

Minimal Nambu-Goldstone-Higgs Model in Supersymmetric SU(5)

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Outline

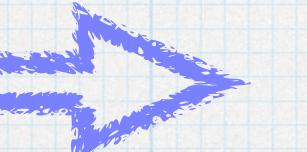
- * 1. Model
- * 2. Matching conditions
- * 3. Results
- * 4. Summary

1. Model

SU(5) GUT

- * SU(5) gauge symmetry

$$* F_i = \mathbf{10} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u_{i3}^c & -u_{i2}^c & u_i^1 & d_i^1 \\ -u_{i3}^c & 0 & u_{i1}^c & u_i^2 & d_i^2 \\ u_{i2}^c & -u_{i1}^c & 0 & u_i^3 & d_i^3 \\ -u_i^1 & -u_i^2 & -u_i^3 & 0 & e_i^c \\ -d_i^1 & -d_i^2 & -d_i^3 & -e_i^c & 0 \end{pmatrix}, \quad \bar{f}_i = \bar{\mathbf{5}} = \begin{pmatrix} d_{i1}^c \\ d_{i2}^c \\ d_{i3}^c \\ e_i \\ -\nu_i \end{pmatrix}, \quad \left(l_i^c = \mathbf{1} = (\nu_i^c) \right)$$

- * $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$  Hypercharge assignments

- Broken by $\Phi = 24 \rightarrow (8,1)_0 + (1,3)_0 + (1,1)_0 + (3,2)_{+5/6} + (\bar{3},2)_{-5/6}$

- $\mathbf{10} \rightarrow (\bar{3},1)_{-2/3} + (3,2)_{+1/6} + (1,1)_{+1}$

- $\bar{\mathbf{5}} \rightarrow (\bar{3},1)_{+1/3} + (1,2)_{-1/2}$

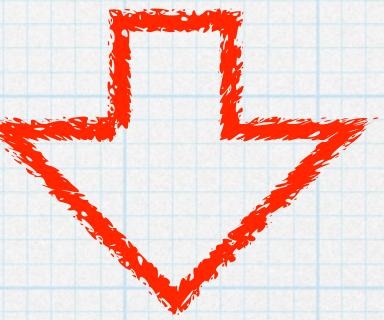
\mathbf{Q}_i	\mathbf{u}_i^c	\mathbf{d}_i^c	\mathbf{L}_i	\mathbf{e}_i^c
 $(3,2)_{+1/6}$	 $(\bar{3},1)_{-2/3}$	 $(\bar{3},1)_{+1/3}$	 $(1,2)_{-1/2}$	 $(1,1)_{+1}$

SU(5) GUT

- * MSSM Higgs: embedded in 5 and $\bar{5}$ in SU(5) theory

$$- H = \begin{pmatrix} \zeta_u \\ H_u \end{pmatrix}, \quad \bar{H} = \begin{pmatrix} \bar{\zeta}_d \\ H_d \end{pmatrix}$$

$\zeta_u, \bar{\zeta}_d$: colored Higgs \rightarrow need to be heavy
 H_u, H_d : MSSM Higgs \rightarrow need to be light



- * Superpotential $W \supset \lambda(H\Phi\bar{H} + MHH\bar{H})$

- VEV of $\Phi = 24$ breaks SU(5) into the SM

$$- \langle \Phi \rangle = \text{diag}(b, b, b, -\frac{3}{2}b, -\frac{3}{2}b)$$

$$- W_{\text{eff}} = \lambda(b + M)\zeta_u\bar{\zeta}_d + \lambda(-\frac{3}{2}b + M)H_uH_d$$

$$- \frac{3}{2}b = M \text{ by hand} \rightarrow \text{fine-tuning problem}$$

Doublet-triplet splitting problem

Nambu-goldstone Higgs

- * **SU(6) global symmetry** (Higgs sector)
 - * MSSM Higgs: in the **adjoint rep.** of the SU(6)

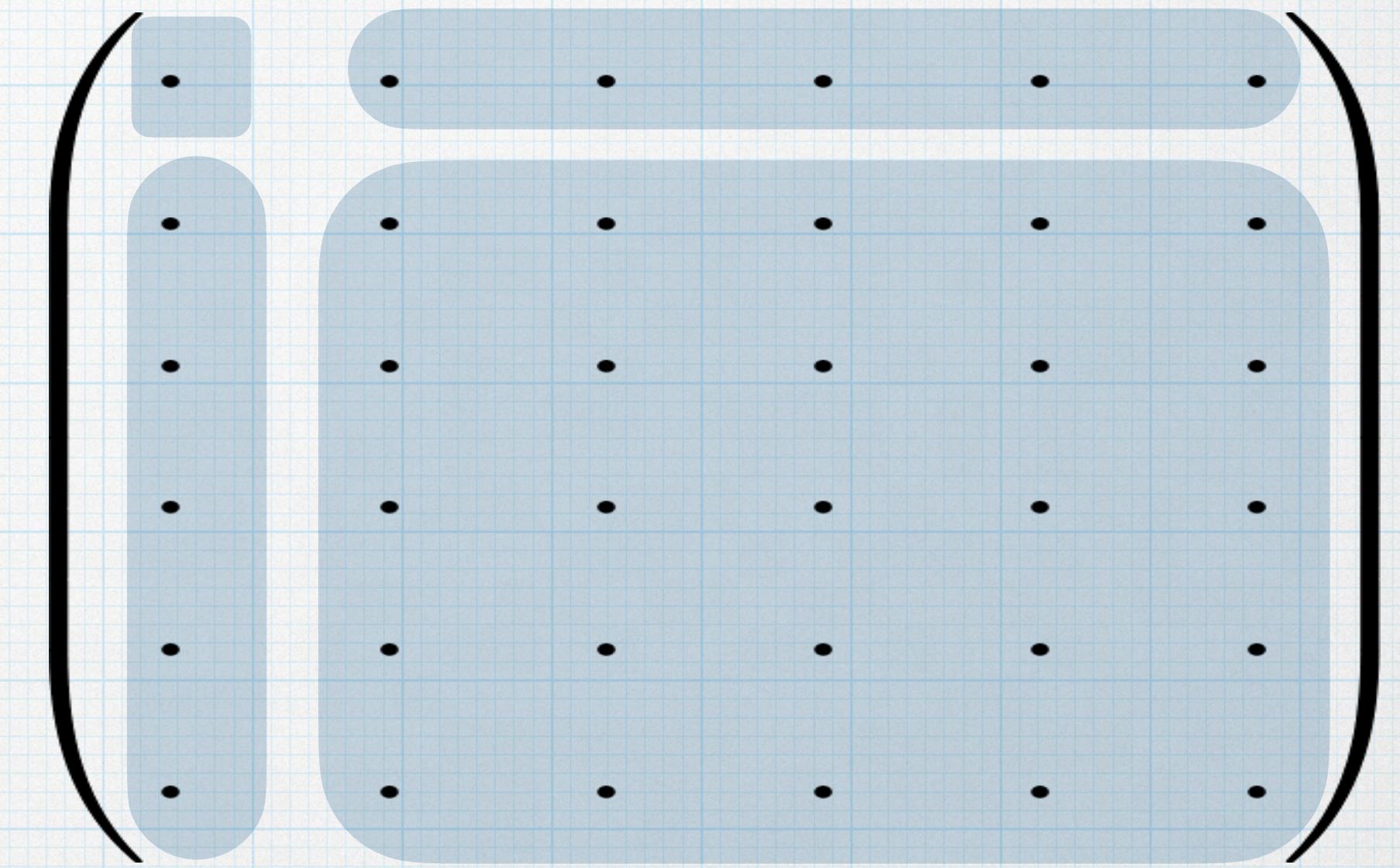
$$* W_{\text{Higgs}}(\hat{\Sigma}) = \frac{1}{3}\lambda \text{Tr}\hat{\Sigma}^3 + \frac{1}{2}M \text{Tr}\hat{\Sigma}^2$$

- * **SU(5) representations**

$$* \hat{\Sigma} = \begin{pmatrix} -5S/\sqrt{60} & \bar{H}/\sqrt{2} \\ H/\sqrt{2} & S1_5/\sqrt{60} + \Sigma \end{pmatrix}$$

$$* W_{\text{Higgs}}(\hat{\Sigma}) = \frac{1}{3}\lambda \text{Tr}(\Sigma^3) + \frac{1}{2}\lambda \bar{H}\Sigma H + \frac{1}{2}M \text{Tr}(\Sigma^2) + \frac{1}{2}M\bar{H}H - \frac{1}{3\sqrt{15}}\lambda S^3 - \frac{1}{\sqrt{15}}\lambda S\bar{H}H + \frac{1}{\sqrt{60}}\lambda S \text{Tr}(\Sigma^2) + \frac{1}{4}MS^2 .$$

- * Relations among the couplings (compared to the usual SU(5) GUT)



Symmetry breaking

- * Symmetry breaking

- * Global $SU(6) \rightarrow SU(4) \times SU(2) \times U(1)$

- * Gauged $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$

- * $\langle \hat{\Sigma} \rangle = \hat{V} \cdot \text{diag}(1, 1, 1, 1, -2, -2), \quad \hat{V} = M/\lambda$

- * $35 - (15 + 3 + 1) = 16$ NG bosons
 $(24 - (8 + 3 + 1)) = 12$ are absorbed by the massive gauge bosons.)

- * Goldstone modes:

$$(3,2) + (\bar{3},2) + (1,2) + (1,2)$$

.	X	X
.	X	X
.	X	X
.	X	X
X	X	X	X	X	.	.
X	X	X	X	X	.	.

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.	X X
.	X X
.	X X
.	X X
.	X X
X X	X X	X X	X X	.	.

Mass spectrum

- * The mass spectrum
- * Higgs fields
 - * $M_\Sigma = M_{\Sigma_8} = M_{\Sigma_3} = \frac{3}{2}\lambda\hat{V}$ $M_{H_C} = \frac{3}{2}\lambda\hat{V}$
 - * MSSM Higgs
 - * $M_H = 0$ massless to all order (non-renormalization theorem)
 - * SU(6)-breaking interactions & soft SUSY breaking \rightarrow radiative EW-symmetry breaking at low energies \rightarrow Higgs mass
 - * The doublet-triplet splitting problem is solved.
 - * SU(5) gauge bosons $M_X = 3\sqrt{2}g_5\hat{V}$

2. Matching conditions

GUT scale matching conditions

- * Coupling unification
- * GUT-scale particles: affect gauge couplings via threshold corrections
- * The one-loop threshold corrections at Q_G near the GUT scale

$$*\frac{1}{g_1^2(Q_G)} = \frac{1}{g_5^2(Q_G)} + \frac{1}{8\pi^2} \left[\frac{2}{5} \ln \frac{Q_G}{M_{H_C}} - 10 \ln \frac{Q_G}{M_X} \right]$$

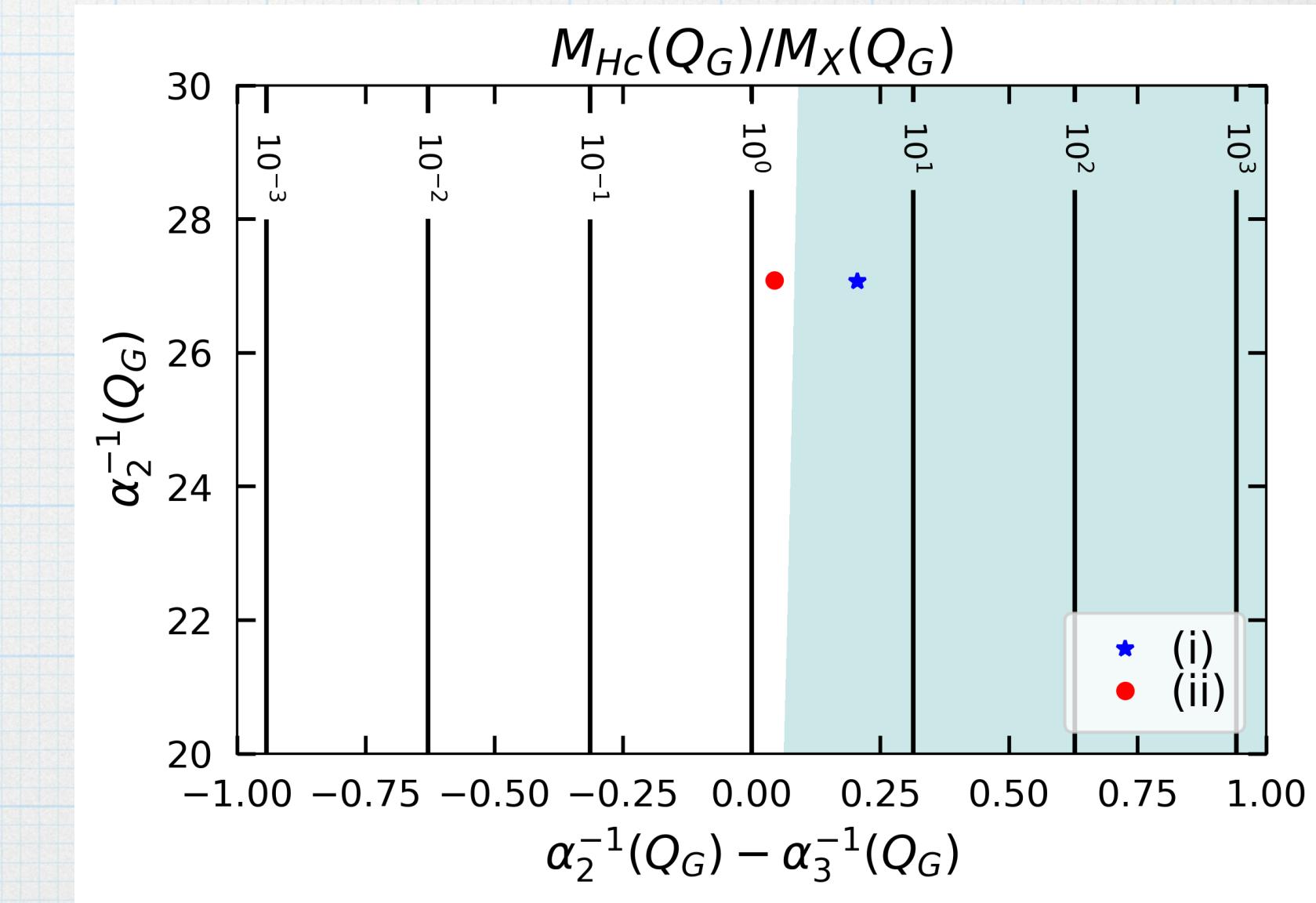
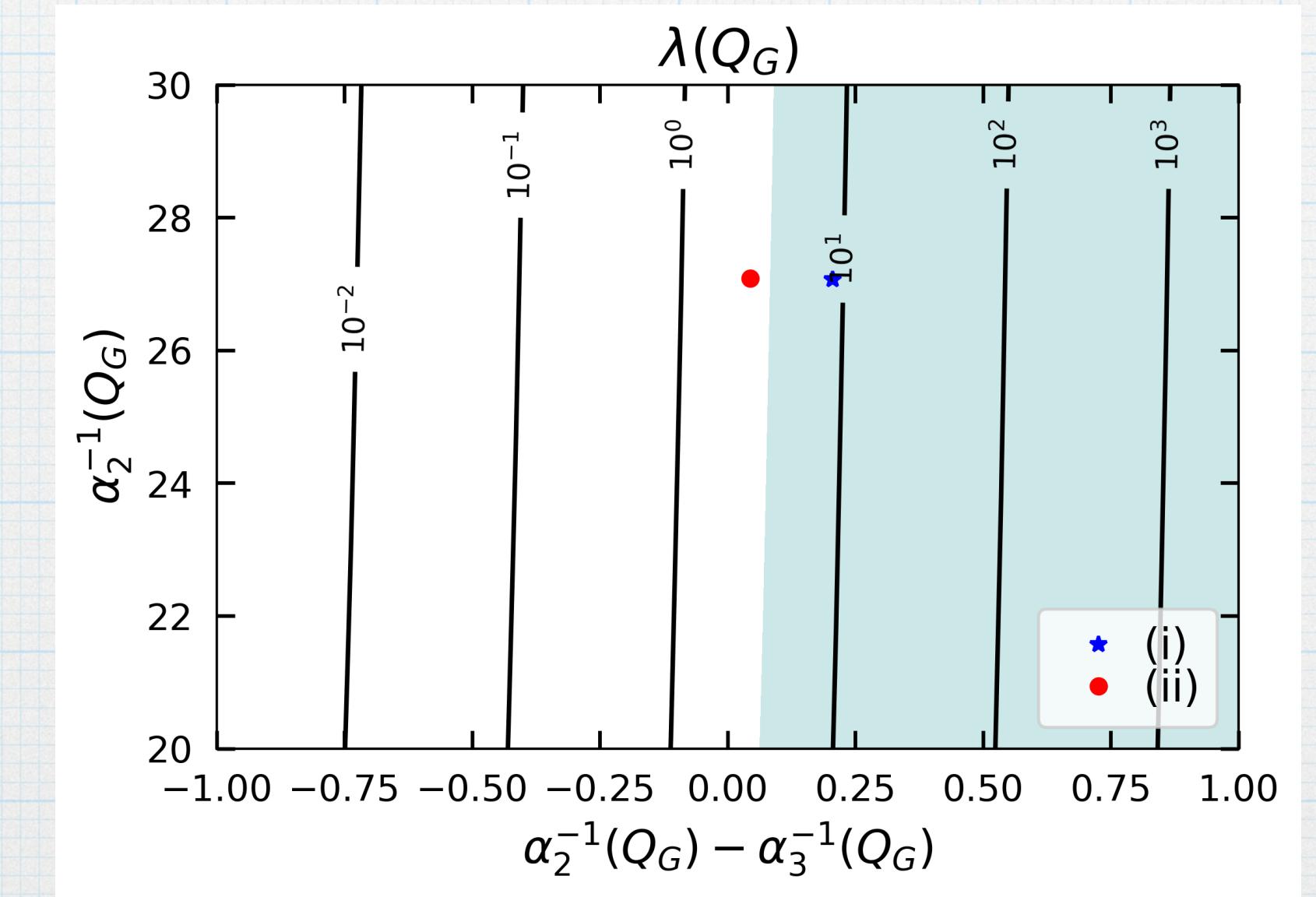
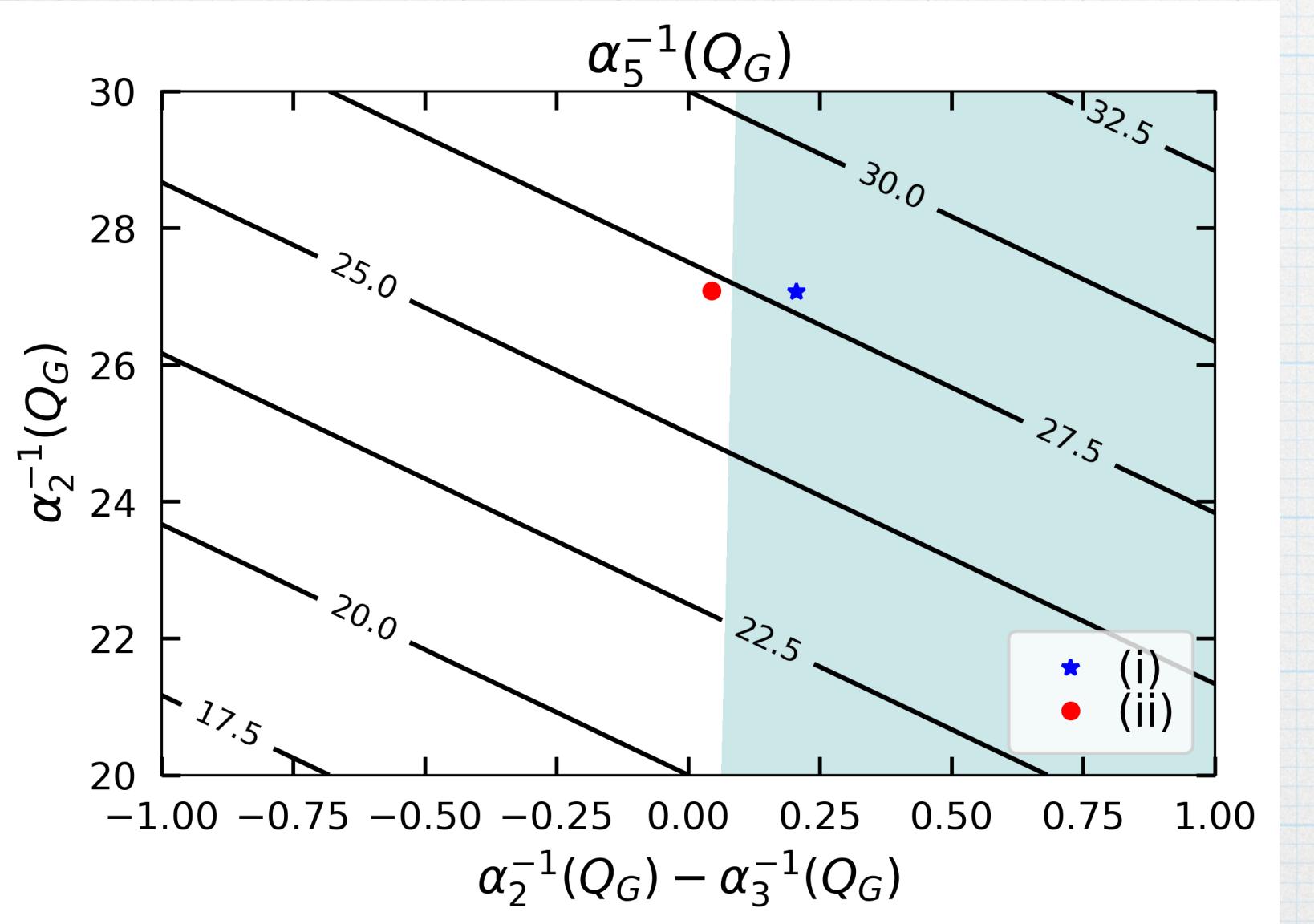
$$*\frac{1}{g_2^2(Q_G)} = \frac{1}{g_5^2(Q_G)} + \frac{1}{8\pi^2} \left[2 \ln \frac{Q_G}{M_\Sigma} - 6 \ln \frac{Q_G}{M_X} \right]$$

$$*\frac{1}{g_3^2(Q_G)} = \frac{1}{g_5^2(Q_G)} + \frac{1}{8\pi^2} \left[\ln \frac{Q_G}{M_{H_C}} + 3 \ln \frac{Q_G}{M_\Sigma} - 4 \ln \frac{Q_G}{M_X} \right]$$

- * In terms of gauge coupling constants at the GUT scale

GUT scale matching conditions

- * $\lambda > \sqrt{4\pi}$ non-perturbative
- * λ increases as $(\alpha_2^{-1}(Q_G) - \alpha_3^{-1}(Q_G))$ gets larger.
- * $\alpha_2(Q_G) \gtrsim \alpha_3(Q_G)$ is favored.
- * $M_{H_C} \lesssim M_X$



Running Couplings

- * The dependence on the low-energy SUSY spectrum

- * Approximate relation (one-loop):

$$\frac{1}{\alpha_2(Q_G)} - \frac{1}{\alpha_3(Q_G)} = -\frac{5}{7} \frac{1}{\alpha_1(m_Z)} + \frac{12}{7} \frac{1}{\alpha_2(m_Z)} - \frac{1}{\alpha_3(m_Z)} + \frac{1}{28\pi} \ln \left(\frac{m_W^{32} \cdot m_H^{12} \cdot m_A^3}{m_Z^{19} \cdot m_g^{28}} \right) + \frac{1}{28\pi} \sum_i \ln \left[\left(\frac{m_{\tilde{Q}_i}^7}{m_{\tilde{u}_i}^5 \cdot m_{\tilde{e}_i}^2} \right) \left(\frac{m_{\tilde{L}_i}^3}{m_{\tilde{d}_i}^3} \right) \right].$$

- * The perturbativity condition

- * Upper limit on $m_{\tilde{W}}$, $m_{\tilde{H}}$ and m_A

- * Lower limit on $m_{\tilde{g}}$

- * We use two-loop RGEs and one-loop threshold corrections in numerical computation

3. Results

Constrained MSSM

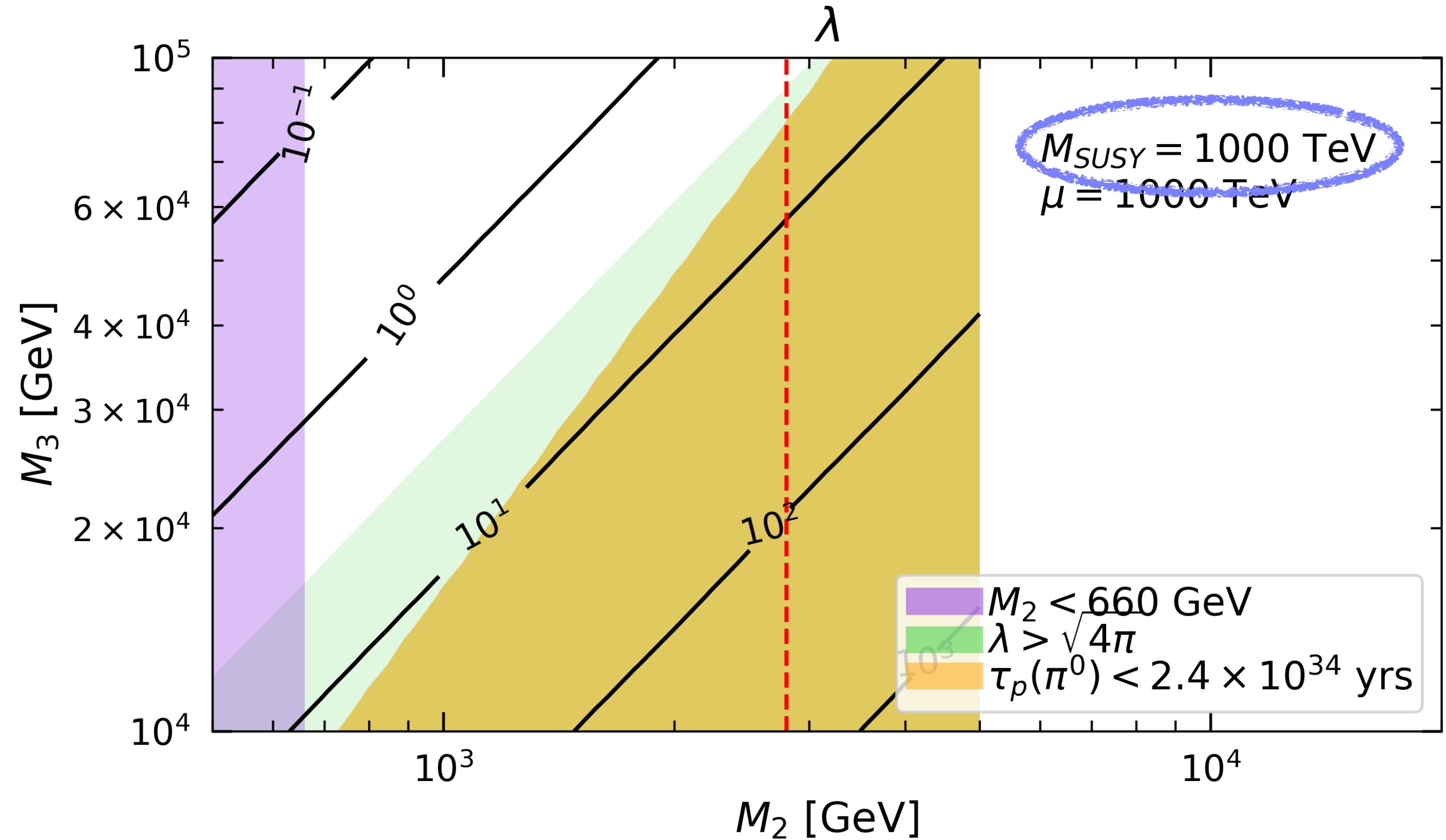
- * Scalar masses m_0 , gaugino masses $m_{1/2}$, A-parameters A_0 , ratio $\tan \beta = v_u/v_d$ and $\text{sgn}(\mu)$
- * Benchmark points [Ellis, J., Evans, L., Nagata, N. Et al. *Supersymmetric proton decay revisited. Eur. Phys. J. C80, 332 (2020).*]
 - * Give the correct Higgs mass and dark matter abundance
 - * Case (i) $m_0 = 14.1 \text{ TeV}$, $m_{1/2} = 9.8 \text{ TeV}$, $A_0 = -3m_0$, $\tan \beta = 5$, $\mu > 0$.
 - * $\alpha_5 = 0.036$, $\lambda = 8.7$, $M_X = 4.8 \times 10^{15} \text{ GeV}$, $M_{H_C} = 2.2 \times 10^{16} \text{ GeV}$.
 - * Case (ii) $m_0 = 27.9 \text{ TeV}$, $m_{1/2} = 9.5 \text{ TeV}$, $A_0 = 0$, $\tan \beta = 4$, $\mu > 0$.
 - * $\alpha_5 = 0.037$, $\lambda = 2.7$, $M_X = 7.6 \times 10^{15} \text{ GeV}$, $M_{H_C} = 1.1 \times 10^{16} \text{ GeV}$.
 - * Perturbative $\lambda \rightarrow M_{H_C} \lesssim 10^{16} \text{ GeV}$: too low to evade the $p \rightarrow K^+ \bar{\nu}$ proton-decay limit if SUSY particles lie around $M_{\text{SUSY}} \sim \mathcal{O}(10) \text{ TeV}$

High scale SUSY

- * Minimal SUSY SU(5) with low-scale SUSY-breaking: rapid proton-decay problem
- * High-scale SUSY
 - * A SUSY-breaking scale of $\mathcal{O}(1)$ PeV
 - * No gauge-singlet SUSY-breaking field in the hidden sector
 - * Gravity mediation
 - * Soft masses of SUSY particles $\mathcal{O}(m_{3/2})$ (gravitino mass)
 - * Gaugino mass: generated only radiatively

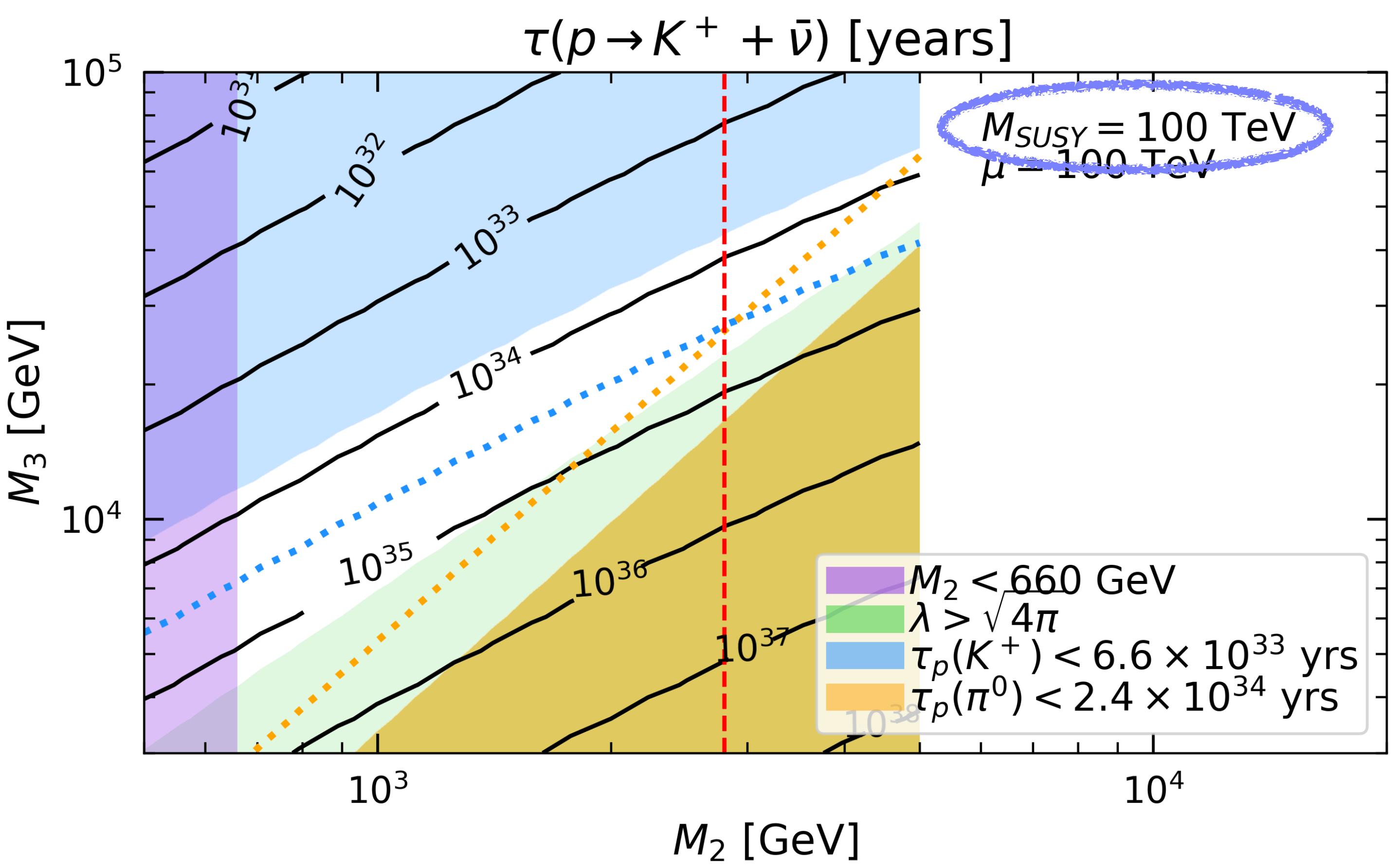
High scale SUSY

- * $M_{SUSY} = 1000 \text{ TeV}$
- * Purple region: LHC limit
- * Red line: $\Omega_{\widetilde{W}} h^2 = \Omega_{DM} h^2$
- * Perturbative region
- * $M_{H_C} \lesssim M_X$
- * $M_{H_C} \lesssim 10^{16} \text{ GeV}$
- * A small wino mass M_2 and a large gluino mass M_3



High scale SUSY

- * Proton decay
- * $\tau(p \rightarrow K^+ \bar{\nu}) > 6.6 \times 10^{33}$ years
- * $\tau(p \rightarrow e^+ \pi^0) > 2.4 \times 10^{34}$ years
- * Lower bound for M_{H_C}
- * $M_{\text{SUSY}} \approx \mathcal{O}(1)$ PeV is favored.



4. Summary

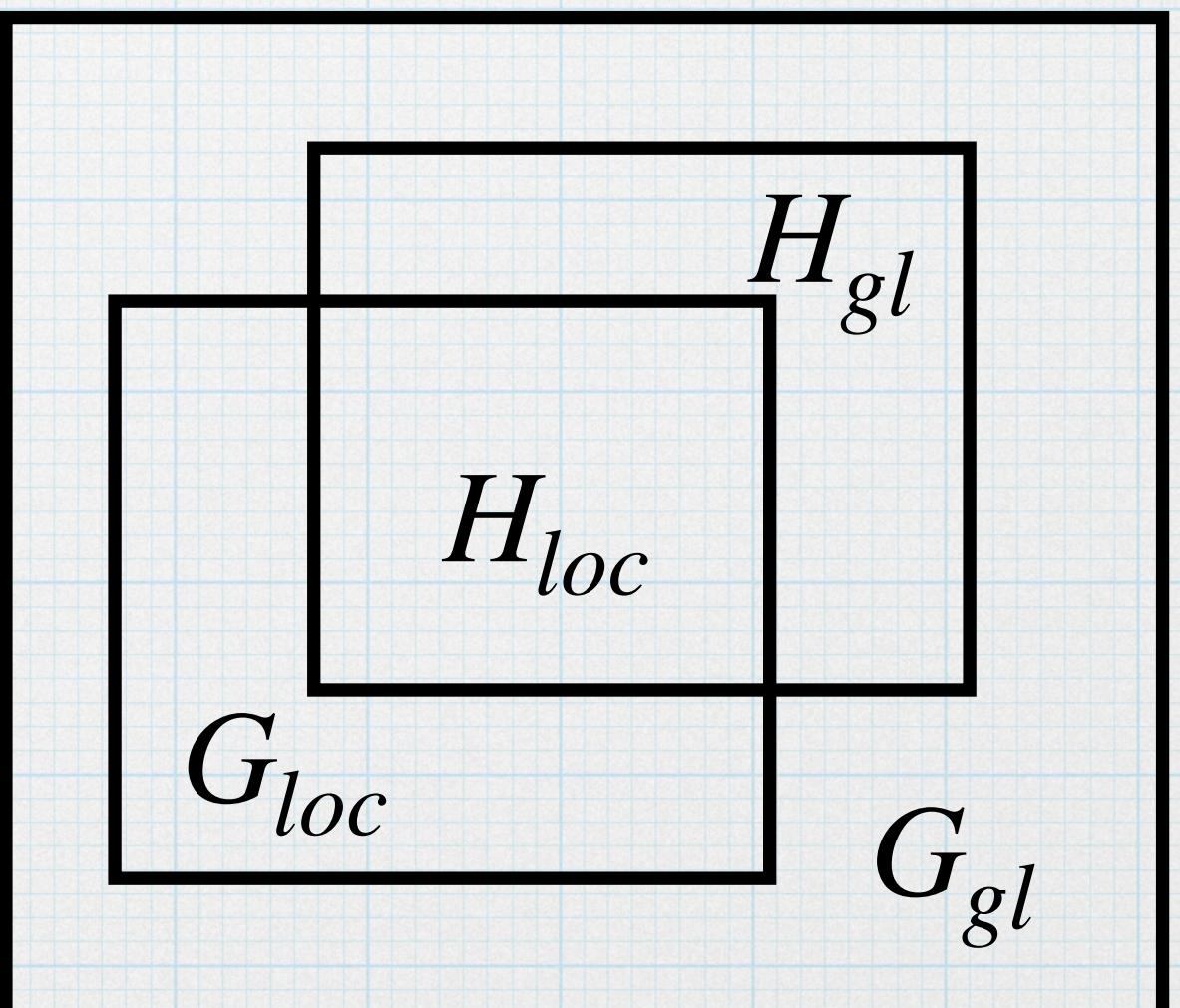
Summary

- * NG Higgs SUSY GUT : a solution to the **doublet-triplet splitting problem (NG boson → naturally light)**
- * In the minimal NG Higgs SUSY SU(5) GUT
 - * **The global symmetry** → non-trivial relations among the GUT parameters, and restricts the SUSY particle masses through the **RGEs**
 - * **The perturbativity condition** suggests $\alpha_2(Q_G) > \alpha_3(Q_G)$
 - * $M_{H_C} \lesssim M_X$
 - * A **small wino mass** ($< \mathcal{O}(1)$ TeV) and a **relatively large gluino mass** ($> \mathcal{O}(1) - \mathcal{O}(100)$ TeV)
 - * **High-scale SUSY** scenario with $M_{\text{SUSY}} \approx \mathcal{O}(1)$ PeV is favored.
 - * Proton decay rates for $(p \rightarrow K^+ \bar{\nu})$ and $(p \rightarrow e^+ \pi^0)$: sizable → probed by future experiments

Backup

Global and Local Symmetry Breaking

- * Spontaneous symmetry breaking
- * Broken symmetry \rightarrow Nambu-goldstone boson
- * Quasi-goldstone bosons from the SSB of an approximate global sym.
- * $W = W_{\text{Higgs}}(\hat{\Sigma}) + W(\hat{\Sigma}, \hat{f})$
- * $W_{\text{Higgs}}(\hat{\Sigma})$: invariant under a global sym. group G_{gl}
- * $H_{loc} = H_{gl} \cap G_{loc}$
- * Goldstone $\hat{P} \sim [\text{adj}(G_{gl}) - \text{adj}(H_{gl})] - [\text{adj}(G_{loc}) - \text{adj}(H_{loc})]$
- * $H_{loc} = SU(3) \times SU(2) \times U(1)$



Global and Local Symmetry Breaking

- * Properties of \hat{P}
 - * Vector-like
 - * Only singlets or doublets under $SU(2)$, and singlets or triplets under $SU(3)$
 - * For $G_{loc} = SO(10)$, $\hat{P} \supset \hat{T}$
 - * \hat{T} and \hat{D} : not from the same multiplet
- * Doublet-triplet splitting

Global and Local Symmetry Breaking

TABLE 1
Content of the \hat{P} supermultiplet under $SU(3) \otimes SU(2) \otimes U(1)$

	G_{loc}	G_{gl}	H_{gl}	\hat{P}
A	$SU(5)$	$SU(6)$	$SU(4) \otimes SU(2) \otimes U(1)$	\hat{D}
B	$SU(5)$	$SU(6)$	$SU(3) \otimes SU(2) \otimes U(1)$	$\hat{D} \oplus \hat{T} \oplus \hat{S}$
C	$SO(10)$	$SO(11)$	$SU(3) \otimes SU(2) \otimes U(1)$	$\hat{D} \oplus \hat{T}$
D	$SO(10)$	$SO(12)$	$SU(4) \otimes SU(2) \otimes U(1)$	$2\hat{D} \oplus \hat{T}$
E	$SO(10)$	$SO(12)$	$SU(3) \otimes SU(2) \otimes U(1)$	$2\hat{D} \oplus 2\hat{T} \oplus \hat{S}$
F	$SO(10)$	E_6	$SU(3) \otimes SU(2) \otimes U(1)$	$2\hat{D} \oplus 2\hat{T} \oplus \hat{S} \oplus \hat{Q}$

$$\hat{D} = (1, 2)_{1/2} \oplus (1, 2)_{-1/2}, \hat{T} = (3, 1)_{-1/3} \oplus (\bar{3}, 1)_{1/3}, \hat{S} = (1, 1)_0, \hat{Q} = (3, 2)_{1/6} \oplus (\bar{3}, 2)_{-1/6}.$$

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Content of the \hat{P} supermultiplet under $SU(3) \otimes SU(2) \otimes U(1)$

	G_{loc}	G_{gl}	H_{gl}	\hat{P}
A	$SU(5)$	$SU(6)$	$SU(4) \otimes SU(2) \otimes U(1)$	D
B	$SU(5)$	$SU(6)$	$SU(3) \otimes SU(2) \otimes U(1)$	$\hat{D} \oplus \hat{T} \oplus \hat{S}$
C	$SO(10)$	$SO(11)$	$SU(3) \otimes SU(2) \otimes U(1)$	$\hat{D} \oplus \hat{T}$
D	$SO(10)$	$SO(12)$	$SU(4) \otimes SU(2) \otimes U(1)$	$2\hat{D} \oplus T$
E	$SO(10)$	$SO(12)$	$SU(3) \otimes SU(2) \otimes U(1)$	$2\hat{D} \oplus 2\hat{T} \oplus \hat{S}$
F	$SO(10)$	E_6	$SU(3) \otimes SU(2) \otimes U(1)$	$2\hat{D} \oplus 2\hat{T} \oplus \hat{S} \oplus \hat{Q}$

$$\hat{D} = (1, 2)_{1/2} \oplus (1, 2)_{-1/2}, \hat{T} = (3, 1)_{-1/3} \oplus (\bar{3}, 1)_{1/3}, \hat{S} = (1, 1)_0, \hat{Q} = (3, 2)_{1/6} \oplus (\bar{3}, 2)_{-1/6}.$$

Nambu-Goldstone mulets

- * SSB of the potential \rightarrow goldstone boson
- * SSB of the superpotential \rightarrow goldstone superfields
- * Our model:
 - * 1 goldstone boson
 - * 1 quasi-goldstone boson
 - * 2 quasi-goldstone fermions

Coupling Constants

- * The scale Q_G such that $g_1(Q_G) = g_2(Q_G)$

- * $\alpha_5(Q_G) = 3 \left[\frac{14}{\alpha_2(Q_G)} - \frac{11}{\alpha_3(Q_G)} \right]^{-1}$

- * $M_X = Q_G \exp \left[\frac{\pi}{6} \left(-\frac{4}{\alpha_2(Q_G)} + \frac{4}{\alpha_3(Q_G)} \right) \right]$

- * $M_{H_C} = Q_G \exp \left[\frac{5\pi}{6} \left(\frac{2}{\alpha_2(Q_G)} - \frac{2}{\alpha_3(Q_G)} \right) \right]$

- * $\lambda = 2\sqrt{2}g_5 \frac{M_{H_C}}{M_X} = 2\sqrt{2}g_5 \exp \left[\frac{7\pi}{3} \left(\frac{1}{\alpha_2(Q_G)} - \frac{1}{\alpha_3(Q_G)} \right) \right]$

- * The NG Higgs GUT is more restrictive than the usual SU(5) (sharper predictions for proton decay)

- * $M_{SUSY} = 1000 \text{ TeV}$
- * Purple region: LHC limit
- * Red line: $\Omega_{\widetilde{W}} h^2 = \Omega_{DM} h^2$
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