TOKYO METROPOLITAN UNIVERSITY
東京都立大学

iPMU INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

Inflation and production of primordial black holes in modified supergravity

by Sergei V. Ketov

17-21 July 2023 SUSY2023, Southampton University, UK

joint work with Y.Aldabergenov, A.Addazi, R. Ishikawa, C.Steinwachs, A.Gundhi

EPJC 80 (2020) 917, PLB 814 (2021) 136069, PRD 103 (2021) 083518,
JCAP 03 (2022) 058, Class. and Quantum Grav. 40 (2023) 065002, etc.

Plan of talk

- From Einstein gravity to modified gravity in 4 spacetime dimensions
- Starobinsky model of inflation and CMB measurements (Planck, BICEP/Keck, LiteBIRD)
- Starobinsky inflation in modified (old-minimal) supergravity
- Production of primordial black holes (PBH) in modified supergravity
- PBH dark matter, induced gravitational waves (GW) and their detection (LISA, DECIGO, etc.), all beyond the Standard Model
- Spontaneous SUSY breaking in modified supergravity with chiral matter
- Conclusion

Modified gravity

• Modified gravity theories are generally-covariant non-perturbative extensions of Einstein-Hilbert gravity theory by the higher-order terms. These terms are irrelevant in the low-curvature regime (Solar system) but are relevant in the high-curvature regime (inflation, black holes).

• A modified gravity action has the higher-derivatives and generically suffers from Ostrogradsky instability and ghosts. However, there are exceptions. For example, in the modified gravity Largrangian quadratic in the spacetime curvature, the only ghost-free term is given by R^2 with a positive coefficient. It leads to the Starobinsky model (1980) of modified gravity with the action

$$S_{\text{Star.}} = \frac{M_{\text{Pl}}^2}{2} \int \mathrm{d}^4x \sqrt{-g} \left(R + \frac{1}{6m^2} R^2 \right) \equiv \frac{M_{\text{Pl}}^2}{2} \int \mathrm{d}^4x \sqrt{-g}\, F(R) \ ,$$

having the only (mass) parameter m, where $M_{\text{Pl}} = 1/\sqrt{8\pi G_{\text{N}}} \approx 2.4 \times 10^{18}$ GeV, the spacetime signature is $(-,+,+,+,)$.

Starobinsky model of inflation

- In the high-curvature regime, the EH term can be ignored and the action becomes scale-invariant.
- Starobinsky gravity has the special (attractor) solution in the FLRW universe with the Hubble function

$$H(t) \approx \left(\frac{m}{6}\right)^2 (t_{\text{end}} - t) \ ,$$

for $m(t_{\text{end}} - t) \gg 0$. This solution spontaneously breaks the scale invariance of R^2-gravity and, hence, implies the existence of the associated Nambu-Goldstone boson called scalaron.

- Scalaron is the physical (scalar) excitation of the higher-derivative gravity. It can be revealed by rewriting the Starobinsky action into the quintessence form by the field redefinition (Legendre-Weyl transform)

$$\varphi = \sqrt{\frac{3}{2}} M_{\text{Pl}} \ln F'(\chi) \quad \text{and} \quad g_{\mu\nu} \to \frac{2}{M_{\text{Pl}}^2} F'(\chi) g_{\mu\nu} \, , \quad \chi = R \, ,$$

which leads to

$$S[g_{\mu\nu}, \varphi] = \frac{M_{\text{Pl}}^2}{2} \int \mathrm{d}^4x \sqrt{-g} R - \int \mathrm{d}^4x \sqrt{-g} \left[\tfrac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + V(\varphi) \right] \, ,$$

with the scalar potential $V(\varphi) = \frac{3}{4} M_{\text{Pl}}^2 m^2 \left[1 - \exp\left(-\sqrt{\tfrac{2}{3}} \varphi / M_{\text{Pl}} \right) \right]^2$.

This potential is perfectly suitable for describing slow-roll inflation with scalaron (NG boson) φ as the inflaton of mass m. *The V is not renormalisable with $\Lambda_{UV} = M_{Pl}$.*

- However, the gravitational origin of inflaton/scalaron and its potential in the quintessence picture is hidden.

Starobinsky model (1980) and CMB measurements (2018)

No phenomenological input was used so far. Nevertheless, Starobinsky model of inflation is still in very good agreement with current CMB measurements.

A duration of inflation is usually measured by the e-foldings number

$$N_e = \int_{t_*}^{t_{\text{end}}} H(t)dt \approx \frac{1}{M_{\text{Pl}}^2} \int_{\varphi_{\text{end}}}^{\varphi_*} \frac{V}{V'} d\varphi \ .$$

The standard slow roll parameters are defined by

$$\varepsilon_{\text{sr}}(\varphi) = \frac{M_{\text{Pl}}^2}{2} \left(\frac{V'}{V}\right)^2 \quad \text{and} \quad \eta_{\text{sr}}(\varphi) = M_{\text{Pl}}^2 \left(\frac{V''}{V}\right) \ .$$

The amplitude of scalar (curvature) perturbations at horizon crossing (with pivot scale $k_* = 0.05~\mathrm{Mpc}^{-1}$) is

$$A_s = \frac{V_*^3}{12\pi^2 M_{\mathrm{Pl}}^6 (V_*')^2} = \frac{3m^2}{8\pi^2 M_{\mathrm{Pl}}^2} \sinh^4\left(\frac{\varphi_*}{\sqrt{6}M_{\mathrm{Pl}}}\right) \approx 1.96 \cdot 10^{-9}$$

that implies (no free parameters!)

$$m \approx 3 \cdot 10^{13}~\mathrm{GeV} \quad \text{or} \quad \frac{m}{M_{\mathrm{Pl}}} \approx 1.3 \cdot 10^{-5}\,, \quad \text{and} \quad H \approx \mathcal{O}(10^{14})~\mathrm{GeV}\,.$$

CMB measurements give the tilt of scalar perturbations $n_s \approx 1 + 2\eta_{\mathrm{sr}} - 6\varepsilon_{\mathrm{sr}} \approx 0.9649 \pm 0.0042~(68\%\mathrm{CL})$ and restrict the tensor-to-scalar ratio as $r \approx 16\varepsilon_{\mathrm{sr}} < 0.032~(95\%\mathrm{CL})$. The Starobinsky inflation gives $r \approx 12/N_e^2 \approx 0.003$ and $n_s \approx 1 - 2/N_e$, with the best fit at $N_e \approx 55$. *The Lyth bound for EFT is satisfied.*

Modified supergravity

Modified supergravity is the (old-minimal) $N = 1$ local SUSY extension of the $(R + \alpha R^2)$ gravity. Manifest SUSY is achieved by using curved superspace. A generic action is given by a sum of D-type and F-type terms,

$$S = \int d^4x d^4\theta E^{-1} N(\mathcal{R}, \bar{\mathcal{R}}) + \left[\int d^4x d^2\Theta 2\mathcal{E} F(\mathcal{R}) + h.c \right] \ ,$$

where the covariantly chiral superfield $\mathcal{R}$ has the spacetime scalar curvature R among its field component. *See also Dalianis, Farakos, Kehagias, Riotto, Unge (2015)*.
The Starobinsky inflation scale $H \sim 10^{14}$ GeV (close to the GUT scale) is the scale where SUSY is expected to play a significant role.
The F-term can be included into the D-term (except a constant). We distinguish them by collecting the R-symmetry preserving terms in the N-potential, and the R-symmetry violating terms in the F-potential.

Field content of modified supergravity

- vierbein e_μ^a, gravitino ψ_μ, complex scalar X, and real vector b_μ,

- form the irreducible (off-shell) supergravity multiplet with linearly realized SUSY and closed SUSY algebra,

- the fields (X, b_μ) are known as the "auxiliary" fields of the old-minimal supergravity (in the textbooks),

- but in modified supergravity (the higher-derivative field theory beyond supergravity textbooks) all these "auxiliary" fields become physical (propagating).

- There are 4 physical scalars in modified supergravity: scalaron φ, complex scalar X and $\widehat{D}_\mu b^\mu / M$ with the nearly equal effective masses of the order M.

Embedding Starobinsky model

Expand the functions N and $\mathcal{F}$ in Taylor series and keep only a few *leading* terms, $(M_{\rm Pl}=1)$,

$$N=\frac{12}{M^2}\mathcal{R}\overline{\mathcal{R}}-\frac{\xi}{2}(\mathcal{R}\overline{\mathcal{R}})^2~, \quad \mathcal{F}=\alpha+3\beta\mathcal{R}~,$$

with real parameters M and ξ, and complex parameters α and β.

- The chiral superfields $\mathcal{R}$ and $\mathcal{E}$ read

$$\begin{aligned}
\mathcal{R} &= X+\Theta\left(-\tfrac{1}{6}\sigma^m\bar{\sigma}^n\psi_{mn}-i\sigma^m\overline{\psi}_m X-\tfrac{i}{6}\psi_m b^m\right)+\\
&\quad +\Theta^2\Big(-\tfrac{1}{12}R-\tfrac{i}{6}\overline{\psi}^m\bar{\sigma}^n\psi_{mn}-4X\overline{X}-\tfrac{1}{18}b_m b^m+\tfrac{i}{6}\nabla_m b^m+\\
&\quad +\tfrac{1}{2}\overline{\psi}_m\overline{\psi}^m X+\tfrac{1}{12}\psi_m\sigma^m\overline{\psi}_n b^n-\tfrac{1}{48}\varepsilon^{abcd}(\overline{\psi}_a\bar{\sigma}_b\psi_{cd}+\psi_a\sigma_b\overline{\psi}_{cd})\Big)~,\\
2\mathcal{E} &= e\left[1+i\Theta\sigma^m\overline{\psi}_m+\Theta^2(6\overline{X}-\overline{\psi}_m\bar{\sigma}^{mn}\overline{\psi}_n)\right]~,
\end{aligned}$$

- The standard supergravity is reproduced when $N=0$ and $\mathcal{F}=-3\mathcal{R}$.
- Starobinsky inflation is realized when $\alpha=0$, $\beta=-3$, and M equals to the scalaron mass, and dynamics of (X,b) is suppressed ($\xi>0$ is needed).

Effective two-scalar field Lagrangian

In the notation

$$\frac{M^4\xi}{144} \equiv \zeta \quad \text{and} \quad |X| \equiv \frac{M}{2\sqrt{6}}\sigma \ ,$$

where σ is the radial part of the complex scalar X, after ignoring its angular part together with $b_m = 0$ for simplicity, the bosonic part of the Lagrangian in our model takes the form

$$e^{-1}\mathcal{L} = \frac{1}{2}f(R,\sigma) - \tfrac{1}{2}(1-\zeta\sigma^2)(\partial\sigma)^2 - U \ ,$$

where we have the specific functions dictated by modified supergravity,

$$f(R,\sigma) = \left(1 + \tfrac{1}{6}\sigma^2 - \tfrac{11}{24}\zeta\sigma^4\right) R + \frac{1}{6M^2}(1-\zeta\sigma^2)R^2 \ ,$$

$$U = \frac{1}{2}M^2\sigma^2\left(1 - \tfrac{1}{6}\sigma^2 + \tfrac{3}{8}\zeta\sigma^4\right) \ .$$

(Standard) transfer to Einstein frame in field components

After introducing the auxiliary field χ and rewriting the Lagrangian as

$$e^{-1}\mathcal{L} = \tfrac{1}{2}\left[f_\chi(R-\chi)+f\right] - \tfrac{1}{2}(1-\zeta\sigma^2)(\partial\sigma)^2 - U ,$$

where $f_\chi \equiv \frac{\partial f}{\partial \chi}$ and in $f \equiv f(\chi,\sigma)$, R was replaced by χ, varying w.r.t. χ gives back the initial Lagrangian. On the other hand, after Weyl rescaling,

$$g_{mn} \to f_\chi^{-1} g_{mn} , \quad e \to f_\chi^{-2} e , \quad e f_\chi R \to eR - \tfrac{3}{2} e f_\chi^{-2}(\partial f_\chi)^2 ,$$

with

$$f_\chi = A + B\chi \quad A \equiv 1 + \tfrac{1}{6}\sigma^2 - \tfrac{11}{24}\zeta\sigma^4 , \quad B \equiv \frac{1}{3M^2}(1-\zeta\sigma^2) ,$$

in terms of the canonically normalized scalaron φ defined by

$$f_\chi = \exp\left[\sqrt{\tfrac{2}{3}}\varphi\right] , \quad \chi = \tfrac{1}{B}\left(e^{\sqrt{\frac{2}{3}}\varphi} - A\right) , \quad f = \frac{1}{2B}\left(e^{2\sqrt{\frac{2}{3}}\varphi} - A^2\right) ,$$

the Lagrangian in Einstein frame takes the form

$$e^{-1}\mathcal{L} = \tfrac{1}{2}R - \tfrac{1}{2}(\partial\varphi)^2 - \tfrac{1}{2}(1-\zeta\sigma^2)e^{-\sqrt{\frac{2}{3}}\varphi}(\partial\sigma)^2 - V ,$$

whose two-field scalar potential reads

$$V = \frac{1}{4B}\left(1 - Ae^{-\sqrt{\frac{2}{3}}\varphi}\right)^2 + e^{-2\sqrt{\frac{2}{3}}\varphi}U =$$

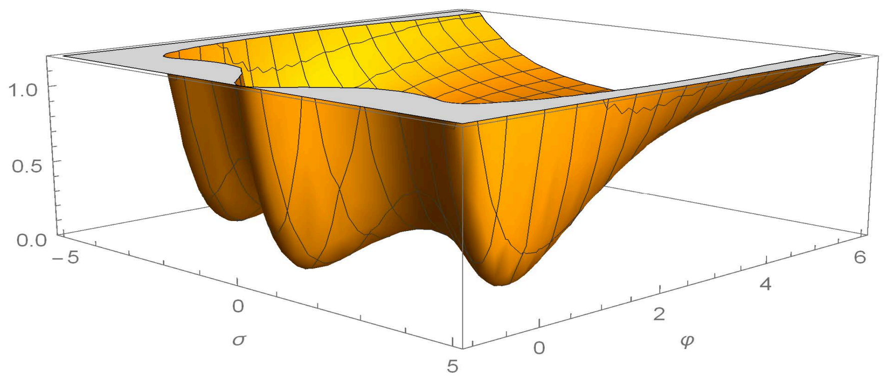
$$= \frac{3M^2}{4(1-\zeta\sigma^2)}\left[1 - e^{-\sqrt{\frac{2}{3}}\varphi} - \frac{\sigma^2}{6}\left(1 - \frac{11}{4}\zeta\sigma^2\right)e^{-\sqrt{\frac{2}{3}}\varphi}\right]^2$$

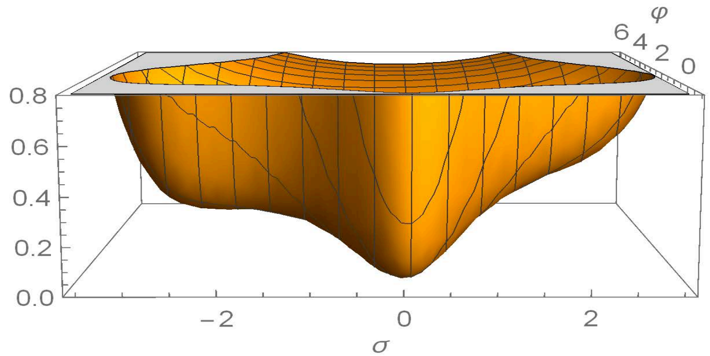
$$+ \frac{M^2}{2}e^{-2\sqrt{\frac{2}{3}}\varphi}\sigma^2\left(1 - \frac{1}{6}\sigma^2 + \frac{3}{8}\zeta\sigma^4\right) \ .$$

When $\sigma^2 > 1/\zeta$, the scalar σ becomes a ghost. However, when approaching $\sigma^2 = 1/\zeta$, the scalar potential becomes singular, so that it would take the infinite amount of energy to turn σ into a ghost (assuming its starting value in the region $\sigma^2 < 1/\zeta$).

Scalar potential in Einstein frame

$$V = \frac{1}{4B}(1 - Ax)^2 + x^2 U\,, \quad e^{-\sqrt{\frac{2}{3}}\varphi} \equiv x\,, \begin{cases} A = 1 + \frac{1}{6}\sigma^2 - \frac{11}{24}\zeta\sigma^4\,, \\ B = \frac{1}{3M^2}(1 - \zeta\sigma^2)\,, \\ U = \frac{M^2}{2}\sigma^2\left(1 - \frac{1}{6}\sigma^2 + \frac{3}{8}\zeta\sigma^4\right)\,. \end{cases}$$





The scalar potential on the left with $\zeta = 1/54 \approx 0.019$ and three Minkowski minima; on the right with $\zeta = 0.027$, a single Minkowski minimum at $\sigma = 0$ and two inflection points. In both cases $M = 1$.

Superfield transfer to Einstein matter-coupled supergravity

After introducing the Lagrange multiplier superfield $\mathbf{T}$ as (Terada and SVK, 2013)

$$\mathcal{L} = \int d^2\Theta 2\mathcal{E} \left\{ -\tfrac{1}{8}(\overline{\mathcal{D}}^2 - 8\mathcal{R})N(\mathbf{S},\bar{\mathbf{S}}) + \mathcal{F}(\mathbf{S}) + 6\mathbf{T}(\mathbf{S}-\mathcal{R}) \right\} + \text{h.c.}\ ,$$

varying the Lagrangian w.r.t. the $\mathbf{T}$ gives back the original Lagrangian. On the other hand, the Lagrangian can be rewritten to the form

$$\mathcal{L} = \int d^2\Theta 2\mathcal{E} \left\{ \tfrac{3}{8}(\overline{\mathcal{D}}^2 - 8\mathcal{R}) \left[\mathbf{T} + \bar{\mathbf{T}} - \tfrac{1}{3}N(\mathbf{S},\bar{\mathbf{S}}) \right] + \mathcal{F}(\mathbf{S}) + 6\mathbf{TS} \right\} + \text{h.c.}$$

that can be put into the standard form in supergravity,

$$\mathcal{L} = \int d^2\Theta 2\mathcal{E} \left[\tfrac{3}{8}(\overline{\mathcal{D}}^2 - 8\mathcal{R})e^{-K/3} + W \right] + \text{h.c.}\ ,$$

where the Kähler potential K takes the no-scale supergravity form

$$K = -3\log(\mathbf{T} + \bar{\mathbf{T}} - \tilde{N})\ , \quad \tilde{N} \equiv \mathbf{S}\bar{\mathbf{S}} - \tfrac{3}{2}\zeta(\mathbf{S}\bar{\mathbf{S}})^2\ ,$$

but the modified supergravity origin of K and W becomes hidden.

See also Ellis, Nanopoulos and Olive (2013); first observed by Cecotti (1987).

Two-field scalar Lagrangian

takes the form of a non-linear sigma-model (NLSM) minimally coupled to gravity,

$$e^{-1}\mathcal{L} = \tfrac{1}{2}R - \tfrac{1}{2}G_{AB}\partial\phi^A\partial\phi^B - V \ ,$$

where $\phi^A = \{\varphi, \sigma\}$, $A = 1, 2$, and the NLSM target space metric is given by

hyperbolic geometry: $$G_{AB} = \begin{pmatrix} 1 & 0 \\ 0 & (1-\zeta\sigma^2)e^{-\sqrt{\frac{2}{3}}\varphi} \end{pmatrix}$$ of negative curvature

With the FLRW spacetime metric $g_{mn} = \mathrm{diag}(-1, a^2, a^2, a^2)$ the EoM read

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{1}{\sqrt{6}}(1-\zeta\sigma^2)e^{-\sqrt{\frac{2}{3}}\varphi}\dot{\sigma}^2 + \partial_\varphi V = 0 \ ,$$

$$\ddot{\sigma} + 3H\dot{\sigma} - \frac{\zeta\sigma\dot{\sigma}^2}{1-\zeta\sigma^2} - \sqrt{\tfrac{2}{3}}\dot{\varphi}\dot{\sigma} + \frac{e^{\sqrt{\frac{2}{3}}\varphi}}{1-\zeta\sigma^2}\partial_\sigma V = 0 \ ,$$

similar to hybrid inflation

Production of primordial black holes (PBH) in inflation

One needs large curvature fluctuations (>10^6 of CMB)!

There are many proposals in the literature:

- gravitational instabilities induced by scalar fields, and collapse of large density fluctuations,
- bubble collisions from first order phase transitions,
- critical topological defects, such as cosmic strings and domain walls.

PBH formation due to amplification of the power spectrum (large peak) of scalar perturbations via tachyonic instabilities of the scalar fields present in modified supergravity, during multi-field inflation. This mechanism is different from the standard mechanism of PBH formation in single-field models of inflation with a near-inflection point in the inflaton scalar potential.

Isocurvature pumping mechanism during inflation

- decompose perturbations into adiabatic Q_a (along inflationary trajectory) and isocurvature Q_s (orthogonal to inflationary trajectory);
- $\ddot{Q}_a + 3H\dot{Q}_a + \Omega Q_a = \hat{f}(d/dt)(\omega Q_s)\ , \quad \ddot{Q}_s + 3H\dot{Q}_s + m_s^2 Q_s = 0$
- When $\ddot{Q}_s \approx 0$, we find the solution $Q_s \approx \exp\left[-\int dt \frac{m_s^2}{3H^2}\right]$
- when the isocurvature mass $m_s^2 < 0$ at the critical point, we get the exp-amplification of Q_s; since Q_a are sourced by Q_s in EoM, we also get an exp-amplification of Q_a when the inflationary trajectory has a sharp turn [Palma, Sypsas, Zenteno (2020); Fumagalli, Renaux-Petel, Ronayne, Witkwoski (2020)];
- after the critical point $m_s^2 > 0$ again, the isocurvature modes get suppressed and, hence, no over-amplification (and no PBH overproduction): [Gundhi, Steinwachs, SVK (2021).

Straightforward generalizations toward PBH

Adding the next-order terms to the modified supergravity potentials yields

$$N = \frac{12}{M^2}|\mathcal{R}|^2 - \frac{72}{M^4}\zeta|\mathcal{R}|^4 - \frac{768}{M^6}\gamma|\mathcal{R}|^6 \ ,$$
$$F = -3\mathcal{R} + \frac{3\sqrt{6}}{M}\delta\mathcal{R}^2 \ .$$

The corresponding Lagrangian in Einstein frame reads

$$e^{-1}\mathcal{L} = \tfrac{1}{2}R - \tfrac{1}{2}(\partial\varphi)^2 - \frac{3M^2}{2}Be^{-\sqrt{\frac{2}{3}}\varphi}(\partial\sigma)^2 - \frac{1}{4B}\left(1 - Ae^{-\sqrt{\frac{2}{3}}\varphi}\right)^2 - e^{-2\sqrt{\frac{2}{3}}\varphi}U \, ,$$

where the functions A, B, U are given by

$$A = 1 - \delta\sigma + \tfrac{1}{6}\sigma^2 - \tfrac{11}{24}\zeta\sigma^4 - \tfrac{29}{54}\gamma\sigma^6 \ ,$$
$$B = \frac{1}{3M^2}(1 - \zeta\sigma^2 - \gamma\sigma^4) \ ,$$
$$U = \frac{M^2}{2}\sigma^2\left(1 + \tfrac{1}{2}\delta\sigma - \tfrac{1}{6}\sigma^2 + \tfrac{3}{8}\zeta\sigma^4 + \tfrac{25}{54}\gamma\sigma^6\right) \ .$$

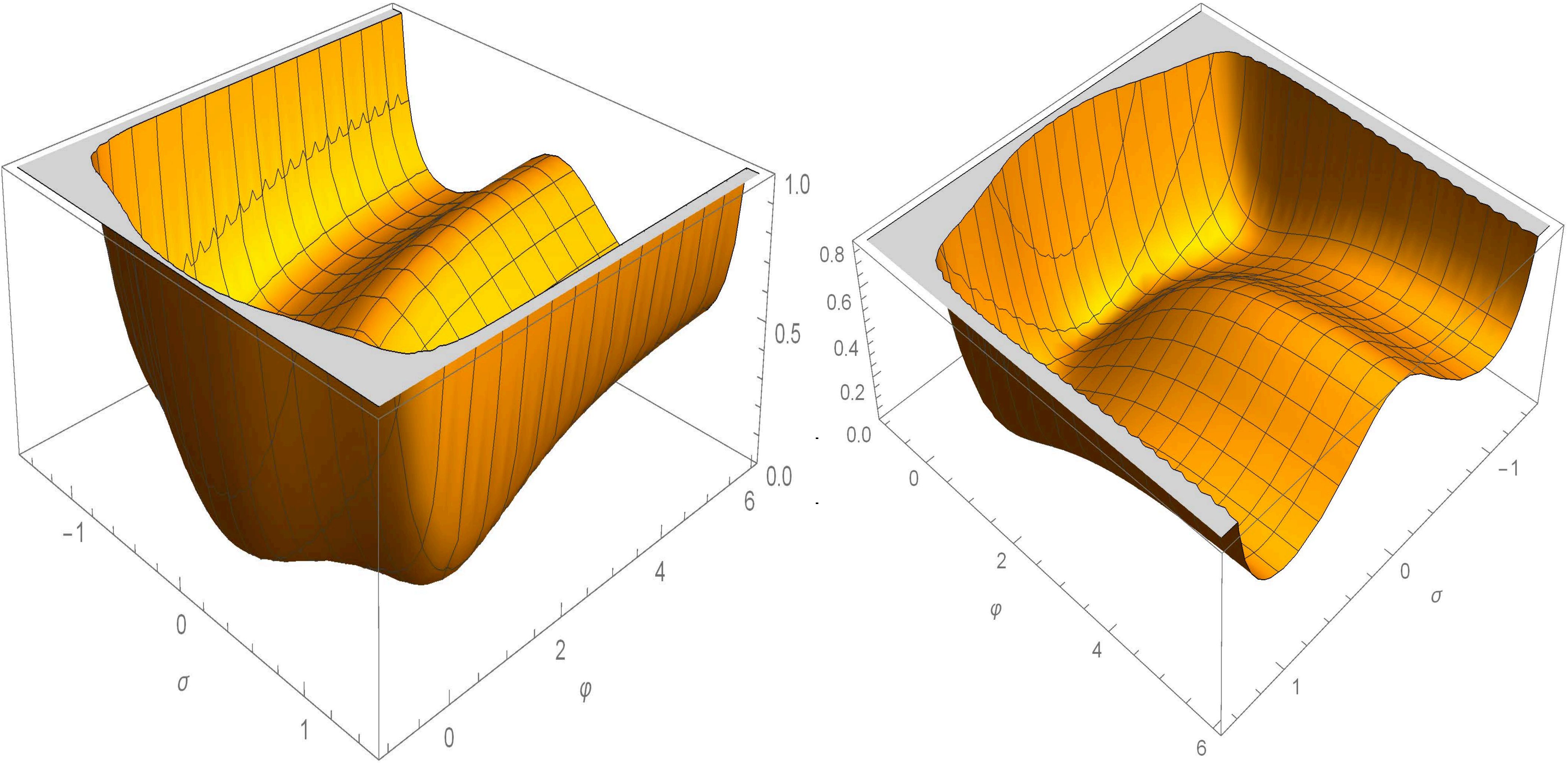
PBH in the γ-model with $\delta = 0$

Let us choose $\gamma = 1$ and $\zeta = -1.7774$ for a numerical analysis. The scalar potential has two valleys and a single Minkowski minimum at $\sigma = \varphi = 0$. The first slow-roll (SR) inflation is possible along either of the valleys. The valleys merge into the Minkowski minimum by passing through the critical points resulting in the so-called ultra-slow-roll (USR) stage.

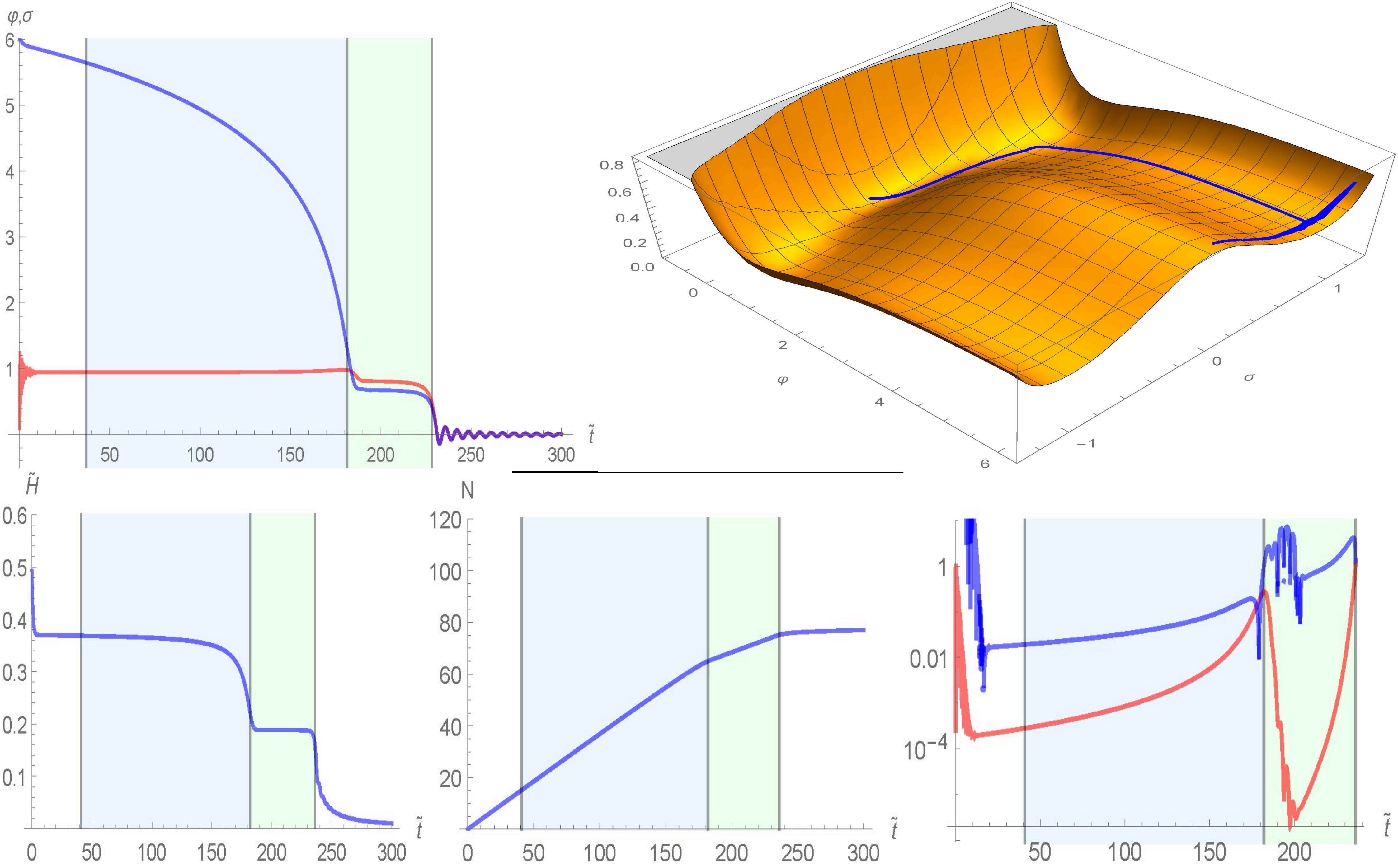
We used the usual (Bunch-Davies) initial conditions.

After solving the equations of motion numerically, we plot the solutions. The total number of e-foldings is set to $\Delta N = 60$. It leads to an enhancement in the scalar power spectrum after fine-tuning the free parameters. With the chosen parameters, the first stage lasts $\Delta N_1 \approx 50$ e-foldings, whereas the second stage lasts for $\Delta N_2 \approx 10$: the first stage of inflation is represented by the blue shaded region, whereas the second stage is marked by the green shaded region. The length of the second stage is controlled by the parameter ζ for a given γ.

The scalar potential of the gamma-model, delta =0



The solution, trajectory, Hubble function, e-foldings, and slow roll parameters

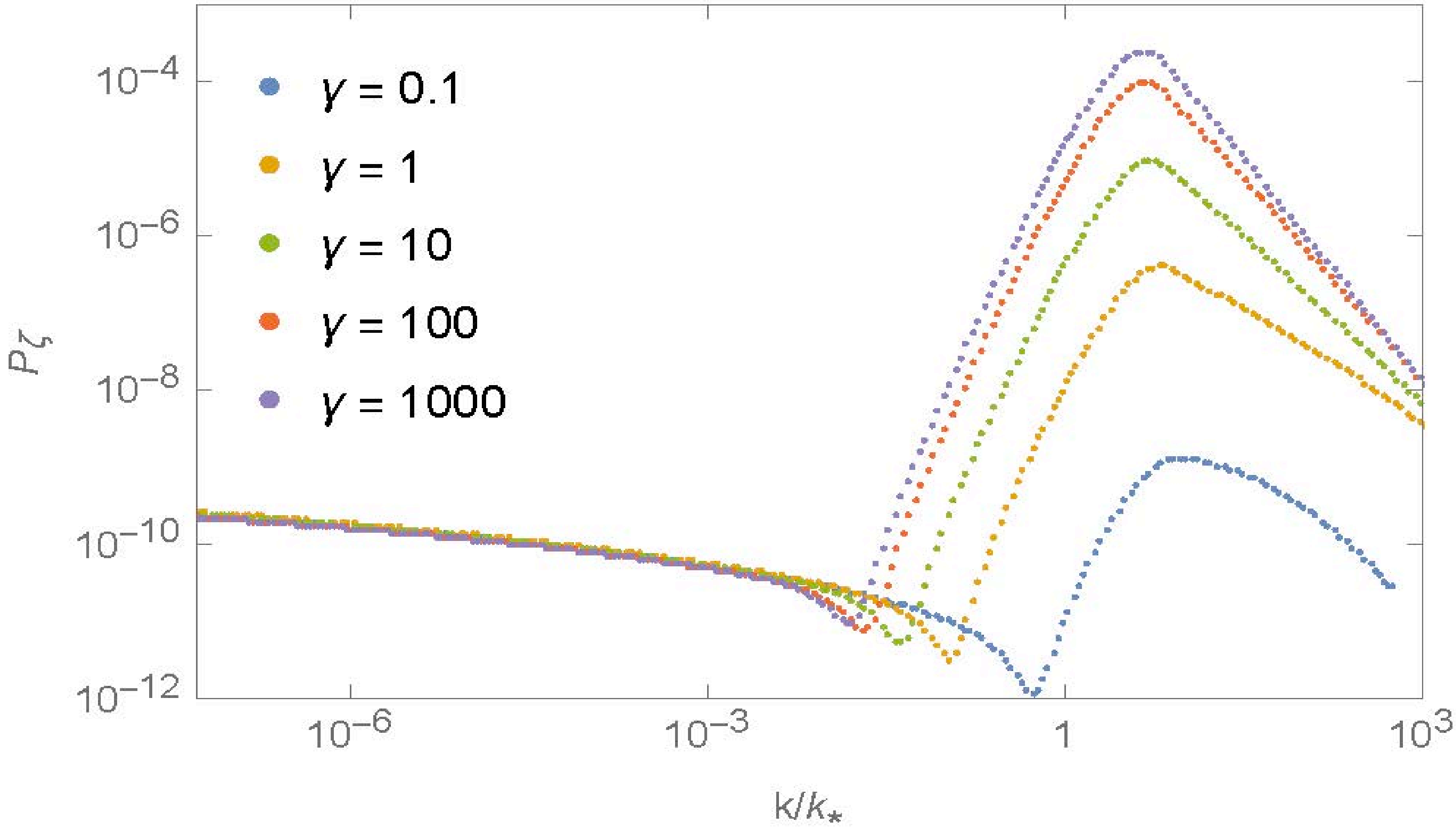


Our computational methods and strategy

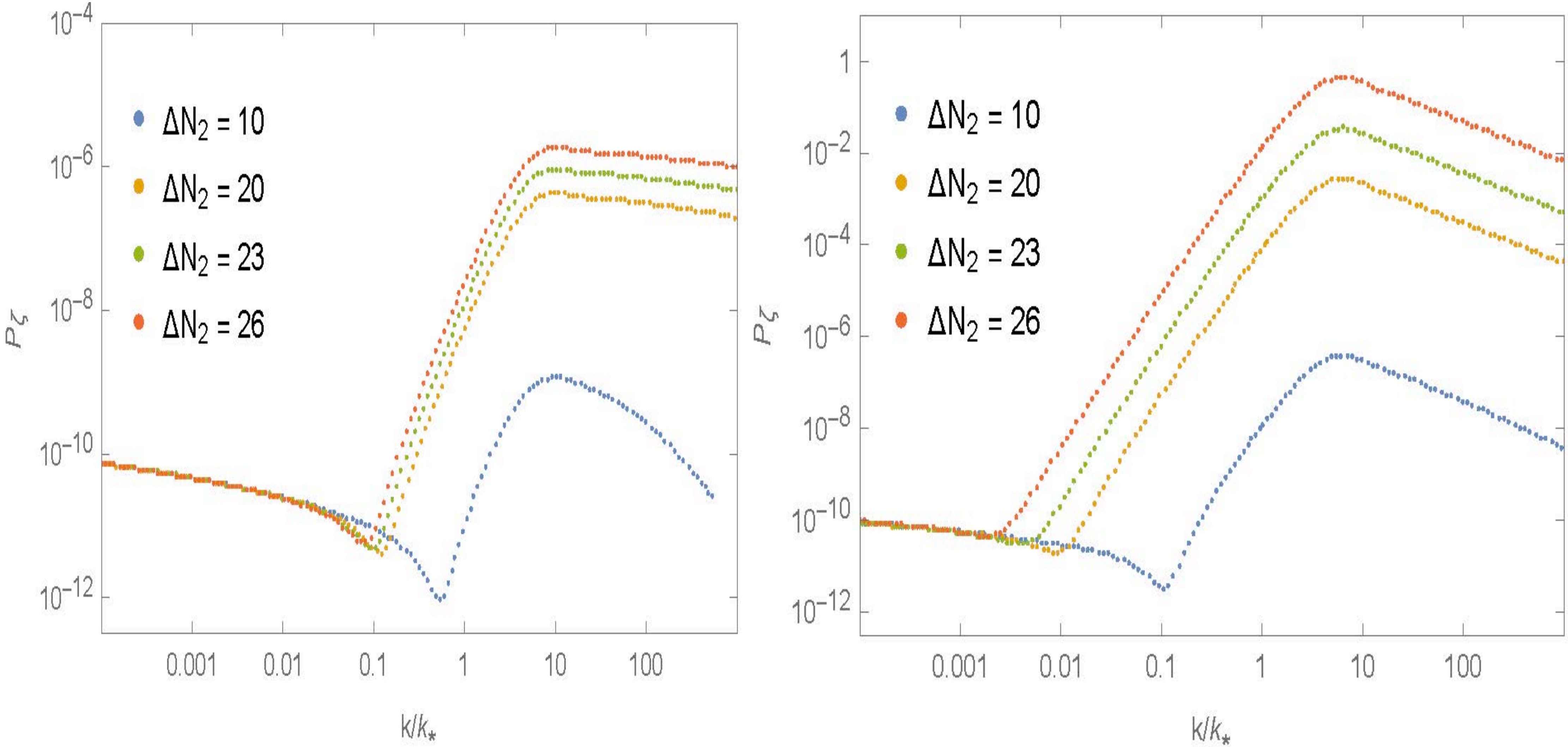
We numerically computed the power spectrum of curvature perturbations by using the *transport method* (Mulryne, 2009-2010) with the *Mathematica* package of Dias (2015), around the pivot scale k_* that leaves the horizon at the end of the first stage, i.e. $\triangle N_2$ e-folds before the end of inflation (let us call this scale $k_{\triangle N_2}$). The inflaton mass was adjusted in each case around $\sim 10^{-5} M_{\mathrm{Pl}}$ by requiring $P_\zeta \approx 2 \times 10^{-9}$ for the mode k_{60}, first studying various values of γ (at fixed $\triangle N_2$), and then various values of $\triangle N_2$ for some values of γ.

$\triangle N_2$	10	20	23	26
n_s	0.96	0.95	0.945	0.94
r_{max}	0.004	0.007	0.008	0.009

Power spectrum at $\Delta N_2 = 10$ for various values of γ



Power spectrum at $\gamma = 0.1$ (left) and $\gamma = 1$ (right) with changing ΔN_2



PBHs masses in the γ-model

The mass of PBH created by late-inflation overdensities was estimated by Pi, Zhang, Huang and Sasaki in arXiv:1712.09896:

$$M_{\rm PBH} \simeq \frac{M_{\rm Pl}^2}{H(t_{\rm peak})} \exp\left[2(N_{\rm end} - N_{\rm peak}) + \int_{t_{\rm peak}}^{t_{60}} \epsilon(t) H(t) dt\right] ,$$

where $t_{\rm peak}$ is the time when the perturbation corresponding to the power spectrum peak ($k_{\rm peak}$) exits the horizon, whereas t_{60} is the time when k_{60} exits the horizon (the beginning of observable inflation). By using this equation, we estimated the values of $M_{\rm PBH}$ for various values of ΔN_2 in our model:

ΔN_2	10	20	23	26
$M_{\rm PBH}$, g	10^8	10^{16}	10^{19}	10^{21}
n_s	0.96	0.95	0.945	0.94

Comments about the PBH masses

Our estimates are universal across the values of $\gamma = 0.1, 1, 10, 100$. PBHs with masses smaller than $\sim 10^{16}g$ would have already evaporated by now via Hawking radiation. Thus, we require $\Delta N_2 > 20$. On the other hand, the lower 3σ limit on the spectral index, $n_s \approx 0.946$, requires $\Delta N_2 < 23$, so that viable PBH masses are restricted by $\mathcal{O}(10^{16}g) < M_{\text{PBH}} < \mathcal{O}(10^{19}g)$ even before considering observational constraints on PBH masses.

As regards the constraints on γ, the obtained power spectrum tells us for $\Delta N_2 > 20$ that it is sufficient to have $\gamma \gtrsim \mathcal{O}(1)$ in order to produce the required enhancement in the spectrum.

PBH density fraction

We numerically estimated the PBH density fraction by using *Press-Schechter* (1973) formalism. The useful formulae include the PBH mass $\tilde{M}_{\text{PBH}}(k)$, the production rate $\beta_f(k)$, and the density contrast $\sigma(k)$ coarse-grained over k:

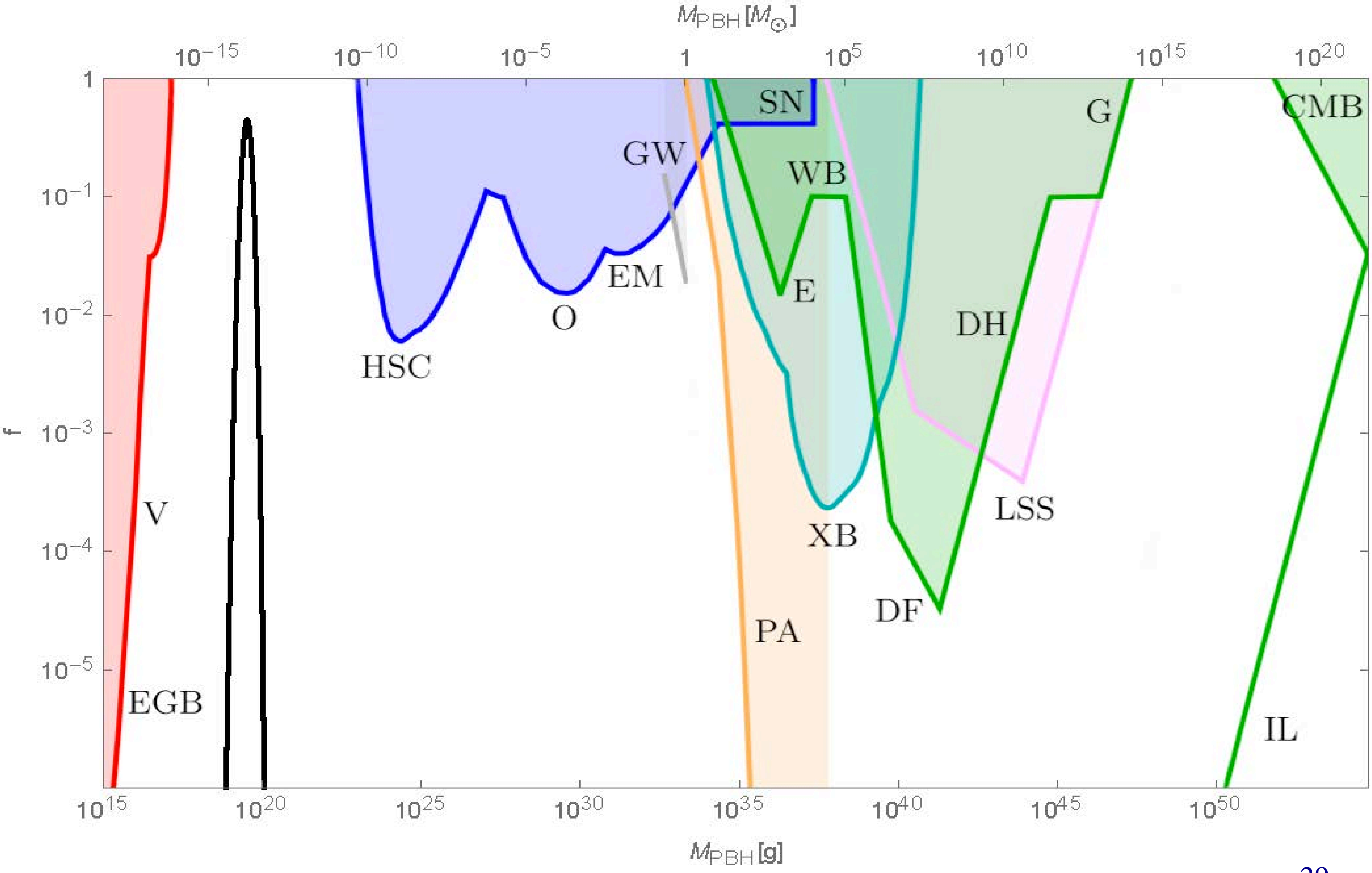
$$\tilde{M}_{\text{PBH}} \simeq 10^{20} \left(\frac{7 \times 10^{12}}{k \text{ Mpc}} \right)^2 \text{g} \,, \quad \beta_f(k) \simeq \frac{\sigma(k)}{\sqrt{2\pi}\delta_c} e^{-\frac{\delta_c^2}{2\sigma^2(k)}} \,,$$

$$\sigma^2(k) = \frac{16}{81} \int \frac{dq}{q} \left(\frac{q}{k} \right)^4 e^{-q^2/k^2} P_\zeta(q) \,.$$

We have chosen the Gaussian window function for the density contrast, and have introduced δ_c is a constant representing the density threshold for PBH formation. According to Carr (1975), the naive estimate is $\delta_c \approx 1/3$, while its more precise value depends upon details of the power spectrum. Then the PBH-to-DM density fraction is

$$\frac{\Omega_{\text{PBH}}(k)}{\Omega_{\text{DM}}} \equiv f(k) \simeq \frac{1.4 \times 10^{24} \beta_f(k)}{\sqrt{\tilde{M}_{\text{PBH}}(k) \text{g}^{-1}}} \,.$$

Comparison with observations based on Kohri et al. (2020), gamma-model



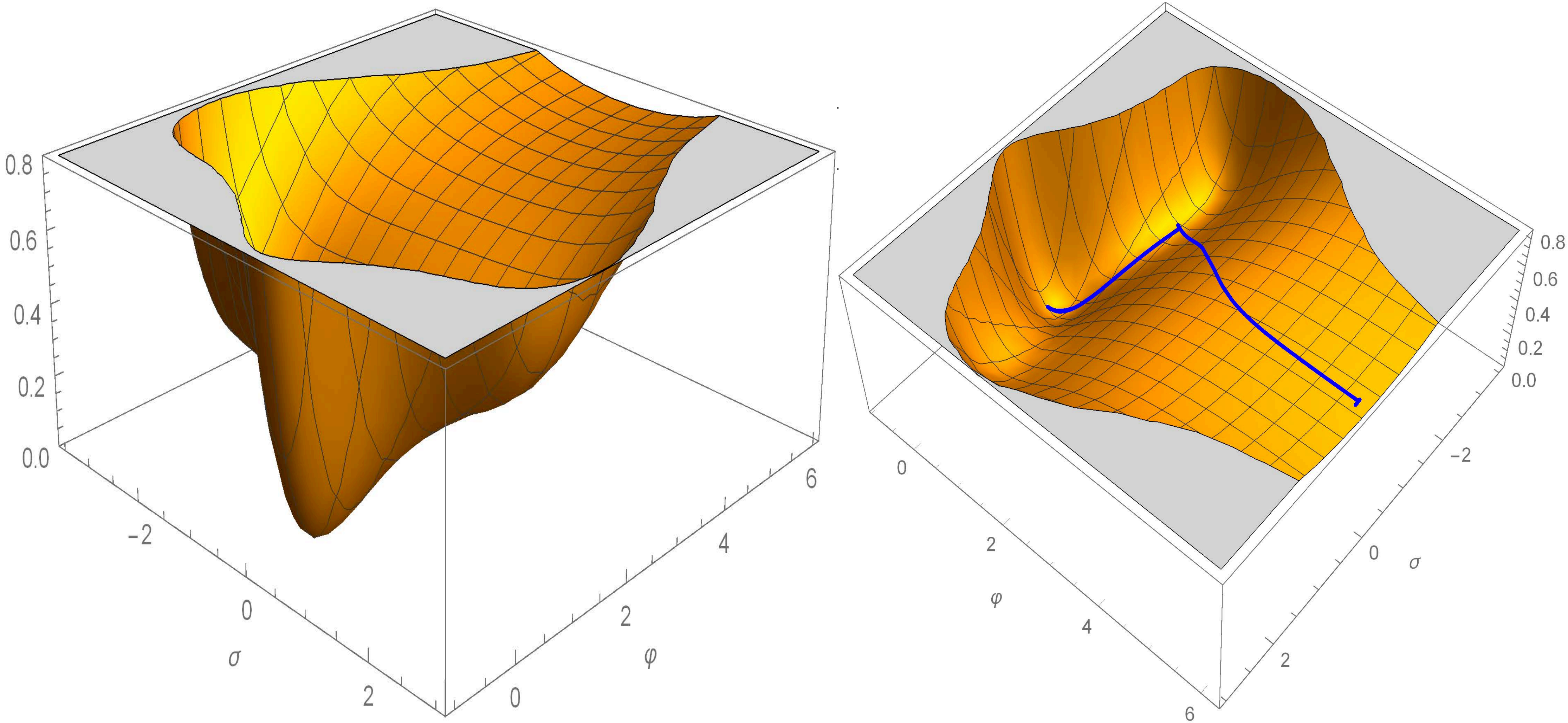
Comments on the comparison

The PBHs fraction was obtained with the parameters $\gamma = 1$, $\Delta N_2 = 22$, and $\delta_c = 0.275$ (black curve). The shaded regions represent the observational constraints: from evaporation (red), lensing (purple), various dynamical effects (green), accretion (light blue), large-scale structure (dark blue), CMB distortions (orange), and background effects (grey). In the relevant regions, the notation F, WD, and NS is used to refer to femtolensing, white dwarfs, and neutron stars, respectively.

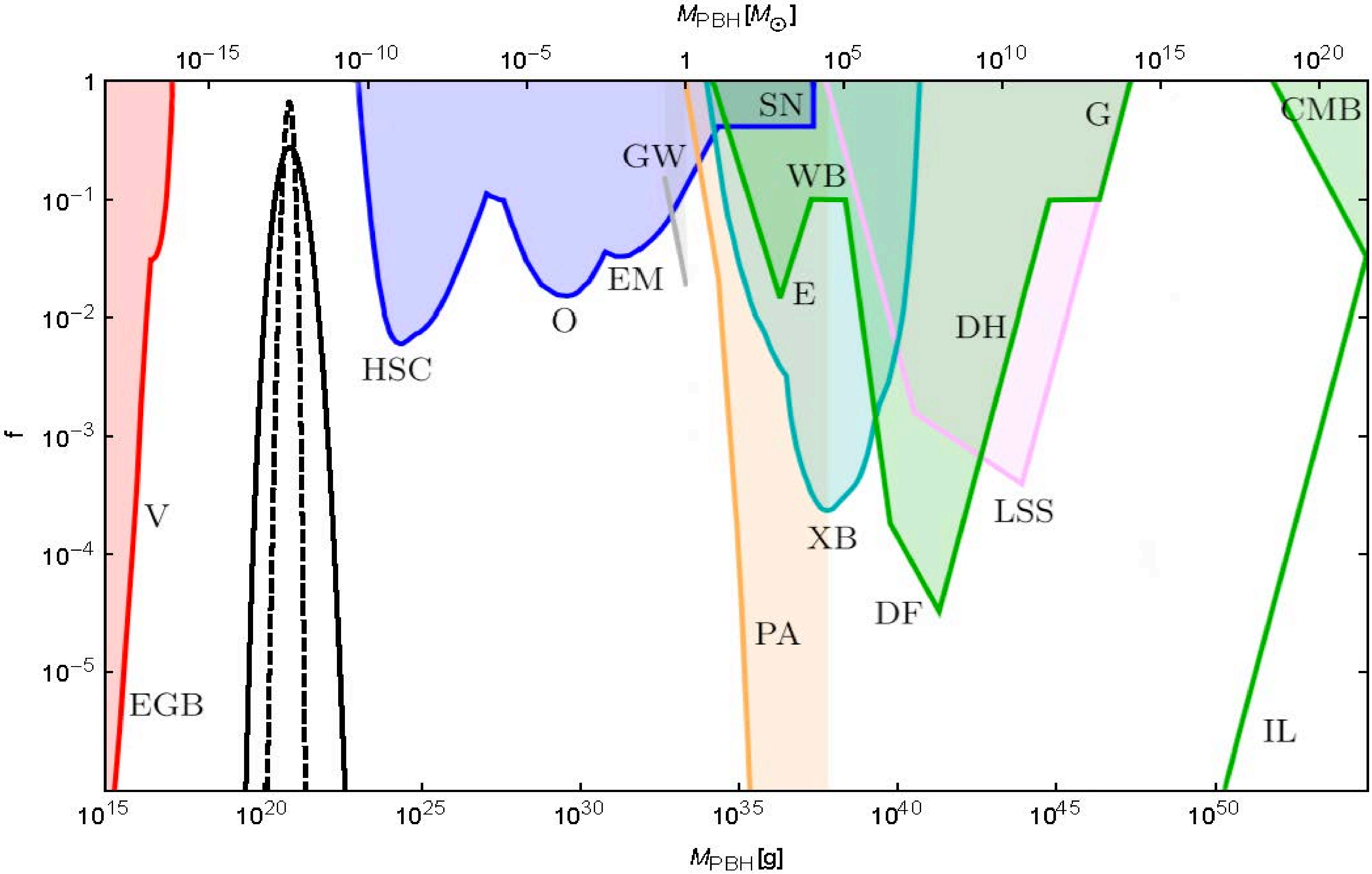
We choose the scale k_{60} to represent the largest observable scale today, which is around 10^{-4} Mpc^{-1}. Our numerical evaluation shows, in order to obtain a substantial density fraction, we need a relatively small δ_c.

Warning: a significant non-Gaussianity may change our results.

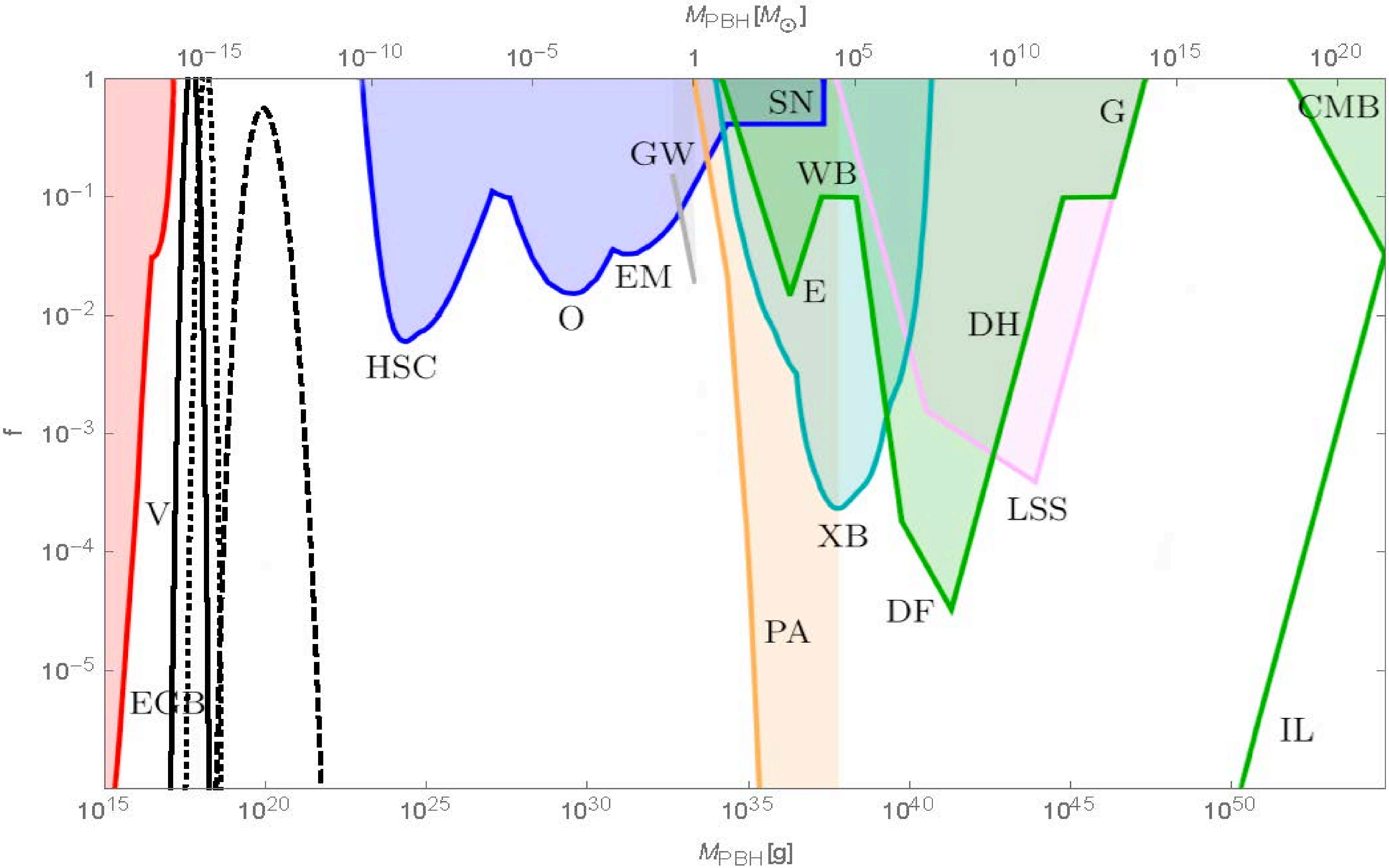
The scalar potential has only a single valley. The trajectories of solutions, Hubble functions, e-folding numbers and the slow-roll parameters are similar, as well as the power spectra, albeit with larger $\delta_c > 1/3$, and larger PBHs masses (up to 10^{23} g).



Comparison with observations (Kohri et al. 2020), the δ-model



The PBH density fraction in the models with γ=1, δ=0, ΔN_2=22.45 (solid line), and δ=0.58 , ΔN_2=23.36 (dotted line). In both cases f_total=1.



Outlook towards observations

The exploration of cosmological predictions from modified supergravity provides a remarkable bridge between quantum gravity on one side and phenomenology of inflation and PBH on the other side.

- PBHs formation necessarily leads to Gravitational Waves (GWs) because large scalar overdensities act as a source for GWs background. Frequencies of those GWs can be directly related to expected PBHs masses and duration of the second stage of inflation. *Supported by the NANOGrav 15 year data (2023).*

- Those GWs may be detected in the future ground-based experiments, such as the Einstein telescope and the *global* network of GWs interferometers including advanced LIGO, Virgo and KAGRA, as well as in the space-based GWs interferometers such as LISA (or eLISA), TAIJI (old ALIA), and DECIGO.

Aldabergenov, Addazi, SVK(2021) for a derivation of GW from modified supergravity.

Energy density of induced GW

The present-day GW density function $\Omega_{\rm GW}$ is given by (*Espinosa, Racco, Riotto, 2018*) in the 2nd order with respect to perturbations:

$$\frac{\Omega_{\rm GW}(k)}{\Omega_r} = \frac{c_g}{72}\int_{-\frac{1}{\sqrt{3}}}^{\frac{1}{\sqrt{3}}} \mathrm{d}d \int_{\frac{1}{\sqrt{3}}}^{\infty} \mathrm{d}s \left[\frac{(s^2-\frac{1}{3})(d^2-\frac{1}{3})}{s^2+d^2}\right]^2 \times P_\zeta(kx)P_\zeta(ky)\left(I_c^2+I_s^2\right) ,$$

where the constant $c_g \approx 0.4$ in the SM, and $c_g \approx 0.3$ in the MSSM.

The present-day value of the radiation density Ω_r is $h^2\Omega_r \approx 2.47\times 10^{-5}$, according to the CMB temperature. Here h is the reduced (present-day) Hubble parameter that we took as $h = 0.67$ (ignoring the Hubble tension).

The variables (x, y) are related to the integration variables (s, d) as

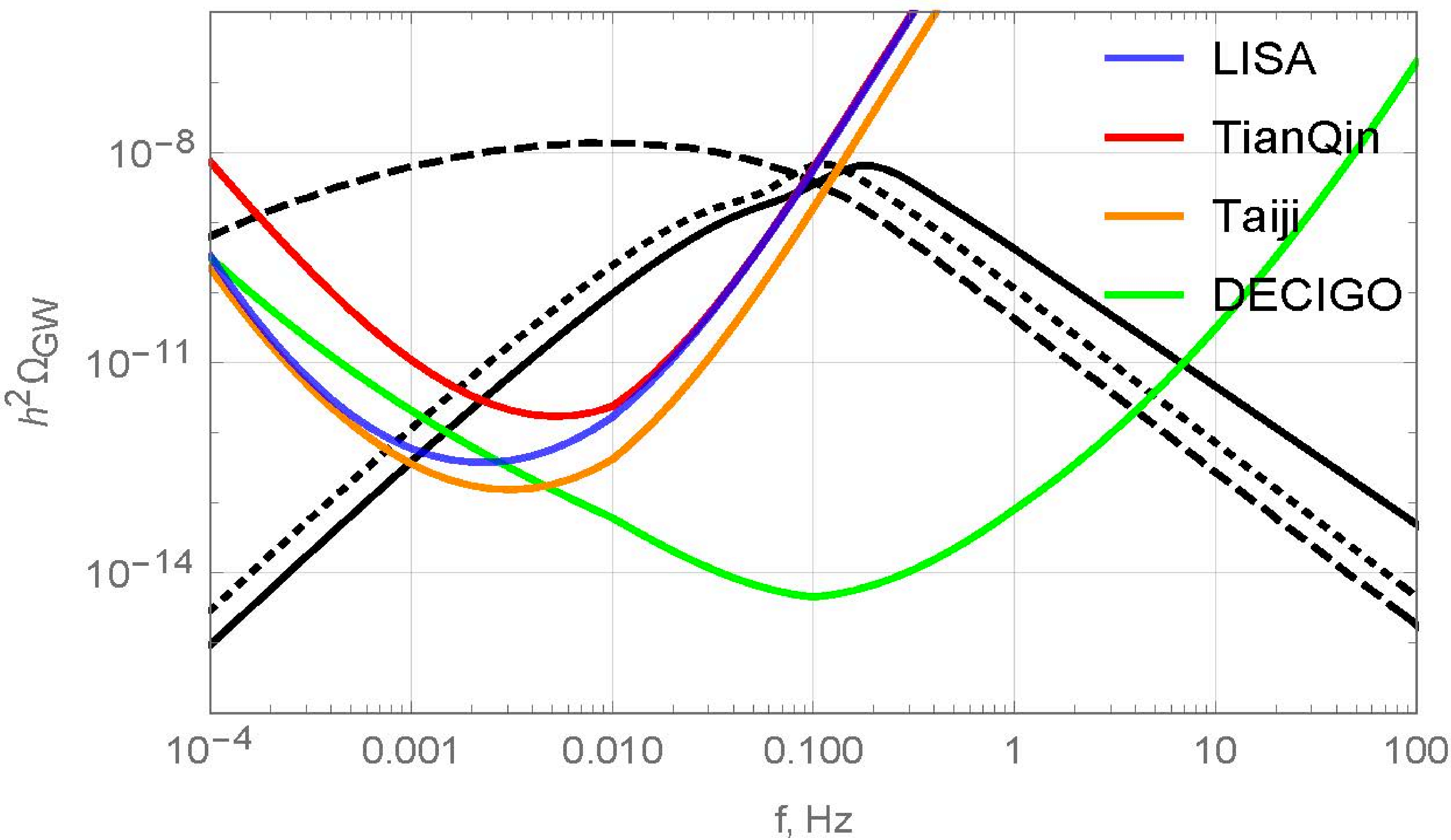
$$x = \frac{\sqrt{3}}{2}(s + d) \ , \quad y = \frac{\sqrt{3}}{2}(s - d) \ .$$

The functions I_c and I_s of $x(s, d)$ and $y(s, d)$ are (*Espinosa, Racco, Riotto, 2018*)

$$I_c = -36\pi \frac{(s^2 + d^2 - 2)^2}{(s^2 - d^2)^3} \theta(s - 1) \ ,$$
$$I_s = -36 \frac{s^2 + d^2 - 2}{(s^2 - d^2)^2} \left[\frac{s^2 + d^2 - 2}{s^2 - d^2} \ln \left| \frac{d^2 - 1}{s^2 - 1} \right| + 2 \right] \ .$$

With these definitions, the GW density can be numerically computed for a given power spectrum. In the pictures, the power-law integrated curves (*Thrane, Romano, 2013*) have been used.

The density of stochastic gravitational waves induced by the power spectrum enhancement in the our supergravity models (solid+dashed+dotted black curves) against the expected sensitivity curves of the space-based GW interferometers.



Spontaneous SUSY breaking after inflation

- can be realized 'built-in" by imposing the nilpotency condition on the chiral goldstino superfield, $S^2 = 0$ (it is equivalent to the Akulov-Volkov theory),

$$S(x, \Theta) = S + \sqrt{2}\Theta\chi + \Theta^2 F^S ,$$

with a solution $S = \chi^2/(2F^S)$, $F^S \neq 0$, which effectively eliminates two scalars (sgoldstino). *Equivalently*, one can impose the nilpotency condition on the scalar curvature chiral superfield of modified supergravity, $\mathcal{R}^2 = 0$, see Antoniadis, Dudas, Ferrara, Sagnotti (2014). However, the origin of the nilpotency condition remains obscure to me. And it can only be used below the SUSY breaking scale.

- It is possible to avoid nilpotent superfields and achieve spontaneous SUSY breaking after inflation in modified supergravity by adding a chiral matter superfield Φ (in the hidden sector), extending the potentials as

$$N(\mathcal{R}, \overline{\mathcal{R}}) \to N(\mathcal{R}, \overline{\mathcal{R}}) + J(\Phi, \overline{\Phi}) \quad \text{and} \quad \mathcal{F}(\mathcal{R}) \to \mathcal{F}(\mathcal{R}) + \Omega(\Phi) \ ,$$

and generalizing the standard (Polonyi, 1977) mechanism of spontaneous SUSY breaking without a cosmological constant, with

$$J = \bar{\Phi}\Phi - \frac{\lambda}{2}(\bar{\Phi}\Phi)^2 \quad \text{and} \quad \Omega = b\Phi + c\Phi^2 + f\Phi^3 \,,$$

while keeping all the already obtained results for inflation, primordial black holes and dark matter (Aldabergenov, SVK, 2022).

- We found it requires the super-high scale of SUSY breaking with the gravitino mass of the order 10^{12} GeV. If this massive gravitino is LSP, then it is the particle dark matter also. Given the composite dark matter (PBH+gravitino), fine-tuning in our supergravity models is significantly relaxed.

Conclusion

- Modified (Starobinsky-like) supergravity is theoretically well-motivated and provides the single source for (i) viable inflation (consistent with the CMB measurements), (ii) production of asteroid-size primordial black holes with the masses up to 10^{21} g, (iii) spontaneous SUSY breaking after inflation and (iv) current dark matter.
- Our approach is based on assuming the supergravitational origin of inflation, primordial black holes and dark matter.
- The GW induced by inflation and PBH formation are within the reach (the tensor-to-scalar-ratio) of the near-future detectors (LiteBIRD, etc.), and the sensitivity curves of the future space-based gravitational interferometers (LISA, DECIGO, etc.), respectively.
- Spontaneous SUSY breaking after inflation is possible without using the nilpotent superfields and without a cosmological constant via an extension of the Polonyi mechanism to modified supergravity.

Thank You for Your Attention!