UV-sensitivity in Kaluza-Klein theories: Higgs mass & vacuum energy

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Introduction

Theories with additional compact dimensions: ubiquitous

- Studied as $4D$ theories w/ infinite towers of states $m_n = f_n m_{\rm tow}$

Surprising UV-softness is found in this framework:

- Paradigm: towers of states contribute $\sim m_{\rm tow}^4$ to vacuum energy / effective potential

For EFTs... extremely surprising... How can it be?

... Let's try to revisit what's behind this ...

Scherk-Schwarz mechanism

5D SUSY theory $\mathcal{S}_{(5)}$ in multiply connected spacetime $(\mathcal{M}^4_{(x)} \times S^1_{(z)})$

- Different R-charge for superpartners $(i = b, f)$

$$\Psi_i(x, z + 2\pi R) = e^{2i\pi R q_i} \Psi_i(x, z) \;\Rightarrow\; \Psi_i(x, z) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{+\infty} \psi_{i,n}(x) e^{i(\frac{n}{R} + q_i)z}$$

$\int dz \to \mathcal{S}_{(4)}$ infinite tower of $4D$ KK fields w/ $m^2_{i,n} \propto \left(\frac{n}{R} + q_i\right)^2$

- 4D "masses" mismatch: effective 4D non-local soft SUSY breaking

Identify Higgs (ϕ) w/: (a) ϕ_0, (b) $4D$ brane field

Effective 4D quadratic operator

$$\boxed{M^2_{i,n}(\phi) = m^2_i(\phi) + \left(\frac{n}{R} + q_i\right)^2}$$

In some cases one can also get $M^2_{i,n}(\phi) = \left(\frac{n}{R} + q_i(\phi)\right)^2$

UV-insensitive Higgs

About 25 years ago: $V_{1l}(\phi)$ & m_H^2 UV-insensitive

Compact dimensions + $(4+n)D$ SUSY: solution to naturalness!

Compact expression for both classes: $M^2(\phi) = m^2 + \left(\frac{n}{R} + q\right)^2$

For each tower:

- Power UV-sensitivity through $m \Longrightarrow$ canceled by SUSY
- No UV-sensitivity through q

$$V_{1l}(\phi) = R\left(\frac{m^2\Lambda^3}{48\pi} - \frac{m^4\Lambda}{64\pi} + \frac{m^5}{60\pi}\right) - \sum_{k=1}^{\infty} \frac{e^{-2\pi k m R}\left(2\pi k m R(2\pi k m R + 3) + 3\right)\cos(2\pi k q)}{64\pi^6 k^5 R^4}$$

How is this obtained?

The "KK regularization"

Result deeply rooted in the interpretation of ∞ KK $4D$ fields

Originally ("KK-regularization") Antoniadis, Dimopoulos, Pomarol, Quiros/Delgado, Pomarol, Quiros/ Barbieri, Hall, Nomura/Arkani-Hamed, Hall, Nomura, Smith, Weiner

- Sum: $]-\infty, \infty[$; Integral: cutoff/proper time ($\Lambda \to \infty$)

Criticism Ghilencea, Nilles/Kim

- Partial result: w/ sum $[-L, L] \to$ UV-sensitive terms

  UV-insensitiveness only for $L \gg R\Lambda$

Countercriticism Delgado, v.Gersdoff, John, Quiros/Contino, Pilo/Barbieri, Hall, Nomura/Masiero, Scrucca, Silvestrini

- L does NOT respect $5D$ symmetries (Lorentz, SUSY): "spurious"
- Thick brane & Pauli-Villars: (apparently) safer derivation

Debate closed in favour of UV-insensitiveness ... but ...

5D origin

$$\mathcal{S}_{(5)} = \int dz\, d^4x \left(\frac{1}{2}\,\partial_a\widehat{\Phi}\,\partial^a\widehat{\Phi} + \partial_a\widehat{\chi}\,\partial^a\widehat{\chi}^\dagger + \frac{m_\Phi^2}{2}\,\widehat{\Phi}^2 + m_\chi^2\,\widehat{\chi}\widehat{\chi}^\dagger + \frac{\widehat{\lambda}}{4!}\,\widehat{\Phi}^4 + \frac{\widehat{g}}{2}\,\widehat{\Phi}^2\widehat{\chi}\widehat{\chi}^\dagger \right)$$

$$\widehat{\Phi}(x, z + 2\pi R) = \widehat{\Phi}(x, z) \quad ; \quad \widehat{\chi}(x, z + 2\pi R) = e^{2i\pi R\, q}\,\widehat{\chi}(x, z)$$

$$\mathcal{V}^{(5)}_{1l}(\widehat{\Phi}) = \frac{1}{2}\mathrm{Tr}_5 \log \frac{p^2 + \frac{n^2}{R^2} + m_\phi^2 + \frac{\widehat{\lambda}}{2}\,\widehat{\Phi}^2}{p^2 + \frac{n^2}{R^2}} + \frac{1}{2}\mathrm{Tr}_5 \log \frac{p^2 + \left(\frac{n}{R} + q\right)^2 + m_\chi^2 + \frac{\widehat{g}}{2}\,\widehat{\Phi}^2}{p^2 + \frac{n^2}{R^2}}$$

$$\hat{p} = (p, p_5 = \tfrac{n}{R}) \to \mathrm{Tr}_5 = \tfrac{1}{2\pi R} \textstyle\sum_n \int \frac{d^4p}{(2\pi)^4}$$

- p & n intertwined: NO hierarchy when including asymptotics
- $\mathcal{V}^{(5)}_{1l}(\widehat{\Phi})$ diverges: signals it is an EFT $\hat{p}^2 \leq \hat{p}^2_{\max} \equiv \Lambda^2$

$$\sum_n \int \frac{d^4p}{(2\pi)^5 R} \to \left(\sum_n \int \frac{d^4p}{(2\pi)^5 R} \right)' \equiv \frac{1}{2\pi R} \sum_{n=-[R\Lambda]}^{[R\Lambda]} \int^{C^n_\Lambda} \frac{d^4p}{(2\pi)^4}$$

$$C^n_\Lambda \equiv \sqrt{\Lambda^2 - \frac{n^2}{R^2}}$$

5D origin

Better formalized in a Wilsonian framework. Fourier expansion similar for $\widehat{\Phi}$

$$\widehat{\chi}(x,z) = \left(\sum_n \int \frac{d^4p}{(2\pi)^5 R}\right)' \widehat{\chi}_{n,p}\, e^{i\left(p\cdot x + \left(\frac{n}{R}+q\right)z\right)}$$

$$\widehat{\chi}(x,z) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-[R\Lambda]}^{[R\Lambda]} \chi_n^{\Lambda}(x)\, e^{i\left(\frac{n}{R}+q\right)z}; \quad \chi_n^{\Lambda}(x) \equiv \frac{1}{\sqrt{2\pi R}} \int^{C_\Lambda^n} \frac{d^4p}{(2\pi)^4} \widehat{\chi}_{n,p}\, e^{ip\cdot x}$$

Performing z integration $\to$ effective $4D$ theory w/ $\phi = \phi_0$

$$V_{1l}(\phi) = \frac{1}{2} \sum_{n=-[R\Lambda]}^{[R\Lambda]} \int^{C_\Lambda^n} \frac{d^4p}{(2\pi)^4} \left(\log \frac{p^2 + \frac{n^2}{R^2} + m_\phi^2 + \frac{\lambda}{2}\phi^2}{p^2 + \frac{n^2}{R^2}} + \log \frac{p^2 + \left(\frac{n}{R}+q\right)^2 + m_\chi^2 + \frac{g}{2}\phi^2}{p^2 + \frac{n^2}{R^2}} \right)$$

$$\lambda \equiv \frac{\widehat{\lambda}}{2\pi R} \quad ; \quad g \equiv \frac{\widehat{g}}{2\pi R}; \quad \widehat{\Phi} = \frac{1}{\sqrt{2\pi R}}\phi \qquad \to \boxed{V_{1l}(\phi) = 2\pi R\, \mathcal{V}_{1l}(\widehat{\Phi})}$$

$+$ The infinite sum/finite integral procedure is mathematically illegitimate

UV-sensitivity and non-trivial topology

$$V_{1l}(\phi) = \frac{5m^2 + 3q^2}{180\pi^2} R\Lambda^3 - \frac{35m^4 + 14m^2q^2 + 3q^4}{840\pi^2} R\Lambda + \frac{m^5 R}{60\pi} - \sum_{k=1}^{\infty} \frac{e^{-2\pi k m R}(2\pi k m R(2\pi k m R + 3) + 3)\cos(2\pi k q)}{64\pi^6 k^5 R^4}$$

New q-dependent UV-sensitive terms:

- Not canceled by SUSY! $\propto \boxed{(q_b^2 - q_f^2)}\, m^2(\phi)\Lambda$
- Topological origin
  1. $= 0$ for $q = 0$ ($q\,\exists$ in multiply connected spacetime ($\cancel{R}$))
  2. R-independent $:\neq 0$ for $R \to \infty$
  3. UV-insensitive terms: opposite, $\to 0$ for $R \to \infty$, $\neq 0$ for $q = 0$

UV-insensitive terms are due to the size of the extra dimension ($R \ll 4D$ box), UV-sensitive terms are solely of topological origin

Allowing the infinite sum: smooth cut-off regularization

According to typical argument: cut on sum $\rightarrow$ spurious "divergences"

But... using

$$V_{1l}(\phi) = \frac{1}{2}\sum_{n=-\infty}^{\infty}\int \frac{d^4p}{(2\pi)^4}\log\left(\frac{p^2+m^2+\left(\frac{n}{R}+q\right)^2}{p^2+\frac{n^2}{R^2}}\right)e^{-\frac{p^2+\frac{n^2}{R^2}}{\Lambda^2}}$$

$\rightarrow$ Same result is found!

UV-sensitive terms are NOT due to the sharp cut of the sum!
They come from a correct treatment of $\hat{p}$ asymptotics

So... why do proper time, thick brane and Pauli-Villars give UV-insensitive result?

The secret liaison of proper time, thick brane & PV

- Thick brane: $\sum_{n=-\infty}^{\infty} \int^{(\Lambda)} \frac{d^4p}{(2\pi)^4} \frac{e^{-\frac{\left(\frac{n}{R}+q\right)^2}{\Lambda^2}}}{p^2+m^2+\left(\frac{n}{R}+q\right)^2}$
- Pauli-Villars: $\sum_{n=-\infty}^{\infty} \int \frac{d^4p}{(2\pi)^4} \frac{(\Lambda R)^4}{(\Lambda R)^4+p^2+(\frac{n}{R}+q)^2} \frac{1}{p^2+m^2+\left(\frac{n}{R}+q\right)^2}$
- Proper Time:

$$V_{1l}(\phi) = -\sum_{n=-\infty}^{\infty} \int \frac{d^4p}{(2\pi)^4} \int_{\frac{1}{\Lambda^2}}^{\infty} \frac{ds}{s} \left\{ e^{-s\left(p^2+m^2+\left(\frac{n}{R}+q\right)^2\right)} - e^{\left(p^2+\frac{n^2}{R^2}\right)} \right\}$$

$$= -\sum_{n=-\infty}^{\infty} \int \frac{d^4p}{(2\pi)^4} \left\{ \Gamma\left(0, \frac{p^2+m^2+\left(\frac{n}{R}+q\right)^2}{\Lambda^2}\right) - \Gamma\left(0, \frac{\frac{n^2}{R^2}+p^2}{\Lambda^2}\right) \right\}$$

They all allow for infinite summation! . . . but most importantly . . .

They introduce a cutoff function of (p_5+q) instead of $p_5 (=\frac{n}{R})$

This + infinite sum performed independently from integral:

Artificial wash-out of UV-sensitive terms!

Vacuum Energy

Compactification w/ gravity $\hat{g}_{MN} = \begin{pmatrix} e^{2\alpha\phi} g_{\mu\nu} - e^{2\beta\phi} A_\mu A_\nu & e^{2\beta\phi} A_\mu \\ e^{2\beta\phi} A_\nu & -e^{2\beta\phi} \end{pmatrix}$

In background configuration $g^0_{\mu\nu} = \eta_{\mu\nu}, A_\mu = 0, \phi = \phi_0$ (hereafter ϕ)

$$\boxed{\begin{aligned} \rho_4 = & \frac{5 \log \frac{\Lambda^2 e^{2\alpha\phi}}{\mu^2} - 2}{300\pi^2} e^{2\alpha\phi} R \Lambda^5 + \frac{5 m^2 + 3 q^2 e^{4\alpha\phi}}{180\pi^2} e^{2\alpha\phi} R \Lambda^3 \\ & - \frac{35 m^4 + 14 m^2 q^2 e^{4\alpha\phi} + 3 q^4 e^{8\alpha\phi}}{840\pi^2} e^{2\alpha\phi} R \Lambda + \frac{m^5}{60\pi} e^{2\alpha\phi} R \\ & + \frac{3 \log \frac{\Lambda^2 e^{2\alpha\phi}}{\mu^2} + 2}{2880\pi^2 R^4} e^{10\alpha\phi} R \Lambda + R_4 + \mathcal{O}(\Lambda^{-1}) = 2\pi R e^{2\alpha\phi} \rho_5 \end{aligned}}$$

$$R_4 = -\frac{x^2 \mathrm{Li}_3\left(r_b e^{-x}\right) + 3x \mathrm{Li}_4\left(r_b e^{-x}\right) + 3\mathrm{Li}_5\left(r_b e^{-x}\right) + 6\zeta(5)}{128\pi^6 R^4} e^{12\alpha\phi} + h.c.$$

$$r \equiv e^{2\pi i q R} \quad , \quad x \equiv 2\pi e^{-2\alpha\phi} R \sqrt{m^2} \implies R_4 \propto \frac{e^{12\alpha\phi}}{R^4} = m_{KK}^4$$

Light tower limit and Dark Dimension

- SUSY: dominant contribution $\rho_4 \sim (q_b^2 - q_f^2)\, e^{6\alpha\phi} R\Lambda^3 = m_{KK}^2 R\Lambda^3$
- Non-SUSY: dominant contribution $\rho_4 \sim e^{2\alpha\phi} R\Lambda^5 = m_{KK}^{\frac{2}{3}} \left(R^{\frac{1}{3}}\Lambda\right)^5$

Even in the light tower limit $\phi \to -\infty$, the UV-insensitive R_4 cannot overthrow these dominating contributions : there is no light tower regime in an EFT where $\rho_4 \sim m_{KK}^4$!

Extremely important for Dark Dimension — Montero, Vafa, Valenzuela

- From EFT side: $\rho_4 \sim m_{KK}^4$ is untenable
- If $\Lambda_{cc}^{\rm exp}$ signals additional dimensions, *must* be outside EFT regime

Cut in tower typical in Swampland: Species scale Λ_S (e.g. emergence proposal)

Grimm, Palti, Valenzuela

Species scale $\Lambda = \left(M_p^2 m_{KK}\right)^{\frac{1}{3}}$: dominant depend on $|\phi|/M_p \lesssim 100$

- SUSY: $\rho_4 \sim (q_b^2 - q_f^2) M_p^2 m_{KK}^3 \longrightarrow \rho_4 \sim -(q_b^2 - q_f^2) m^2 M_p^{2/3} m_{KK}^{7/3}$
- Non-SUSY: $\rho_4 \sim M_p^{10/3} m_{KK}^{7/3} \longrightarrow \rho_4 \sim m^5 m_{KK}^{2/3}$

Global picture: EFTs with compact dimensions

Recent argument: EFT not applicable as it requires large hierarchy between last included and first excluded KK mode Burgess, Quevedo

$\rightarrow$ Rely on usual interpretation of KK modes as massive $4D$ states

- Start: $\mathcal{S}_\Lambda^{(5)}$ w/ "Wilsonian" mode expansion $\hat{p} \in [0, \Lambda]$
- Integrating out modes in $[k, \Lambda] \rightarrow \mathcal{S}_k^{(5)}$ k Wilsonian running scale

Due to $p_5 = n/R$ discreteness, p_5 eigenmodes contribution is stepwise

- For $k < 1/R$ no p_5 eigenmodes anymore: RG evolution becomes effectively of $4D$ type

> It is **only in this sense** that the $4D$ theory emerges from the $5D$ one: **by no means it has an** *infinite* **tower of states**

Summary & Conclusions

- UV-insensitive Higgs/vacuum energy paradigm must be revisited
- Usual calculations implement physically and mathematically illegitimate procedure
- Correct treatment of the asymptotics of loop momentum shows the presence of UV-sensitive terms
- Usual interpretation of the theory as a $4D$ theory w/ ∞ number of fields needs to be taken w/ a grain of salt
- The UV-sensitive terms are of topological origin, R-independent
- Same is true for vacuum energy: dark dimension untenable from EFT side
- Raises some warnings on the use of proper time in multiply connected manifolds