

# $(p - 1)$ -Bracket for D $p$ -branes in Large R-R Field Background

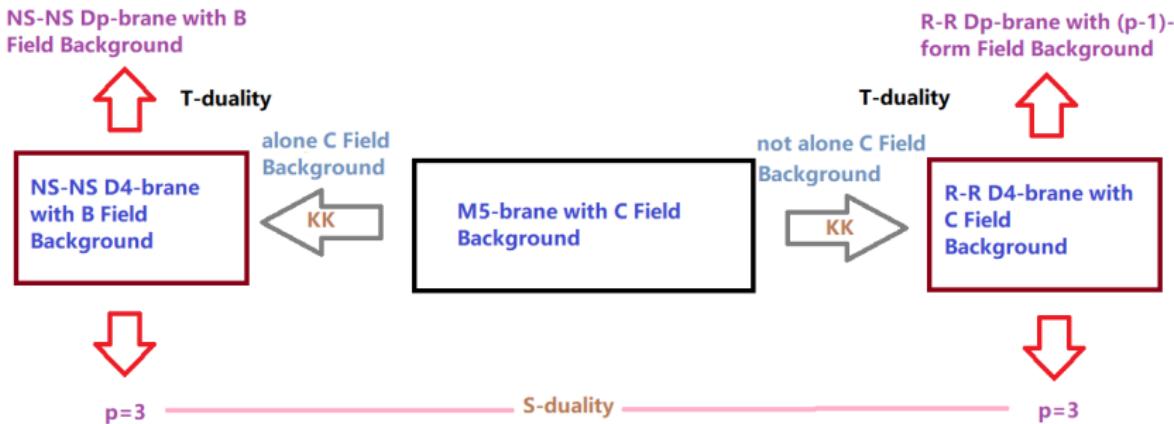
Chen-Te Ma (APCTP)

Pei-Ming Ho (NTU)  
JHEP 07 (2023) 002;  
JHEP 05 (2021) 081; JHEP 11 (2014) 142; JHEP 05 (2013) 056

July 18, 2023

# Duality

[Ho, Imamura, Matsuo, and Shiba 2008; Ho and Yeh 2011; Ho and Ma 2013; Ho and Ma 2014]



# Large $(p - 1)$ -Form Field Background

- Lagrangian [Ho and Ma 2013]

$$\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2,$$

where

$$\mathcal{L}_1 \equiv -\frac{1}{2}(\mathcal{D}_\alpha X^I)^2 + \frac{1}{2g}\epsilon^{\alpha\beta}\mathcal{F}_{\alpha\beta} + \frac{1}{2}\mathcal{F}_{\alpha\dot{\mu}}^2$$

and

$$\begin{aligned}\mathcal{L}_2 &\equiv -\frac{1}{2} \sum_{n,m,l \in S} \frac{g^{2(p-2-m)}}{(n!)(m!)^2(l!)} \\ &\quad \{X^{\dot{\mu}_1}, \dots, X^{\dot{\mu}_n}, a_{\dot{\nu}_1}, \dots, a_{\dot{\nu}_m}, y^{\dot{\nu}_1}, \dots, y^{\dot{\nu}_m}, X^h, \dots, X^{l_I}\}^2 \\ S &\equiv \{(n, m, l) \mid n, m, l \geq 0 ; n + 2m + l = p - 1\}\end{aligned}$$

# Gauge Symmetry

- gauge symmetry, VPD,  $\delta y^{\dot{\mu}} = \hat{\kappa}^{\dot{\mu}}$  preserves the **volume-form**  
 $dy^{\dot{1}} dy^{\dot{2}} \cdots dy^{\dot{\mu}_{p-1}}$  if  $\partial_{\dot{\mu}} \hat{\kappa}^{\dot{\mu}} = 0$
- VPD **covariant**

$$\hat{\delta}_{\hat{\Lambda}} \Phi = \hat{\kappa}^{\dot{\mu}} \partial_{\dot{\mu}} \Phi \quad (1)$$

- Field Content for R-R D-branes:  $\hat{b}$ ,  $\hat{a}$
- gauge transformation is

$$\begin{aligned} \hat{\delta}_{\hat{\Lambda}} \hat{b}^{\dot{\mu}} &= \hat{\kappa}^{\dot{\mu}} + i[\hat{\lambda}, \hat{b}^{\dot{\mu}}] + g \hat{\kappa}^{\dot{\nu}} \partial_{\dot{\nu}} \hat{b}^{\dot{\mu}}; \\ \hat{\delta}_{\hat{\Lambda}} \hat{a}_{\dot{\mu}} &= \partial_{\dot{\mu}} \hat{\lambda} + i[\hat{\lambda}, \hat{a}_{\dot{\mu}}] + g(\hat{\kappa}^{\dot{\nu}} \partial_{\dot{\nu}} \hat{a}_{\dot{\mu}} + \hat{a}_{\dot{\nu}} \partial_{\dot{\mu}} \hat{\kappa}^{\dot{\nu}}); \\ \hat{\delta}_{\hat{\Lambda}} \hat{a}_{\alpha} &= \partial_{\alpha} \hat{\lambda} + i[\hat{\lambda}, \hat{a}_{\alpha}] + g(\hat{\kappa}^{\dot{\nu}} \partial_{\dot{\nu}} \hat{a}_{\alpha} + \hat{a}_{\dot{\nu}} \partial_{\alpha} \hat{\kappa}^{\dot{\nu}}), \end{aligned} \quad (2)$$

where  $g = 1/C_{\dot{\mu}_1 \cdots \dot{\mu}_{p-1}}$

- R-R field background: dominant in the U(1) sector

## ( $p - 1$ )-Bracket

- **( $p - 1$ )-Bracket**  $\{f_1, f_2, \dots, f_{p-1}\}_{(p-1)}$  ( $\partial_{\dot{\mu}} \rightarrow \mathcal{D}_{\dot{\mu}}$ )

$$\epsilon^{\dot{\mu}_1 \dot{\mu}_2 \dots \dot{\mu}_{p-1}} (\mathcal{D}_{\dot{\mu}_1} f_1) (\mathcal{D}_{\dot{\mu}_2} f_2) \dots (\mathcal{D}_{\dot{\mu}_{p-1}} f_{p-1}), \quad (3)$$

where

$$\mathcal{D}_{\dot{\mu}} \hat{X}^{\dot{\nu}} \equiv \partial_{\dot{\mu}} \hat{X}^{\dot{\nu}} - i[\hat{a}_{\dot{\mu}}, \hat{X}^{\dot{\nu}}] \equiv D_{\dot{\mu}} \hat{X}^{\dot{\nu}}; \quad \mathcal{D}_{\dot{\mu}} \hat{a}_{\dot{\nu}} \equiv (\partial_{\dot{\mu}} - i\hat{a}_{\dot{\mu}}) \hat{a}_{\dot{\nu}}$$

- R-R D9-Branes  $\mathcal{L}_2^{(p=9)}$

$$\sim \text{Str} \left[ \left( \left\{ \hat{X}^{\dot{\mu}_1}, \dots, \hat{X}^{\dot{\mu}_n}, \hat{a}_{\dot{\nu}_1}, \dots, \hat{a}_{\dot{\nu}_m}, y^{\dot{\nu}_1}, \dots, y^{\dot{\nu}_m} \right\}_{(p-1)} \right)^2 \right]$$

# Covariant Field Strength



$$\hat{\mathcal{H}}^{\dot{\mu}_1 \dot{\mu}_2 \cdots \dot{\mu}_{p-1}} \equiv g^{p-2} \{ \hat{X}^{\dot{\mu}_1}, \hat{X}^{\dot{\mu}_2}, \dots, \hat{X}^{\dot{\mu}_{p-1}} \}_{(p-1)} - \frac{1}{g} \epsilon^{\dot{\mu}_1 \dot{\mu}_2 \cdots \dot{\mu}_{p-1}},$$

$$\hat{F}_{\dot{\mu}\dot{\nu}} = \frac{g^{p-3}}{(p-3)!} \epsilon_{\dot{\mu}\dot{\nu}\dot{\mu}_1 \cdots \dot{\mu}_{p-3}} \{ \hat{X}^{\dot{\mu}_1}, \dots, \hat{X}^{\dot{\mu}_{p-3}}, \hat{a}_{\dot{\rho}}, \hat{y}^{\dot{\rho}} \};$$

$$\hat{F}_{\alpha\dot{\mu}} \equiv (\hat{V}^{-1})_{\dot{\mu}}{}^{\dot{\nu}} (\hat{F}_{\alpha\dot{\nu}} + g \hat{F}_{\dot{\nu}\dot{\delta}} \hat{B}_{\alpha}{}^{\dot{\delta}});$$

$$\begin{aligned} \hat{F}_{\alpha\beta} &\equiv \hat{F}_{\alpha\dot{\mu}} \hat{B}_{\beta}{}^{\dot{\mu}} + \hat{F}_{\dot{\mu}\beta} \hat{B}_{\alpha}{}^{\dot{\mu}} \\ &+ \frac{g^2}{2} \hat{F}_{\dot{\mu}\dot{\nu}} (\hat{B}_{\alpha}{}^{\dot{\mu}} \hat{B}_{\beta}{}^{\dot{\nu}} + \hat{B}_{\beta}{}^{\dot{\mu}} \hat{B}_{\alpha}{}^{\dot{\nu}}), \end{aligned}$$

where

$$\hat{V}_{\dot{\nu}}{}^{\dot{\mu}} \equiv \delta_{\dot{\nu}}{}^{\dot{\mu}} + g D_{\dot{\nu}} \hat{b}^{\dot{\mu}}; \quad \hat{X}^{\dot{\mu}} \equiv \frac{y^{\dot{\mu}}}{g} + \hat{b}^{\dot{\mu}}, \quad (4)$$

$$\hat{V}_{\dot{\mu}}{}^{\dot{\nu}} (D^{\alpha} \hat{b}_{\dot{\nu}} - \hat{V}^{\dot{\rho}}{}_{\dot{\nu}} \hat{B}^{\alpha}{}_{\dot{\rho}}) + \epsilon^{\alpha\beta} \hat{F}_{\beta\dot{\mu}} + g \epsilon^{\alpha\beta} \hat{F}_{\dot{\mu}\dot{\nu}} \hat{B}_{\beta}{}^{\dot{\nu}} = 0 \quad (5)$$

## Discussion and Outlook

- obtain YM theory after integrating out  $\hat{b}$
- $(p - 1)$ -bracket in non-Abelian Theory
- extend to all orders from T-duality (or D3-brane theory)
- consistent with multiple M5-branes [Chu and Ko 2012]

Low-Energy Brane Theory  
oo

Multiple Branes  
ooo

Discussion and Outlook  
oo

Thank you!