Uplift and towers of states in warped throat

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Metastable dS vacuum and swampland

- The ΛCDM model with the constant and tiny cosmological constant $\Lambda \sim 10^{-120} m_{Pl}^4$ seems to be consistent with the observational cosmology, suggesting that our universe is in the **metastable dS vacuum**.
- However, the string model for the metastable dS vacuum is quite involved:
  Tuning between various ingredients
  1) flux compactification, non-perturbative effect for the moduli stabilization
  2) uplift for the dS spacetime

Any special reason in string theory / quantum gravity which makes dS space ‘unnatural’ (so the metastable dS vacuum is in the swampland) ?

Distance conjecture and cosmological constant

- Many of conjectured criteria distinguishing the landscape from the swampland rely on the **distance conjecture** :

the infinite distance limit of the scalar moduli space is a particular corner of the landscape, beyond which the EFT breaks down by the descent of an infinite tower of states

H. Ooguri, C. Vafa, Nucl. Phys. B**766** (2007) 21 [hep-th/0605264]

Example : KK modes associated with the radion

- AdS distance conjecture D. Lust, E. Palti, C. Vafa, Nucl. Phys. B**797** (2019) 134867 [1906.05225]

$$\Delta m \sim \left(\frac{|\Lambda|}{m_{\rm Pl}^4}\right)^{\alpha} m_{\rm Pl} \qquad (\alpha > 0)$$

(Maybe extended to dS)

1) discontinuity between Minkowski ($\Lambda = 0$) and (A)dS in the $\Lambda \to 0$ limit
(different branches of the space of vacua cannot be interpolated by EFT)

2) if $\Delta m = m_{KK}$, $\frac{1}{4} < \alpha < \frac{1}{2}$

$\frac{1}{4}$: observational bound on large extra dimension

M. Montero, C. Vafa, I. Valenzuela, JHEP **02** (2023) 022 [2205.12293]

$\frac{1}{2}$: no-ghost condition (Higuchi bound)

- Gravitino distance conjecture N.Cribiori, D. Lust, M. Scalisi, JHEP **06** (2021) 071 [2104.08288],

A. Castellano, A. Font, A. Heraaez, L. E. Ibanez, JHEP **08** (2021) 092 [2104.10181],

$|\Lambda|$ in AdS is smaller than 3 $m_{Pl}^2 m_{3/2}^2$: it may be $m_{3/2}$ rather than Λ that satisfies the scaling behavior with respect to the tower mass scale

Question

- In a model building point of view, which of ingredients satisfies the scaling behavior with respect to the tower mass scale?

Model under consideration : Type IIB flux compactifications containing the warped deformed conifold, with the uplift potential produced by $\overline{D3}$-brane at the tip of the throat

M.-S. Seo, JHEP **07** (2023) 082 [2303.10237]

Strong/weak warping

- Under the metric ansatz

$$ds^2 = e^{2A(y)} e^{2\Omega(x)} g_{\mu\nu} dx^\mu dx^\nu + e^{-2A(y)} \sigma(x)^{1/2} g_{mn} dy^m dy^n$$

$$e^{2\Omega(x)} = \frac{\mathcal{V}_0 \ell_s^6}{\sigma(x)^{3/2} \int d^6 y \sqrt{g_6} e^{-4A}} = \frac{\langle \sigma(x)^{3/2} \rangle}{\sigma(x)^{3/2}}$$

Warp factor : $$e^{-4A(y)} = 1 + \frac{e^{-4A_0(y)}}{\sigma(x)}$$

$$e^{-4A_0(y)} = 2^{2/3} \frac{(\alpha' g_s M)^2}{\epsilon^{8/3}} I(\eta)$$

$$I(\eta) = \int_\eta^\infty dx \frac{x \coth x - 1}{\sinh^2 x} (\sinh(2x) - 2x)^{1/3}$$

$$|z| = \epsilon^2 / \ell_s^3$$
$$= \Lambda_0^3 \exp[-\frac{2\pi K}{g_s M}]$$

$$e^{-4A} \simeq \frac{e^{-4A_0}}{\langle\sigma\rangle} \simeq 2^{2/3} I(0) \frac{(g_s M)^2}{(2\pi)^4 |z|^{4/3} \mathcal{V}_0^{2/3}} \gg 1$$

$$e^{-4A} \simeq 1 \qquad \left(\frac{e^{-4A_0}}{\langle\sigma\rangle} \ll 1\right)$$

$$\eta_{\rm UV} < \frac{(g_s M)^2}{(2\pi)^4 \mathcal{V}_0^{2/3} |z|^{4/3}}$$

Strong warping

$$\eta_{\rm UV} > \frac{(g_s M)^2}{(2\pi)^4 \mathcal{V}_0^{2/3} |z|^{4/3}}$$

Weak warping

$$\eta_{\rm UV} < \frac{(g_s M)^2}{(2\pi)^4 \mathcal{V}_0^{2/3} |z|^{4/3}}$$

Extremely weak warping

$$K^{z\bar{z}} \propto \left(\log \frac{\Lambda_0^3}{|z|} + \frac{c'(g_s M)^2}{(2\pi)^4 \mathcal{V}_0^{2/3} |z|^{4/3}}\right)^{-1}$$

$$\eta_{\rm UV} = \log \frac{\Lambda_0^3}{|z|} \simeq \frac{2\pi K}{g_s M}$$

$$e^{-2A} \simeq [2^{2/3} I(0)]^{1/2} \frac{(g_s M)}{(2\pi)^2 |z|^{2/3} \langle \sigma^{1/2} \rangle} \gg 1$$

- The combination

$$e^{-2A(y)} \sigma(x)^{1/2} g_{mn} dy^m dy^n$$

with

$$g_{mn} dy^m dy^n \simeq \ell_s^2 \frac{|z|^{2/3}}{4} \left(\frac{2}{3}\right)^{1/3} \left[d\eta^2 + \frac{1}{2}((g^1)^2 + (g^2)^2) + 2\left(\frac{1}{2}(g^5)^2 + (g^3)^2 + (g^4)^2\right) \right]$$

Becomes independent of both $|z|$ and $\sigma^{3/2} = V_0$.

Towers of states in the model

- String excitations : $m_s = \frac{g_s}{\sqrt{4\pi\mathcal{V}_0}} m_{\rm Pl}$ $\qquad \mathcal{V}_0 = \langle\sigma\rangle^{3/2}$

- KK modes : when they are localized in the throat region,

$$ds^2 = e^{2\Omega_4(x,y)} g_{\mu\nu} dx^\mu dx^\nu + e^{2\Omega_6(x,y)} g_{mn} dy^m dy^n$$

$$e^{2\Omega_4(x,y)} = e^{2A(y)} e^{2\Omega(x)}, \qquad e^{2\Omega_6(x,y)} = e^{-2A(y)} \sigma(x)^{1/2}$$

$$m_{\rm KK} = \langle e^{\Omega_4}\rangle \frac{1}{\langle e^{\Omega_6}\rangle R} = \langle e^A\rangle \frac{1}{\langle e^{\Omega_6}\rangle R}$$

$$m_{\mathrm{KK}} = \langle e^{\Omega_4} \rangle \frac{1}{\langle e^{\Omega_6} \rangle R} = \langle e^{A} \rangle \frac{1}{\langle e^{\Omega_6} \rangle R} \qquad e^{2\Omega_6(x,y)} = e^{-2A(y)} \sigma(x)^{1/2}$$

$$g_{mn} dy^m dy^n \simeq \ell_s^2 \frac{|z|^{2/3}}{4} \left(\frac{2}{3}\right)^{1/3} \left[d\eta^2 + \frac{1}{2}((g^1)^2 + (g^2)^2) + 2\left(\frac{1}{2}(g^5)^2 + (g^3)^2 + (g^4)^2\right) \right] \Longrightarrow R \sim \eta_{\mathrm{UV}} \epsilon^{2/3} = \eta_{\mathrm{UV}} |z|^{1/3} \ell_s$$

1. When $e^{-4A} \simeq \frac{e^{-4A_0}}{\langle \sigma \rangle} \simeq 2^{2/3} I(0) \frac{(g_s M)^2}{(2\pi)^4 |z|^{4/3} \mathcal{V}_0^{2/3}} \gg 1$ $\quad \langle e^{\Omega_6} \rangle R$ is independent of both $|z|$ and $\sigma^{3/2} = V_0$.

   Then

$$m_{\mathrm{KK}}^{\mathrm{w}} \propto \langle e^{A} \rangle m_s \sim |z|^{1/3} \mathcal{V}_0^{1/6} m_s$$

$$m_{\mathrm{KK}}^{\mathrm{w}} = \frac{2^{1/2} 3^{1/6} \pi^{3/2}}{I(0)^{1/2}} \frac{|z|^{1/3}}{M \eta_{\mathrm{UV}} \mathcal{V}_0^{1/3}} m_{\mathrm{Pl}} \sim \frac{|z|^{1/3}}{M \eta_{\mathrm{UV}} \mathcal{V}_0^{1/3}} m_{\mathrm{Pl}}$$

As $|z| \ll 1$ and $\eta_{UV} > 1$, it is typically the lowest tower mass scale.

$$m_{\rm KK} = \langle e^{\Omega_4}\rangle \frac{1}{\langle e^{\Omega_6}\rangle R} = \langle e^{A}\rangle \frac{1}{\langle e^{\Omega_6}\rangle R} \qquad e^{2\Omega_6(x,y)} = e^{-2A(y)}\sigma(x)^{1/2}$$

$$g_{mn}dy^m dy^n \simeq \ell_s^2 \frac{|z|^{2/3}}{4}\left(\frac{2}{3}\right)^{1/3}\left[d\eta^2 + \frac{1}{2}((g^1)^2+(g^2)^2) + 2\left(\frac{1}{2}(g^5)^2+(g^3)^2+(g^4)^2\right)\right] \Longrightarrow R \sim \eta_{\rm UV}\epsilon^{2/3} = \eta_{\rm UV}|z|^{1/3}\ell_s$$

2. When $e^{-4A} \simeq 1$ $\left(\frac{e^{-4A_0}}{\langle\sigma\rangle} \ll 1\right)$ $\langle e^{\Omega_6}\rangle R$ is no longer independent of both $|z|$ and $\sigma^{3/2} = V_0$.

Then $m_{\rm KK}^{\rm ew} \propto |z|^{-1/3}\mathcal{V}_0^{-1/6} m_s$

$$m_{\rm KK}^{\rm ew} \sim \frac{g_s}{\eta_{\rm UV}|z|^{1/3}\mathcal{V}_0^{2/3}} m_{\rm Pl}$$

This is heavier than the bulk KK mass scale $m_{\rm KK} = \frac{2\pi m_s}{\mathcal{V}_0^{1/6}} = \sqrt{\pi}\frac{g_s}{\mathcal{V}_0^{2/3}} m_{\rm Pl}$ $\langle e^{-4A}\rangle = 1$ and $2\pi R = \ell_s = m_s^{-1}$

- Typically, the KK mass scale is the lowest tower mass scale :

1. When $e^{-4A} \simeq \frac{e^{-4A_0}}{\langle\sigma\rangle} \simeq 2^{2/3} I(0) \frac{(g_s M)^2}{(2\pi)^4 |z|^{4/3} \mathcal{V}_0^{2/3}} \gg 1$, the KK modes localized at the throat provide the lowest tower mass scale

$$m_{\rm KK}^{\rm w} = \frac{2^{1/2} 3^{1/6} \pi^{3/2}}{I(0)^{1/2}} \frac{|z|^{1/3}}{M \eta_{\rm UV} \mathcal{V}_0^{1/3}} m_{\rm Pl} \sim \frac{|z|^{1/3}}{M \eta_{\rm UV} \mathcal{V}_0^{1/3}} m_{\rm Pl}.$$

2. When $e^{-4A} \simeq 1$ $\left(\frac{e^{-4A_0}}{\langle\sigma\rangle} \ll 1\right)$, the bulk KK mass scale is the lowest tower mass scale

$$m_{\rm KK} = \frac{2\pi m_s}{\mathcal{V}_0^{1/6}} = \sqrt{\pi} \frac{g_s}{\mathcal{V}_0^{2/3}} m_{\rm Pl}$$

Uplift by $\overline{D3}$-branes

- For $\overline{D3}$-branes extended over noncompact 4-dimensional spacetime, the induced metric is given by

$$ds^2_{\overline{D3}} = e^{2\Omega_4(x,y)} g_{\mu\nu} dx^\mu dx^\nu = e^{2A(y)} e^{2\Omega(x)} g_{\mu\nu} dx^\mu dx^\nu$$

From the DBI+CS action, the uplift potential is given by

$$V_{\rm up} = 2p \frac{T_3}{g_s} e^{4\Omega_4(x,y)} = 4\pi p \frac{m_s^4}{g_s} e^{4A(y)} e^{4\Omega(x)}$$

(p : the number of $\overline{D3}$-branes)

- In the uplift potential

$$V_{\rm up} = \frac{g_s^3}{4\pi}\frac{p}{\sigma^3}m_{\rm Pl}^4\Big[1+\frac{2^{2/3}(g_sM)^2 I(\eta)}{(2\pi)^4|z|^{4/3}\sigma}\Big]^{-1}$$

η is the position where the branes are stabilized : minimized at $\eta = 0$

In the extremely weakly warped case, we may have η close zero (almost vanishing throat length) : brane may not be localized.

But as we can find in the finite potential well, even if the depth of the potential is very small, at least one bound state exists, with the probability to find the brane is large compared to other region.

1. When $e^{-4A} \simeq \frac{e^{-4A_0}}{\langle\sigma\rangle} \simeq 2^{2/3} I(0) \frac{(g_s M)^2}{(2\pi)^4 |z|^{4/3} \mathcal{V}_0^{2/3}} \gg 1$

$$V_{\rm up} = 2p \frac{T_3}{g_s} e^{4\Omega_4(x,y)} = 4\pi p \frac{m_s^4}{g_s} e^{4A(y)} e^{4\Omega(x)}$$

$$e^{4\Omega} = \frac{\langle\sigma^3\rangle}{\sigma^3}$$

$$V_{\rm up}^{\rm w} = \frac{2^{4/3}\pi^3}{I(0)} \frac{g_s p}{M^2} \frac{|z|^{4/3}}{\sigma(x)^2} m_{\rm Pl}^4 \qquad \text{or} \quad \langle V_{\rm up}^{\rm w} \rangle \sim (g_s p/M^2)(|z|^{4/3}/\mathcal{V}_0^{4/3}) m_{\rm Pl}^4$$

then the scaling behavior

$$m_{\rm KK}^{\rm w} \sim \frac{1}{g_s^{1/4} \eta_{\rm UV} M^{1/2} p^{1/4}} \langle V_{\rm up}^{\rm w} \rangle^{1/4}$$

is satisfied

R. Blumenhagen, M. Brinkmann, A. Makridou, Phys. Lett. B838 (2023) 137699 [2208.01057]

2. When $e^{-4A} \simeq 1$ $\left(\frac{e^{-4A_0}}{\langle\sigma\rangle} \ll 1\right)$

$$V_{\rm up} = 2p\frac{T_3}{g_s}e^{4\Omega_4(x,y)} = 4\pi p\frac{m_s^4}{g_s}e^{4A(y)}e^{4\Omega(x)}$$

$$e^{4\Omega} = \frac{\langle\sigma^3\rangle}{\sigma^3}$$

$$V_{\rm up}^{\rm ew} = \frac{g_s^3}{4\pi}\frac{p}{\sigma(x)^3}m_{\rm Pl}^4\left[1+\frac{2^{2/3}(g_sM)^2I(0)}{(2\pi)^4|z|^{4/3}\sigma(x)}\right]^{-1} \simeq \frac{g_s^3}{4\pi}\frac{p}{\sigma(x)^3}m_{\rm Pl}^4$$

$$\langle V_{\rm up}^{\rm ew}\rangle/m_{\rm Pl}^4 = [g_s^3/(4\pi)]p/\mathcal{V}_0^2$$

in this case, the warping is not important but the volume dependence allows two scaling behaviors :

(a) string scale : even though not the lowest tower scale, the exponent is given by $\frac{1}{4}$

$$m_s \sim \left(\frac{g_s}{4\pi p}\right)^{1/4}\langle V_{\rm up}^{\rm ew}\rangle^{1/4}$$

(b) bulk KK scale (the lowest tower scale)

$$m_{\rm KK} \sim \frac{1}{p^{1/3}} \left\langle \frac{V_{\rm up}^{\rm ew}}{m_{\rm Pl}^4} \right\rangle^{1/3} m_{\rm Pl}.$$

- Discussion so far shows that the uplift potential produced by $\overline{D3}$-branes satisfies the scaling behavior with respect to the lowest tower mass scale (KK mass scale)

- Away from this, there exists a tower mass scale with the exponent in the scaling behavior is given by ¼ (warped KK scale or the string scale)

- In the model for the metastable dS space, it is identified with the AdS scale, but this is a result of tuning so we need to distinguish them

  cf. the model for the inflation

- The scaling behavior makes sense provided the number of $\overline{D3}$-branes is nonzero : discontinuity between the exactly vanishing uplift potential (with all the tower states decoupled) and nonzero but tiny uplift potential realizing the same amount of the cosmological constant.

- For the validity of the EFT, the lowest tower mass scale (KK mass scale here) is required to be heavier than the masses in the EFT. (cf. conifold modulus)

R. Blumenhagen, D. Kläwer, L. Schlechter, JHEP 05 (2019) 152 [1902.07724]

Example : gravitino mass $m_{3/2} = e^{K/2}|W| < m_{\rm KK} \sim V_{\rm up}^{\alpha}$

- On the other hand, too large uplift potential for the simple model (say, dominated by the single non-perturbative term) may spoil the moduli stabilization as the total potential shows the runaway behavior :

$$V_{\rm up} \lesssim \mathcal{O}(10) \times |V_{\rm AdS}| \leq \mathcal{O}(10) \times \Lambda_{\rm SUSY} = \mathcal{O}(10) \times 3m_{\rm Pl}^2 m_{3/2}^2$$

➡ Constraints on the uplift and the superpotential, depending on the model (for details see M.-S. Seo, JHEP **07** (2023) 082 [2303.10237])