# Investigating the Catalytic Effect on Metastable Vacuum Decay in String Theory

Based on 2305.00781 [hep-th]

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- In GUT and gravitational theory, monopoles and black holes are well known for acting as catalysts to enhance the instability of the vacuum.

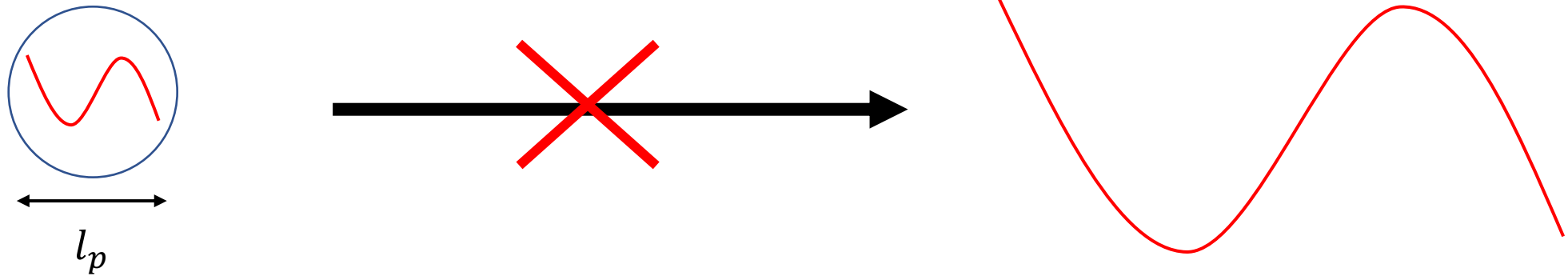[P. J. Steinhardt, Nucl. Phys. B 190, 583-616 (1981); Phys. Rev. D 24, 842 (1981)]

[Y. Hosotani, Phys. Rev. D 27, 789 (1983)]  [U. A. Yajnik, Phys. Rev. D 34, 1237-1240 (1986)]

e.g. Black hole catalysis  [R. Gregory, I. G. Moss and B. Withers, JHEP 03, 081 (2014)]



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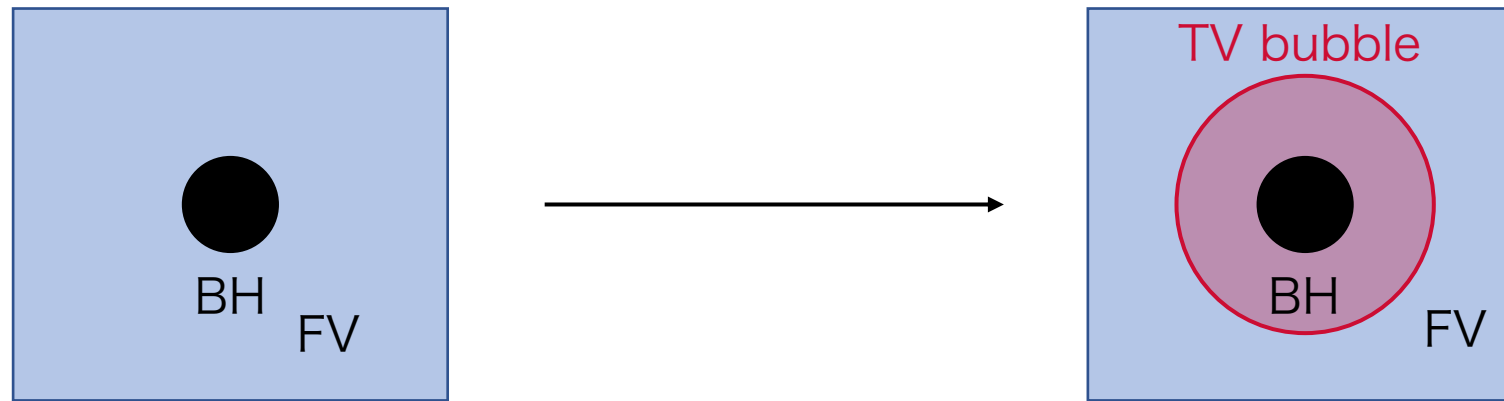
- TCC forbids enlargement of sub-Planckian fluctuation to classical one during inflation. [Vafa et al., JHEP 09 123 (2020)]



- It gives strong constraints on lifetimes of metastable de-Sitter vacua.

[Bedroya et al.,arXiv:2008.07555[hep-th]]

$$\tau < \frac{1}{H} \log \frac{M_{pl}}{H}$$  TCC condition

- Can metastable vacua with impurities circumvent swampland??

- Comparing the critical lifetime with the TCC condition, we discussed a specific restriction on the string scale.

- We determined the complete expression of the 1-loop pre-factor for the noncanonical system and found a behavior different from canonical theories.

- Unfortunately, 1-loop analysis breaks down when the potential barrier completely vanished. We used variation perturbation method instead and obtain a finite lifetime.

# Contents

1. Introduction and Summary

2. 1-loop analysis
   - Comment on zeromode
   - Functional determinant

3. Reduction to cubic oscillator and comparison to TCC

- D5 and antiD5-branes wrapped to conifolds form a metastable state.
  [F. Cachazo et al., Nucl. Phys. B 603, 3-41 (2001);C. Vafa, J. Math. Phys. 42, 2798-2817 (2001);M. Aganagic et al., Nucl. Phys. B 789, 382-412 (2008)]

- D3-branes which are wrapped to the internal space dissolve into D5-brane and become magnetic flux. ⟶ Catalyst!
  [A. Kasai et al., Phys. Rev. D 91, no.12, 126002 (2015)]

Table 1. brane configuration

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| D5/antiD5 | ◯ | ◯ | ◯ | ◯ | △ | △ | △ | × | × | × |
| DWD5 | ◯ | △ | △ | △ | △ | △ | △ | △ | × | × |
| D3 | ◯ | × | × | × | △ | △ | △ | △ | × | × |

$S^2$    $S^3$

$$L_{\text{total}} = -T_{\text{DW}} 4\pi \sqrt{(R^4 + b^2)\left(1 - \dot{R}^2\right)} + 2T_{D5} rb \left[ 4\pi R \times {}_2F_1\left(-\frac{1}{2}, \frac{1}{4}, \frac{5}{4}, -\frac{R^4}{b^2}\right) \right]$$

$$T_{\text{DW}} = T_{D5} \left[ 2\pi^2 L^2 \int_0^\pi d\psi_I \frac{2}{\pi} \sqrt{\sin^4 \psi_I + \left(\frac{b_{\text{NS}}}{L^2}\right)^2} \right]$$
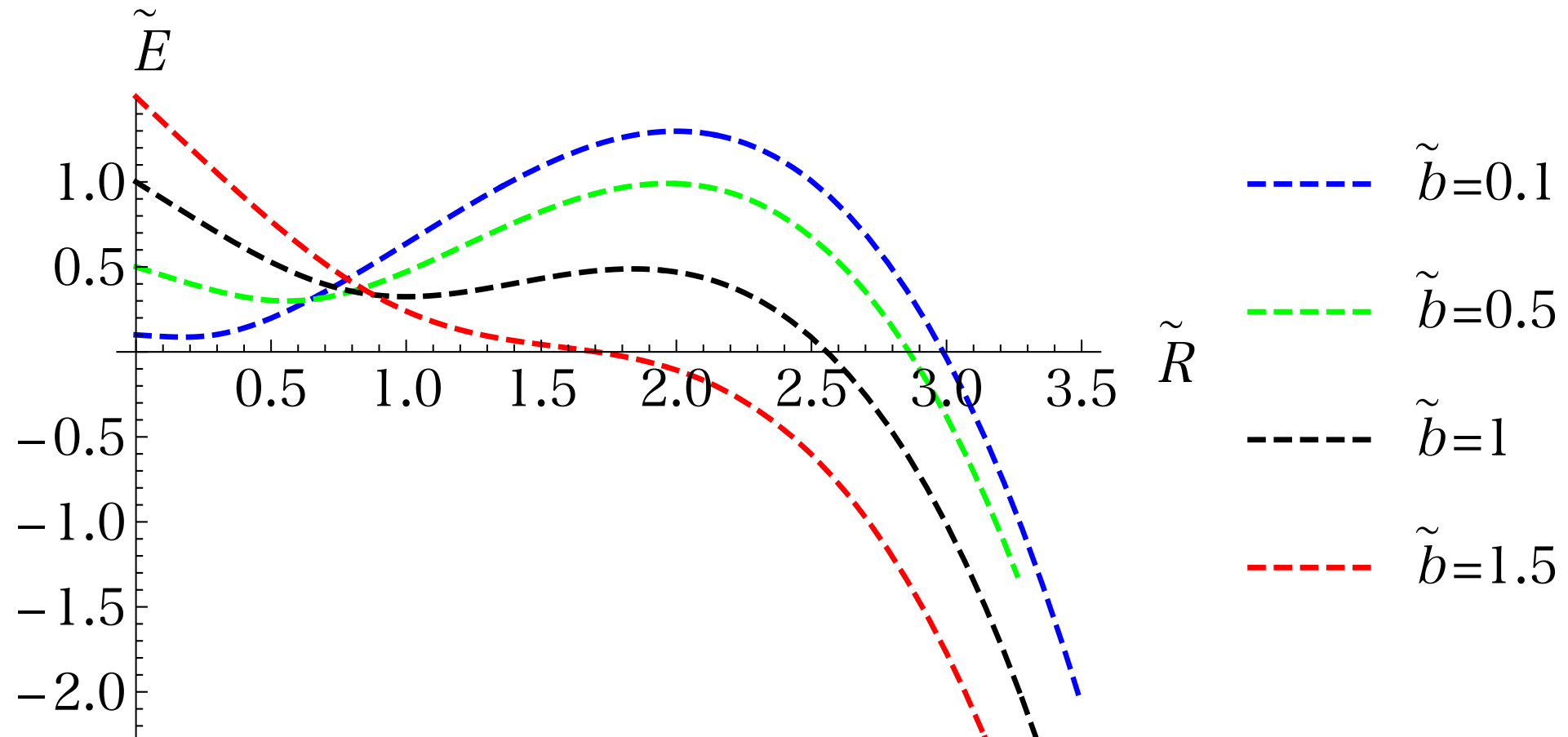
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- To get decay rate (lifetime), we must calculate bounce action, zero mode normalization constant and functional determinant.

[S. R. Coleman, Phys. Rev. D 15, 2929-2936 (1977) [erratum: Phys. Rev. D 16, 1248 (1977)] ]

$$\Gamma = \sqrt{\frac{N_{\mathrm{b}}}{2\pi\hbar}} \left| \frac{\det' S''[\bar{x}_{\mathrm{b}}]}{\det S''[\bar{x}_{\mathrm{triv}}]} \right|^{-1/2} e^{-B/\hbar} = \sqrt{\frac{B}{2\pi\hbar}} \left| \frac{\det' S''[\bar{x}_{\mathrm{b}}]}{\det S''[\bar{x}_{\mathrm{triv}}]} \right|^{-1/2} e^{-B/\hbar}$$

- We must note that $N_b$ doesn't correspond to $B$ in noncanonical theories.

$$\tilde{B} = \tilde{S}_{\mathrm{b}} - \tilde{S}_{\mathrm{sub}}$$

$$= 2 \int_{\tilde{R}_{\min}}^{\tilde{R}_{\max}} d\tilde{R}_{\mathrm{b}} \sqrt{\tilde{R}_{\mathrm{b}}^4 + \tilde{b}^2 - \left[ C + \tilde{b}\tilde{R}_{\mathrm{b}\,2}F_1 \left( -\frac{1}{2}, \frac{1}{4}, \frac{5}{4}, -\frac{\tilde{R}_{\mathrm{b}}^4}{\tilde{b}^2} \right) \right]^2}$$

$$\tilde{N}_{\mathrm{b}} = \int ds \left( \frac{d\tilde{R}_{\mathrm{b}}}{ds} \right)^2$$

$$= 2 \int_{\tilde{R}_{\min}}^{\tilde{R}_{\max}} \frac{d\tilde{R}_{\mathrm{b}}}{C + \tilde{b}\tilde{R}_{\mathrm{b}\,2}F_1 \left( -\frac{1}{2}, \frac{1}{4}, \frac{5}{4}, -\frac{\tilde{R}_{\mathrm{b}}^4}{\tilde{b}^2} \right)} \times \sqrt{\tilde{R}_{\mathrm{b}}^4 + \tilde{b}^2 - \left[ C + \tilde{b}\tilde{R}_{\mathrm{b}\,2}F_1 \left( -\frac{1}{2}, \frac{1}{4}, \frac{5}{4}, -\frac{\tilde{R}_{\mathrm{b}}^4}{\tilde{b}^2} \right) \right]^2}$$
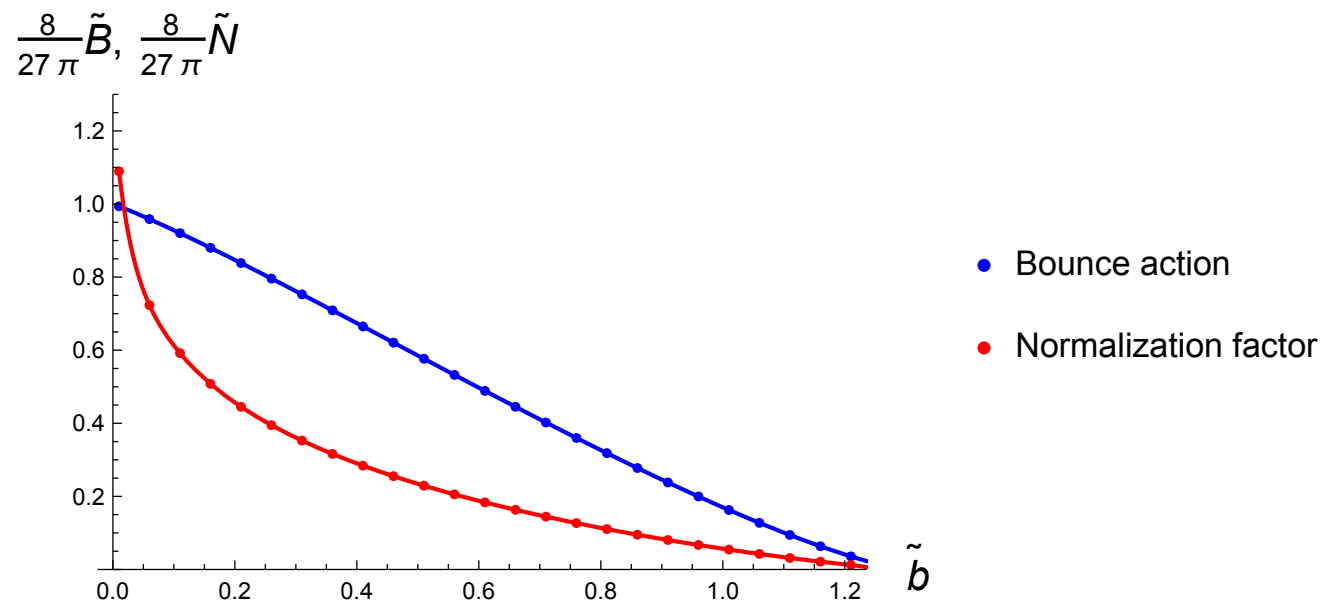
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$$S''[x] = -\frac{d}{dt}\left(\frac{\partial^2 L}{\partial \dot{x}^2}\frac{d}{dt}\right) + \frac{\partial^2 L}{\partial x^2} - \frac{d}{dt}\frac{\partial^2 L}{\partial x \partial \dot{x}} = -\frac{d}{dt}\left(\cancel{P}[x]\frac{d}{dt}\right) + Q[x]$$

canonical $\longrightarrow$ $m$

- Functional determinant=infinite products of eigenvalues ${\prod_n}'\lambda_n$ .

- One can treat the UV divergence via zeta function regularization.

$$\text{det}'\, S''[x] = \exp\left(-\zeta'(s=0; S'')\right) \qquad \zeta(s; S'') = \sum_n \frac{1}{\lambda_n^s}$$

- The regularization procedure is well known for canonical case .
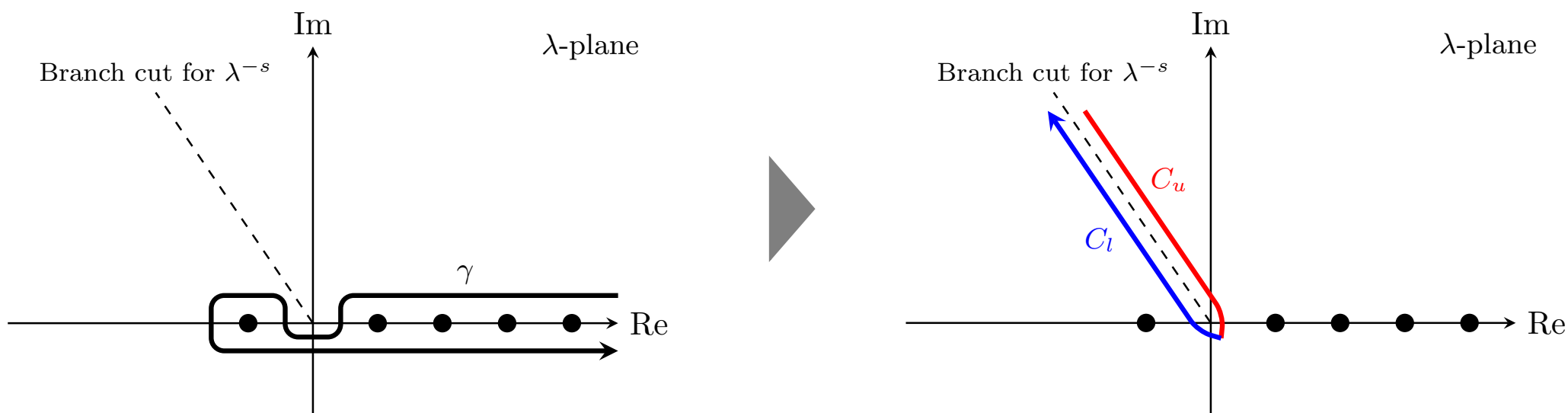  How about non-canonical case?

need some developments !!

- Spectral zeta function can be written by a contour integration of the meromorphic function which has simple poles at eigenvalues.

## Contour deformation method [K. Kirsten et al., (2003);K. Kirsten et al., (2004)]



$$\zeta(s; L_{A,B}) = e^{is(\pi - \psi)} \frac{\sin(\pi s)}{\pi} \int_0^\infty dt\, t^{-s} \frac{d}{dt} \ln \left( F_{A,B}(te^{i\psi}) t^{-m_0} e^{-im_0\psi} \right) \qquad \frac{1}{2} < s < 1$$

- The asymptotic expansion of characteristic function at large $\lambda$ is order $\sqrt{\lambda}$.
- We can perform analytic continuation by subtracting N terms and add them back. [F. Gesztesy and K. Kirsten, (2019); G. Fucci et al., (2021)]

$$\zeta'\left(0; L_{A,B}\right) = i\pi n - \ln\left(2c\left|\frac{F_{m_0}}{\Gamma_{k_0}}\right|\right)$$

$n$ : number of negative modes        $F_{m_0}$ :  1st order coefficient of small $\lambda$ expansion

$$\mathrm{det}' S''[R_b] = -\left|F_{m_0}\right| = \frac{N_b}{P_{\min}\dot{R}_b(-\beta/2)\dot{\chi}(-\beta/2)} \cdot \frac{\chi(\beta/2) - \chi(-\beta/2)}{\dot{R}_b(-\beta/2)}$$

- We need to extract a scattering mode which diverge at $\beta \to \infty$.

[M.Marino, "*Instanton and Large N*" (2015);ST, (2023)]

$$\frac{\chi(\beta/2) - \chi(-\beta/2)}{\dot{R}_b(-\beta/2)} \propto \exp\left[\sqrt{\frac{Q_{\min}}{P_{\min}}}\beta\right]$$
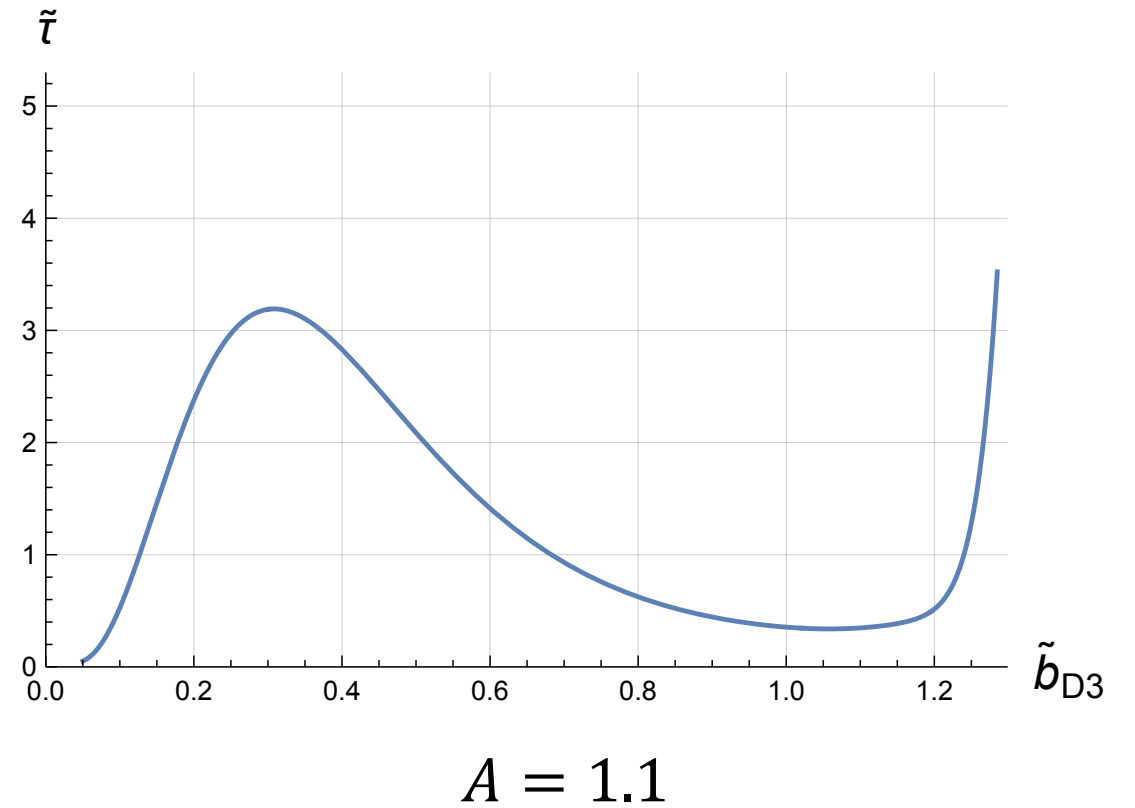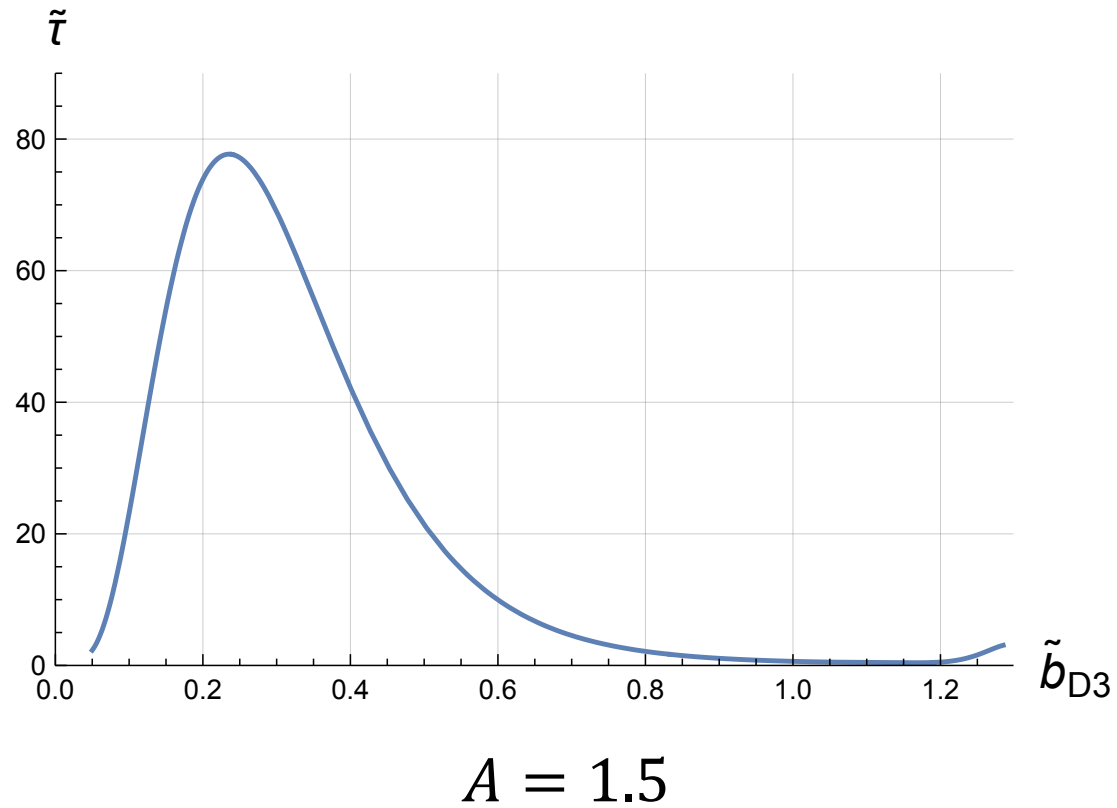
- The scattering mode can be regularized by reference determinant.

$$\frac{\det'\left[S''[\widetilde{R}_b]\right]}{\det\left[S''[\widetilde{R}_{\min}]\right]} = -\frac{N_b}{2\left(\widetilde{R}_{\max} - \widetilde{R}_{\min}\right)^2}\frac{P_{\min}^{1/2}}{Q_{\min}^{3/2}}\exp\left[-2\sqrt{\frac{Q_{\min}}{P_{\min}}}\int_{\widetilde{R}_{\min}}^{\widetilde{R}_{\max}}\left\{\frac{1}{\sqrt{(\dot{\widetilde{R}}_b)^2}} - \sqrt{\frac{P_{\min}}{Q_{\min}}}\frac{1}{\widetilde{R} - \widetilde{R}_{\min}}\right\}d\widetilde{R}_b\right]$$
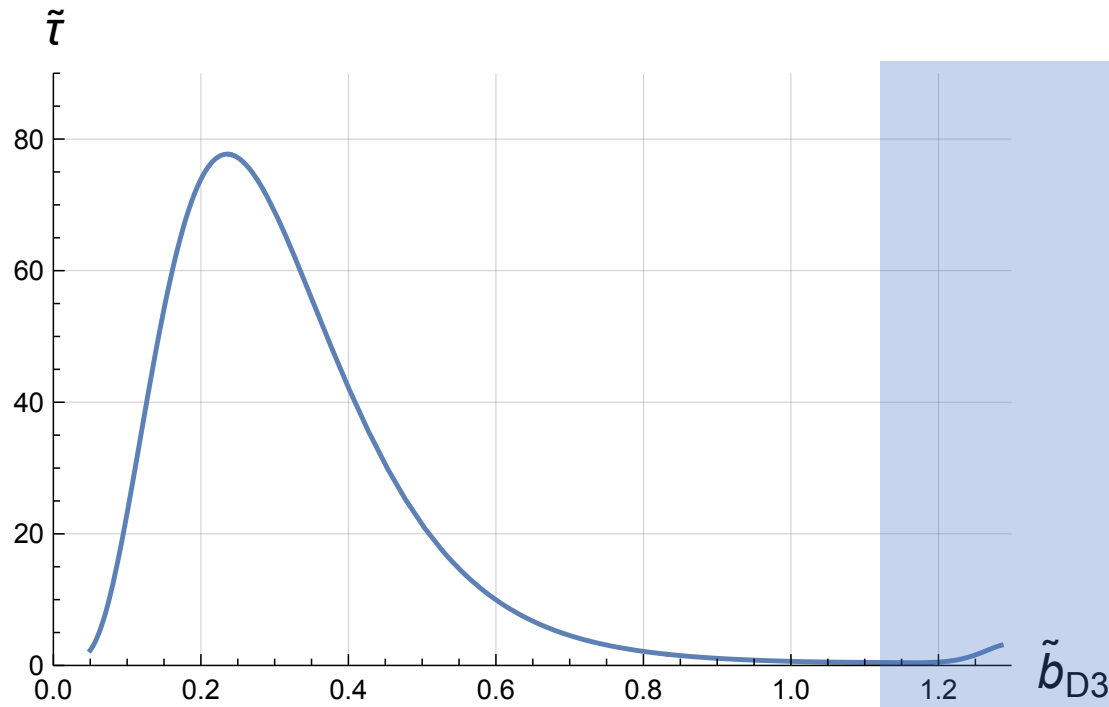
- We get a complete 1-loop result of the lifetime.

$$\tau = \frac{\sqrt{\pi}}{\left(\widetilde{R}_{\max} - \widetilde{R}_{\min}\right)}\frac{P_{\min}^{1/4}}{Q_{\min}^{3/4}}\exp\left[-\sqrt{\frac{Q_{\min}}{P_{\min}}}\int_{\widetilde{R}_{\min}}^{\widetilde{R}_{\max}}\left\{\frac{1}{\sqrt{(\dot{\widetilde{R}}_b)^2}} - \sqrt{\frac{P_{\min}}{Q_{\min}}}\frac{1}{\widetilde{R} - \widetilde{R}_{\min}}\right\}d\widetilde{R}_b\right]e^B .$$
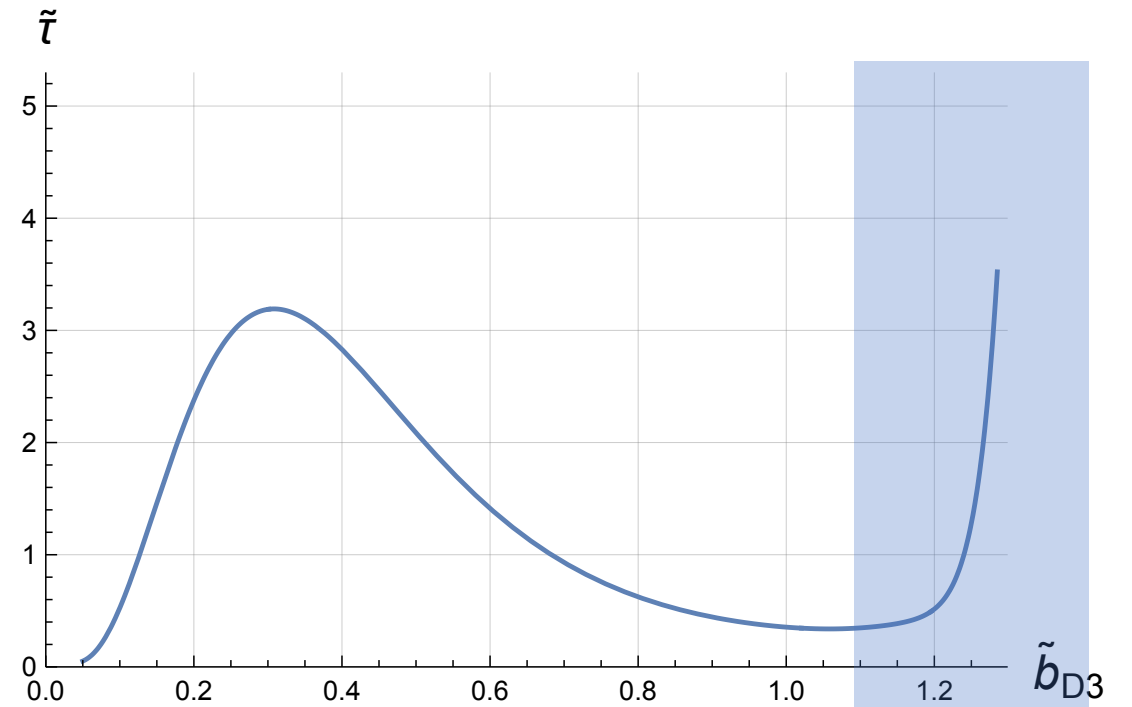
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$$A = 1.5$$



$$A = 1.1$$

$$A = 1.5$$

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- For nearly flat potential, we can expand our DBI Lagrangian and approximate by the anharmonic oscillator.

$$L_E \simeq A \left[ \sqrt{\tilde{R}_v^4 + \tilde{b}_{\text{crit}}^2} \frac{\dot{\tilde{y}}^2}{2} - \alpha(\tilde{R}_v, \tilde{b}_{\text{crit}})\tilde{y} - \beta(\tilde{R}_v, \tilde{b}_{\text{crit}})\tilde{y}^3 + V(\tilde{R}_v, \tilde{b}_{\text{crit}}) \right]$$

- For the cubic oscillator, the decay rate for low potential barriers is well investigated by variational perturbation method.

[H.Kleinert and I. Mustapic, *Int.J.Mod.Phys.A* 11 (1996) 4383-4400]

$$L_{KM} = \frac{m}{2}\dot{x}^2 + \frac{m\omega^2}{2}x^2 - \lambda x^3$$

$$\blacktriangleright \quad \Gamma = -2\text{Im}\, E_0 \simeq 2 \times 0.448 \left( \frac{\lambda^2}{m^3} \right)^{1/5} \left( 1 - 0.186 \left( \frac{m^2\omega^5}{\lambda^2} \right)^{2/5} \right)$$

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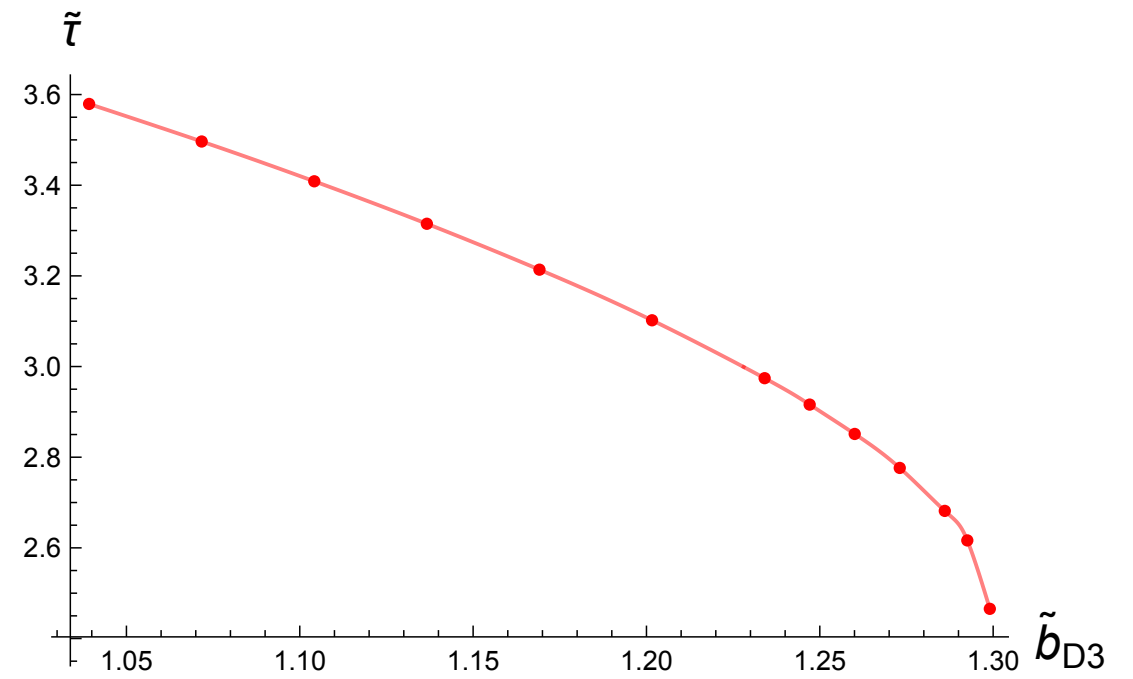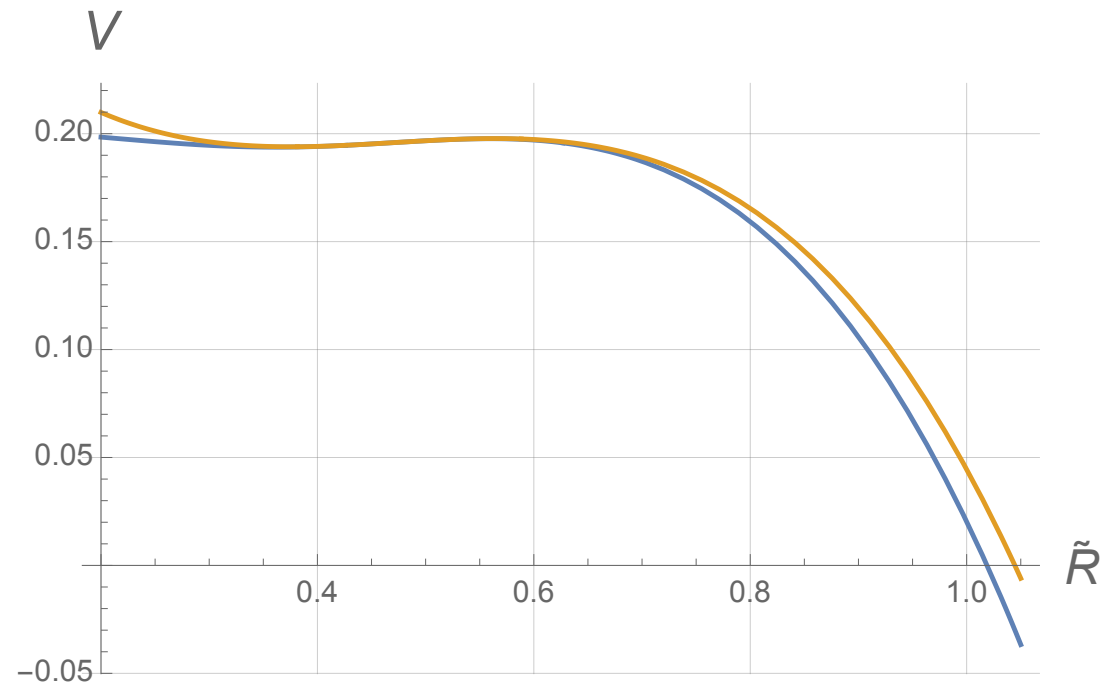
- We compare the critical lifetime for completely flat potential to TCC.
- All parameters are rewritten by string scale $M_{st}$, Planck scale $M_{pl}$ and string coupling constant $g_s$.
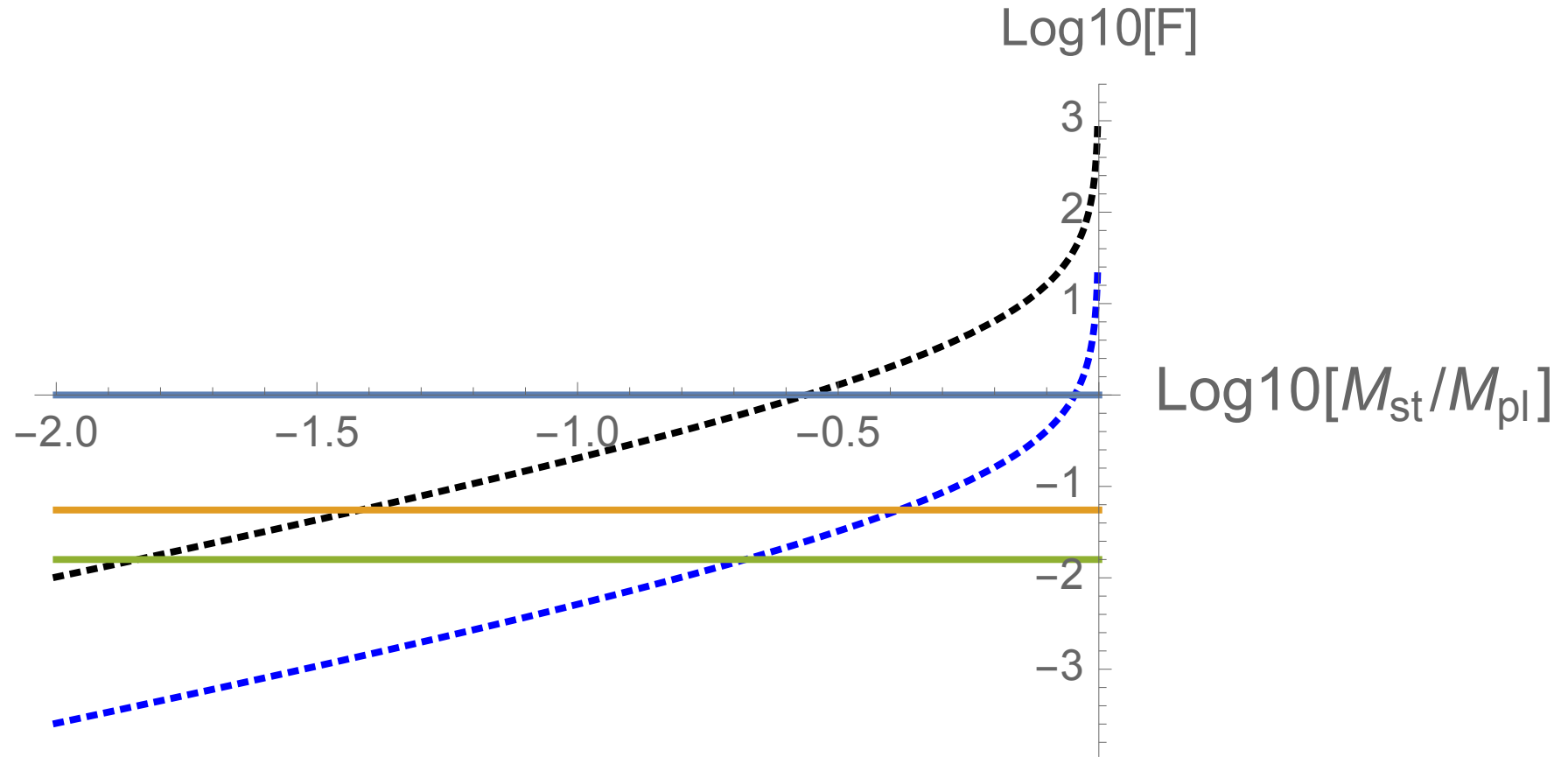
$$\tau_{\text{crit}} \simeq \frac{1}{2 \times 0.448} \frac{T_{DW}}{2 T_{D5} r} A^{1/5} \left( \frac{\beta(\tilde{R}_v, \tilde{b}_{\text{crit}})}{(\tilde{R}_v^4 + \tilde{b}_{\text{crit}}^2)^{3/2}} \right)^{-1/5} \simeq 2.46 A^{1/5} \frac{T_{DW}}{2 T_{D5} r} \leq \frac{1}{H_I} \log \frac{M_{\text{pl}}}{H_I}$$
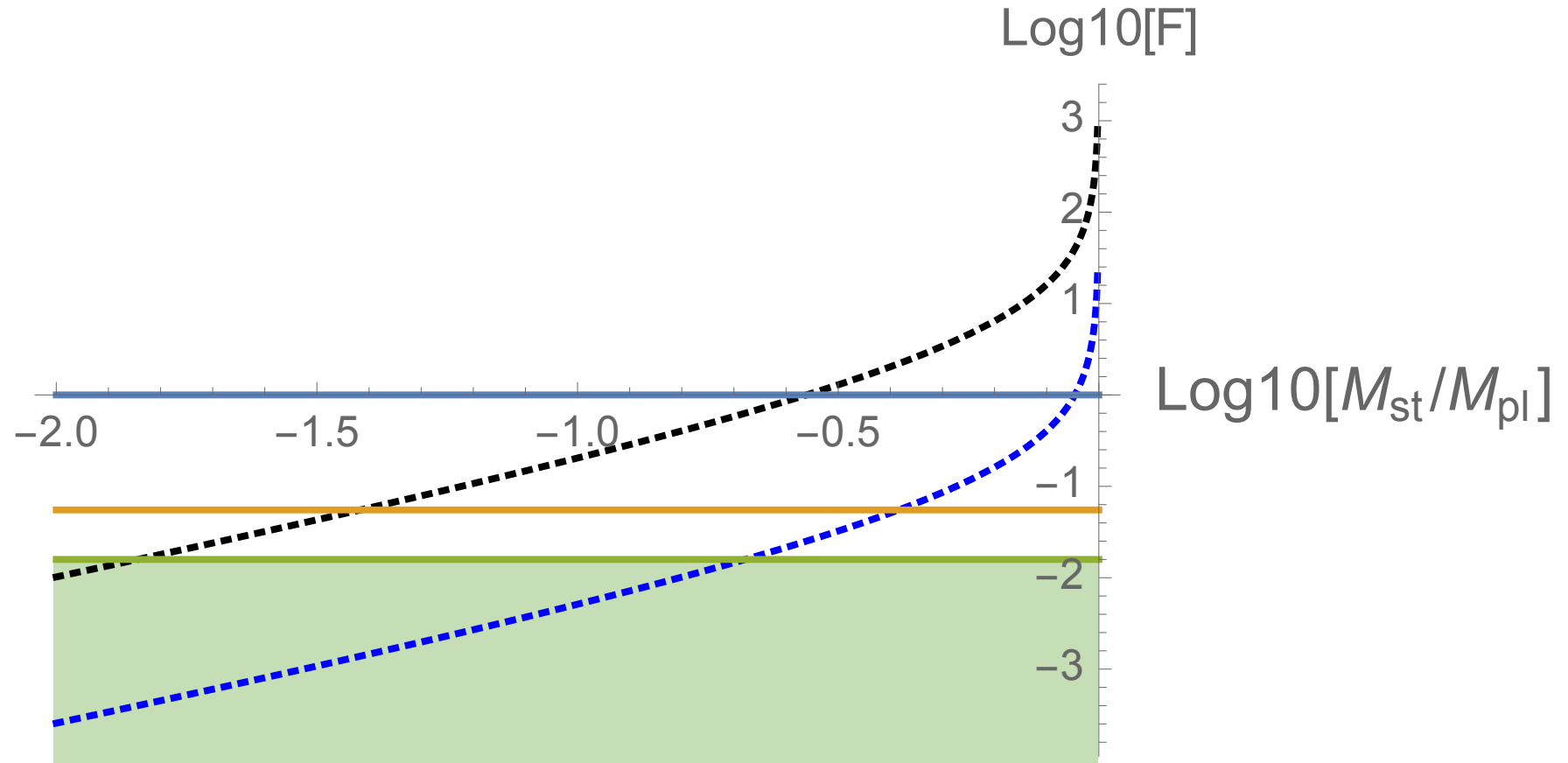
$$\log F \leq \frac{9}{5} \log g_s$$

$$F = \frac{2.46}{2} \left( \frac{4\pi}{n^8} \right)^{1/5} \left( \frac{M_{\text{st}}}{M_{\text{pl}}} \right) \left( \log \frac{M_{\text{st}}}{M_{\text{pl}}} \right)^{-1}$$

Satisfies TCC for $g_s = 1/10$.

- Construct a metastable state as domain wall/monopole bound state.

- Complete expression for the determinant factor valid for noncanonical theories (DBI, Nambu-Goto).

- Show that zeromode normalization factor does not always match to bounce action.

- Finite lifetime via VPM for the critical point.

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# Thank you!
## ご清聴ありがとうございました

# BACKUP

$$\text{EoM :} \quad \partial_s \left( \sqrt{\frac{\tilde{R}^4 + \tilde{b}^2}{1 + \dot{\tilde{R}}^2}} - \tilde{b}\tilde{R}_2 F_1 \left( -\frac{1}{2}, \frac{1}{4}, \frac{5}{4}; -\frac{\tilde{R}^4}{\tilde{b}^2} \right) \right) = 0$$

$\tilde{B} = \tilde{S}_{\mathrm{b}} - \tilde{S}_{\mathrm{sub}}$ ⟵ Difference between the instanton action and the background.

$$= 2 \int_{\tilde{R}_{\min}}^{\tilde{R}_{\max}} d\tilde{R}_{\mathrm{b}} \sqrt{\tilde{R}_{\mathrm{b}}^4 + \tilde{b}^2 - \left[ C + \tilde{b}\tilde{R}_{\mathrm{b}2} F_1 \left( -\frac{1}{2}, \frac{1}{4}, \frac{5}{4}, -\frac{\tilde{R}_{\mathrm{b}}^4}{\tilde{b}^2} \right) \right]^2}$$

$$\tilde{N}_{\mathrm{b}} = \int ds \left( \frac{d\tilde{R}_{\mathrm{b}}}{ds} \right)^2 \quad \text{⟵ Normalization constant of the zeromode.}$$

$$= 2 \int_{\tilde{R}_{\min}}^{\tilde{R}_{\max}} \boxed{\frac{d\tilde{R}_{\mathrm{b}}}{C + \tilde{b}\tilde{R}_{\mathrm{b}2} F_1 \left( -\frac{1}{2}, \frac{1}{4}, \frac{5}{4}, -\frac{\tilde{R}_{\mathrm{b}}^4}{\tilde{b}^2} \right)}} \times \sqrt{\tilde{R}_{\mathrm{b}}^4 + \tilde{b}^2 - \left[ C + \tilde{b}\tilde{R}_{\mathrm{b}2} F_1 \left( -\frac{1}{2}, \frac{1}{4}, \frac{5}{4}, -\frac{\tilde{R}_{\mathrm{b}}^4}{\tilde{b}^2} \right) \right]^2}$$

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$$\left[ -\frac{d^2}{dx^2} + V(x) \right] \psi_n(x) = \lambda_n \psi_n(x) \ , \quad M \begin{pmatrix} \psi_n(0) \\ \psi_n'(0) \end{pmatrix} + N \begin{pmatrix} \psi_n(L) \\ \psi_n'(L) \end{pmatrix} = 0$$

$$\psi_0^1(0) = 1 \ , \quad \psi_0^{1\prime}(0) = 0 \ ,$$

$$\psi_0^2(0) = 0 \ , \quad \psi_0^{2\prime}(0) = 1 \ .$$

- $M$ and $N$ are 2×2 matrices, which determine boundary conditions.

- $\psi_0^i$ $(i = 1, 2)$ are independent solutions for zeromode equations.

$$\blacktriangleright \quad \det \left[ -\frac{d^2}{dx^2} + V(x) \right] = \det \left[ M + N \begin{pmatrix} \psi_0^1(L) & \psi_0^2(L) \\ \psi_0^{1\prime}(L) & \psi_0^{2\prime}(L) \end{pmatrix} \right]$$