MSSM-inflation revisited: Towards a coherent description of high-energy physics and cosmology

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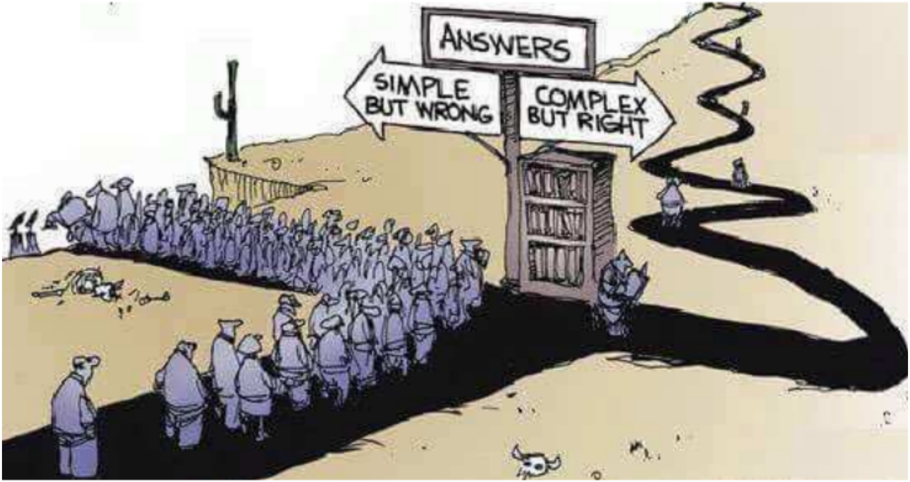
Outline

- Introductory motivations
- MSSM flat directions
- The inflection point (e)MSSM-Inflation model
- CMB constraints & possible implications on the MSSM spectrum
- Conclusions & outlook

Introductory motivations

Where is –Is there– (TeV) New Physics ??

message from (the) BSM at the LHC (?)



Too early to give up on Supersymmetry!

$\rightarrow$ deep connection between internal and space-time symmetries

$\rightarrow$ unification with Gravity...

$\rightarrow$ SUSY DM candidates still viable, even for (relatively) light LSP, despite direct / indirect search limits and LHC constraints.

$\rightarrow$ ubiquity of flat directions of the scalar potential
$\rightarrow$ potentially viable inflationary sectors?!

$\rightarrow$ In its (next-to-)minimal versions, (N)MSSM, possible relations between constraints from inflation (prediction of the scalar spectral index, the power spectrum normalization, etc.), and constraints from particle physics searches ?

MSSM flat directions

$$V_{\mathrm{SUSY}} = \sum_k |F_{\phi_k}|^2 + \frac{1}{2}\sum_A g_A^2 \vec{D}^A \cdot \vec{D}^A$$

$$F_{\phi_k} := \frac{\partial W}{\partial \phi_k}, \quad \vec{D}^A := \Phi^\dagger \vec{T}^A \Phi, \ \ (\Phi \text{ multiplet of } \phi_k's)$$

$\rightarrow$ An MSSM flat direction $\rightarrow$ $V = 0$

$\rightarrow$ find simultaneous solutions to $F_{\phi_k} = 0$ and $\vec{D}^A = 0$

$\rightarrow$ in the space of 49 complex scalar fields ϕ_k (squarks, sletons, Higgses).

$\rightarrow$ one-to-one correspondence with gauge invariant monomials

$\rightarrow$ The (R_p-conserving)MSSM potential has $\mathcal{O}(300)$ flat directions!

a promising paradise for Inflation!

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BUT...

MSSM flat directions

a flat direction:

- has to be lifted...
- and by not too much $\rightarrow$ slow-roll/enough e-folding to fit observations

$\rightarrow$ three main lifting sources:

- renormalizable superpotential (MSSM)
- soft SUSY breaking masses (MSSM)

$\Rightarrow$ effective non-renormalizable superpotential (beyond the MSSM)

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e.g. $\rightarrow$ *LLe* or *udd* flat directions lifted by

$$W_6^{LLe} = \frac{\widetilde{\lambda_6}}{M_{\mathrm{Pl}}^3}(LLe)(LLe) \text{ resp. } W_6^{udd} = \frac{\widetilde{\lambda_6}}{M_{\mathrm{Pl}}^3}(udd)(udd)$$

Collecting the pieces...

We follow R. Allahverdi, et al., Phys. Rev. Lett. 97, 191304 (2006) and literature in line
...however, partly at variance:

- $V_{\rm SUSY} \supset \sum_i \left| \frac{\partial W_6}{\partial \varphi_i} \right|^2 + \cdots V_{\rm D}$
- SUGRA-mediated SUSY breaking:

$$V_{\rm soft} = |m_{3/2}|^2 \sum_i |\varphi_i|^2 + \left\{ m_{3/2} \left[\sum_i \varphi_i \frac{\partial W}{\partial \varphi_i} + (a-3) W \right] + h.c. \right\},$$

$$V_{\rm soft} \supset \widetilde{A}_n W_n + h.c. \Rightarrow \widetilde{A}_6 = \frac{3+a}{a} \widetilde{A}_3, \text{ e.g. } \widetilde{\rm A}_6(\rm M_{SUSY}) = \left(\frac{3-\sqrt{3}}{6-\sqrt{3}} \right)^{-1} \rm A_t(M_{SUSY})$$

- $\sqrt{3}\varphi = L_i^{\uparrow} = L_i^{\downarrow} = e_k$ or $\sqrt{3}\varphi = u_i^a = d_j^b = d_k^c$, all other MSSM scalar fields $\to 0$.

$$\boxed{V_{\rm tree}^{\rm Inflaton}(\phi) = \frac{1}{2} m_\phi^2 \phi^2 - \sqrt{2} A_6 \frac{\lambda_6 \phi^6}{6 M_{\rm Pl}^3} + \lambda_6^2 \frac{\phi^{10}}{M_{\rm Pl}^6}}, \ \phi = \pm\sqrt{2}\,|\varphi|\,, \ A_6 = \left|\widetilde{A}_6\right|, \lambda_6 = \frac{\widetilde{\lambda_6}}{18\sqrt{2}} > 0.$$

$$m_\phi^2 = \frac{1}{3}(m_{soft_1}^2 + m_{soft_2}^2 + m_{soft_3}^2)$$

$\Rightarrow$ A_6 could be rescaled, BUT effects propagate on the PP side!

$\Rightarrow$ $\widetilde{\lambda_6} \sim \mathcal{O}(1) \to \lambda_6 \ll \mathcal{O}(1)$

$n_s, A_s, \ldots$

Hubble-flow params./slow-roll approx.

$$\varepsilon_1 \simeq \frac{M_{\rm Pl}^2}{2}\left(\frac{V_\phi}{V}\right)^2$$

$$\varepsilon_2 \simeq 2M_{\rm Pl}^2\left[\left(\frac{V_\phi}{V}\right)^2 - \frac{V_{\phi\phi}}{V}\right]$$

$$\varepsilon_3 \simeq \frac{2}{\epsilon_2}M_{\rm Pl}^4\left[\frac{V_{\phi\phi\phi}V_\phi}{V^2} - 3\frac{V_{\phi\phi}}{V}\left(\frac{V_\phi}{V}\right)^2 + 2\left(\frac{V_\phi}{V}\right)^4\right]$$

$$\vdots$$

Parameter	Value and error
$\ln(10^{10}A_{\rm s})$	3.047 ± 0.014
$n_{\rm s}$	0.9665 ± 0.0038
[$n_{\rm s,run}$	-0.0042 ± 0.0067]
[r	< 0.032]

(Planck 2020)

Scalar perturbations power spectrum

$$\mathcal{P}_s(k) = A_s\left(\frac{k}{k*}\right)^{n_s-1}$$

Amplitude

$$A_s = \frac{H_*^2}{8\pi^2\varepsilon_{1*}M_p} \simeq \frac{V_*}{24\pi^2 M_{\rm Pl}^4\varepsilon_{1*}}$$

Scalar spectral index

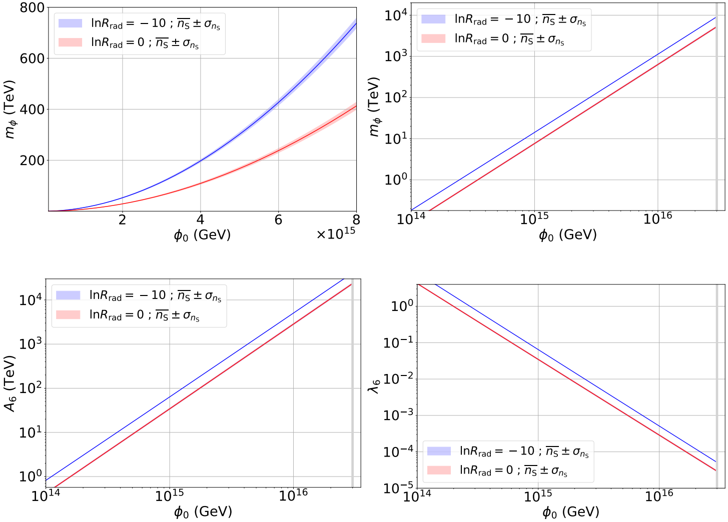
$$n_s \simeq 1 - 2\varepsilon_{1*} - \varepsilon_{2*}$$

its running

$$n_{\rm s,run} \simeq -2\epsilon_{1*}\epsilon_{2*} - \epsilon_{2*}\epsilon_{3*}$$

$$\Delta N_* \simeq -\frac{1}{M_{\rm Pl}^2}\int_{\phi_\star}^{\phi_{end}}\frac{V(\phi)}{V_\phi(\phi)}d\phi$$

$$= \ln R_{\rm rad} - \ln\left(\frac{k_*}{a_0\,\rho_\gamma^{\frac{1}{4}}}\right) - \frac{1}{4}\ln\left(\frac{9V_{end}}{\epsilon_{1*}(3-\epsilon_{1end})V_*}\right) + \frac{1}{4}\ln(8\pi^2 A_s)$$



$\ln R_{rad} = -10$; $\overline{n_S} \pm \sigma_{n_S}$
$\ln R_{rad} = 0$; $\overline{n_S} \pm \sigma_{n_S}$
m_ϕ (TeV)
ϕ_0 (GeV)
$\times 10^{15}$
A_6 (TeV)
λ_6

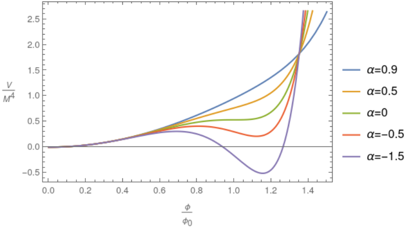
quasi-flat inflection point

at $\phi = \phi_0 = \left(\frac{M_p^3 m_\phi \sqrt{1-\alpha}}{\lambda\sqrt{10}}\right)^{1/4}$ with $1 - \alpha \equiv \frac{A_6^2}{20 m_\phi^2}$

$V_{\phi\phi}(\phi_0) = 0$ and
$V_\phi(\phi_0) \equiv \nu = m_\phi^2 \phi_0 \alpha + \mathcal{O}(\alpha^2)$
$0 \leq \alpha \ll 1$

- $\alpha = 0 \rightarrow$ too low $n_s \simeq 1 - \frac{4}{\Delta N_*}$
- $\alpha \neq 0 \rightarrow \alpha \lesssim 10^{-5}$ for $\varepsilon_1 \lesssim 1$

  and $\alpha \lesssim 10^{-22}$ to get, e.g. $\Delta N \sim 50$



huge fine-tuning, challenging numerical evaluation.
a new methodology:

$$V_{\text{tree},\phi}(\phi) = (1+\Delta_4)^{\frac{1}{4}} \left\{\nu + \Delta_4 \lambda_6 \frac{\phi_0^5}{M_p^3}\left[10(2+\Delta_4)\lambda_6 \frac{\phi_0^4}{M_p^3} - \sqrt{2}A_6\right]\right\}$$

$$V_{\text{tree},\phi\phi}(\phi) = 5\Delta_4\lambda_6 \frac{\phi_0^4}{M_p^3}\left[18(2+\Delta_4)\lambda_6 \frac{\phi_0^4}{M_p^3} - \sqrt{2}A_6\right],\ \Delta_4 \equiv \phi^4/\phi_0^4 - 1$$

$\rightarrow$ m_ϕ^2 and λ_6 become functions of ϕ_0, A_6, ν.
$\rightarrow$ a well-defined methodology circumventing numerical high precision issues.

RGE 'improved' inflationary potential

$$V^{\text{Inflaton}}_{\text{tree}}(\phi) = \frac{1}{2} m_\phi^2 \phi^2 - \sqrt{2} A_6 \frac{\lambda_6 \phi^6}{6 M_{\text{Pl}}^3} + \lambda_6^2 \frac{\phi^{10}}{M_{\text{Pl}}^6}$$

$m_\phi^2 \to \overline{m}_\phi^2(\phi)$, $\widetilde{A}_6 \to \overline{A}_6(\phi)$

$\lambda_6 \to \overline{\lambda}_6(\phi)$, $\phi \to \overline{\phi}(\phi)$

$\rightarrow \overline{m}_\phi^2$ from the running of the soft masses

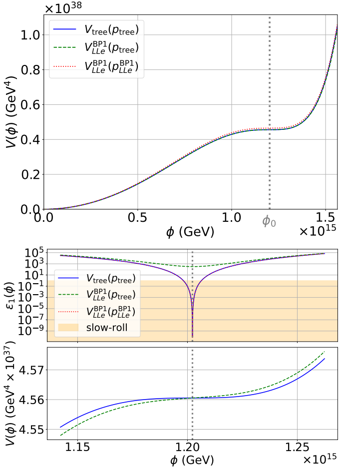
$\rightarrow$ the RGEs for $\overline{\lambda}_6$ and $\overline{A}_6$ extracted from (renormalizable) RPV operators. Leading β-functions depend only on running g_i, M_i. The quasi-flat conditions can still be determined from $V^{\text{Inflaton}}_{\text{loop}}(\phi)$:

$$\overline{m}_\phi^2(\phi_0) = \frac{\mathcal{A}^2(\phi_0, \nu, \overline{g}_i, \overline{M}_i, \overline{A}_6)}{20}$$

$$\overline{\lambda}_6(\phi_0) = \mathcal{B}(\phi_0, \nu, \overline{g}_i, \overline{M}_i, \overline{A}_6)$$

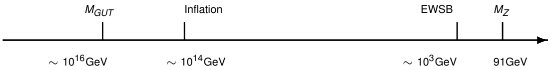
define: $$1 - \alpha^{(\text{loop})} \equiv \frac{\mathcal{A}^2(\phi_0, \nu = 0)}{20 m_\phi^2(\phi_0)}$$

$\rightarrow$same methodology as at tree-level!



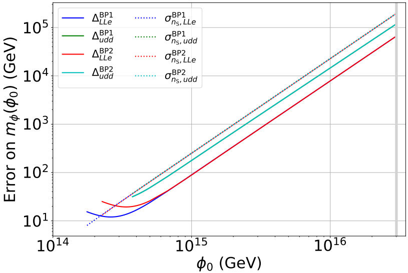
Improvements

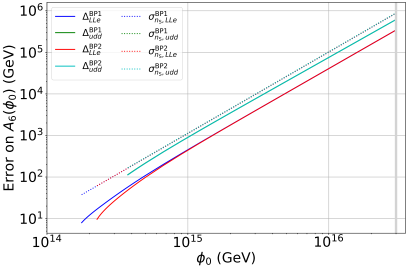
- propagation of (non-trivial) normalizations
- RGE improved one-loop potential
- 'exact' one-loop fine-tuning
- inclusion of Yukawa effects in the RGEs
- effects of reheating on the number of e-folds
- tree-level versus 1-loop for given ϕ_0 / given $A_6(M_{\rm GUT})$
- relating to the MSSM spectrum (SUSPECT3: J.L.Kneur et al. CPC 291 (2023) 108805, [2211.16956 [hep-ph]])



$$\Delta_i^{\mathrm{BP}_j}[p] = p^{V_{\mathrm{tree}}|\phi_0}(Q=\phi_0, n_S=\overline{n_S}) - p^{V_{\mathrm{RGE}}|\phi_0}(Q=\phi_0, n_S=\overline{n_S})$$

$$\sigma_{n_S,i}^{\mathrm{BP}_j}[p] = \frac{1}{2}\left| p^{V_{\mathrm{RGE}}|\phi_0}(Q=\phi_0, n_S=\overline{n_S}+\sigma_{n_S}) - p^{V_{\mathrm{RGE}}|\phi_0}(Q=\phi_0, n_S=\overline{n_S}-\sigma_{n_S})\right|,$$





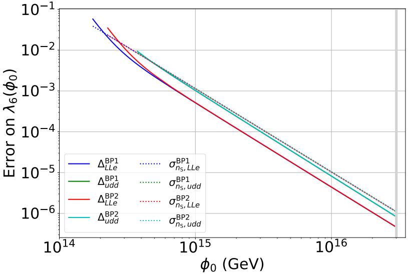


TABLE V. Fitted benchmark MSSM spectra and their status regarding the observables. The parameters are given at $M_{\rm EWSB}$. All the masses and energy scales are given in GeV. The ✗ symbol highlights the points excluded by direct LHC searches (see Sec. VI B 1).

ID	H_1	H_2	h_1	h_2	A_1	A_2	A_3
DM channel	Higgsino		h-funnel		A-funnel		
inflaton	$u_1d_1d_2$	$L_1L_2e_1$	$u_1d_1d_2$	$L_1L_2e_1$	$u_1d_1d_2$	$L_1L_2e_1$	$L_1L_2e_1$
EWSB scale	2556.9	2556.8	2713.1	2713.0	4008.9	4008.9	4009.7
$\tan\beta$	29		26.6		18.3		24.7
sgn(μ)	+		+		−		
$M_{\rm GUT}$	1.237×10^{16}		3×10^{16}		1.295×10^{16}		
M_1	1571		61		400		362
M_2	2917		967		1515		1662
M_3	1931		1934		1898		1098
$M_{\tilde{\mu}_L}$	2864	12108	4210	9892	2564	10567	4773
$M_{\tilde{\mu}_R}$	2785	12108	4221	9892	3976	10400	4723
$M_{\tilde{\tau}_L}$	3342		4068		2039		
$M_{\tilde{\tau}_R}$	2064		3931		2801		
$M_{\tilde{q}_{2L}}$	11959	9090	9860	7370	10452	7299	2984
$M_{\tilde{q}_{3L}}$	2176		3155		3950		
$M_{\tilde{t}_R}$	3003		2330		4066		
$M_{\tilde{b}_R}$	3474		1952		2422		
A_τ	−3499		−2564		−3010		
A_t	3180	3223	2985	3020	3450	3493	1190
A_b	148		143		187		
$m(\tilde{\chi}_1^0)$	1108	1108	60.0	60.1	397	398	357
$m(\tilde{\chi}_2^0)$	−1113	−1112	497	496	766	766	760
$m(\tilde{\chi}_3^0)$	1578	1587	−504	−503	−769	−770	−763
$m(\tilde{\chi}_4^0)$	2992	3005	1041	1041	1589	1592	1710
$m(\tilde{\chi}_1^\pm)$	1111	1111	496	495	765	766	760
$m(\tilde{\chi}_2^\pm)$	2992	3005	1041	1041	1589	1592	1711
$m(\tilde{g})$	2371	2333	2351	2310	2382	2332	1389
$m(\tilde{q}_L)$	12072	9173	9956	7442	10557	7376	3036
$m(\tilde{q}_R)$	12056	9162	9939	7429	10540	7364	3024
$m(\tilde{b}_1)$	2243	2244	2031	2031	2499	2499	2453
$m(\tilde{b}_2)$	3512	3512	3177	3177	3974	3974	3971
$m(\tilde{t}_1)$	2244	2244	2347	2347	3946	3945	3972
$m(\tilde{t}_2)$	2992	2992	3188	3188	4074	4075	4059
$m(\tilde{\tau}_1^-)$	2066	2066	3931	3931	2055	2055	2055
m_h	125.3	125.2	125.3	125.2	125.4	125.3	122.2
m_A	784.9	783.9	3625.9	3625.8	782.2	784.4	757.2
$m_{3/2}$	12596	12557	10352	10383	11005	11010	4886
ϕ_0	1.25×10^{15}	1.26×10^{15}	1.13×10^{15}	1.14×10^{15}	1.16×10^{15}	1.17×10^{15}	7.68×10^{14}
$m_\phi(\phi_0)$	11982	11973	9847	9892	10459	10487	4661
$A_6(\phi_0)$	53757	53593	44181	44312	46970	46990	20855
$\lambda_6(\phi_0)$	0.0224	0.0218	0.0278	0.0269	0.0260	0.0252	0.0609
$\chi^2_{\rm HEP}$ (d.o.f. = 78)	50.9	51.7	46.1	46.5	47.5	47.0	49.2
LHC searches			✗	✗			✗

ASPIC-inspired

+

SUSPECT3

+

SFITTER

Conclusions & Outlook

- we have revisited the (e)MSSM inflection-point scenario of inflation, both at tree- and one-loop levels, and shown that combined fits of particle and cosmological observables are already sensitive to the quantum corrections to the inflationary potential.
- careful theoretical scrutiny of the model is thus called for:
  - $\rightarrow$ we propagated correct normalizations $\rightarrow$ effects on the theoretical consistency and the MSSM spectrum.
  - $\rightarrow$ derived full one-loop RGEs including Yukawa effects for the parameters of the inflationary potential.
  - $\rightarrow$ determined the quas-flatness conditions for the full one-loop non-polynomial potential $\rightarrow$ fine-tuning is not reduced wrt to tree-level!
- the details of the reheating era are important (instantaneous or not) affecting the viable parameter space regions.
- global fit combining cosmological and particle physics observables/constaints (interfacing `ASPIC`-like with `SuSpect3`, `SFitter`)

Further investigations still needed:

- SUGRA mediation assumptions
- complete one-loop effecitve potential
- explicit $T_{\rm reh}$ calculations
- lower effective scales e.g. $M_{\rm pl} \rightarrow M_{\rm GUT}$,...