Phase transitions and light scalars in bottom-up holography

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Based on [arXiv:2212.07954] and [arXiv:2303.00541] Daniel Elander, AF, Maurizio Piai
Data sets released on Zenodo:7477647, Zenodo:7705408

SUSY 2023, University of Southampton

July 17, 2023

Overview

1. Introduction
2. Background
   - Conformal transition
3. The Model
   - Classes of solutions
4. Results
5. Summary and Outlook

Composite Higgs Models

$$\frac{m_H^2}{\Lambda_{\rm SM}^2} \sim 10^{-28} \lll 1 \quad \rightarrow \text{Naturalness problem}$$

- Investigations into the fundamental nature and origin of the Higgs boson are among the topical subjects in theoretical physics.
- Because of the approximate (explicitly and spontaneously broken) scale-invariant nature of the SM theory, the Higgs boson itself is in fact a dilaton, the pseudo-Nambu-Goldstone boson associated with dilatations.
- The Higgs being a composite object with a compositeness scale of TeV order, is one of the few options for "Naturally" generating its mass.

Scale transformations

$$\mathcal{L} = \sum_i g_i(\mu)\mathcal{O}_i(x), \tag{1}$$

with $[\mathcal{O}_i] = d_i$. The content of the RG equations is summarized by assigning the transformations laws

$$\begin{aligned} \mathcal{O}_i(x) &\rightarrow e^{\lambda d_i}\mathcal{O}_i(e^{\lambda}x), \\ \mu &\rightarrow e^{-\lambda}\mu, \end{aligned}$$

under scale transformations $x^\mu \rightarrow e^\lambda x^\mu$.

If $d_i = 4$ and $\beta_i = 0$, the theory is scale invariant.

A simple way of incorporating non-linearly realized scale invariance is to add a field $\chi(x)$ that serves as a conformal compensator.

$$\chi(x) \rightarrow e^{\lambda}\chi(e^{\lambda}x),$$

We need to make the replacement

$$g_i(\mu) \to g_i \left(\mu \frac{\chi}{f}\right) \left(\frac{\chi}{f}\right)^{4-d_i},$$

Here $f = \langle \chi \rangle$ is the order parameter for scale symmetry breaking, determined by the dynamics of the underlying strong sector.

$$\chi(x) = f e^{\sigma(x)/f},$$

which transforms non-linearly under scale transformations,
$\lambda : \sigma(x)/f \to \sigma(e^{\lambda} x)/f + \lambda$. One can also define $\bar{\chi}(x) = \chi(x) - f$.

Conformal transition

In AdS/CFT correspondence, there is the notion of BF bound. An operator with a complex dimension in CFT corresponds to fields in AdS with mass below the BF band.

Following *[Kaplan et al. '09]* and *[Gorbenko et al. '18]*, there may be a range of values in the parameter space of the theory that the operator $\mathcal{O}_g$, dual to a field in the gravity side, becomes marginal, and develops a complex dimension. This will be a signal for the end of conformality.

1 Introduction

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6d garvity action

$$\mathcal{S}_6 = \mathcal{S}_6^{(bulk)} + \sum_{i=1,2} \mathcal{S}_{5,i}\,,$$

$$\mathcal{S}_6^{(bulk)} = \int \mathrm{d}^6x\sqrt{-\hat{g}_6}\left\{\frac{\mathcal{R}_6}{4} - \frac{1}{2}\hat{g}^{\hat{M}\hat{N}}\partial_{\hat{M}}\phi\partial_{\hat{N}}\phi - \mathcal{V}_6(\phi)\right\},$$

$$\mathcal{S}_{5,i} = (-)^i\int \mathrm{d}^5x\sqrt{-\tilde{\hat{g}}}\left\{\frac{\mathcal{K}}{2} + \lambda_i(\phi) + f_i\left(\tilde{\hat{g}}_{\hat{M}\hat{N}}\right)\right\}\Big|_{\rho=\rho_i},$$

$\rho_1 < \rho < \rho_2$ and the he space-time index is $\hat{M} = 0, 1, 2, 3, 5, 6$.

$$\mathcal{V}_6(\phi) = -5 - \frac{\Delta(5-\Delta)}{2}\phi^2 - \frac{5\Delta^2}{16}\phi^4\,,$$

Asymptotically in the UV, the dual field theory flows towards a CFT in 5 dimensions, deformed by the insertion of an operator $\mathcal{O}$ with scaling dimension given by $\max(\Delta, 5-\Delta)$.
The two parameters appearing in the solution of the corresponding second-order classical equations correspond in field-theory terms to the coupling and condensate associated with $\mathcal{O}$.

Dimensional reduction to five dimensions

$$\mathrm{d}s_6^2 = e^{-2\chi}\mathrm{d}x_5^2 + e^{6\chi}\left(\mathrm{d}\eta + \chi_M \mathrm{d}x^M\right)^2 ,$$

where the space-time index is $M = 0, 1, 2, 3, 5$. The reduced action then becomes

$$\mathcal{S}_5^{(bulk)} = \int \mathrm{d}^5x\sqrt{-g_5}\left\{\frac{R}{4} - \frac{1}{2}g^{MN}\left[6\partial_M\chi\partial_N\chi + \partial_M\phi\partial_N\phi\right] - e^{-2\chi}\mathcal{V}_6(|\phi|) - \frac{1}{16}e^{8\chi}g^{MP}g^{NQ}F_{MN}^{(\chi)}F_{PQ}^{(\chi)}\right\} ,$$

We consider background solutions in which $\chi_M = 0$, while the metric g_{MN}, ϕ, and χ depend on the radial coordinate only. The metric in five dimensions takes the domain-wall (DW) form

$$\mathrm{d}s_5^2 = \mathrm{d}r^2 + e^{2A(r)}\mathrm{d}x_{1,3}^2 = e^{2\chi(\rho)}\mathrm{d}\rho^2 + e^{2A(\rho)}\mathrm{d}x_{1,3}^2 ,$$

with $\mathrm{d}\rho = e^{-\chi}\mathrm{d}r$.

UV expansions

$$\rho \to +\infty, \quad \phi = 0 \quad \text{critical point of } \mathcal{V}_6, \quad \chi \simeq \frac{1}{3}\rho, \quad A \simeq \frac{4}{3}\rho$$

We classify all the solutions in terms of a power expansion in the small parameter $z \equiv e^{-\rho}$. The expansion depends on five free parameters.

$$\begin{aligned}
\phi(z) &= \phi_J z^{\Delta_J} + \cdots + \phi_V z^{\Delta_V} + \cdots, \\
\chi(z) &= \chi_U - \frac{1}{3}\log(z) + \cdots + (\chi_5 + \cdots) z^5 + \cdots, \\
A(z) &= A_U - \frac{4}{3}\log(z) + \cdots.
\end{aligned}$$

Background solutions

Confining solutions The confining solutions are such that the circle parametrised by η shrinks to zero size at some point ρ_o of the radial direction ρ and there is no conical singularity. For small $(\rho - \rho_o)$, we find that such solutions have the following form

$$\phi(\rho) = \phi_I - \frac{1}{16}\Delta\phi_I \left(20 + \Delta\left(5\phi_I^2 - 4\right)\right)(\rho-\rho_o)^2 + \mathcal{O}\left((\rho-\rho_o)^2\right) ,$$
$$\chi(\rho) = \chi_I + \frac{1}{3}\log(\rho-\rho_o) + O\left((\rho-\rho_o)^4\right) ,$$
$$A(\rho) = A_I + \frac{1}{3}\log(\rho-\rho_o) + \mathcal{O}\left((\rho-\rho_o)^2\right) ,$$

Singular domain-wall solutions They obey the DW ansatz $A = 4\chi = \frac{4}{3}\mathcal{A}$. In six dimensions, they take the form of Poincaré domain walls.

$$\begin{aligned}\phi(\rho) &= \phi_I - \sqrt{\frac{2}{5}}\log(\rho-\rho_o) + \mathcal{O}((\rho-\rho_o)^2)\,,\\ \mathcal{A}(\rho) &= \frac{1}{5}\log(\rho-\rho_o) + \mathcal{O}((\rho-\rho_o)^2)\,.\end{aligned}$$

The system of equations is symmetric under the change $\phi \to -\phi$, hence a second branch of solutions can be obtained by just changing the sign of ϕ. These solutions are singular at the end of space.

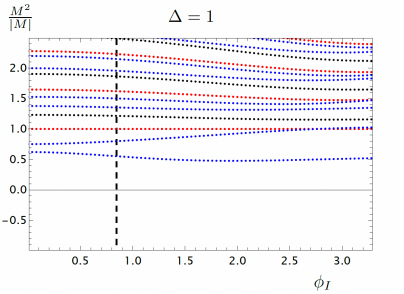
The free energy density is given by the following expression:

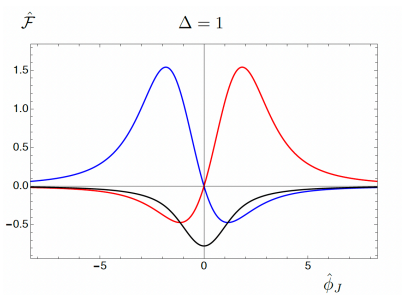
$$\mathcal{F} \;=\; -\lim_{\rho_2\to+\infty} e^{4A-\chi}\left(\frac{3}{2}\partial_\rho A+\mathcal{W}_2\right)\Big|_{\rho_2},$$

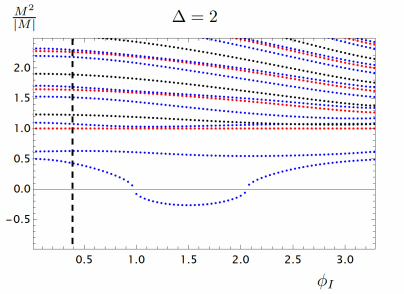
For example, for $\Delta < 5/2$, the free energy density is

$$\mathcal{F} \;=\; -\frac{1}{40}e^{4A_U-\chi_U}\left(16\Delta\left(\frac{5}{2}-\Delta\right)\phi_J\phi_V \;-75\chi_5\right),$$

Results


$\frac{M^2}{|M|}$
$\Delta = 1$
2.0
1.5
1.0
0.5
0.0
−0.5
0.5
1.0
1.5
2.0
2.5
3.0
ϕ_I


$\hat{\mathcal{F}}$
$\Delta = 1$
1.5
1.0
0.5
0.0
−0.5
−5
0
5
$\hat{\phi}_J$


$\frac{M^2}{|M|}$
$\Delta = 2$
2.0
1.5
1.0
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0.0
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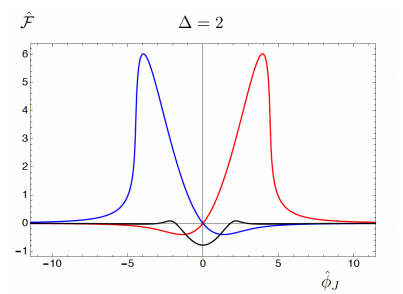
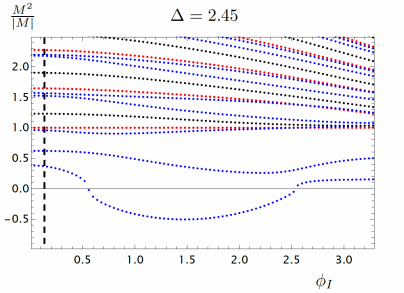
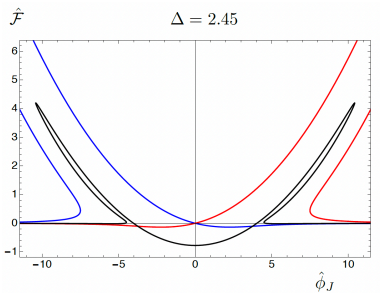
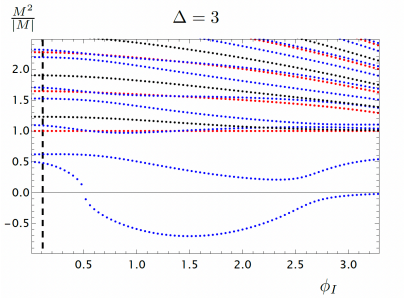
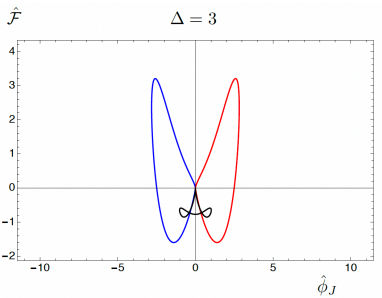
0.5
1.0
1.5
2.0
2.5
3.0
ϕ_I

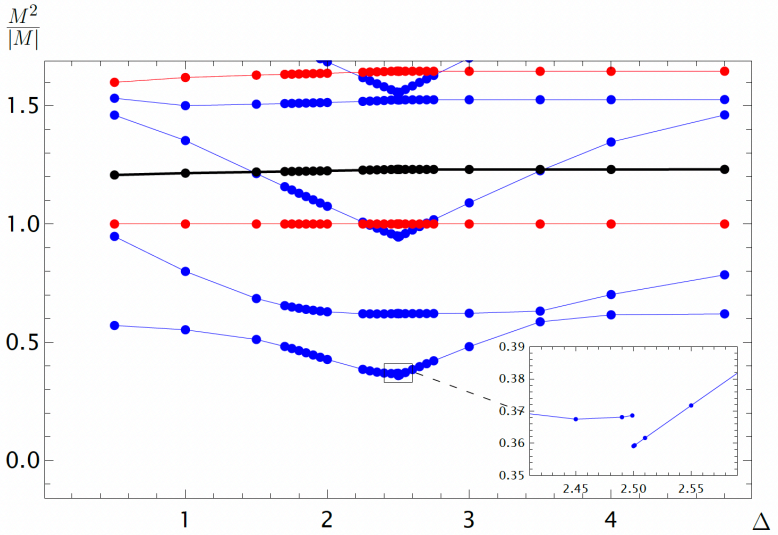

$\hat{\mathcal{F}}$
$\Delta = 2$
6
5
4
3
2
1
0
−1
−10
−5
0
5
10
$\hat{\phi}_J$


$\frac{M^2}{|M|}$
$\Delta = 2.45$
2.0
1.5
1.0
0.5
0.0
−0.5
0.5
1.0
1.5
2.0
2.5
3.0
ϕ_I


$\hat{\mathcal{F}}$
$\Delta = 2.45$
6
5
4
3
2
1
0
−1
−10
−5
0
5
10
$\hat{\phi}_J$


$\frac{M^2}{|M|}$
$\Delta = 3$
2.0
1.5
1.0
0.5
0.0
−0.5
0.5
1.0
1.5
2.0
2.5
3.0
ϕ_I


$\hat{\mathcal{F}}$
$\Delta = 3$
4
3
2
1
0
−1
−2
−10
−5
0
5
10
$\hat{\phi}_J$

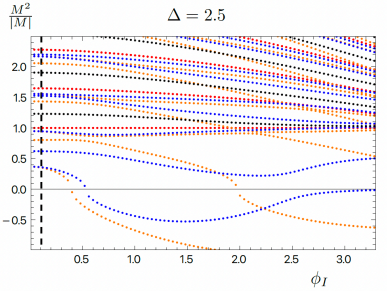

$\frac{M^2}{|M|}$
1.5
1.0
0.5
0.0
1
2
3
4
Δ
0.39
0.38
0.37
0.36
0.35
2.45
2.50
2.55

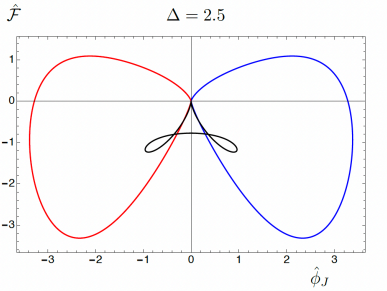
Summary and Outlook

- We studied the mass spectrum of bosonic states in the field theory. The lightest state in this spectrum is a scalar particle.
- Along the confining branch of solutions, we find the presence of a tachyonic instability. In a region of parameter space nearby the tachyonic one, the lightest scalar particle can be interpreted as an approximate dilaton,
- We also computed the free energy. We find that both the dilatonic and tachyonic regions are hidden behind a first-order phase transition,
- The (approximate) dilaton appears in metastable solutions. Yet, the mass of the lightest state, computed close to the phase transition, is (mildly) suppressed when Δ is near $5/2$.
- The class of models analysed here does not cover all possible realisations of confining field theories and can be improved by considering top-down models
- We can further extend this six-dimensional model in the complementary context of composite Higgs models.

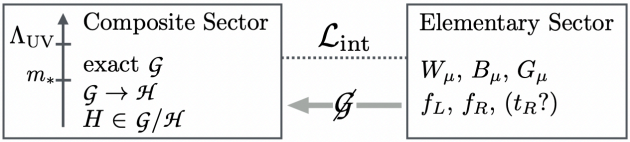
Thank you

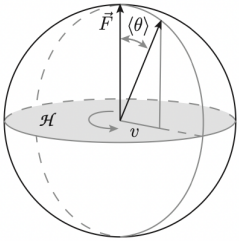
Probe Approximation





Vacuum misalignment





$\mathcal{G} \to \mathcal{H}$ spontaneous breaking $\to$ massless NGB's in the coset $\mathcal{G}/\mathcal{H}$

$$G_{\rm EW} = {\rm SU}(2)_L \times {\rm U}(1)_Y \subseteq \mathcal{H}$$

G is large enough to contain at least one Higgs doublet in the coset

[Panico, Wulzer '15]