Littlest Modular Seesaw

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PROGRAMA OPERACIONAL POTENCIAL HUMANO

Outline

- Littlest Seesaw
  - Numerical Results
- Modular Framework
- Multiple Modular Symmetries
- Domain(s)
  - Fundamental *vs.* Full Domain
  - Fixed Points
- Littlest Modular Seesaw
- Charged-Lepton Hierarchies
  - The Weighton
- Quarks?
  - *SU(5)* Extension: WIP

Littlest Seesaw

Littlest Seesaw (disclaimers)

- Left-Right Convention
- Only *b* is complex
- Type-I Seesaw
- (Effective) 2 RH Neutrino Framework
- Basis: $(a \otimes b)_1 = (a_1 b_1 + a_2 b_3 + a_3 b_2)$
- Bottom-Up

Littlest Seesaw

- Dirac Neutrino Mass Matrix

$$M_D = v_u \begin{pmatrix} 0 & b \\ a & b\,n \\ -a & b\,(2-n) \end{pmatrix}$$

- Diagonal Charged Leptons

$$M_\ell = \mathrm{diag}\left(m_e\,,\, m_\mu\,,\, m_\tau\right)$$

- Diagonal RH Neutrinos

$$M_R = \mathrm{diag}\left(M_A\,,\, M_B\right)$$

Littlest Seesaw

- Effective Neutrino Mass Matrix

$$m_\nu = M_D \cdot M_R^{-1} \cdot M_D^T = v_u^2 \begin{pmatrix} \frac{b^2}{M_B} & \frac{b^2 n}{M_B} & \frac{b^2(2-n)}{M_B} \\ . & \frac{a^2}{M_A} + \frac{b^2 n^2}{M_B} & -\frac{a^2}{M_A} + \frac{b^2 n(2-n)}{M_B} \\ . & . & \frac{a^2}{M_A} + \frac{b^2(2-n)^2}{M_B} \end{pmatrix}$$

Littlest Seesaw

$$m_\nu$$

Analytics

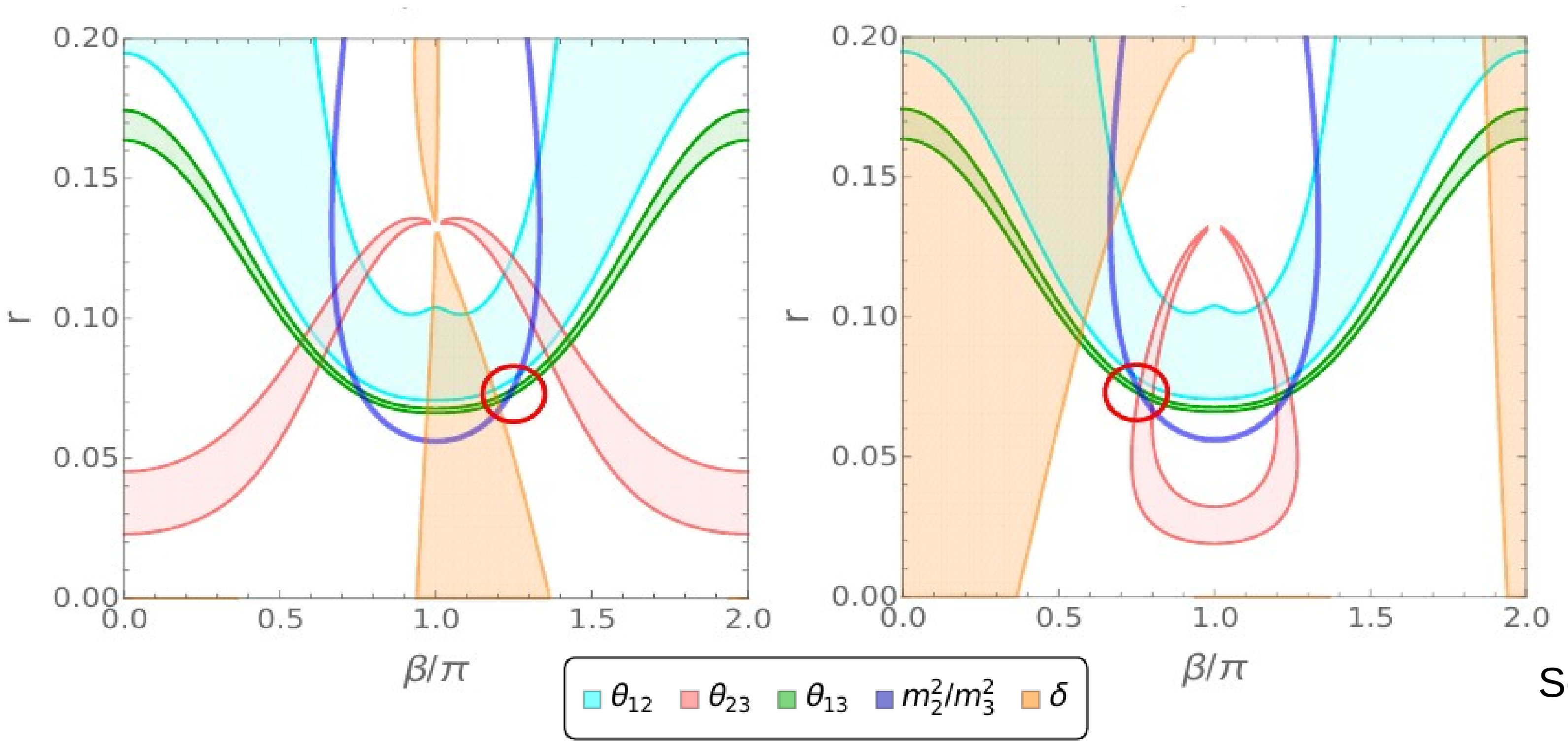
$$\mathcal{U}_\nu^T \cdot m_\nu \cdot \mathcal{U}_\nu = \mathrm{diag}(0, |m_1|, |m_2|)$$

$$\mathcal{U}_\nu \equiv (\mathcal{U}_{\mathrm{TBM}} \mathcal{U}_\alpha P_\nu) = \begin{pmatrix} -\sqrt{\frac{2}{3}} & \frac{c_\alpha}{\sqrt{3}} & e^{i\gamma}\frac{s_\alpha}{\sqrt{3}} \\ \sqrt{\frac{1}{6}} & \frac{c_\alpha}{\sqrt{3}} - e^{-i\gamma}\frac{s_\alpha}{\sqrt{2}} & \frac{c_\alpha}{\sqrt{2}} + e^{i\gamma}\frac{s_\alpha}{\sqrt{3}} \\ \sqrt{\frac{1}{6}} & \frac{c_\alpha}{\sqrt{3}} + e^{-i\gamma}\frac{s_\alpha}{\sqrt{2}} & -\frac{c_\alpha}{\sqrt{2}} + e^{i\gamma}\frac{s_\alpha}{\sqrt{3}} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & e^{i\phi_3} \end{pmatrix}$$

Littlest Seesaw (Numerical Results)

1σ **with** SK atmospheric data

$n = 1 + \sqrt{6}$ $\qquad$ $n = 1 - \sqrt{6}$



$$r = \left(\frac{a^2}{M_A}\right) / \left(\frac{|b|^2}{M_B}\right)$$

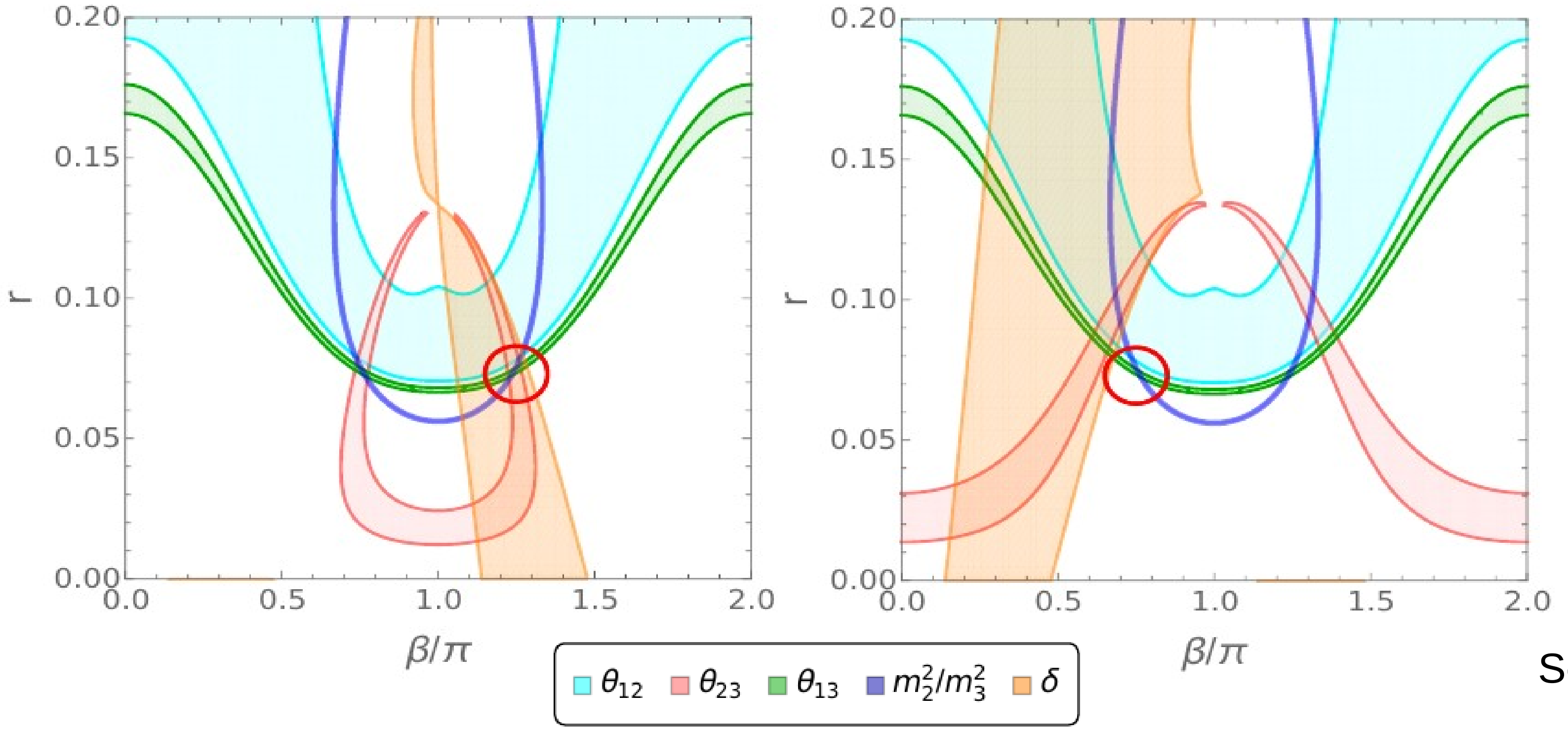
$$b = |b| e^{i\beta}$$

See also 1910.03460

Littlest Seesaw (Numerical Results)

1σ **without** SK atmospheric data

$$n = 1 + \sqrt{6} \qquad\qquad n = 1 - \sqrt{6}$$



$$r = \left(\frac{a^2}{M_A}\right) \Big/ \left(\frac{|b|^2}{M_B}\right)$$

$$b = |b|e^{i\beta}$$

See also 1910.03460

Littlest Seesaw

$$n = 1 \pm \sqrt{6}$$

$$M_D = v_u \begin{pmatrix} 0 & b \\ a & b\,n \\ -a & b\,(2-n) \end{pmatrix}$$

$$M_\ell = \mathrm{diag}\left(m_e\,,\; m_\mu\,,\; m_\tau\right)$$

$$M_R = \mathrm{diag}\left(M_A\,,\; M_B\right)$$

Works
(as expected)

Modular Framework

Modular Framework

- Modular Action

$$\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma \, : \, \tau \to \gamma\tau = \frac{a\tau + b}{c\tau + d}$$

- Weighted Representations

$$\psi \sim (\mathbf{r}_i \, , k_i)$$

- Transformation

$$\psi_i \to (c\tau + d)^{-k_i} \, \rho_{ij}(\gamma) \, \psi_j$$

Modular Framework

- Modular Action

$$\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma \ : \ \tau \to \gamma\tau = \frac{a\tau + b}{c\tau + d}$$

- Weighted Representations

$$\psi \sim (\mathbf{r}_i \, , \boxed{k_i}) \longrightarrow \boxed{\text{Modular Action}}$$

- Transformation

$$\psi_i \to \boxed{(c\tau + d)^{-k_i}} \rho_{ij}(\gamma) \psi_j$$

Modular Framework

- Modular Action

$$\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma \; : \; \tau \to \gamma\tau = \frac{a\tau + b}{c\tau + d}$$

- Weighted Representations

$$\psi \sim \left(\boxed{\mathbf{r}_i \,,} \; k_i\right)$$

Select Γ_N

- Transformation

$$\psi_i \to (c\tau + d)^{-k_i} \boxed{\rho_{ij}(\gamma)} \, \psi_j$$

Modular Framework

- Superpotential

$$W\left(\tau,\psi_I\right)=\sum\left(Y_{I_1\ldots I_n}\left(\tau\right)\psi_{I_1}\ldots\psi_{I_n}\right)_{\mathbf{1}}$$

Modular Framework

- Superpotential

$$W\left(\tau, \psi_I\right) = \sum \left(Y_{I_1 \ldots I_n}\left(\tau\right) \underbrace{\psi_{I_1} \ldots \psi_{I_n}}_{\text{Invariant}} \right)_{\mathbf{1}}$$

Modular Framework

- Superpotential

$$W\left(\tau, \psi_I\right) = \sum \left(\underbrace{Y_{I_1 \ldots I_n}(\tau)}_{\text{Trivial}} \underbrace{\boxed{\psi_{I_1} \ldots \psi_{I_n}}}_{\text{Invariant}} \right)_{\mathbf{1}}$$

Modular Framework

- Superpotential

$$W(\tau,\psi_I)=\sum\left(\underbrace{Y_{I_1\ldots I_n}(\tau)}_{\text{Trivial}}\underbrace{\boxed{\psi_{I_1}\ldots\psi_{I_n}}}_{\text{Invariant}}\right)_{\mathbf{1}}$$

(Business as usual)

Modular Framework

- Superpotential

$$W\left(\tau,\psi_I\right)=\sum\left(Y_{I_1\ldots I_n}\left(\tau\right)\underbrace{\boxed{\psi_{I_1}\ldots\psi_{I_n}}}_{\text{Not-Invariant}}\right)_{\mathbf{1}}$$

Modular Framework

- Superpotential

$$W(\tau, \psi_I) = \sum \left(\underbrace{Y_{I_1 \ldots I_n}(\tau)}_{?} \, \underbrace{\boxed{\psi_{I_1} \ldots \psi_{I_n}}}_{\text{Not-Invariant}} \right)_{\mathbf{1}}$$

Modular Framework

- Cancels Out Transformation

$$Y_{I_1 \ldots I_n}(\tau) \to Y_{I_1 \ldots I_n}(\gamma\tau) = (c\tau + d)^{k_Y}\, \rho(\gamma)\, Y_{I_1 \ldots I_n}(\tau)$$

- If $\quad k_Y = k_{I_1} + \cdots + k_{I_n} \quad$ & $\quad \rho \otimes \rho_{I_1} \otimes \cdots \otimes \rho_{I_n} \supset \mathbf{1}$

Modular Framework

- Cancels Out Transformation

$$Y_{I_1 \ldots I_n}(\tau) \to Y_{I_1 \ldots I_n}(\gamma\tau) = (c\tau + d)^{k_Y} \rho(\gamma) Y_{I_1 \ldots I_n}(\tau)$$

- If $\quad k_Y = k_{I_1} + \cdots + k_{I_n} \quad$ & $\quad \rho \otimes \rho_{I_1} \otimes \cdots \otimes \rho_{I_n} \supset \mathbf{1}$

$$\boxed{\text{Modular Forms!}}$$

Modular Framework

- Modular Forms (Yukawas):
  - "Weighted-representations" (-*ish)*
  - Representations (and weights) constrained by Γ_N
  - Specific functions of τ
  - $$Y_{\mathbf{r}}^{(k)}(\tau) = \left(\bigotimes_{n=1}^{k} Y_{\mathbf{r_1}}^{(1)}(\tau)\right)_{\mathbf{r}}$$

Multiple Modular Symmetries

Multiple Modular Symmetries

- $\Gamma_N \to \Gamma_N^A \times \Gamma_N^B \times \ldots$
- Invariance Required *Individually* (Symmetry by Symmetry)
- $W\left(\tau, \psi_I\right) = \sum \left(Y_{I_1 \ldots I_n}\left(\tau_A\right) Y_{I_1 \ldots I_n}\left(\tau_B\right) \ldots \psi_{I_1} \ldots \psi_{I_n} \right)_{\mathbf{1}}$

Multiple Modular Symmetries

- $\Gamma_N \to \Gamma_N^A \times \Gamma_N^B \times \ldots$

- Invariance Required *Individually* (Symmetry by Symmetry)

- $W(\tau, \psi_I) = \sum \left(Y_{I_1 \ldots I_n}(\tau_A) \, Y_{I_1 \ldots I_n}(\tau_B) \ldots \psi_{I_1} \ldots \psi_{I_n} \right)_{\mathbf{1}}$

- *Nothing Changes*

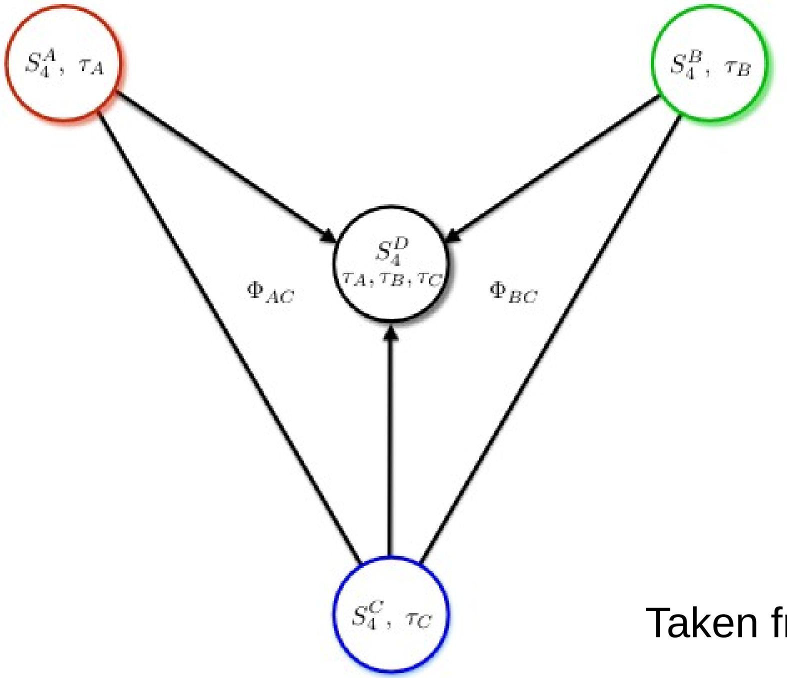
Multiple Modular Symmetries

- (Spontaneously) Breaking the Multiple Modular Symmetries

- Introduce *N*-reps

vevs

"Diagonal" Subgroup



Taken from 1906.02208

Multiple Modular Symmetries

- Example:
  - Forbidden

ψ_1	ψ_2	$Y(\tau_A)$	$Y(\tau_B)$
$(\mathbf{3}_0, \mathbf{1}_0)$	$(\mathbf{1}_0, \mathbf{1}_4)$	$(\mathbf{3}_0, -)$	$(-, \mathbf{1}_4)$

Multiple Modular Symmetries

- Example:
  - Forbidden

$$\begin{array}{cccc} \psi_1 & \psi_2 & Y(\tau_A) & Y(\tau_B) \\ (\mathbf{3}_0, \mathbf{1}_0) & (\mathbf{1}_0, \mathbf{1}_4) & (\mathbf{3}_0, -) & (-, \mathbf{1}_4) \end{array}$$

  - Possible

$$\begin{array}{ccccc} \psi_1 & \Phi_{AB} & \psi_2 & Y(\tau_A) & Y(\tau_B) \\ (\mathbf{3}_0, \mathbf{1}_0) & (\mathbf{3}_0, \mathbf{3}_0) & (\mathbf{1}_0, \mathbf{1}_4) & (\mathbf{1}_0, -) & (-, \mathbf{3}_4) \end{array}$$

  - Carefully chosen vevs

$$\langle \Phi_{AB} \rangle \, \psi_1 \, \psi_2 \, Y_{\mathbf{1}}^{(0)}(\tau_A) \, Y_{\mathbf{3}}^{(4)}(\tau_B)$$

Multiple Modular Symmetries

- Example:
  - Carefully chosen vevs:
    - Mimics a single Modular Symmetry Exchanging the Representation
    - In Practice: Change Value of τ

$$\langle \Phi_{AB}\rangle \; \psi_1 \, \psi_2 \, Y^{(0)}_{\mathbf{1}}(\tau_A) \, Y^{(4)}_{\mathbf{3}}(\tau_B) \sim \begin{array}{cccc} \psi_1 & \psi_2 & Y(\tau_A) & Y(\tau_B) \\ (\mathbf{1}_0, \mathbf{3}_0) & (\mathbf{1}_0, \mathbf{1}_4) & (\mathbf{1}_0, -) & (-, \mathbf{3}_4) \end{array}$$

(Fundamental) Domain

(Fundamental) Domain

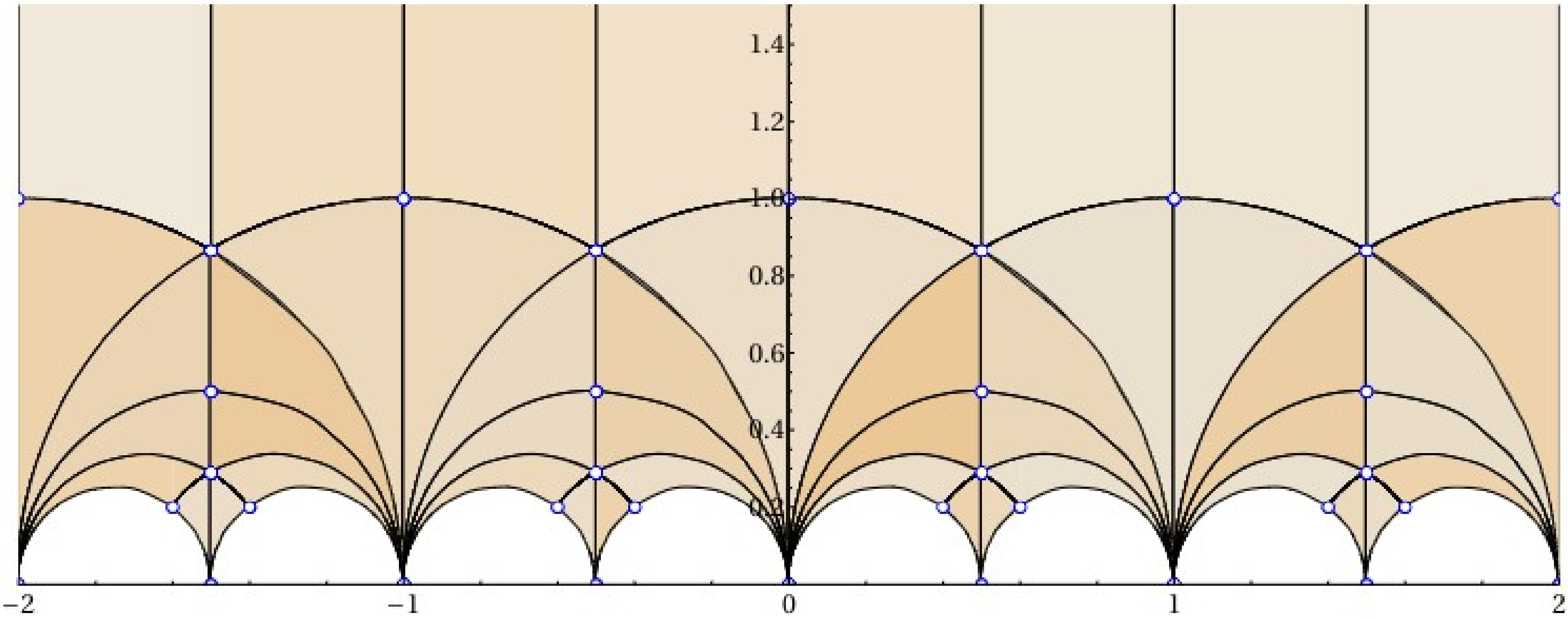
- No Flavons
  (Invariance under Full Γ_N)
  - We Can Send τ to Fundamental Domain via a Modular Action

$$Y_{I_1 \ldots I_n}(\tau) \to Y_{I_1 \ldots I_n}(\gamma\tau)$$

  - Redefine Couplings:
    Same Physical Theory!

- Flavons
  (Bi-Triplets w/ vevs)
  - No Longer Have the Freedom
  - We Can Take Advantage of the Full Domain of Γ_N

Fixed Points (Γ_4)



Taken from 2008.05329

Fixed Points (Γ_4)

$$\tau_C = \omega \ : \qquad \begin{aligned} Y^{(2)}_{\mathbf{3'}}(\tau_C) &= (0\,,1\,,0) \\ Y^{(4)}_{\mathbf{3'}}(\tau_C) &= (0\,,0\,,1) \\ Y^{(6)}_{\mathbf{3'}}(\tau_C) &= (1\,,0\,,0) \end{aligned}$$

$$\tau_A = \frac{1}{2} + \frac{i}{2} \ : \qquad Y^{(4)}_{\mathbf{3'}}(\tau_C) = (0\,,-1\,,1)$$

$$\tau_B = \frac{3}{2} + \frac{i}{2} \ : \qquad Y^{(2)}_{\mathbf{3'}}(\tau_C) = \left(1\,,1-\sqrt{6}\,,1+\sqrt{6}\right)$$

$$\tau_B = -\frac{1}{2} + \frac{i}{2} \ : \qquad Y^{(2)}_{\mathbf{3'}}(\tau_C) = \left(1\,,1+\sqrt{6}\,,1-\sqrt{6}\right)$$

Fixed Points (Γ_4)

$$\tau_C = \omega \;: \qquad \begin{aligned} Y^{(2)}_{\mathbf{3}'}(\tau_C) &= (0\,,1\,,0) \\ Y^{(4)}_{\mathbf{3}'}(\tau_C) &= (0\,,0\,,1) \\ Y^{(6)}_{\mathbf{3}'}(\tau_C) &= (1\,,0\,,0) \end{aligned}$$

$$M_\ell = \mathrm{diag}\left(m_e\,,\, m_\mu\,,\, m_\tau\right)$$

Fixed Points (Γ_4)

$$M_D = v_u \begin{pmatrix} 0 & b \\ a & b\,n \\ -a & b\,(2-n) \end{pmatrix} \qquad n = 1 \pm \sqrt{6}$$

$$\tau_A = \frac{1}{2} + \frac{i}{2} \; : \qquad Y_{\mathbf{3}'}^{(4)}(\tau_C) = (0\,,-1\,,1)$$

$$\tau_B = \frac{3}{2} + \frac{i}{2} \; : \qquad Y_{\mathbf{3}'}^{(2)}(\tau_C) = \left(1\,,1-\sqrt{6}\,,1+\sqrt{6}\right)$$

$$\tau_B = -\frac{1}{2} + \frac{i}{2} \; : \qquad Y_{\mathbf{3}'}^{(2)}(\tau_C) = \left(1\,,1+\sqrt{6}\,,1-\sqrt{6}\right)$$

Fixed Points (Γ_4)

Different Moduli

$$\tau_C = \omega \; :$$

$$Y^{(2)}_{\mathbf{3}'}(\tau_C) = (0\,,1\,,0)$$
$$Y^{(4)}_{\mathbf{3}'}(\tau_C) = (0\,,0\,,1)$$
$$Y^{(6)}_{\mathbf{3}'}(\tau_C) = (1\,,0\,,0)$$

$$\tau_A = \frac{1}{2} + \frac{i}{2} \; :$$

$$Y^{(4)}_{\mathbf{3}'}(\tau_C) = (0\,,-1\,,1)$$

$$\tau_B = \frac{3}{2} + \frac{i}{2} \; :$$

$$Y^{(2)}_{\mathbf{3}'}(\tau_C) = \left(1\,,1-\sqrt{6}\,,1+\sqrt{6}\right)$$

$$\tau_B = -\frac{1}{2} + \frac{i}{2} \; :$$

$$Y^{(2)}_{\mathbf{3}'}(\tau_C) = \left(1\,,1+\sqrt{6}\,,1-\sqrt{6}\right)$$

Modular Littlest Seesaw

Modular Littlest Seesaw

How?

Modular Littlest Seesaw (Superpotential)

Field	S_4^A	S_4^B	S_4^C	$2k_A$	$2k_B$	$2k_C$
L	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	0	0	0
e^c	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}'$	0	0	-6
μ^c	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}'$	0	0	-4
τ^c	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}'$	0	0	-2
N_A^c	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{1}$	-4	0	0
N_B^c	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{1}$	0	-2	0
Φ_{AC}	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{3}$	0	0	0
Φ_{BC}	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	0	0	0

Modular Littlest Seesaw (Superpotential)

Field	S_4^A	S_4^B	S_4^C	$2k_A$	$2k_B$	$2k_C$
L	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	0	0	0
e^c	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}'$	0	0	-6
μ^c	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}'$	0	0	-4
τ^c	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}'$	0	0	-2
N_A^c	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{1}$	-4	0	0
N_B^c	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{1}$	0	-2	0
Φ_{AC}	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{3}$	0	0	0
Φ_{BC}	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	0	0	0

$$
\begin{aligned}
w_\ell = \Big\{ & Y^{(6)}_{\mathbf{3}'}(\tau_C) Y^{(0)}_{\mathbf{1}}(\tau_A) Y^{(0)}_{\mathbf{1}}(\tau_B)\, L e^c \\
& + Y^{(4)}_{\mathbf{3}'}(\tau_C) Y^{(0)}_{\mathbf{1}}(\tau_A) Y^{(0)}_{\mathbf{1}}(\tau_B)\, L \mu^c \\
& + Y^{(2)}_{\mathbf{3}'}(\tau_C) Y^{(0)}_{\mathbf{1}}(\tau_A) Y^{(0)}_{\mathbf{1}}(\tau_B)\, L \tau^c \Big\} H_d
\end{aligned}
$$

Fixed Points

$$\tau_C = \omega \ : \qquad \begin{aligned} Y_{\mathbf{3}'}^{(2)}(\tau_C) &= (0\,,1\,,0) \\ Y_{\mathbf{3}'}^{(4)}(\tau_C) &= (0\,,0\,,1) \\ Y_{\mathbf{3}'}^{(6)}(\tau_C) &= (1\,,0\,,0) \end{aligned}$$

$$M_\ell = \mathrm{diag}\left(m_e\,,\ m_\mu\,,\ m_\tau\right)$$

Modular Littlest Seesaw (Superpotential)

Field	S_4^A	S_4^B	S_4^C	$2k_A$	$2k_B$	$2k_C$
L	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	0	0	0
e^c	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}'$	0	0	-6
μ^c	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}'$	0	0	-4
τ^c	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}'$	0	0	-2
N_A^c	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{1}$	-4	0	0
N_B^c	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{1}$	0	-2	0
Φ_{AC}	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{3}$	0	0	0
Φ_{BC}	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	0	0	0

$$w_\ell = \left\{ Y_{\mathbf{3}'}^{(6)}(\tau_C) Y_{\mathbf{1}}^{(0)}(\tau_A) Y_{\mathbf{1}}^{(0)}(\tau_B)\, L e^c + Y_{\mathbf{3}'}^{(4)}(\tau_C) Y_{\mathbf{1}}^{(0)}(\tau_A) Y_{\mathbf{1}}^{(0)}(\tau_B)\, L \mu^c + Y_{\mathbf{3}'}^{(2)}(\tau_C) Y_{\mathbf{1}}^{(0)}(\tau_A) Y_{\mathbf{1}}^{(0)}(\tau_B)\, L \tau^c \right\} H_d$$

$$\boxed{w_D = \left\{ Y_{\mathbf{1}}^{(0)}(\tau_C) Y_{\mathbf{1}}^{(0)}(\tau_B) Y_{\mathbf{3}'}^{(4)}(\tau_A)\, L \langle \Phi_{AC} \rangle N_A^c + Y_{\mathbf{1}}^{(0)}(\tau_C) Y_{\mathbf{1}}^{(0)}(\tau_A) Y_{\mathbf{3}'}^{(2)}(\tau_B)\, L \langle \Phi_{BC} \rangle N_B^c \right\} \frac{H_u}{\Lambda}}$$

Fixed Points

$$M_D = v_u \begin{pmatrix} 0 & b \\ a & b\,n \\ -a & b\,(2-n) \end{pmatrix} \qquad n = 1 \pm \sqrt{6}$$

$$\boxed{\begin{array}{ll} \tau_A = \dfrac{1}{2} + \dfrac{i}{2} \;: & Y^{(4)}_{\mathbf{3'}}(\tau_C) = (0\,, -1\,, 1) \\[2em] \tau_B = \dfrac{3}{2} + \dfrac{i}{2} \;: & Y^{(2)}_{\mathbf{3'}}(\tau_C) = \left(1\,, 1-\sqrt{6}\,, 1+\sqrt{6}\right) \\ \tau_B = -\dfrac{1}{2} + \dfrac{i}{2} \;: & Y^{(2)}_{\mathbf{3'}}(\tau_C) = \left(1\,, 1+\sqrt{6}\,, 1-\sqrt{6}\right) \end{array}}$$

Modular Littlest Seesaw (Superpotential)

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$$
\begin{aligned}
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+ & Y^{(4)}_{\mathbf{3}'}(\tau_C) Y^{(0)}_{\mathbf{1}}(\tau_A) Y^{(0)}_{\mathbf{1}}(\tau_B)\, L \mu^c \\
+ & Y^{(2)}_{\mathbf{3}'}(\tau_C) Y^{(0)}_{\mathbf{1}}(\tau_A) Y^{(0)}_{\mathbf{1}}(\tau_B)\, L \tau^c \Big\} H_d
\end{aligned}
$$

$$
\begin{aligned}
w_D = \Big\{ & Y^{(0)}_{\mathbf{1}}(\tau_C) Y^{(0)}_{\mathbf{1}}(\tau_B) Y^{(4)}_{\mathbf{3}'}(\tau_A)\, L \langle \Phi_{AC} \rangle N^c_A \\
+ & Y^{(0)}_{\mathbf{1}}(\tau_C) Y^{(0)}_{\mathbf{1}}(\tau_A) Y^{(2)}_{\mathbf{3}'}(\tau_B)\, L \langle \Phi_{BC} \rangle N^c_B \Big\} \frac{H_u}{\Lambda}
\end{aligned}
$$

$$
\begin{aligned}
w_M = \frac{1}{2} \Big\{ & Y^{(0)}_{\mathbf{1}}(\tau_C) Y^{(0)}_{\mathbf{1}}(\tau_B) Y^{(8)}_{\mathbf{1}}(\tau_A)\, N^c_A N^c_A \\
+ & Y^{(0)}_{\mathbf{1}}(\tau_C) Y^{(0)}_{\mathbf{1}}(\tau_A) Y^{(4)}_{\mathbf{1}}(\tau_B)\, N^c_B N^c_B \\
+ & Y^{(0)}_{\mathbf{1}}(\tau_C) Y^{(4)}_{\mathbf{1}'}(\tau_A) Y^{(2)}_{\mathbf{1}'}(\tau_B)\, N^c_A N^c_B \Big\}
\end{aligned}
$$

Modular Littlest Seesaw

- Dirac Neutrino Mass Matrix

$$M_D = v_u \begin{pmatrix} 0 & b \\ a & b\,n \\ -a & b\,(2-n) \end{pmatrix}$$

$$n = 1 \pm \sqrt{6}$$

- Diagonal Charged Leptons

$$M_\ell = \mathrm{diag}\left(m_e\,,\, m_\mu\,,\, m_\tau\right)$$

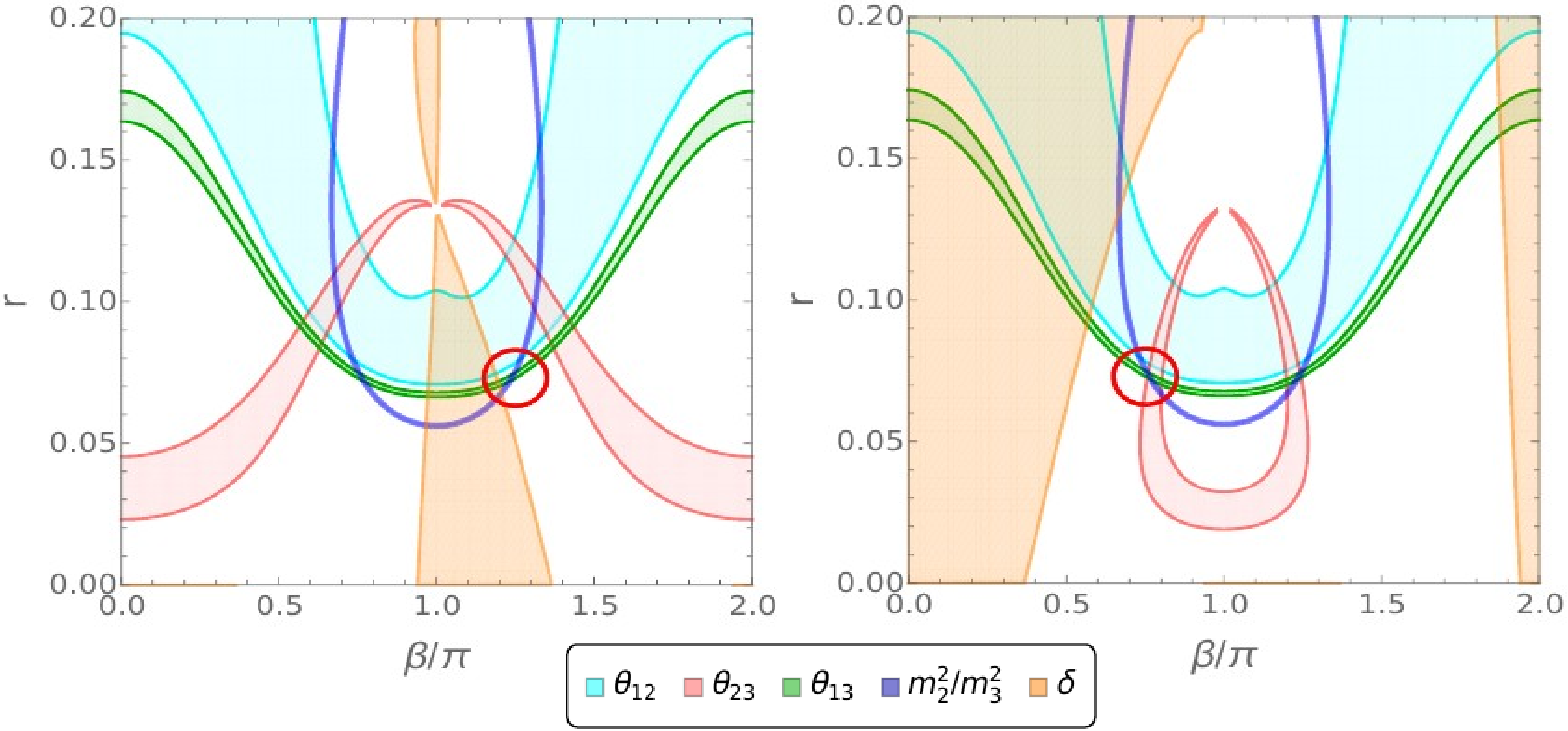
- Diagonal RH Neutrinos

$$M_R = \mathrm{diag}\left(M_A\,,\, M_B\right)$$

Littlest Seesaw (Numerical Results)

1σ **with** SK atmospheric data

$n = 1+\sqrt{6}$ $\qquad$ $n = 1-\sqrt{6}$

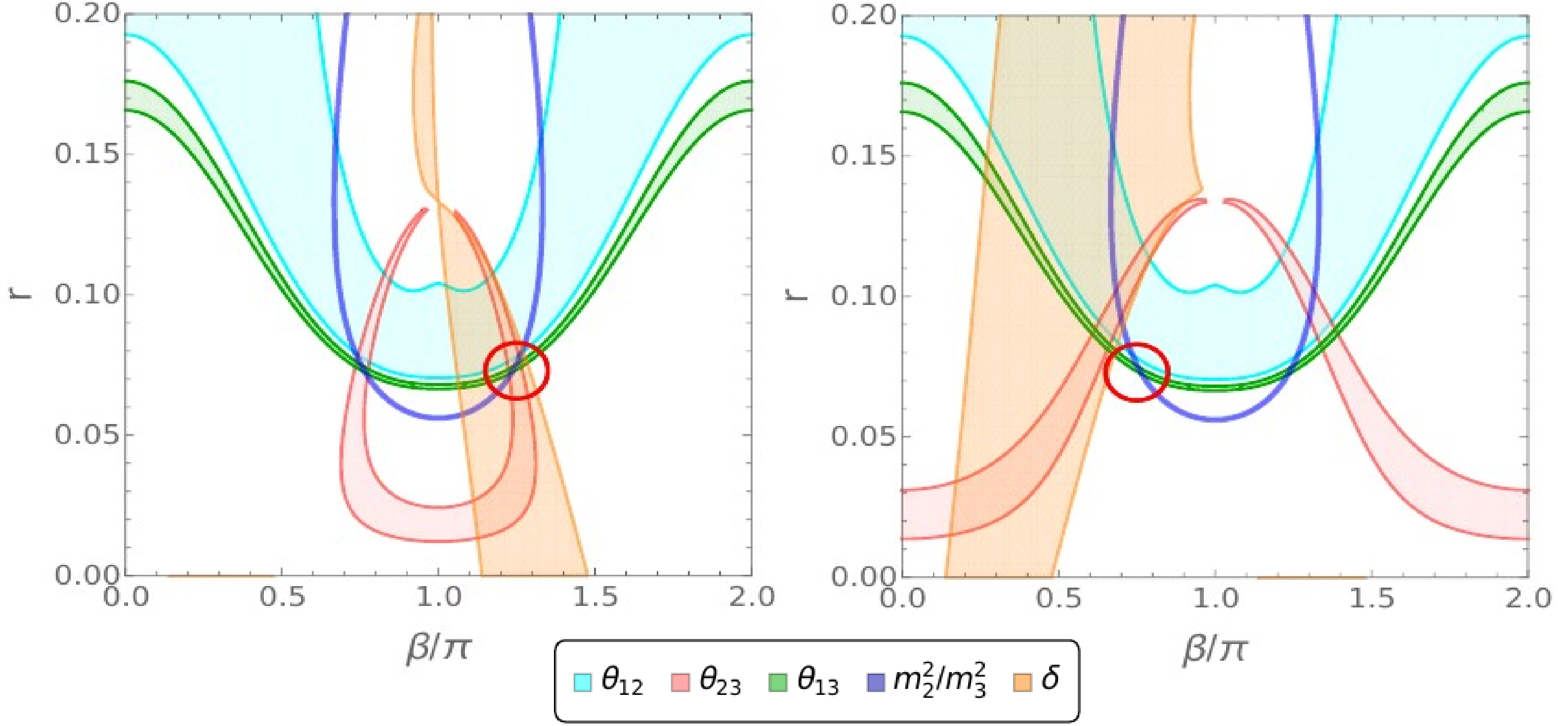


$$r = \left(\frac{a^2}{M_A}\right) / \left(\frac{|b|^2}{M_B}\right)$$

$$b = |b|e^{i\beta}$$

Littlest Seesaw
(Numerical Results)

$$n = 1 + \sqrt{6}$$ $$n = 1 - \sqrt{6}$$



1σ **without** SK atmospheric data

$$r = \left(\frac{a^2}{M_A}\right) \Big/ \left(\frac{|b|^2}{M_B}\right)$$

$$b = |b| e^{i\beta}$$

```
if(time) then
    keep talking
else
    Thank you!
end if
```

Charged-Lepton Mass Hierarchies

The Weighton

Weightons

- Modular Symmetries:
  - Γ_N ~ “Traditional” Flavour Symmetry
  - Weights ~ *U(1)* Symmetry
  - (Yukawas need not be singlets)

Weightons

- Modular Symmetries:
  - Γ_N ~ "Traditional" Flavour Symmetry
  - Weights ~ *U(1)* Symmetry → Extra Use?
  - (Yukawas need not be singlets)

Weightons

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  - Γ_N ~ "Traditional" Flavour Symmetry
  - Weights ~ *U(1)* Symmetry
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- Weighton:
  - Singlet under Flavour
  - Non-trivial Weight
  - Acts as a FN-type *U(1)*
  - *U(1)* not imposed

Weightons

- Modular Symmetries:
  - Γ_N ~ “Traditional” Flavour Symmetry
  - Weights ~ *U(1)* Symmetry
  - (Yukawas need not be singlets)
- Weighton:
  - Singlet under Flavour
  - Non-trivial Weight
  - Acts as a FN-type *U(1)*
  - *U(1)* not imposed
  - Can lead to infinite corrections?

Weightons

$$
\begin{array}{rcl}
Y_{\mathbf{r}}^{(k)} & & \psi_1 \ldots \psi_n \\
Y_{\mathbf{r}}^{(k+k_\phi)} & \phi & \psi_1 \ldots \psi_n \\
Y_{\mathbf{r}}^{(k+2k_\phi)} & \phi^2 & \psi_1 \ldots \psi_n \\
 & \ldots &
\end{array}
$$

- Weighton:
  - Singlet under Flavour
  - Non-trivial Weight
  - Acts as a FN-type *U(1)*
  - *U(1)* not imposed
  - Can lead to infinite corrections?

Weighton Model #1

- Change the Irrep

Field	$S_4'^A$	$S_4'^B$	$S_4'^C$	$2k_A$	$2k_B$	$2k_C$
L	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	0	0	0
e^c	$\hat{\mathbf{1}}$	$\hat{\mathbf{1}}$	$\mathbf{1}'$	0	0	0
μ^c	$\hat{\mathbf{1}}'$	$\hat{\mathbf{1}}'$	$\mathbf{1}'$	0	0	-2
τ^c	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}'$	0	0	-2
N_A^c	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{1}$	-4	0	0
N_B^c	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{1}$	0	-2	0
Φ_{AC}	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{3}$	0	0	0
Φ_{BC}	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	0	0	0
ϕ	$\hat{\mathbf{1}}$	$\hat{\mathbf{1}}$	$\hat{\mathbf{1}}$	0	0	-2

Yuk/Mass	$S_4'^A$	$S_4'^B$	$S_4'^C$	$2k_A$	$2k_B$	$2k_C$
$Y_e(\tau_C)$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}'$	0	0	6
$Y_\mu(\tau_C)$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}'$	0	0	4
$Y_\tau(\tau_C)$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}'$	0	0	2
$Y_A(\tau_A)$	$\mathbf{3}'$	$\mathbf{1}$	$\mathbf{1}$	4	0	0
$Y_B(\tau_B)$	$\mathbf{1}$	$\mathbf{3}'$	$\mathbf{1}$	0	2	0
$M_A(\tau_A)$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	8	0	0
$M_B(\tau_B)$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	4	0

Weighton Model #1

- Why?

$$(\hat{\mathbf{1}} \otimes \hat{\mathbf{1}}) = \mathbf{1}'$$
$$(\mathbf{1}' \otimes \mathbf{1}') = \mathbf{1}$$
$$\Rightarrow (\hat{\mathbf{1}}^4) = \mathbf{1}$$

ϕ^4 Corrections

S_4	S_4'
$\mathbf{1}$	$\mathbf{1}$ and $\hat{\mathbf{1}}$
$\mathbf{1}'$	$\mathbf{1}'$ and $\hat{\mathbf{1}}'$
$\mathbf{2}$	$\mathbf{2}$ and $\hat{\mathbf{2}}$
$\mathbf{3}$	$\mathbf{3}$ and $\hat{\mathbf{3}}$
$\mathbf{3}'$	$\mathbf{3}'$ and $\hat{\mathbf{3}}'$

Weighton Model #1

	ϕ^0	ϕ^1	ϕ^2	ϕ^3	ϕ^4
Le^c	$(\hat{\mathbf{1}}_0, \hat{\mathbf{1}}_0, \hat{\mathbf{3}}'_0)$	$(\mathbf{1}'_0, \mathbf{1}'_0, \mathbf{3}_{-2})$	$(\hat{\mathbf{1}}'_0, \hat{\mathbf{1}}'_0, \hat{\mathbf{3}}_{-4})$	$(\mathbf{1}_0, \mathbf{1}_0, \mathbf{3}'_{-6})$	$(\hat{\mathbf{1}}_0, \hat{\mathbf{1}}_0, \hat{\mathbf{3}}'_{-8})$
$L\mu^c$	$(\hat{\mathbf{1}}'_0, \hat{\mathbf{1}}'_0, \hat{\mathbf{3}}_{-2})$	$(\mathbf{1}_0, \mathbf{1}_0, \mathbf{3}'_{-4})$	$(\hat{\mathbf{1}}_0, \hat{\mathbf{1}}_0, , \hat{\mathbf{3}}'_{-6})$	$(\mathbf{1}'_0, \mathbf{1}'_0, \mathbf{3}_{-8})$	$(\hat{\mathbf{1}}'_0, \hat{\mathbf{1}}'_0, \hat{\mathbf{3}}_{-10})$
$L\tau^c$	$(\mathbf{1}_0, \mathbf{1}_0, \mathbf{3}'_{-2})$	$(\hat{\mathbf{1}}_0, \hat{\mathbf{1}}_0, \hat{\mathbf{3}}'_{-4})$	$(\mathbf{1}'_0, \mathbf{1}'_0, \mathbf{3}_{-6})$	$(\hat{\mathbf{1}}'_0, \hat{\mathbf{1}}'_0, \hat{\mathbf{3}}_{-8})$	$(\mathbf{1}_0, \mathbf{1}_0, \mathbf{3}'_{-10})$
$L\Phi_{AC}N_A^c$	$(\mathbf{3}'_{-4}, \mathbf{1}_0, \mathbf{1}_0)$	$(\hat{\mathbf{3}}'_{-4}, \hat{\mathbf{1}}_0, \hat{\mathbf{1}}_{-2})$	$(\mathbf{3}_{-4}, \mathbf{1}'_0, \mathbf{1}'_{-4})$	$(\hat{\mathbf{3}}_{-4}, \hat{\mathbf{1}}'_0, \hat{\mathbf{1}}'_{-6})$	$(\mathbf{3}'_{-4}, \mathbf{1}_0, \mathbf{1}_{-8})$
$L\Phi_{BC}N_B^c$	$(\mathbf{1}_0, \mathbf{3}'_{-2}, \mathbf{1}_0)$	$(\hat{\mathbf{1}}_0, \hat{\mathbf{3}}'_{-2}, \hat{\mathbf{1}}_{-2})$	$(\mathbf{1}'_0, \mathbf{3}_{-2}, \mathbf{1}'_{-4})$	$(\hat{\mathbf{1}}'_0, \hat{\mathbf{3}}_{-2}, \hat{\mathbf{1}}'_{-6})$	$(\mathbf{1}_0, \mathbf{3}'_{-2}, \mathbf{1}_{-8})$
$N_A^cN_A^c$	$(\mathbf{1}_{-8}, \mathbf{1}_0, \mathbf{1}_0)$	$(\hat{\mathbf{1}}_{-8}, \hat{\mathbf{1}}_0, \hat{\mathbf{1}}_{-2})$	$(\mathbf{1}'_{-8}, \mathbf{1}'_0, \mathbf{1}'_{-4})$	$(\hat{\mathbf{1}}'_{-8}, \hat{\mathbf{1}}'_0, \hat{\mathbf{1}}'_{-6})$	$(\mathbf{1}_{-8}, \mathbf{1}_0, \mathbf{1}_{-8})$
$N_B^cN_B^c$	$(\mathbf{1}_0, \mathbf{1}_{-4}, \mathbf{1}_0)$	$(\hat{\mathbf{1}}_0, \hat{\mathbf{1}}_{-4}, \hat{\mathbf{1}}_{-2})$	$(\mathbf{1}'_0, \mathbf{1}'_{-4}, \mathbf{1}'_{-4})$	$(\hat{\mathbf{1}}'_0, \hat{\mathbf{1}}'_{-4}, \hat{\mathbf{1}}'_{-6})$	$(\mathbf{1}_0, \mathbf{1}_{-4}, \mathbf{1}_{-8})$
$N_A^cN_B^c$	$(\mathbf{1}'_{-4}, \mathbf{1}'_{-2}, \mathbf{1}_0)$	$(\hat{\mathbf{1}}'_{-4}, \hat{\mathbf{1}}'_{-2}, \hat{\mathbf{1}}_{-2})$	$(\mathbf{1}_{-4}, \mathbf{1}_{-2}, \mathbf{1}'_{-4})$	$(\hat{\mathbf{1}}_{-4}, \hat{\mathbf{1}}_{-2}, \hat{\mathbf{1}}'_{-6})$	$(\mathbf{1}'_{-4}, \mathbf{1}'_{-2}, \mathbf{1}_{-8})$
$N_A^c\Phi_{AC}N_A^c$	$(\mathbf{3}_{-8}, \mathbf{1}_0, \mathbf{3}_0)$	$(\hat{\mathbf{3}}_{-8}, \hat{\mathbf{1}}_0, \hat{\mathbf{3}}_{-2})$	$(\mathbf{3}'_{-8}, \mathbf{1}'_0, \mathbf{3}'_{-4})$	$(\hat{\mathbf{3}}'_{-8}, \hat{\mathbf{1}}'_0, \hat{\mathbf{3}}'_{-6})$	$(\mathbf{3}_{-8}, \mathbf{1}_0, \mathbf{3}_{-8})$
$N_B^c\Phi_{AC}N_B^c$	$(\mathbf{3}_0, \mathbf{1}_{-4}, \mathbf{3}_0)$	$(\hat{\mathbf{3}}_0, \hat{\mathbf{1}}_{-4}, \hat{\mathbf{3}}_{-2})$	$(\mathbf{3}'_0, \mathbf{1}'_{-4}, \mathbf{3}'_{-4})$	$(\hat{\mathbf{3}}'_0, \hat{\mathbf{1}}'_{-4}, \hat{\mathbf{3}}'_{-6})$	$(\mathbf{3}_0, \mathbf{1}_{-4}, \mathbf{3}_{-8})$
$N_A^c\Phi_{AC}N_B^c$	$(\mathbf{3}'_{-4}, \mathbf{1}'_{-2}, \mathbf{3}_0)$	$(\hat{\mathbf{3}}'_{-4}, \hat{\mathbf{1}}'_{-2}, \hat{\mathbf{3}}_{-2})$	$(\mathbf{3}_{-4}, \mathbf{1}_{-2}, \mathbf{3}'_{-4})$	$(\hat{\mathbf{3}}_{-4}, \hat{\mathbf{1}}_{-2}, \hat{\mathbf{3}}'_{-6})$	$(\mathbf{3}'_{-4}, \mathbf{1}'_{-2}, \mathbf{3}_{-8})$
$N_A^c\Phi_{BC}N_A^c$	$(\mathbf{1}_{-8}, \mathbf{3}_0, \mathbf{3}_0)$	$(\hat{\mathbf{1}}_{-8}, \hat{\mathbf{3}}_0, \hat{\mathbf{3}}_{-2})$	$(\mathbf{1}'_{-8}, \mathbf{3}'_0, \mathbf{3}'_{-4})$	$(\hat{\mathbf{1}}'_{-8}, \hat{\mathbf{3}}'_0, \hat{\mathbf{3}}'_{-6})$	$(\mathbf{1}_{-8}, \mathbf{3}_0, \mathbf{3}_{-8})$
$N_B^c\Phi_{BC}N_B^c$	$(\mathbf{1}_0, \mathbf{3}_{-4}, \mathbf{3}_0)$	$(\hat{\mathbf{1}}_0, \hat{\mathbf{3}}_{-4}, \hat{\mathbf{3}}_{-2})$	$(\mathbf{1}'_0, \mathbf{3}'_{-4}, \mathbf{3}'_{-4})$	$(\hat{\mathbf{1}}'_0, \hat{\mathbf{3}}'_{-4}, \hat{\mathbf{3}}'_{-6})$	$(\mathbf{1}_0, \mathbf{3}_{-4}, \mathbf{3}_{-8})$
$N_A^c\Phi_{BC}N_B^c$	$(\mathbf{1}'_{-4}, \mathbf{3}'_{-2}, \mathbf{3}_0)$	$(\hat{\mathbf{1}}'_{-4}, \hat{\mathbf{3}}'_{-2}, \hat{\mathbf{3}}_{-2})$	$(\mathbf{1}_{-4}, \mathbf{3}_{-2}, \mathbf{3}'_{-4})$	$(\hat{\mathbf{1}}_{-4}, \hat{\mathbf{3}}_{-2}, \hat{\mathbf{3}}'_{-6})$	$(\mathbf{1}'_{-4}, \mathbf{3}'_{-2}, \mathbf{3}_{-8})$

Weighton Model #2

- Still, there are corrections…
  - Mandatory?

Weighton Model #2

Field	S'^A_4	S'^B_4	S'^C_4	$2k_A$	$2k_B$	$2k_C$
L	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	0	0	0
e^c	$\hat{\mathbf{1}}$	$\hat{\mathbf{1}}$	$\mathbf{1}'$	0	0	-12
μ^c	$\hat{\mathbf{1}}'$	$\hat{\mathbf{1}}'$	$\mathbf{1}'$	0	0	-6
τ^c	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}'$	0	0	-2
N^c_A	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{1}$	-4	0	0
N^c_B	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{1}$	0	-2	0
Φ_{AC}	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{3}$	0	0	0
Φ_{BC}	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	0	0	0
ϕ	$\hat{\mathbf{1}}$	$\hat{\mathbf{1}}$	$\hat{\mathbf{1}}$	0	0	$+2$

Yuk/Mass	S'^A_4	S'^B_4	S'^C_4	$2k_A$	$2k_B$	$2k_C$
$Y_e(\tau_C)$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}'$	0	0	6
$Y_\mu(\tau_C)$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}'$	0	0	4
$Y_\tau(\tau_C)$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}'$	0	0	2
$Y_A(\tau_A)$	$\mathbf{3}'$	$\mathbf{1}$	$\mathbf{1}$	4	0	0
$Y_B(\tau_B)$	$\mathbf{1}$	$\mathbf{3}'$	$\mathbf{1}$	0	2	0
$M_A(\tau_A)$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	8	0	0
$M_B(\tau_B)$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	4	0

	ϕ^0	ϕ^1	ϕ^2	ϕ^3	ϕ^4
Le^c	$(\hat{\mathbf{1}}_0, \hat{\mathbf{1}}_0, \hat{\mathbf{3}}'_{-12})$	$(\mathbf{1}'_0, \mathbf{1}'_0, \mathbf{3}_{-10})$	$(\hat{\mathbf{1}}'_0, \hat{\mathbf{1}}'_0, \hat{\mathbf{3}}_{-8})$	$(\mathbf{1}_0, \mathbf{1}_0, \mathbf{3}'_{-6})$	$(\hat{\mathbf{1}}_0, \hat{\mathbf{1}}_0, \hat{\mathbf{3}}'_{-4})$
$L\mu^c$	$(\hat{\mathbf{1}}'_0, \hat{\mathbf{1}}'_0, \hat{\mathbf{3}}_{-6})$	$(\mathbf{1}_0, \mathbf{1}_0, \mathbf{3}'_{-4})$	$(\hat{\mathbf{1}}_0, \hat{\mathbf{1}}_0, \hat{\mathbf{3}}'_{-2})$	$(\mathbf{1}'_0, \mathbf{1}'_0, \mathbf{3}_0)$	$(\hat{\mathbf{1}}'_0, \hat{\mathbf{1}}'_0, \hat{\mathbf{3}}_{+2})$
$L\tau^c$	$(\mathbf{1}_0, \mathbf{1}_0, \mathbf{3}'_{-2})$	$(\hat{\mathbf{1}}_0, \hat{\mathbf{1}}_0, \hat{\mathbf{3}}'_0)$	$(\mathbf{1}'_0, \mathbf{1}'_0, \mathbf{3}_{+2})$	$(\hat{\mathbf{1}}'_0, \hat{\mathbf{1}}'_0, \hat{\mathbf{3}}_{+4})$	$(\mathbf{1}_0, \mathbf{1}_0, \mathbf{3}'_{+6})$

Weighton Model(s)

- Same Littlest Seesaw Structure
- Automatic Hierarchies for the Charged Fermions

	$\boxed{\phi^0}$	$\boxed{\phi^1}$	ϕ^2	$\boxed{\phi^3}$	ϕ^4
Le^c	$(\hat{\mathbf{1}}_0, \hat{\mathbf{1}}_0, \hat{\mathbf{3}}'_{-12})$	$(\mathbf{1}'_0, \mathbf{1}'_0, \mathbf{3}_{-10})$	$(\hat{\mathbf{1}}'_0, \hat{\mathbf{1}}'_0, \hat{\mathbf{3}}_{-8})$	$(\mathbf{1}_0, \mathbf{1}_0, \mathbf{3}'_{-6})$	$(\hat{\mathbf{1}}_0, \hat{\mathbf{1}}_0, \hat{\mathbf{3}}'_{-4})$
$L\mu^c$	$(\hat{\mathbf{1}}'_0, \hat{\mathbf{1}}'_0, \hat{\mathbf{3}}_{-6})$	$(\mathbf{1}_0, \mathbf{1}_0, \mathbf{3}'_{-4})$	$(\hat{\mathbf{1}}_0, \hat{\mathbf{1}}_0, \hat{\mathbf{3}}'_{-2})$	$(\mathbf{1}'_0, \mathbf{1}'_0, \mathbf{3}_0)$	$(\hat{\mathbf{1}}'_0, \hat{\mathbf{1}}'_0, \hat{\mathbf{3}}_{+2})$
$L\tau^c$	$(\mathbf{1}_0, \mathbf{1}_0, \mathbf{3}'_{-2})$	$(\hat{\mathbf{1}}_0, \hat{\mathbf{1}}_0, \hat{\mathbf{3}}'_0)$	$(\mathbf{1}'_0, \mathbf{1}'_0, \mathbf{3}_{+2})$	$(\hat{\mathbf{1}}'_0, \hat{\mathbf{1}}'_0, \hat{\mathbf{3}}_{+4})$	$(\mathbf{1}_0, \mathbf{1}_0, \mathbf{3}'_{+6})$

What about the Quarks?

What about the Quarks?

Trivial!

```
if(time) then
    keep talking
else
    Thank you!
end if
```

What about the Quarks?

SU(5) Extension!

(Hopefully)

SU(5) Extension

- Work in Progress
  - Goal:

$$Y_u \sim \begin{pmatrix} \lambda^8 & \lambda^5 & \lambda^4 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & \lambda^0 \end{pmatrix}, \qquad Y_d^T \sim \begin{pmatrix} \lambda^8 & 0 & 0 \\ \lambda^7 & \lambda^6 & 0 \\ \lambda^7 & \lambda^5 & \lambda^3 \end{pmatrix}, \qquad Y_\ell \sim \begin{pmatrix} \lambda^9 & 0 & 0 \\ - & \lambda^5 & 0 \\ - & - & \lambda^3 \end{pmatrix}$$

  - Because:

$$\begin{aligned} &y_e \sim \lambda^{8.9}, \quad y_\mu \sim \lambda^{5.3}, \quad y_\tau \sim \lambda^{3.3}, \\ &y_u \sim \lambda^{8.6}, \quad y_c \sim \lambda^{4.4}, \quad y_t \sim \lambda^{0.4}, \\ &y_d \sim \lambda^{8.2}, \quad y_s \sim \lambda^{6.2}, \quad y_b \sim \lambda^{3.4}, \\ &\theta_{12} \sim \lambda^1, \quad \theta_{23} \sim \lambda^{2.3}, \quad \theta_{13} \sim \lambda^{3.9}. \end{aligned}$$

SU(5) Extension

- Work in Progress
  - Goal:

$$Y_u \sim \begin{pmatrix} \lambda^8 & \lambda^5 & \lambda^4 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & \lambda^0 \end{pmatrix}, \qquad Y_d^T \sim \begin{pmatrix} \lambda^8 & 0 & 0 \\ \lambda^7 & \lambda^6 & 0 \\ \lambda^7 & \lambda^5 & \lambda^3 \end{pmatrix}, \qquad Y_\ell \sim \begin{pmatrix} \lambda^9 & 0 & 0 \\ - & \lambda^5 & 0 \\ - & - & \lambda^3 \end{pmatrix}$$

  - Two Weightons

$$Y_u \sim \begin{pmatrix} \phi_T^4 & \phi_T^3 & \phi_T^2 \\ & \phi_T^2 & \phi_T \\ & & 1 \end{pmatrix} \qquad Y_d \sim \begin{pmatrix} \phi_F^3 & \phi_F^2\phi_T & \phi_F\phi_T^2 \\ 0 & \phi_F^2 & \phi_F\phi_T \\ 0 & 0 & \phi_F \end{pmatrix}$$

SU(5) Extension

- Work in Progress
  - Goal:

$$Y_u \sim \begin{pmatrix} \lambda^8 & \lambda^5 & \lambda^4 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & \lambda^0 \end{pmatrix}, \qquad Y_d^T \sim \begin{pmatrix} \lambda^8 & 0 & 0 \\ \lambda^7 & \lambda^6 & 0 \\ \lambda^7 & \lambda^5 & \lambda^3 \end{pmatrix}, \qquad Y_\ell \sim \begin{pmatrix} \lambda^9 & 0 & 0 \\ - & \lambda^5 & 0 \\ - & - & \lambda^3 \end{pmatrix}$$

  - “Automatic” Quark Masses and Mixings
    (Order 1 coefficients)

SU(5) Extension

- Leptonic Sector:
  - Lower-Triangular

$$Y_\ell = \begin{pmatrix} y_{ee}\phi_F^3 & 0 & 0 \\ y_{\mu e}\phi_F^2\phi_T & y_{\mu\mu}\phi_F^2 & 0 \\ y_{\tau e}\phi_F\phi_T^2 & y_{\tau\mu}\phi_F\phi_T & y_{\tau\tau}\phi_F \end{pmatrix}$$

  - "Same" Neutrino Structure

$$Y_D = \phi_T \begin{pmatrix} 0 & b \\ a & b\left(1 \pm \sqrt{6}\right) \\ -a & b\left(1 \mp \sqrt{6}\right) \end{pmatrix} \qquad M_M = \begin{pmatrix} M_A & 0 \\ 0 & M_B \end{pmatrix}$$

SU(5) Extension

- Small Corrections to Results