

TRILINEAR HIGGS SELF-COUPPLINGS AT $\mathcal{O}(\alpha_t^2)$ IN THE CP-VIOLATING NMSSM

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with Thi Nhung Dao, Martin Gabelmann, Margarete Mühlleitner, Heidi Rzehak



30TH INTERNATIONAL CONFERENCE ON SUPERSYMMETRY
AND UNIFICATION OF FUNDAMENTAL INTERACTIONS

Southampton, 19 July 2023

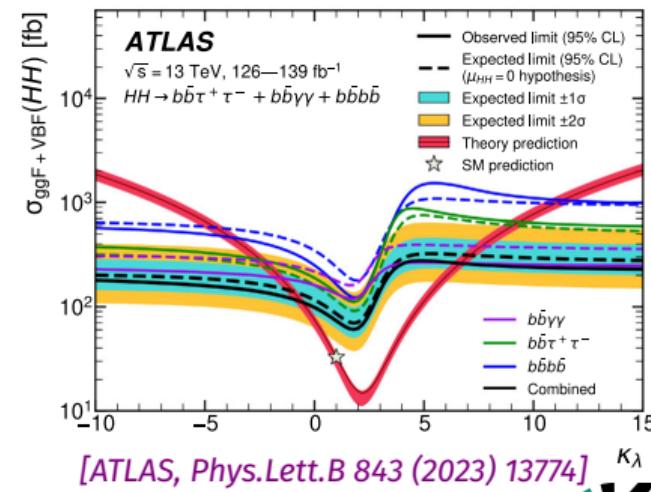
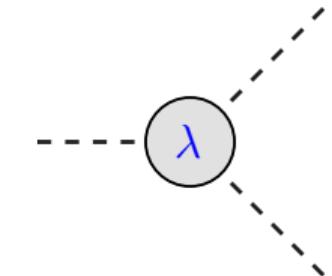
Trilinear Higgs self-coupling

Trilinear Higgs coupling

- ▶ Probes EWSB mechanism: determines shape of the potential

$$V_{\text{SM}} = \frac{1}{2} m_h^2 h^2 + \frac{1}{3!} \lambda_{hhh}^{\text{SM}} \kappa_\lambda h^3 + \frac{1}{4!} \lambda_{hhhh}^{\text{SM}} \kappa_{2\lambda} h^4$$

with $\lambda_{hhh}^{\text{SM}} = \frac{3m_h^2}{v} = 191 \text{ GeV}$



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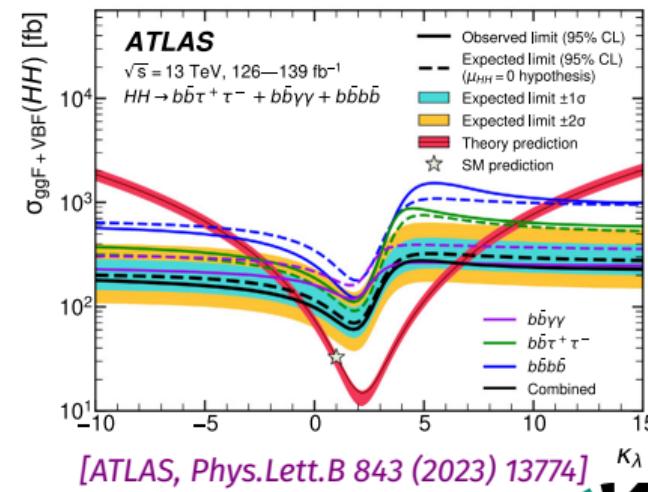
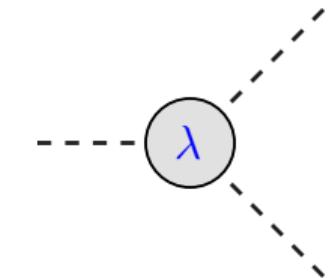
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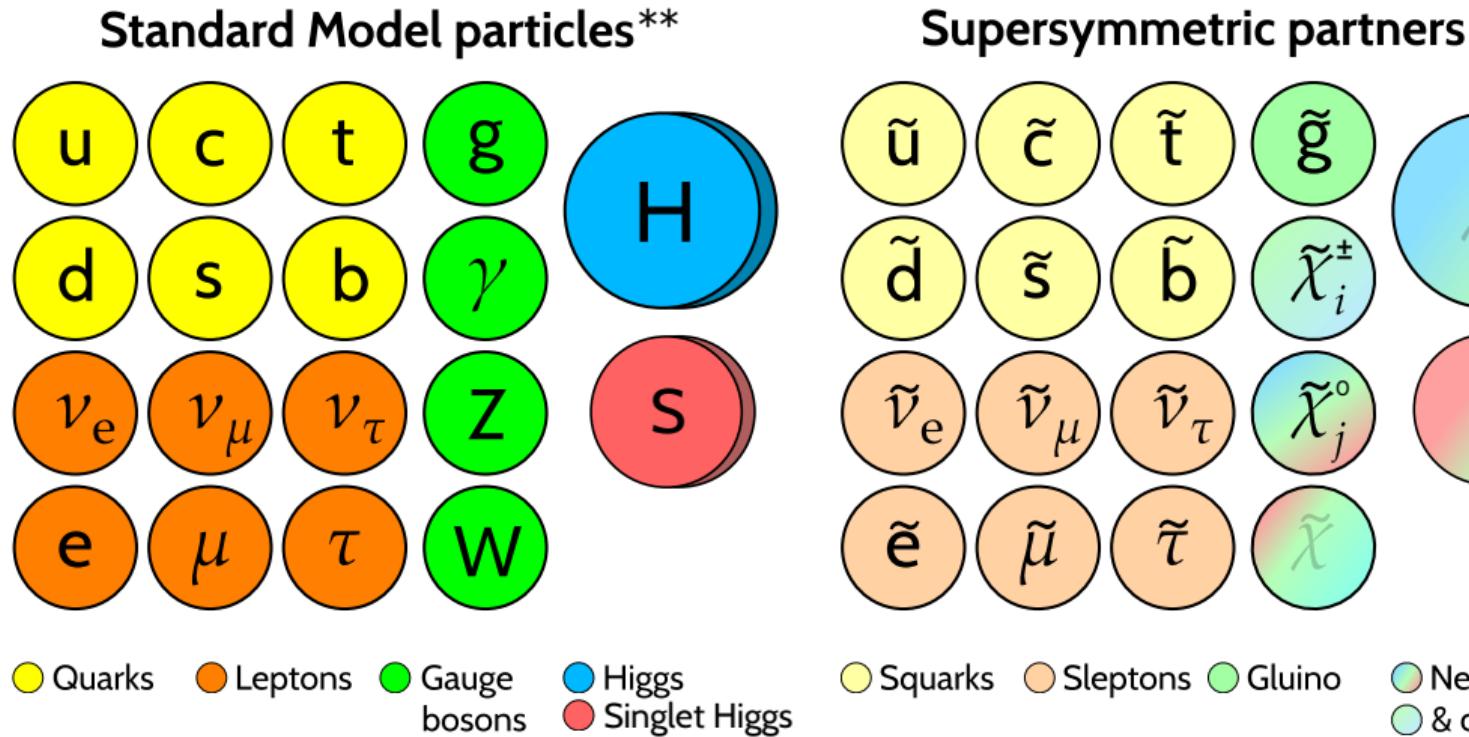
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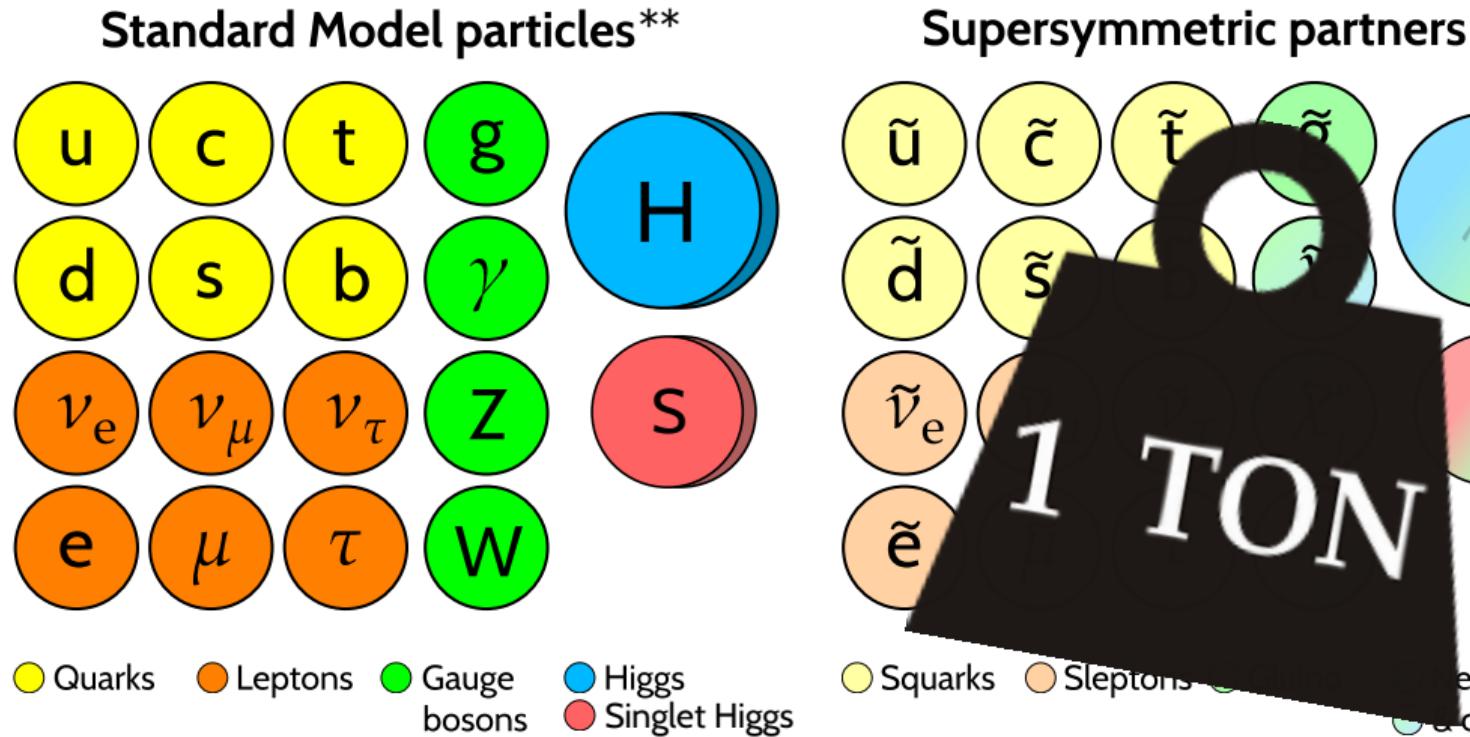
- ▶ Very sensitive to physics beyond the SM
- ▶ Important input for hh production
- ▶ Important input for $h \rightarrow hh$ decays
- ▶ Important input for phase transitions



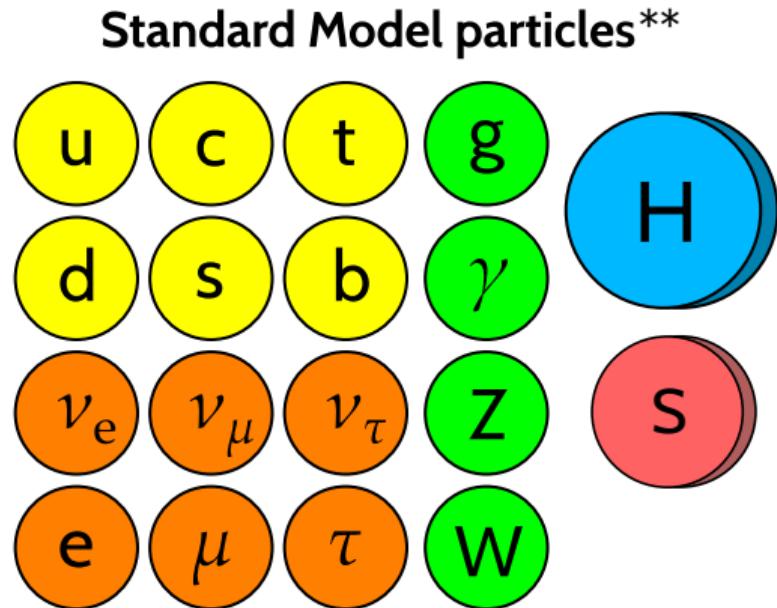
Next-to-Minimal Supersymmetric Standard Model



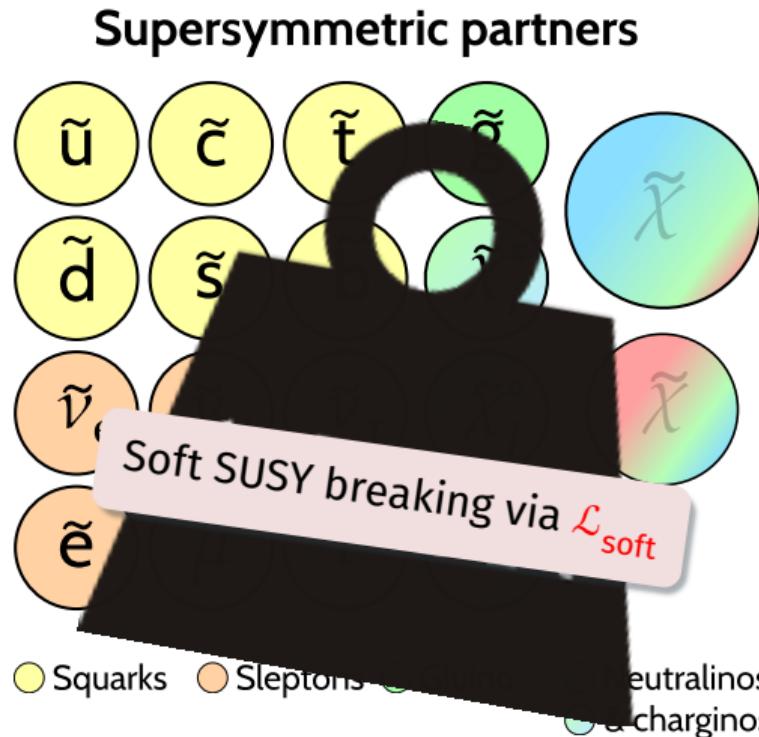
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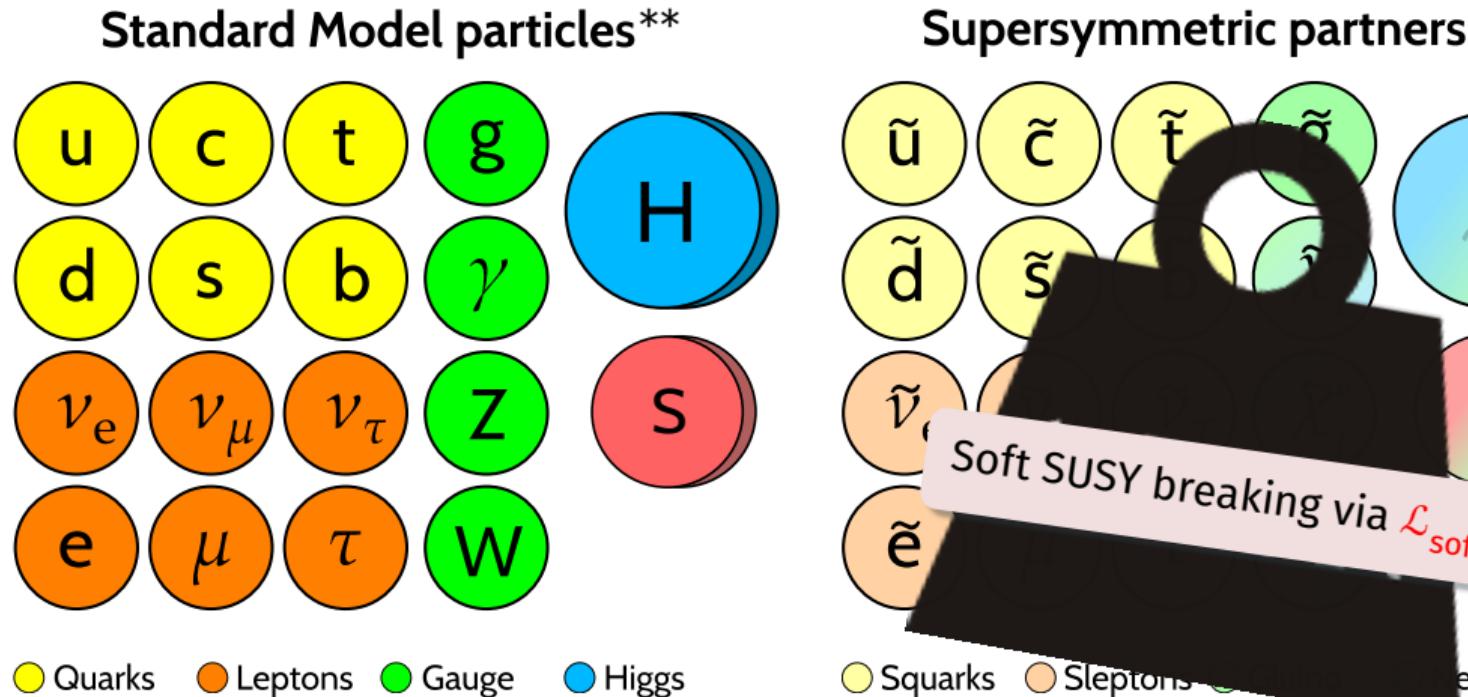
Next-to-Minimal Supersymmetric Standard Model



● Quarks ● Leptons ● Gauge
bosons ● Higgs
● Singlet Higgs



Next-to-Minimal Supersymmetric Standard Model



Tomorrow (plenary): M. Mühlleitner: “Recent Developments of the NMSSM”

The CP-Violating NMSSM

Complex Next-to-Minimal Supersymmetric Standard Model

- Complex scalar singlet extension of the MSSM with rich Higgs sector:

$$H_d = \begin{pmatrix} v_d + \textcolor{blue}{h_d} + i\textcolor{red}{a_d} \\ \sqrt{2} \\ \textcolor{violet}{h_d^-} \end{pmatrix}, \quad H_u = e^{i\varphi_u} \begin{pmatrix} \textcolor{blue}{h_u^+} \\ \frac{v_u + \textcolor{blue}{h_u} + i\textcolor{red}{a_u}}{\sqrt{2}} \end{pmatrix}, \quad S = \frac{e^{i\varphi_S}}{\sqrt{2}}(v_S + \textcolor{blue}{h_S} + i\textcolor{red}{a_S})$$

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- $\phi_i = \mathbf{h}_d, \mathbf{h}_u, \mathbf{h}_S, \mathbf{a}_d, \mathbf{a}_u, \mathbf{a}_S$ and $\mathbf{h}_d^\pm, \mathbf{h}_u^\pm$ mixing to $\mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_3, \mathbf{h}_4, \mathbf{h}_5, \mathbf{G}^0$ and $\mathbf{h}^\pm, \mathbf{G}^\pm$

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Scalar Higgs boson potential

$$\begin{aligned} V_H = & \left(|\lambda S|^2 + m_{H_d}^2 \right) H_d^\dagger H_d + \left(|\lambda S|^2 + m_{H_u}^2 \right) H_u^\dagger H_u + m_S^2 |S|^2 + |\kappa S^2 - \lambda H_d \cdot H_u|^2 \\ & + \left[\frac{1}{3} \kappa A_\kappa S^3 - \lambda A_\lambda S H_d \cdot H_u + \text{h.c.} \right] + \frac{1}{8} (g_1^2 + g_2^2) (H_d^\dagger H_d - H_u^\dagger H_u)^2 + \frac{1}{2} g_2^2 |H_d^\dagger H_u|^2 \end{aligned}$$

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Scalar Higgs boson potential

$$V_H = \left(|\lambda S|^2 + \right. \\ \left. + \left[\frac{1}{3} \textcolor{brown}{K} A_K \right] \right) |H_d| \cdot |H_u|^2$$

\Rightarrow Trilinear Higgs couplings: $\lambda_{ijk} = \frac{\partial^3 V_H}{\partial \phi_i \partial \phi_j \partial \phi_k} \Big|_{\phi=0}$ $\frac{1}{2} g_2^2 |H_d^\dagger H_u|^2$

Theoretical status on accuracy in the NMSSM

	Mass corrections δm_h	Coupling corrections $\delta \lambda_{hhh}$
► full 1-loop	✓ [lots of independent contribs.]	✓ [Dao et al. '13]
► 2-loop $\mathcal{O}(\alpha_t \alpha_s)$	✓ [Dao et al. '14]	✓ [Dao et al. '15]
► 2-loop $\mathcal{O}(\alpha_t^2)$	✓ [Dao et al. '19]	✓ [CB, Dao, Gabelmann, Mühlleitner, Rzehak '22]
► 2-loop $\mathcal{O}(\alpha_\lambda^2)$	✓ [Goodsell et al. '16][Dao et al. '21]	✗
► full 2-loop	✗	✗
► 3-loop $\mathcal{O}(\alpha_t \alpha_s^2)$	(✓ [Kant et al. '10][Reyes, Fazio '19])	✗

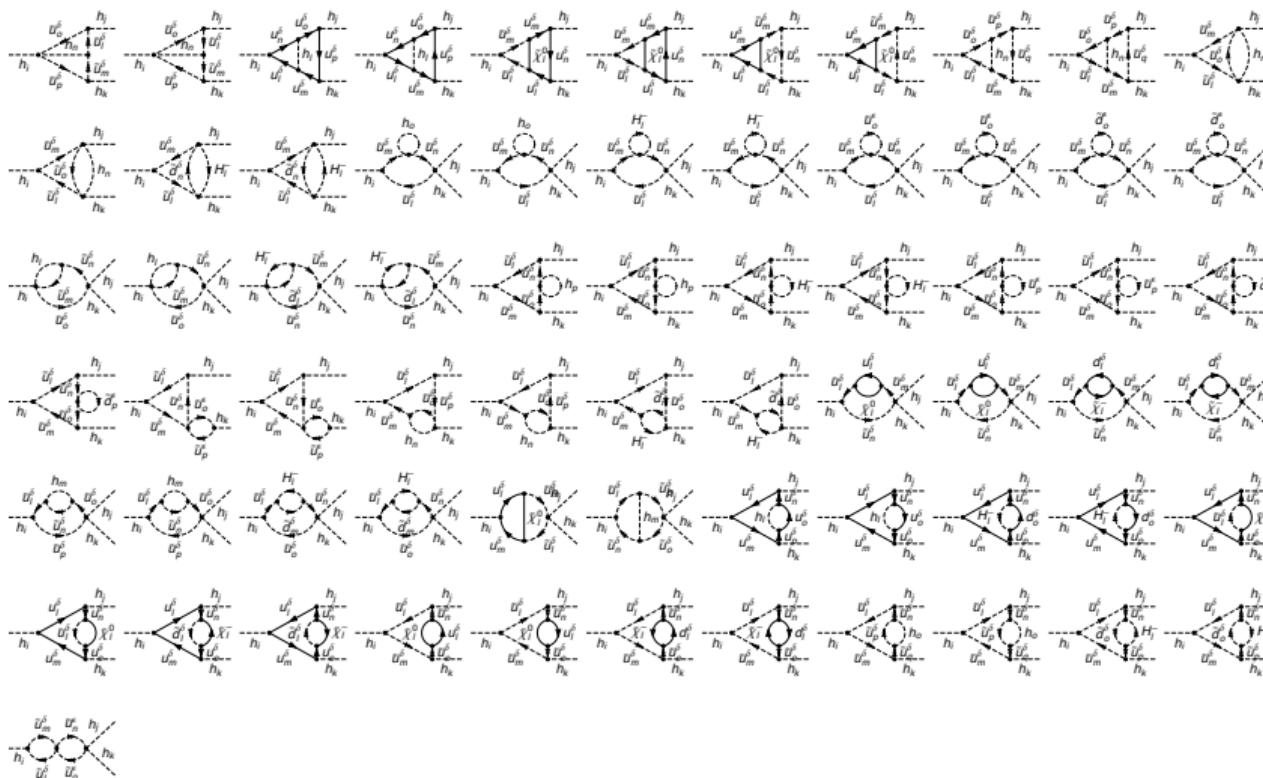
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► full 2-loop	✗	✗
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Goal of coupling corrections

- Catch up with δm_h and take into account dominant α_t corrections
- Study effects on Higgs-pair production and Higgs-to-Higgs decays (backup)

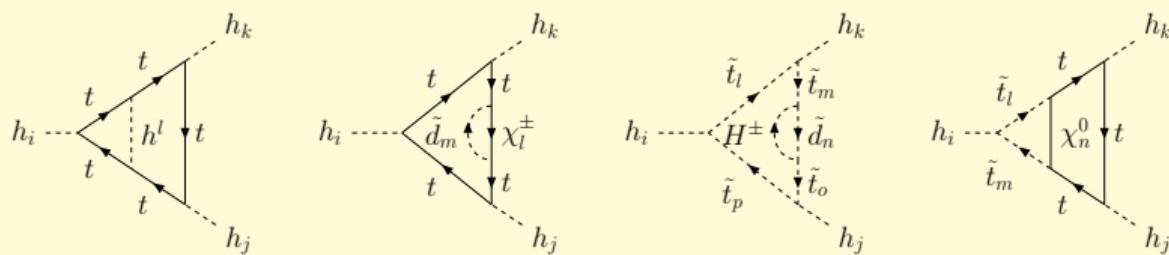
Trilinear Higgs couplings at two-loop $\mathcal{O}(\alpha_t^2)$



Trilinear Higgs couplings at two-loop $\mathcal{O}(\alpha_t^2)$



New corrections at $\mathcal{O}(\alpha_t^2)$: all two-loop diagrams with top/stops and at most one Higgs field, e.g.



i.e. proportional to top mass m_t and trilinear stop-Higgs coupling A_t (from $\mathcal{L}_{\text{soft}}$)

- ▶ Gaugeless limit: $g_1, g_2 \rightarrow 0$ (keeping $\tan \theta_W = g_2/g_1$ fixed)
- ▶ Approximation of vanishing external momenta → effective coupling



NMSSMCALC

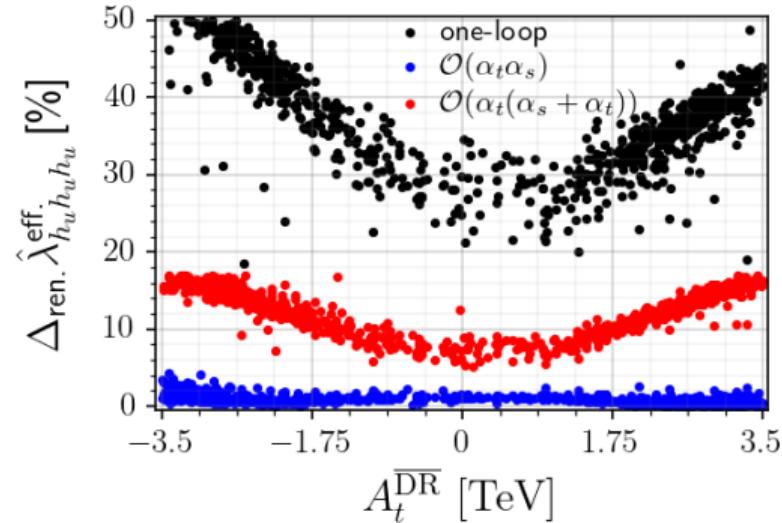
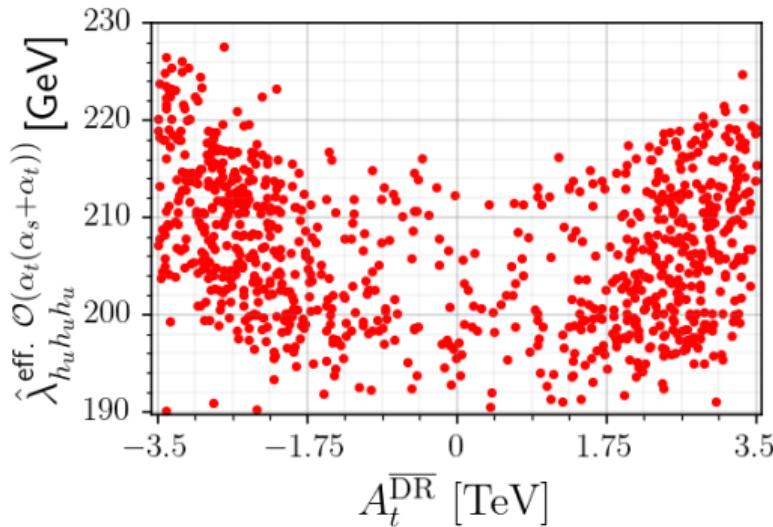
NMSSMCALC – Calculator of 1-loop and $\mathcal{O}((\alpha_t + \alpha_\lambda + \alpha_\kappa)^2 + \alpha_t \alpha_s)$ 2-loop Higgs mass corrections and Higgs decay widths

Features

- ▶ In the CP-conserving and -violating NMSSM
- ▶ SLHA input
- ▶ Use $M_{H^\pm}^2$ (OS scheme) or $\text{Re } A_\lambda$ ($\overline{\text{DR}}$ scheme) as independent input
- ▶ Possibility to choose OS or $\overline{\text{DR}}$ scheme in top/stop sector for
$$m_t, m_{\tilde{Q}_3}, m_{\tilde{t}_R} \quad \text{and} \quad A_t$$
- ▶ Now also calculation of Higgs boson self couplings up to $\mathcal{O}(\alpha_s \alpha_t + \alpha_t^2)$
- ▶ ...and more! (EDMs, $g - 2$, ...)

Download from <https://www.itp.kit.edu/~maggie/NMSSMCALC/>

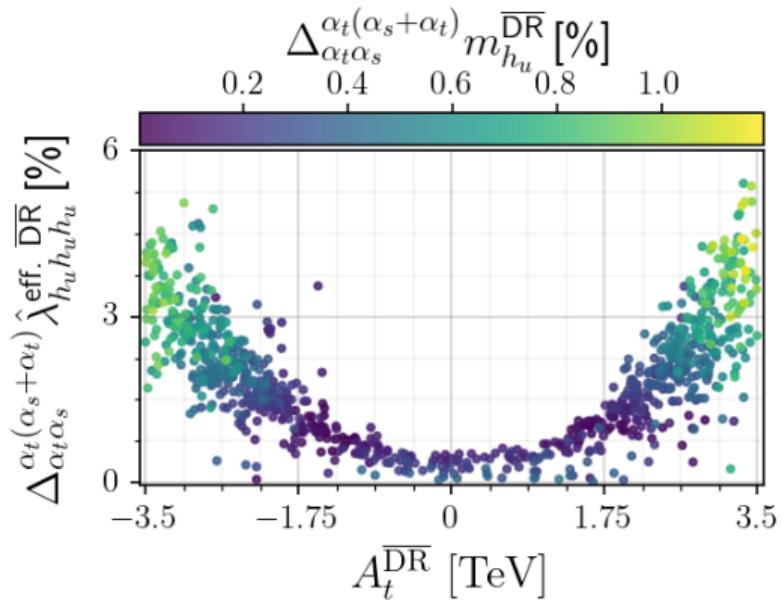
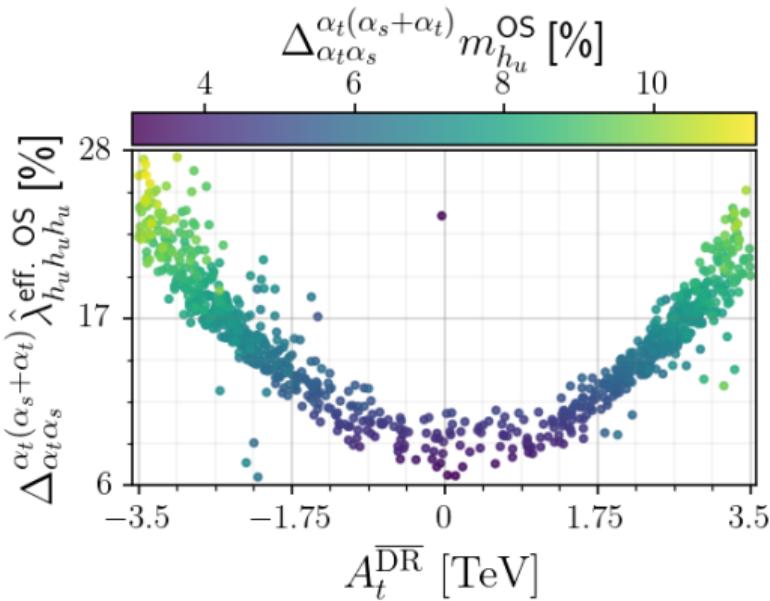
Trilinear couplings: scans



$$\text{with } \Delta_{\text{ren.}} \hat{\lambda}^{\text{eff.}} = \left[\lambda(m_t^{\overline{\text{DR}}}, A_t^{\overline{\text{DR}}}) - \lambda(m_t^{\text{OS}}, A_t^{\text{OS}}) \right] / \lambda(m_t^{\overline{\text{DR}}}, A_t^{\overline{\text{DR}}})$$

- Within theoretical uncertainties, agreement for most points with the value of the SM, $\lambda_{hhh}^{\text{SM}} = 191$ GeV

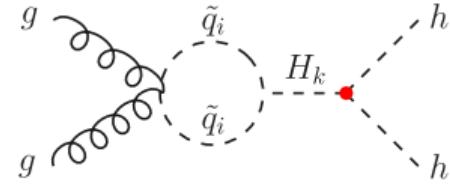
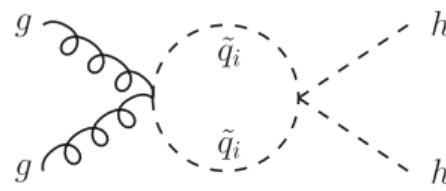
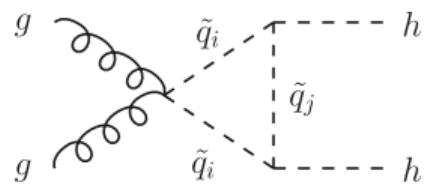
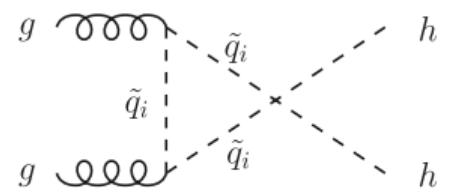
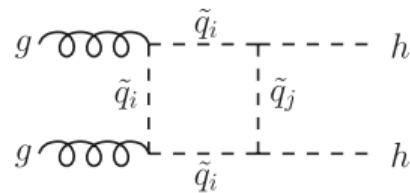
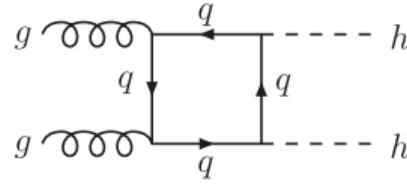
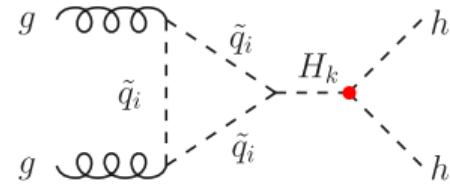
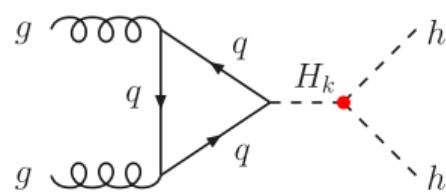
Trilinear couplings: size of corrections



with $\Delta_{\alpha_i}^{\alpha_{i+1}} \lambda = |\lambda^{\alpha_{i+1}} - \lambda^{\alpha_i}| / \lambda^{\alpha_i}$

- ▶ Correlation with size of mass corrections
- ▶ Smaller \overline{DR} corrections due to partial resummation of higher-order terms

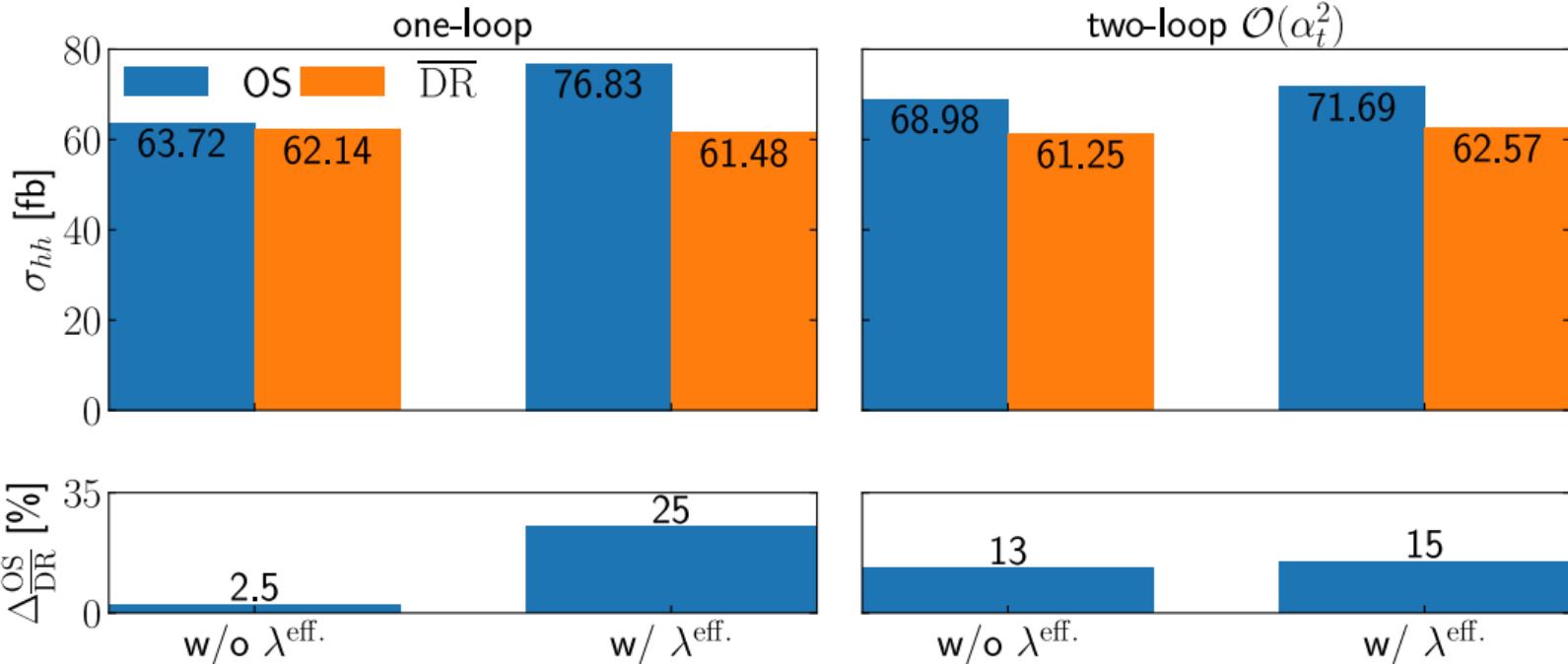
Higgs-pair production



Code to calculate Higgs-pair cross sections at hadron colliders: HPAIR [Spira et al.]

- ▶ Use $\lambda_{hhh}^{\text{eff.}, \mathcal{O}(\alpha_t(\alpha_s + \alpha_t))}$ as input to estimate higher-order effects in cross section σ_{HH}

Effect on Higgs-pair production for a benchmark point



- ▶ w/o $\lambda^{\text{eff.}}$: loop corrections to **input parameters only** (masses, mixing angles)
- ▶ w/ $\lambda^{\text{eff.}}$: loop corrections **also to Higgs-pair process/trilinear Higgs coupling**

Summary and conclusions

First calculation of two-loop $\mathcal{O}(\alpha_t^2)$ corrections to the effective trilinear Higgs couplings $\hat{\lambda}_{hhh}^{\text{eff.}}$ in the CP-violating NMSSM

- ▶ Generally larger than corrections to Higgs masses, but correlated
- ▶ Size of corrections between 0–30% in OS (0–6% in $\overline{\text{DR}}$) scheme wrt. $\mathcal{O}(\alpha_t \alpha_s)$

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- ◀ Outlook: Further EW corrections, non-vanishing p_{ext} , catch up with δm_h

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THANK YOU FOR YOUR ATTENTION! 😊

Backup

μ problem of the MSSM

Supersymmetry needs to be broken (no light sparticles found so far)

- ▶ Add **soft SUSY-breaking terms** $\mathcal{L}_{\text{soft}}$ (only masses and couplings with *positive* mass dimension), SUSY-breaking scale around $\mathcal{O}(10^3)$ GeV

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However:

Superpotential of the MSSM

$$\mathcal{W}_{\text{MSSM}} = [y_e \hat{H}_d \cdot \hat{L} \hat{E}^c + y_d \hat{H}_d \cdot \hat{Q} \hat{D}^c - y_u \hat{H}_u \cdot \hat{Q} \hat{U}^c] - \mu \hat{H}_d \cdot \hat{H}_u$$

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- ▶ μ is a **SUSY-respecting** parameter
- ▶ EWSB requires μ to be near the EW scale ~ **SUSY-breaking** scale
- ⇒ Large cancellations necessary between μ and **soft SUSY-breaking** parameters or mechanism to relate μ to **SUSY breaking**

The CP-Violating NMSSM

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Superpotential of the \mathbb{Z}_3 -symmetric NMSSM

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- ▶ $m_{h_1} < m_{h_2} < m_{h_3} < m_{h_4} < m_{h_5}$, \mathbf{G}^0 and \mathbf{G}^\pm would-be Goldstone bosons

Soft-SUSY breaking terms in the NMSSM

Soft-SUSY breaking terms

$$\begin{aligned}\mathcal{L}_{\text{soft, NMSSM}} = & -m_{H_d}^2 H_d^\dagger H_d - m_{H_u}^2 H_u^\dagger H_u - m_{\tilde{Q}}^2 \tilde{Q}^\dagger \tilde{Q} - m_{\tilde{L}}^2 \tilde{L}^\dagger \tilde{L} - m_{\tilde{u}_R}^2 \tilde{u}_R^* \tilde{u}_R - m_{\tilde{d}_R}^2 \tilde{d}_R^* \tilde{d}_R \\ & - m_{\tilde{e}_R}^2 \tilde{e}_R^* \tilde{e}_R - ([y_e A_e H_d \cdot \tilde{L} \tilde{e}_R^* + y_d A_d H_d \cdot \tilde{Q} \tilde{d}_R^* - y_u A_u H_u \cdot \tilde{Q} \tilde{u}_R^*] + \text{h.c.}) \\ & - \frac{1}{2}(M_1 \tilde{B} \tilde{B} + M_2 \tilde{W}_i \tilde{W}_i + M_3 \tilde{G} \tilde{G} + \text{h.c.}) \\ & - m_S^2 |S|^2 + (\lambda A_\lambda S H_d \cdot H_u - \frac{1}{3} \kappa A_\kappa S^3 + \text{h.c.})\end{aligned}$$

The CPV-NMSSM Higgs sector

Mass basis

$$H_d = \begin{pmatrix} v_d + \mathbf{h}_d + i\mathbf{a}_d \\ \sqrt{2} \\ \mathbf{h}_d^- \end{pmatrix}, \quad H_u = e^{i\varphi_u} \begin{pmatrix} \mathbf{h}_u^+ \\ \frac{v_u + \mathbf{h}_u + i\mathbf{a}_u}{\sqrt{2}} \end{pmatrix}, \quad S = \frac{e^{i\varphi_S}}{\sqrt{2}}(v_S + \mathbf{h}_S + i\mathbf{a}_S) \quad (\tan \beta = \frac{v_u}{v_d})$$

$\mathbf{h}_d, \mathbf{h}_u, \mathbf{h}_S, \mathbf{a}_d, \mathbf{a}_u, \mathbf{a}_S$ and $\mathbf{h}_d^\pm, \mathbf{h}_u^\pm$

mixing to

$\mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_3, \mathbf{h}_4, \mathbf{h}_5, \mathbf{G}^0$ and $\mathbf{h}^\pm, \mathbf{G}^\pm$

- ▶ $m_{h_1} < m_{h_2} < m_{h_3} < m_{h_4} < m_{h_5}$, \mathbf{G}^0 and \mathbf{G}^\pm would-be Goldstone bosons
- ▶ h_1 or h_2 is the “SM-like” Higgs boson, $m_h^2 \text{“SM”} \leq m_Z^2 \cos^2 2\beta + |\lambda| v_S^2 \sin^2 2\beta$
- ▶ CP violation possible at tree level (unlike in the MSSM)

Trilinear Higgs couplings at tree level

Scalar Higgs boson potential (from $\mathcal{L}_{\text{soft}}$, $\mathcal{W}_{\text{NMSSM}}$, and gauge sector)

$$\begin{aligned} V_H = & \left(|\lambda S|^2 + m_{H_d}^2 \right) H_d^\dagger H_d + \left(|\lambda S|^2 + m_{H_u}^2 \right) H_u^\dagger H_u + m_S^2 |S|^2 \\ & + \frac{1}{8} (g_1^2 + g_2^2) (H_d^\dagger H_d - H_u^\dagger H_u)^2 + \frac{1}{2} g_2^2 |H_d^\dagger H_u|^2 \\ & + |\kappa S^2 - \lambda H_d \cdot H_u|^2 + \left[\frac{1}{3} \kappa A_\kappa S^3 - \lambda A_\lambda S H_d \cdot H_u + \text{h.c.} \right] \end{aligned}$$

Then, the trilinear Higgs couplings are

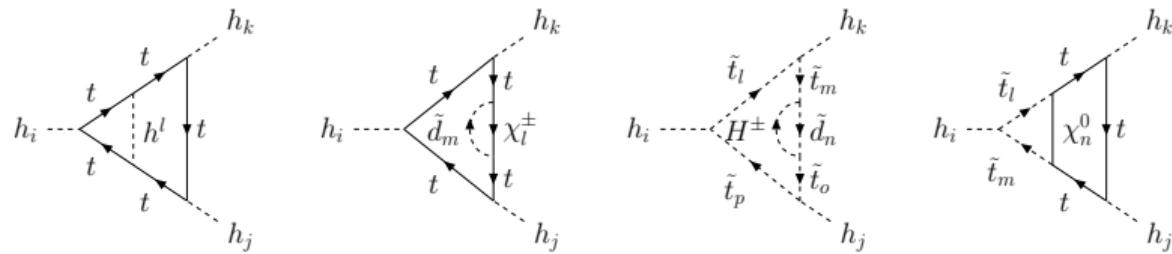
$$\lambda_{ijk} = \left. \frac{\partial^3 V_H}{\partial \phi_i \partial \phi_j \partial \phi_k} \right|_{\phi=0}$$

with the gauge eigenstates

$$\phi = (\mathbf{h}_d, \mathbf{h}_u, \mathbf{h}_S, \mathbf{a}_d, \mathbf{a}_u, \mathbf{a}_S)^T$$

Trilinear Higgs couplings at two-loop $\mathcal{O}(\alpha_t^2)$

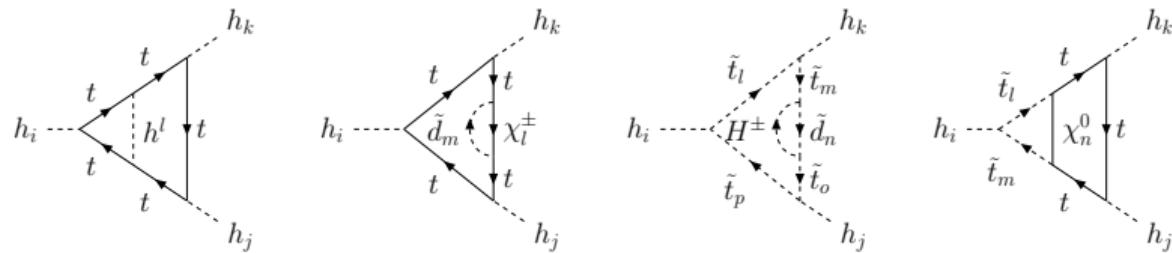
New corrections at $\mathcal{O}(\alpha_t^2)$: all two-loop diagrams with top/stops and at most one Higgs field, e.g.



i.e. proportional to top mass m_t and trilinear stop-Higgs coupling A_t (from $\mathcal{L}_{\text{soft}}$)

Trilinear Higgs couplings at two-loop $\mathcal{O}(\alpha_t^2)$

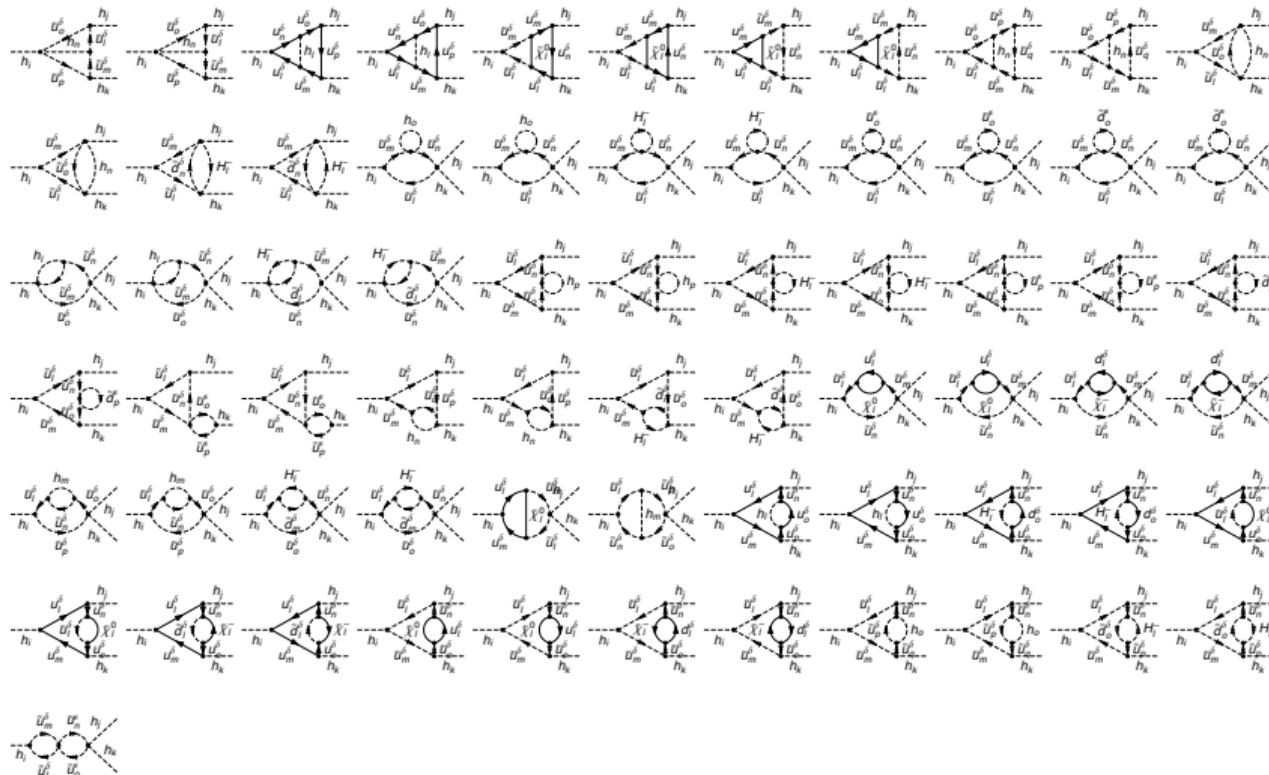
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i.e. proportional to top mass m_t and trilinear stop-Higgs coupling A_t (from $\mathcal{L}_{\text{soft}}$)

- ▶ Gaugeless limit: $g_1, g_2 \rightarrow 0$ (keeping $\tan \theta_W = g_2/g_1$ fixed)
- ▶ Approximation of vanishing external momenta
⇒ Two-loop integrals can be reduced to **products of one-loop integrals** and the **two-loop tadpole integral** (known analytically [Ford, Jack, Jones '01])

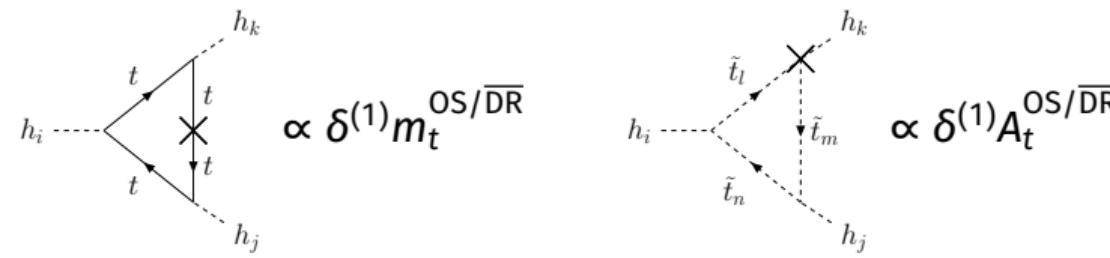
Trilinear Higgs couplings at two-loop $\mathcal{O}(\alpha_t^2)$: all 2L diagrams



Trilinear Higgs couplings at two-loop $\mathcal{O}(\alpha_t^2)$: counterterms

Choice of renormalisation schemes

- ▶ Use on-shell (OS) or $\overline{\text{DR}}$ renormalisation scheme in top/stop sector to estimate part of the theory uncertainty



- ▶ $M_{H^\pm}^2$ as input: $\underbrace{\text{tadpoles}, M_{H^\pm}^2, v}_{\text{OS}}, \underbrace{\tan \beta, |\lambda|, v_S, |\kappa|, \text{Re } A_\kappa, \varphi_\lambda, \varphi_\kappa, \varphi_u, \varphi_S}_{\overline{\text{DR}}}$
- ▶ $\text{Re } A_\lambda$ as input: $\underbrace{\text{tadpoles}, v}_{\text{OS}}, \underbrace{\tan \beta, |\lambda|, v_S, |\kappa|, \text{Re } A_\lambda, \text{Re } A_\kappa, \varphi_\lambda, \varphi_\kappa, \varphi_u, \varphi_S}_{\overline{\text{DR}}}$

Effective trilinear couplings in the mass basis

$$\hat{\lambda}_{ijk} = \lambda_{ijk} + \Delta^{(1)}\lambda_{ijk} + \Delta^{(2)}\lambda_{ijk} \quad \text{in basis } (\mathbf{h}_d, \mathbf{h}_u, \mathbf{h}_s, \mathbf{a}_d, \mathbf{a}_u, \mathbf{a}_s) \text{ with}$$

- ▶ $\Delta^{(1)}\lambda_{ijk}$: $\mathcal{O}(\alpha_t)$ corrections (full corrections for Higgs-to-Higgs decays)
- ▶ $\Delta^{(2)}\lambda_{ijk} = \Delta^{\alpha_t \alpha_s} \lambda_{ijk} + \Delta^{\alpha_t^2} \lambda_{ijk}$

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Two-fold rotation procedure

- ▶ Single out Goldstone boson (rotate with $\mathcal{R}^G(\beta_n)$) $\rightarrow (\mathbf{h}_d, \mathbf{h}_u, \mathbf{h}_s, \mathbf{a}, \mathbf{a}_s, \mathbf{G}^0)$:

$$\hat{\tilde{\lambda}}_{nmq} = \mathcal{R}_{ni}^G \mathcal{R}_{mj}^G \mathcal{R}_{qk}^G \hat{\lambda}_{ijk}$$

- ▶ Rotate into mass eigenstates with $\mathcal{R}^{l,\text{eff}}$ $\rightarrow (\mathbf{H}_1, \mathbf{H}_2, \mathbf{H}_3, \mathbf{H}_4, \mathbf{H}_5, \mathbf{G}^0)$:

$$\hat{\lambda}_{abc}^{\text{eff}} = \mathcal{R}_{an}^{l,\text{eff}} \mathcal{R}_{bm}^{l,\text{eff}} \mathcal{R}_{cq}^{l,\text{eff}} \hat{\tilde{\lambda}}_{nmq}$$

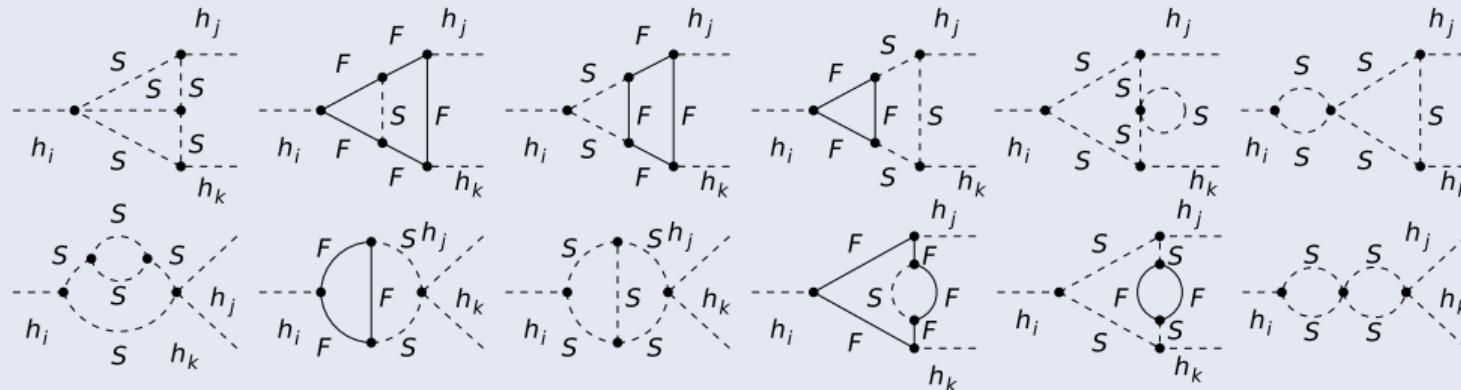
$\mathcal{R}^{l,\text{eff}}$ diagonalises the loop-corrected Higgs mass matrix

Steps of the calculation

Work with *generic diagrams*, assume most general couplings and masses, simplify, insert fields and numerics at the very end

Toolchain

- ▶ Use FeynArts to generate generic diagrams [Hahn '01]



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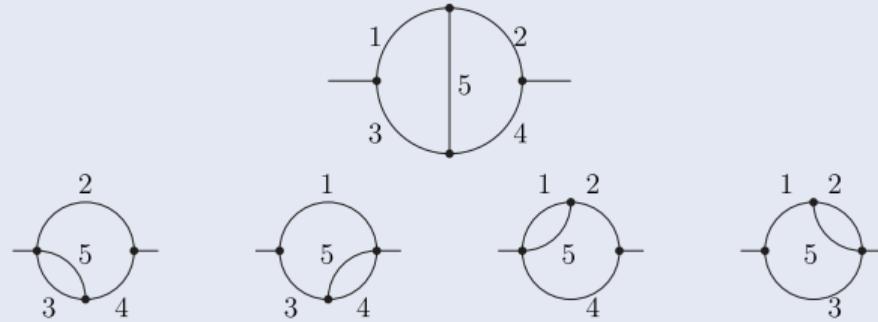
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- ▶ Handle cases with e.g. vanishing Gram determinants separately, e.g.
 - ▶ $m_1 = m_2 = m_3 = m_4$
 - ▶ $m_1 = m_2 = m_3, m_4$
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 - ▶ ...
 - ▶ $m_1 \neq m_2 \neq m_3 \neq m_4$
- ▶ Evaluate numerically → NMSSMCALC

Numerical set-up

Scan parameters and ranges

parameter	scan range (TeV)	parameter	scan range
M_{H^\pm}	[0.5, 1]	$\tan \beta$	[1, 10]
M_1, M_2	[0.4, 1]	λ	[0.01, 0.7]
M_3	2	κ	$\lambda \cdot \xi$
μ_{eff}	[0.1, 1]	ξ	[0.1, 1.5]
$m_{\tilde{Q}_3}, m_{\tilde{t}_R}$	[0.4, 3]	A_t	[-3.5, 3.5] TeV
$m_{\tilde{\chi} \neq \tilde{Q}_3, \tilde{t}_R}$	3	$A_{i \neq t}$	[-2, 2] TeV

Points are checked with HiggsBounds & HiggsSignals [Bechtle et al. '08-'13]

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Furthermore, discard points with any of the following mass configurations:

- (i) $m_{\chi_i^{(\pm)}}^{}, m_{h_i}^{} > 1 \text{ TeV}, m_{\tilde{t}_2}^{} > 2 \text{ TeV},$
- (ii) $m_{h_i}^{} - m_{h_j}^{} < 0.1 \text{ GeV}, m_{\chi_i^{(\pm)}}^{} - m_{\chi_j^{(\pm)}}^{} < 0.1 \text{ GeV}$
- (iii) $m_{\chi_1^\pm}^{} < 94 \text{ GeV}, m_{\tilde{t}_1}^{} < 1 \text{ TeV}$

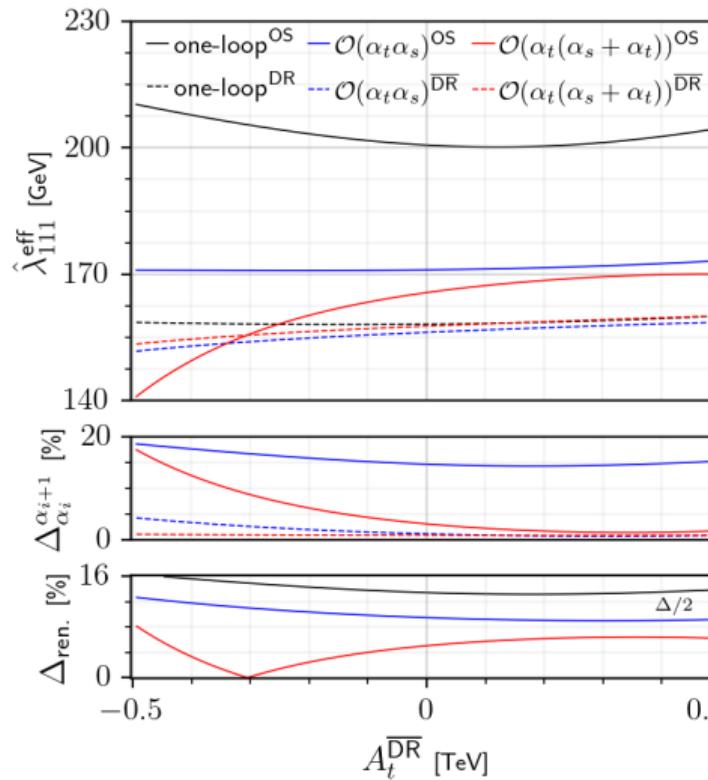
BP10: Higgs masses

	$h_1 [h_u]$	$h_2 [h_s]$	$h_3 [h_d]$	$a_1 [a_s]$	$a_2 [a_d]$
tree-level	97.21	307.80	626.13	556.71	617.22
one-loop	131.46 (114.81)	299.65 (299.28)	625.96 (625.52)	543.58 (543.69)	615.82 (616.01)
two-loop $\mathcal{O}(\alpha_t \alpha_s)$	118.90 (120.36)	299.40 (299.38)	625.78 (625.58)	543.73 (543.60)	615.90 (615.96)
two-loop $\mathcal{O}(\alpha_t (\alpha_s + \alpha_t))$	123.53 (120.14)	299.44 (299.38)	625.89 (625.57)	543.73 (543.60)	615.90 (615.96)
two-loop $\mathcal{O}(\alpha_{\lambda K}^2)$	122.36 (119.97)	300.27 (299.90)	625.94 (625.65)	543.34 (543.47)	615.91 (616.01)

P20S: Higgs masses

	$h_1 [h_u]$	$h_2 [h_s]$	$h_3 [h_d]$	$a_1 [a_s]$	$a_2 [a_d]$
tree-level	96.86	112.10	926.25	511.34	925.86
one-loop	129.01 (116.3)	135.09 (130.1)	926.69 (926.33)	512.55 (512.66)	925.08 (925.18)
two-loop $\mathcal{O}(\alpha_t \alpha_s)$	121.36 (121.65)	129.7 (130.39)	926.37 (926.46)	512.62 (512.61)	925.11 (925.15)
two-loop $\mathcal{O}(\alpha_t (\alpha_s + \alpha_t))$	126.09 (121.54)	130.04 (130.38)	926.49 (926.45)	512.62 (512.61)	925.11 (925.15)
two-loop $\mathcal{O}(\alpha_{\lambda_K}^2)$	125.25 (121.67)	129.91 (130.20)	926.62 (926.52)	511.91 (512.12)	925.07 (925.14)

Trilinear couplings - benchmark point P2OS



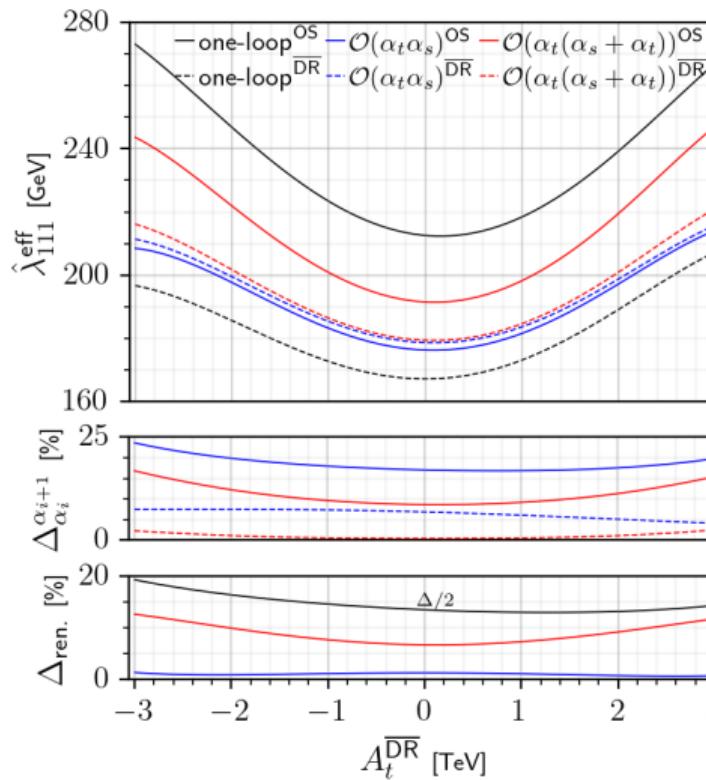
- Benchmark point P2OS: large singlet admixture to SM-like Higgs
- With

$$\Delta_{\alpha_i}^{\alpha_{i+1}} = \frac{|\lambda^{\alpha_{i+1}} - \lambda^{\alpha_i}|}{\lambda^{\alpha_i}}$$

and

$$\Delta_{\text{ren.}} = \frac{\lambda(m_t^{\overline{\text{DR}}}, A_t^{\overline{\text{DR}}}) - \lambda(m_t^{\text{OS}}, A_t^{\text{OS}})}{\lambda(m_t^{\overline{\text{DR}}}, A_t^{\overline{\text{DR}}})}$$

Trilinear couplings - benchmark point BP10



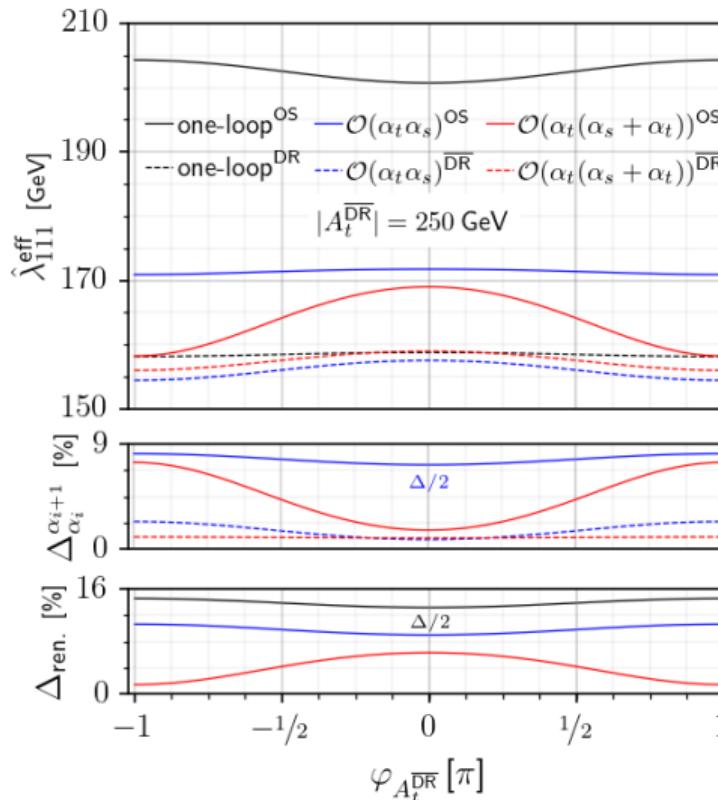
- Benchmark point BP10: resonantly enhanced Higgs-pair production in gluon fusion
- With

$$\Delta_{\alpha_i}^{\alpha_{i+1}} = \frac{|\lambda^{\alpha_{i+1}} - \lambda^{\alpha_i}|}{\lambda^{\alpha_i}}$$

and

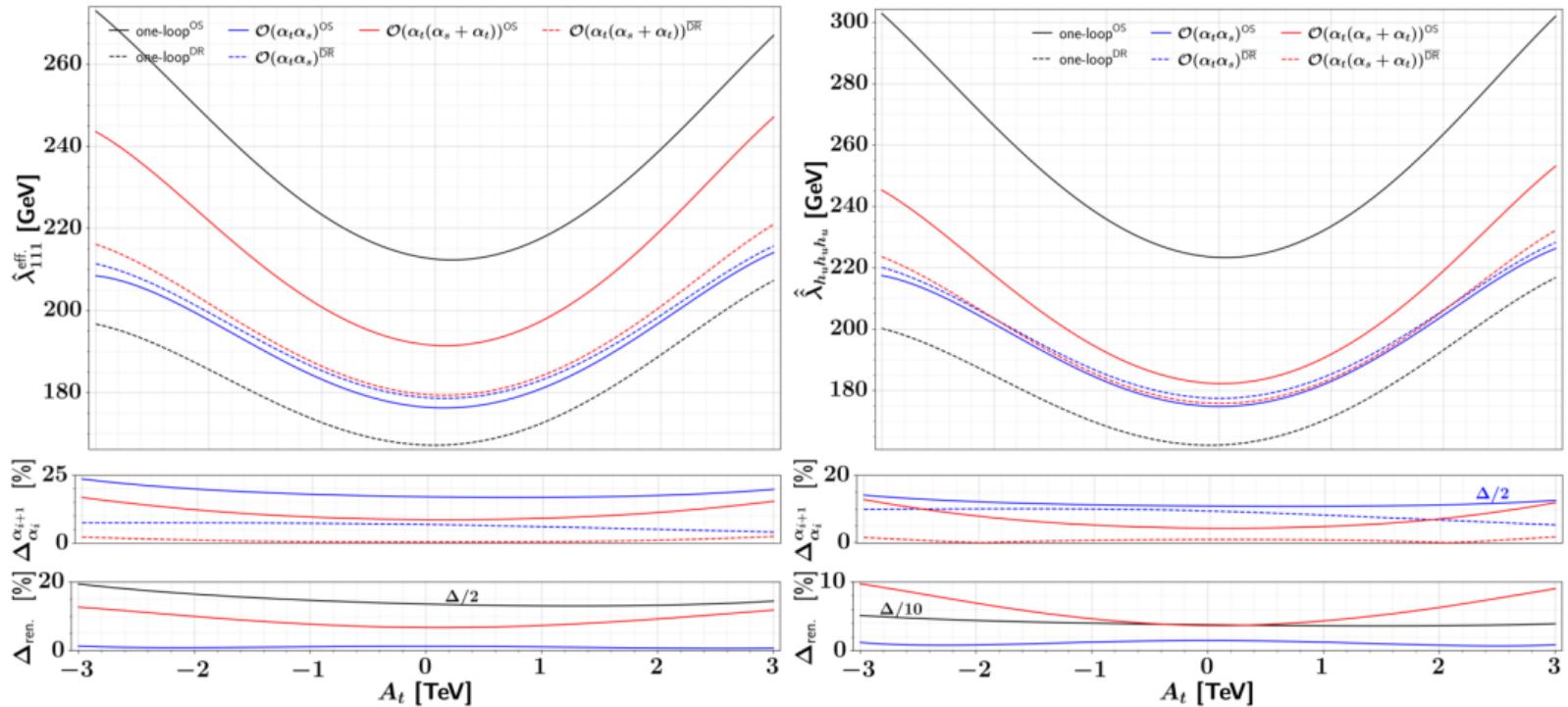
$$\Delta_{\text{ren.}} = \frac{\lambda(m_t^{\overline{DR}}, A_t^{\overline{DR}}) - \lambda(m_t^{\text{OS}}, A_t^{\text{OS}})}{\lambda(m_t^{\overline{DR}}, A_t^{\overline{DR}})}$$

Trilinear couplings - CP violation

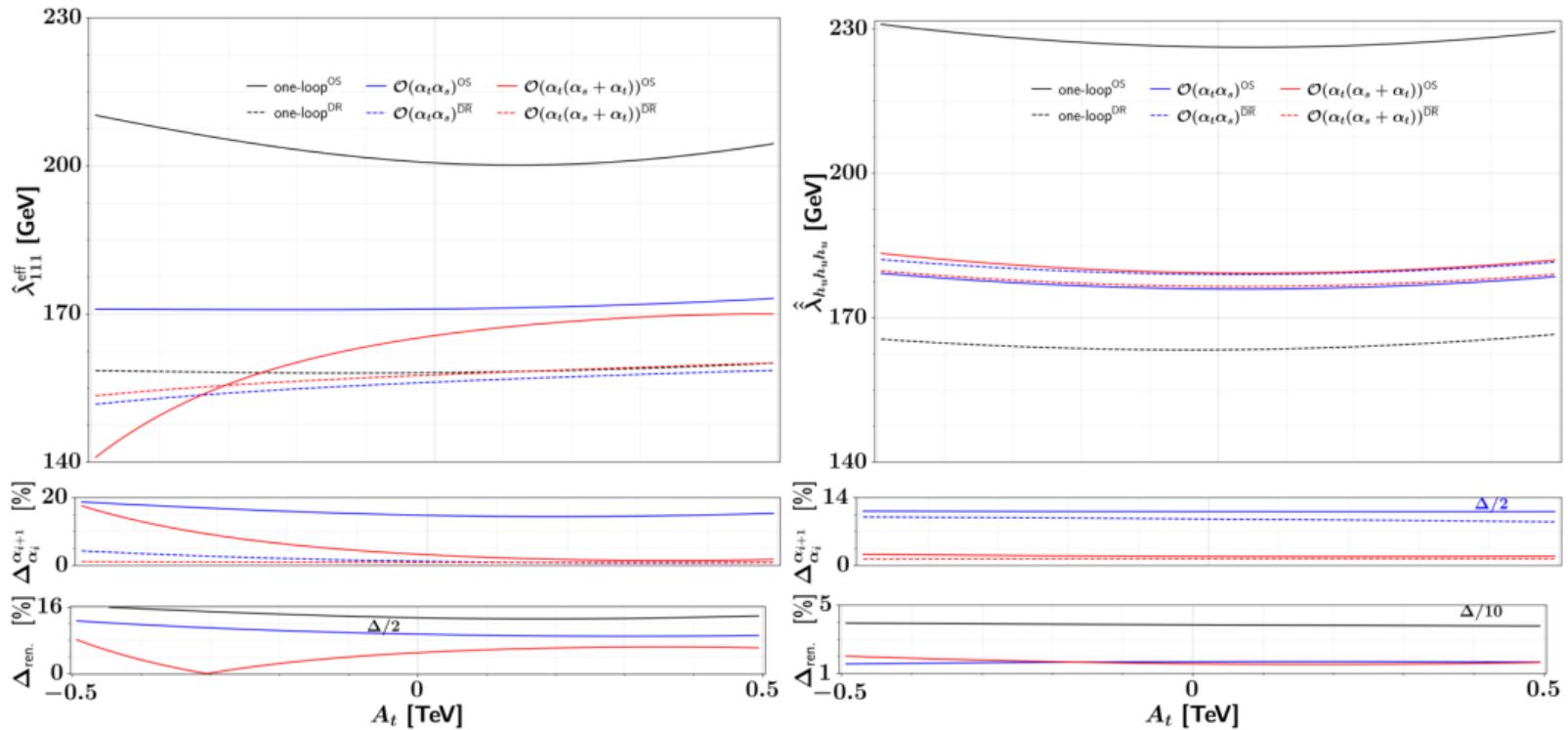


- Benchmark point P2OS
- CP-violating phase $A_t = |A_t| e^{i\varphi_{A_t}}$
- Loop-induced effect
- Accidental cancellations for almost flat OS curve at $\mathcal{O}(\alpha_t \alpha_s)$
- ⇒ Stringent constraints from EDMs: effect of CP-violating phases on masses and mixing angles marginal

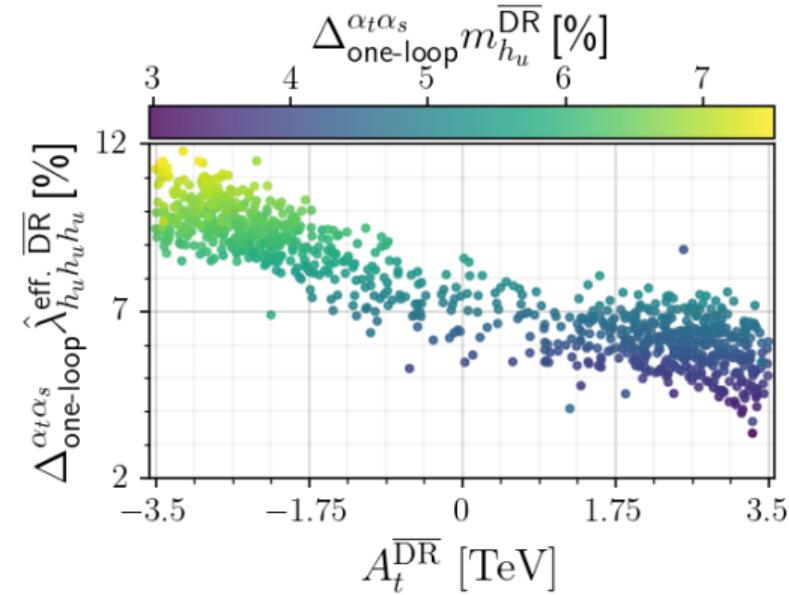
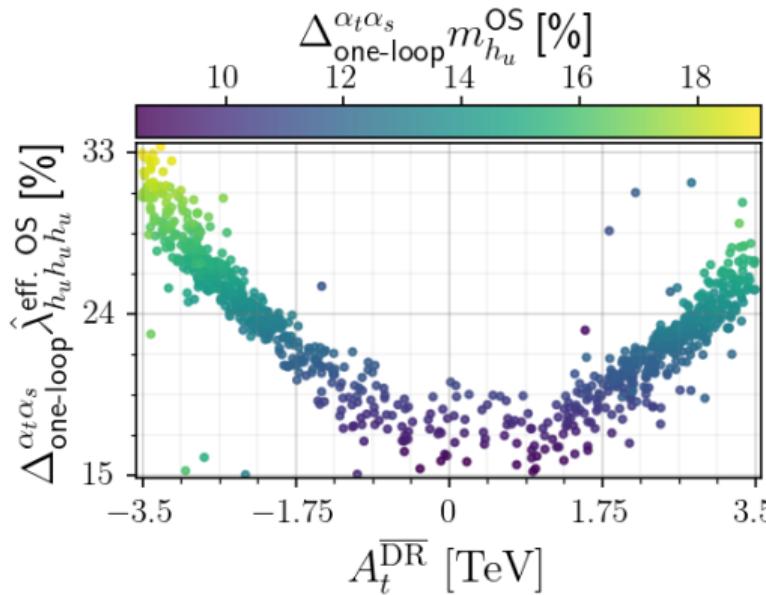
$\lambda^{\text{eff.}}$: mass vs. gauge basis for BP10 (MSSM-like point)



$\lambda^{\text{eff.}}$: mass vs. gauge basis for P2OS (large singlet admixture)



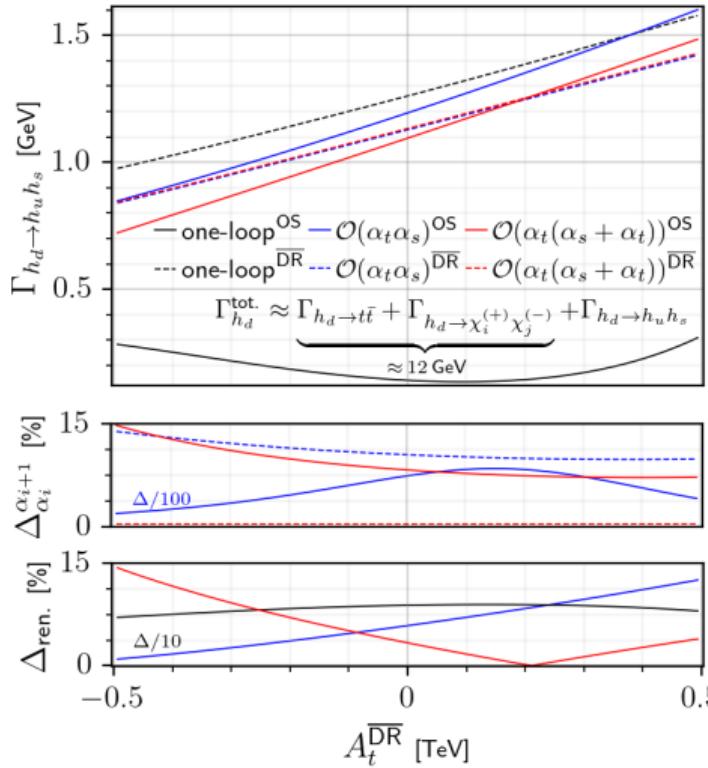
Trilinear couplings: size of corrections (1)



with $\Delta_{\alpha_i}^{\alpha_{i+1}} \lambda = |\lambda^{\alpha_{i+1}} - \lambda^{\alpha_i}| / \lambda^{\alpha_i}$

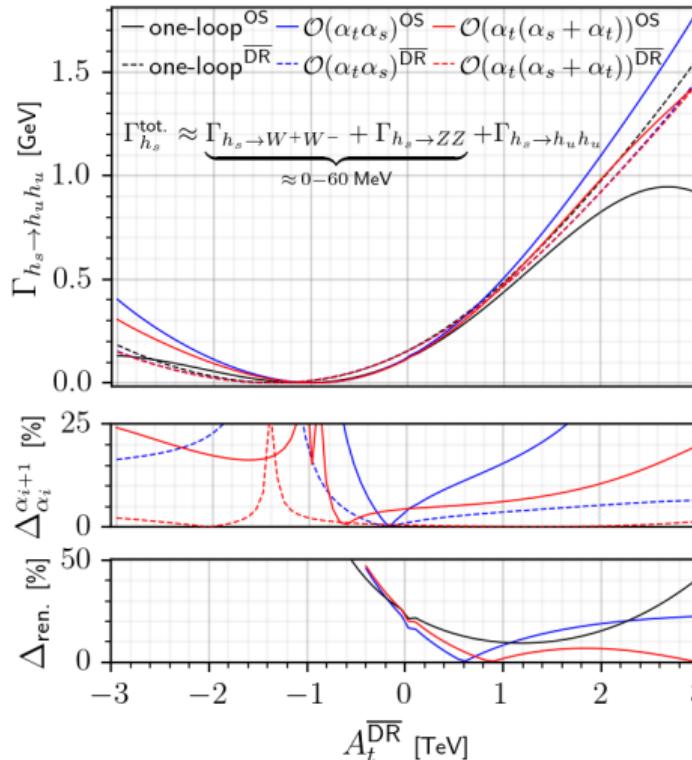
- ▶ Correlation with size of mass corrections
- ▶ Smaller $\overline{\text{DR}}$ corrections due to partial resummation of higher-order terms

Higgs-to-Higgs decays: $h_d \rightarrow h_u h_s$ -like



- Benchmark point P2OS
- Large relative corrections from $\mathcal{O}(\alpha_t \alpha_s)$ due to small one-loop decay width in the OS scheme
- Branching ratio of around 12% at most

Higgs-to-Higgs decays: $h_s \rightarrow h_u h_u$ -like



- Benchmark point BP10
- Resonant contribution to h_u -like Higgs-pair production
- Large branching ratio, dominant decay channel