

Towards minimal $SU(5)$ model*

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*I.D. and Shaikh Saad, Phys.Rev.D 101 (2020) 1, 015009, arXiv:1910.09008.

*I.D., Emina Džaferović-Mašić, and Shaikh Saad, Phys.Rev.D 104 (2021) 1, 015023, arXiv:2105.01678.

OUTLINE

- **NOVEL $SU(5)$ MODEL PROPOSAL**

- **PARAMETER SPACE ANALYSIS**

- **CONCLUSIONS**

GEORGI-GLASHOW MODEL*

$SU(5)$	$SU(3) \times SU(2) \times U(1)$	$SU(5)$	$SU(3) \times SU(2) \times U(1)$
$5_H \equiv \Lambda$	$\Lambda_1 (1, 2, \frac{1}{2})$ $\Lambda_3 (3, 1, -\frac{1}{3})$	$\bar{5}_{F_i} \equiv F_i$	$L_i (1, 2, -\frac{1}{2})$ $d_i^c (\bar{3}, 1, \frac{1}{3})$
$24_H \equiv \phi$	$\phi_0 (1, 1, 0)$ $\phi_1 (1, 3, 0)$ $\phi_3 (3, 2, -\frac{5}{6})$ $\phi_{\bar{3}} (\bar{3}, 2, \frac{5}{6})$ $\phi_8 (8, 1, 0)$	$10_{F_i} \equiv T_i$	$Q_i (3, 2, \frac{1}{6})$ $u_i^c (\bar{3}, 1, -\frac{2}{3})$ $e_i^c (1, 1, 1)$

*H. Georgi and S.L. Glashow, Phys. Rev. Lett. 32 (1974) 438–441.

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$\langle \Lambda_1 \rangle \equiv v_5$ - electroweak VEV

$\langle \phi_0 \rangle \equiv v_{24}$ - $SU(5)$ breaking VEV

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- NEUTRINOS ARE MASSLESS;
- UNIFICATION OF THE STANDARD MODEL GAUGE COUPLINGS DOES NOT TAKE PLACE;
- $m_e = m_d, m_\mu = m_s, m_\tau = m_b$;

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A NOVEL $SU(5)$ MODEL PROPOSAL*

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		$\bar{15}_F \equiv \bar{\Sigma}$	$\bar{\Sigma}_1 (1, 3, -1)$ $\bar{\Sigma}_3 (\bar{3}, 2, -\frac{1}{6})$ $\bar{\Sigma}_6 (\bar{6}, 1, \frac{2}{3})$

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- $m_e \neq m_d, m_\mu \neq m_s, m_\tau \neq m_b;$

*N. Oshimo, Phys.Rev.D 80 (2009) 075011, arXiv:0907.3400.

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- TWO NEUTRINOS BECOME MASSIVE;

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- NEUTRINOS HAVE NORMAL MASS ORDERING;

*I.D. and Shaikh Saad, Phys.Rev.D 101 (2020) 1, 015009, arXiv:1910.09008.

A NOVEL $SU(5)$ MODEL PROPOSAL

$$\begin{aligned}
 \mathcal{L} \supset & \left\{ +Y_{ij}^u T_i^{\alpha\beta} T_j^{\gamma\delta} \Lambda^\rho \epsilon_{\alpha\beta\gamma\delta\rho} + Y_{ij}^d T_i^{\alpha\beta} F_{\alpha j} \Lambda_\beta^* + Y_i^a \Sigma^{\alpha\beta} F_{\alpha i} \Lambda_\beta^* + Y_i^b \bar{\Sigma}_{\beta\gamma} F_{\alpha i} \Phi^{*\alpha\beta\gamma} \right. \\
 & \left. + Y_i^c T_i^{\alpha\beta} \bar{\Sigma}_{\beta\gamma} \phi_\alpha^\gamma + \text{h.c.} \right\} + M_\Sigma \bar{\Sigma}_{\alpha\beta} \Sigma^{\alpha\beta} + y \bar{\Sigma}_{\alpha\beta} \Sigma^{\beta\gamma} \phi_\gamma^\alpha \\
 & - \mu_\Lambda^2 (\Lambda_\alpha^* \Lambda^\alpha) + \lambda_0^\Lambda (\Lambda_\alpha^* \Lambda^\alpha)^2 + \mu_1 \Lambda_\alpha^* \Lambda^\beta \phi_\beta^\alpha + \lambda_1^\Lambda (\Lambda_\alpha^* \Lambda^\alpha) (\phi_\gamma^\beta \phi_\beta^\gamma) + \lambda_2^\Lambda \Lambda_\alpha^* \Lambda^\beta \phi_\beta^\gamma \phi_\gamma^\alpha \\
 & - \mu_\phi^2 (\phi_\gamma^\beta \phi_\beta^\gamma) + \mu_2 \phi_\beta^\alpha \phi_\gamma^\beta \phi_\alpha^\gamma + \lambda_0^\phi (\phi_\gamma^\beta \phi_\beta^\gamma)^2 + \lambda_1^\phi \phi_\beta^\alpha \phi_\gamma^\beta \phi_\delta^\gamma \phi_\alpha^\delta + \mu_\Phi^2 (\Phi^{*\alpha\beta\gamma} \Phi_{\alpha\beta\gamma}) \\
 & + \lambda_0^\Phi (\Phi^{*\alpha\beta\gamma} \Phi_{\alpha\beta\gamma})^2 + \lambda_1^\Phi \Phi^{*\alpha\beta\gamma} \Phi_{\alpha\beta\delta} \Phi^{*\delta\rho\sigma} \Phi_{\rho\sigma\gamma} + \lambda_0 (\Phi^{*\alpha\beta\gamma} \Phi_{\alpha\beta\gamma}) (\phi_\rho^\delta \phi_\delta^\rho) \\
 & + \lambda_0' (\Phi^{*\alpha\beta\gamma} \Phi_{\alpha\beta\gamma}) (\Lambda_\rho^* \Lambda^\rho) + \lambda_0'' \Phi^{*\alpha\beta\gamma} \Phi_{\beta\gamma\delta} \Lambda^\delta \Lambda_\alpha^* + \mu_3 \Phi^{*\alpha\beta\gamma} \Phi_{\beta\gamma\delta} \phi_\alpha^\delta \\
 & + \lambda_1 \Phi^{*\alpha\beta\gamma} \Phi_{\alpha\delta\rho} \phi_\beta^\delta \phi_\gamma^\rho + \lambda_2 \Phi^{*\alpha\beta\rho} \Phi_{\alpha\beta\delta} \phi_\rho^\gamma \phi_\gamma^\delta + \{ \lambda' \Lambda^\alpha \Lambda^\beta \Lambda^\gamma \Phi_{\alpha\beta\gamma} + \text{h.c.} \}
 \end{aligned}$$

A NOVEL $SU(5)$ MODEL PROPOSAL

PARTICLE CONTENT:

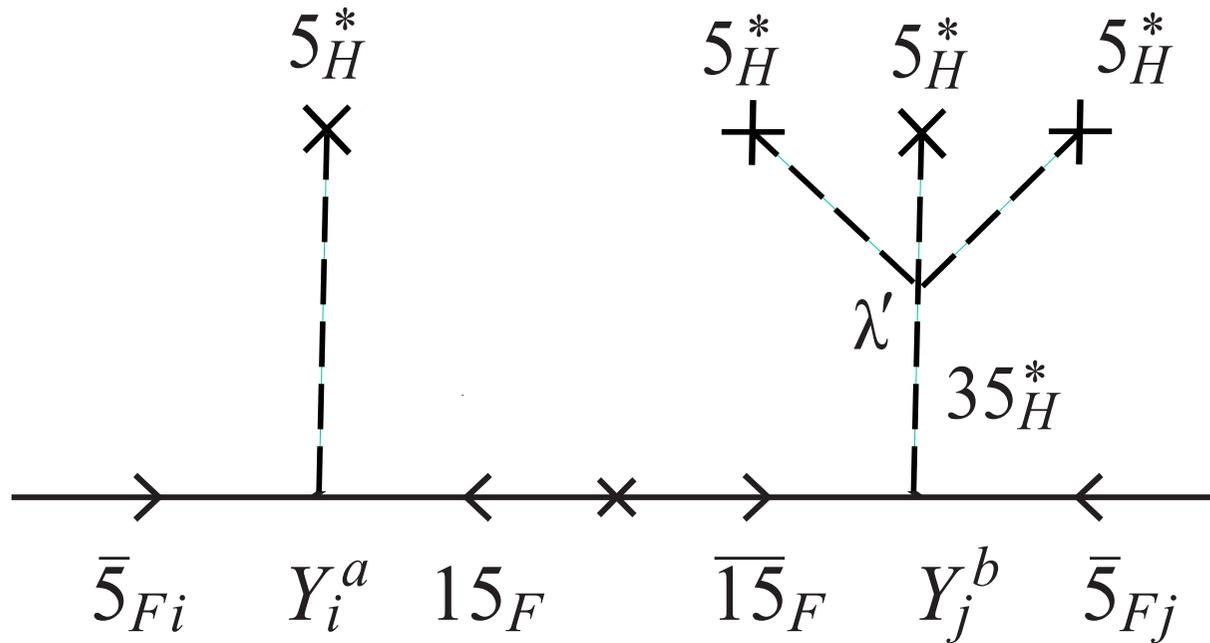
$$5_H, 24_H, 35_H, \bar{5}_{F_i}, 10_{F_i}, 15_F, \bar{15}_F \quad i = 1, 2, 3$$

YUKAWA COUPLINGS:

$$Y_i^a, Y_i^b, Y_i^c, Y_{ij}^u \equiv Y_{ji}^u, Y_{ij}^d \equiv \delta_{ij} Y_i^d, y \quad i, j = 1, 2, 3$$

NEUTRINO MASSES *

$d=7$ CONTRIBUTION

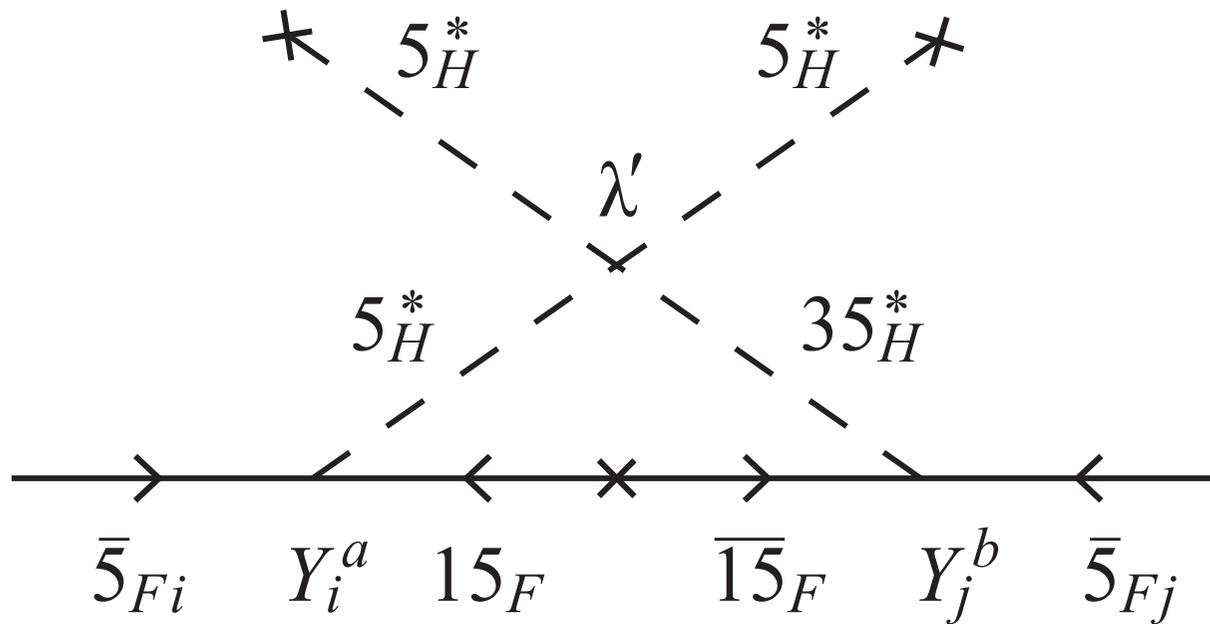


$$\mathcal{L} \supset \lambda' 5_H 5_H 5_H 35_H + Y_i^a 15_F \bar{5}_{Fi} 5_H^* + Y_j^b \bar{15}_F \bar{5}_{Fj} 35_H^*$$

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NEUTRINO MASSES *

$d=5$ CONTRIBUTION

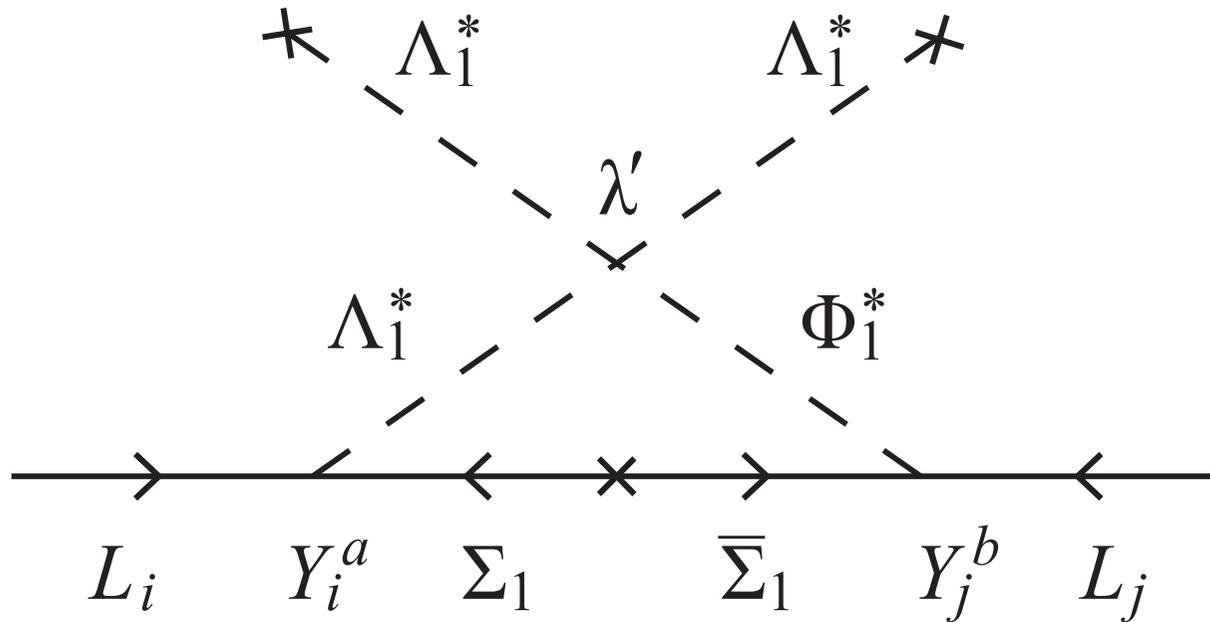


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NEUTRINO MASSES *

$d=5$ CONTRIBUTION



$$(M_N)_{ij} = m_0 (Y_i^a Y_j^b + Y_i^b Y_j^a) \quad m_0 = \frac{\lambda' v_5^2}{16\pi^2} \frac{M_{\Sigma_1}}{M_{\Sigma_1}^2 - M_{\Phi_1}^2} \log \left(\frac{M_{\Sigma_1}^2}{M_{\Phi_1}^2} \right)$$

*K.S. Babu, S. Nandi, and Z. Tavartkiladze, Phys.Rev.D 80 (2009) 071702, arXiv:0905.2710.

CHARGED FERMION MASSES

GEORGI-GLASHOW MODEL: *

$$(M_U)_{ij} = 4v_5(Y_{ij}^u + Y_{ji}^u)$$

$$(M_D)_{ij} = v_5 Y_{ij}^d$$

$$(M_E)_{ij} = v_5 Y_{ji}^d$$

*H. Georgi and S.L. Glashow, Phys. Rev. Lett. 32 (1974) 438–441.

CHARGED FERMION MASSES

NOVEL PROPOSAL:*

$$(M_U)_{ij} = 4v_5(Y_{ij}^u + Y_{ji}^u)$$

$$(M_D)_{ij} = v_5(Y_{ij}^d + \delta' Y_i^c Y_j^a)$$

$$(M_E)_{ij} = v_5 Y_{ji}^d$$

$$\delta' \equiv \sqrt{10/3} v_{24} / (4M_{\Sigma_3})$$

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FERMION MASSES *

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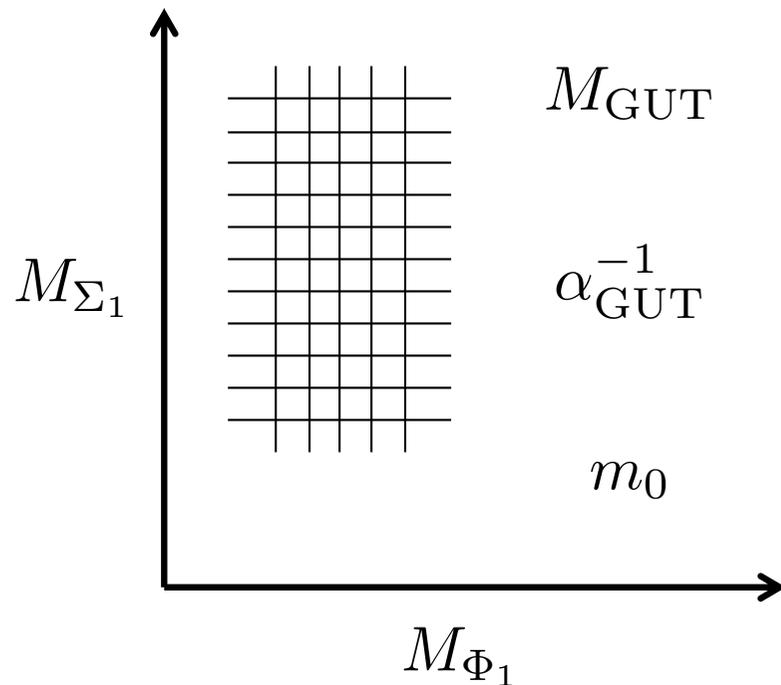
$$(M_N)_{ij} = m_0(Y_i^a Y_j^b + Y_i^b Y_j^a)$$

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PARAMETER SPACE ANALYSIS *

- GAUGE COUPLING UNIFICATION ($\max(M_{\text{GUT}})$)

$$M \equiv \min(M_J), \quad J = \Phi_1, \Phi_3, \Phi_6, \Phi_{10}, \Sigma_1, \Sigma_3, \Sigma_6, \phi_1, \phi_8, \Lambda_3$$



$$\Sigma_1 (1, 3, 1)$$

$$\Phi_1 \left(1, 4, -\frac{3}{2} \right)$$

- MASS RELATION(S)

$$M_{\Sigma_6} + M_{\Sigma_1} = 2M_{\Sigma_3}$$

SCENARIO (a):

$$M_{\Sigma_6} \approx M_{\Sigma_1} \approx M_{\Sigma_3}$$

SCENARIO (b):

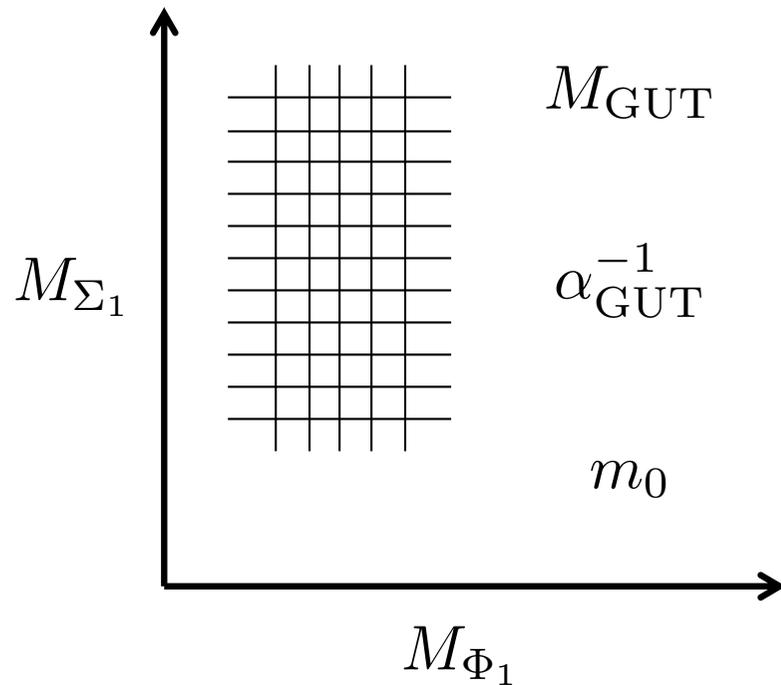
$$M_{\Sigma_6} = -M_{\Sigma_1}$$

*I.D., Emina Džaferović-Mašić, and Shaikh Saad, Phys.Rev.D 104 (2021) 1, 015023, arXiv:2105.01678.

PARAMETER SPACE ANALYSIS

- GAUGE COUPLING UNIFICATION

$$M \equiv \min(M_J), \quad J = \Phi_1, \Phi_3, \Phi_6, \Phi_{10}, \Sigma_1, \Sigma_3, \Sigma_6, \phi_1, \phi_8, \Lambda_3$$



$$\Sigma_1 (1, 3, 1)$$

$$\Phi_1 \left(1, 4, -\frac{3}{2} \right)$$

- FERMION MASS FIT

$$(M_U)_{ij} = 4v_5(Y_{ij}^u + Y_{ji}^u)$$

$$(M_D)_{ij} = v_5 (Y_{ij}^d + \delta' Y_i^c Y_j^a)$$

$$(M_E)_{ij} = v_5 Y_{ji}^d$$

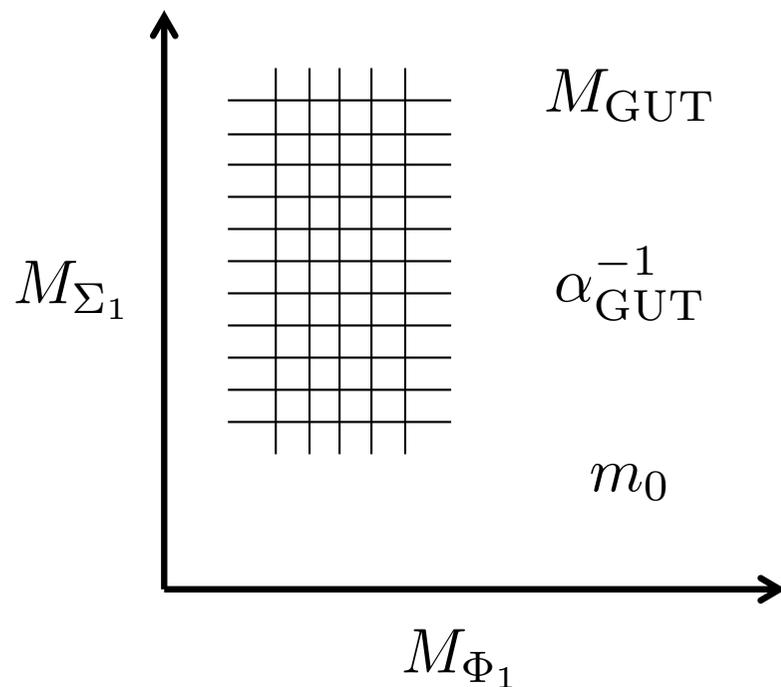
$$(M_N)_{ij} = m_0 (Y_i^a Y_j^b + Y_i^b Y_j^a)^*$$

*I. Cordero-Carrión, M. Hirsch, and A. Vicente, Phys.Rev.D 101 no. 7, (2020) 075032, arXiv:1912.08858.

PARAMETER SPACE ANALYSIS *

- GAUGE COUPLING UNIFICATION

$$M \equiv \min(M_J), \quad J = \Phi_1, \Phi_3, \Phi_6, \Phi_{10}, \Sigma_1, \Sigma_3, \Sigma_6, \phi_1, \phi_8, \Lambda_3$$



$$\Sigma_1 (1, 3, 1)$$

$$\Phi_1 \left(1, 4, -\frac{3}{2} \right)$$

- PROTON DECAY ANALYSIS

$$M_U = U_L M_U^{\text{diag}} U_R^\dagger$$

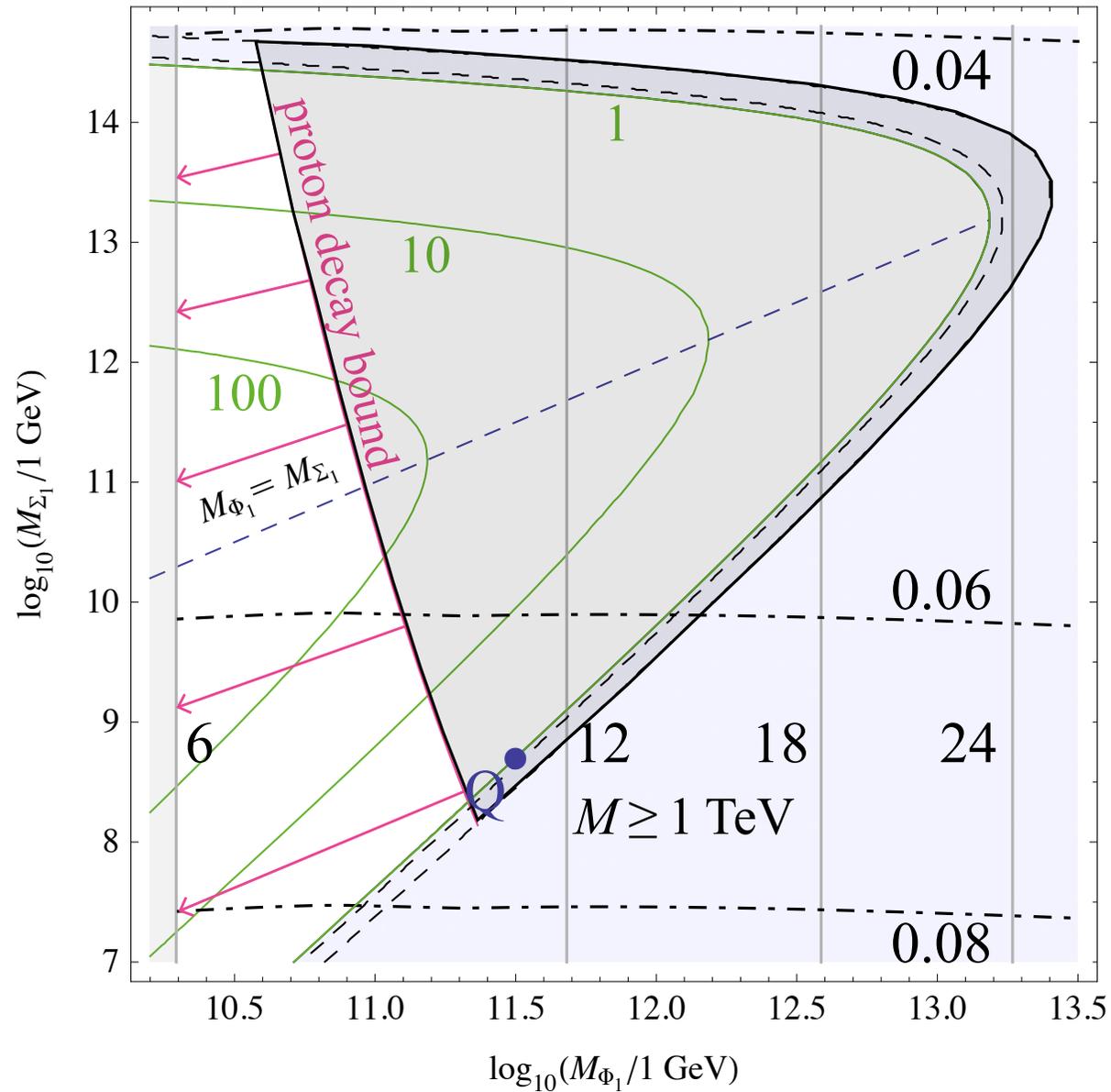
$$M_D = D_L M_D^{\text{diag}} D_R^\dagger$$

$$M_E \equiv E_L M_E^{\text{diag}} E_R^\dagger$$

$$M_N = N M_N^{\text{diag}} N^T$$

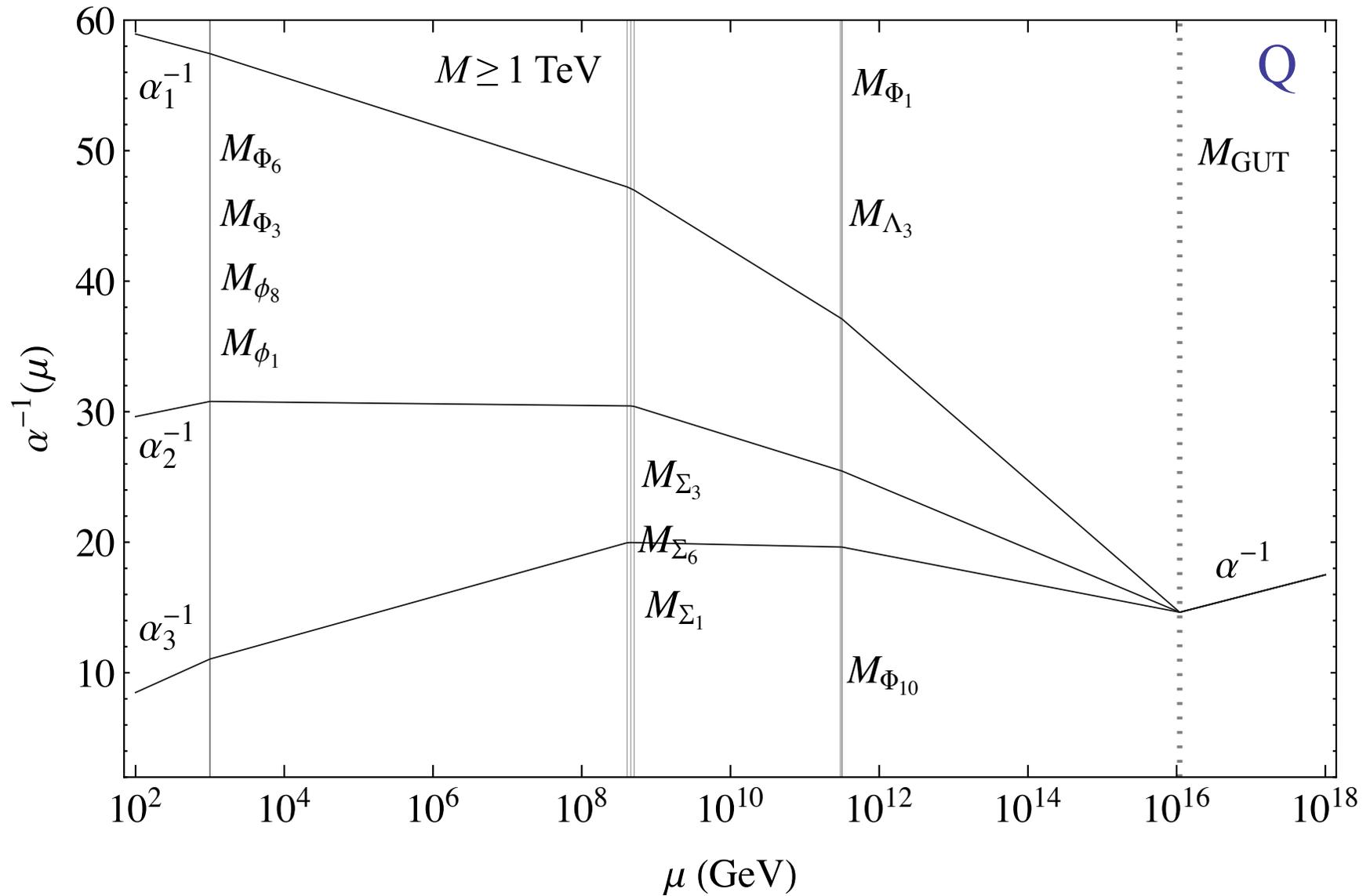
*I.D., Emina Džaferović-Mašić, and Shaikh Saad, Phys.Rev.D 104 (2021) 1, 015023, arXiv:2105.01678.

PARAMETER SPACE ANALYSIS - SCENARIO (a)*



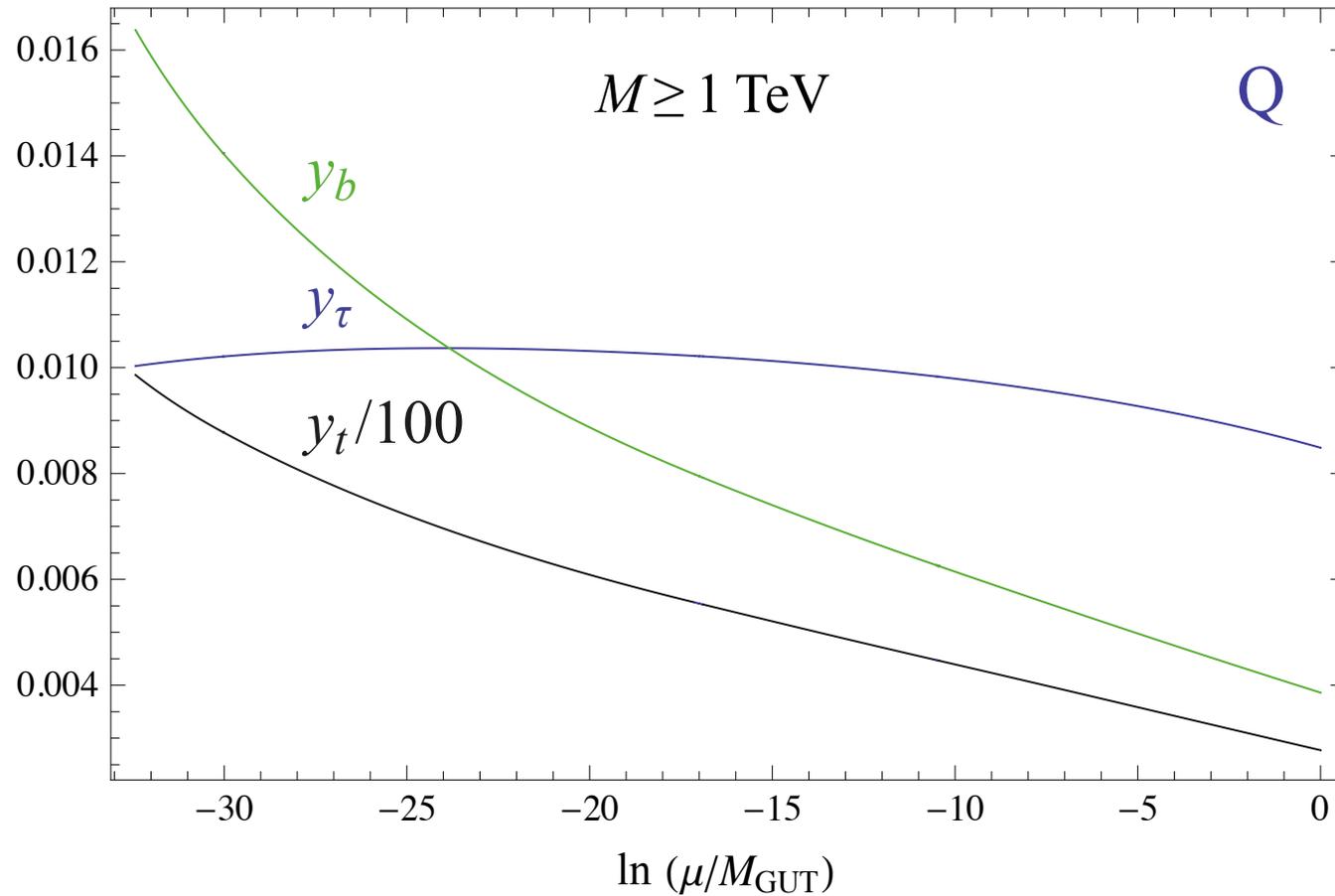
*I.D., Emina Džaferović-Mašić, and Shaikh Saad, Phys.Rev.D 104 (2021) 1, 015023, arXiv:2105.01678.

PARAMETER SPACE ANALYSIS - SCENARIO (a)*



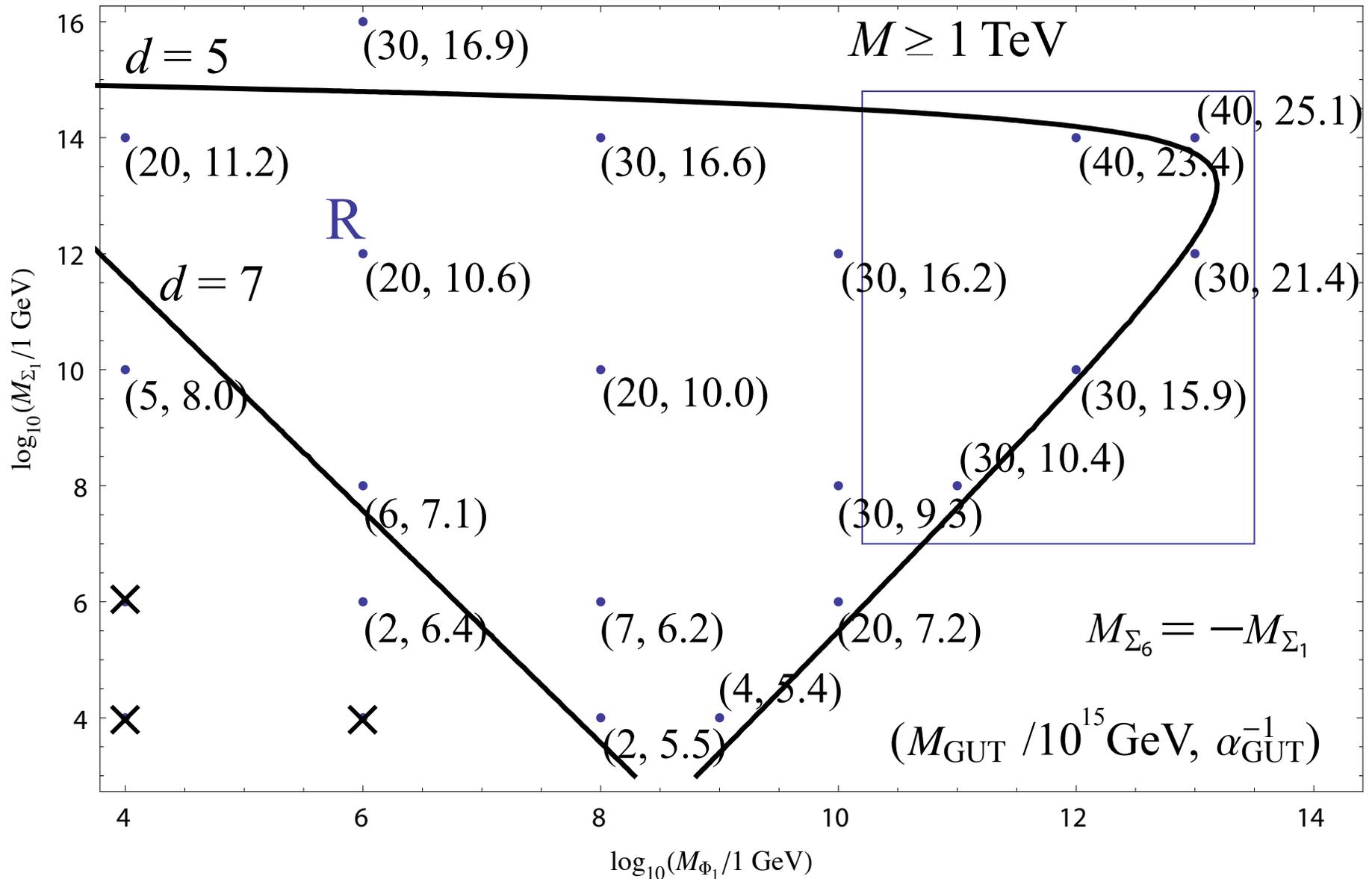
*I.D., Emina Džaferović-Mašić, and Shaikh Saad, Phys.Rev.D 104 (2021) 1, 015023, arXiv:2105.01678.

PARAMETER SPACE ANALYSIS - SCENARIO (a)*



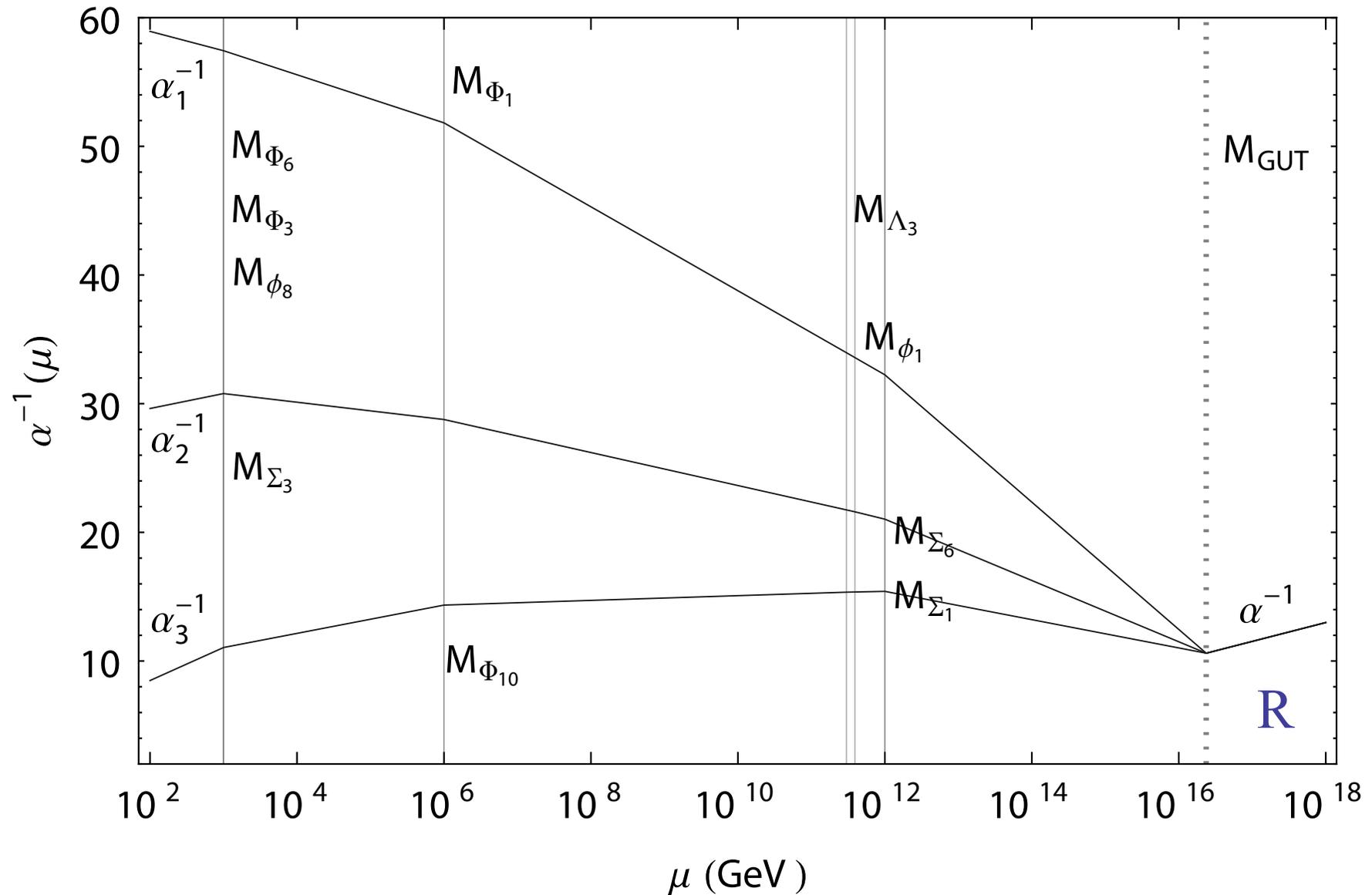
*I.D., Emina Džaferović-Mašić, and Shaikh Saad, Phys.Rev.D 104 (2021) 1, 015023, arXiv:2105.01678.

PARAMETER SPACE ANALYSIS - SCENARIO (b)*



*I.D., Emina Džaferović-Mašić, Svjetlana Fajfer, and Shaikh Saad, work in progress.

PARAMETER SPACE ANALYSIS - SCENARIO (b)*



*I.D., Emina Džaferović-Mašić, Svjetlana Fajfer, and Shaikh Saad, work in progress.

CONCLUSIONS

PARTICLE CONTENT:

$$5_H, 24_H, 35_H, \bar{5}_{Fi}, 10_{Fi}, 15_F, \bar{15}_F \quad i = 1, 2, 3$$

YUKAWA COUPLINGS:

$$Y_i^a, Y_i^b, Y_i^c, Y_{ij}^u \equiv Y_{ji}^u, Y_{ij}^d \equiv \delta_{ij} Y_i^d, y \quad i, j = 1, 2, 3$$

- NEUTRINOS ARE MAJORANA PARTICLES:
- ONE NEUTRINO IS MASSLESS;
- NEUTRINO MASSES HAVE NORMAL ORDERING;

$$\begin{aligned}
\mathcal{L} \supset & \left\{ +Y_{ij}^u T_i^{\alpha\beta} T_j^{\gamma\delta} \Lambda^\rho \epsilon_{\alpha\beta\gamma\delta\rho} + Y_{ij}^d T_i^{\alpha\beta} F_{\alpha j} \Lambda_\beta^* + Y_i^a \Sigma^{\alpha\beta} F_{\alpha i} \Lambda_\beta^* + Y_i^b \bar{\Sigma}_{\beta\gamma} F_{\alpha i} \Phi^{*\alpha\beta\gamma} \right. \\
& \left. + Y_i^c T_i^{\alpha\beta} \bar{\Sigma}_{\beta\gamma} \phi_\alpha^\gamma + \text{h.c.} \right\} + M_\Sigma \bar{\Sigma}_{\alpha\beta} \Sigma^{\alpha\beta} + y \bar{\Sigma}_{\alpha\beta} \Sigma^{\beta\gamma} \phi_\gamma^\alpha \\
& - \mu_\Lambda^2 (\Lambda_\alpha^* \Lambda^\alpha) + \lambda_0^\Lambda (\Lambda_\alpha^* \Lambda^\alpha)^2 + \mu_1 \Lambda_\alpha^* \Lambda^\beta \phi_\beta^\alpha + \lambda_1^\Lambda (\Lambda_\alpha^* \Lambda^\alpha) (\phi_\gamma^\beta \phi_\beta^\gamma) + \lambda_2^\Lambda \Lambda_\alpha^* \Lambda^\beta \phi_\beta^\gamma \phi_\gamma^\alpha \\
& - \mu_\phi^2 (\phi_\gamma^\beta \phi_\beta^\gamma) + \mu_2 \phi_\beta^\alpha \phi_\gamma^\beta \phi_\alpha^\gamma + \lambda_0^\phi (\phi_\gamma^\beta \phi_\beta^\gamma)^2 + \lambda_1^\phi \phi_\beta^\alpha \phi_\gamma^\beta \phi_\delta^\gamma \phi_\alpha^\delta + \mu_\Phi^2 (\Phi^{*\alpha\beta\gamma} \Phi_{\alpha\beta\gamma}) \\
& + \lambda_0^\Phi (\Phi^{*\alpha\beta\gamma} \Phi_{\alpha\beta\gamma})^2 + \lambda_1^\Phi \Phi^{*\alpha\beta\gamma} \Phi_{\alpha\beta\delta} \Phi^{*\delta\rho\sigma} \Phi_{\rho\sigma\gamma} + \lambda_0 (\Phi^{*\alpha\beta\gamma} \Phi_{\alpha\beta\gamma}) (\phi_\rho^\delta \phi_\delta^\rho) \\
& + \lambda_0' (\Phi^{*\alpha\beta\gamma} \Phi_{\alpha\beta\gamma}) (\Lambda_\rho^* \Lambda^\rho) + \lambda_0'' \Phi^{*\alpha\beta\gamma} \Phi_{\beta\gamma\delta} \Lambda^\delta \Lambda_\alpha^* + \mu_3 \Phi^{*\alpha\beta\gamma} \Phi_{\beta\gamma\delta} \phi_\alpha^\delta \\
& + \lambda_1 \Phi^{*\alpha\beta\gamma} \Phi_{\alpha\delta\rho} \phi_\beta^\delta \phi_\rho^\gamma + \lambda_2 \Phi^{*\alpha\beta\gamma} \Phi_{\alpha\beta\delta} \phi_\rho^\gamma \phi_\gamma^\delta + \{ \lambda' \Lambda^\alpha \Lambda^\beta \Lambda^\gamma \Phi_{\alpha\beta\gamma} + \text{h.c.} \}
\end{aligned}$$

THANK YOU

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