## (Updated) Global bounds on heavy neutrino mixing

Based on:
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## Motivation

Neutrinos are massive $\qquad$ need a mechanism to generate their (tiny) masses

O Seesaw mechanism via heavy neutrinos

## Searches for heavy neutrinos

Plethora of searches for heavy neutrinosHowever, experimental bounds die off for $M_{N}>M_{W}$

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Plethora of searches for heavy neutrinos
© However, experimental bounds die off for $M_{N}>M_{W}$

## Why update the global fit?

Updates on key observables:* New measurements of $M_{W}$ (CDF-II, ATLAS)
$\star$ Anomaly $(\sim 2-3 \sigma)$ in the extraction of CKM elements $\left|V_{u d}\right|$ and $\left|V_{u s}\right|$
$\star$ LEP anomaly ( $\sim 2 \sigma$ ) in $N_{\nu}$ is now gone
( Improvement of the analysis:
$\star$ Correlations
* Deviations from Wilks' theorem: Bootstrapping


## Non-unitarity in general

Precision observables are modified by leptonic non-unitarityIn general:$$
N=(1-\eta) \underset{\text { Diagonalises } m_{\nu}}{U,} \quad \eta^{\dagger}=\eta
$$

O Convenient: $\eta$ has flavor indices

OGenerally, mass eigenstates are summed over:

$$
\sum_{i=1}^{3}(N)_{\alpha i}\left(N^{\dagger}\right)_{i \beta}=\delta_{\alpha \beta}-2 \eta_{\alpha \beta}+O\left(\eta^{2}\right)
$$

## Heavy neutrinos and non-unitarity

O In general:

$$
N=(1-\eta) \underline{U}, \quad \eta^{\dagger}=\eta
$$

Diagonalises $m_{\nu}$
O In the context of heavy neutrinos $N \sim(1,1,0)$ : $-\mathscr{L} \supset Y_{\nu} \bar{L}_{L} \tilde{H} N+\frac{1}{2} M_{M} \overline{N^{c}} N$

$$
\mathscr{M}=\left(\begin{array}{cc}
0 & Y_{\nu} \nu / \sqrt{2} \\
Y_{\nu}^{T} \nu / \sqrt{2} & M_{M}
\end{array}\right)
$$

Diagonalised by:

$$
V=\left(\begin{array}{cc}
1-\frac{1}{2} \Theta \Theta^{\dagger} & \Theta \\
-\Theta^{\dagger} & 1-\frac{1}{2} \Theta^{\dagger} \Theta
\end{array}\right)\left(\begin{array}{cc}
U & 0 \\
0 & U^{\prime}
\end{array}\right)
$$

## Heavy neutrinos and non-unitarity

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N=(1-\eta) \frac{U,}{} \quad \eta^{\dagger}=\eta
$$

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Y_{\nu}^{T} \nu / \sqrt{2} & M_{M}
\end{array}\right)
$$

Diagonalised by:


## Heavy neutrinos and non-unitarity

© In the Type-I seesaw:

$$
N=\left(1-\frac{1}{2} \Theta \Theta^{\dagger}\right) U
$$

$$
\underbrace{\eta=\frac{1}{2} \Theta^{\dagger}}_{\text {Mass-independent }}
$$

$\eta$ is positive-definite $\left\{\begin{array}{l}\eta_{\alpha \alpha} \geq 0 \\ \left|\eta_{\alpha \beta}\right| \leq \sqrt{\eta_{\alpha \alpha} \eta_{\beta \beta}} \text { (Schwarz inequality) }\end{array}\right.$
© Additionally: $m_{\nu} \simeq-\Theta M_{M} \Theta^{T}$ can impose correlations within $\eta$

## Precision observables and non-unitarity

$\bigcirc$ SM inputs $\left\{\begin{array}{c}\alpha \\ M_{Z} \\ G_{F}\end{array}\right.$
$\bigcirc G_{F}$ is extracted from $\mu$-decay $\longrightarrow$ Modified by lepton non-unitarity

$$
\begin{aligned}
& \Gamma_{\mu}=\frac{G_{F}^{2} m_{\mu}^{5}}{192 \pi^{3}} \sum_{i=1}^{3}\left|N_{\mu i}\right|^{2} \sum_{j=1}^{3}\left|N_{e j}\right|^{2} \simeq \frac{G_{F}^{2} m_{\mu}^{5}}{192 \pi^{3}}\left(1-2 \eta_{e e}-2 \eta_{\mu \mu}\right) \equiv \frac{G_{\mu}^{2} m_{\mu}^{5}}{192 \pi^{3}}, \\
& G_{F} \simeq G_{\mu}\left(1+\eta_{e e}+\eta_{\mu \mu}\right) \longrightarrow \text { Modifies all EWPO }
\end{aligned}
$$

## $M_{W}$ and $s_{\text {eff }}^{2}$

© We consider only tree-level $\eta$-dependence and loop-level SM corrections

$$
M_{W}=M_{Z} \sqrt{\frac{1}{2}+\sqrt{\frac{1}{4}-\frac{\pi \alpha\left(1+\eta_{e e}+\eta_{\mu \mu}\right)}{\sqrt{2} G_{\mu} M_{Z}^{2}(1-\Delta r)}}}
$$Similarly with $s_{\text {eff }}^{2}$

## Z-pole observables

Z-boson partial widths also modified$\bigcirc \Gamma(Z \rightarrow f \bar{f})$ modified by $G_{F}$ and $s_{e f f}^{2}$
$\bigcirc \Gamma_{i n v}$ modified by $G_{F}$ and by $Z \rightarrow \nu \nu$ vertex

## Z-pole observables

Z-boson partial widths also modified$\Gamma(Z \rightarrow f \bar{f})$ modified by $G_{F}$ and $s_{e f f}^{2}$© $\Gamma_{i n \nu}$ modified by $G_{F}$ and by $Z \rightarrow \nu \nu$ vertex


## LEP precision measurements also constrain $\eta$

## Z-pole observables

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## Lepton Flavor Universality (LFU)

$\bigcirc \rightarrow l \nu$ vertex modified by $\eta$

$$
\sum_{i=1}^{3}\left|N_{\alpha i}\right|^{2}=1-2 \eta_{\alpha \alpha}
$$

© Weak interactions are no longer flavor universal
(O) Ratios of $\pi, K$ and $\tau$ decays constrain the universality of weak interactions

## CKM unitarity

The CKM matrix remains unitary:$$
1=\left|V_{u d}\right|^{2}+\left|V_{u s}\right|^{2}+\left|V_{u b}\right|^{2},
$$

© But the extraction of $\left|V_{u d}\right|$ and $\left|V_{u s}\right|$ is affected

$$
\left|V_{u d}\right|=\sqrt{1-\left\lvert\, \frac{\left|V_{u s}\right|^{2}}{}\right.}
$$

Nuisance parameter (minimized over)
© $\left|V_{u d}\right|$ extracted from superallowed $\beta$-decays

## CKM unitarity

© $\left|V_{u s}\right|$ extracted from $K$ and $\tau$ semileptonic decays
© Cabibbo anomaly: $\left|V_{u d}\right|<\sqrt{1-\left|V_{u s}\right|^{2}}$ at $2-3 \sigma$ level
© Only worsened in presence of $\eta_{\mu \mu}>0$

## Charged Lepton Flavor Violation (cLFV)

Previous observables are LFC: depend on $\eta_{\alpha \alpha}$O The off-diagonal elements $\eta_{\alpha \beta}$ induces LFV processes $\left\{\begin{array}{l}l_{\alpha} \rightarrow l_{\beta} \gamma \\ l_{\alpha} \rightarrow l_{\beta} l_{\beta} l_{\beta} \\ \mu-e\end{array}\right.$
For example: $\quad B R\left(l_{\alpha} \rightarrow l_{\beta \gamma}\right) \simeq \frac{3 \alpha}{2 \pi}\left|\eta_{\alpha \beta}\right|^{2}, \quad$ for $\quad M_{N} \gg M_{W}$The off-diagonal elements alternatively constrained via the Schwarz inequality:

$$
\left|\eta_{\alpha \beta}\right| \leq \sqrt{\eta_{\alpha \alpha} \eta_{\beta \beta}}
$$

## The preference of the data

© $M_{W}, s_{e f f}^{2}$ showcase $\sim 1-2 \sigma$ preference for $\eta_{e e}+\eta_{\mu \mu}>0$LFU prefers $\eta_{e e}>\eta_{\mu \mu}$ at $\sim 1 \sigma$
( Cabibbo anomaly disfavors $\eta_{\mu \mu}>0$
© Observables constraining $\eta_{\tau \tau}$ show good agreement with SMSumming up: data prefers $\eta_{e e}>0, \eta_{\mu \mu}=0, \eta_{\tau \tau}=0$

## Cases under study

Minimal scenario with 2 heavy neutrinos: 2 N -SS(Previously missing in the literature)

O Next-to-minimal scenario with 3 heavy neutrinos: 3 N -SS

O General scenario with arbitrary number of heavy neutrinos: G-SS

## Cases under study

Minimal scenario with 2 heavy neutrinos: $2 \mathrm{~N}-\mathrm{SS}$(Previously missing in the literature)
$\star$ Correlations from $m_{\nu}$
$\star\left|\eta_{\alpha \beta}\right|=\sqrt{\eta_{\alpha \alpha} \eta_{\beta \beta}}$
$\star$ LFV with LFC
( Next-to-minimal scenario with 3 heavy neutrinos: 3 N -SSGeneral scenario with arbitrary number of heavy neutrinos: G-SS

## Cases under study

Minimal scenario with 2 heavy neutrinos: 2 N -SS(Previously missing in the literature)
$\star$ Correlations from $m_{\nu}$
$\star\left|\eta_{\alpha \beta}\right|=\sqrt{\eta_{\alpha \alpha} \eta_{\beta \beta}}$
$\star \mathrm{LFV}$ with LFC
O Next-to-minimal scenario with 3 heavy neutrinos: 3 N -SSGeneral scenario with arbitrary number of heavy neutrinos: G-SS
$\star \eta_{e e}, \eta_{\mu \mu}$ and $\eta_{\tau \tau}$ independent
$\star\left|\eta_{\alpha \beta}\right| \leq \sqrt{\eta_{\alpha \alpha} \eta_{\beta \beta}}$

* LFV decoupled from LFC


## Results for the $\mathbf{2}$ heavy neutrino case

Very restrictive flavor structure

O cLFV bounds play a very important role

## Results for the $\mathbf{2}$ heavy neutrino case

© Stringent bounds $\sim 10^{-5}-10^{-4}$

| 2N-SS | Normal Ordering |  | Inverted Ordering |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $68 \% \mathrm{CL}$ | $95 \% \mathrm{CL}$ | $68 \% \mathrm{CL}$ | $95 \% \mathrm{CL}$ |
| $\left.\eta_{e e}=\frac{\left\|\theta_{e}\right\|^{2}}{2} \right\rvert\,$ | $6.4 \cdot 10^{-6}$ | $9.4 \cdot 10^{-6}$ | $[0.98,4.4] \cdot 10^{-4}$ | $5.5 \cdot 10^{-4}$ |
| $\eta_{\mu \mu}=\frac{\left\|\theta_{\mu}\right\|^{2}}{2}$ | $6.9 \cdot 10^{-5}$ | $1.3 \cdot 10^{-4}$ | $[0.20,1.0] \cdot 10^{-6}$ | $3.2 \cdot 10^{-5}$ |
| $\left.\eta_{\tau \tau}=\frac{\left\|\theta_{\tau}\right\|^{2}}{2} \right\rvert\,$ | $8.6 \cdot 10^{-5}$ | $2.1 \cdot 10^{-4}$ | $[0.94,2.8] \cdot 10^{-5}$ | $4.5 \cdot 10^{-5}$ |
| $\operatorname{Tr}[\eta]=\frac{\|\theta\|^{2}}{2}$ | $1.6 \cdot 10^{-4}$ | $2.9 \cdot 10^{-4}$ | $[1.1,4.8] \cdot 10^{-4}$ | $6.0 \cdot 10^{-4}$ |
| $\left\|\eta_{e \mu}\right\|=\frac{\left\|\theta_{e} \theta_{\mu}^{*}\right\|}{2}$ | $8.3 \cdot 10^{-6}$ | $1.2 \cdot 10^{-5}$ | $[0.37,1.0] \cdot 10^{-5}$ | $1.3 \cdot 10^{-5}$ |
| $\left\|\eta_{e \tau}\right\|=\frac{\left\|\theta_{e} \theta_{\tau}^{*}\right\|}{2}$ | $1.5 \cdot 10^{-5}$ | $2.2 \cdot 10^{-5}$ | $[0.25,1.2] \cdot 10^{-4}$ | $1.4 \cdot 10^{-4}$ |
| $\left\|\eta_{\mu \tau}\right\|=\frac{\left\|\theta_{\mu} \theta_{\tau}^{*}\right\|}{2}$ | $7.2 \cdot 10^{-5}$ | $1.3 \cdot 10^{-4}$ | $[0.38,3.0] \cdot 10^{-6}$ | $3.5 \cdot 10^{-5}$ |Non-zero best-fit for IO, unlike NO

## Results for the $\mathbf{3}$ heavy neutrino case

More flexible flavor structure
© Easier to accommodate data and survive cLFV bounds

## Results for the $\mathbf{3}$ heavy neutrino case

( $\sim 10^{-3}$ bounds on $\eta_{e e}, \eta_{\tau \tau}$ and $\sim 10^{-5}$ bound on $\eta_{\mu \mu}$

| 3N-SS | Normal Ordering |  | Inverted Ordering |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $68 \% \mathrm{CL}$ | $95 \% \mathrm{CL}$ | $68 \% \mathrm{CL}$ | $95 \% \mathrm{CL}$ |
| $\eta_{e e}=\frac{\left\|\theta_{e}\right\|^{2}}{2}$ | $[0.28,0.99] \cdot 10^{-3}$ | $1.3 \cdot 10^{-3}$ | $[0.31,1.0] \cdot 10^{-3}$ | $1.4 \cdot 10^{-3}$ |
| $\eta_{\mu \mu}=\frac{\left\|\theta_{\mu}\right\|^{2}}{2}$ | $1.3 \cdot 10^{-7}$ | $1.1 \cdot 10^{-5}$ | $1.2 \cdot 10^{-7}$ | $1.0 \cdot 10^{-5}$ |
| $\eta_{\tau \tau}=\frac{\left\|\theta_{\tau}\right\|^{2}}{2}$ | $[0.3,3.9] \cdot 10^{-4}$ | $1.0 \cdot 10^{-3}$ | $1.7 \cdot 10^{-4}$ | $8.1 \cdot 10^{-4}$ |
| $\operatorname{Tr}[\eta]=\frac{\left\|\theta^{2}\right\|^{2}}{2}$ | $[0.35,1.3] \cdot 10^{-3}$ | $1.9 \cdot 10^{-3}$ | $[0.33,1.0] \cdot 10^{-3}$ | $1.5 \cdot 10^{-3}$ |
| $\left\|\eta_{e \mu}\right\|=\frac{\left\|\theta_{e} \theta_{\mu}^{*}\right\|}{2}$ | $8.5 \cdot 10^{-6}$ | $1.2 \cdot 10^{-5}$ | $8.5 \cdot 10^{-6}$ | $1.2 \cdot 10^{-5}$ |
| $\left\|\eta_{e \tau}\right\|=\frac{\left\|\theta_{e} \theta_{\tau}^{*}\right\|}{2}$ | $[1.3,5.1] \cdot 10^{-4}$ | $9.0 \cdot 10^{-4}$ | $3.3 \cdot 10^{-4}$ | $8.0 \cdot 10^{-4}$ |
| $\left\|\eta_{\mu \tau}\right\|=\frac{\left\|\theta_{\mu} \theta_{\tau}^{*}\right\|}{2}$ | $5.0 \cdot 10^{-6}$ | $5.7 \cdot 10^{-5}$ | $3.8 \cdot 10^{-6}$ | $1.8 \cdot 10^{-5}$ |cLFV in $\mu-e$ sector strongly constrains $\eta_{\mu \mu}$

## Results for arbitrary number of heavies

( $\sim 10^{-3}$ bounds on $\eta_{e e}, \eta_{\tau \tau}$ and $\sim 10^{-4}$ bound on $\eta_{\mu \mu}$



© Physical boundary $\eta_{\alpha \alpha} \geq 0$ induces deviations from Wilks' theorem

## Results for arbitrary number of heavies

© LFC bounds on $\left|\eta_{e \tau}\right|$ and $\left|\eta_{\mu \tau}\right|$ much stronger than the LFV ones

| G-SS | LFC Bound |  | LFV Bound |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $68 \% \mathrm{CL}$ | $95 \% \mathrm{CL}$ | $68 \% \mathrm{CL}$ | $95 \% \mathrm{CL}$ |
| $\eta_{e e}$ | $[0.33,1.0] \cdot 10^{-3}$ | $[0.081,1.4] \cdot 10^{-3}$ | - | - |
| $\eta_{\mu \mu}$ | $1.5 \cdot 10^{-5}$ | $1.4 \cdot 10^{-4}$ | - | - |
| $\eta_{\tau \tau}$ | $1.6 \cdot 10^{-4}$ | $8.9 \cdot 10^{-4}$ | - | - |
| $\operatorname{Tr}[\eta]$ | $[0.28,1.2] \cdot 10^{-3}$ | $2.1 \cdot 10^{-3}$ | - | - |
| $\left\|\eta_{e \mu}\right\|$ | $1.4 \cdot 10^{-4}$ | $3.4 \cdot 10^{-4}$ | $\mathbf{8 . 4} \cdot \mathbf{1 0}^{-\mathbf{6}}$ | $\mathbf{1 . 2 \cdot \mathbf { 1 0 } ^ { - \mathbf { 5 } }}$ |
| $\left\|\eta_{e \tau}\right\|$ | $\mathbf{4 . 2} \cdot \mathbf{1 0}^{-\mathbf{4}}$ | $\mathbf{8 . 8} \cdot \mathbf{1 0}^{-\mathbf{4}}$ | $5.7 \cdot 10^{-3}$ | $8.1 \cdot 10^{-3}$ |
| $\left\|\eta_{\mu \tau}\right\|$ | $\mathbf{9 . 4 \cdot \mathbf { 1 0 } ^ { - \mathbf { 6 } }}$ | $\mathbf{1 . 8} \cdot \mathbf{1 0}^{-\mathbf{4}}$ | $6.6 \cdot 10^{-3}$ | $9.4 \cdot 10^{-3}$ |

## Conclusions

( (Updated) Bounds obtained for different setups ( $2 \mathrm{~N}-\mathrm{SS}, 3 \mathrm{~N}-\mathrm{SS}, \mathrm{G}-\mathrm{SS}$ )
© Bounds substantially change between setups
© Quantified tension between CDF-II $M_{W}$ and other observables: irreconcilableQuantified deviations from Wilks' theorem

## Thanks for your attention!

## Backup

| Observable | SM prediction | Experimental value |  |
| :---: | :---: | :---: | :---: |
| $\begin{gathered} M_{W} \simeq M_{W}^{\mathrm{SM}}\left(1+0.20\left(\eta_{e e}+\eta_{\mu \mu}\right)\right) \\ s_{\mathrm{eff}}^{2 \text { Tev }} \simeq s_{\mathrm{eff}}^{2 \mathrm{SM}}\left(1-1.40\left(\eta_{e e}+\eta_{\mu \mu}\right)\right) \\ s_{\mathrm{eff}}^{2 \mathrm{LHC}} \simeq s_{\mathrm{eff}}^{2 \mathrm{SM}}\left(1-1.40\left(\eta_{e e}+\eta_{\mu \mu}\right)\right) \end{gathered}$ | $\begin{gathered} \hline 80.356(6) \mathrm{GeV} \\ 0.23154(4) \\ 0.23154(4) \end{gathered}$ | $\begin{gathered} \hline 80.373(11) \mathrm{GeV} \\ 0.23148(33) \\ 0.23129(33) \end{gathered}$ | [76] [76] |
| $\begin{gathered} \Gamma_{\text {inv }}^{\mathrm{LHC}} \simeq \Gamma_{\text {inv }}^{\mathrm{SM}}\left(1-0.33\left(\eta_{e e}+\eta_{\mu \mu}\right)-1.33 \eta_{\tau \tau}\right) \\ \Gamma_{Z} \simeq \Gamma_{Z}^{\mathrm{SM}}\left(1+1.08\left(\eta_{e e}+\eta_{\mu \mu}\right)-0.27 \eta_{\tau \tau}\right) \\ \sigma_{\text {had }}^{0} \simeq \sigma_{\text {had }}^{0 \mathrm{SM}}\left(1+0.50\left(\eta_{e e}+\eta_{\mu \mu}\right)+0.53 \eta_{\tau \tau}\right) \\ R_{e} \simeq R_{e}^{\mathrm{SM}}\left(1+0.27\left(\eta_{e e}+\eta_{\mu \mu}\right)\right) \\ R_{\mu} \simeq R_{\mu}^{\mathrm{SM}}\left(1+0.27\left(\eta_{e e}+\eta_{\mu \mu}\right)\right) \\ R_{\tau} \simeq R_{\tau}^{\mathrm{SM}}\left(1+0.27\left(\eta_{e e}+\eta_{\mu \mu}\right)\right) \end{gathered}$ | $\begin{gathered} 0.50145(5) \mathrm{GeV} \\ 2.4939(9) \mathrm{GeV} \\ 41.485(8) \mathrm{nb} \\ 20.733(10) \\ 20.733(10) \\ 20.780(10) \end{gathered}$ | $\begin{gathered} 0.523(16) \mathrm{GeV} \\ 2.4955(23) \mathrm{GeV} \\ 41.481(33) \mathrm{nb} \\ 20.804(50) \\ 20.784(34) \\ 20.764(45) \end{gathered}$ | $[77]$ $[76]$ $[76]$ $[76]$ $[76]$ $[76]$ |
| $\begin{aligned} & R_{\mu e}^{\pi} \simeq\left(1-\left(\eta_{\mu \mu}-\eta_{e e}\right)\right) \\ & R_{\tau \mu}^{\pi} \simeq\left(1-\left(\eta_{\tau \tau}-\eta_{\mu \mu}\right)\right) \\ & R_{\mu e}^{K} \simeq\left(1-\left(\eta_{\mu \mu}-\eta_{e e}\right)\right) \\ & R_{\mu e}^{\tau} \simeq\left(1-\left(\eta_{\mu \mu}-\eta_{e e}\right)\right) \\ & R_{\tau \mu}^{\tau} \simeq\left(1-\left(\eta_{\tau \tau}-\eta_{\mu \mu}\right)\right) \end{aligned}$ | $\begin{aligned} & 1 \\ & 1 \\ & 1 \end{aligned}$ | $\begin{gathered} \hline 1.0010(9) \\ 0.9964(38) \\ 0.9978(18) \\ 1.0018(14) \\ 1.0010(14) \end{gathered}$ | [78] <br> [78] <br> [78] <br> [78] <br> [78] |
| $\begin{aligned} &\left\|V_{u d}^{\beta}\right\| \simeq \sqrt{1-\left\|V_{u s}\right\|^{2}}\left(1+\eta_{\mu \mu}\right) \\ &\left\|V_{u s}^{\tau \rightarrow K \nu}\right\| \simeq\left\|V_{u s}\right\|\left(1+\eta_{e e}+\eta_{\mu \mu}-\eta_{\tau \tau}\right) \\ &\left\|V_{u s}^{\tau \rightarrow K, \pi}\right\| \simeq\left\|V_{u s}\right\|\left(1+\eta_{\mu \mu}\right) \\ &\left\|V_{u s}^{K_{L} \rightarrow \pi e \nu}\right\| \simeq\left\|V_{u s}\right\|\left(1+\eta_{\mu \mu}\right) \\ &\left\|V_{u s}^{K_{L} \rightarrow \pi \mu \nu}\right\| \simeq\left\|V_{u s}\right\|\left(1+\eta_{e e}\right) \\ &\left\|V_{u s}^{K_{s} \rightarrow \pi e \nu}\right\| \simeq\left\|V_{u s}\right\|\left(1+\eta_{\mu \mu}\right) \\ &\left\|V_{u s}^{K_{s} \rightarrow \pi \mu \nu}\right\| \simeq\left\|V_{u s}\right\|\left(1+\eta_{e e}\right) \\ &\left\|V_{u s}^{K^{ \pm} \rightarrow \pi e \nu}\right\| \simeq\left\|V_{u s}\right\|\left(1+\eta_{\mu \mu}\right) \\ &\left\|V_{u s}^{K^{ \pm} \rightarrow \pi \mu \nu}\right\| \simeq\left\|V_{u s}\right\|\left(1+\eta_{e e}\right) \\ &\left\|\frac{V_{u s}}{V_{u d}}\right\|^{K, \pi \rightarrow \mu \nu} \simeq \frac{\left\|V_{u s}\right\|}{\sqrt{1-\left\|V_{u s}\right\|^{2}}} \end{aligned}$ | $\begin{gathered} \hline \sqrt{1-\left\|V_{u s}\right\|^{2}} \\ \left\|V_{u s}\right\| \\ \left\|V_{u s}\right\| \\ \left\|V_{u s}\right\| \\ \left\|V_{u s}\right\| \\ \left\|V_{u s}\right\| \\ \left\|V_{u s}\right\| \\ \left\|V_{u s}\right\| \\ \left\|V_{u s}\right\| \\ \left\|V_{u s}\right\| \\ \sqrt{1-\left\|V_{u s}\right\|^{2}} \end{gathered}$ | $\begin{gathered} 0.97373(31) \\ 0.2236(15) \\ 0.2234(15) \\ 0.2229(6) \\ 0.2234(7) \\ 0.2220(13) \\ 0.2193(48) \\ 0.2239(10) \\ 0.2238(12) \\ 0.23131(53) \end{gathered}$ | $[76]$ $[79]$ $[76]$ $[76]$ $[76]$ $[76]$ $[76]$ $[76]$ $[76]$ $[76]$ |

## Backup



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