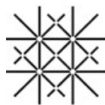


Singling out $SO(10)$ GUT models using recent PTA results

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University of Southampton, 10-14 July 2023

[arXiv:2307.04595](https://arxiv.org/abs/2307.04595) (S. Antusch, K. Hinze, S. Saad, J. Steiner)

Outline

- Motivations
- Promising SUSY SO(10) GUT models
- Gravitational wave signals

SO(10) GUT: Most Elegant Candidate?

$$16 = \begin{pmatrix} \nu \\ u \\ u \\ u \\ e^- \\ d \\ d \\ d \\ d^c \\ d^c \\ d^c \\ e^+ \\ u^c \\ u^c \\ u^c \\ \nu^c \end{pmatrix}_L$$

all fermions + neutrino mass

Motivations

Promising models:

- Gauge coupling unification
- Cosmic inflation
- Doublet-Triplet splitting
- Fermion mass
- Proton decay bounds
- ...

Features of the models

- Lower-dimensional reps.: 10, 16, 45

- Superpotential:

$$W = W_{\text{GUT-breaking}} + \underbrace{W_{\text{Inflation}} + W_{\text{Mixed}}}_{W_{\text{Intermediate-breaking}}} + W_{\text{DTS}} + W_{\text{Yukawa}}$$

DTS problem

- Doublets & Triplets

$$\begin{aligned}10_H &= (2_H + 3_H) + (\bar{2}_H + \bar{3}_H) \\ &= (1, 2, 1/2) + (3, 1, -1/3) + \text{c.c.}\end{aligned}$$

- $\langle 45_H \rangle \propto i\tau_2 \otimes \text{diag}(a_1, a_2, a_3, a_4, a_5)$

- GUT scale mass:

$$10_{1H} \langle 45_H \rangle 10_{2H} = \bar{2}_{1H} 2_{2H} + \bar{2}_{2H} 2_{1H} + \bar{3}_{1H} 3_{2H} + \bar{3}_{2H} 3_{1H}$$

Dimopoulos-Wilczek mechanism

Missing VEV mechanism: [S. Dimopoulos, F. Wilczek 1981](#), [M. Srednicki 1982](#)

- $\langle 45_H \rangle \propto B - L \propto iT_2 \otimes \text{diag}(a, a, a, 0, 0)$

- $\langle 45_H \rangle \propto I_{3R} \propto iT_2 \otimes \text{diag}(0, 0, 0, b, b)$

- $\langle 45_H \rangle \propto B - L \quad \& \quad \langle 45'_H \rangle \propto I_{3R}$

K.S. Babu, S. M. Barr, Z. Berezhiani, R. N. Mohapatra, J. C. Pati, S. Raby, ...

$B - L$ -case: Monopole problem

- $\langle 45_H \rangle \propto i\tau_2 \otimes \text{diag}(a, a, a, 0, 0)$

$$\begin{aligned} SO(10) &\xrightarrow[45_H]{M_{\text{GUT}}} SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\ &\xrightarrow[16_H + \overline{16}_H]{M_I} SU(3)_C \times SU(2)_L \times U(1)_Y \end{aligned}$$

- Monopole problem.
- (no cosmic strings)

B – L-case: Inflation

$$\begin{aligned} SO(10) &\xrightarrow[45_H]{M_{\text{GUT}}} SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\ &\xrightarrow[16_H + \overline{16}_H]{M_I} SU(3)_C \times SU(2)_L \times U(1)_Y \end{aligned}$$

- Hybrid inflation: [A. Linde 1991](#), [G. R. Dvali et. al. 1994](#)

$$W_{\text{Inflation}} \supset \kappa \underbrace{S}_{\text{inflaton}} (\overline{16}_H 16_H - m_{16}^2)$$

- Vacuum energy $V \sim \kappa^2 m_{16}^4$
- Waterfall at the end of the inflation

B – L-case: DTS

- $\langle 45_H \rangle \propto i\tau_2 \otimes \text{diag}(a, a, a, 0, 0)$:

$$10_{1H} \langle 45_H \rangle 10_{2H} = \bar{2}_{1H} \overrightarrow{2}_{2H}^0 + \bar{2}_{2H} \overrightarrow{2}_{1H}^0 + \bar{3}_{1H} 3_{2H} + \bar{3}_{2H} 3_{1H}$$

- Single pair of light doublets:

$$10_{2H} 10_{2H} = \bar{2}_{2H} 2_{2H} + \bar{3}_{2H} 3_{2H}.$$

- $d = 5$ proton decay:

$$\bar{3}_{1H} \langle 45_H \rangle 3_{2H} \quad \bar{3}_{2H} m_{10} 3_{2H} \quad \bar{3}_{2H} \langle 45_H \rangle 3_{1H}$$

\uparrow \uparrow \uparrow
 $\bar{3}_{1H} \langle 45_H \rangle 3_{2H}$ $\bar{3}_{2H} m_{10} 3_{2H}$ $\bar{3}_{2H} \langle 45_H \rangle 3_{1H}$

Effective triplet mass $\sim M_{\text{GUT}}^2 / m_{10}$.

I_{3R} -case: Symmetry breaking

- $\langle 45_H \rangle \propto i\tau_2 \otimes \text{diag}(0, 0, 0, b, b)$

$$\begin{aligned} SO(10) &\xrightarrow[45_H]{M_{\text{GUT}}} SU(4)_C \times SU(2)_L \times U(1)_R \\ &\xrightarrow[16_H + \overline{16}_H]{M_I} SU(3)_C \times SU(2)_L \times U(1)_Y \end{aligned}$$

- Monopole problem...
- (no cosmic strings) ...
- Inflation ...

I_{3R} -case: DTS

- $\langle 45_H \rangle \propto i\tau_2 \otimes \text{diag}(0, 0, 0, b, b)$:

$$10_{1H} \langle 45_H \rangle 10_{2H} = \bar{2}_{1H} 2_{2H} + \bar{2}_{2H} 2_{1H} + \bar{3}_{1H} 3_{2H} + \bar{3}_{2H} 3_{1H}$$

- $\bar{16}_H \langle 45_H \rangle 16_H$ ✗ $\bar{16}'_H \langle 45_H \rangle 16'_H$ ✓

$$\bar{16}'_H \langle 45_H \rangle 16'_H = \bar{2}'_H 2'_H + \bar{3}'_H 3'_H$$

- Masses to all the color-triplets:

$$W_{\text{DTS}} \supset \lambda_3 \bar{16}' \cdot 45 \cdot 16' + \lambda_4 10 \cdot 16 \cdot 16 + \lambda_5 10 \cdot \bar{16} \cdot \bar{16}$$

$B - L$ & I_{3R} -case

- $\langle 45_H \rangle \propto i\tau_2 \otimes \text{diag}(a, a, a, 0, 0)$, $\langle 45'_H \rangle \propto i\tau_2 \otimes \text{diag}(0, 0, 0, b, b)$
- (a) $\langle 45_H \rangle > \langle 45'_H \rangle > \langle 16_H \rangle, \langle \overline{16}_H \rangle$:

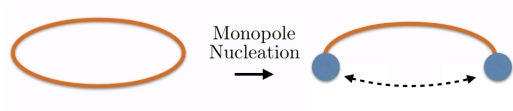
$$\begin{aligned} SO(10) &\xrightarrow[45_H]{M_{\text{GUT}}} SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\ &\xrightarrow[45'_H]{M_I} SU(3)_C \times SU(2)_L \times U(1)_R \times U(1)_{B-L} \\ &\xrightarrow[16_H + \overline{16}_H]{M_{II}} SU(3)_C \times SU(2)_L \times U(1)_Y \end{aligned}$$

- cosmic string network

Metastable strings

P. Langacker and S. Y. Pi, 1980

- Example: $SU(2) \xrightarrow{M_m} U(1) \xrightarrow{M_s} \text{broken}$



- lifetime determined by:

$$\kappa_m = \frac{m^2}{\mu} \sim \frac{8\pi}{g^2} \left(\frac{M_m}{M_s} \right)^2$$

$$\left(\kappa_m^{1/2} < 9 \text{ metastable} \right)$$

$B - L$ & I_{3R} -case

- $\langle 45_H \rangle \propto i\tau_2 \otimes \text{diag}(a, a, a, 0, 0)$, $\langle 45'_H \rangle \propto i\tau_2 \otimes \text{diag}(0, 0, 0, b, b)$
- (b) $\langle 45'_H \rangle > \langle 45_H \rangle > \langle 16_H \rangle, \langle \overline{16}_H \rangle$:

$$\begin{aligned} SO(10) &\xrightarrow[45'_H]{M_{\text{GUT}}} SU(4)_C \times SU(2)_L \times U(1)_R \\ &\xrightarrow[45_H]{M_I} SU(3)_C \times SU(2)_L \times U(1)_R \times U(1)_{B-L} \\ &\xrightarrow[16_H + \overline{16}_H]{M_{II}} SU(3)_C \times SU(2)_L \times U(1)_Y \end{aligned}$$

- cosmic string network

$B - L$ & I_{3R} -case

- $\langle 45_H \rangle \propto i\tau_2 \otimes \text{diag}(a, a, a, 0, 0)$, $\langle 45'_H \rangle \propto i\tau_2 \otimes \text{diag}(0, 0, 0, b, b)$

- (c) $\langle 45_H \rangle = \langle 45'_H \rangle > \langle 16_H \rangle, \langle \overline{16}_H \rangle$:

$$\begin{aligned} SO(10) &\xrightarrow[45_H+45'_H]{M_{\text{GUT}}} SU(3)_C \times SU(2)_L \times U(1)_R \times U(1)_{B-L} \\ &\xrightarrow[16_H+\overline{16}_H]{M_I} SU(3)_C \times SU(2)_L \times U(1)_Y \end{aligned}$$

- cosmic string network

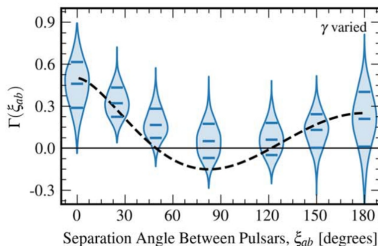
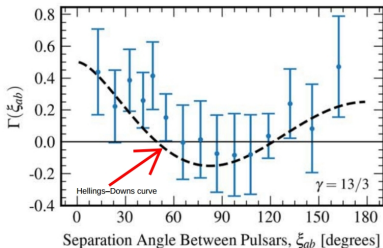
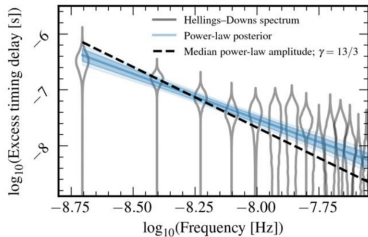
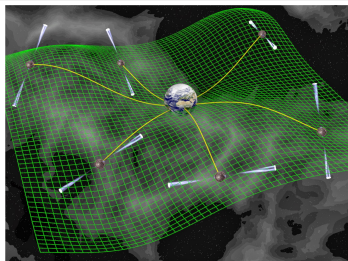
Fermion mass

- Charged fermions: higher dimensional operators
- Right-handed neutrino mass:

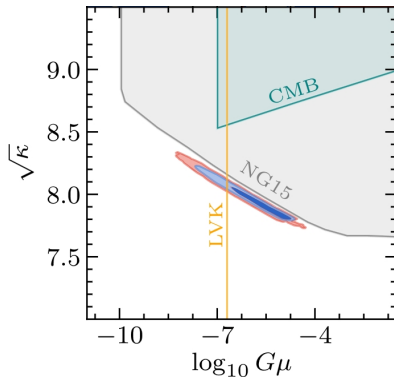
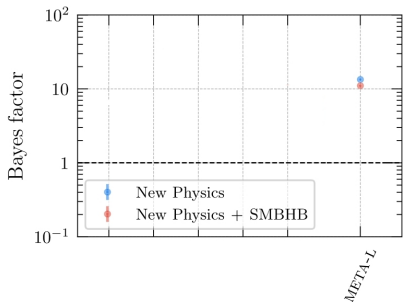
$$W_{\text{Yukawa}} \supset Y_R 16_i 16_j \frac{\overline{16}_H \overline{16}_H}{\Lambda} \sim Y_R \frac{v_R^2}{\Lambda} \nu^c \nu^c$$

- Neutrino mass: Type-I seesaw

Pulsar timing data

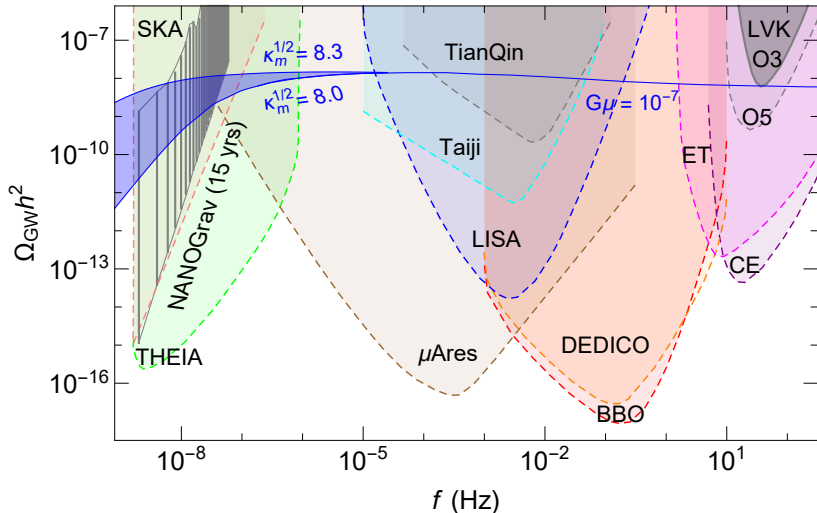


PTA data: Metastable strings



Adeela Afzal et al 2023 ApJL 951 L11

Gravitational Wave signal



$m_S \sim 10^{15}$ GeV (string scale = seesaw scale = inflation scale)

Summary

- ❄ Promising models towards SO(10) GUT
- ❄ Inflation, DTS, Unification, Fermion mass, Gravitational waves
- ❄ Minimal models: $10_H, 16_H, 45_H$
- ❄ $\langle 45_H \rangle \propto B - L$ and $\langle 45_H \rangle \propto I_{3R}$ no stable/metastable strings
- ❄ $\langle 45_H^{(\prime)} \rangle \propto B - L, I_{3R}$ metastable strings
- ❄ GW/PTAs: $v_m \sim v_s \sim 10^{15}$ GeV ($v_s = v_R = M_{\text{inflation}}$)
- ❄ Fully testable in a number of gravitational wave observatories

THANK YOU!

B – L-case

- $$W_{\text{GUT-breaking}} \supset \frac{m_{45}}{2} \text{Tr}[45_H^2] + \frac{\lambda}{4\Lambda} \text{Tr}[45_H^4],$$

- $$45_H, 16_H, \overline{16}_H \supset (1, 1, 1) + (3, 2, 1/6) + (\overline{3}, 1, -2/3) + \text{c.c.},$$

- $$W_{\text{Mixed}} \supset$$

$$\overline{16}_H(\lambda_1 45_H + \lambda'_1 1_H) 16'_H + \overline{16}'_H(\lambda_2 45_H + \lambda'_2 1'_H) 16_H.$$

- $$W_{\text{DTS}} \supset \gamma 10_{1H} 45_H 10_{2H} + m_{10} 10_{2H} 10_{2H}$$

B - L & I_{3R} -case: DTS

- $10_{2H}10_{2H} \times$ (forbidding effective triplet mass)

$$10_{2H}45_H'^2 10_{2H} =$$

$$\bar{2}_{2H}2_{2H} + \bar{2}_{2H}2_{2H} + \bar{3}_{2H}3_{2H} \xrightarrow{0} + \bar{3}_{2H}3_{2H} \xrightarrow{0}.$$

- To give mass to (rest) one pair of color-triplets and an additional pair of weak doublets:

$$W_{\text{DTS}} \supset$$

$$\gamma_1 10_{1H}45_H 10_{2H} + \frac{\gamma_2}{\Lambda} 10_{2H}45_H'^2 10_{2H} + \omega_{16} \bar{16}_H'' 16_H'$$