

Unveiling BSM Physics: Multi-scalar coupling modifiers

(Based on work done in 23XX.XXXXX with C. Englert and D. Sutherland)

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July 17, 2023

The κ -Framework

- Ratios with respect to SM couplings:

$$\kappa_i = \frac{g_i}{g_i^{SM}}$$

[LHC Higgs Cross-Section WG '13]

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Precision (2σ)	$\delta\kappa_V$	$\delta\kappa_{2V}$	$\delta\kappa_\lambda$
HL-LHC	2.5%	30%	100%

[ATLAS '23, CMS '22]

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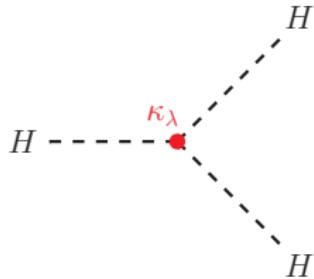
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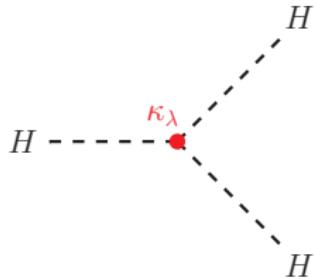


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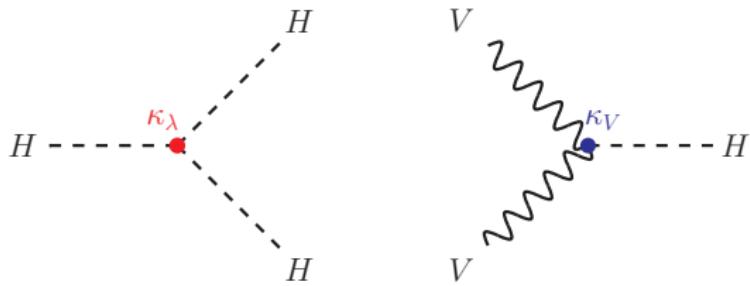


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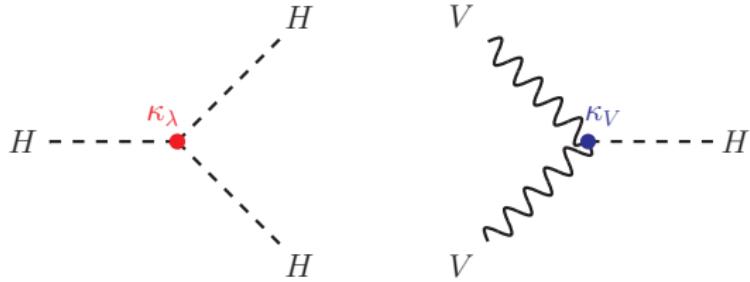
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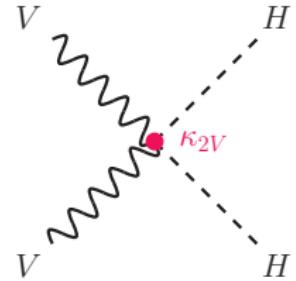
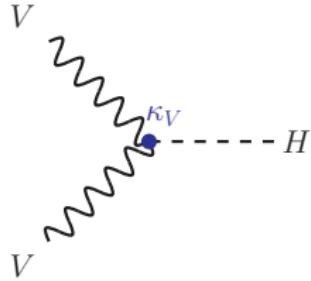
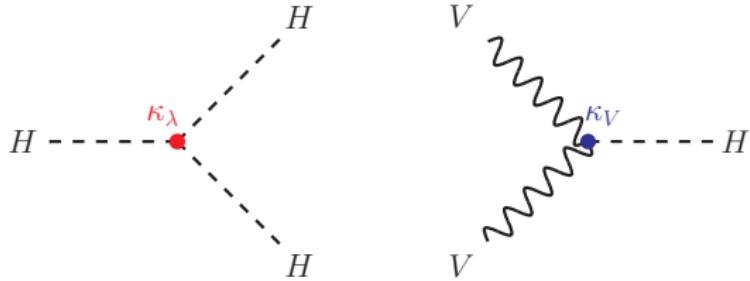
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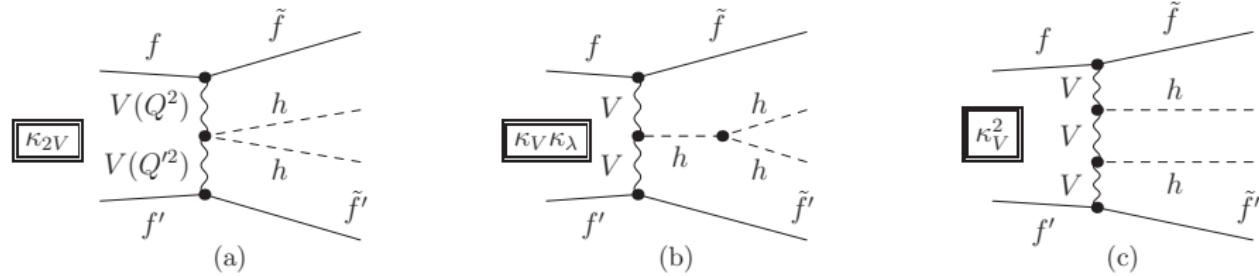


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- Probe these couplings through WBF Higgs pair production.
[Figy et al '03, Figy et al '08, Dreyer et al '18, ATLAS, CMS]

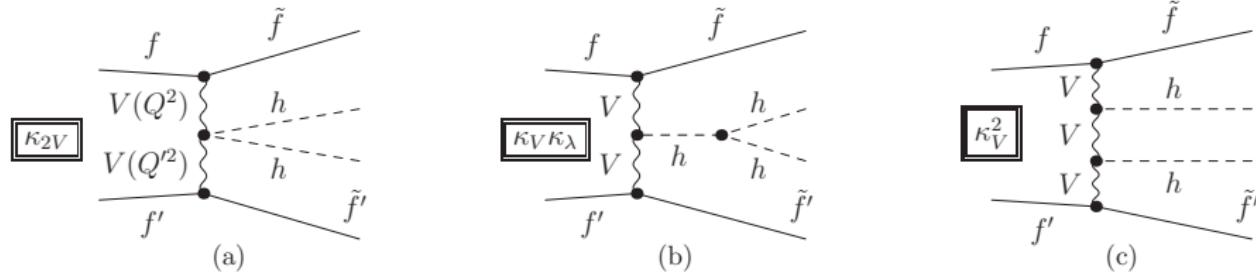
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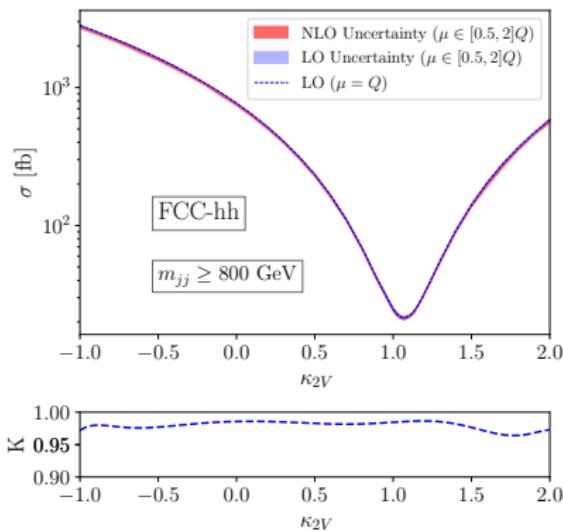
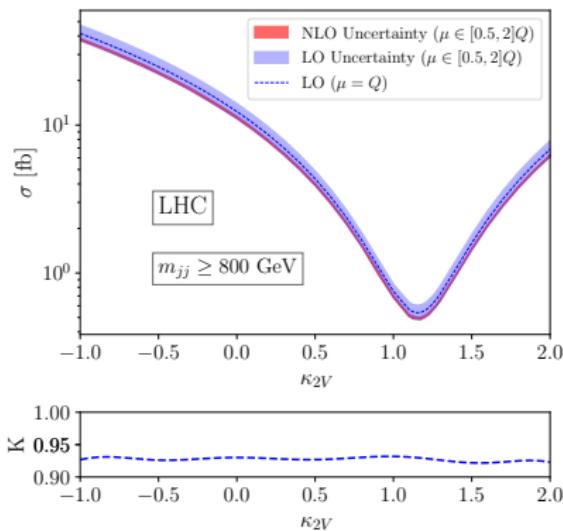
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[VBFNLO '08]

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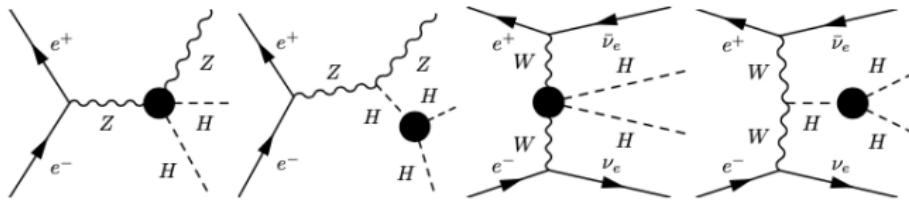
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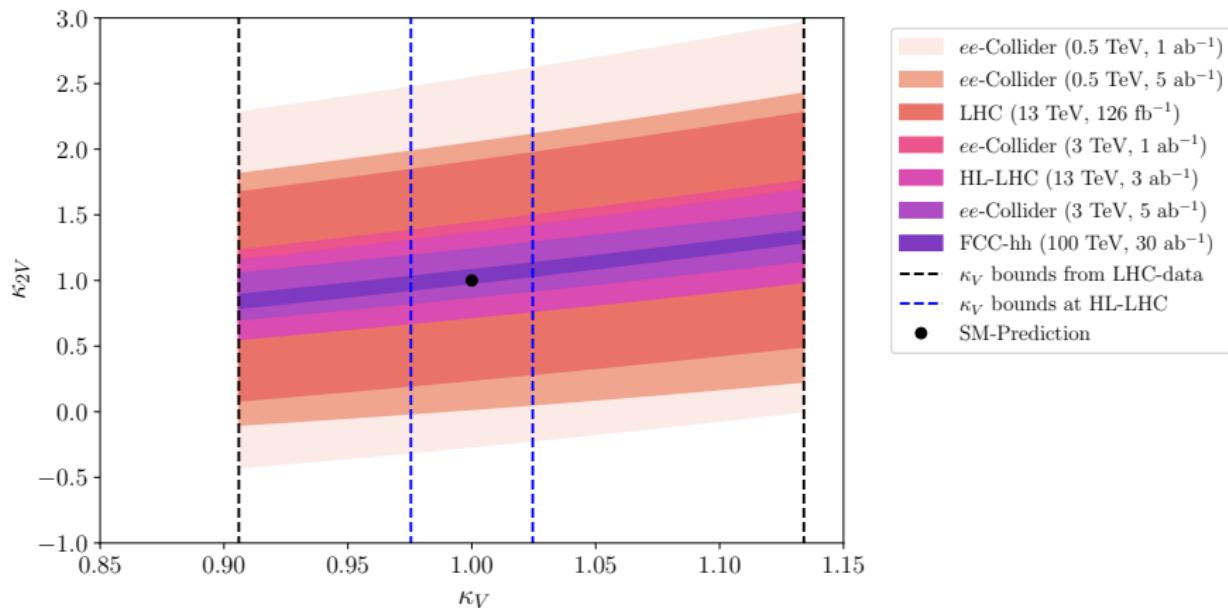
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$$\chi^2(\kappa_V, \kappa_{2V}) = (b_{\text{BSM}}^i(\kappa_V, \kappa_{2V}) - b_{\text{SM}}^i)V_{ij}^{-1}(b_{\text{BSM}}^j(\kappa_V, \kappa_{2V}) - b_{\text{SM}}^j)$$

- $V_{ij} = \varepsilon_{\text{stat.}}^2 \delta_{ij} + \varepsilon_{\text{rel.}}^2 b_{\text{SM}}^i b_{\text{SM}}^j + \varepsilon_{\text{syst.}}^2.$

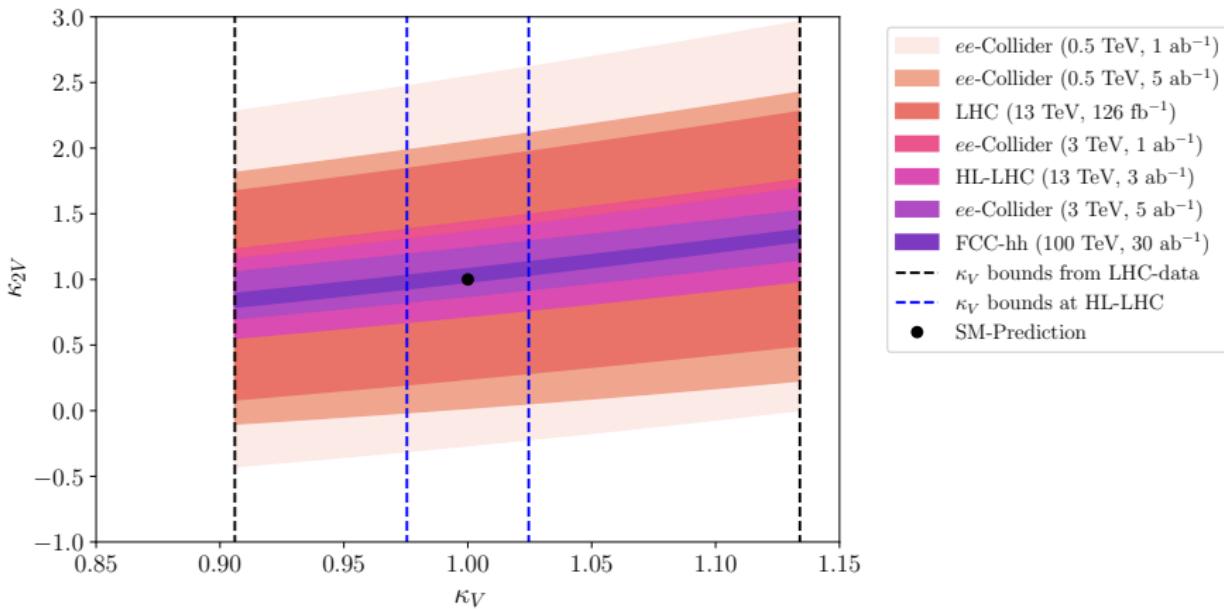
Collider Constraints on κ



- κ_V limits from Single Higgs data.

[ATLAS Collaboration '22]

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This serves as a motivation to check what signals the myriad of BSM models have on the κ parameter space!

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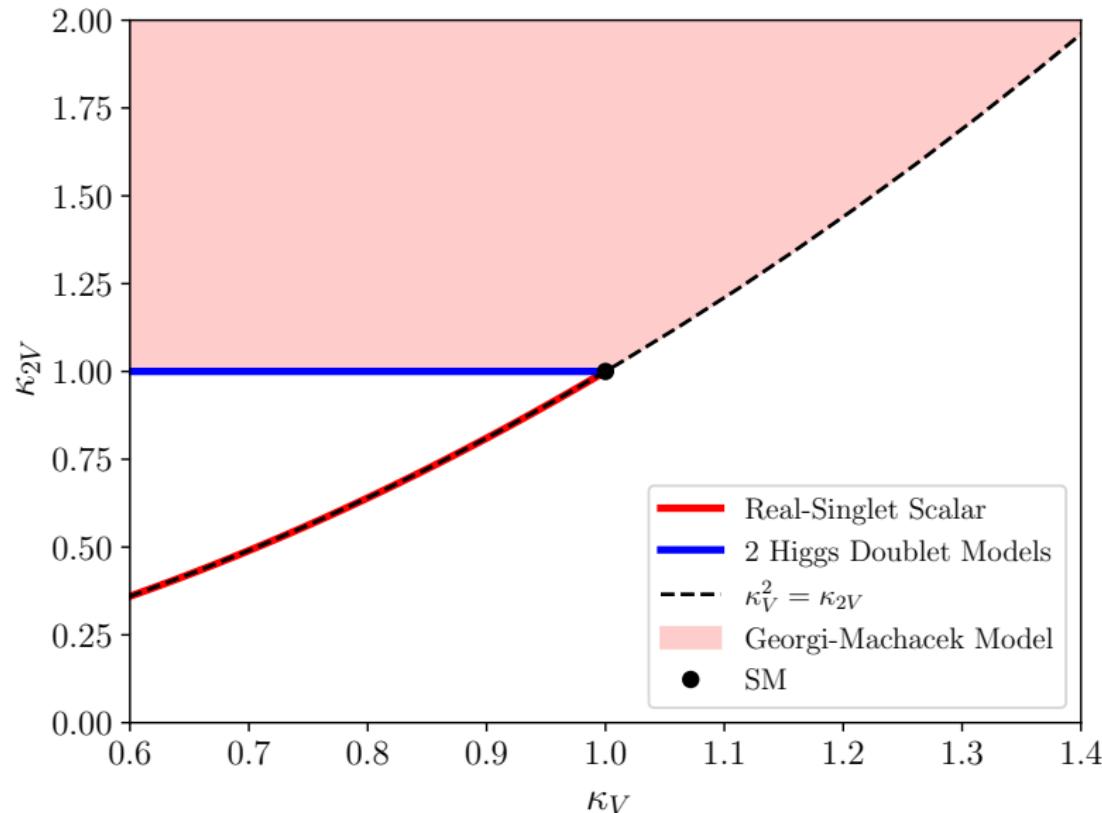
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- Therefore, $\kappa_{2V} \geq \kappa_V^2$, equal in the alignment limit.

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- In their absence, if ϵ_a is a small angle denoting mixing into non-SM Higgses, masses m_a^2 , looking at the decoupling limit:

$$\kappa_V \approx 1 - O(\epsilon^2)$$

$$\kappa_{2V} \approx 1 - O(\epsilon^2)$$

$$\kappa_\lambda \approx 1 - 2 \sum \epsilon_a^2 \left(\frac{m_a^2}{m_h^2} - \frac{1}{4} \right) .$$

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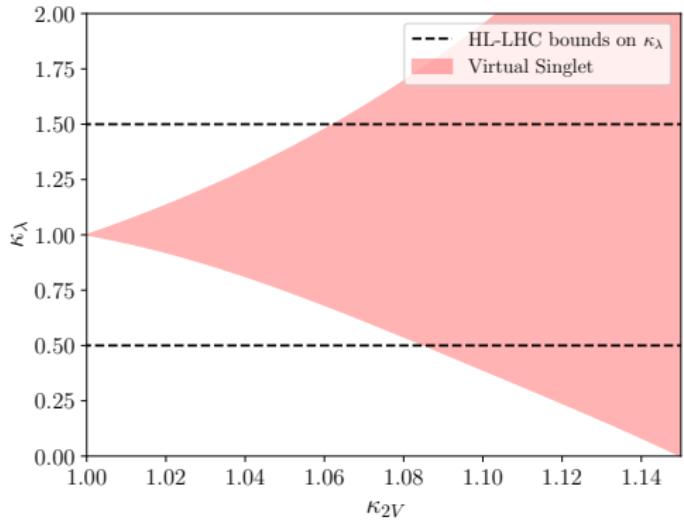
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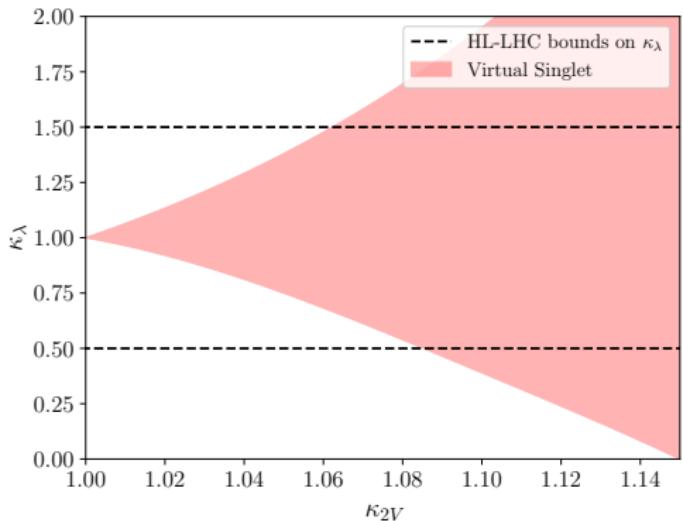
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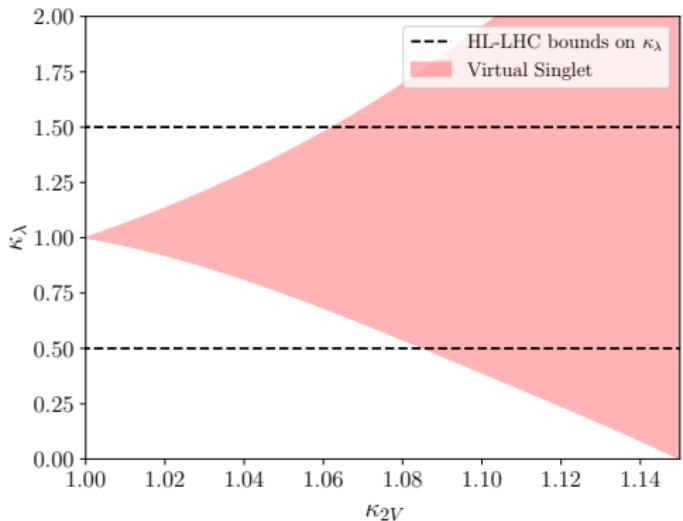
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- κ_λ enhanced in the non-decoupling limit $\lambda v^2 \sim m_\varphi^2 > m_h^2$.

Composite Higgs Models

[Agashe et al '04, Contino et al '07]

- MCHM4/5

$$\kappa_V = \sqrt{1 - \xi}$$

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[Alonso, West '21]

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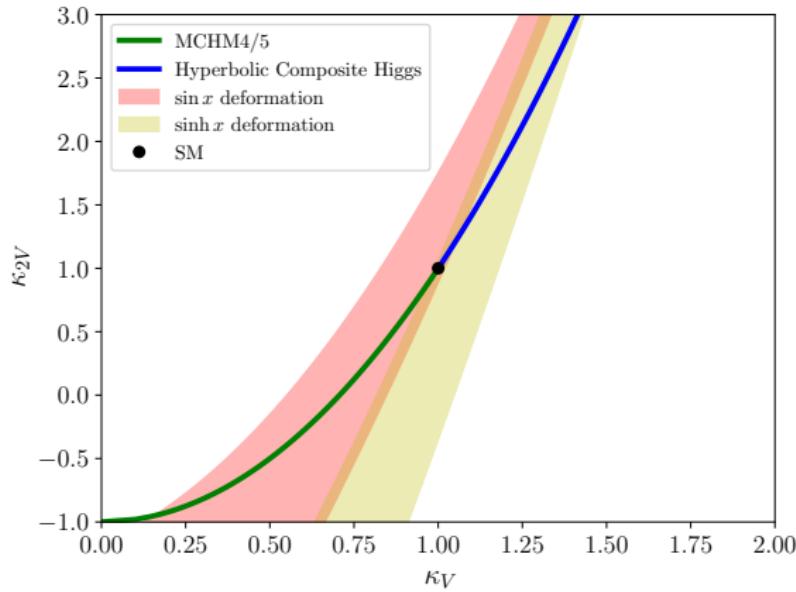
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[Alonso, West '21]

Composite Higgs-Dilaton Mixing

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[Brugisser et al '22, Goldberger et al '07]

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$$\kappa_\lambda \approx \kappa_\lambda^{\text{MCHM}} c_\phi^3 - 4c_\phi^2 s_\phi \sqrt{\zeta}$$

$$(\xi = \frac{v^2}{f^2}, \zeta = \frac{v^2}{\langle \chi^2 \rangle}, \phi \rightarrow h-\chi \text{ mixing angle})$$

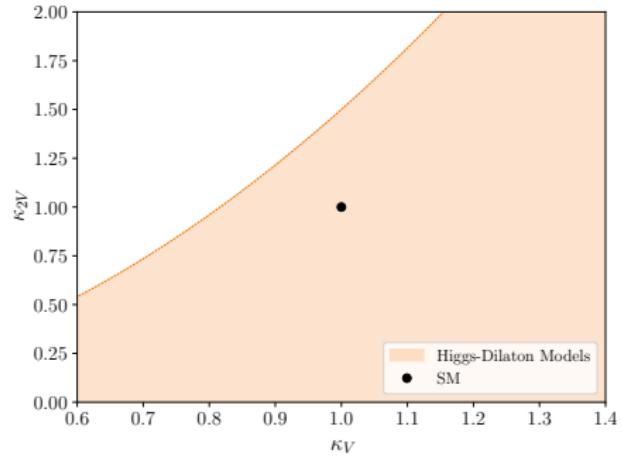
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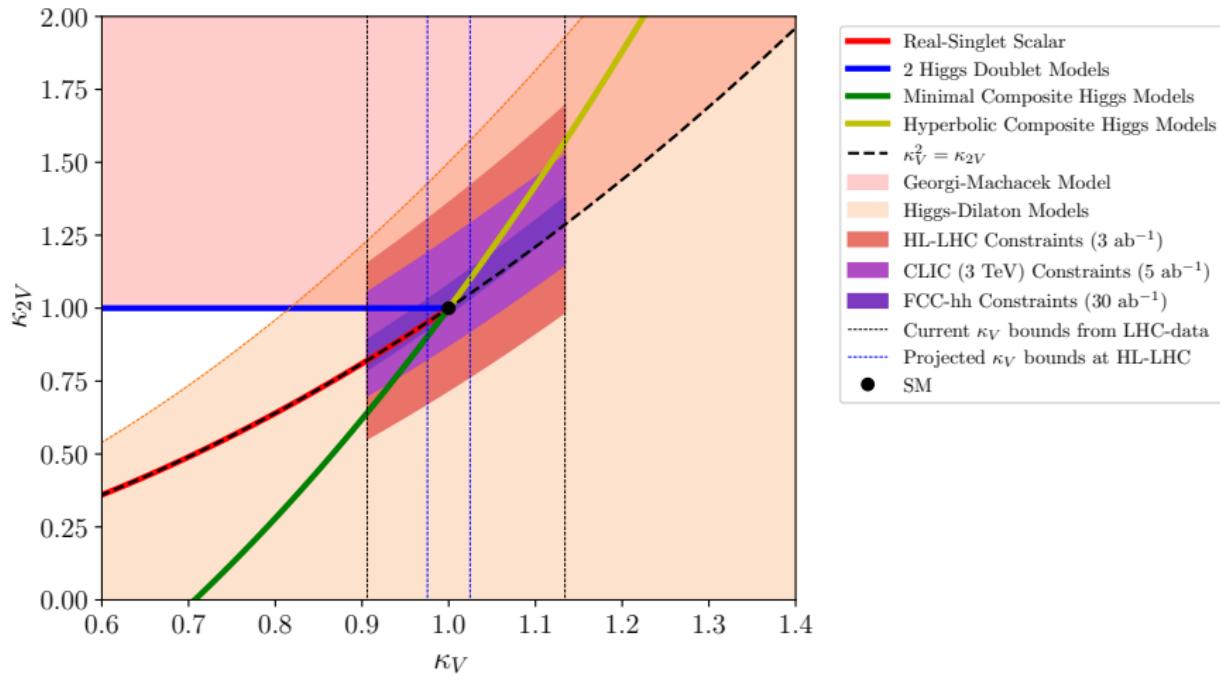
[Brugisser et al '22, Goldberger et al '07]

$$\begin{aligned}\kappa_V &\approx \kappa_V^{\text{MCHM}} c_\phi - s_\phi \sqrt{\zeta} \\ \kappa_{2V} &\approx \kappa_{2V}^{\text{MCHM}} c_\phi^2 - 2\sqrt{\zeta(1-\xi)} s_{2\phi} \\ \kappa_\lambda &\approx \kappa_\lambda^{\text{MCHM}} c_\phi^3 - 4c_\phi^2 s_\phi \sqrt{\zeta}\end{aligned}$$

($\xi = \frac{v^2}{f^2}, \zeta = \frac{v^2}{\langle \chi^2 \rangle}$, $\phi \rightarrow h-\chi$ mixing angle)



Results



Summary and Conclusions

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- κ_λ serves as an independent test, particularly for renormalisable scalar extensions (significant enhancement).

Thank You Questions?

Backup Slides

Beyond Tree-level: RG Evolution of κ s

- Linearising κ s about the SM point ($\kappa_V = \kappa_{2V} = \kappa_\lambda = 1$):

$$\delta\kappa_V = \kappa_V^{\text{eff}} - 1,$$

$$\delta K_{2V} = \kappa_{2V} - (\kappa_V^{\text{eff}})^2,$$

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- The RGE equations (upto $O(\delta\kappa^2)$ corrections):

$$16\pi^2 \frac{d}{d \log \mu^2} \delta\kappa_V = \delta\kappa_V \left(2 \frac{m_h^2}{v^2} - \frac{3(m_W^2 + m_Z^2)}{2v^2} \right) + \delta K_{2V} \left(\frac{3m_W^2}{v^2} \right)$$

$$16\pi^2 \frac{d}{d \log \mu^2} \delta K_{2V} = \delta\kappa_V \left(-4 \frac{m_h^2}{v^2} \right) + \delta K_{2V} \left(-\frac{3m_h^2}{2v^2} - \frac{40m_W^2}{3v^2} \right)$$

$$16\pi^2 \frac{d}{d \log \mu^2} \delta\kappa_\lambda = \delta\kappa_V \left(-\frac{27m_h^2}{2v^2} - \frac{9(m_W^2 + m_Z^2)}{2v^2} \right) + \delta K_{2V} \left(\frac{2m_h^2}{v^2} \right) + \delta\kappa_\lambda \left(\frac{15m_h^2}{2v^2} \right)$$