# (Updated) Global bounds on heavy neutrino mixing

#### Based on:

M. Blennow, E. Fernández Martínez, J. Hernández-García, J. López-Pavón, X. Marcano, DN [2306.01040]







#### Motivation

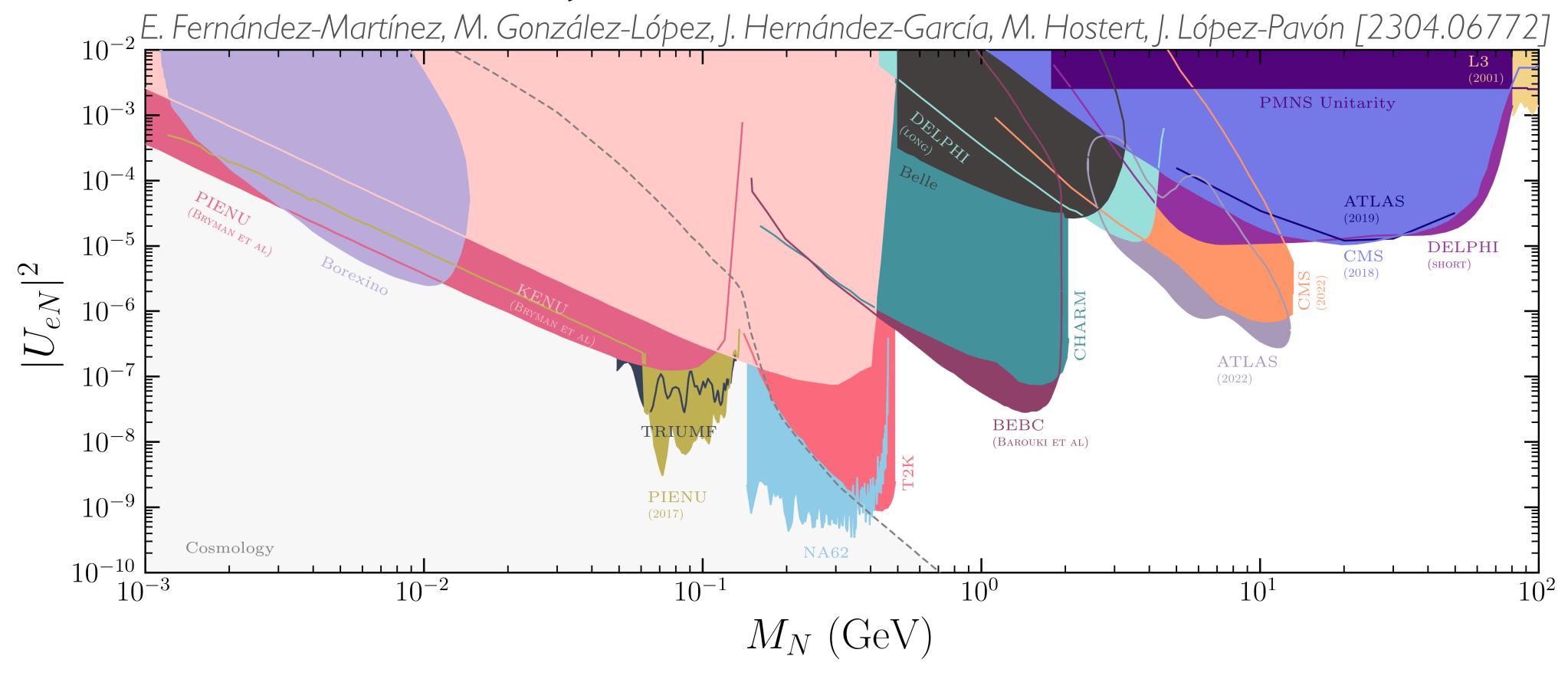
Neutrinos are massive — heed a mechanism to generate their (tiny) masses



Seesaw mechanism via heavy neutrinos

#### Searches for heavy neutrinos

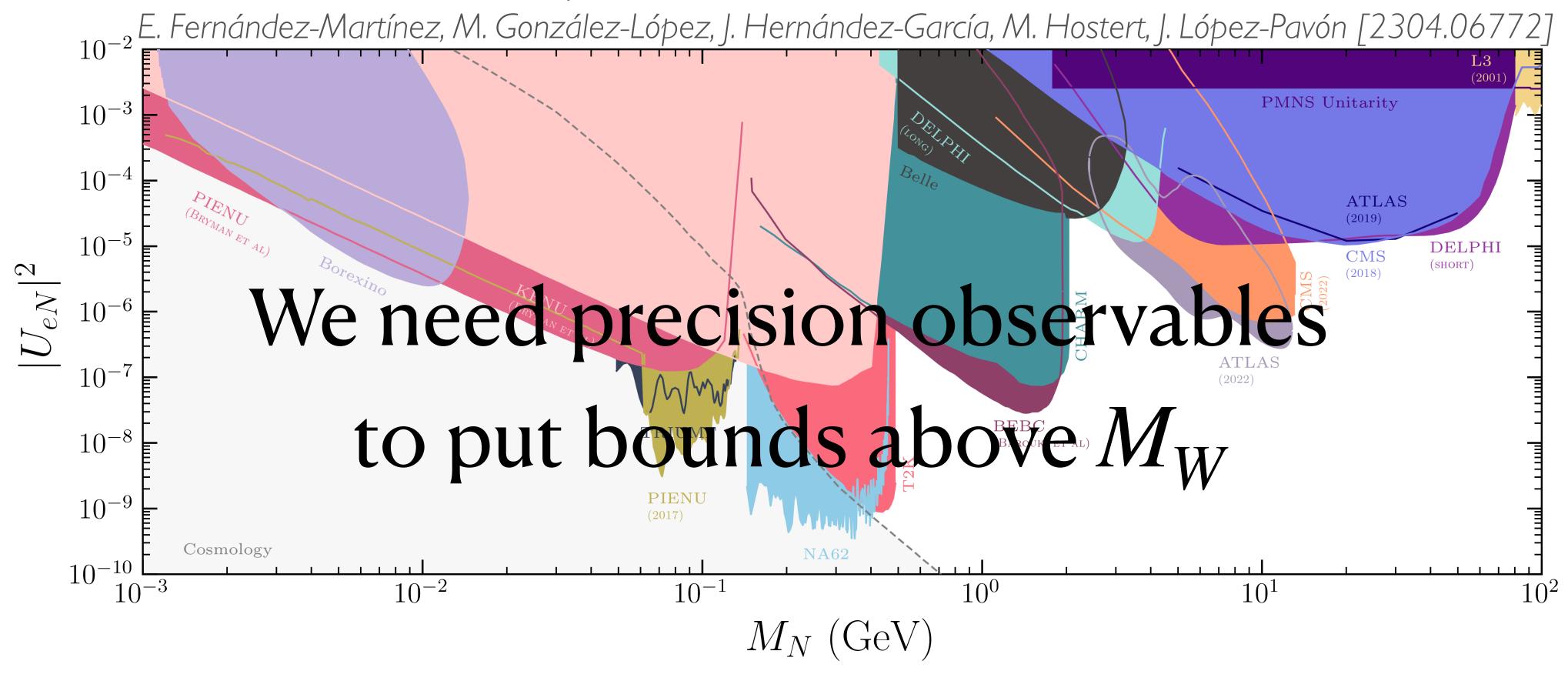
Plethora of searches for heavy neutrinos



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#### Why update the global fit?

- Updates on key observables:
  - $\star$  New measurements of  $M_W$  (CDF-II, ATLAS)
  - $\star$  Anomaly (  $\sim 2-3\sigma$ ) in the extraction of CKM elements  $|V_{ud}|$  and  $|V_{us}|$
  - $\star$  LEP anomaly (  $\sim 2\sigma$ ) in  $N_{\nu}$  is now gone

- Improvement of the analysis:
  - \* Correlations
  - ★ Deviations from Wilks' theorem: Bootstrapping

#### Non-unitarity in general

Precision observables are modified by leptonic non-unitarity

In general:

$$N = (1 - \eta) \underline{U}, \quad \eta^{\dagger} = \eta$$
Diagonalises  $m_{\nu}$ 

 $\bigcirc$  Convenient:  $\eta$  has flavor indices

© Generally, mass eigenstates are summed over:

$$\sum_{i=1}^{3} (N)_{\alpha i} (N^{\dagger})_{i\beta} = \delta_{\alpha\beta} - 2\eta_{\alpha\beta} + O(\eta^{2})$$

#### Heavy neutrinos and non-unitarity

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Diagonalises  $m_{\nu}$ 

○ In the context of heavy neutrinos  $N \sim (1,1,0)$ :  $-\mathcal{L} \supset Y_{\nu} \overline{L}_{L} \tilde{H} N + \frac{1}{2} M_{M} \overline{N^{c}} N$ 

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$$\Theta \equiv \frac{v}{\sqrt{2}} Y_{\nu} M_M^{-1}$$

$$V = \begin{pmatrix} 1 - \frac{1}{2}\Theta\Theta^{\dagger} & \Theta \\ -\Theta^{\dagger} & 1 - \frac{1}{2}\Theta^{\dagger}\Theta \end{pmatrix} \begin{pmatrix} U & 0 \\ 0 & U' \end{pmatrix}$$

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#### Heavy neutrinos and non-unitarity

• In the Type-I seesaw:

$$N = \left(1 - \frac{1}{2}\Theta\Theta^{\dagger}\right)U,$$

$$\frac{1}{2} \Theta \Theta^{\dagger}$$
Mass-independent

O Additionally:  $m_{\nu} \simeq -\Theta M_M \Theta^T$  can impose correlations within  $\eta$ 

#### Precision observables and non-unitarity

$$\odot$$
 SM inputs  $egin{array}{c} lpha \ M_Z \ G_F \ \end{array}$ 

 $\bigcirc$   $G_F$  is extracted from  $\mu$ -decay  $\longrightarrow$  Modified by lepton non-unitarity

$$\Gamma_{\mu} = \frac{G_F^2 m_{\mu}^5}{192\pi^3} \sum_{i=1}^3 \left| N_{\mu i} \right|^2 \sum_{j=1}^3 \left| N_{ej} \right|^2 \simeq \frac{G_F^2 m_{\mu}^5}{192\pi^3} \left( 1 - 2\eta_{ee} - 2\eta_{\mu\mu} \right) \equiv \frac{G_{\mu}^2 m_{\mu}^5}{192\pi^3},$$

$$G_F \simeq G_\mu \left( 1 + \eta_{ee} + \eta_{\mu\mu} \right)$$
 Modifies all EWPO

## $M_W$ and $s_{eff}^2$

 $\odot$  We consider only tree-level  $\eta$ -dependence and loop-level SM corrections

$$M_{W} = M_{Z} \sqrt{\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\pi \alpha \left(1 + \eta_{ee} + \eta_{\mu\mu}\right)}{\sqrt{2} G_{\mu} M_{Z}^{2} (1 - \Delta r)}}},$$

 $\bullet$  Similarly with  $s_{eff}^2$ 

#### Z-pole observables

Z-boson partial widths also modified

 $lackbox{}{ullet}$   $\Gamma_{inv}$  modified by  $G_F$  and by  $Z \rightarrow \nu \nu$  vertex

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#### Z-pole observables

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lacksquare  $\Gamma_{inv}$  modified by  $G_F$  and by  $Z \to \nu \nu$  vertex

LEP precision measurements also constrain  $\eta$ 

Z-pole observables are in strong tension with CDF-II  $M_W$  measurement

#### Lepton Flavor Universality (LFU)

 $\bigcirc W \rightarrow l\nu$  vertex modified by  $\eta$ 

$$\sum_{i=1}^{3} |N_{\alpha i}|^2 = 1 - 2\eta_{\alpha \alpha}$$

Weak interactions are no longer flavor universal

 $\bigcirc$  Ratios of  $\pi$ , K and  $\tau$  decays constrain the universality of weak interactions

#### CKM unitarity

• The CKM matrix remains unitary:

$$1 = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2,$$

$$|V_{ud}| = \sqrt{1 - |V_{us}|^2},$$

 $\bigcirc$  But the extraction of  $|V_{ud}|$  and  $|V_{us}|$  is affected

Nuisance parameter (minimized over)

 $|V_{ud}|$  extracted from superallowed  $\beta$ -decays

$$|V_{ud}^{\beta}| = |V_{ud}| \left(1 + \eta_{ee} + \eta_{\mu\mu}\right) \left(1 - \eta_{ee}\right) \simeq \sqrt{1 - |V_{us}|^2} \left(1 + \eta_{\mu\mu}\right)$$

$$G_F \qquad W \to e\nu$$
vertex

#### CKM unitarity

 $|V_{us}|$  extracted from K and  $\tau$  semileptonic decays

© Cabibbo anomaly:  $|V_{ud}| < \sqrt{1 - |V_{us}|^2}$  at  $2 - 3\sigma$  level

Only worsened in presence of  $\eta_{\mu\mu} > 0$ 

#### Charged Lepton Flavor Violation (cLFV)

lacksquare Previous observables are LFC: depend on  $\eta_{\alpha\alpha}$ 

• The off-diagonal elements  $\eta_{\alpha\beta}$  induces LFV processes  $\begin{cases} l_{\alpha} \to l_{\beta}\gamma \\ l_{\alpha} \to l_{\beta}l_{\beta}l_{\beta} \\ \mu - e \end{cases}$ 

For example: 
$$BR\left(l_{\alpha} \to l_{\beta}\gamma\right) \simeq \frac{3\alpha}{2\pi} |\eta_{\alpha\beta}|^2$$
, for  $M_N \gg M_W$ 

The off-diagonal elements alternatively constrained via the Schwarz inequality:

$$|\eta_{\alpha\beta}| \leq \sqrt{\eta_{\alpha\alpha}\eta_{\beta\beta}}$$

#### The preference of the data

•  $M_W$ ,  $s_{eff}^2$  showcase  $\sim 1-2\sigma$  preference for  $\eta_{ee}+\eta_{\mu\mu}>0$ 

• LFU prefers  $\eta_{ee} > \eta_{\mu\mu}$  at  $\sim 1\sigma$ 

 $\bigcirc$  Cabibbo anomaly disfavors  $\eta_{\mu\mu} > 0$ 

Observables constraining  $\eta_{\tau\tau}$  show good agreement with SM

O Summing up: data prefers  $\eta_{ee} > 0$ ,  $\eta_{\mu\mu} = 0$ ,  $\eta_{\tau\tau} = 0$ 

#### Cases under study

Minimal scenario with 2 heavy neutrinos: 2N-SS (Previously missing in the literature)

Next-to-minimal scenario with 3 heavy neutrinos: 3N-SS

© General scenario with arbitrary number of heavy neutrinos: G-SS

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- ★ Correlations from  $m_{\nu}$ ★  $|\eta_{\alpha\beta}| = \sqrt{\eta_{\alpha\alpha}\eta_{\beta\beta}}$ ★ LFV with LFC

General scenario with arbitrary number of heavy neutrinos: G-SS

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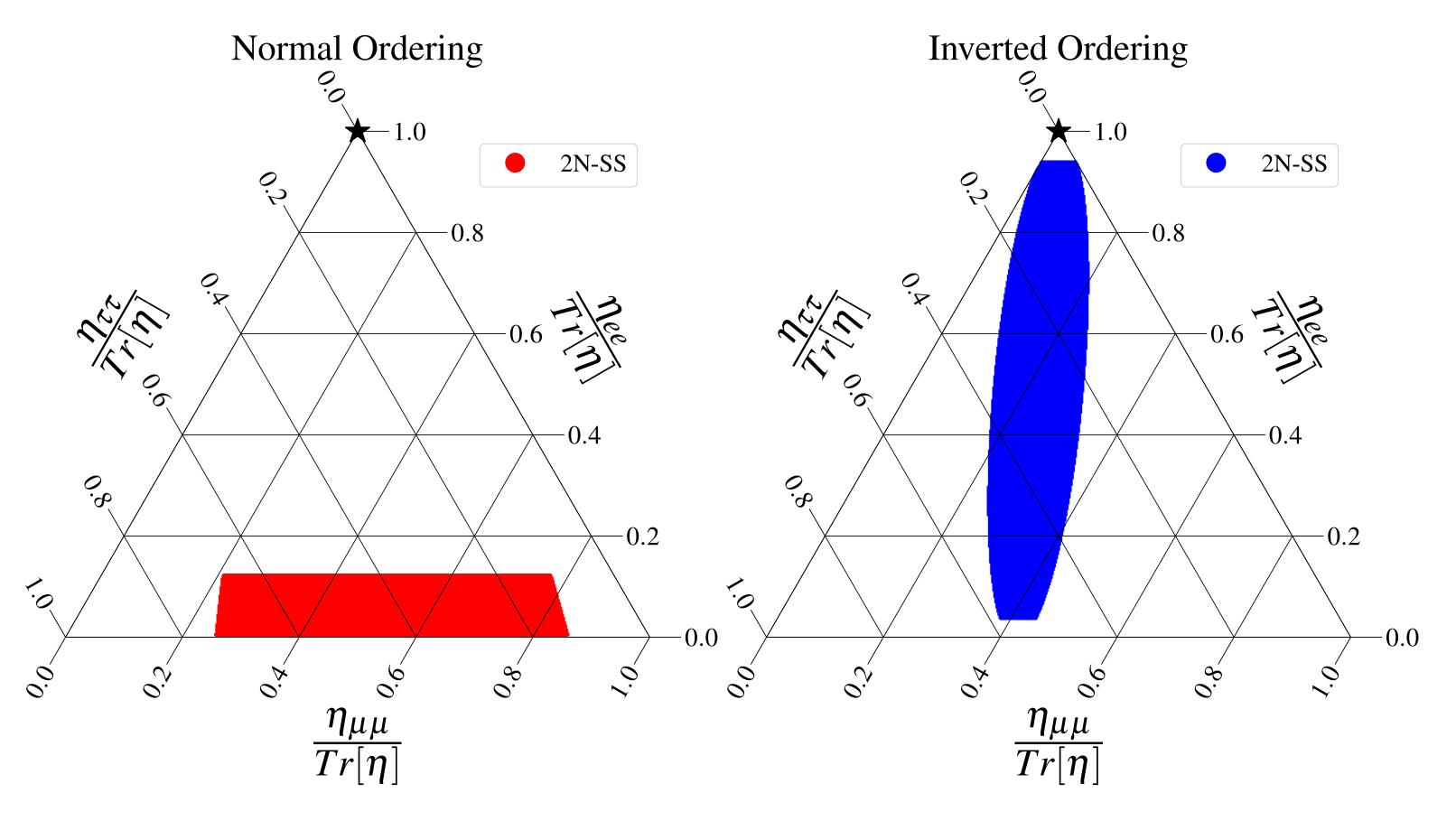
Next-to-minimal scenario with 3 heavy neutrinos: 3N-SS

- ★ Correlations from  $m_{\nu}$ ★  $|\eta_{\alpha\beta}| = \sqrt{\eta_{\alpha\alpha}\eta_{\beta\beta}}$ ★ LFV with LFC

- © General scenario with arbitrary number of heavy neutrinos: G-SS
  - $\star$   $\eta_{ee}$ ,  $\eta_{\mu\mu}$  and  $\eta_{\tau\tau}$  independent
  - $+ |\eta_{\alpha\beta}| \leq \sqrt{\eta_{\alpha\alpha}\eta_{\beta\beta}}$
  - ★ LFV decoupled from LFC

#### Results for the 2 heavy neutrino case

Very restrictive flavor structure



cLFV bounds play a very important role

#### Results for the 2 heavy neutrino case

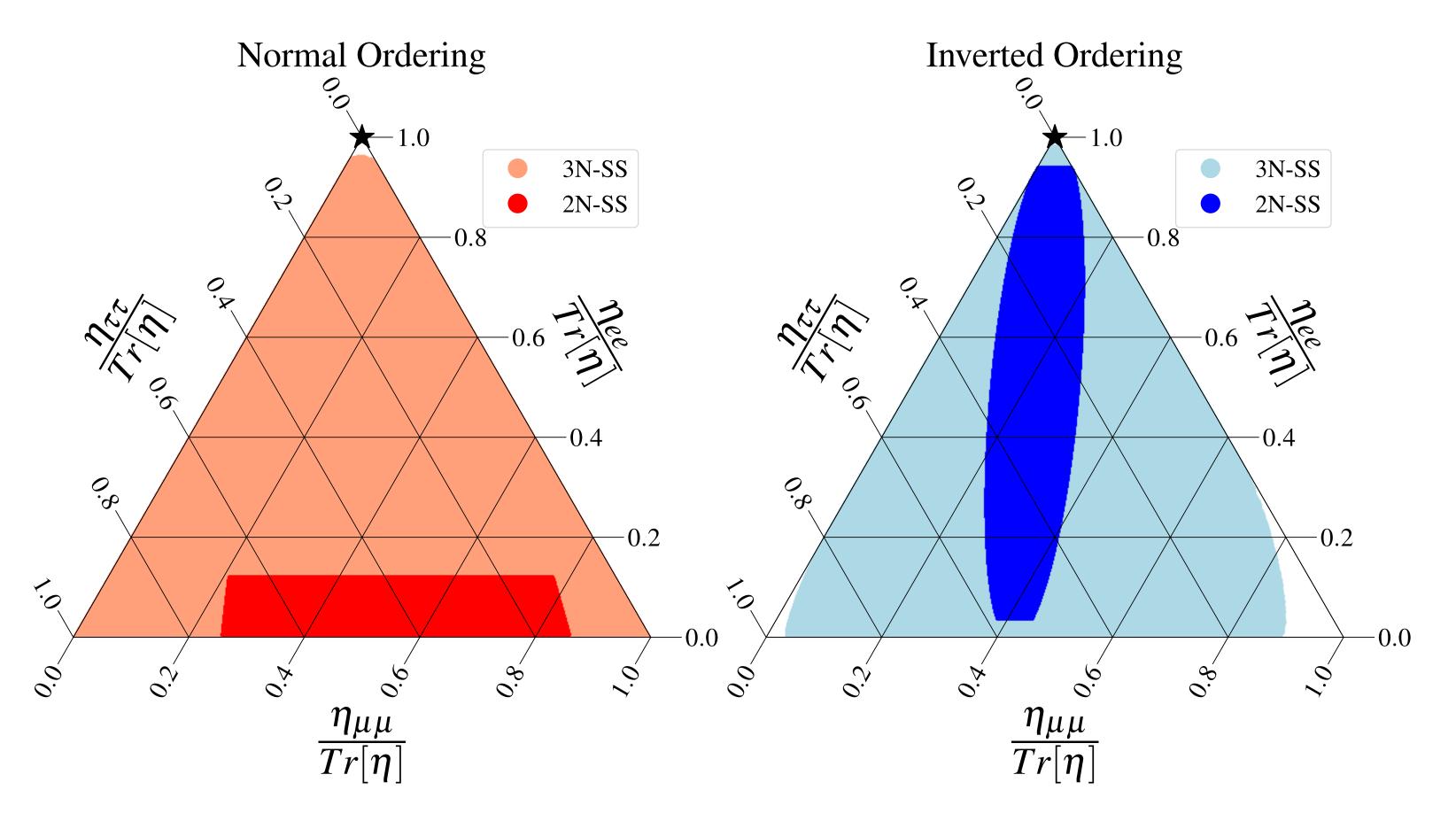
 $\odot$  Stringent bounds  $\sim 10^{-5} - 10^{-4}$ 

2N-SS	Normal Ordering		Inverted Ordering	
211-55	$68\%\mathrm{CL}$	$95\%\mathrm{CL}$	68%CL	$95\%\mathrm{CL}$
$\eta_{ee} = rac{  heta_e ^2}{2}$	$6.4\cdot10^{-6}$	$9.4\cdot10^{-6}$	$[0.98, 4.4] \cdot 10^{-4}$	$5.5\cdot 10^{-4}$
$\eta_{\mu\mu}=rac{\leftert heta_{\mu} ightert^{2}}{2}$	$6.9\cdot10^{-5}$	$1.3\cdot 10^{-4}$	$[0.20, 1.0] \cdot 10^{-6}$	$3.2\cdot 10^{-5}$
$\eta_{ au au}=rac{\left  heta_{ au} ight ^{2}}{2}$	$8.6\cdot10^{-5}$	$2.1\cdot 10^{-4}$	$[0.94, 2.8] \cdot 10^{-5}$	$4.5\cdot 10^{-5}$
$ ext{Tr}\left[\eta ight] = rac{\left  heta ight ^2}{2}$	$1.6\cdot 10^{-4}$	$2.9\cdot 10^{-4}$	$[1.1, 4.8] \cdot 10^{-4}$	$6.0\cdot 10^{-4}$
$ \eta_{e\mu} =rac{\left  heta_e^{-}\overline{ heta}_\mu^* ight }{2}$	$8.3\cdot 10^{-6}$		$[0.37, 1.0] \cdot 10^{-5}$	
$ \eta_{e au} =rac{  heta_e heta_ au^* }{2}$	$1.5 \cdot 10^{-5}$	$2.2\cdot 10^{-5}$	$ \left  \ [0.25, 1.2] \cdot 10^{-4} \right  $	$\begin{array}{ c c c c c }\hline 1.4\cdot 10^{-4} \end{array}$
$ \eta_{\mu au} =rac{  heta_{\mu} heta_{ au}^{*} }{2}$	$7.2 \cdot 10^{-5}$	$1.3\cdot 10^{-4}$	$\begin{bmatrix} 0.25, 1.2 \end{bmatrix} \cdot 10^{-4}$ $[0.38, 3.0] \cdot 10^{-6}$	$3.5\cdot 10^{-5}$

Non-zero best-fit for IO, unlike NO

#### Results for the 3 heavy neutrino case

More flexible flavor structure



Easier to accommodate data and survive cLFV bounds

#### Results for the 3 heavy neutrino case

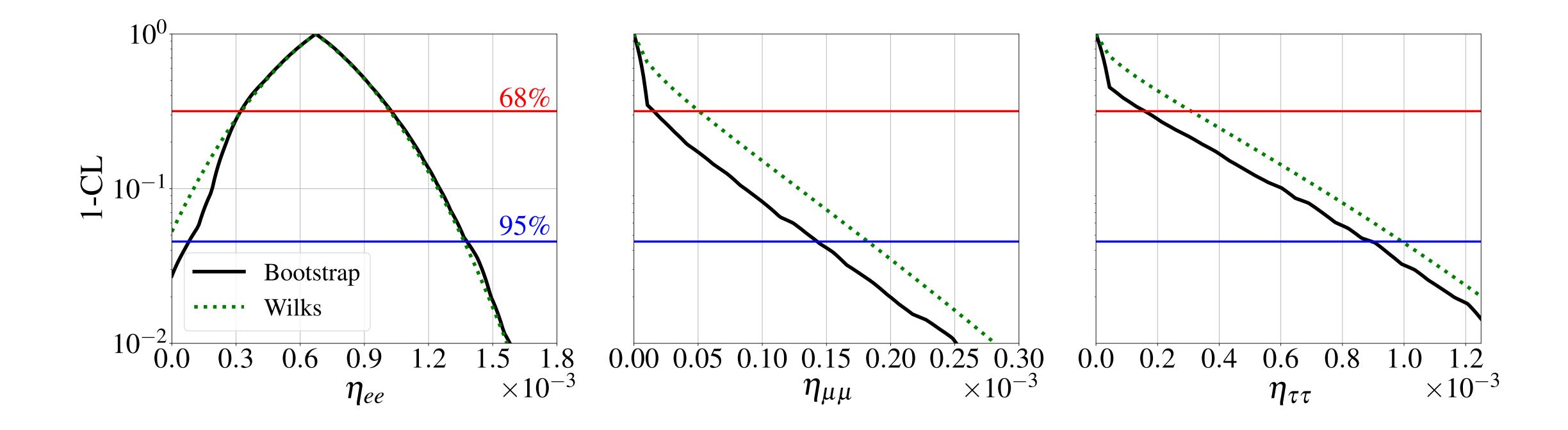
 $\bullet$  ~  $10^{-3}$  bounds on  $\eta_{ee}$ ,  $\eta_{\tau\tau}$  and ~  $10^{-5}$  bound on  $\eta_{\mu\mu}$ 

3N-SS	Normal Orde	ering	Inverted Ordering		
	68%CL	$95\%\mathrm{CL}$	68%CL	$95\%\mathrm{CL}$	
$\eta_{ee} = rac{  heta_e ^2}{2}$	$[0.28, 0.99] \cdot 10^{-3}$	$1.3\cdot 10^{-3}$	$[0.31, 1.0] \cdot 10^{-3}$	$1.4\cdot 10^{-3}$	
$\eta_{\mu\mu}=rac{ ec{ heta_{\mu}} ^2}{2}$	$1.3\cdot 10^{-7}$	$1.1\cdot 10^{-5}$	$1.2\cdot 10^{-7}$	$1.0\cdot 10^{-5}$	
$\left  \eta_{ au au} = rac{  heta_{ au} ^2}{2}  ight $	$[0.3, 3.9] \cdot 10^{-4}$	$1.0\cdot 10^{-3}$	$1.7\cdot 10^{-4}$	$8.1\cdot 10^{-4}$	
$\mathrm{Tr}\left[\eta ight]=rac{\left  heta ight ^{2}}{2}$	$[0.35, 1.3] \cdot 10^{-3}$	$1.9\cdot 10^{-3}$	$[0.33, 1.0] \cdot 10^{-3}$	$1.5\cdot 10^{-3}$	
$\left  \  \eta_{e\mu}  = rac{\left   heta_e  ilde{ heta}_\mu^*  ight }{2}  ight $	$8.5\cdot 10^{-6}$	$1.2\cdot 10^{-5}$	$8.5 \cdot 10^{-6}$	$1.2\cdot 10^{-5}$	
$ \eta_{e au} =rac{  heta_e heta_ au^* }{2}$	$[1.3, 5.1] \cdot 10^{-4}$	$9.0\cdot 10^{-4}$	$3.3\cdot 10^{-4}$	$8.0\cdot 10^{-4}$	
$ \eta_{\mu au} =rac{  heta_{\mu} heta_{ au}^* }{2}$	$5.0\cdot 10^{-6}$	$5.7\cdot 10^{-5}$	$3.8\cdot 10^{-6}$	$1.8\cdot 10^{-5}$	

• cLFV in  $\mu - e$  sector strongly constrains  $\eta_{\mu\mu}$ 

#### Results for arbitrary number of heavies

 $\bullet$  ~  $10^{-3}$  bounds on  $\eta_{ee}$ ,  $\eta_{\tau\tau}$  and ~  $10^{-4}$  bound on  $\eta_{\mu\mu}$ 



• Physical boundary  $\eta_{\alpha\alpha} \ge 0$  induces deviations from Wilks' theorem

#### Results for arbitrary number of heavies

• LFC bounds on  $|\eta_{e\tau}|$  and  $|\eta_{\mu\tau}|$  much stronger than the LFV ones

G-SS	LFC Bound		LFV Bound	
G-55	$68\%\mathrm{CL}$	$95\%\mathrm{CL}$	68%CL	$95\%\mathrm{CL}$
$\eta_{ee}$	$[0.33, 1.0] \cdot 10^{-3}$	$[0.081, 1.4] \cdot 10^{-3}$	_	_
$\eta_{\mu\mu}$	$1.5\cdot 10^{-5}$	$1.4\cdot 10^{-4}$	_	_
$\eta_{ au au}$	$1.6 \cdot 10^{-4}$	$8.9 \cdot 10^{-4}$	_	_
${ m Tr}\left[\eta ight]$	$[0.28, 1.2] \cdot 10^{-3}$	$2.1\cdot 10^{-3}$	_	_
$ \eta_{e\mu} $	$1.4\cdot 10^{-4}$	$3.4\cdot 10^{-4}$	$8.4\cdot 10^{-6}$	$\mathbf{1.2\cdot 10^{-5}}$
$ \eta_{e au} $	$\mathbf{4.2\cdot 10^{-4}}$	$\mathbf{8.8\cdot 10^{-4}}$	$5.7 \cdot 10^{-3}$	$8.1 \cdot 10^{-3}$
$ \eta_{\mu au} $	$\mathbf{9.4\cdot 10^{-6}}$	$1.8\cdot 10^{-4}$	$6.6 \cdot 10^{-3}$	$9.4 \cdot 10^{-3}$

#### Conclusions

(Updated) Bounds obtained for different setups (2N-SS, 3N-SS, G-SS)

Bounds substantially change between setups

 $\bigcirc$  Quantified tension between CDF-II  $M_W$  and other observables: irreconcilable

Quantified deviations from Wilks' theorem

#### Thanks for your attention!

Observable	SM prediction	Experimental value	
$M_W \simeq M_W^{\rm SM} (1 + 0.20 (\eta_{ee} + \eta_{\mu\mu}))$	80.356(6) GeV	80.373(11) GeV	-
$s_{ m eff}^{2 { m Tev}} \simeq s_{ m eff}^{2 { m SM}} \left(1 - 1.40 \left(\eta_{ee} + \eta_{\mu\mu}\right)\right)$	0.23154(4)	0.23148(33)	[76]
$s_{\rm eff}^{2 \ \rm LHC} \simeq s_{\rm eff}^{2 \ \rm SM} \left(1 - 1.40 \left(\eta_{ee} + \eta_{\mu\mu}\right)\right)$	0.23154(4)	0.23129(33)	[76]
$\Gamma_{ m inv}^{ m LHC} \simeq \Gamma_{ m inv}^{ m SM} \left(1 - 0.33 \left(\eta_{ee} + \eta_{\mu\mu}\right) - 1.33 \eta_{ au au} ight)$	0.50145(5)  GeV	0.523(16)  GeV	[77]
$\Gamma_Z \simeq \Gamma_Z^{ m SM} \left( 1 + 1.08 \left( \eta_{ee} + \eta_{\mu\mu} \right) - 0.27 \eta_{ au au}  ight)$	2.4939(9)  GeV	2.4955(23)  GeV	[76]
$\sigma_{ m had}^0 \simeq \sigma_{ m had}^{0~{ m SM}} \left( 1 + 0.50 \left( \eta_{ee} + \eta_{\mu\mu} \right) + 0.53 \eta_{ au au} \right)$	41.485(8) nb	41.481(33) nb	[76]
$R_e \simeq R_e^{\rm SM} (1 + 0.27 (\eta_{ee} + \eta_{\mu\mu}))$	20.733(10)	20.804(50)	[76]
$R_{\mu} \simeq R_{\mu}^{ m SM} \left( 1 + 0.27 \left( \eta_{ee} + \eta_{\mu\mu} \right) \right)$	20.733(10)	20.784(34)	[76]
$R_{\tau} \simeq R_{\tau}^{\rm SM} \left( 1 + 0.27 \left( \eta_{ee} + \eta_{\mu\mu} \right) \right)$	20.780(10)	20.764(45)	[76]
$R^{\pi}_{\mu e} \simeq (1 - (\eta_{\mu\mu} - \eta_{ee}))$	1	1.0010(9)	[78]
$R^{\pi}_{ au\mu} \simeq (1 - (\eta_{ au au} - \eta_{\mu\mu}))$	1	0.9964(38)	[78]
$R_{\mu e}^K \simeq \left(1 - \left(\eta_{\mu\mu} - \eta_{ee}\right)\right)$	1	0.9978(18)	[78]
$R^{ au}_{\mu e} \simeq \left(1 - \left(\eta_{\mu\mu} - \eta_{ee} ight) ight)$	1	1.0018(14)	[78]
$R^{ au}_{ au\mu} \simeq (1 - (\eta_{ au au} - \eta_{\mu\mu}))$	1	1.0010(14)	[78]
$\left V_{ud}^{eta}\right  \simeq \sqrt{1-\left V_{us}\right ^2} \left(1+\eta_{\mu\mu} ight)$	$\sqrt{1-\left V_{us} ight ^{2}}$	0.97373(31)	[76]
$\left V_{us}^{ au  o K u}\right  \simeq \left V_{us}\right  \left(1 + \eta_{ee} + \eta_{\mu\mu} - \eta_{ au au}\right)$	$ V_{us} $	0.2236(15)	[79]
$\left V_{us}^{\tau \to K,\pi}\right  \simeq \left V_{us}\right  (1 + \eta_{\mu\mu})$	$ V_{us} $	0.2234(15)	[76]
$\left V_{us}^{K_L \to \pi e \nu}\right  \simeq \left V_{us}\right  (1 + \eta_{\mu\mu})$	$ V_{us} $	0.2229(6)	[76]
$\left V_{us}^{K_L \to \pi \mu \nu}\right  \simeq \left V_{us}\right  (1 + \eta_{ee})$	$ V_{us} $	0.2234(7)	[76]
$\left V_{us}^{K_S \to \pi e \nu}\right  \simeq \left V_{us}\right  (1 + \eta_{\mu\mu})$	$ V_{us} $	0.2220(13)	[76]
$\left V_{us}^{K_S  o \pi \mu  u}\right  \simeq \left V_{us}\right  (1 + \eta_{ee})$	$ V_{us} $	0.2193(48)	[76]
$\left V_{us}^{K^{\pm}  o \pi e  u}\right  \simeq \left V_{us}\right  (1 + \eta_{\mu\mu})$	$ V_{us} $	0.2239(10)	[76]
$\left V_{us}^{K^{\pm}  ightarrow \pi \mu  u}\right  \simeq \left V_{us}\right  (1 + \eta_{ee})$	$ V_{us} $	0.2238(12)	[76]
$\left  \frac{V_{us}}{V_{ud}} \right ^{K,\pi  o \mu  u} \simeq \frac{ V_{us} }{\sqrt{1 -  V_{us} ^2}}$	$\frac{ V_{us} }{\sqrt{1-\left V_{us}\right ^2}}$	0.23131(53)	[76]

