|^2 + |\lambda|^2 \frac{|\phi|^{2(n-1)}}{M_{\text{Pl}}^{2n-6}} + \lambda a_M \frac{m_{3/2} \phi^n}{n M_{\text{Pl}}^{n-3}} + \text{h.c.}$$

Hubble induced term Non-renormalizable term V_{NR} A-term V_A

ϕ : Leptonic flat direction in MSSM (=AD field) Ex.) $L_1 L_2 \bar{e}_2$

$m_{3/2}$: gravitino mass, H : Hubble parameter, $a_M = \mathcal{O}(1)$, $c(> 0) = \mathcal{O}(1)$

Time



(Hubble induced term), (Non-renormalizable term) \gg (A-term)

$$H \sim m_{3/2} (\equiv H_{\text{osc}})$$



Kicked by A-term
 $\rightarrow \dot{\theta} \neq 0$

Resultant lepton number :

$$n_L = i (\phi^* \dot{\phi} - \dot{\phi}^* \phi) \simeq \dot{\theta} |\phi|^2$$

$$\simeq \epsilon m_{3/2} \phi_{\text{osc}}^2$$

ϵ : efficiency parameter
determined by $\arg(a_M)$, λ , etc.

Formation of “Q-ball”s

We consider the situation where

$$\min [V(\varphi)/\varphi^2] < m_\phi^2 \quad (\text{after coherent AD field oscillation})$$

The fluctuation with certain scale ($\equiv k_{\text{NL}}$) grows and solitons (“Q-ball”s) with scale $\sim 2\pi k_{\text{NL}}^{-1}$ are formed.

Q-ball : Spherically symmetric soliton configuration confining lepton charge

S. Coleman (1985)

Coherent AD field oscillation

If $\min [V(\varphi)/\varphi^2] < m_\phi^2$

Where $\varphi \equiv |\phi|$

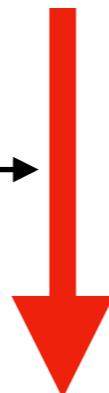
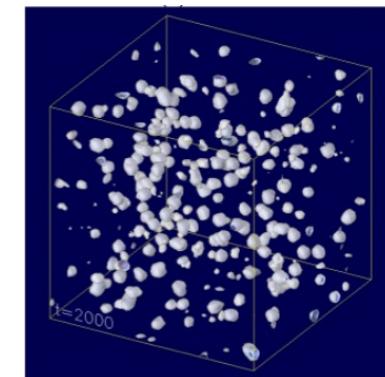


Figure:

T. Hiramatsu, M. Kawasaki, F. Takahashi,
JCAP (2010)



“Q-ball” formation

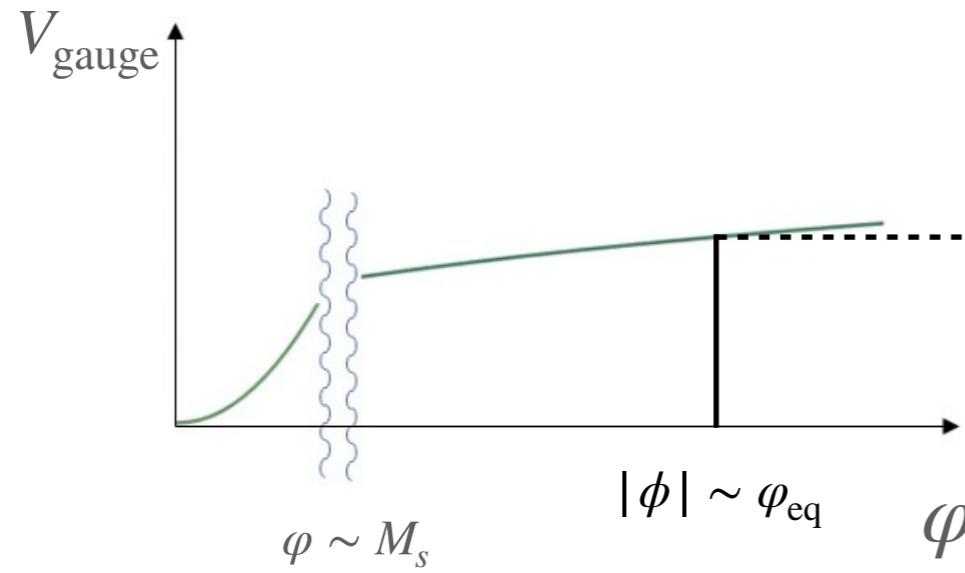
Delayed-type Q-ball scenario

$$V(\phi) = V_{\text{gauge}}(\phi) + V_{\text{grav}}(\phi)$$

“Gauge mediation” potential

$$V_{\text{gauge}}(\phi) = \begin{cases} m_\phi^2 |\phi|^2 & (|\phi| \ll M_s) \\ M_F^4 \left(\ln \left(\frac{|\phi|^2}{M_s^2} \right) \right)^2 \sim M_F^4 & (|\phi| \gg M_s) \end{cases}$$

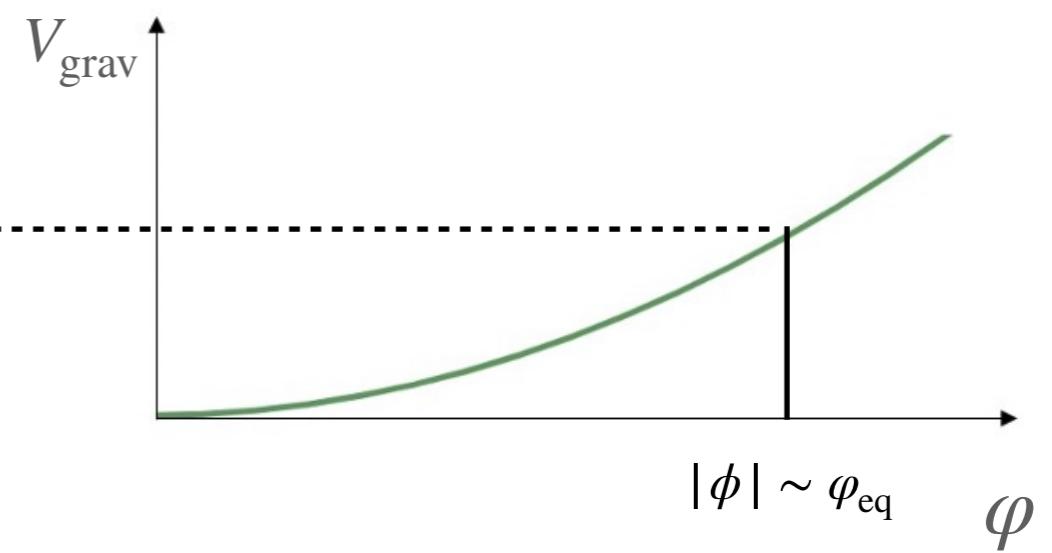
M_s : Messenger scalar mass



“Gravity mediation” potential

$$V_{\text{grav}}(\phi) \simeq m_{3/2}^2 |\phi|^2 \left[1 + K \ln \left(\frac{|\phi|^2}{M_{\text{pl}}^2} \right) \right]$$

$K > 0$: Assumption



$V_{\text{gauge}} \sim V_{\text{grav}}$ when $|\phi| \sim \varphi_{\text{eq}} \equiv M_F^2/m_{3/2}$

■ Delayed-Type Q-ball

S. Kasuya, M. Kawasaki, (2002)

The case with $K > 0$, $\varphi_{\text{osc}} \gtrsim \varphi_{\text{eq}}$. Q-balls are formed right after V_{gauge} becomes dominant i.e. $\varphi \sim \varphi_{\text{eq}}$ is satisfied.

Why Q-ball scenario?

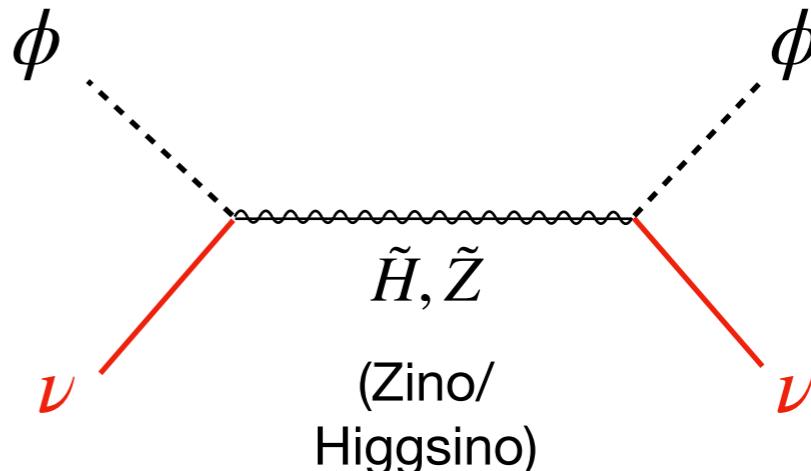
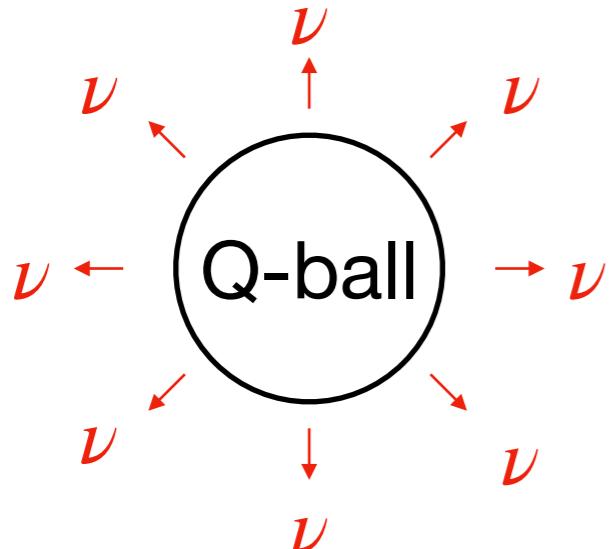
M. Kawasaki, F. Takahashi, M. Yamaguchi, (2002)

Lepton charge inside the Q-balls is protected from sphaleron process

There is small effect of evaporation into SM neutrinos due to coupling to SM neutrinos at high temperature
→ small baryon asymmetry

Q-balls finally decay due to coupling to SM neutrinos below the electroweak scale
→ large lepton asymmetry

Decay/evaporation rate of Q-balls



(Decay/evaporation rate) \propto (Surface area of Q-balls)

Decay rate

$$\Gamma_Q \simeq \frac{N_l}{Q} \frac{\omega_Q^3}{12\pi^2} 4\pi R_Q^2$$

G. Cohen, S. R. Coleman,
H. Georgi, A. Monohar, (1986)

M. Kawasaki, M. Yamada (2012)

N_l : number of neutrino species, Q : lepton charge inside each Q-ball,

ω_Q : energy per unit lepton charge, R_Q : radius of each Q-ball

Evaporation rate

M. Laine, M. Shaposhnikov, (1998)

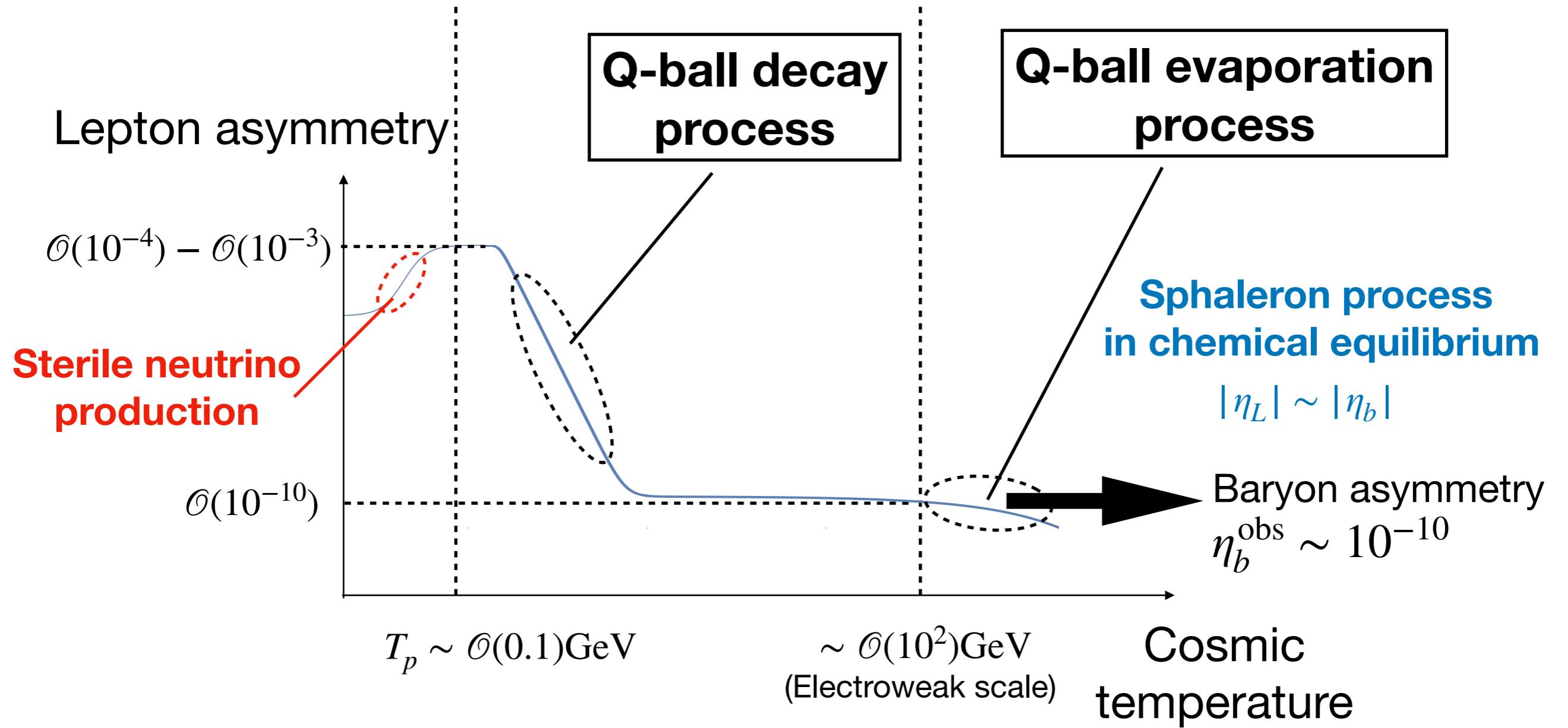
$$\frac{dQ}{dt} = -D(\mu_Q - \mu_{\text{plasma}})T^2 4\pi R_Q^2$$

$\mu_Q \simeq \omega_Q$: chemical potential of slepton field in Q-balls

μ_{plasma} ($\ll \omega_Q$) : chemical potential in thermal plasma

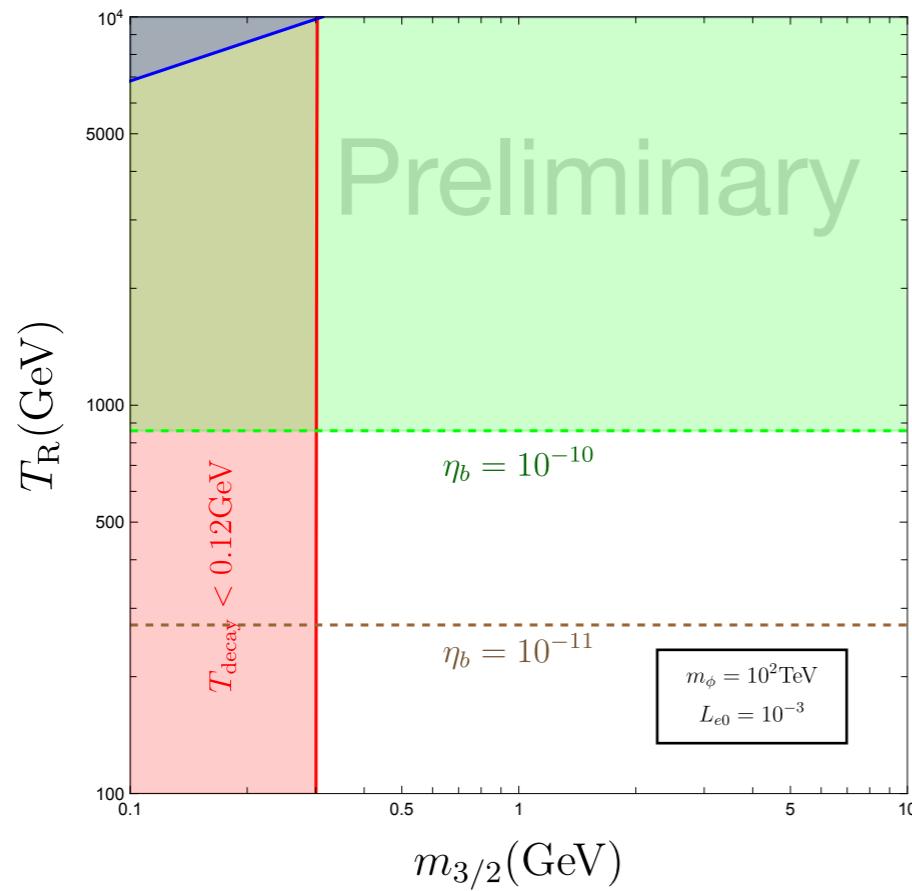
$D \sim gT^2/m_\phi^2$: diffusion coefficient

Why Q-ball scenario?



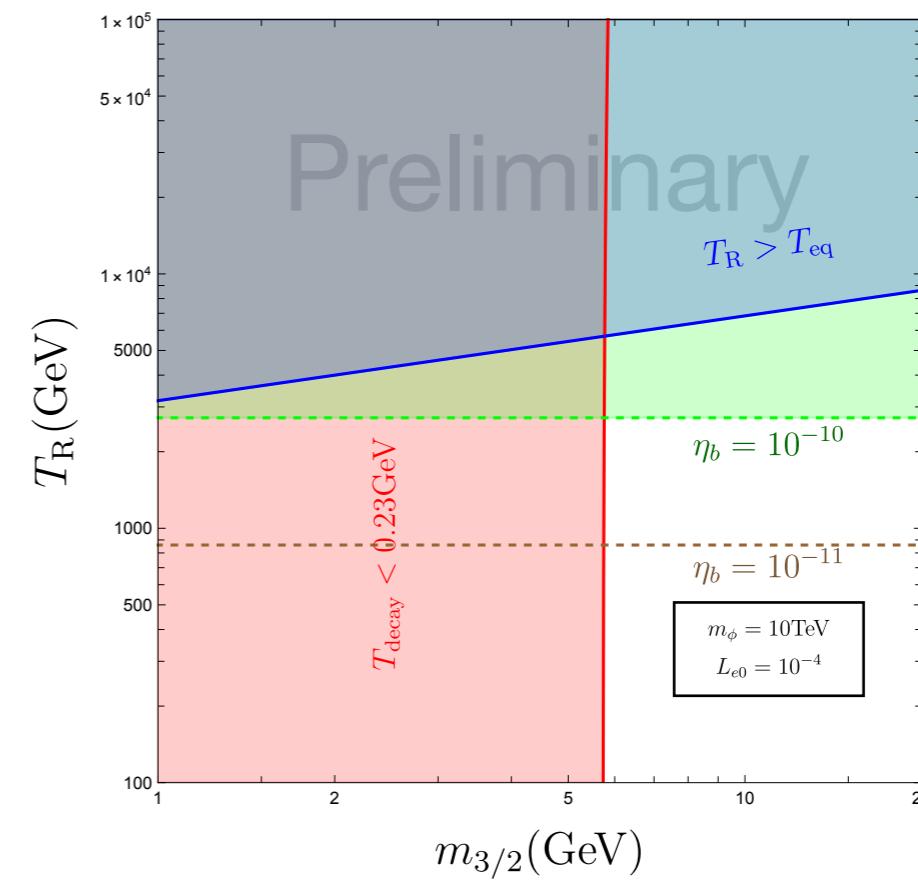
Delayed-type Q-ball scenario : Parameter space

$$L_e = 10^{-3} \text{ (fixed)}$$



$$m_\phi = 100 \text{ TeV}, \epsilon = 10^{-2}$$

$$L_e = 10^{-4} \text{ (fixed)}$$



$$m_\phi = 10 \text{ TeV}, \epsilon = 10^{-2}$$

(Decay temperature) $\gtrsim T_p \rightarrow$ Lower bound of gravitino mass

Q-ball evaporation explain baryon asym. \rightarrow Reheating temperature

Conclusion

1. We revisited the calculation of Shi-Fuller mechanism and confirmed that initial lepton asymmetry is required to be $L_e \gtrsim \mathcal{O}(10^{-4})$ (for $m_s \lesssim \mathcal{O}(10)\text{keV}$) to evade the current X-ray constraints.
2. We searched for the parameter space in AD leptogenesis scenario to realize the above conditions.

Gravitino mass with

$m_{3/2} \gtrsim \mathcal{O}(0.1)\text{GeV}$ (for $L_e = 10^{-3}$, $m_\phi = 10^2\text{TeV}$),

$m_{3/2} \gtrsim \mathcal{O}(1)\text{GeV}$ (for $L_e = 10^{-4}$, $m_\phi = 10\text{TeV}$)

and Reheating temperature $T_R \sim \mathcal{O}(1)\text{TeV}$ is favored.

Supplementary materials

Neutrino oscillation in vacuum

Effective Hamiltonian in flavor basis

$$H = U \cdot \text{diag} \left(\frac{m_1^2}{2E_\nu}, \frac{m_2^2}{2E_\nu} \right) \cdot U^\dagger, \quad U = \begin{pmatrix} \cos \theta_\alpha & \sin \theta_\alpha \\ -\sin \theta_\alpha & \cos \theta_\alpha \end{pmatrix}$$

Oscillation amplitude

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_s) \\ = |\langle \nu_s | \exp(iHt) | \nu_a \rangle|^2 = \left| \langle \nu_s | U \cdot \text{diag} \left(\exp \left(i \frac{m_1^2}{2E_\nu} t \right), \exp \left(i \frac{m_2^2}{2E_\nu} t \right) \right) \cdot U^\dagger | \nu_a \rangle \right|^2 \\ = \sin^2 2\theta_a \sin^2 \frac{m_1^2 - m_2^2}{4E_\nu} t \end{aligned}$$

Average over oscillation period :

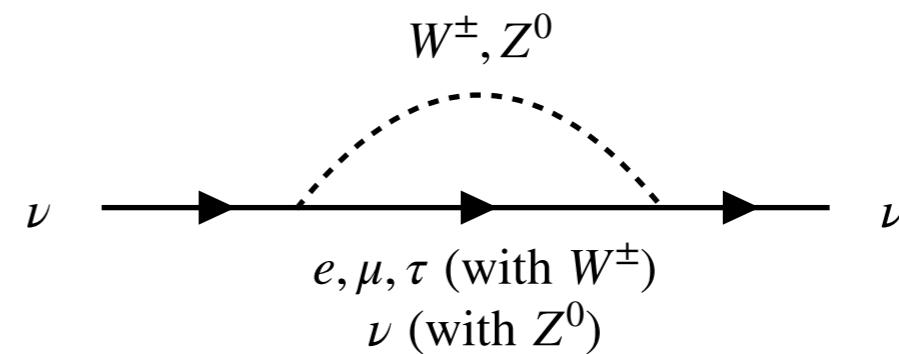
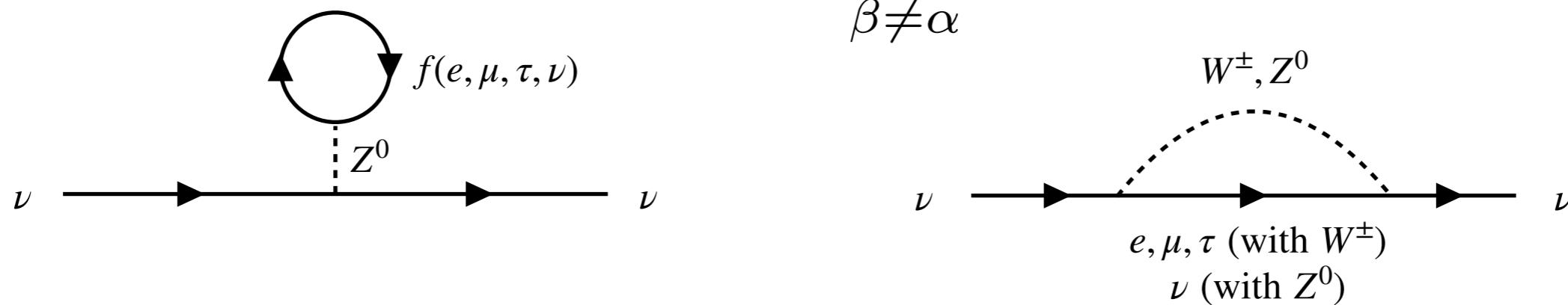
$$\langle P(\nu_a \rightarrow \nu_s) \rangle = \frac{1}{2} \sin^2 2\theta_a$$

Neutrino oscillation in thermal plasma : Matter effect

D. Notzold, G. Raffelt, (1988)

In the early universe, the active neutrinos acquire thermal self energy through weak interaction :

$$V_{\alpha\alpha} \simeq 2\sqrt{2}G_F \left(2(n_{\nu_\alpha}^L - n_{\bar{\nu}_\alpha}^L) + \sum_{\beta \neq \alpha} (n_{\nu_\beta}^L - n_{\bar{\nu}_\beta}^L) \right) - r_\alpha G_F^2 E_\nu T^4$$

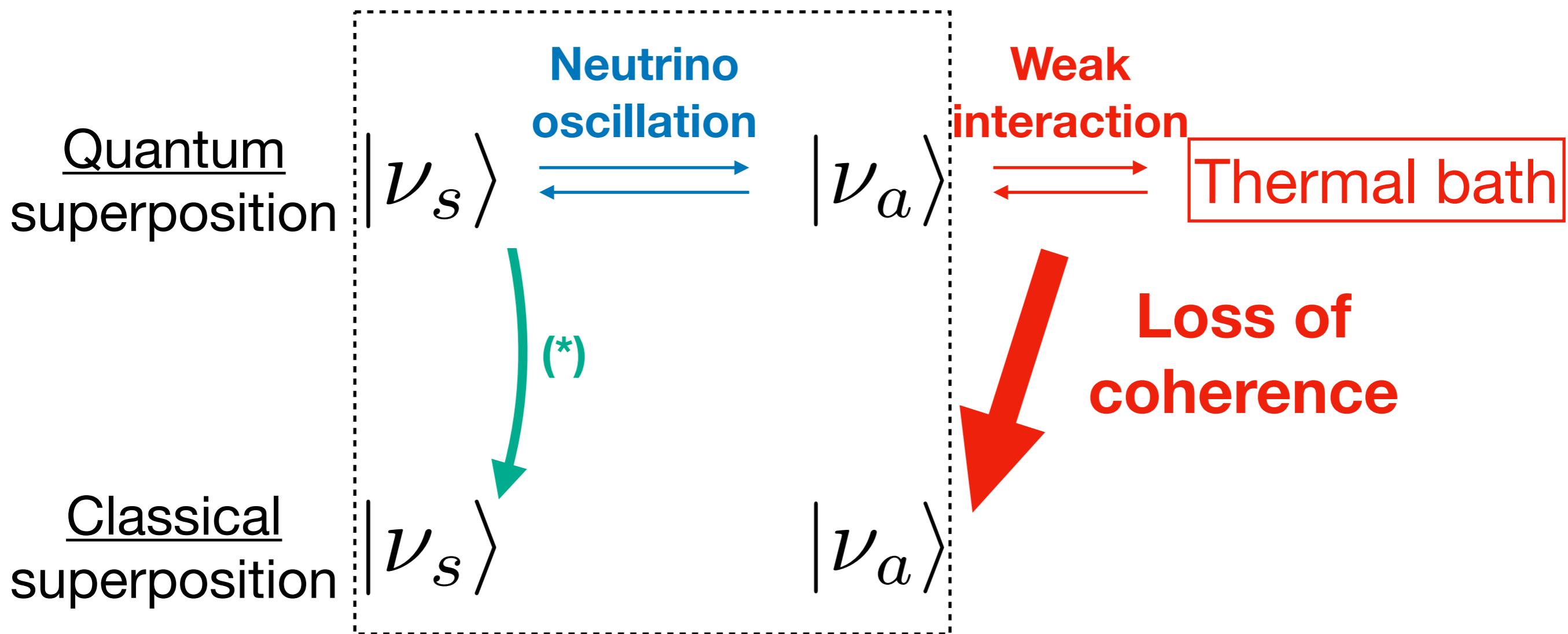


$$H = U \cdot \text{diag}\left(\frac{m_1^2}{2E_\nu}, \frac{m_2^2}{2E_\nu}\right) \cdot U^\dagger + V_{\text{int}}, \quad V_{\text{int}} = \begin{pmatrix} V_{\alpha\alpha} & 0 \\ 0 & 0 \end{pmatrix}$$

Oscillation amplitude averaged over oscillation period :

$$\langle P(\nu_a \rightarrow \nu_s) \rangle \simeq \frac{1}{2} \sin^2(2\theta_a^{\text{mat}}) \equiv \frac{1}{2} \frac{\sin^2 2\theta_a}{\sin^2 2\theta_a + (\cos 2\theta_a - V_{aa} \cdot 2E_\nu/m_s^2)^2}$$

Production mechanism of sterile neutrino dark matter



(*) : Transition rate

\propto

(scattering rate), (effective mixing angle)

Production mechanism of sterile neutrino dark matter

A. D. Dolgov, (2002)

Effective description of density operator equation:

$$\dot{\rho} = -i[H, \rho] - \underbrace{\{\Gamma, (\rho - \rho_{\text{eq}})\}}_{\textcircled{2}}$$

$$\Gamma = \begin{pmatrix} \Gamma_a & 0 \\ 0 & 0 \end{pmatrix} \quad \Gamma_a: \text{scattering rate of active neutrinos with thermal plasma}$$

$$\rho = \begin{pmatrix} \rho_{aa} & \rho_{as} \\ \rho_{sa} & \rho_{ss} \end{pmatrix} \quad \rho_{\text{eq}} = \begin{pmatrix} \frac{1}{\exp((E_\nu - \mu)/T) + 1} & 0 \\ 0 & \frac{1}{\exp((E_\nu - \mu)/T) + 1} \end{pmatrix}$$

: density matrix : thermal distribution

① : Vacuum oscillation

② : Equilibrates the distribution ρ into ρ_{eq} in time scale $\Delta t \sim \Gamma_0^{-1}$

Production mechanism of sterile neutrino dark matter

$$\dot{\rho}_{aa} = -i \frac{m_s^2}{4E_\nu} \sin 2\theta_a (\rho_{sa} - \rho_{as}) - 2\Gamma_a (\rho_{aa} - f_{eq})$$

$$\dot{\rho}_{ss} = i \frac{m_s^2}{4E_\nu} \sin 2\theta_a (\rho_{sa} - \rho_{as})$$

$$\dot{\rho}_{as} = -i \frac{m_s^2}{4E_\nu} \sin 2\theta_a (\rho_{ss} - \rho_{aa}) + \left(-i \left(\frac{m_s^2}{2E_\nu} \cos 2\theta_a - V_{aa} \right) - \Gamma_a / 2 \right) \rho_{as}$$

$$\dot{\rho}_{sa} = i \frac{m_s^2}{4E_\nu} \sin 2\theta_a (\rho_{ss} - \rho_{aa}) + \left(i \left(\frac{m_s^2}{2E_\nu} \cos 2\theta_a - V_{aa} \right) - \Gamma_a / 2 \right) \rho_{sa}$$

Assuming (Oscillation period) $\ll \Gamma_a^{-1}$, we can set $\langle \dot{\rho}_{sa} \rangle = \langle \dot{\rho}_{as} \rangle \simeq 0$.

Then,

$$\dot{\rho}_{ss} \simeq \frac{1}{4} \Gamma_a \sin^2 2\theta_a^{\text{mat}} (\rho_{aa} - \rho_{ss}) = \frac{1}{4} \Gamma_a \sin^2 2\theta_a^{\text{mat}} (f_{eq} - f_s)$$

Where $\Gamma_a \ll m_s^2/E_\nu$, $\rho_{aa} \simeq f_{eq} \gg \rho_{ss}$

Production mechanism of sterile neutrino dark matter : Master equation

C. T. Kishimoto, G. M. Fuller
(2008)

$$\dot{f}_{\nu_s} \simeq \frac{1}{4} \Gamma_a \sin^2 2\theta_a^{\text{mat}} (f_{\text{eq}} - f_{\nu_s}) \times \frac{1}{1 + l_m^2 \Gamma_a^2 / 2}$$

“Dumping factor”

Where

$$l_m \equiv \frac{2E_\nu}{m_s^2} \cdot \frac{\sin 2\theta_M}{\sin 2\theta_0} : \text{Effective neutrino oscillation length}$$

Dumping factor

If $l_m \gg \Gamma_a^{-1}$: significantly suppress the sterile neutrino production rate

If $l_m \ll \Gamma_a^{-1}$: $\simeq 1$

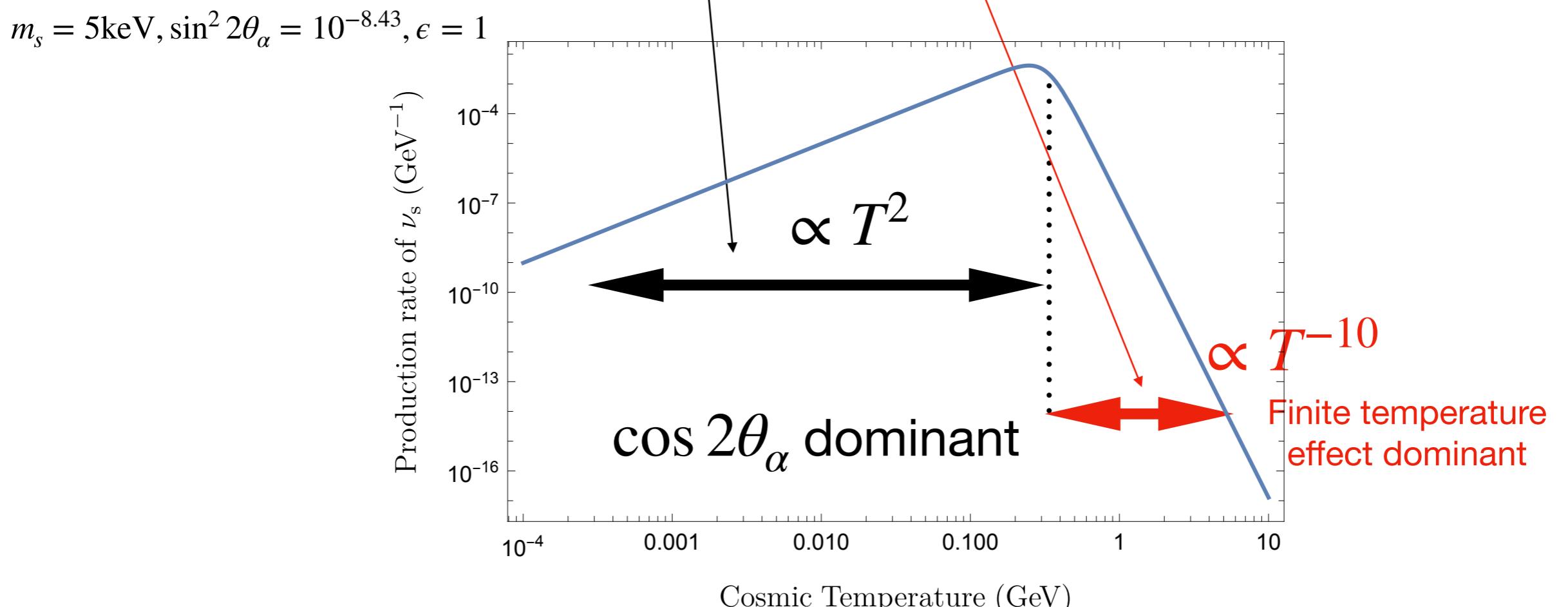
Sterile neutrino dark matter 1. Dodelson-Widrow mechanism

S. Dodelson, L. M. Widrow, (1993)

Consider the situation where there is no lepton asymmetry.

$$P_{\nu_\alpha \rightarrow \nu_s} \sim \frac{1}{4} \Gamma_\alpha(\epsilon, t) \frac{\sin^2 2\theta_\alpha}{\sin^2 2\theta_\alpha + (\cos 2\theta_\alpha + \frac{2r_\alpha G_F^2 \epsilon^2 T^6 / m_s^2}{})^2} \left(f_{\text{eq}}(\epsilon, T) - f_{\nu_s}(\epsilon, T) \right) \quad (\times \text{ Dumping factor})$$

Where $\epsilon \equiv E_\nu/T$

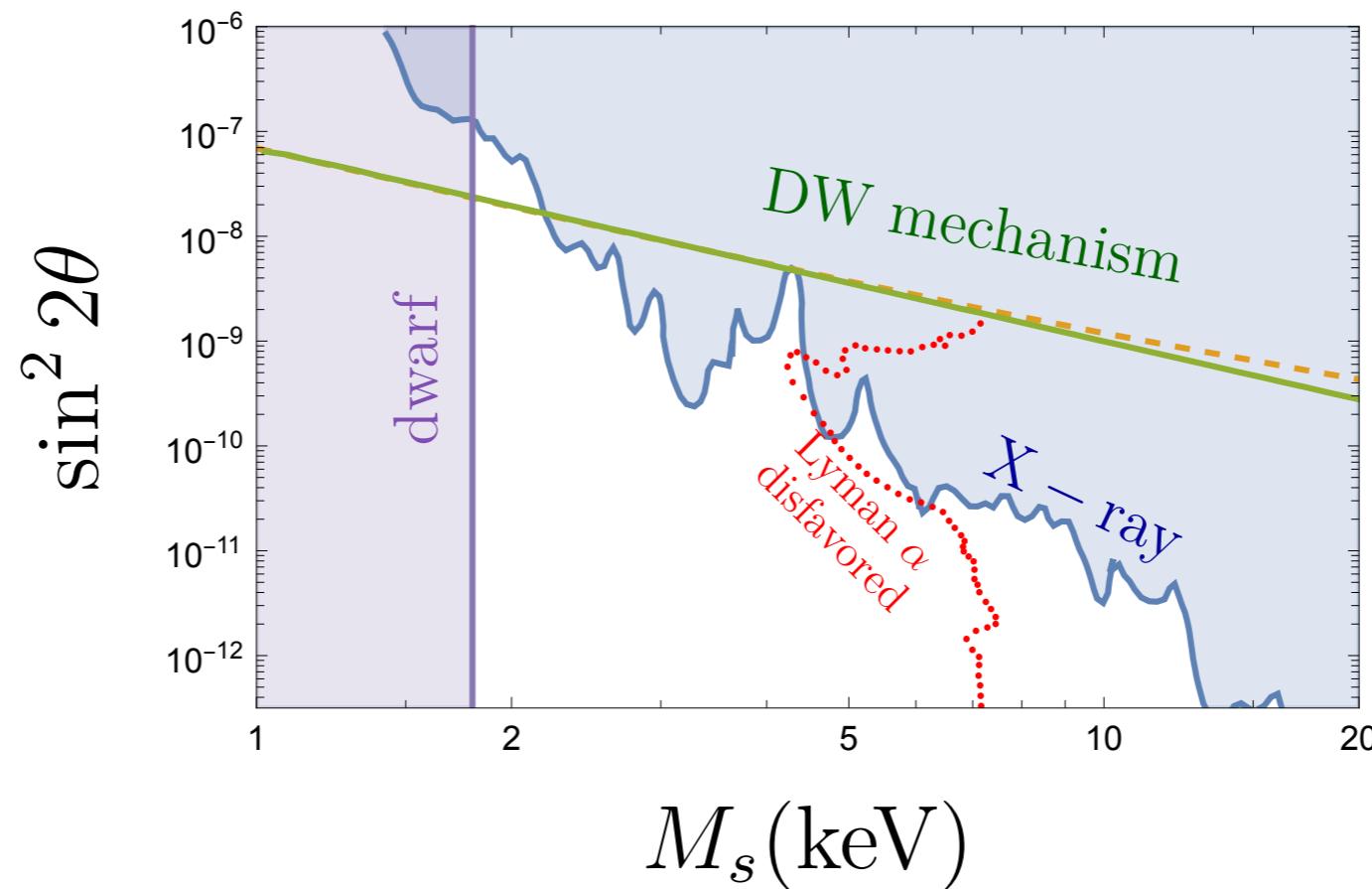


Production peak : $\cos 2\theta_\alpha \simeq 2r_\alpha G_F^2 \epsilon^2 T^6 / m_s^2 \rightarrow T \simeq 108\text{MeV} \cdot \epsilon^{-1/3} \left(\frac{m_s}{1\text{keV}} \right)^{1/3}$

Constraints on Dodelson-Widrow mechanism

- $m_s - \sin^2 2\theta_0$ contour plot to explain all dark matter

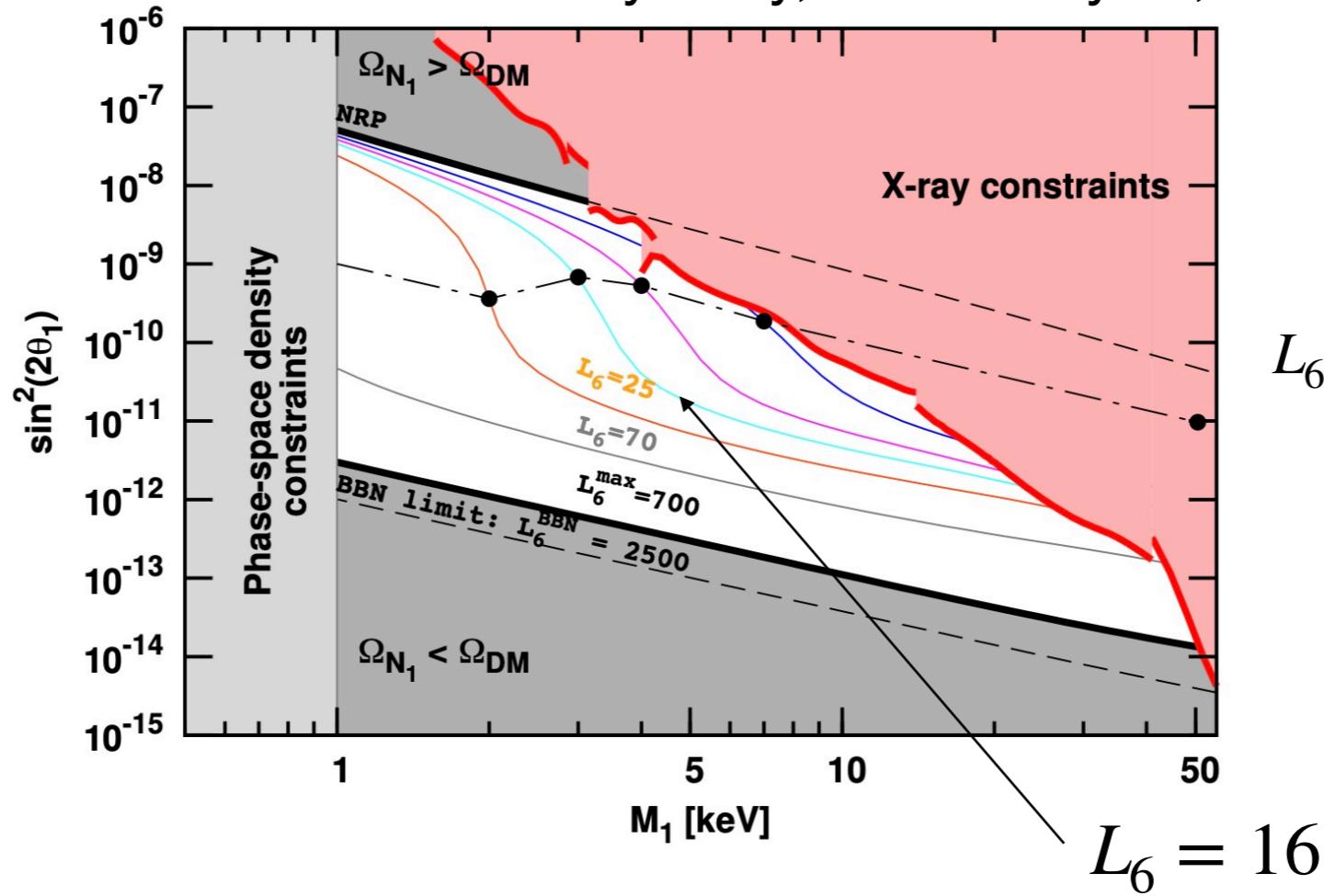
A. Gouvea, M. Sen, W. Tangarife, Y. Zhang, (2020)



Simple Dodelson-Widrow mechanism is
almost inconsistent with X-ray observations
and phase space constraints.

Shi-Fuller mechanism : Previous work

A. Boyarsky, O. Ruchayski, M. Shaposhnikov (2009)



$$L_6 \equiv 10^6 \frac{n_{\nu_e} - n_{\bar{\nu}_e}}{s_{\text{tot}}}$$

$$L_6 = 16$$

With parameters in which Shi-Fuller mechanism occurs efficiently, $\Omega_{\nu_s} \lesssim \frac{m_{\nu_s} L_{e,\text{init}} s_{\text{tot}}}{\rho_0^{\text{crit}}}$ must be satisfied.

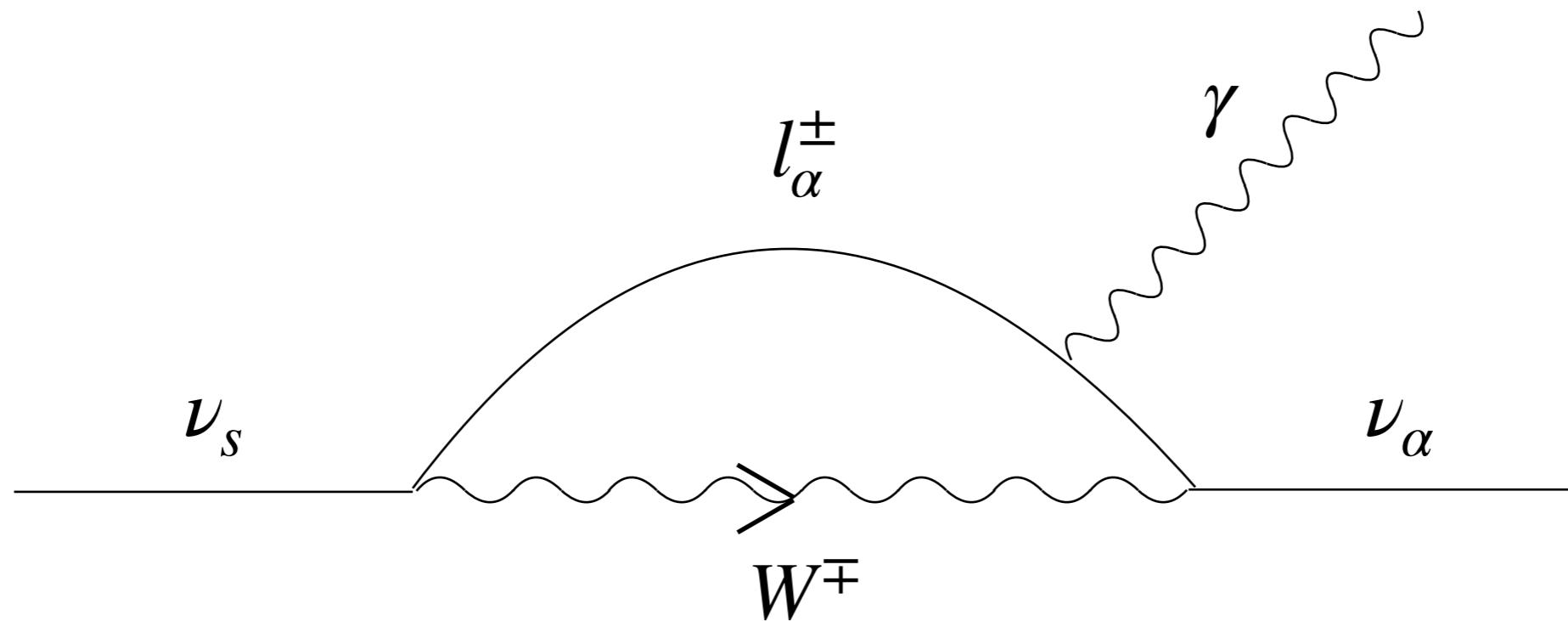
We must properly take into account **total entropy conservation** and **time-evolution of lepton asymmetry** in active neutrino sector.

Constraints on sterile neutrino dark matter : X-ray observations, phase space constraint

Simple Dodelson-Widrow mechanism is already excluded by

1. X-rays from $\nu_s \rightarrow \nu_\alpha + \gamma$ (through active-sterile mixing) K. C. Y. Ng et al., (2019)
2. Upper bound of phase-space density due to Pauli blocking effect
A. Boyarsky et al., (2009)
3. Effect on structure formation (Lyman α) C. Yeh et.al., (2017)

1.



Shi-Fuller mechanism : Calculation

Resonance condition : $\cos 2\theta_0 \simeq 16\sqrt{2}G_F E_\nu s_{\text{tot}} L_\alpha / m_s^2 = \frac{32\sqrt{2}\pi^2}{45} g_{s,*} G_F E_\nu L_e T^3 / m_s^2$

$$T_{\text{res}} \propto m_s^{1/4} L_e^{-1/2} \epsilon^{-1/4}$$

Example : $m_s = 5\text{keV}$, $L_{e,\text{init}} = 10^{-4}$

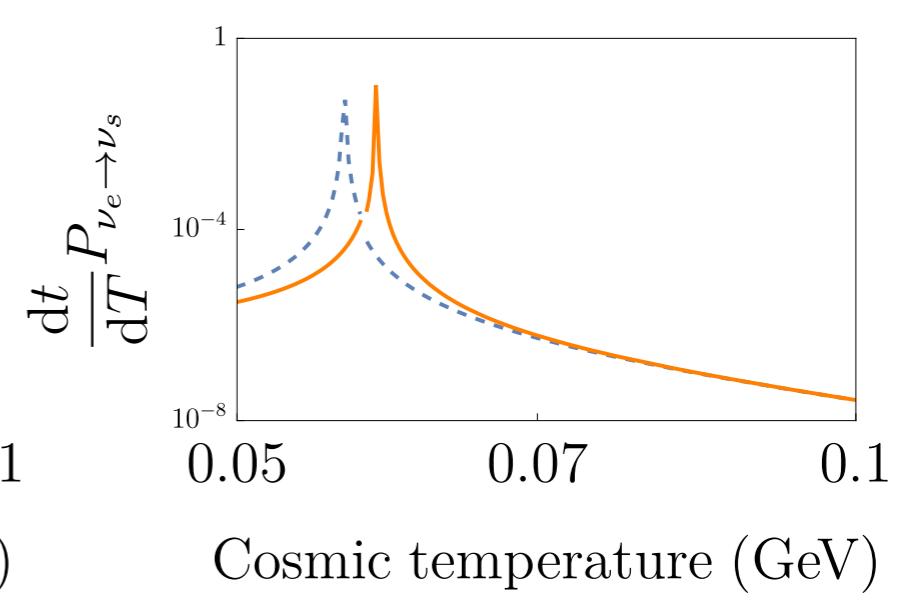
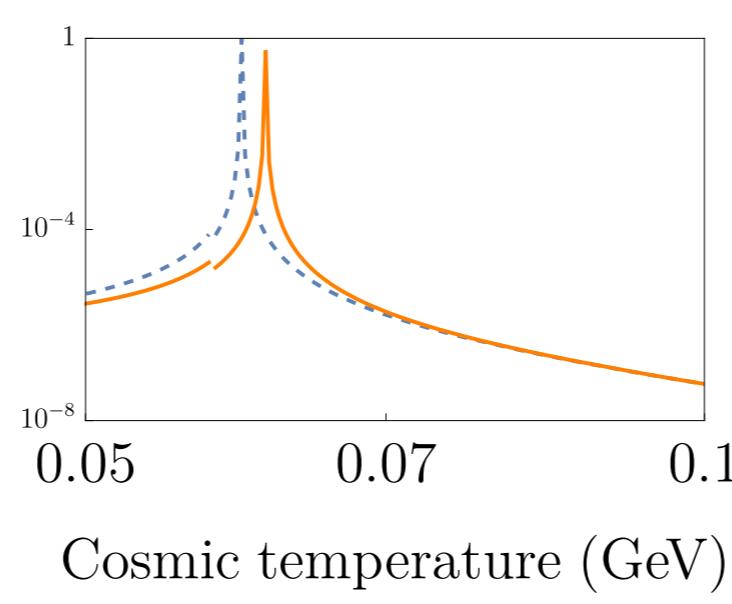
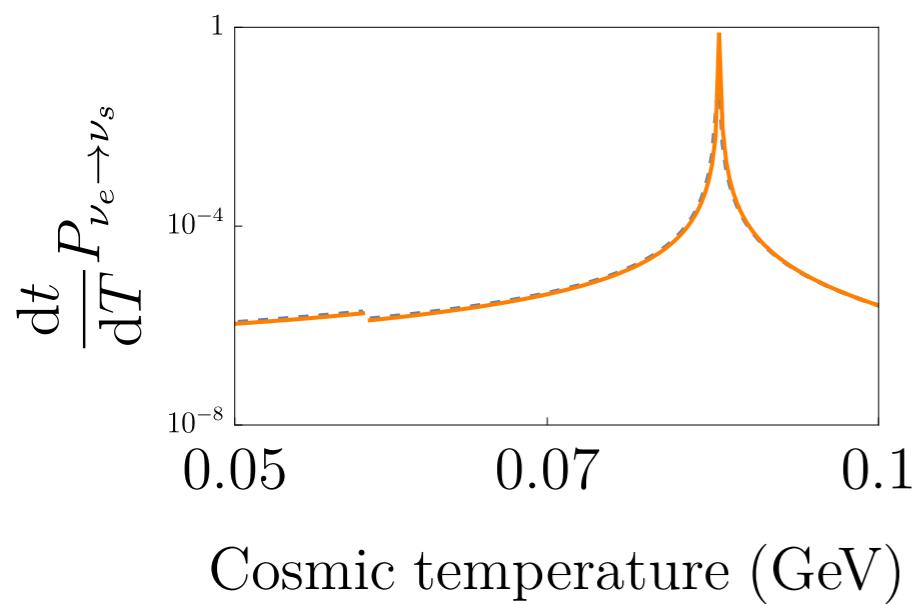
Substituting $L_e = L_{e,\text{init}}$
in effective mixing angle

Actual value

$$\epsilon = 1$$

$$\epsilon = 4$$

$$\epsilon = 5$$



$$T_{\text{false}} \equiv T(g_*/g_{\text{init}})^{1/3} (\text{GeV})$$

Shi-Fuller mechanism : Calculation setup

$$\begin{cases} \frac{d}{dt} L_e = \frac{15}{4\pi^4 g_i} \int d\epsilon \epsilon^2 (P_{\nu_e \rightarrow \nu_s}(\epsilon, T) - P_{\bar{\nu}_e \rightarrow \bar{\nu}_s}(\epsilon, T)) \\ \dot{f}_{\nu_s}(\epsilon, T) = P_{\nu_e \rightarrow \nu_s}(\epsilon, T) \\ \dot{f}_{\bar{\nu}_s}(\epsilon, T) = P_{\bar{\nu}_e \rightarrow \bar{\nu}_s}(\epsilon, T) \end{cases}$$

where

$$\epsilon \equiv \left(\frac{g_i}{g_*} \right)^{1/3} \frac{E_\nu}{T} : \text{dimensionless constant} \quad (\text{Assuming } E_\nu \propto a^{-1}, s_{\text{tot}} = \frac{2\pi^2}{45} g_{s,*} T^3 \propto a^{-3})$$

$$P_{\nu_e \rightarrow \nu_s} \simeq \frac{1}{4} \Gamma_e(\epsilon, t) \frac{\sin^2 2\theta_0}{\sin^2 2\theta_0 + (\cos 2\theta_0 - 8\sqrt{2}G_F s_{\text{tot}} L_e + r_e G_F^2 E_\nu T^4)^2} (f_{\text{eq}}(\epsilon, T) - f_{\nu_s}(\epsilon, T))$$

$$\cdot \frac{1}{1 + \Gamma_e^2 l_m^2 / 2}$$

$$P_{\bar{\nu}_e \rightarrow \bar{\nu}_s} \simeq \frac{1}{4} \Gamma_e(\epsilon, t) \frac{\sin^2 2\theta_0}{\sin^2 2\theta_0 + (\cos 2\theta_0 + 8\sqrt{2}G_F s_{\text{tot}} L_e + r_e G_F^2 E_\nu T^4)^2} (f_{\text{eq}}(\epsilon, T) - f_{\bar{\nu}_s}(\epsilon, T))$$

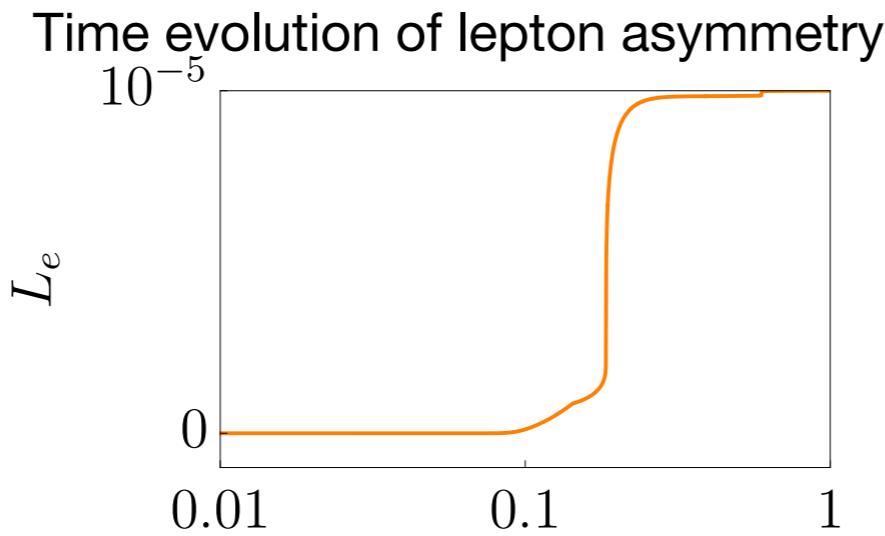
$$\cdot \frac{1}{1 + \Gamma_e^2 l_m^2 / 2}$$

Discretization:

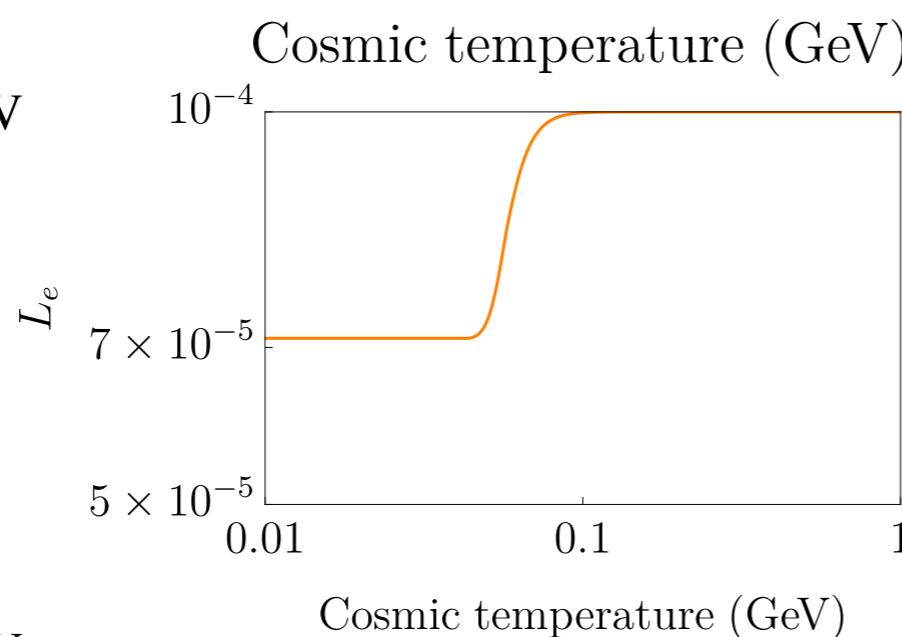
$$\epsilon_n = \frac{n}{N_{\text{bin}}} \epsilon_{\text{max}} \quad (n = 1, 2, \dots, N_{\text{bin}})$$

Shi-Fuller mechanism: Result 1

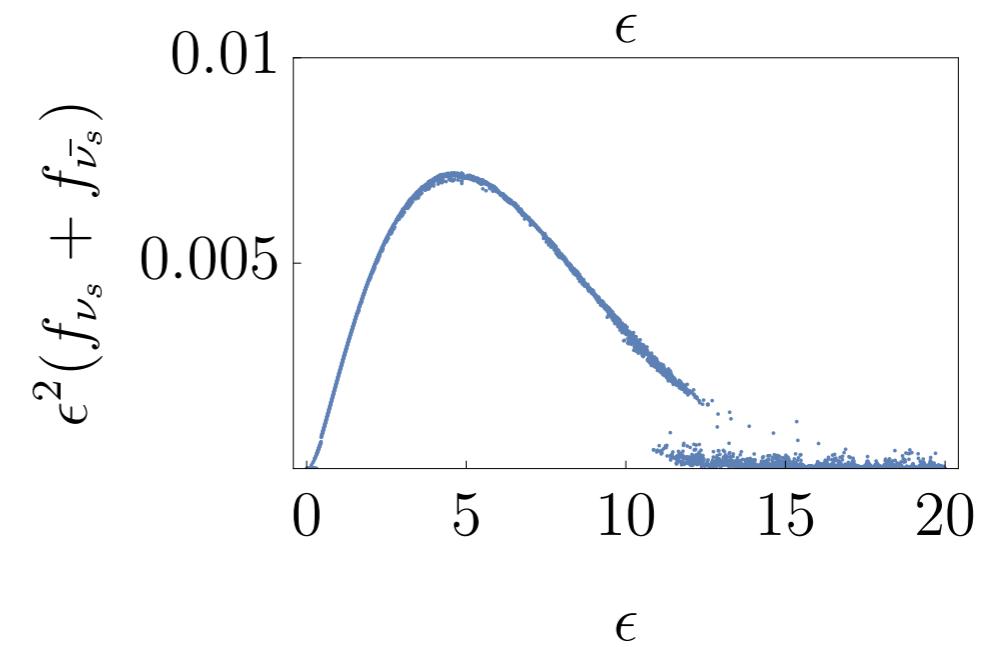
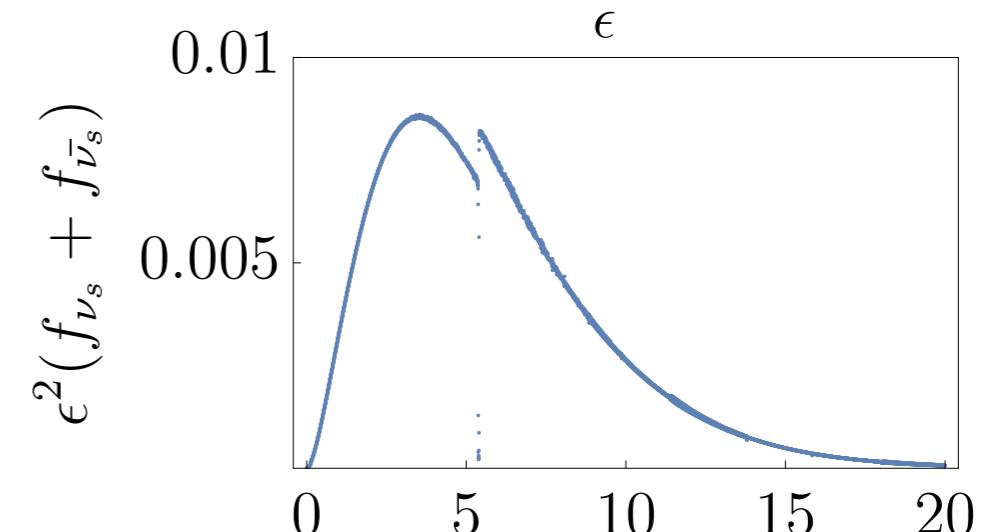
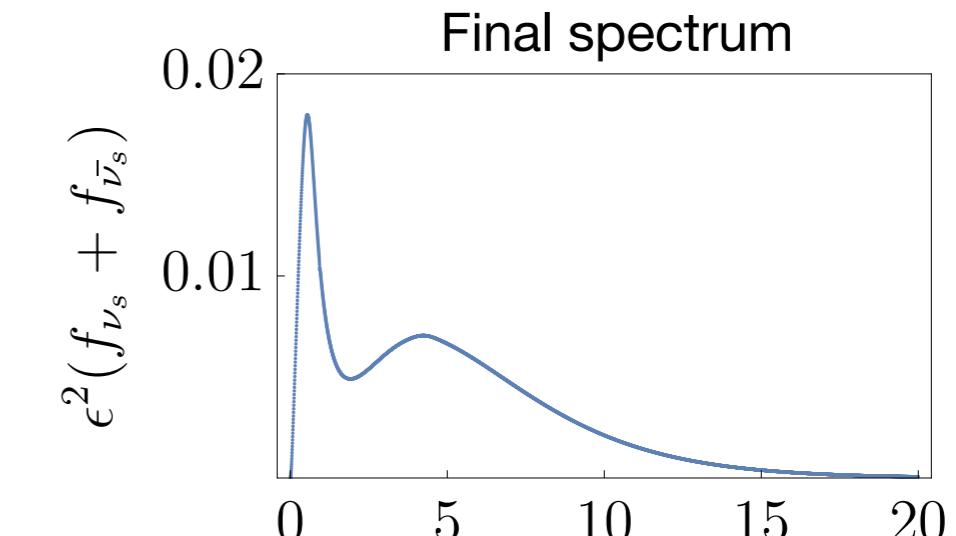
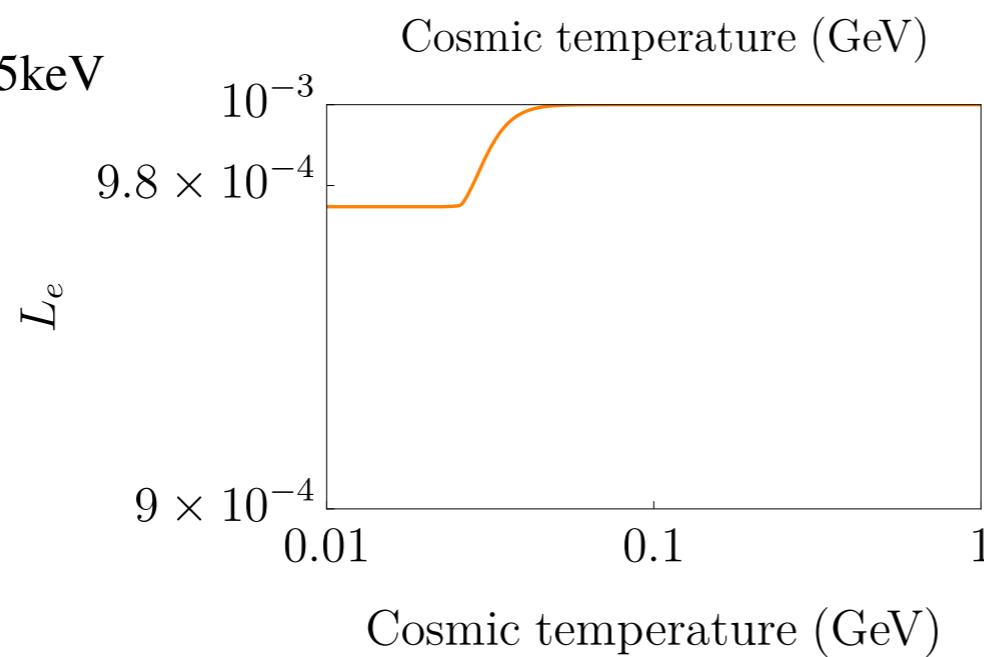
1. $L_{e,\text{init}} = 10^{-5}$, $m_s = 5\text{keV}$



2. $L_{e,\text{init}} = 10^{-4}$, $m_s = 5\text{keV}$



3. $L_{e,\text{init}} = 10^{-3}$, $m_s = 5\text{keV}$



Supersymmetry, MSSM

■ SUSY boson \leftrightarrow fermion

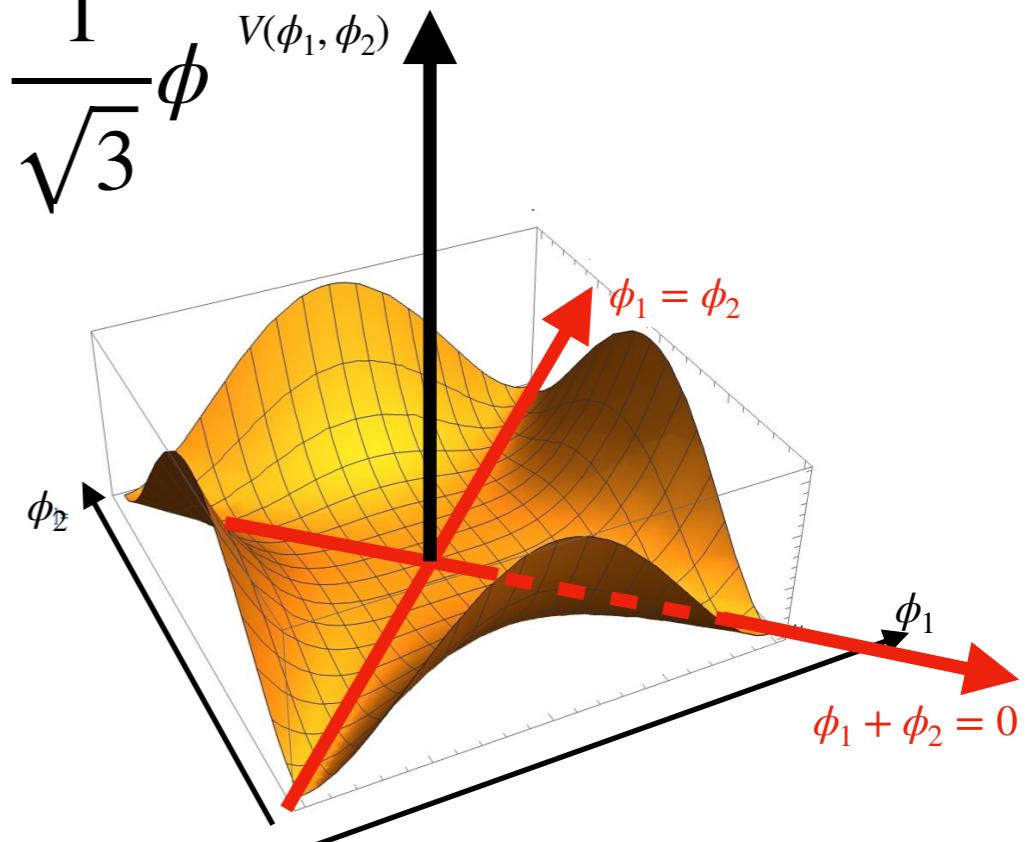
■ MSSM Ex.) squark (spin 0) \leftrightarrow quark (spin 1/2)
slepton (spin 0) \leftrightarrow lepton (spin 1/2)

■ Flat directions in MSSM T. Gherghetta, C. Kolda and S. P. Martin, Nucl. Phys. (1996)
: There exist flat directions in supersymmetric limit

Ex.) $L_1^\alpha = \frac{1}{\sqrt{3}} \begin{pmatrix} \phi \\ 0 \end{pmatrix}, L_2^\alpha = \frac{1}{\sqrt{3}} \begin{pmatrix} 0 \\ \phi \end{pmatrix}, \bar{e}_2 = \frac{1}{\sqrt{3}} \phi^{\alpha} e^{\alpha}$

\leftrightarrow gauge invariant operator : $L_1 L_2 \bar{e}_2$
 $(B - L = -1)$

ϕ : “Affleck-Dine (AD) field”



Lifting up MSSM flat directions

Local SUSY = supergravity

“Gauge field” : **gravitino** (spin 3/2)

- Pattern 1: Effect of soft SUSY breaking $m_{3/2}^2 |\phi|^2$
($m_{3/2}$: gravitino mass)
- Pattern 2: Effect of inflaton coupling in supergravity $cH^2 |\phi|^2$
(Derived from inflaton coupling , c : constant depending on coupling with inflaton)
- Pattern 3 : Effect of non-renormalizable interaction
$$V_{\text{NR}} \equiv |\lambda|^2 \frac{|\phi|^{2(n-1)}}{M_{\text{Pl}}^{2n-6}} \quad n(\geq 3) \text{ : integer}$$
- Pattern 4 : Effect of supergravity & non-renormalizable interaction
$$V_A \equiv \lambda a_M \frac{m_{3/2} \phi^n}{n M_{\text{Pl}}^{n-3}}$$
 Called A-term

Lifting up MSSM flat directions

LLe

(ex.

$$L_1 = \frac{1}{\sqrt{3}} \begin{pmatrix} \phi \\ 0 \end{pmatrix} \quad L_2 = \frac{1}{\sqrt{3}} \begin{pmatrix} 0 \\ \phi \end{pmatrix} \quad \bar{e}_1 = \frac{1}{\sqrt{3}} \phi$$

Lifted by

$$W = \frac{1}{M_*^3} LLeLLe.$$

($n = 6$)

Affleck-Dine (AD) mechanism

I. Affleck, M. Dine, *Nucl.Phys.B* (1985), M. Dine, L. Randall, S. D. Thomas, *Nucl.Phys.B* (1996)

$$V(\phi) \simeq m_{3/2}^2 |\phi|^2 - \frac{cH^2 |\phi|^2}{\text{Hubble induced term}} + |\lambda|^2 \frac{|\phi|^{2(n-1)}}{M_{\text{Pl}}^{2n-6}} + \lambda a_M \frac{m_{3/2} \phi^n}{n M_{\text{Pl}}^{n-3}} + \text{h.c.}$$

Hubble induced term Non-renormalizable term V_{NR} A-term V_A

$m_{3/2}$: **gravitino mass**, H : **Hubble parameter**, $a_M = \mathcal{O}(1)$, $c(>0) = \mathcal{O}(1)$

Negative Hubble induced term, V_{NR} dominant
 \rightarrow Wine-bottle potential

.....
 Origin of field space is stabilized.
 The AD field is kicked in phase direction by A-term.
 \rightarrow non-zero baryon/lepton number

Lepton number :

$$n_L = i (\phi^* \dot{\phi} - \dot{\phi}^* \phi) \simeq \dot{\theta} |\phi|^2$$

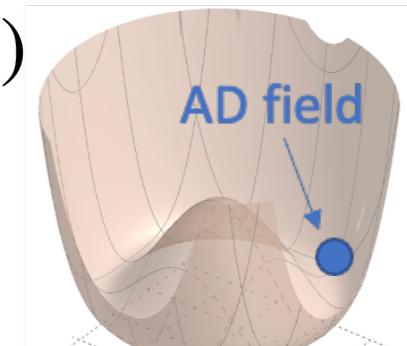
$$\simeq \epsilon m_{3/2} \varphi_{\text{osc}}^2$$

ϵ : efficiency parameter determined by $\arg(a_M)$, λ , etc.

$$H \gtrsim m_{3/2}$$

$$H \sim m_{3/2} (\equiv H_{\text{osc}})$$

$$H \lesssim m_{3/2}$$



Time

Q-balls

S. Coleman (1985)

- **Q-ball:** A spherically symmetric field configuration under $U(1)$ symmetric potential

$$\mathcal{L} = \sum_i \partial_\mu \phi_i^* \partial^\mu \phi_i - V(|\phi_i|), \quad Q = -i \sum_i q_i \int d^3x (\phi_i^* \dot{\phi}_i - \phi_i \dot{\phi}_i^*)$$

- For fixed Q , there exists spherically symmetric configuration which minimizes

$$E = \int d^3x (|\partial_t \phi_i|^2 + |\nabla \phi_i|^2 + V(\phi_i))$$

- Condition that Q-ball solution minimizes the energy :

$$\min [V(\varphi)/\varphi^2] < m_\phi^2$$

ポテンシャルの例と解析解くらいは
どこかのsupplemental materialで載せておく

Q-balls : non-linear growth of fluctuations

A. Kusenko, M. E. Shaposhnikov, Phys. Lett. B 418 (1998)

$$\varphi = Re^{i\Omega}$$

$$\begin{cases} \ddot{\Omega} + 3H\dot{\Omega} - \frac{1}{a^2(t)}\Delta\Omega + \frac{2\dot{R}}{R}\dot{\Omega} - \frac{2}{a^2(t)R}(\partial_i\Omega)(\partial^i R) = 0 \\ \ddot{R} + 3H\dot{R} - \frac{1}{a^2(t)}\Delta R - \dot{\Omega}^2 R + \frac{1}{a^2(t)}(\partial_i\Omega)^2 R + \frac{\partial V}{\partial R} = 0 \end{cases}$$

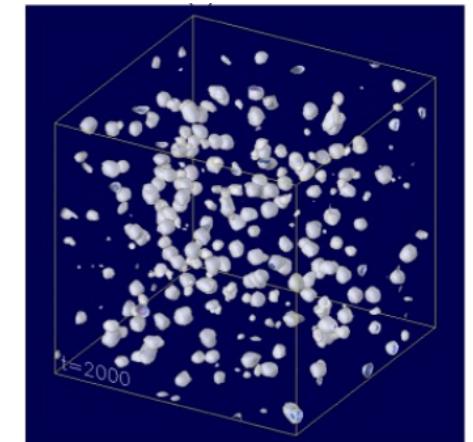
$$\begin{aligned} R &= \varphi(t) + \delta\varphi(x) \\ \Omega &= \theta(t) + \delta\theta(x). \end{aligned}$$

$$\begin{cases} \ddot{\delta\varphi} + 3H\dot{\delta\varphi} - \left(2\dot{\theta}(t)\varphi(t)\dot{\delta\theta} + \delta\varphi\dot{\theta}(t)\right) - \frac{1}{a(t)^2}\Delta\delta\varphi + V''(\varphi(t))\delta\varphi = 0 \\ \varphi(t)\ddot{\delta\theta} + 3H\left(\dot{\theta}(t)\delta\varphi + \dot{\delta\theta}\varphi(t)\right) + 2\left(\dot{\varphi}(t)\dot{\delta\theta} + \dot{\delta\varphi}\dot{\theta}(t)\right) - \frac{1}{a(t)^2}\varphi(t)\Delta\delta\theta = 0 \end{cases}$$

$$\begin{aligned} \varphi(t) &= \left(\frac{a_{\text{osc}}}{a(t)}\right)^{3/2} \varphi_{\text{osc}} \\ \delta\varphi &= \left(\frac{a_{\text{osc}}}{a(t)}\right)^{3/2} \delta\varphi_{\text{osc}} e^{i(S(t)+\mathbf{k}\cdot\mathbf{x})} \\ \delta\theta &= \delta\theta_{\text{osc}} e^{i(S(t)+\mathbf{k}\cdot\mathbf{x})} \end{aligned}$$

Figure:

T. Hiramatsu, M. Kawasaki, F. Takahashi,
JCAP (2010)



Instability band :

$$\frac{\mathbf{k}_{\max}^2}{a^2} = \frac{1}{16\dot{\theta}^2}(-V''^2 - 6V''\dot{\theta}^2 + 7\dot{\theta}^4)$$

The negative Hubble-induced term

Scalar potential in supergravity

$$V \supset e^{K(I^*, I)/M_{\text{Pl}}^2} \left[K_I^I (K^{-1})_I^I (K^{-1})_I^I \left(W_I^* + W(I)^* \cdot \frac{K_I}{M_{\text{Pl}}^2} \right) \left(W^I + \frac{W(I)K^I}{M_{\text{Pl}}^2} \right) - 3 \frac{W(I)W(I)^*}{M_{\text{Pl}}^2} \right] \quad (1)$$

$$\simeq \frac{1-a}{M_{\text{Pl}}^2} \phi^* \phi V(I) \quad (2)$$

I : Inflaton, K : Kahler function, W : Superpotential

$$K = \phi^* \phi + I^* I + \frac{a}{M_{\text{Pl}}^2} \phi^* \phi I^* I$$

$$V(I) \simeq M_{\text{Pl}}^2 H^2 \quad \rightarrow \quad V(\phi) \supset -\mathcal{O}(H^2) \phi^* \phi$$

$$(a \gtrsim 1)$$

The A-term

$$\delta K = -\frac{c}{M_{\text{Pl}}} X \phi^* \phi + \text{h.c.}$$

X : SUSY breaking field, δK : Kahler function, W : Superpotential

$$V \supset e^{K(X^*, X)/M_{\text{Pl}}^2} \left[K_X^X (K^{-1})_X^\phi (W_\phi^* + W^* K_\phi / M_{\text{Pl}}^2) (K^{-1})_X^X (W^X + W K^X / M_{\text{Pl}}^2) \right. \\ \left. + K_X^X (K^{-1})_X^X (W_X^* + W^* K_X / M_{\text{Pl}}^2) (K^{-1})_\phi^X (W^\phi + W K^\phi / M_{\text{Pl}}^2) \right] \\ \simeq c \frac{\phi}{M_{\text{Pl}}} W^\phi W_X^* + \text{h.c.}$$

$$m_{3/2} \sim \frac{\langle F_X \rangle}{M_{\text{Pl}}} \sim \frac{|W_X|}{M_{\text{Pl}}} \rightarrow V \sim c m_{3/2} W(\phi)$$

Gauge mediation model

Gravitino mass:

$$m_{3/2} \simeq \frac{\langle F_X \rangle}{\sqrt{3} M_{\text{Pl}}}$$

Slepton mass, Gaugino mass

$$m_\phi \sim M_g \sim \frac{M_F^2}{M_s}$$

Where

M_s : Messenger scalar mass

$$M_F \equiv \frac{1}{4\pi} g^{1/2} k^{1/2} \langle F_X \rangle^{1/2}$$

(k : coupling between messenger sector and SUSY breaking sector (X))

The property of delayed-type Q-ball

Initial charge

$$Q_G \simeq \beta_G \frac{\varphi_{\text{eq}}^4}{M_F^4} \simeq \beta_G \left(\frac{M_F}{m_{3/2}} \right)^4,$$

Mass, Radius, energy per unit lepton charge

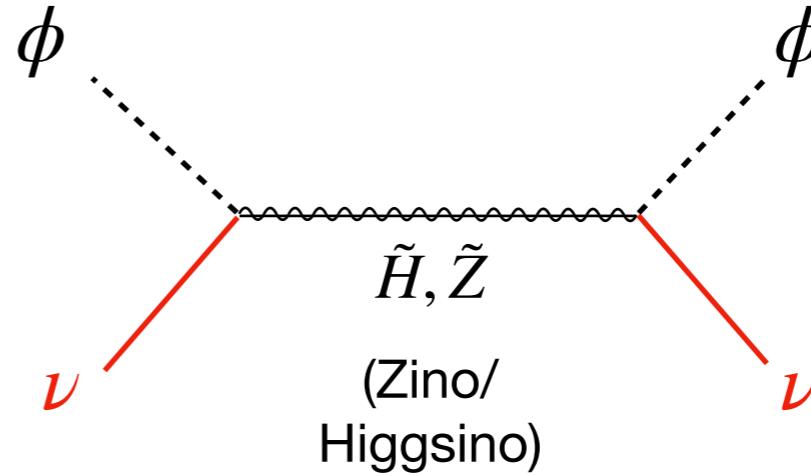
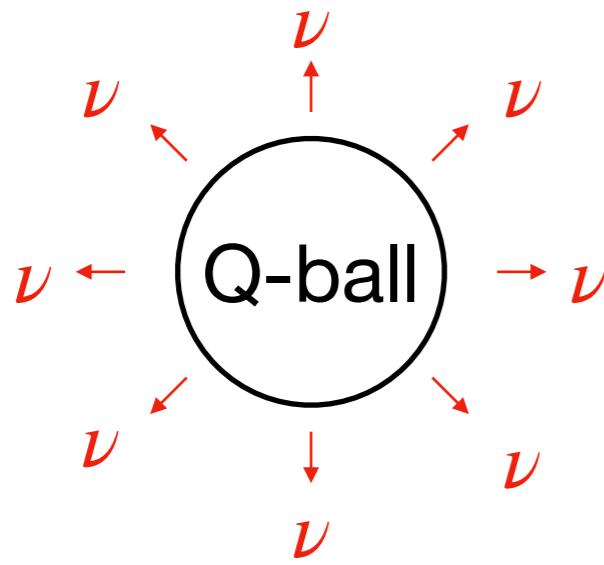
$$M_Q \simeq \frac{4\sqrt{2}\pi}{3} \zeta M_F Q_G^{\frac{3}{4}},$$

$$R_Q \simeq \frac{1}{\sqrt{2}\zeta} M_F^{-1} Q_G^{\frac{1}{4}},$$

$$\omega_Q \simeq \sqrt{2}\pi \zeta M_F Q_G^{-\frac{1}{4}}.$$

Decay rate of Q-balls

A. G. Cohen, S. R. Coleman, H. Georgi, A. Monohar, (1986)
M. Kawasaki, M. Yamada (2012)



The decay rate is bounded by Pauli blocking effect of final states.

(Decay rate) \propto (Surface area of Q-balls)

$$\Gamma_Q \simeq \frac{N_l}{Q} \frac{\omega_Q^3}{12\pi^2} 4\pi R_Q^2$$

このスライ
は1つにまとめ

N_l : number of neutrino species, Q : lepton charge inside each Q-ball,

ω_Q : energy per unit lepton charge, R_Q : radius of each Q-ball

In case with $g |\phi| \gg \omega_0$, the decay rate is almost saturated by the bound.

Thermodynamical effect of Q-balls

M. Laine, M. Shaposhnikov, (1998)

Q-balls thermalize due to interactions with neutrinos in thermal plasma via gaugino/Higgsino exchange

$$\text{if } T_{\text{form}} \gtrsim \frac{m_\phi^{3/2}}{M_{\text{Pl}}^{1/2}} Q^{1/4}$$

The evaporation rate of Q-balls :

$$\frac{dQ}{dt} = -D(\mu_Q - \mu_{\text{plasma}})T^2 4\pi R_Q^2$$

Where

$\mu_Q \simeq \omega_Q$: chemical potential of slepton field in Q-balls

μ_{plasma} ($\ll \omega_Q$) : chemical potential in thermal plasma

$D \sim gT^2/m_\phi^2$: diffusion coefficient

Condition for Shi-Fuller mechanism to work

We consider the 2 cases : $\eta_L = 3 \times 10^{-3}, 3 \times 10^{-4}$ (fixed)

- Condition 1.

The decay temperature must be consistent with sterile neutrino production

$$T_{\text{decay}} \gtrsim T_p(m_s = 10\text{keV})$$

- Condition 2.

The Q-ball evaporation process explain $\eta_b \sim 10^{-10}$

Delayed-type Q-ball scenario : Calculations

We consider Q-balls dominate over the universe at decay.

$$f_Q \equiv \frac{m_{3/2}^2 \varphi_{\text{osc}}^2}{3M_{\text{Pl}}^2 H_{\text{osc}}^2} \frac{T_{\text{R}}}{T_{\text{decay}}} \gg 1.$$

Decay temperature of Q-balls :

$$T_{\text{decay}} \simeq \left(\frac{90}{\pi^2 g_*(T_{\text{decay}})} \right)^{1/4} \sqrt{M_{\text{Pl}} \Gamma_Q}$$

Resultant lepton asymmetry :

$$\eta_L \simeq \frac{n_L}{\rho_{Q,\text{decay}}} \times \frac{3\rho_{Q,\text{decay}}}{4T_{\text{decay}}} = \frac{3n_{L,\text{osc}}}{4T_{\text{decay}}} \simeq \epsilon \frac{3T_{\text{decay}}}{4m_{3/2}}$$

Resultant baryon asymmetry :

$$\eta_b \simeq \eta_L \left(\frac{\Delta Q_b}{Q_G} \right) \sim \frac{128\sqrt{2}}{207} \pi^2 \zeta^{-1} \eta_L \frac{M_{\text{Pl}} T_{\text{R}}^2}{m_\phi^2 M_F} Q_G^{-\frac{3}{4}}$$

Delayed-type Q-ball scenario : Calculations

We analyze for two possible case about the value of
resultant η_L : $\eta_L = 3 \times 10^{-3}, 3 \times 10^{-4}$

The decay temperature must be consistent with sterile neutrino production

$$T_{\text{decay}} \simeq 0.067 \text{GeV} \times \left(\frac{\epsilon}{10^{-2}} \right)^{-1} \left(\frac{m_{3/2}}{0.5 \text{GeV}} \right) \left(\frac{\eta_L}{10^{-3}} \right) \gtrsim T_p$$

We fix the resultant baryon asymmetry:

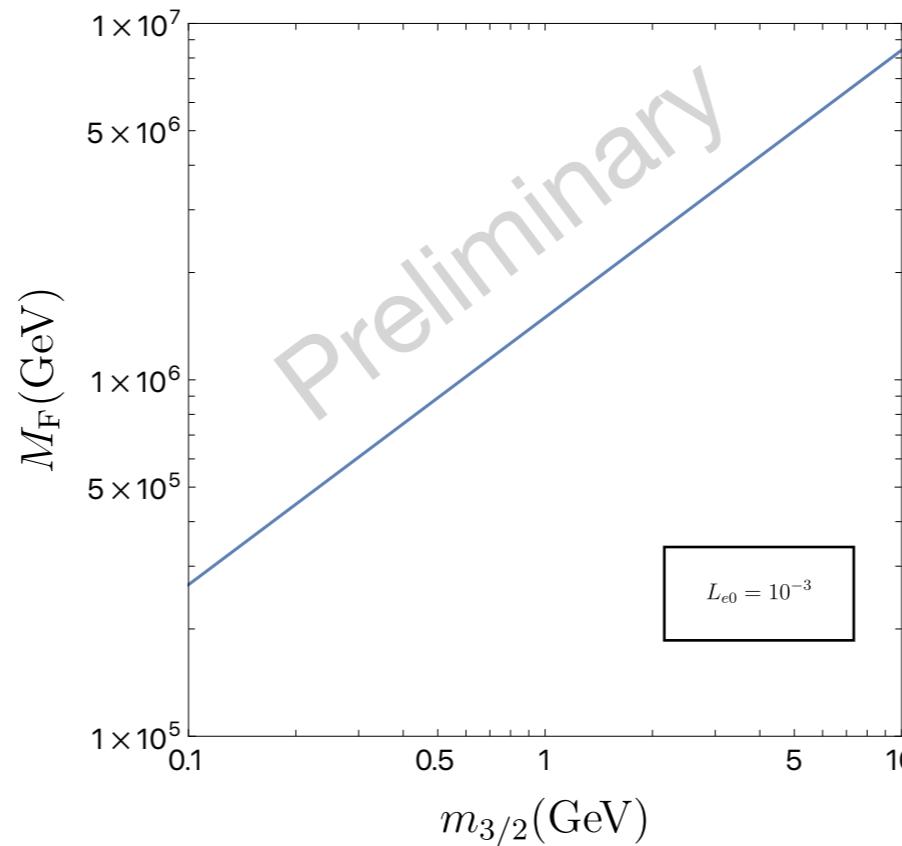
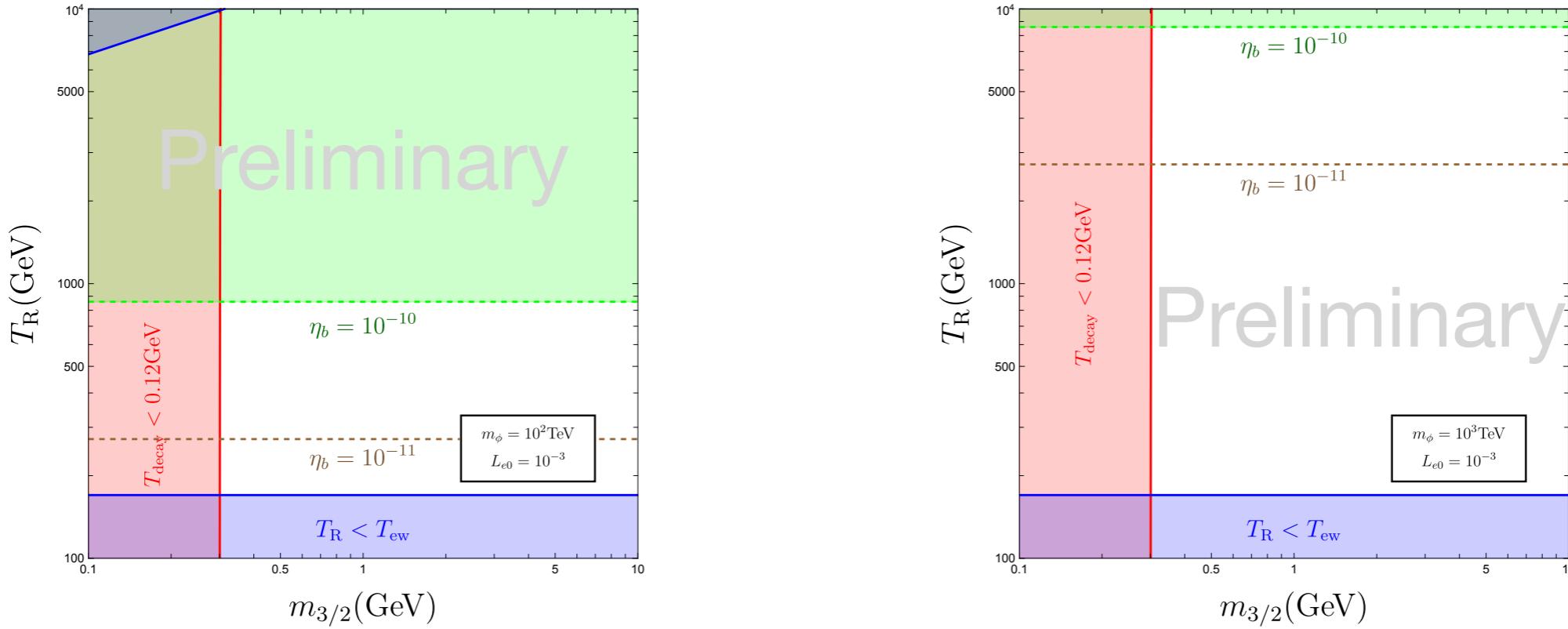
$$\eta_b \sim 1.3 \times 10^{-10} \left(\frac{\zeta}{2.5} \right)^{-1} \left(\frac{\epsilon}{10^{-2}} \right)^{-2} \left(\frac{\eta_L}{3 \times 10^{-3}} \right)^3 \left(\frac{m_\phi}{10^5 \text{GeV}} \right)^{-2} \left(\frac{T_R}{10^3 \text{GeV}} \right)^2 \sim 10^{-10}$$

(The energy density of the gravitino should not dominate over dark matter:

$$\Omega_{3/2} h^2 \simeq 0.71 \left(\frac{m_{3/2}}{0.5 \text{GeV}} \right)^{-1} \left(\frac{M_{\tilde{g}}}{10^4 \text{GeV}} \right)^2 \left(\frac{T_R}{10^5 \text{GeV}} \right) f_Q^{-3/4} \ll 1$$

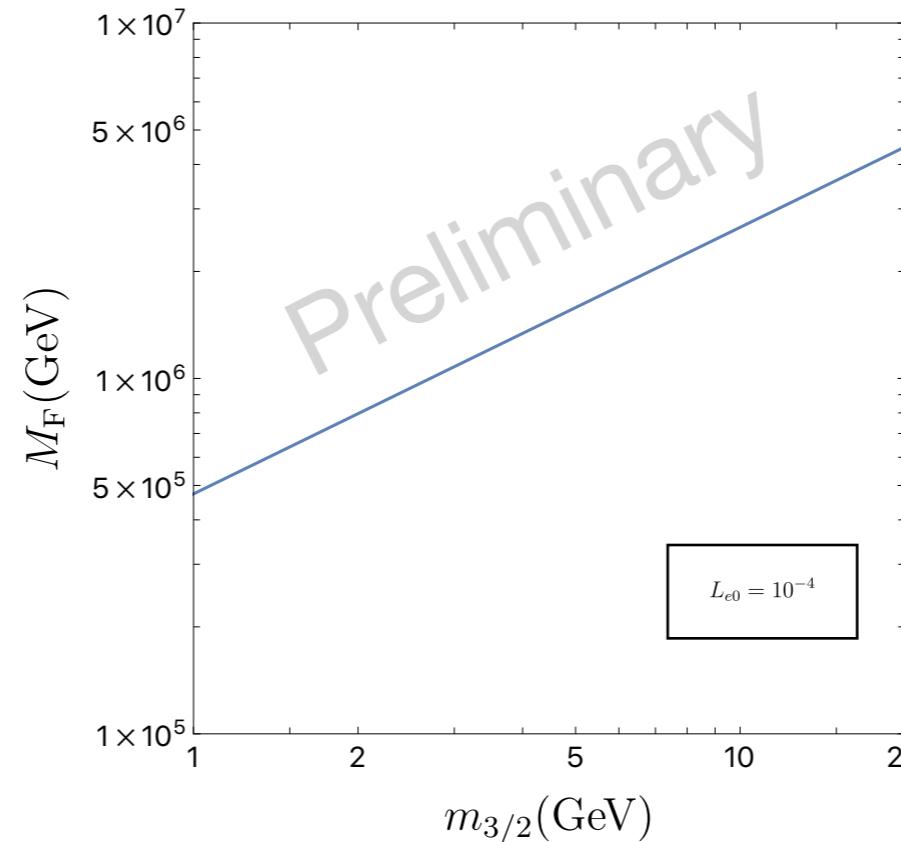
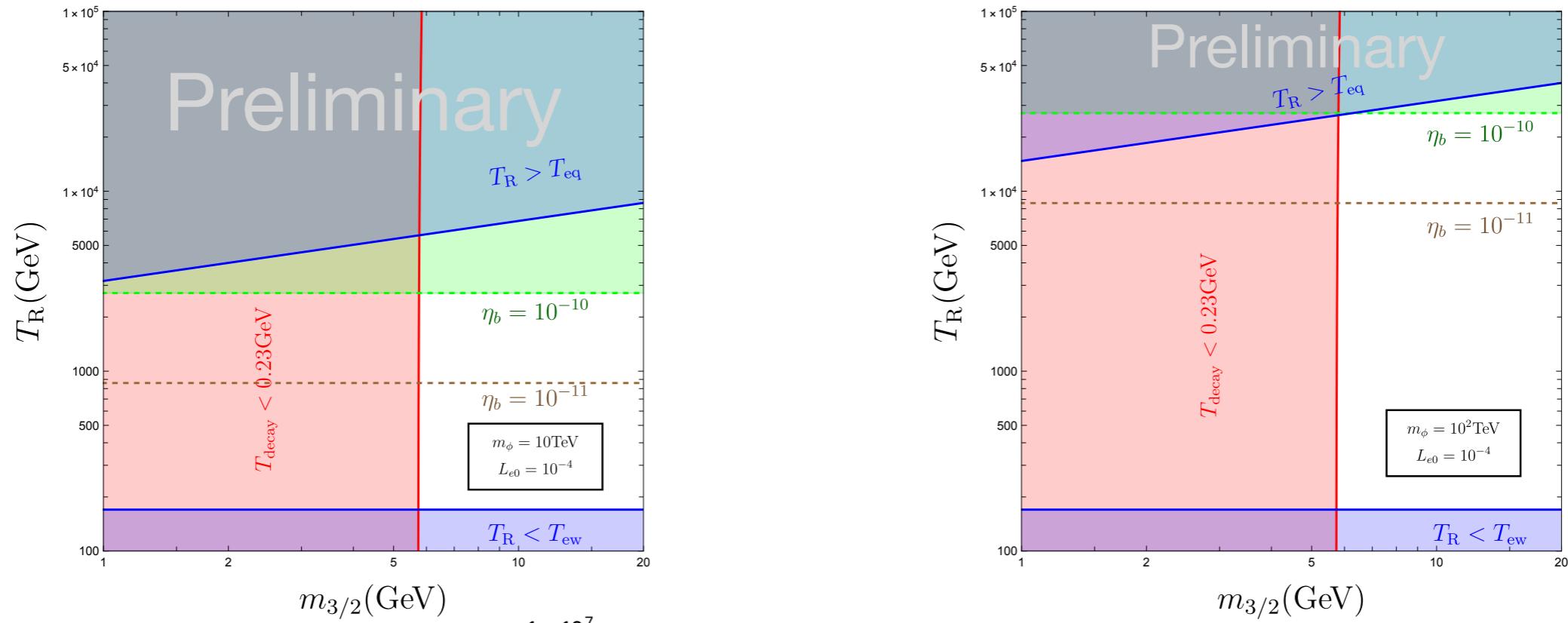
Delayed-type Q-ball scenario

: Parameter space ($L_{e,\text{init}} = 10^{-3}$, $\epsilon = 10^{-2}$)



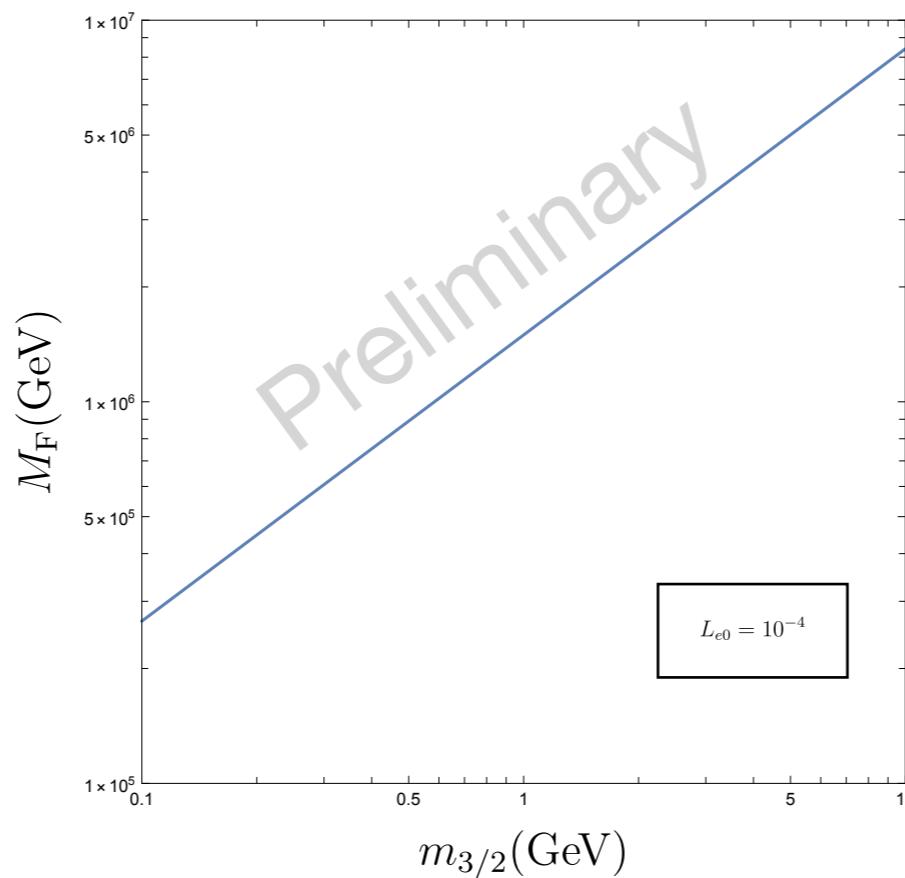
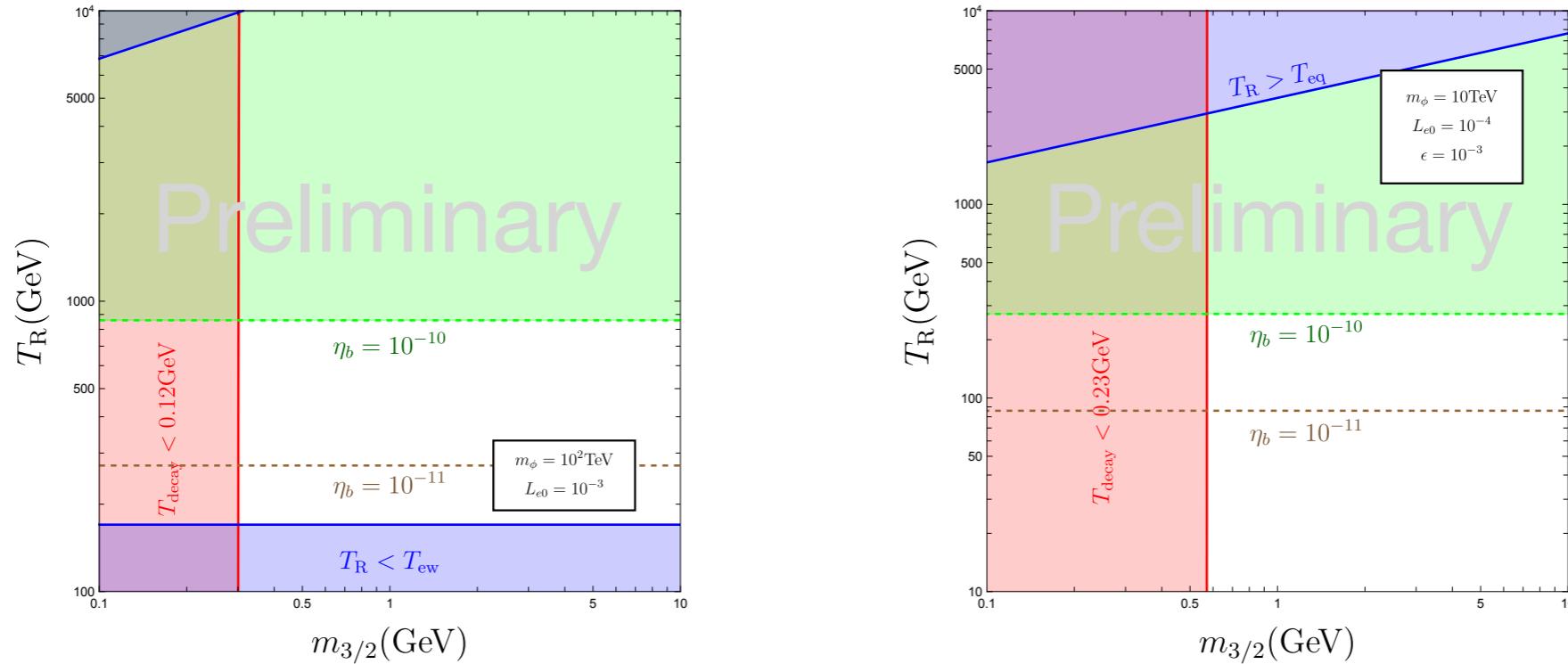
Delayed-type Q-ball scenario

: Parameter space ($L_{e,\text{init}} = 10^{-4}$, $\epsilon = 10^{-2}$)



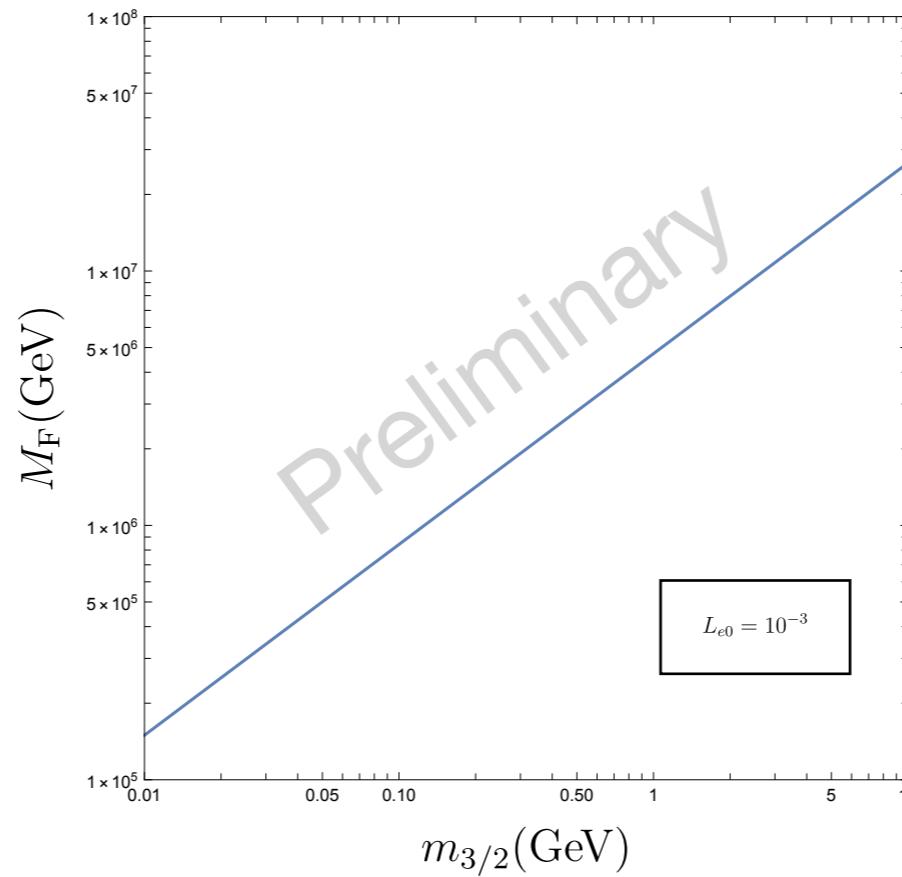
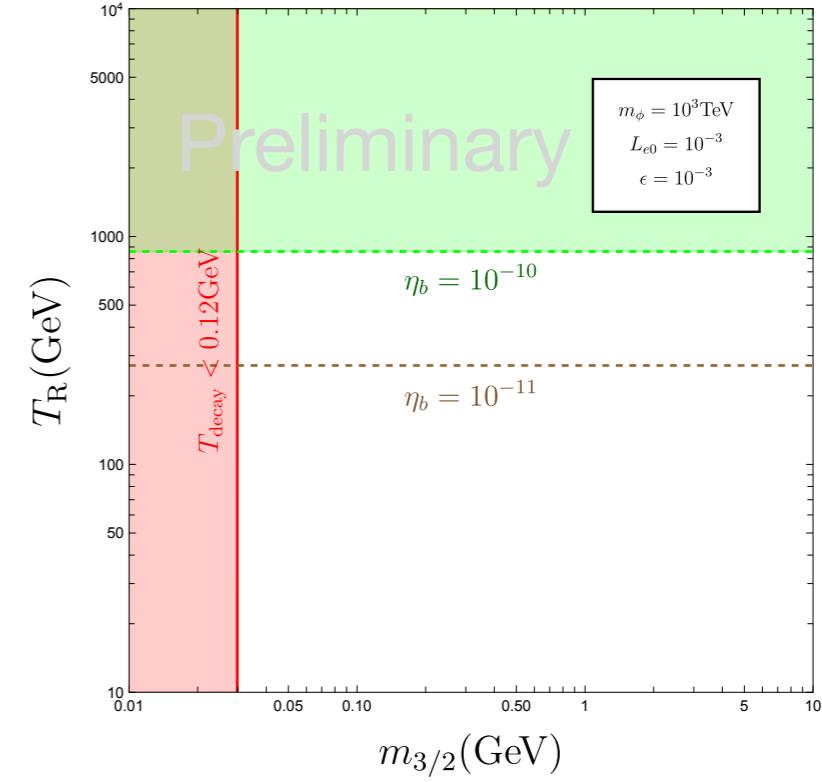
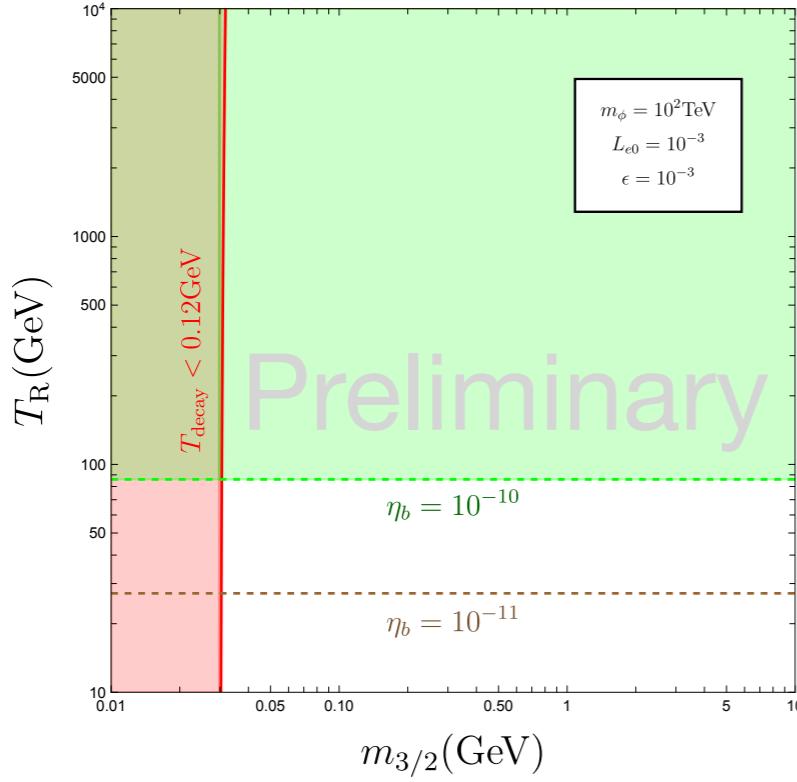
Delayed-type Q-ball scenario

: Parameter space ($L_{e,\text{init}} = 10^{-4}$, $\epsilon = 10^{-3}$)



Delayed-type Q-ball scenario

: Parameter space ($L_{e,\text{init}} = 10^{-3}$, $\epsilon = 10^{-3}$)



Future work : isocurvature perturbation from sterile neutrino dark matter

Final abundance of sterile neutrino is determined by $m_s, \theta_0, L_{e,\text{init}}$.

In Affleck-Dine scenario, $L_{e,\text{init}}$ has fluctuations which is independent from inflaton fluctuations.

→ In principle, this can induce isocurvature perturbations from sterile neutrino dark matter.

$$\begin{aligned}\frac{\delta\rho_{\nu_s}}{\rho_{\nu_s}} &= \left| \left(\frac{\partial \ln \rho_{\nu_s}}{\partial L_{e,\text{init}}} \right) \cdot \frac{\delta L_{e,\text{init}}}{L_{e,\text{init}}} \right| \\ &\sim \left| \frac{\partial \ln \rho_{\nu_s}}{\partial L_{e,\text{init}}} \cdot \frac{H_I}{\varphi_{\text{osc}}} \right|\end{aligned}$$