

A Generalised Missing Partner Mechanism for $SU(5)$ GUT Inflation

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arXiv:23xx.xxxx (S. Antusch, K. Hinze, S. Saad and JS)

Introduction

Motivation:

- Explaining cosmic inflation in a SUSY GUT
→ Solving the Monopole Problem
- “Desirable” -features $SU(5)$ GUT
 - Unification of gauge couplings
 - Realistic fermion masses
 - No prompt nucleon decay
 - “No fine-tuning” (e.g. DT-splitting)
 - ...

Outline:

- ① Unification of Gauge Couplings
- ② Natural Multiplet Splitting
- ③ Fermion Masses
- ④ Phenomenology of Light Relics

Inflation in SUSY GUTs



$$W = S(f_1(\Phi) - M^2) + f_2(\Phi)f_3(N_i)$$

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Examples:

- ‘New Inflation’ $W = S(\Phi^n - M^2)$ [Rehman, Abid, Ejaz ’18]
- ‘Pseudosmooth Tribrid’ $W = S(\Phi^5 - M^2) + \Phi^3 N_i N_j$ [Masoud, Rehman, Shafi ’19]
- Other variants as ‘Shifted’ or ‘Smooth Hybrid’ & more...

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The structure is often protected by a $U(1)_R$:

$$W \rightarrow e^{i2\varphi} W, \quad S \rightarrow e^{i2\varphi} S, \quad f_1(\Phi) \rightarrow f_1(\Phi) \quad \text{and}$$

$$f_2(\Phi)f_3(N_i) \rightarrow e^{i2\varphi} f_2(\Phi)f_3(N_i)$$

But $U(1)_R$ symmetric GUTs always have mass-less states [Barr, Kye, Shafi ’05 & Fallbacher, Ratz, Vaudrevange ’11]

Light Octet and Triplet



In the case of $SU(5)$ GUTs: $\mathcal{O}(8,1)_0 \oplus \mathcal{T}(1,3)_0 \subset \Phi(24)$

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SUSY breaking effects [Dvali, Lazarides, Shafi '97]:

$$\mathcal{L} \sim m_{3/2} (\tilde{\mathcal{O}}\tilde{\mathcal{O}} + \tilde{\mathcal{T}}\tilde{\mathcal{T}}) + H.c.$$

Light Octet and Triplet

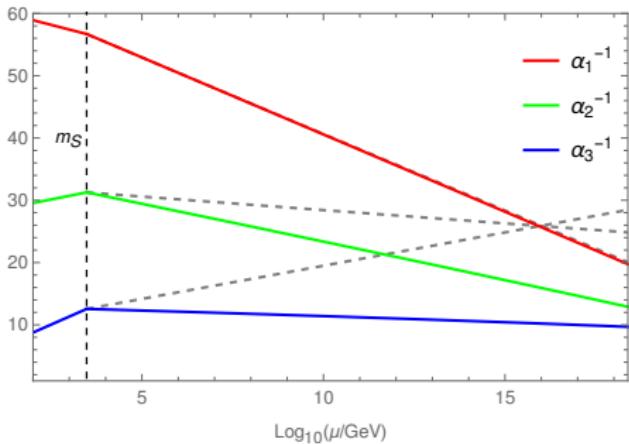
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This changes the one-loop beta-function above the SUSY-scale:

$$\beta_{MSSM}^{(1)} = \begin{pmatrix} 33/5 \\ 1 \\ -3 \end{pmatrix} \rightarrow$$

$$\beta_{MSSM}^{(1)} + \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 33/5 \\ 3 \\ 0 \end{pmatrix}$$

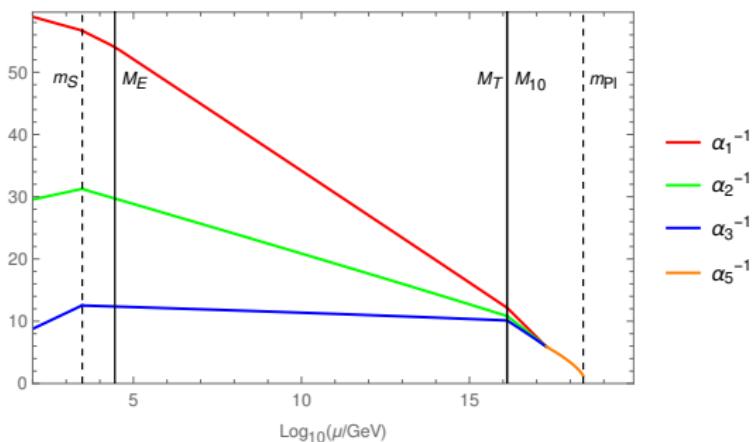


Gauge Coupling Unification (GCU)

Solution: Adding new particles, e.g. [Khalil, Rehman, Shafi, Zaakouk '10]

$$\begin{aligned}\bar{5}' + 5' &= \{\bar{D}'(1,2)_{-1/2} \oplus \bar{T}'(\bar{3},1)_{1/3}\} + \{D'(1,2)_{1/2} \oplus T'(3,1)_{-1/3}\}, \\ \bar{10}_I + 10_I &= \{\bar{E}_I^c(1,1)_{-1} \oplus \bar{Q}_I(\bar{3},2)_{-1/6} \oplus \bar{U}_I^c(3,1)_{2/3}\} \\ &\quad + \{E_I^c(1,1)_1 \oplus Q_I(3,2)_{1/6} \oplus U_I^c(\bar{3},1)_{-2/3}\}\end{aligned}$$

with $I = 1, 2$, and additionally $\bar{T}_H(\bar{3},1)_{1/3} + T_H(3,1)_{-1/3} \subset \bar{5}_H + 5_H$



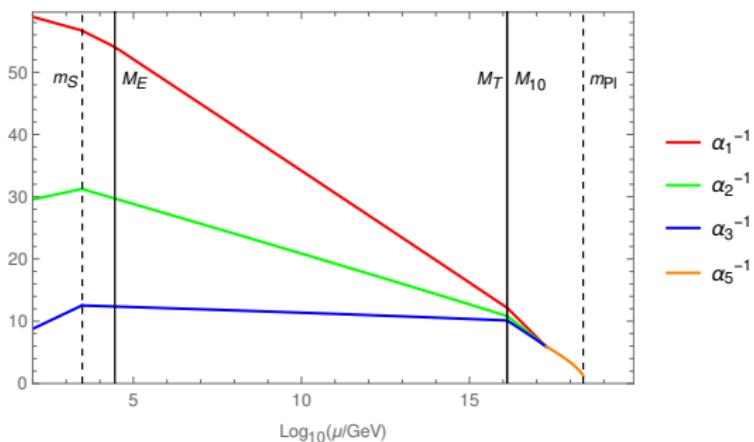
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[Antusch, Hinze, Saad, JS '23]

The Generalised Missing Partner Mechanism (GMPM)



Strategy: Missing Partner Mechanism [Masiero et al.'82 & Grinstein'82]

We generalise [Antusch, Hinze, Saad, JS '23]:

$$10_I \rightarrow E_I^c(1,1)_1 \oplus Q_I(3,2)_{1/6} \oplus U_I^c(\bar{3},1)_{-2/3}$$

Introduce 40's:

$$\begin{aligned}\overline{40}_J \rightarrow & \overline{Q}_J^{40}(\bar{3},2)_{-1/6} \oplus \overline{U}_J^{c,40}(3,1)_{2/3} \oplus \Pi_J(3,3)_{2/3} \\ & \oplus \Sigma_J(8,1)_{-1} \oplus \Psi_J(6,2)_{-1/6} \oplus \Delta_J(1,2)_{3/2}\end{aligned}$$

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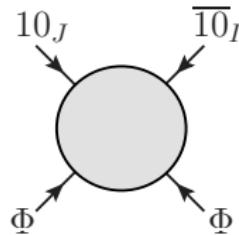
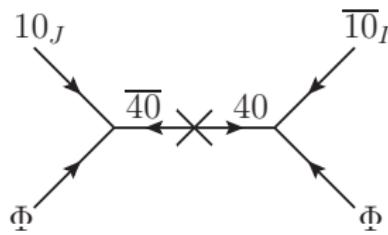
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No Partner for E^c !

$$10_J \Phi \overline{40}_I \longrightarrow v_{24} (\overline{Q}_I^{40} Q_J^{10} + \overline{U}_I^{c,40} U_J^{c,10})$$



Splitting 10-plets with GMPM

$$W_{GMPM} = (Y_{40})_{IJ} 40_I \Phi \overline{10}_J + (Y_{\overline{40}})_{IJ} \overline{40}_I \Phi 10_J + \overline{40}_I (M_{40})_{IJ} 40_J + \delta W$$

with $I, J = 1, 2$ and δW μ -term like:

$$\delta W = \overline{10}_I (m_E)_{IJ} 10_J, \quad m_{3/2} \lesssim (m_E)_{IJ} \ll M_{GUT} \ll M_{40}$$

Thus

$$\begin{aligned} W \supset & \begin{pmatrix} (\overline{Q}^{10})^T & (\overline{Q}^{40})^T \end{pmatrix} \begin{pmatrix} m_E & \nu_{24} Y_{40}^T \\ \nu_{24} Y_{\overline{40}} & M_{40} \end{pmatrix} \begin{pmatrix} Q^{10} \\ Q^{40} \end{pmatrix} \\ & + (\overline{E}^c)^T m_E E^c + \Pi^T M_{40} \overline{\Pi} + \dots \end{aligned}$$

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- Two Vector-like electrons with mass $\sim m_E$

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- Two Vector-like $U^c \oplus Q$ states with masses $\sim M_{GUT}^2 / M_{40}$

Doublet Triplet Splitting with DMPPM



Idea: Repurpose the Double Missing Partner Mechanism [Yanagida et al.'94 & Berezhiani, Tavartkiladze '96 & Antusch et al.'14]

$$\begin{aligned} W_{DMPPM} = & \frac{\alpha_2}{\Lambda} 5'(\Phi^2)_{75} \overline{50}_X + \frac{\alpha_4}{\Lambda} \overline{5}'(\Phi^2)_{75} 50_Y \\ & + \frac{\alpha_1}{\Lambda} 5_H(\Phi^2)_{75} \overline{50}_Y + \frac{\alpha_3}{\Lambda} \overline{5}_H(\Phi^2)_{75} 50_X \\ & + M_X 50_X \overline{50}_X + M_Y 50_Y \overline{50}_Y + \underbrace{\delta W}_{\mu_H \overline{5}_H 5_H + \mu' \overline{5}' 5'} \end{aligned}$$

- H_u , H_d , D' and \overline{D}' light.
- Two T with masses $\overline{\alpha}^2 M_{\text{GUT}}^4 / (\Lambda^2 M_{X,Y})$
- All other states around $M_{X,Y}$
- $\tau_p \sim 10^{56}$ years via $D = 5$

Fermion Masses (I)

Key point: 10_a instead of 10_i and 10_j

$$W \supset \begin{pmatrix} (\bar{Q}^{10})^T & (\bar{Q}^{40})^T \end{pmatrix} \underbrace{\begin{pmatrix} m_E & v_{24} Y_{40}^T \\ v_{24} Y_{\overline{40}} & M_{40} \end{pmatrix}}_{\in \text{Mat}_{4 \times 7}(\mathbb{C})} \begin{pmatrix} Q^{10} \\ Q^{40} \end{pmatrix} + (\bar{E}^c)^T \underbrace{\begin{matrix} m_E \\ E^c \end{matrix}}_{\in \text{Mat}_{2 \times 5}(\mathbb{C})}$$

and

$$W_Y = (Y_u)_{ab} 10_a 10_b 5_H + (Y_d)_{ia} \bar{5}_i 10_a \bar{5}_H + W_\nu$$

Blockdiagonalise:

$$\begin{pmatrix} m_E & v_{24} Y_{40}^T \\ v_{24} Y_{\overline{40}} & M_{40} \end{pmatrix} = \left(\begin{array}{c|cc} \eta' & \eta & v_{24} Y_{40}^T \\ \textcolor{red}{0}_{2 \times 3} & \tilde{v}_{24} Y_{\overline{40}} & M_{40} \end{array} \right) U_{40}$$

and

$$m_E = \begin{pmatrix} \textcolor{red}{0}_{1 \times 3} & M_{E,1} & 0 \\ \textcolor{red}{0}_{1 \times 3} & M_{E,2} & M_{E,3} \end{pmatrix} U_E$$

Fermion Masses (II)

$$\left(Y_d^{MSSM} | * \right) \equiv Y_d U_{\overline{40}}, \quad \left((Y_e^{MSSM})^T | * \right) \equiv Y_d U_E$$

thus

$$Y_d^{MSSM} = \begin{pmatrix} y_1 e^{i(\varphi_{11}^{\overline{40}} + \varphi_{21}^{\overline{40}})} c_{14}^{\overline{40}} c_{15}^{\overline{40}} & \mathcal{O}(y_1) & \mathcal{O}(y_1) \\ 0 & y_2 e^{i(\varphi_{12}^{\overline{40}} + \varphi_{22}^{\overline{40}})} c_{24}^{\overline{40}} c_{25}^{\overline{40}} & \mathcal{O}(y_2) \\ 0 & 0 & y_3 e^{i(\varphi_{13}^{\overline{40}} + \varphi_{23}^{\overline{40}})} c_{34}^{\overline{40}} c_{35}^{\overline{40}} \end{pmatrix}$$

and

$$Y_e^{MSSM} = \begin{pmatrix} y_1 e^{i(\varphi_{11}^\eta + \varphi_{21}^\eta)} c_{14}^\eta c_{15}^\eta & 0 & 0 \\ \mathcal{O}(y_1) & y_2 e^{i(\varphi_{12}^\eta + \varphi_{22}^\eta)} c_{24}^\eta c_{25}^\eta & 0 \\ \mathcal{O}(y_1) & \mathcal{O}(y_2) & y_3 e^{i(\varphi_{13}^\eta + \varphi_{23}^\eta)} c_{34}^\eta c_{35}^\eta \end{pmatrix}$$



On top of standard SUSY-pheno:

1. Colour Octet $\mathcal{O}(8, 1)_0$:

Combined bounds from Cosmology [*Kang, Luty, Nasri, '06*] and Colliders [*ATLAS '22*]:

$$1250 - 2000 \text{ GeV} \lesssim m_{\mathcal{O}} \lesssim 2500 \text{ GeV}^*$$

2. Weak Triplet $\mathcal{T}(1, 3)_0$

Not over-closing the universe [*Strumia et al. '05, '07 & '09*]:

$$m_{\mathcal{T}} \lesssim 2000 \text{ (2400) GeV}^*$$

*: Assuming standard cosmology and no soft mixing with gauginos.

Phenomenology of Light Relics (II)

3. Extra Doublets $D'(1,2)_{1/2} \oplus \overline{D}'(1,2)_{-1/2}$:

Not over-closing the universe [Strumia et al. '05, '07 & '09]:

$$m_{D'} \lesssim 540 \text{ (1100) GeV}$$

Or decay to H_u & H_d

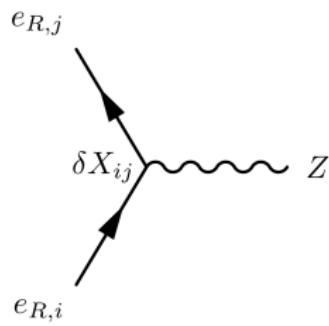
4. Electron-like states $\overline{E}^c(1,1)_{-1} \oplus E^c(1,1)_1$

Collider-bounds [CMS '22]:

$$\text{Excluded for } m_E \in [125, 150] \text{ GeV}$$

Z^μ -Physics:

$$\mathcal{L} \supset (\delta X_{ij}) e_{R,i}^\dagger \bar{\sigma}^\mu e_{R,j} Z_\mu \quad \Longleftrightarrow$$



Conclusion

$$\begin{aligned}
 W = & S(f_1(\Phi) - M^2) + f_2(\Phi)f_3(N_i) \\
 & + (Y_u)_{ab}10_a10_b5_H + (Y_d)_{ia}\bar{5}_i10_a\bar{5}_H + W_\nu \\
 & + \frac{\alpha_2}{\Lambda}5'(\Phi^2)_{75}\bar{50}_X + \frac{\alpha_4}{\Lambda}\bar{5}'(\Phi^2)_{75}50_Y + \frac{\alpha_1}{\Lambda}5_H(\Phi^2)_{75}\bar{50}_Y \\
 & + \frac{\alpha_3}{\Lambda}\bar{5}_H(\Phi^2)_{75}50_X + M_X50_X\bar{50}_X + M_Y50_Y\bar{50}_Y \\
 & + (Y_{40})_{IJ}40_I\Phi\bar{10}_J + (Y_{\bar{40}})_{IJ}\bar{40}_I\Phi10_J + \bar{40}_I(M_{40})_{JI}40_J + \delta W
 \end{aligned}$$

- Inflation
- no monopoles
- GCU can be achieved without fine-tuning
- States *required* for GCU can suppress proton decay and correct fermion masses
- ‘Window’ of light energy relics

Questions



Questions?



Backup

Why are \mathcal{O} and \mathcal{T} Light?

$$W = S(f_1(\Phi) - M^2) + f_2(\Phi)f_3(N_i)$$

Typical Vacuum:

$$-F_S^* = f_1(\langle \Phi \rangle) - M^2 = 0, \quad \langle S \rangle = 0 \quad \text{and} \quad f_3(\langle N_i \rangle) = 0$$

Decomposition of the adjoint:

$$\Phi(24) \rightarrow \mathcal{O}(8,1)_0 \oplus \mathcal{T}(1,3)_0 \oplus X(3,2)_{-5/6} \oplus \overline{X}(\bar{3},2)_{5/6} \oplus \phi(1,1)_0$$

Triplet and Octet masses:

$$\begin{aligned} \int d^2\theta W \supset & S \left[\tilde{f}_{1,\mathcal{O}}(\langle \Phi \rangle) \tilde{\mathcal{O}} \tilde{\mathcal{O}} + \tilde{f}_{1,\mathcal{T}}(\langle \Phi \rangle) \tilde{\mathcal{T}} \tilde{\mathcal{T}} \right] \\ & + f_3(N_i) \left[\tilde{f}_{2,\mathcal{O}}(\langle \Phi \rangle) \tilde{\mathcal{O}} \tilde{\mathcal{O}} + \tilde{f}_{2,\mathcal{T}}(\langle \Phi \rangle) \tilde{\mathcal{T}} \tilde{\mathcal{T}} \right] \end{aligned}$$

\Rightarrow mass-less \mathcal{O} and \mathcal{T}

Doublet Triplet Splitting with DMPPM (Detailed)



$$\begin{aligned}
 W_{DMPPM} = & \frac{\alpha_2}{\Lambda} 5'(\Phi^2)_{75} \overline{50}_X + \frac{\alpha_4}{\Lambda} \overline{5}'(\Phi^2)_{75} 50_Y \\
 & + \frac{\alpha_1}{\Lambda} 5_H (\Phi^2)_{75} \overline{50}_Y + \frac{\alpha_3}{\Lambda} \overline{5}_H (\Phi^2)_{75} 50_X \\
 & + M_X 50_X \overline{50}_X + M_Y 50_Y \overline{50}_Y + \underbrace{\delta W}_{\mu_H \overline{5}_h 5 + \mu' \overline{5}' 5'}
 \end{aligned}$$

$W \supset$

$$(H_u \quad D') \begin{pmatrix} \mu_H & 0 \\ 0 & \mu' \end{pmatrix} \begin{pmatrix} H_d \\ \overline{D}' \end{pmatrix}$$

$$+ (T_H \quad T' \quad T_X \quad T_Y) \begin{pmatrix} \mu_H & 0 & 0 & \alpha_1 v_{24}^2 / \Lambda \\ 0 & \mu' & \alpha_2 v_{24}^2 / \Lambda & 0 \\ \alpha_3 v_{24}^2 / \Lambda & 0 & M_X & 0 \\ 0 & \alpha_4 v_{24}^2 / \Lambda & 0 & M_Y \end{pmatrix} \begin{pmatrix} \overline{T}_H \\ \overline{T}' \\ \overline{T}_X \\ \overline{T}_Y \end{pmatrix}$$

Proton Decay



D=5

$$\mathcal{M}_T = \begin{pmatrix} \mu_H & 0 & 0 & \alpha_1 v_{24}^2 / \Lambda \\ 0 & \mu' & \alpha_2 v_{24}^2 / \Lambda & 0 \\ \alpha_3 v_{24}^2 / \Lambda & 0 & M_X & 0 \\ 0 & \alpha_4 v_{24}^2 / \Lambda & 0 & M_Y \end{pmatrix}$$

$$\Gamma_p \approx |\mathcal{A}|^2 m_p^5, \quad \mathcal{A}_{D=5} \approx \frac{g_5^2}{(4\pi)^2} \frac{y_u y_d}{M_T^{eff} m_S}$$

with

$$(\mathcal{M}_T^{-1})_{11} = M_T^{eff} \sim \frac{v_{24}^8 \bar{\alpha}^4}{\mu' M_X M_Y \Lambda^4} \approx M_T \frac{M_T}{M_{D'}}$$

D=6

$$\Gamma \sim \frac{m_p^5}{M_{GUT}^4} \quad \Rightarrow \quad \tau_p \sim 10^{37} \text{ years}$$

Shaping Symmetries (I)



	$SU(5)$	$U(1)_R$	\mathbb{Z}_6	\mathbb{Z}_6	\mathbb{Z}_6	\mathbb{Z}_2	\mathbb{Z}_5
S	1	2
Φ	24	.	1
5_H	5	.	4
$\bar{5}_H$	$\bar{5}$.	1	.	.	1	3
$\bar{5}_i$	$\bar{5}$	1	4	.	.	1	2
10_a	10	1	1
$5'$	5	2	.	1	.	.	3
$\bar{5}'$	$\bar{5}$	2	.	1	.	.	1
50_X	50	2	3	.	.	1	2
$\bar{50}_X$	$\bar{50}$.	4	5	.	.	2
50_Y	50	.	4	5	.	.	4

Shaping Symmetries (II)



	$SU(5)$	$U(1)_R$	\mathbb{Z}_6	\mathbb{Z}_6	\mathbb{Z}_6	\mathbb{Z}_2	\mathbb{Z}_5
$\overline{50}_Y$	$\overline{50}$	2
Ω_X	1	.	5	1	.	1	1
Ω_Y	1	.	2	1	.	.	1
$\overline{10}_I$	$\overline{10}$	1	3	.	1	.	1
40_I	40	1	2	.	5	.	4
$\overline{40}_I$	$\overline{40}$	1	4
Ξ	1	.	.	.	1	.	1
Z_H	1	.	5	.	.	1	3
Z_E	1	2	4	.	1	.	1
Z'	1	4	.	2	.	.	4

Shaping Symmetries (III)

$$\begin{aligned}
 W = & S(\Phi^6 - M^2) \\
 & + (Y_u)_{ab} 10_a 10_b 5_H + (Y_d)_{ia} \bar{5}_i 10_a \bar{5}_H + W_\nu \\
 & + \frac{\alpha_2}{\Lambda} 5'(\Phi^2)_{75} \bar{50}_X + \frac{\alpha_4}{\Lambda} \bar{5}'(\Phi^2)_{75} 50_Y + \frac{\alpha_1}{\Lambda} 5_H(\Phi^2)_{75} \bar{50}_Y \\
 & + \frac{\alpha_3}{\Lambda} \bar{5}_H(\Phi^2)_{75} 50_X + \Omega_X 50_X \bar{50}_X + \Omega_Y 50_Y \bar{50}_Y \\
 & + (Y_{40})_{IJ} 40_I \Phi \bar{10}_J + (Y_{\bar{40}})_{IJ} \bar{40}_I \Phi 10_J + \Xi \bar{40}_I (y_{40})_{JI} 40_J + ...
 \end{aligned}$$

and

$$K = K_0 + Z_H^\dagger 5_H \bar{5}_H + Z_E^\dagger \bar{10}_I 10_a + Z'^\dagger 5' \bar{5}' + c.c. + ...$$