

Electroweak Phase Transition and Prospects of Detecting Gravitational Waves in an Extended Supersymmetric Model

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Based on: [arXiv:2301.05061](https://arxiv.org/abs/2301.05061)

SUSY 2023

University of Southampton Highfield Campus
July 17-21, 2023

What Standard Model of Particle Physics cannot answer

- No explanation for non-zero neutrino masses and mixing.
- No viable Dark Matter candidate.
- More matter than antimatter.

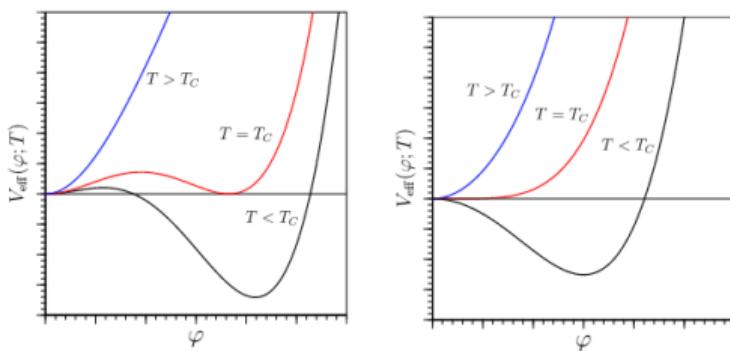


Image courtesy: online sources

Electroweak Phase Transition (EWPT)

↓
(strong) 1st order

Electroweak Baryogenesis (EWBG)

↓

Baryon Asymmetry of the Universe (BAU)

- In the SM, EWPT is smooth cross-over.

K. Kajantie et al., Phys. Rev. Lett. 77 (1996) 2887

- $V(\phi, T) = \frac{1}{2}(-\mu_h^2 + cT^2)\phi^2 + \frac{\lambda}{4}\phi^4 - ET\phi^3$,
 $ET\phi^3 \propto \sum_i m_i^3(\phi)$ → sum over all bosons which couple to the SM-Higgs.

■ For Higgs mass > 72 GeV, no 1st order phase transition within the SM!

Motivation for Beyond the Standard Model

What is the nature of the Electroweak Phase Transition in the early Universe?

⇒ Predictions depend on the Particle Physics Model.

- Not MSSM

- Not MSSM
- For a strong first order EWPT (SFOEWPT), it requires one lighter stop with mass $\mathcal{O}(100 \text{ GeV})$, [A. Menon and D. E. Morrissey, Phys. Rev. D 79 \(2009\)](#)

- NMSSM, nMSSM, $\mu\nu$ SSM... ?

- SFOEWPT possible → single- or multi-step.
- Detectable Gravitational Wave (GW) signature: possible in some variants of NMSSM, e.g., split NMSSM, nMSSM; not fully realized in Z_3 -NMSSM, $\mu\nu$ SSM, etc. [Huber, Stephan J. et al JCAP 05 \(2008\) 017](#), [Chung, D. J. H. and Long, A. J. Phys.Rev.D 81 \(2010\) 123531](#), [Kozaczuk, J. et al JHEP 01 \(2015\) 144](#), [Demidov, S. V. et al Phys.Lett.B 779 \(2018\) 191-194](#), [Chatterjee, A. et al JHEP 06 \(2022\) 108](#) and many more...

✗ The above mentioned models **cannot** incorporate non-zero neutrino masses and mixing (except $\mu\nu$ SSM). [Chung, D. J. H. and Long, A. J. Phys.Rev.D 81 \(2010\) 123531](#)

NMSSM + a Right Handed Neutrino (RHN)

NMSSM + RHN is a well motivated model. [Kitano and Oda, Phys.Rev.D 61 \(2000\)](#)

- ✓ neutrino masses and mixing [Das, D. and Roy, S. Phys.Rev.D 82 \(2010\) 035002](#)
- ✓ viable DM candidate (e.g., R-handed sneutrino) [Cerdeno, D. G. et al Phys.Rev.D 79 \(2009\) 023510, etc.](#)
- EWPT and GW signature → *in this work.*

NMSSM + an RHN Superpotential

$$\begin{aligned} W &= W'_{\text{MSSM}}(\mu = 0) + \lambda \widehat{S} \cdot \widehat{H}_u \cdot \widehat{H}_d + \frac{\kappa}{3} \widehat{S}^3 + Y_N^i \widehat{N} \cdot \widehat{L}_i \cdot \widehat{H}_u + \frac{\lambda_N}{2} \widehat{S} \cdot \widehat{N} \cdot \widehat{N} \\ -\mathcal{L}_{\text{soft}} &= -\mathcal{L}'_{\text{soft}} + m_S^2 S^* S + M_N^2 \widetilde{N}^* \widetilde{N} + (\lambda A_\lambda S H_u \cdot H_d + h.c.) \\ &\quad + \left(\frac{\kappa A_\kappa}{3} S^3 + (A_N Y_N)^i \widetilde{L}_i \cdot H_u \cdot \widetilde{N} + \frac{A_{\lambda_N} \lambda_N}{2} S \widetilde{N} \widetilde{N} + h.c. \right) \end{aligned}$$

- Along with H_u , H_d and S , the RH-sneutrino, \widetilde{N} also develops a VEV (v_N) in our setup ⇒ R_P spontaneously broken.

A few Remarks

- To study the EWPT within a model, the central object to consider is the finite-temperature effective potential.

$$V_{\text{eff}}^T = V_{\text{tree}} + \Delta V + V'_{\text{CW}}^{1\text{-loop}} + V_{T \neq 0}^{1\text{-loop}} + V_{\text{ct}}$$

- ▶ Assuming all trilinear A -terms, soft-squared mass terms $\sim \mathcal{O}(TeV)$ and $\lambda_N \sim \mathcal{O}(1) \rightarrow RHN$ mass in the TeV regime.
- ▶ Neutrino mass generation via the TeV scale seesaw mechanism \rightarrow constraints $Y_N^i \sim \mathcal{O}(10^{-6} - 10^{-7})$ and $\langle \tilde{\nu}_i \rangle = \frac{4v_N Y_N^i(v_u(A_N + \lambda_N v_S) + \lambda v_d v_S)}{(g_1^2 + g_2^2)(v_d^2 - v_u^2) + 4m_{L_i}^2}$
 $\rightarrow \langle \tilde{\nu}_i \rangle \sim \mathcal{O}(10^{-3} - 10^{-4}) \text{ GeV}.$
- ▶ Tiny R_P violation ($\sim \mathcal{O}(10^{-3} - 10^{-4}) \text{ GeV}$); **weak mixing of the left-handed leptons and sleptons (neutral and charged).**

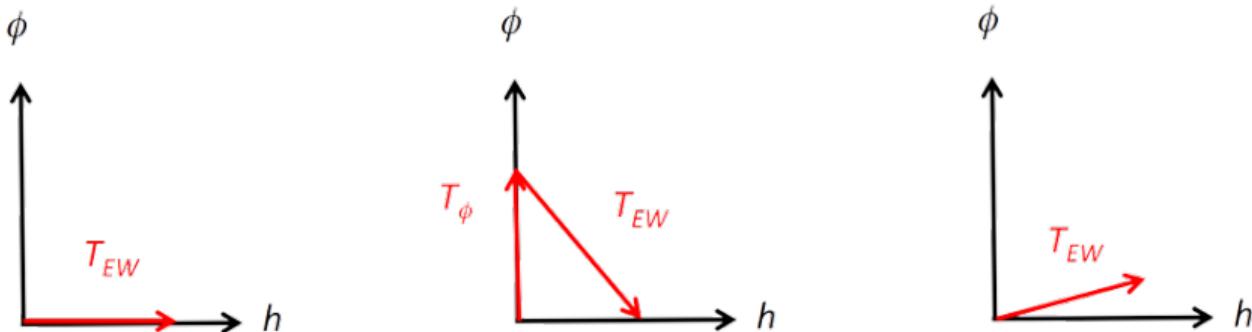
Dynamical fields \Rightarrow

H_u, H_d, S, \tilde{N}

instead of

$H_u, H_d, S, \tilde{N}, \tilde{L}_i$

Phase Transition Patterns



Representative thermal histories of EWSB in the presence of the Higgs field h and additional neutral scalar ϕ .

PT Patterns in NMSSM + an RHN

■ A few possible PT patterns:

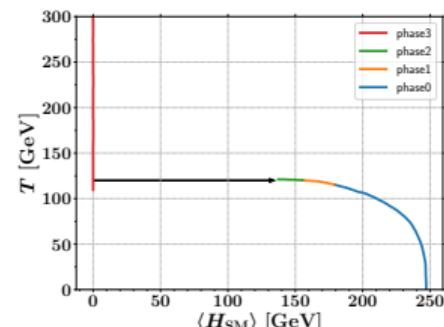
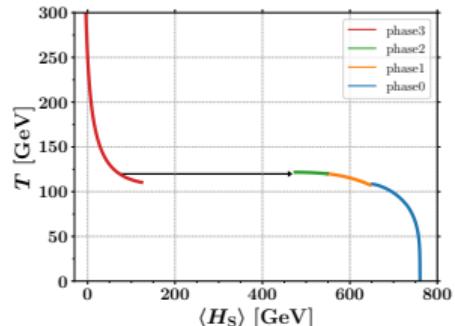
- ◆ **Type-I:** $\Omega_0 \xrightarrow{\text{PT}} \Omega_{H_{\text{SM}}}$
- ◆ **Type-IIa:** $\Omega_0 \rightarrow \Omega'_{H_S} \xrightarrow{\text{PT}} \Omega_{H_{\text{SM}}} + \Omega_{H_S}$
- ◆ **Type-IIb:** $\Omega_0 \rightarrow \Omega'_{H_S} \xrightarrow{\text{PT}} \Omega_{H_S}$
- ◆ **Type-IIc:** $\Omega_0 \rightarrow \Omega'_{H_S} \xrightarrow{\text{PT}} \Omega_{H_S} + \Omega_{\tilde{N}}$
- ◆ **Type-IIIa:** $\Omega_0 \rightarrow \Omega_{H'_{\text{SM}}} + \Omega'_{H_S} \xrightarrow{\text{PT}} \Omega_{H_{\text{SM}}} + \Omega_{H_S}$
- ◆ **Type-IIIb:** $\Omega_0 \rightarrow \Omega'_{H_{\text{SM}}} + \Omega'_{H_S} \xrightarrow{\text{PT}} \Omega_{H_{\text{SM}}} + \Omega_{H_S} + \Omega_{\tilde{N}}$

	BP-I	BP-IV
$\tan \beta$	2.90	5.77
λ	0.416	0.384
κ	0.022	0.012
λ_N	0.146	0.130
$Y_N^i \times 10^7$	0.9	1.0
A_λ [GeV]	775.48	1184.87
A_κ [GeV]	-62.74	-107.1
A_{λ_N} [GeV]	-349.68	-363.16
μ_{eff} [GeV]	224.56	203.12
v_N [GeV]	308.79	386.45
$v_i \times 10^4$ [GeV]	1.0	1.2
A_N [GeV]	-16000.0	-12000.0
$m_{h_{125}}$ [GeV]	126.02	125.63
m_H [GeV]	772.36	1213.76
m_{H_S} [GeV]	83.60	109.54
$m_{\tilde{N}}$ [GeV]	48.60	27.65
$\text{BR}(B \rightarrow X_s \gamma) \times 10^4$	3.61	3.47
$\text{BR}(B_s^0 \rightarrow \mu^+ \mu^-) \times 10^9$	3.24	3.19
$\text{BR}(\mu \rightarrow e \gamma) \times 10^{30}$	394	51.4
$\text{BR}(\mu \rightarrow eee) \times 10^{29}$	113.0	53.9
$\text{CR}(\mu N \rightarrow e N^*) \times 10^{28}$	1.81	4.43
$\Delta m_{\text{atm}}^2 \times 10^3$ eV ²	2.51	2.54
$\Delta a_\mu \times 10^{10}$	3.88	1.94

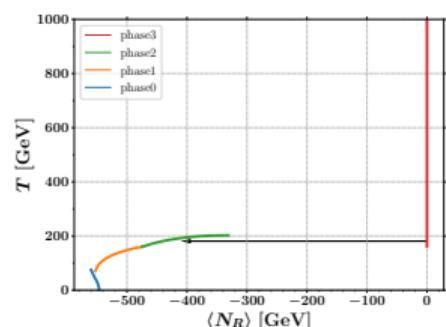
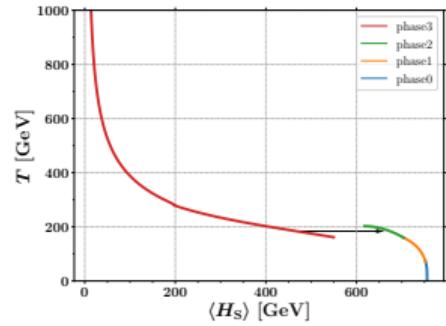
PT Results

	BP-I	BP-IV
Transition Type	Type-IIa	Type-IIc
v_c / T_c	1.30 (In) ; 0 (Out)	0.0 (In) ; 0.0 (Out)
$\Delta\phi_{SU(2)} / T_n$	1.58	0
$\Delta\phi_s / T_n$	4.70	1.01
$\Delta\phi_{\tilde{N}} / T_n$	0	2.81
T_c [GeV]	117.8	184.5
T_n [GeV]	109.9	165.8
high- T_n VEV	(0,0,113.8,0)	(0,0,529.9,0)
low- T_n VEV	(173.1,9.5,631.3,0)	(0,0,696.6,-465.28)
high- T_c VEV	(0,0,72.6,0)	(0,0,459.9,0)
low- T_c VEV	(152.9,11.8,572.5,0)	(0,0,671.2,-429.5)

- SFOPT: $\frac{v_c}{T_c} > 1.0$,
 $\frac{\Delta\phi_X}{T_n} > 1.0$; $X = SU(2), \phi_s, \phi_{\tilde{N}}$
- BP-I: Transition from high-temperature (hT) SM-like Higgs VEV, $\langle H_{SM}^{hT} \rangle = 0$ to low-temperature (lT) VEV, $\langle H_{SM}^{lT} \rangle \neq 0$



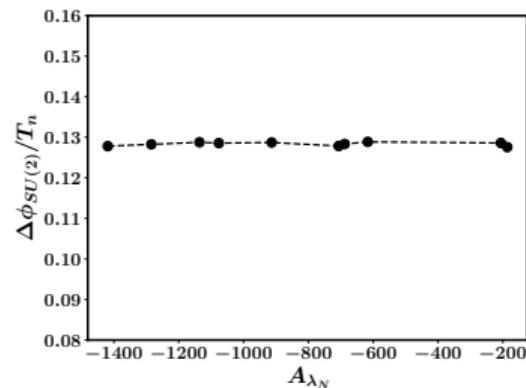
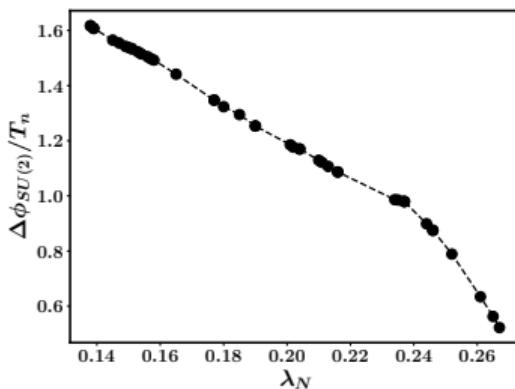
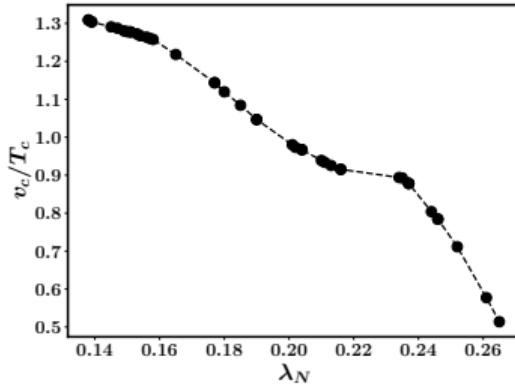
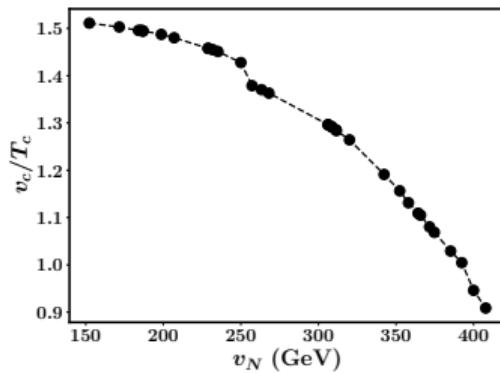
Type-IIa
SFOPT along H_{SM}, H_S direction



Type-IIc
SFOPT along H_S, \tilde{N} direction

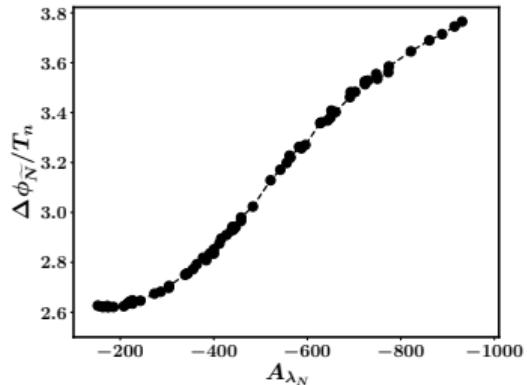
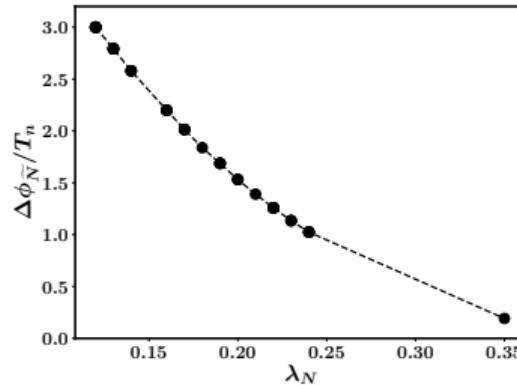
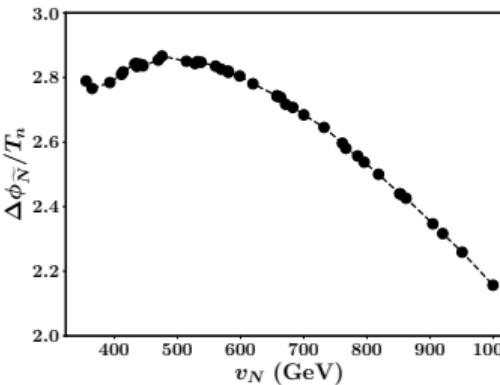
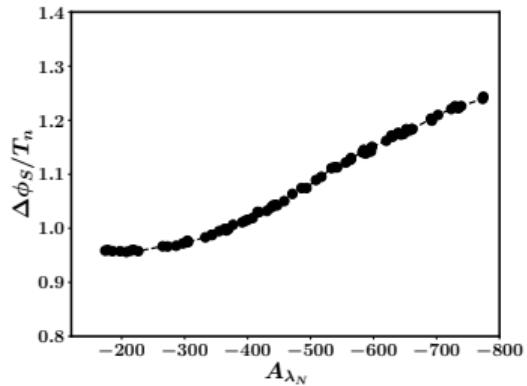
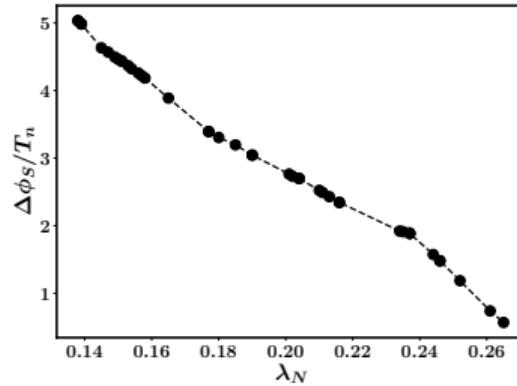
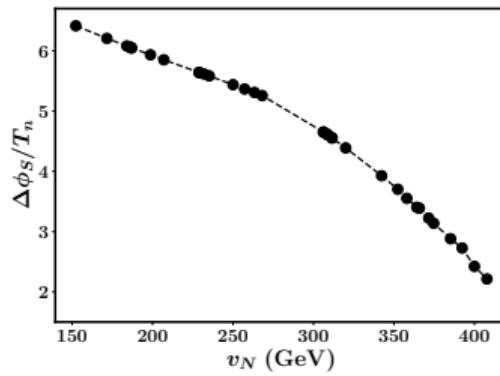
Role of the new parameters $(\lambda_N, v_N, A_{\lambda_N})$ in PT dynamics

Along $SU(2)_L$ Higgs direction



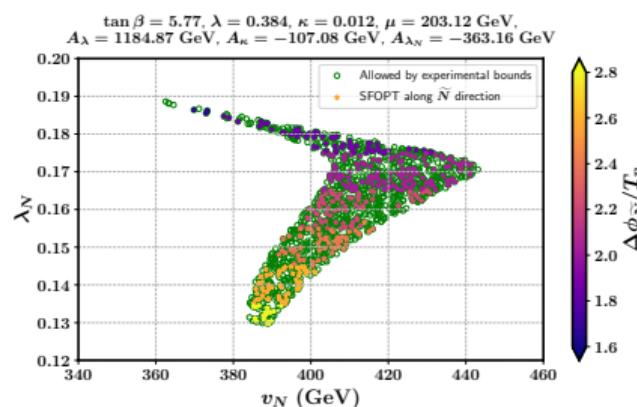
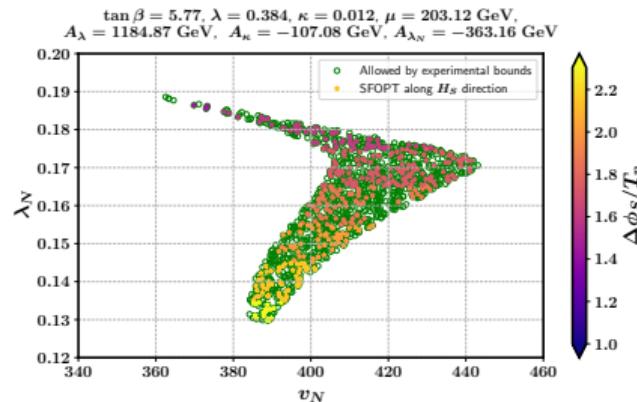
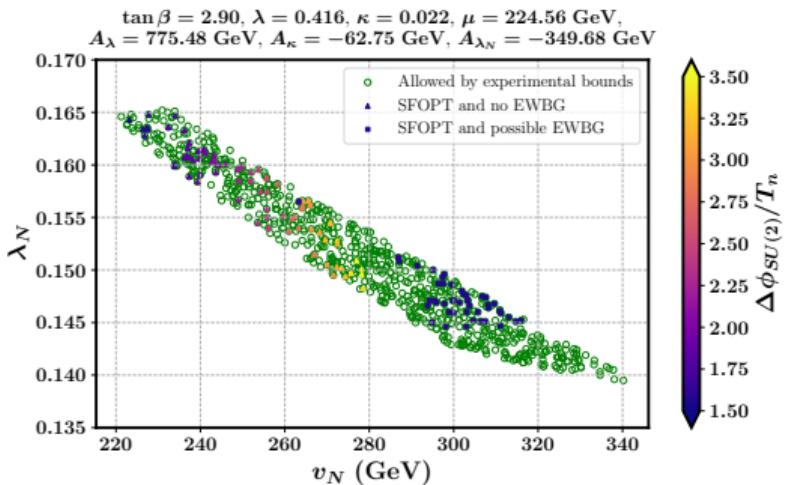
Role of the new parameters in PT dynamics

Along H_S, \tilde{N} direction



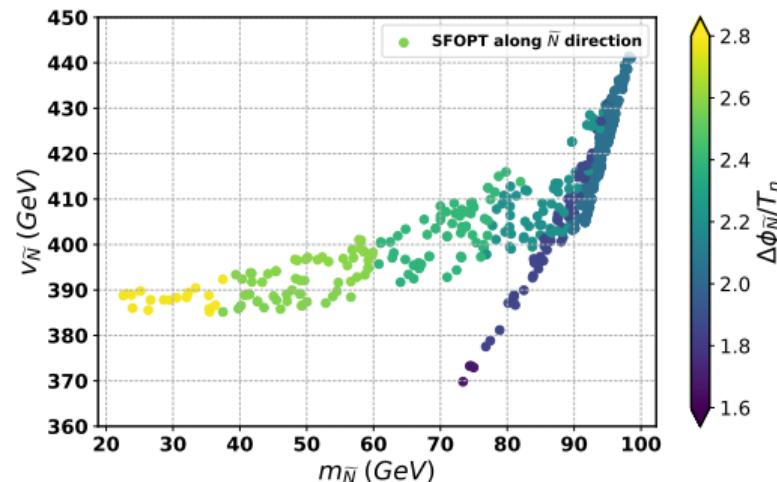
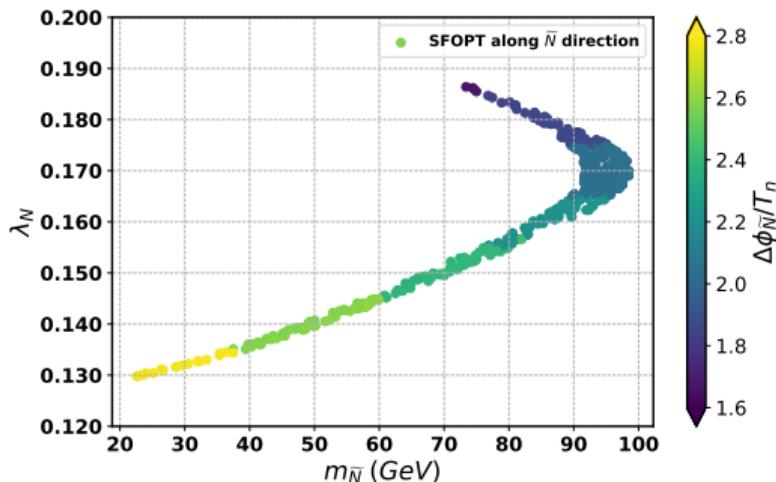
Role of the new parameters in PT dynamics

Correlations between λ_N and v_N



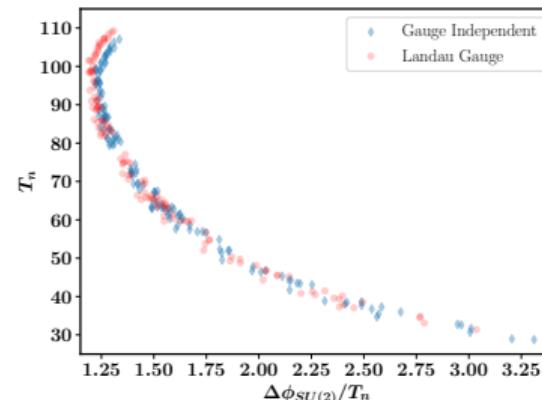
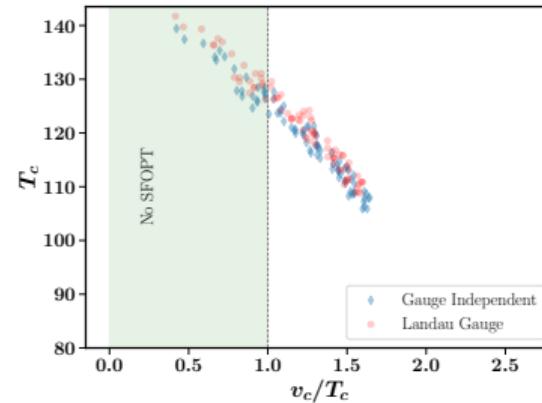
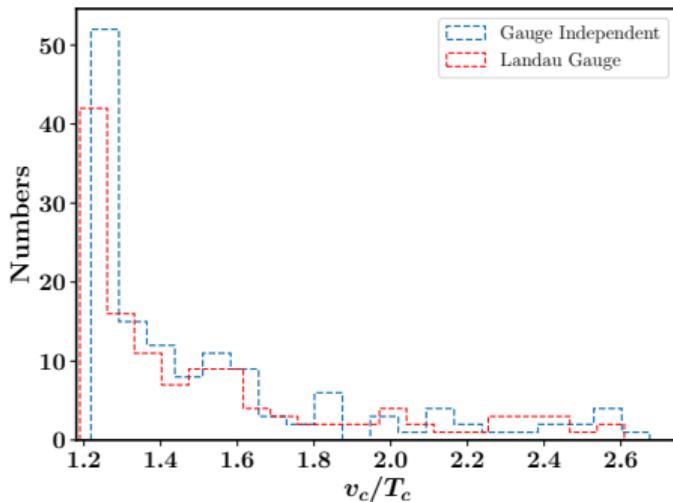
Role of the new parameters in PT dynamics

Lighter $m_{\tilde{N}}$ states and PT along \tilde{N}



- In the extended Higgs basis, $m_{\tilde{N}} \propto \lambda_N v_N$. Small λ_N and $v_N \rightarrow$ light $m_{\tilde{N}}$ states.
- We find, in our numerical scan, $m_{\tilde{N}}$ states to be below 125 GeV.
- We checked lab and theoretical constraints by modifying SPheno, HiggsBounds and CosmoTransitions.

Remarks on Gauge Dependence



- The comparison of results obtained using the gauge dependent and the gauge invariant approaches.
- We find, the estimation of T_c gets reduced by $\sim 3\%$ at most whereas v_c/T_c shows an enhancement of $\lesssim 0.1\%$ only from earlier results.

A. Katz et al., Phys. Rev. D 92 (2015) 095019

GW spectrum from SFOPT

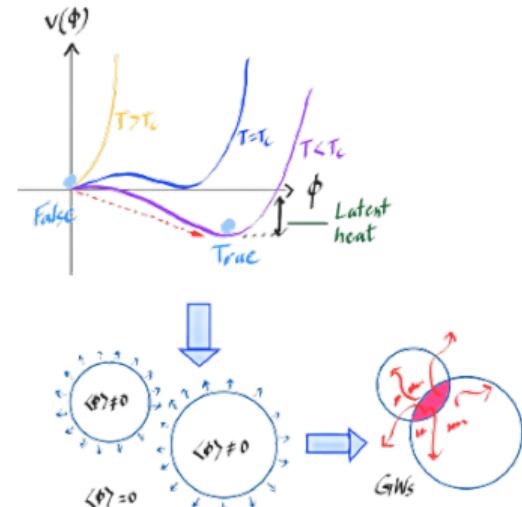
- The GWs are generated by a SFOPT in the early universe. The main production processes are,

- ▶ **Bubble collisions**
- ▶ **Sound waves left behind in thermal plasma**
- ▶ **Turbulence**

- The total GW energy budget can be obtained from 3 sources,

$$\Omega_{\text{GW}} h^2 \approx \Omega_{\text{col}} h^2 + \Omega_{\text{sw}} h^2 + \Omega_{\text{tur}} h^2$$

- $\Omega_{\text{col}} h^2 \propto \left(\frac{\beta}{H_*}\right)^{-2}$, $\Omega_{\text{sw}} h^2 \propto \left(\frac{\beta}{H_*}\right)^{-1}$, $\Omega_{\text{tur}} h^2 \propto \left(\frac{\beta}{H_*}\right)^{-1}$



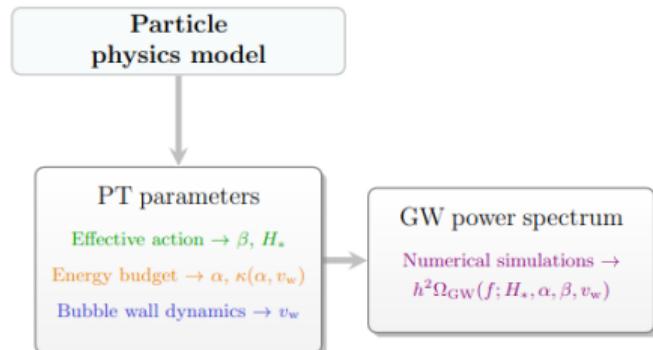
The key PT quantities for GW

Analyse the tunneling probabilities and nucleation temperatures (T_n),

- ▶ $\alpha = \frac{\Delta\rho}{\rho_{rad}}$, latent heat released by the PT process, with $\rho_{rad} = \frac{g^* \pi^2}{30} T_n^2$
- ▶ $\beta = 1/\text{duration of PT}$
- ▶ $\frac{\beta}{H_*} = \left[T \cdot \frac{d(S_E/T)}{dT} \right]_{T=T_n}$
- ▶ $v_w \rightarrow 1$

■ α and β/H_* depends on particle physics model,

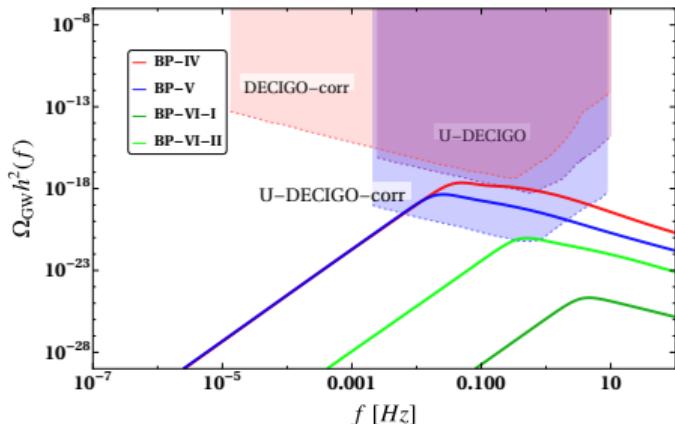
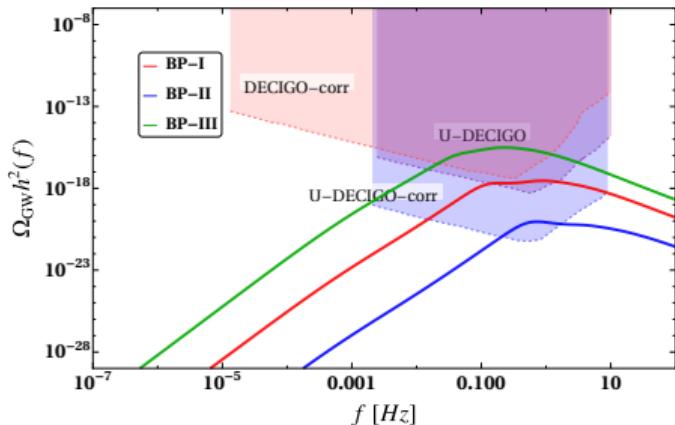
$$\Delta\rho = \left[V_{\text{eff}}^T(\phi_0, T) - T \frac{dV_{\text{eff}}^T(\phi_0, T)}{dT} \right]_{T=T_n} - \left[V_{\text{eff}}^T(\phi_n, T) - T \frac{dV_{\text{eff}}^T(\phi_n, T)}{dT} \right]_{T=T_n}$$



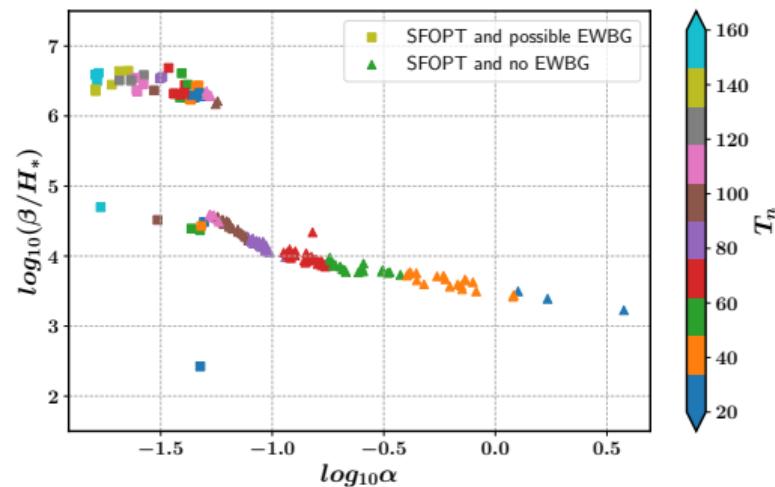
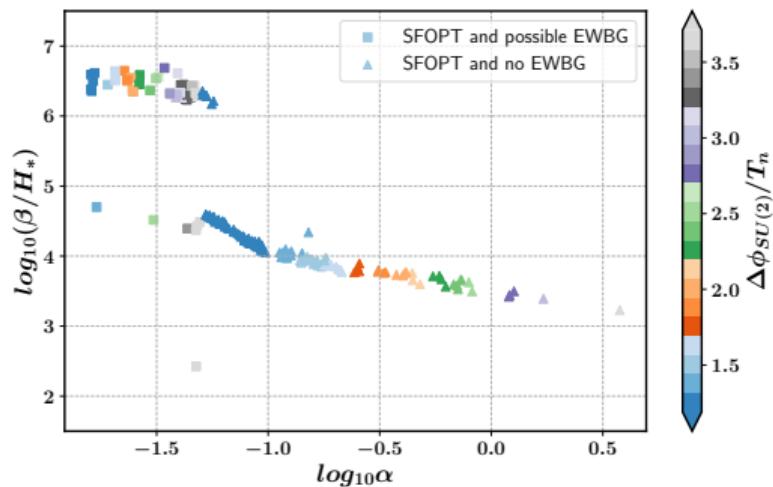
GW spectrum in NMSSM + an RHN

BP _s	α	β/H_*
BP I	0.0456	37535.2
BP IV	0.0101	7596.0

- BP-I is interesting from **EWBG** viewpoint.
→ A complementary search for BSM.
- BP-IV is due to SFOPT along \tilde{N} .

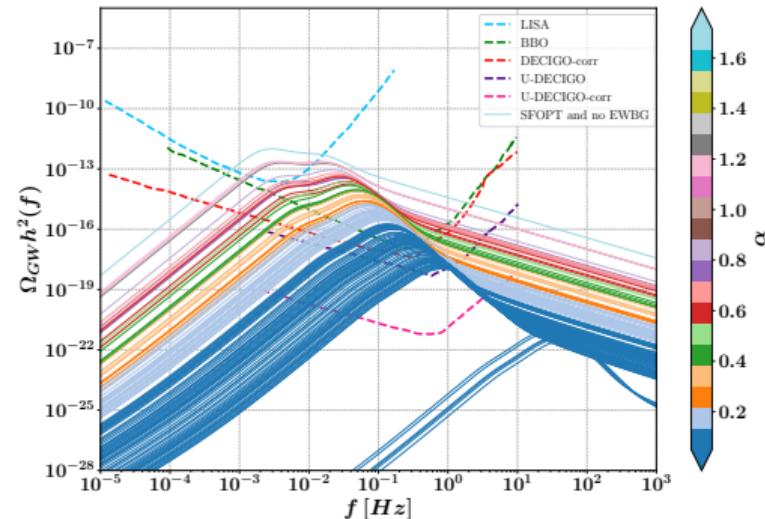
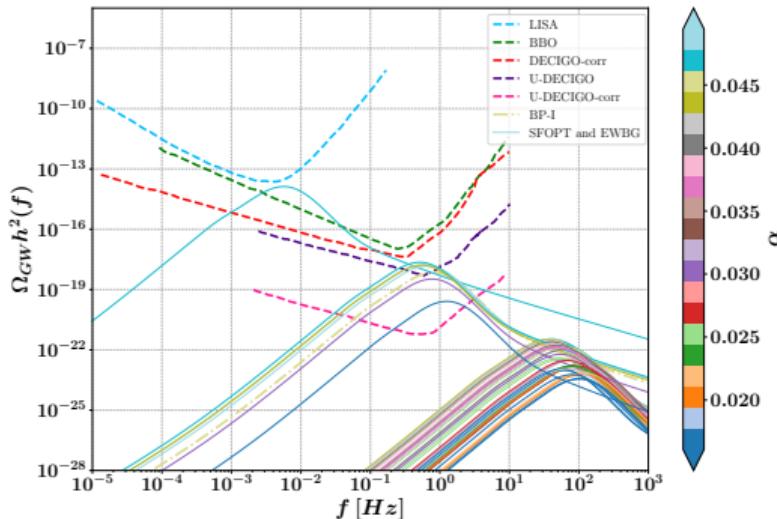


The parameters α and β/H_*



- The points, favoured for EWBG have relatively higher β/H_* and lower α values compared to the points that do not favour EWBG.

Detectability of GWs



- Parameter points that favours EWBG mostly falls within the U-DECIGO and U-DECIGO-corr sensitivity.
- Majority of the points that may not lead to successful EWBG are within the reach of LISA, BBO, DECIGO-corr, U-DECIGO and U-DECIGO-corr.

Summary and Conclusion

- ❖ A Right Handed Neutrino (RHN) superfield is introduced in the NMSSM framework and the possibility of SFOPT is analysed.
- ❖ SFOPT along $SU(2)_L$ Higgs, singlet Higgs (H_S) and \tilde{N} is possible with richer PT patterns compared to Z_3 -NMSSM in different corner of parameter space along with the possibility of EWBG in some points.
- ❖ In all of the parameter regions, comparatively low values of λ_N (< 0.4) and v_N ($\lesssim \mathcal{O}(1\text{TeV})$) is preferred for a SFOPT which also indicates the presence of a light RH-sneutrino-like state $m_{\tilde{N}}$ ($< m_{h_{125}}$).
- ❖ GW signals within reach of LISA, DECIGO-corr, U-DECIGO, U-DECIGO-corr. For LISA, SNR calculation is needed to check the threshold mark.

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THANK YOU FOR YOUR ATTENTION!
(*email: pankajborah316@gmail.com*)

Back Ups

Scalar Potential

$$\begin{aligned}
 V_F &= \left| -\lambda H_u^0 H_d^0 + \kappa S^2 + \frac{\lambda_N}{2} \tilde{N}^2 \right|^2 + |Y_N^i|^2 |H_u^0|^2 |\tilde{N}|^2 + |\lambda|^2 |S|^2 |H_u^0|^2 + \left| \sum_{i=1}^3 Y_N^i \tilde{\nu}_i H_u^0 + \lambda_N S \tilde{N} \right|^2 + \left| \sum_{i=1}^3 Y_N^i \tilde{\nu}_i \tilde{N} - \lambda S H_d^0 \right|^2, \\
 V_D &= \frac{g_1^2 + g_2^2}{8} \left(|H_d^0|^2 + \sum_{i=1}^3 |\tilde{\nu}_i|^2 - |H_u^0|^2 \right)^2, \quad V_{\text{soft}} = -\mathcal{L}_{\text{soft}}
 \end{aligned} \tag{1}$$

$$\begin{aligned}
 V_{\text{scalar}}^{\text{uncolored}} &= \left| \sum_{i=1}^3 Y_N^i \tilde{\nu}_i \tilde{N} - \lambda S H_d^0 \right|^2 + \left| \sum_{i,j=1}^3 Y_e^{ij} \tilde{l}_i \tilde{e}_j^c - \lambda S H_u^0 \right|^2 + \left| Y_N^i H_u^0 \tilde{N} - \sum_{j=1}^3 Y_e^{ij} H_d^- \tilde{e}_j^c \right|^2 \\
 &\quad + \left| \lambda H_u \cdot H_d + \kappa S^2 + \frac{\lambda_N}{2} \tilde{N}^2 \right|^2 + \left| \sum_{i=1}^3 Y_N^i \tilde{l}_i \cdot H_u + \lambda_N S \tilde{N} \right|^2 + \left| \sum_{i=1}^3 Y_e^{ij} H_d \cdot \tilde{l}_i \right|^2 \\
 &\quad + \left| \lambda S H_u^+ - \sum_{i,j=1}^3 Y_e^{ij} \tilde{\nu}_i \tilde{e}_j^c \right|^2 + \left| \lambda S H_d^- - \sum_{i=1}^3 Y_N^i \tilde{l}_i \tilde{N} \right|^2 + \left| \sum_{j=1}^3 Y_e^{ij} H_d^0 \tilde{e}_j^c - Y_N^i H_u^+ \tilde{N} \right|^2 \\
 &\quad + \frac{g_1^2}{8} (|H_d|^2 - |H_u|^2 + |\tilde{l}_i|^2 - 2|\tilde{e}_j^c|^2)^2 + \frac{g_2^2}{2} \sum_{a=1}^3 (H_d^\dagger \frac{\tau^a}{2} H_d + H_u^\dagger \frac{\tau^a}{2} H_u + \tilde{l}_i^\dagger \frac{\tau^a}{2} \tilde{l}_i)^2 \\
 &\quad + m_{H_d}^2 |H_d|^2 + m_{H_u}^2 |H_u|^2 + m_S^2 |S|^2 + M_N^2 |\tilde{N}|^2 + \sum_{i,j=1}^3 m_{\tilde{l}_{ij}}^2 \tilde{l}_i^m \tilde{l}_j^m + \sum_{i,j=1}^3 m_{\tilde{e}_{ij}^c}^2 \tilde{e}_i^c \tilde{e}_j^m \\
 &\quad + \sum_{i=1}^3 (A_e Y_e)^{ij} H_d \cdot \tilde{l}_i \tilde{e}_j^c + \lambda A_\lambda S H_u \cdot H_d + (A_N Y_N)^i \tilde{l}_i \cdot H_u \tilde{N} + \frac{\kappa A_\kappa}{3} S^3 + \frac{\lambda_N A_\lambda N}{2} S \tilde{N}^2 + h.c
 \end{aligned} \tag{2}$$

Back Ups

Higgs Basis

■ Extended Higgs basis

$$\begin{aligned} H_d &= \begin{pmatrix} \frac{1}{\sqrt{2}}(c_\beta H_{\text{SM}} - s_\beta H_{\text{NSM}}) + \frac{i}{\sqrt{2}}(-c_\beta G^0 + s_\beta A_{\text{NSM}}) \\ -c_\beta G^- + s_\beta H^- \end{pmatrix}, \\ H_u &= \begin{pmatrix} s_\beta G^+ + c_\beta H^+ \\ \frac{1}{\sqrt{2}}(s_\beta H_{\text{SM}} + c_\beta H_{\text{NSM}}) + \frac{i}{\sqrt{2}}(s_\beta G^0 + c_\beta A_{\text{NSM}}) \end{pmatrix}, \\ S &= \frac{1}{\sqrt{2}}(H_S + iA_S), \\ \tilde{N} &= \frac{1}{\sqrt{2}}(N_R + iN_I), \end{aligned} \tag{3}$$

■ Zero-temperature VEVs

$$\langle H_u^0 \rangle = v_u, \quad \langle H_d^0 \rangle = v_d, \quad \langle \tilde{\nu}_i \rangle = v_i, \quad \langle S \rangle = v_S, \quad \langle \tilde{N} \rangle = v_N, \quad i = 1, 2, 3 \quad \text{or} \quad e, \mu, \tau. \tag{4}$$

■ Counter-term potential

$$\begin{aligned} V_{ct} &= \delta_{m_{H_d}^2} |H_d|^2 + \delta_{m_{H_u}^2} |H_u|^2 + \delta_{m_S^2} |S|^2 + \delta_{M_N^2} |\tilde{N}|^2 + \delta_{\lambda A_\lambda} (S H_u \cdot H_d + h.c.) \\ &\quad + \delta_{\lambda_N A_{\lambda_N}} (S \tilde{N} \tilde{N} + h.c.) + \frac{\delta \lambda_2}{2} |H_u|^4, \end{aligned} \tag{5}$$

Back Ups (BPs)

	BP-I	BP-II	BP-III	BP-IV	BP-V	BP-VI
$\tan \beta$	2.90	2.74	2.90	5.77	4.79	5.86
λ	0.416	0.412	0.416	0.384	0.118	0.111
κ	0.022	0.019	0.022	0.012	0.013	0.051
λ_N	0.146	0.142	0.146	0.130	0.260	0.238
$Y_N^1 \times 10^7$	0.9	0.65	1.1	1.0	3.6	4.3
$Y_N^2 \times 10^7$	0.9	0.65	1.1	1.0	3.6	4.3
$Y_N^3 \times 10^7$	0.9	0.65	1.1	1.0	3.6	4.3
A_λ [GeV]	775.48	705.32	775.48	1184.87	988.08	920.08
A_κ [GeV]	-62.75	-25.37	-95.61	-107.08	-11.70	-41.61
A_{λ_N} [GeV]	-349.68	-337.77	-326.60	-363.16	-1358.30	-1528.57
A_N [GeV]	-16000.0	-12000.0	-8500.0	-12000.0	-6500.0	-5000.0
μ [GeV]	224.56	220.86	224.56	203.12	153.59	162.64
v_N [GeV]	308.80	325.21	284.50	386.45	136.57	355.66
$v_1 \times 10^4$ [GeV]	1.0	0.55	1.0	1.2	1.0	1.0
$v_2 \times 10^4$ [GeV]	1.0	0.55	1.0	1.2	1.0	1.0
$v_3 \times 10^4$ [GeV]	1.0	0.55	1.0	1.2	1.0	1.0
$m_{h_{125}}$ [GeV]	126.02	124.80	125.64	125.63	126.28	124.05
m_H [GeV]	772.36	718.07	772.73	1213.76	897.40	1012.14
m_{H_S} [GeV]	83.60	88.98	69.48	109.54	97.31	195.41
$m_{\tilde{N}}$ [GeV]	48.60	51.65	51.89	27.65	65.18	115.63
$\text{BR}(B \rightarrow X_s \gamma) \times 10^4$	3.61	3.70	3.62	3.47	3.59	3.55
$\text{BR}(B_s^0 \rightarrow \mu^+ \mu^-) \times 10^9$	3.24	3.26	3.24	3.19	3.20	3.19
$\text{BR}(\mu \rightarrow e \gamma) \times 10^{30}$	394	0.61	4.98	51.4	404	173
$\text{BR}(\mu \rightarrow eee) \times 10^{29}$	113.0	363.7	44.6	53.9	2.04	2.04
$\text{CR}(\mu N \rightarrow e N^*) \times 10^{28}$	1.81	0.11	2.49	4.43	4.85	7.31
$\Delta m_{\text{atm}}^2 \times 10^3$ eV ²	2.51	2.57	2.58	2.54	2.58	2.46
$\Delta a_\mu \times 10^{10}$	3.88	0.75	3.42	1.94	1.54	3.24

Back Ups

GW equations

$$\Omega_{\text{GW}} h^2 \approx \Omega_{\text{col}} h^2 + \Omega_{\text{sw}} h^2 + \Omega_{\text{tur}} h^2 ,$$

$$\Omega_{\text{col}} h^2 = 1.67 \times 10^{-5} \left(\frac{H_*}{\beta} \right)^2 \left(\frac{\kappa_c \alpha}{1+\alpha} \right)^2 \left(\frac{100}{g^*} \right)^2 \left(\frac{0.11 v_w^3}{0.42 + v_w^2} \right) \frac{3.8(f/f_{\text{col}})^{2.8}}{1+2.8(f/f_{\text{col}})^{3.8}} ,$$

$$\Omega_{\text{sw}} h^2 = 2.65 \times 10^{-6} \Upsilon(\tau_{\text{sw}}) \left(\frac{\beta}{H_*} \right)^{-1} v_w \left(\frac{\kappa_{\text{sw}} \alpha}{1+\alpha} \right)^2 \left(\frac{g^*}{100} \right)^{1/3} \left(\frac{f}{f_{\text{sw}}} \right)^3 \left[\frac{7}{4+3(f/f_{\text{sw}})^2} \right]^{7/2} ,$$

$$\Omega_{\text{tur}} h^2 = 3.35 \times 10^{-4} \left(\frac{\beta}{H_*} \right)^{-1} v_w \left(\frac{\kappa_{\text{tur}} \alpha}{1+\alpha} \right) \left(\frac{100}{g^*} \right) \left[\frac{(f/f_{\text{tur}})^3}{[1+(f/f_{\text{tur}})]^{11/3} \left(1 + \frac{8\pi f}{h_*} \right)} \right] ,$$

$$f_{\text{col}} = 16.5 \times 10^{-6} \left(\frac{f_*}{\beta} \right) \left(\frac{\beta}{H_*} \right) \left(\frac{T_n}{100 \text{ GeV}} \right) \left(\frac{g^*}{100} \right)^{1/6} \text{Hz},$$

$$f_{\text{sw}} = 1.9 \times 10^{-5} \left(\frac{1}{v_w} \right) \left(\frac{\beta}{H_*} \right) \left(\frac{T_n}{100 \text{ GeV}} \right) \left(\frac{g^*}{100} \right)^{1/6 \text{Hz}} ,$$

$$f_{\text{tur}} = 2.7 \times 10^{-5} \frac{1}{v_w} \left(\frac{\beta}{H_*} \right) \left(\frac{T_n}{100 \text{ GeV}} \right) \left(\frac{g^*}{100} \right)^{1/6} \text{Hz}.$$

Back Ups

Other equations

$$E_{sp}(T) = \frac{8\pi v(T)}{g} f(m_h/m_w),$$

$$\Gamma_{sp} \simeq A(\alpha_w T)^4 \left(\frac{E_{sp}(T)}{T} \right)^4 e^{-E_{sp}(T)/T},$$

$$\mathcal{M}_{S,11}^2 = \lambda^2 v^2 \sin^2 2\beta + \frac{(g_1^2 + g_2^2)v^2}{2} \cos^2 2\beta, \quad \mathcal{M}_{S,12}^2 = \frac{\lambda^2 v^2}{2} \sin 4\beta - \frac{(g_1^2 + g_2^2)v^2}{4} \sin 4\beta,$$

$$\mathcal{M}_{S,13}^2 = 2\lambda^2 v v_S - \lambda v(A_\lambda + 2\kappa v_S) \sin 2\beta, \quad \mathcal{M}_{S,14}^2 = -\lambda \lambda_N v_N \sin 2\beta,$$

$$\mathcal{M}_{S,22}^2 = 2\lambda v_S(A_\lambda + \kappa v_S) \csc 2\beta + \lambda \lambda_N v_N^2 \csc 2\beta - \lambda^2 v^2 \sin^2 2\beta + \frac{(g_1^2 + g_2^2)v^2}{2} \sin^2 2\beta,$$

$$\mathcal{M}_{S,23}^2 = -\lambda v(A_\lambda + 2\kappa v_S) \cos 2\beta, \quad \mathcal{M}_{S,24}^2 = -\lambda \lambda_N v v_N \cos 2\beta,$$

$$\mathcal{M}_{S,33}^2 = \kappa v_S(A_\kappa + 4\kappa v_S) + \frac{\lambda v^2 A_\lambda}{2v_S} \sin 2\beta - \frac{\lambda_N v_N^2 A_{\lambda_N}}{2v_S},$$

$$\mathcal{M}_{S,34}^2 = \lambda_N v_N A_{\lambda_N} + 2\lambda_N \kappa v_S v_N + 2\lambda_N^2 v_S v_N, \quad \mathcal{M}_{S,44}^2 = \lambda_N^2 v_N^2, \quad (6)$$