Accidentally light scalars

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Scalars are heavy.*

*Conditions apply.

The natural mass scale of an elementary scalar field is the mass of the heaviest states it couples to (directly or indirectly).

- SM Higgs mass: electroweak hierarchy problem
- How to protect inflaton potential from radiative corrections?

Zoology of light scalar fields

Nambu–Goldstone bosons (NGB)
(exactly massless)

SUSY moduli
(exactly massless)

pseudo–NGBs

pseudo–moduli

accidentally
massless
scalars

Terminology

An accidentally light scalar is a scalar field which obtains no tree-level mass from the most general renormalizable potential compatible with symmetry.

- **Counterexample 1**:

$$V = \lambda_\Phi(\Phi^2 - v^2)^2 + \kappa(\Phi^2 - v^2)\varphi^2 + \lambda_\varphi \varphi^4$$

  has $m_\varphi = 0$. Here φ is light not by accident but by fine-tuning.

- **Counterexample 2**: (symmetry of V) $>$ (symmetry of full model)

  e.g. $\phi = \mathbf{3}_1$ of gauged $SU(2) \times U(1)$
  $\Rightarrow V = -\mu^2|\phi|^2 + \lambda|\phi|^4$ has "custodial" SO(6) symmetry

  2 tree-level massless scalars (3 more eaten by gauge bosons): not accidentally light but pseudo-NGBs of SO(6) $\to$ SO(5) $\to$ Weinberg '73

- **Actual examples**: historic ones $\to$ Bars&Lane '73, Georgi&Pais '75, … tend to be complicated

This talk

Accidentally light scalars can be found in simple models

- continuous symmetry = $SU(N) \times U(1)$
- no ad-hoc discrete symmetries
- a single scalar multiplet

The trick is to consider (slightly) large $SU(N)$ representations:
e.g. $\square\square\square\square$ of SU(2) or $\square\square\square$ of $SU(N \geq 3)$

Tree-level vacuum manifolds have an interesting structure:

- enhanced symmetry points (with extra massless scalars)
- calculable one-loop lifting of flat directions
  $\rightarrow$ scalars not massless but "light"

Applications: DM, Higgs (?), cosmology (?) — go find some!

The simplest Accident® model

$G = \mathrm{SU}(2) \times \mathrm{U}(1)$ (gauged or global), with $\phi = \mathbf{5}_1$

- Parameterize V as

$$V = -\mu^2 S + \frac{1}{2}\left(\lambda\, S^2 + \kappa(S^2 - |S'|^2) + \delta A^a A^a\right)$$

  where
  - $S = \phi^\dagger \phi$
  - $S' = \phi^T \phi$
  - $A^a = \phi^\dagger T^a \phi \quad (a = 1, 2, 3), \quad T^a = \mathrm{SU}(2)$ 5-plet generators
- This is the most general G-invariant potential (up to dim-4).
- For $\lambda, \kappa, \delta > 0$ you can minimize it in your head:

$$\langle\phi\rangle = c\,\hat{\varphi}$$

  with $\hat{\varphi}$ = any real 5-component unit vector, $\quad c \in \mathbb{C}, \quad |c|^2 = \mu^2/\lambda$.

Counting flat directions

$G = \mathrm{SU}(2) \times \mathrm{U}(1)$ (gauged or global), with $\phi = \mathbf{5}_1$

$$\langle \phi_j \rangle = v_j \, e^{i\theta} \,, \quad v_j \in \mathbb{R} \,, \quad \vec{v}^2 = \mu^2/\lambda$$

- 5 flat directions: rotate $\vec{v}$, shift θ
- 4 of these are Goldstone directions (eaten if G is gauged) since $G \to \emptyset$ for generic $\vec{v}$: $3 + 1$ broken generators
- One flat direction goes unexplained.
  This is not a pNGB since V has no enhanced symmetry $> G$.
- This is what we call an Accident. 

What happened?

The most general SU(2) $\times$ U(1)-invariant quartic potential does not give a mass to all states that should get one.

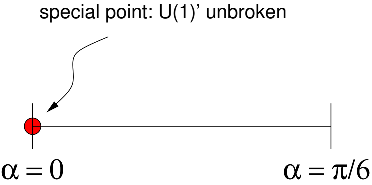
Violation of naive expectation from symmetry selection rules.

Intuitively: many d.o.f. (10 real scalars) but "not enough invariants for all of them"

Tree-level vacuum manifold

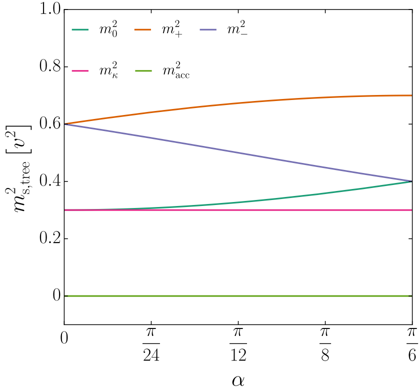
Assume G is gauged.

Then there is a tree-level flat direction: a one-parameter family of physically inequivalent vacua parameterized by $\alpha \in [0, \pi/6]$.

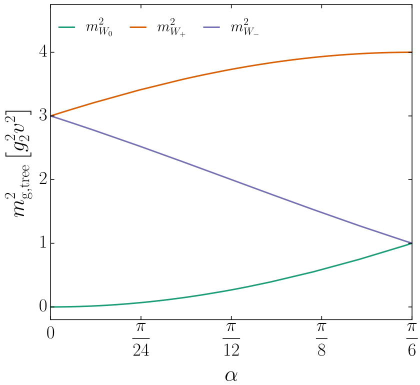


At $\alpha = 0$, $\vec{v}$ is a null eigenvector of T^3 $\Rightarrow$ $U(1)' \subset SU(2)$ unbroken

Tree-level mass spectrum



scalars



gauge bosons

Sum rules: $\sum m^2_{\text{scalar}}$ = constant, $\sum m^2_{\text{vector}}$ = constant

One-loop lifting of the flat direction

The Coleman-Weinberg effective potential can be used to compute one-loop corrections to the Accident mass.

$$\Delta V_{\mathrm{CW}} = \frac{1}{64\pi^2}\,\mathrm{Str}\left(\mathcal{M}^4 \log \frac{\mathcal{M}^2}{\Lambda^2}\right)$$

- Subtract one-loop tadpole for radial mode; fixes Λ
- Resulting correction to Accident mass is finite
- The symmetry-enhanced point $\alpha = 0$ becomes a minimum

$$m_{\mathrm{acc}}^2 = \frac{1}{4\pi^2}\left[3\,g_2^2\,m_{W+}^2 + \delta\,m_+^2\;f\left(\frac{m_0^2}{m_+^2}\right)\right]\Bigg|_{\alpha=0}\,,$$

$$f(x) = 1 - x + x\log x \quad (\geq 0)\,.$$

The opposite point $\alpha = \pi/6$ becomes a saddle point

Summary of $G = \mathrm{SU}(2) \times \mathrm{U}(1)$ model

With ϕ = complex SU(2) 5-plet:

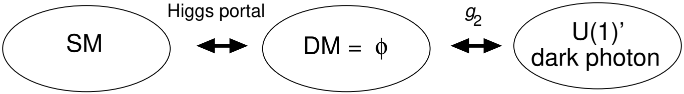
- 10 real scalars
- 5 are massive at the tree level
- away from $\alpha = 0$:
  4 are NGBs of $\mathrm{SU}(2) \times \mathrm{U}(1) \to \emptyset$ (eaten if G is gauged)
  and one is an Accident
- at $\alpha = 0$:
  3 are NGBs of $\mathrm{SU}(2) \times \mathrm{U}(1) \to \mathrm{U}(1)'$ (eaten if G is gauged)
  and two form a light complex scalar with $\mathrm{U}(1)'$ charge 2
- At one loop, the enhanced-symmetry point $\alpha = 0$ is revealed to be the true vacuum.
  EFT around this point = $\mathrm{U}(1)'$ gauge theory + light charged scalar

Application I: Dark Matter

If this is a dark sector:

- the Accident is a natural DM candidate: lightest charged state under $U(1)'$
- interactions with SM thermal bath through Higgs portal
- coupling to $U(1)'$ dark photon $\Rightarrow$ interesting thermal history possible (reannihilation/dark-sector freeze-out...)

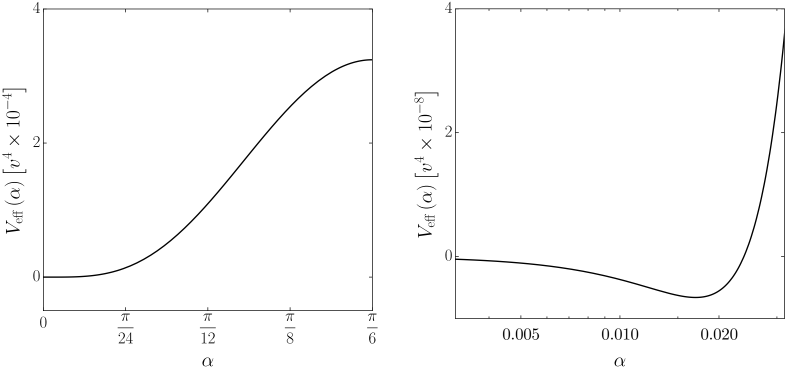
  $\rightarrow$ Chu/Hambye/Tytgat '12, Hambye et al. '19, Bharucha/FB/Desai/Mutzel '22, Frigerio/Grinbaum-Yamamoto/Hambye '22...



Application II: Poor person's EWSB

The Accident as an abelian Higgs: destabilize the enhanced-symmetry point $\alpha = 0$ by coupling to fermions (= mock top quarks)

By ~~fine-tuning~~ judiciously choosing parameters one can get this:



It would be extremely interesting to find an accident model where U(1)′ becomes SU(2) $\times$ U(1) or SO(4). $\rightarrow$ composite Higgs, little Higgs models

Application III: Inflation

Can the inflaton be an Accident?

In the simplest model, Accident effective potential $\approx$ natural inflation, disfavoured by Planck.

Other, more complicated models feature multiple Accidents and more complicated effective potentials.

To be explored. Also, the minimum is typically at an enhanced symmetry point where gauge bosons become massless and the number of Accidents is increased $\Rightarrow$ relevance for particle production?

More Accidents

The next-to-minimal Accident model has $\phi = \mathbf{10}_1$ of $SU(3) \times U(1)$:

- 2-dimensional tree-level vacuum manifold, 2 Accidents 
- At a generic point, $SU(3) \times U(1) \to \emptyset$
- At a special point,
  - $SU(3) \times U(1) \to U(1)^2$
  - this is the minimum of the one-loop effective potential
  - now 6 light scalars

Can generalize to $\phi = \square\square\square$ of $SU(N)$:

- many more Accidents, and $U(1)$s at the symmetry-enhanced point…

Are there models with non-abelian residual symmetries?

Conclusions

- Accidents (accidentally light scalars) are light scalar fields whose mass suppression does not follow (obviously?) from selection rules/NDA
- They appear in models with large representations because of the restrictive structure of the scalar potential
- Many features in common with pNGBs (in particular, they may also get part of their mass from gauge or fermion loops)
- It would be worthwile to better understand what precisely are the conditions to find Accidents in some given model
- Phenomenology of Accidents remains to be explored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