The implication of Yukawa Unification in SO(10) GUTs

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Based on:

1. 2106.15822 Djouadi, RO, Raidal ’21
2. 2212.11315 Djouadi, Fonseca, RO, Raidal ’22

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Outline

1. Introduction
2. Non-SUSY SO(10) grand unified theory
3. Constraints from unification of fundamental couplings
4. Conclusions

1. Introduction

Motivations for new physics

- Standard Model is a very delicate mathematical model for particles physics. We can solve it to very high accuracy, but we don't understand its' origin. The key question is: why it works so well?

- Two most important questions naturally arises concerning:

- 1. What is the origin of parameters in the Standard Model?

- 2. What are the conditions under which the SM can be applied?

Principles for EFT model building

Agmon, Bedroya, Kang, Vafa ’22

- Symmetry principle: all terms allowed by symmetries are allowed. Renormalizability is certainly not required. The symmetry $\mathcal{G}_{\mathrm{Lorentz}} \times \mathcal{G}_{\mathrm{Gauge}}$ is a free parameter.

- UV/IR decoupling principle: low-energy physics can be effectively described independently of high-energy physics within the EFT framework. (Wilson’s Renormalization group)

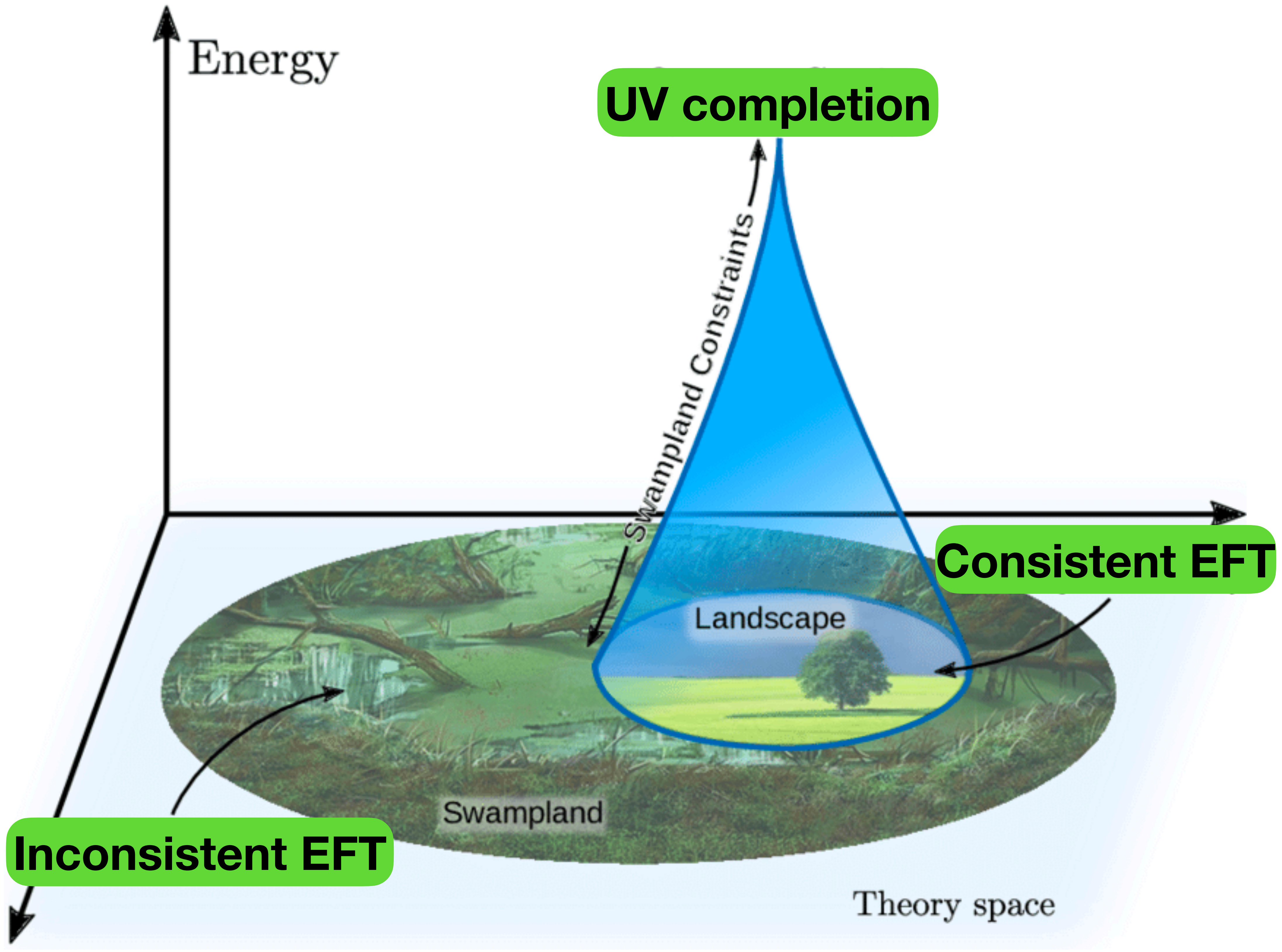
- Naturalness principle: coupling constants in a theory are of order one in the appropriate mass scale. Therefore, if any parameter is unusually small or large, a good explanation, such as an underlying symmetry, is require.

Constraints on BSM model building

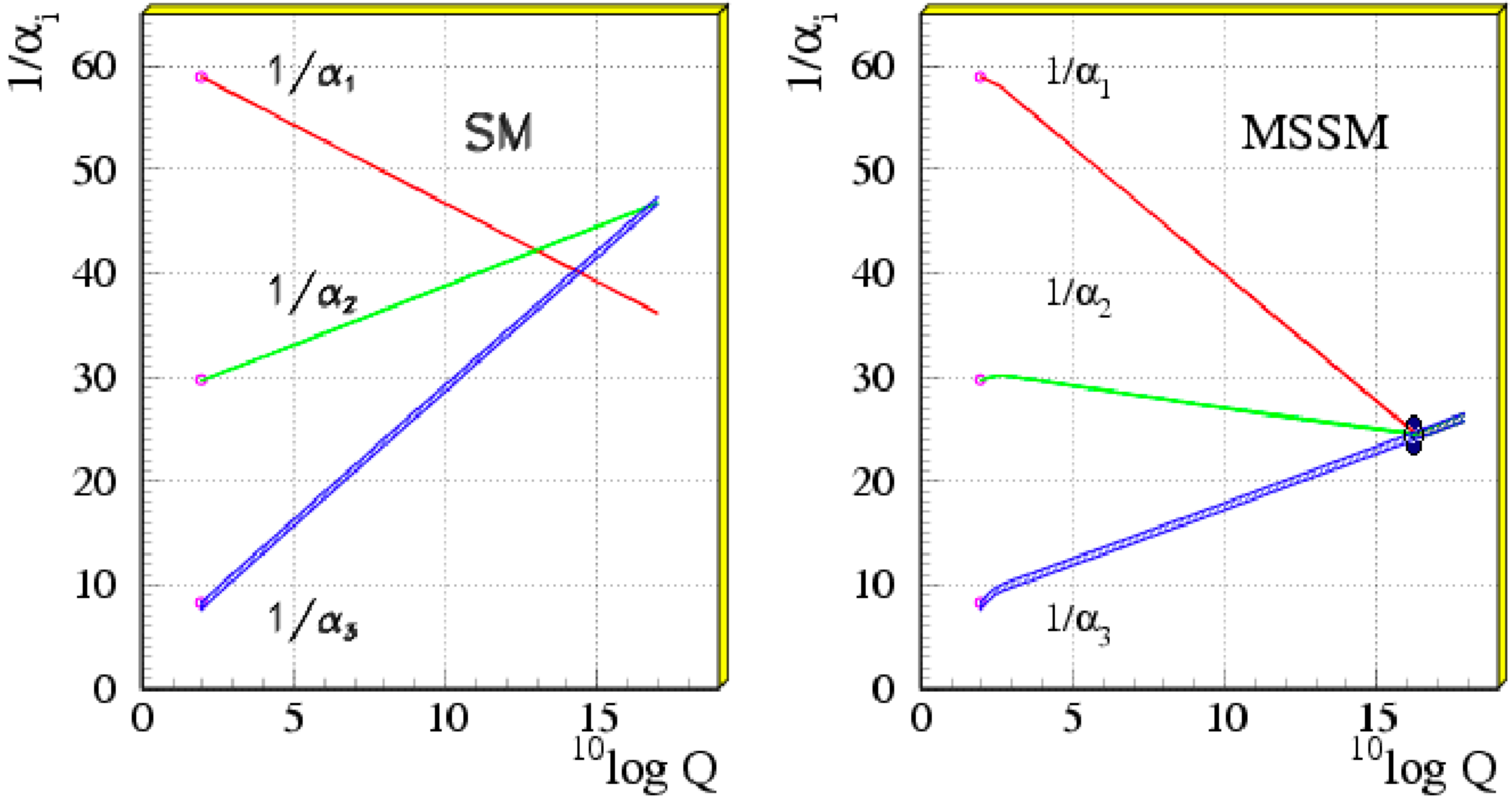
Consider building a BSM model where the symmetry group and representations are free parameters. For the model to be a phenomenologically-consistent EFT it must satisfy certain constraints, for examples:

Valenzuela, et al. ’21

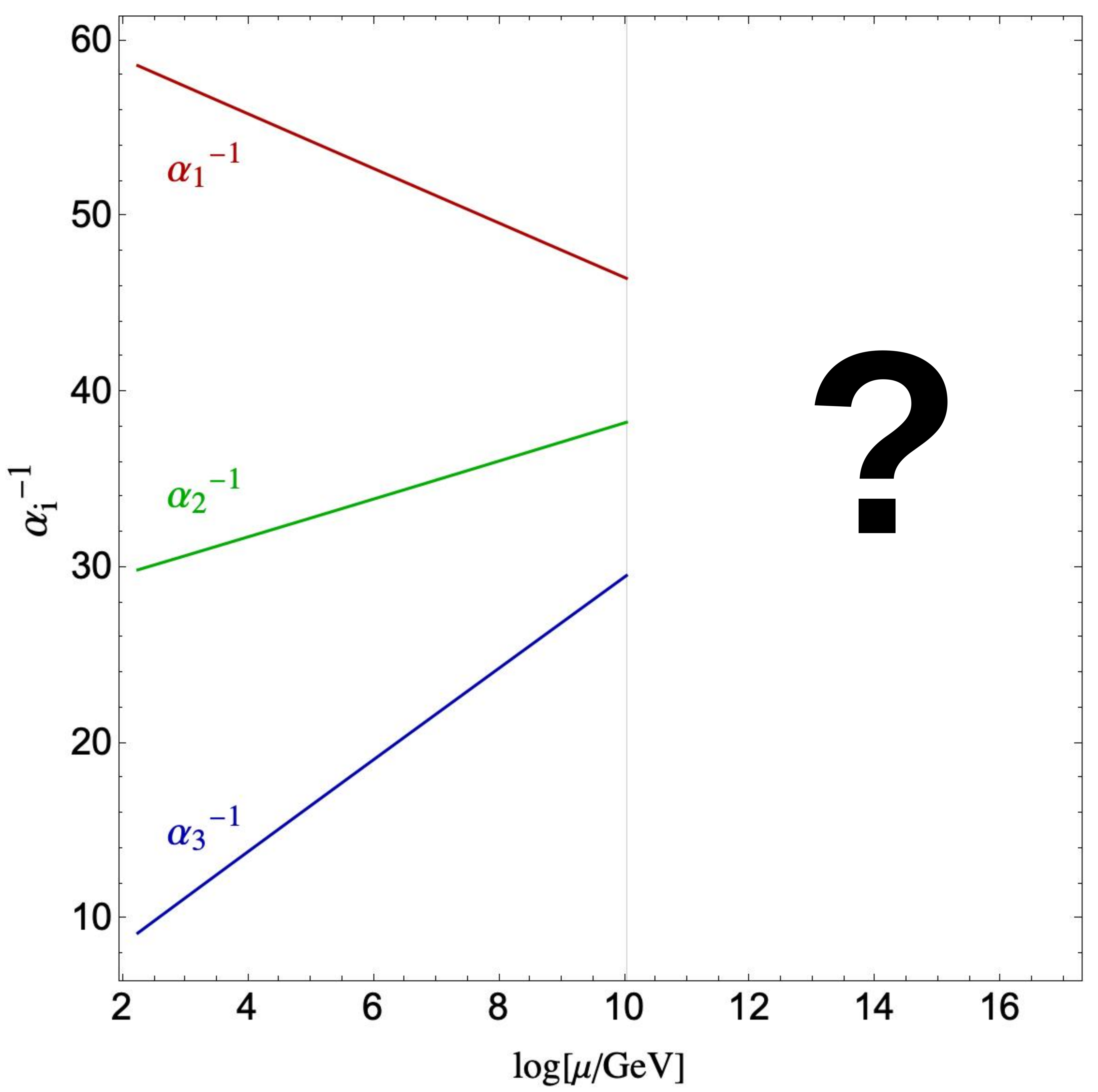
1. Anomaly cancellation
2. Alignments
3. Stable (long-lived) vacuum
4. UV completion: asymptotic free/safe
5. Unification of fundamental couplings



Unification of fundamental couplings



Why we expect unification?



2. Non-SUSY SO(10) Grand Unified Theory

Fermion representations of SO(10)

- Counting of SM chiral fermions of a single generation:
  8 Left handed fermions: $u_L^{c_1}, d_L^{c_1}, u_L^{c_2}, d_L^{c_2}, u_L^{c_3}, d_L^{c_3}, \ell_L, \nu_L^{\ell}$
  7 Right handed fermions: $u_R^{c_1}, d_R^{c_1}, u_R^{c_2}, d_R^{c_2}, u_R^{c_3}, d_R^{c_3}, \ell_R$

- We can put all these fermion contents into a fundamental 16-dimensional spinor representation on SO(10) group: $\mathbf{16_F}$, with an additional right-handed fields identified as the right-handed neutrino: ν_R^{ℓ}

$$\mathbf{16_F} \supset \left(u_L^{c_1}, d_L^{c_1}, u_R^{c_1}, d_R^{c_1}, u_L^{c_2}, d_L^{c_2}, u_R^{c_2}, d_R^{c_2}, u_L^{c_3}, d_L^{c_3}, u_R^{c_3}, d_R^{c_3}, \ell_L, \nu_L^{\ell}, \ell_R, \nu_R^{\ell}\right)$$

Scalar representations of SO(10)

- With the fermions representations $\mathbf{16_F}$ for each generation, we can imagine the gauge sector of SO(10) model as a YM theory with only a single fermion field $\mathbf{16_F}$.

- The next key question is: how to control the scalar degrees of freedom?

- By SO(10) invariance, the the allowed Yukawa couplings of the scalar bosons to pairs of these fermions belong to the direct product of fermion representation decomposed as:

$$\mathbf{16_F} \times \mathbf{16_F} = \mathbf{10_H} + \overline{\mathbf{126}}_\mathbf{H} + \mathbf{120_H}$$

$$\longrightarrow \quad -\mathscr{L}_{\text{Yukawa}} = \mathbf{16_F}(Y_{10}\mathbf{10_H} + Y_{126}\overline{\mathbf{126}}_\mathbf{H} + \cancel{Y_{120}\mathbf{120_H}})\mathbf{16_F}$$

Unification of fundamental couplings

$$\mathbf{SO(10)}$$

$$\mathbf{SO(10)} \xrightarrow{(\mathbf{210_H})} \mathbf{SU(4)\times SU(2)_L\times SU(2)_R}$$

$$\mathbf{SO(10)} \xrightarrow{(\mathbf{45_H})} \mathbf{SU(3)\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}}$$

$$\mathbf{SU(4)\times SU(2)_L\times SU(2)_R} \longrightarrow \mathbf{SU(3)\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}}$$

$$\mathbf{SU(4)\times SU(2)_L\times SU(2)_R} \longrightarrow \mathbf{SU(3)\times SU(2)_L\times U(1)_Y}$$

$$\mathbf{SU(3)\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}} \xrightarrow{(\overline{\mathbf{126}}_\mathbf{H})} \mathbf{SU(3)\times SU(2)_L\times U(1)_Y}$$

$$\mathbf{SU(3)\times SU(2)_L\times U(1)_Y} \xrightarrow{(\mathbf{10_H})} \mathbf{SU(3)\times U(1)_{em}}$$

PS : $\mathrm{SO}(10)|_{M_U} \xrightarrow{\langle \mathbf{210_H}\rangle} \mathcal{G}_{422}|_{M_I} \xrightarrow{\langle \overline{\mathbf{126}}_\mathbf{H}\rangle} \mathcal{G}_{321}|_{M_Z} \xrightarrow{\langle \mathbf{10_H}\rangle} \mathcal{G}_{31}$

LR : $\mathrm{SO}(10)|_{M_U} \xrightarrow{\langle \mathbf{45_H}\rangle} \mathcal{G}_{3221}|_{M_I} \xrightarrow{\langle \overline{\mathbf{126}}_\mathbf{H}\rangle} \mathcal{G}_{321}|_{M_Z} \xrightarrow{\langle \mathbf{10_H}\rangle} \mathcal{G}_{31}$

Decomposition of scalar representations

- The spontaneous breaking of SO(10) symmetry can be triggered with multiple steps, each corresponds to a different gauge symmetry at the intermediate scale:

$$\mathrm{SO}(10)(M_U) \longrightarrow \mathrm{EFT}(M_I) \longrightarrow \mathrm{SM}$$

- So we can decompose the SO(10) scalar representation and figure out which one corresponds to the Higgs doublet field we see in SM:

- For example, under $\mathscr{G}_{3221} = \mathrm{SU}(3)_\mathrm{c} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R} \times \mathrm{U}(1)_{\mathrm{B-L}}$:

$$\mathbf{10_H} \supset \underset{(\Phi_{10})}{(\mathbf{1}, \mathbf{2}, \mathbf{2}, \mathbf{0})} \oplus \ldots; \;\; \overline{\mathbf{126}}_\mathbf{H} \supset \underset{(\Sigma_{126})}{(\mathbf{1}, \mathbf{2}, \mathbf{2}, \mathbf{0})} \oplus \underset{(\Delta_R)}{(\mathbf{1}, \mathbf{1}, \mathbf{3}, \mathbf{2})} \oplus \underset{(\Delta_L)}{(\mathbf{1}, \mathbf{3}, \mathbf{1}, \mathbf{2})} \oplus \ldots$$

The survival hypothesis

- The survival hypothesis: scalars should have masses of order 1 at the symmetry breaking scale (the GUT scale), unless there are symmetries to protect their masses. (Again motivated by Naturalness)

- Only certain scalar components from $\mathbf{10_H}$ and $\overline{\mathbf{126}}_\mathbf{H}$ representations can acquire small vevs, so they can stay light below the GUT scale;

The EFT at intermediate scale

- The EFT at the intermediate scale should be left-right symmetric in the discussed breaking chains: it is a left-right model where the left-handed and right-handed fermions are coupled via a bi-doublet scalar field as

$$\bar{F}_L(Y_{10}\Phi_{10} + Y_{126}\Sigma_{126})F_R + Y_R F_R^T C\overline{\Delta_R}F_R + \mathrm{h.c.}$$

- The $\mathrm{SU}(2)_R$ right-handed symmetry will be broken by the right-handed triplet field Δ_R, which acquires an intermediate scale masses.

- Below the intermediate scale, we can integrate out the heavy gauge bosons and decouple most scalars except for the (two) Higgs doublet fields. So we should end up with a two Higgs doublet model (2HDM) at lower energy.

SO(10) as BSM model

- SO(10) models generalize the gauge group of SM to a larger gauge symmetry. The vacuum structure is much more complicated with many different phases. We can have different intermediate breaking patterns.

- The fermion within one generation plus a right-handed neutrino can all be embedded into a single representation $\mathbf{16_F}$ of SO(10).

- The SM Higgs field, with hypercharge +1/2, come from a decomposition of the SO(10) scalar field (can be a mixing of Φ_{10} and Σ_{126}).

- At the intermediate scale, we will have a left-right model, which is broken by the vev of Δ_R. The right-handed neutrinos can thus get Majorana masses at the scale Δ_R, and triggers the seesaw mechanism in this scenario.

Step 1: Adding missing ingredients (ν_R^ℓ)

Step 2: controlling the scalar sector

Step 3: getting the low energy EFT

Step 4: Study the model-independent effects of the constraints from unification of couplings

3. Constraints from Unification of fundamental couplings

Unification of gauge couplings

- The RGEs are a set of differential equations that takes the form:

$$\frac{\mathrm{d}\alpha_i^{-1}(\mu)}{\mathrm{d}\ln\mu} = -\frac{a_i}{2\pi} - \sum_j \frac{b_{ij}}{8\pi^2\alpha_j^{-1}(\mu)}$$

- It has an approximate solution:

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(\mu_0) - \frac{a_i}{2\pi}\ln\frac{\mu}{\mu_0} - \frac{1}{4\pi}\sum_j \frac{b_{ij}}{a_j}\ln\frac{\alpha_j(\mu)}{\alpha_j(\mu_0)} + \Delta_Y^i$$

- All three gauge couplings unify at a scale implies:

$$\alpha_1^{-1}(\Lambda_G) = \alpha_2^{-1}(\Lambda_G) = \alpha_3^{-1}(\Lambda_G) = \alpha_U^{-1}(\Lambda_G)$$

Unification of gauge couplings

- In particular in non-SUSY SO(10) with only one intermediate scale, we derived the following analytical solutions as a good approximation:

$$\ln\left(\frac{M_I}{M_Z}\right) = \frac{(\alpha^{-1}_{1_{\rm EW}} - \alpha^{-1}_{3_{\rm EW}}) - C_{\mathcal{G}_I}(\alpha^{-1}_{2_{\rm EW}} - \alpha^{-1}_{3_{\rm EW}}) + D_{\mathcal{G}_I}}{C_{\mathcal{G}_I}\Delta^{\mathcal{G}_{321}}_{32} - \Delta^{\mathcal{G}_{321}}_{31}}$$

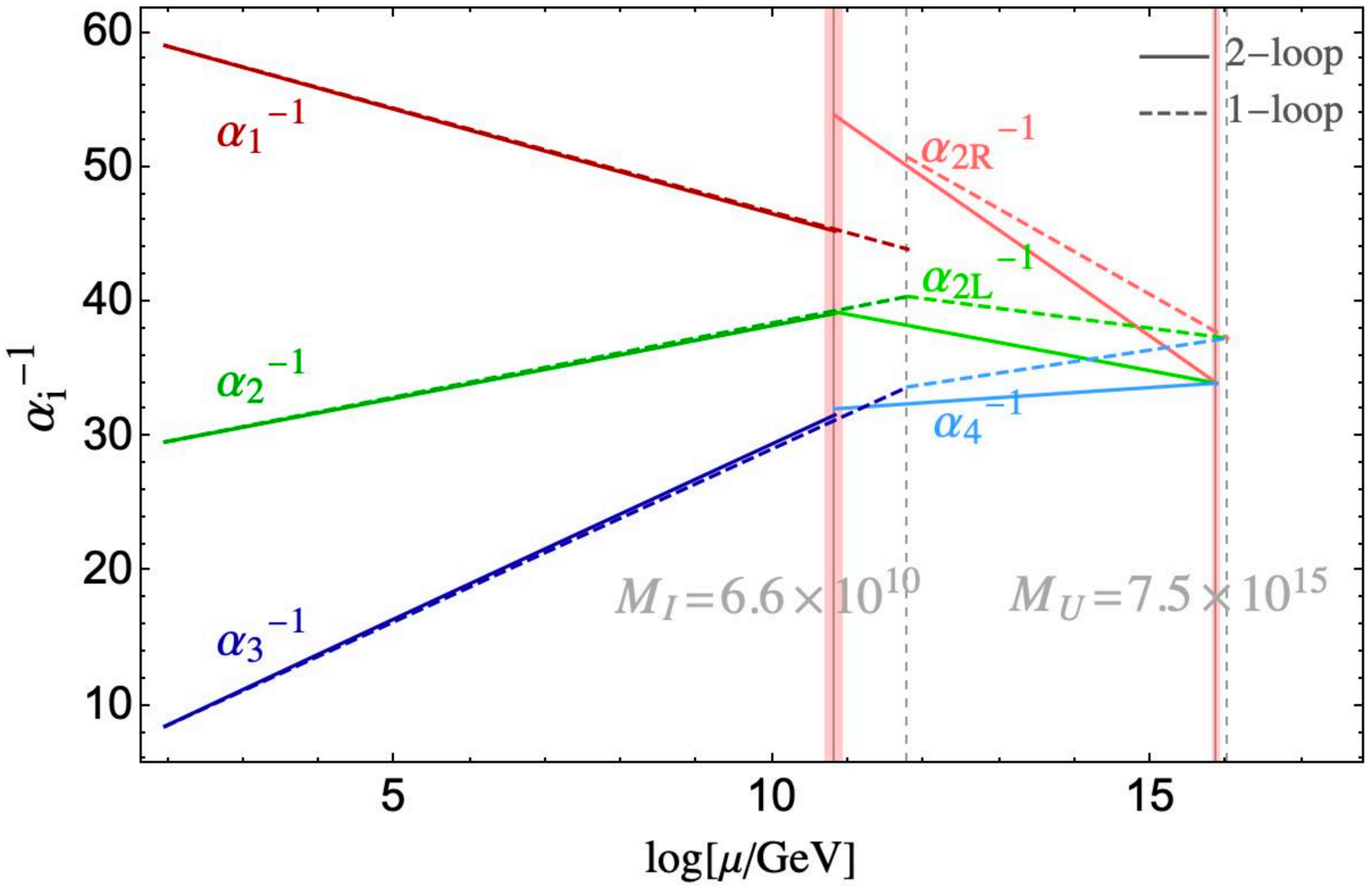
$$\ln\left(\frac{M_U}{M_I}\right) = -\frac{\alpha^{-1}_{2_{\rm EW}} - \alpha^{-1}_{3_{\rm EW}}}{\Delta^{\mathcal{G}_I}_{3_I 2L_I}} - \frac{\Delta^{\mathcal{G}_{321}}_{32}}{\Delta^{\mathcal{G}_I}_{3_I 2L_I}}\ln\left(\frac{M_I}{M_Z}\right) - \frac{D'_{\mathcal{G}_I}}{\Delta^{\mathcal{G}_I}_{3_I 2L_I}}$$

Unification of gauge couplings

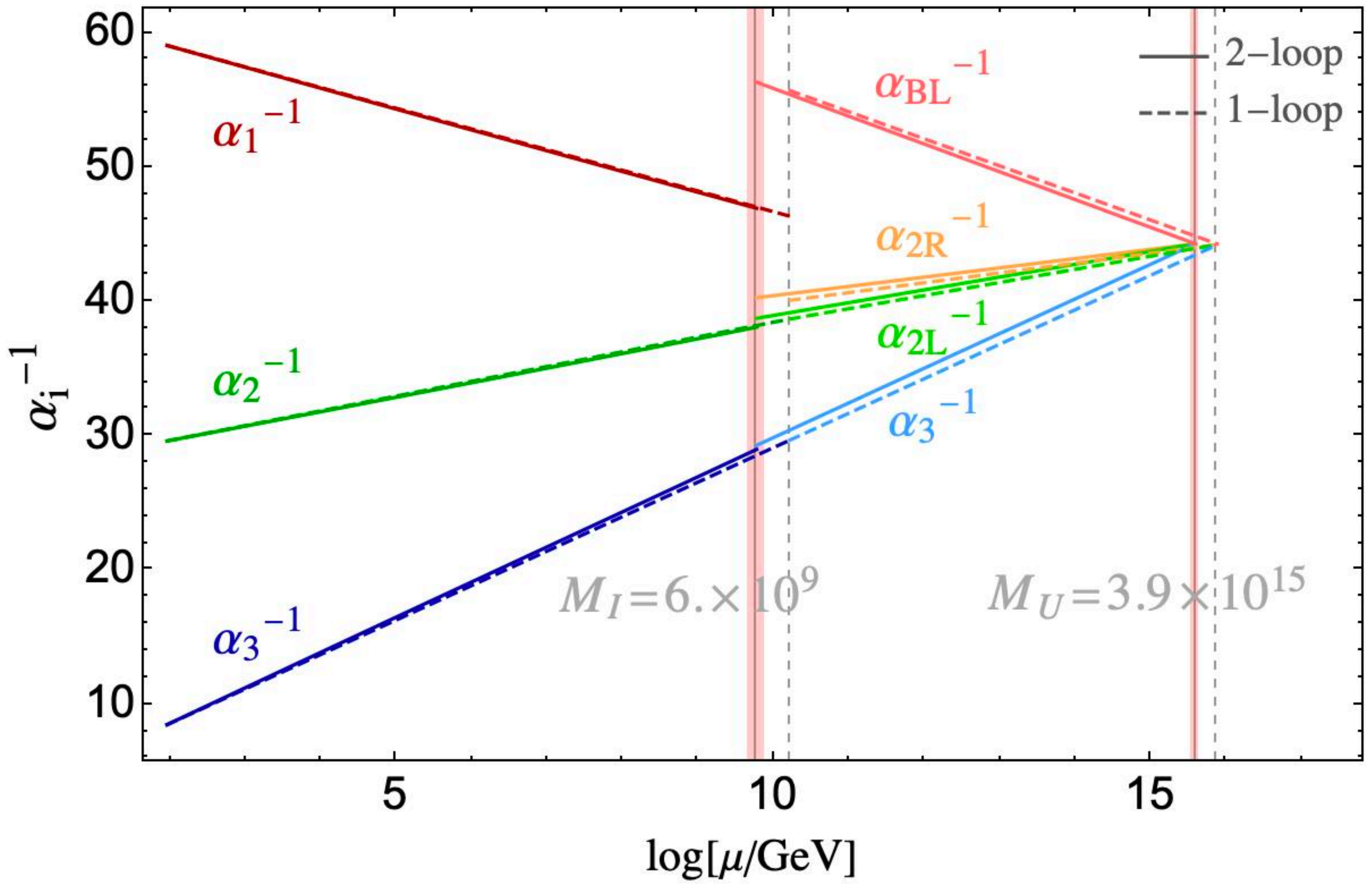
- And also the universal SO(10) coupling $\alpha_U^{-1}(M_U)$:

$$
\begin{aligned}
\alpha_U^{-1} \quad &\simeq \alpha_{3_{\rm EW}}^{-1} - \frac{1}{C_{\mathcal{G}_I}\Delta_{32}^{\mathcal{G}_{321}} - \Delta_{31}^{\mathcal{G}_{321}}} \Bigg[\\
&\left(\frac{a_3^{\mathcal{G}_{321}}}{2\pi} - \frac{\Delta_{32}^{\mathcal{G}_{321}}}{\Delta_{3_I 2L_I}^{\mathcal{G}_I}} \frac{a_{3_I}^{\mathcal{G}_I}}{2\pi} + \mathcal{O}\left(\frac{\alpha_U \theta_i^{\mathcal{G}}}{8\pi^2}\right)\right)(\alpha_{1_{\rm EW}}^{-1} - \alpha_{3_{\rm EW}}^{-1}) \\
- \quad &\left(C_{\mathcal{G}_I}\frac{a_3^{\mathcal{G}_{321}}}{2\pi} - \frac{\Delta_{31}^{\mathcal{G}_{321}}}{\Delta_{3_I 2L_I}^{\mathcal{G}_I}} \frac{a_{3_I}^{\mathcal{G}_I}}{2\pi} + \mathcal{O}\left(\frac{\alpha_U \theta_i^{\mathcal{G}}}{8\pi^2}\right)\right)(\alpha_{2_{\rm EW}}^{-1} - \alpha_{3_{\rm EW}}^{-1}) \Bigg] \; .
\end{aligned}
$$

Unification of gauge couplings



PS



LR

Constraints from gauge unification

- The constraints from unification of gauge couplings reduce the number of parameters (free gauge couplings).

- The original dimesionless parameters (gauge couplings) can be rewritten by the ratio of a few dimensionful parameters (Λ_{GUT} , Λ_I , …) and a universal gauge coupling (α_U)

- The physics behind it is simply the renormalization, followed from the principle of RG (UV-IR decoupling).

Unification of Yukawa couplings

- In a SUSY SO(10), if the Yukawa coupling is also unified:

For SUSY GUTs, see e.g.
PDG '22
Ananthanarayan, Lazarides, Shafi '91
Rattazzi, Sarid, Hall '94
Baer, Ferrandis '01
Blazek, Dermisek, Raby '02
Hebbar, Leontaris, Shafi '16
…

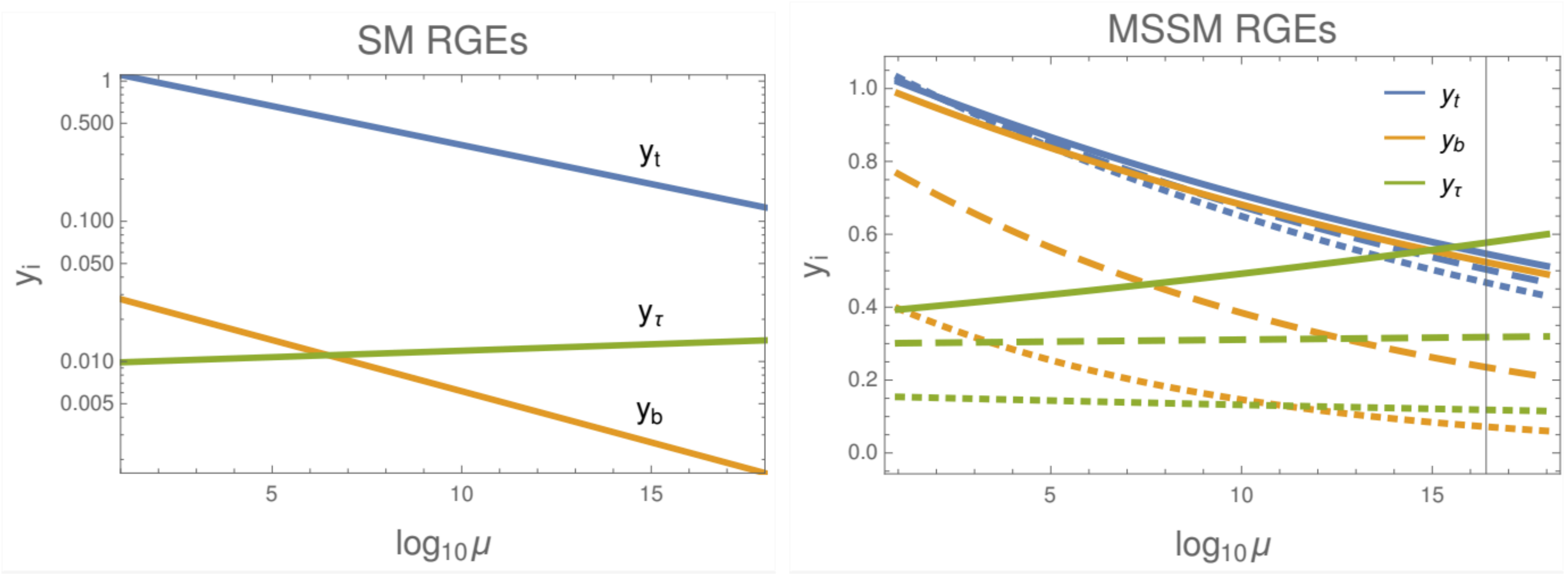


Figure 5: *One loop renormalisation group flow of the SM (left) and MSSM (right) Yukawa couplings, with* $m_0 = 2$ *TeV*, $m_{1/2} = 3$ *TeV*, $A_0 = 0$ *and* $\tan\beta = 40$ *(solid)*, $\tan\beta = 30$ *(dashed) and* $\tan\beta = 15$ *(dotted).*

Croon, Gonzalo, Graf, Košnik, White '19

What it means for Yukawa unification?

- Both the two scalar representations can be embedded into a single representation, so that all Yukawa couplings are originate from a single Yukawa couplings between scalars and fermions at UV scale.

$$\frac{Y_{10}}{Y_{126}} = \frac{c_{10}Y}{c_{126}Y} = \frac{c_{10}}{c_{126}}$$

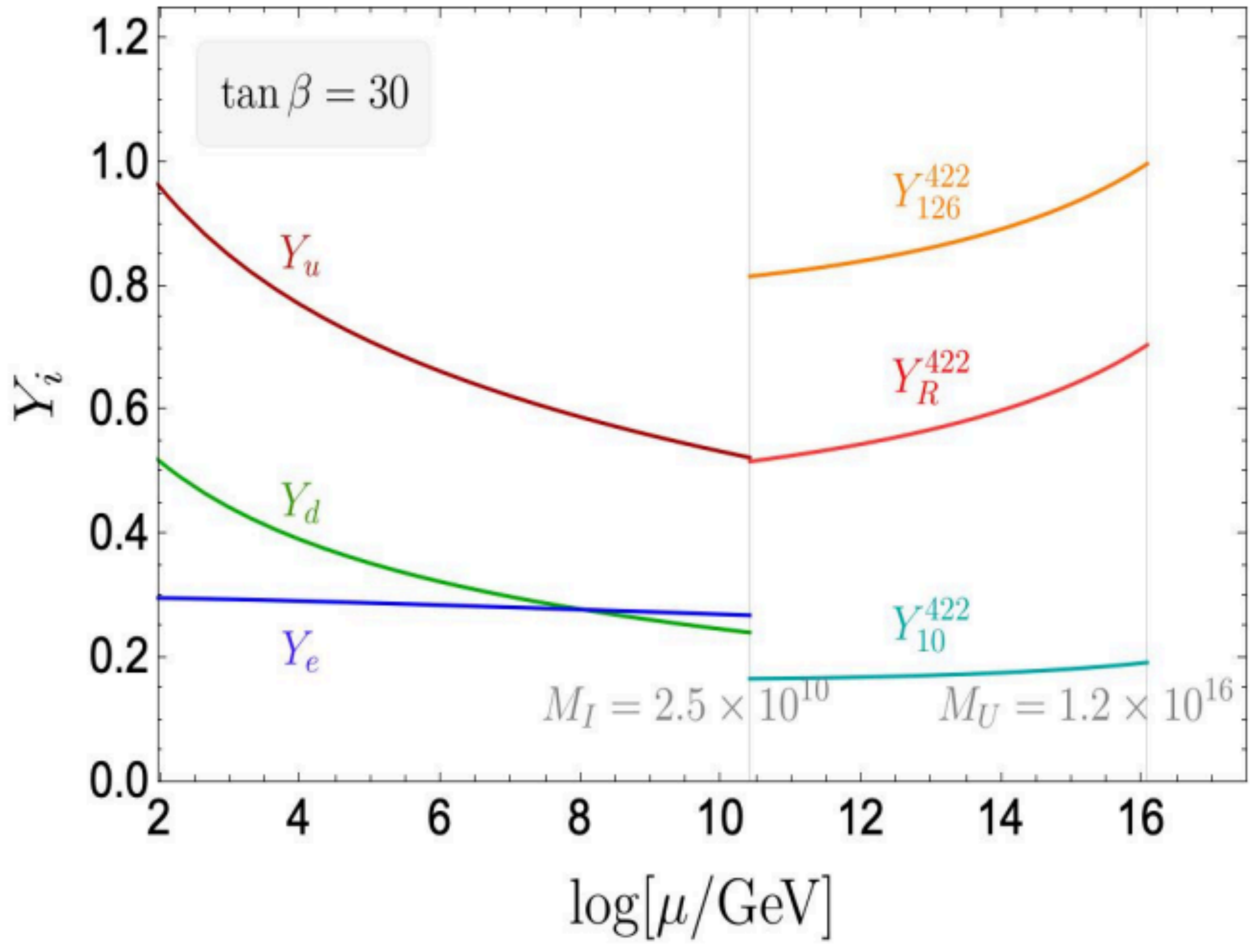
- For example, if they both come from the E6 representation, we can have

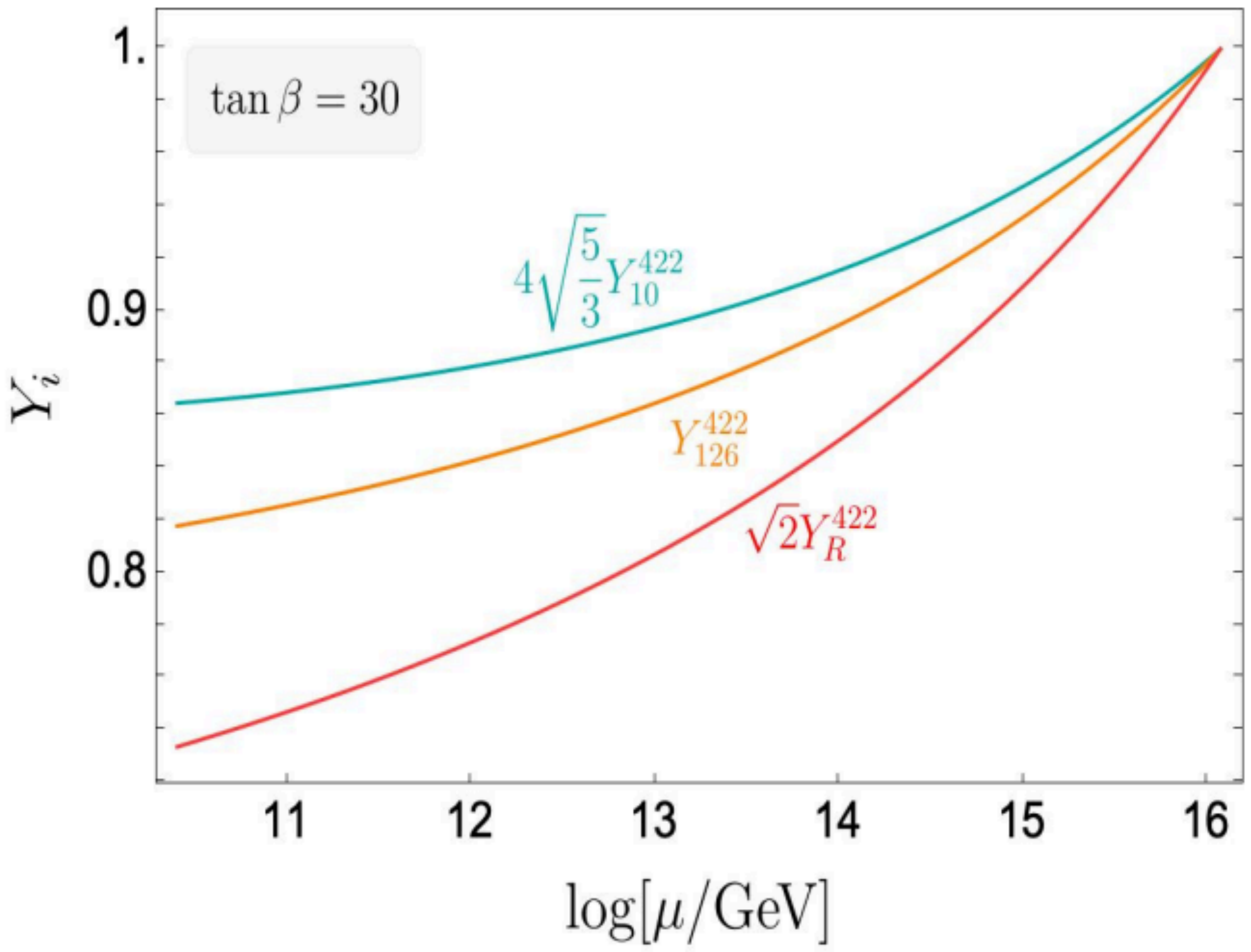
$$Y \times \mathbf{27_F} \cdot \mathbf{27_F} \cdot \mathbf{351}'_H \supset c_{10} Y \times \mathbf{16_F} \cdot \mathbf{16_F} \cdot \mathbf{10_H} + c_{126} Y \times \mathbf{16_F} \cdot \mathbf{16_F} \cdot \overline{\mathbf{126}}_\mathbf{H} + \cdots$$

- In the E6 case, $c_{10}/c_{126} = \sqrt{3/5}$

Unification of fundamental couplings

- In non-SUSY SO(10) case, the Yukawa coupling can also be unified with two Higgs doublets in the low energy:





What happens at the intermediate scale?

- We assume that the mass should be continuous at the intermediate scale. We will then have a matching conditions coming from the mass relations from low-energy EFT and intermediate scale models:

  In 422 intermediate scale model:

$$m_t = \frac{v_{10}^u}{\sqrt{2}} Y_{10}^{422} + \frac{v_{126}^u}{4\sqrt{2}} Y_{126}^{422}, \; m_b = \frac{v_{10}^d}{\sqrt{2}} Y_{10}^{422} + \frac{v_{126}^d}{4\sqrt{2}} Y_{126}^{422}, \; m_\tau = \frac{v_{10}^d}{\sqrt{2}} Y_{10}^{422} - \frac{3v_{126}^d}{4\sqrt{2}} Y_{126}^{422},$$

  In 2HDM:

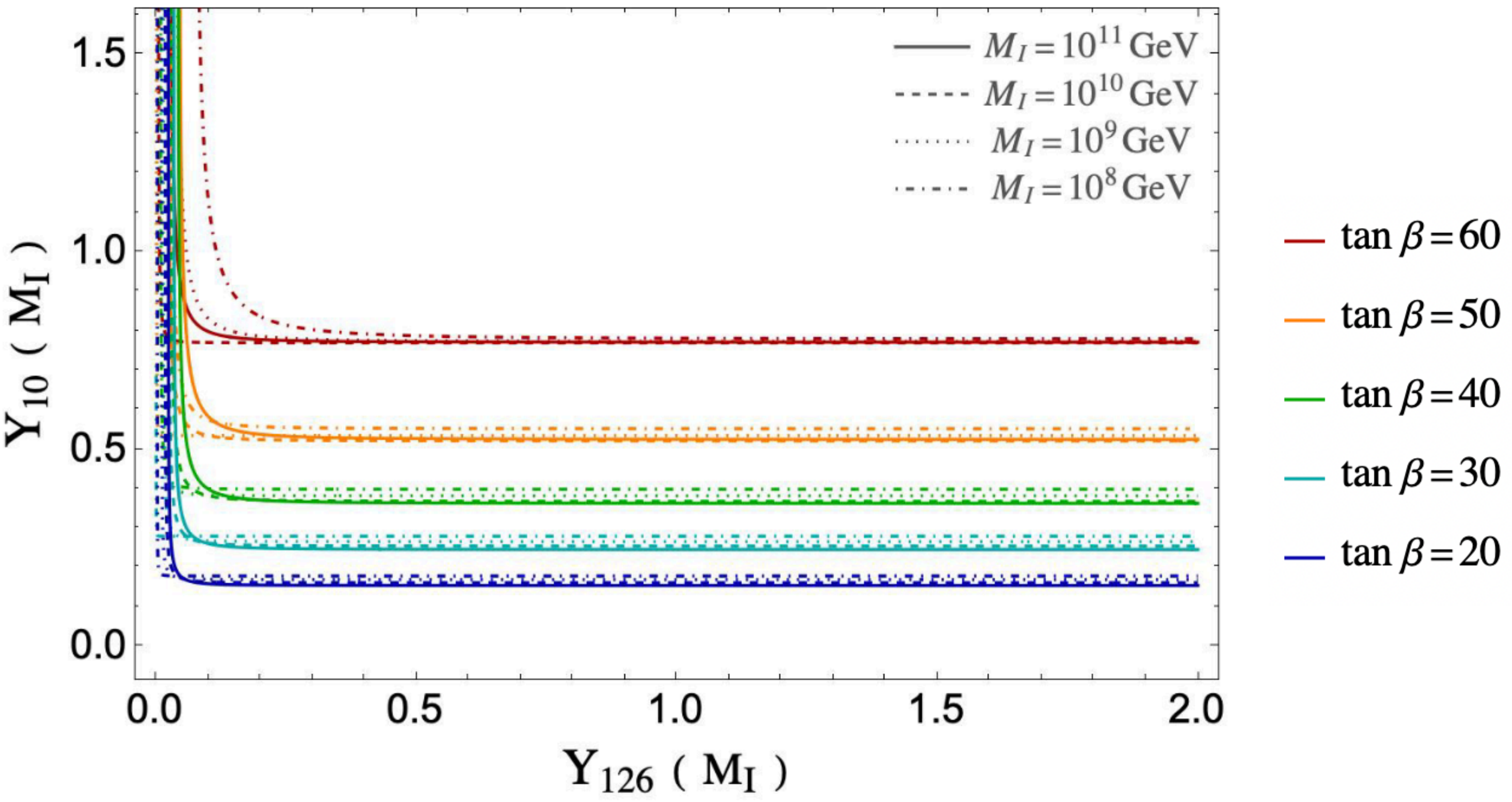
$$m_t = \frac{1}{\sqrt{2}} Y_t v_u , \quad m_b = \frac{1}{\sqrt{2}} Y_b v_d , \quad m_\tau = \frac{1}{\sqrt{2}} Y_\tau v_d .$$

What happens at the intermediate scale?

- These relations can be simplified to be (assuming no tree-level FCNCs):

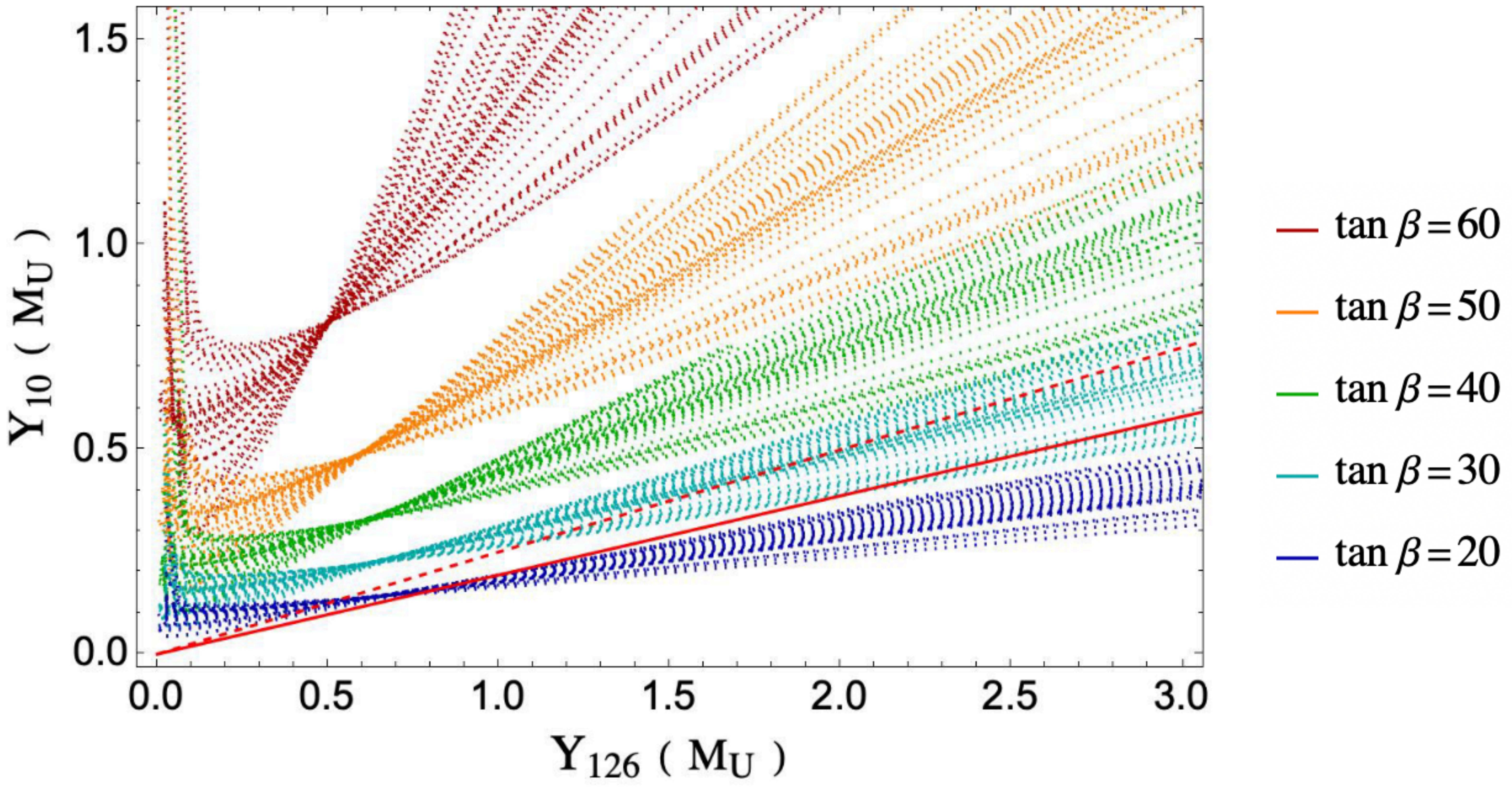
$$\left(Y_{10}^{422}(M_I)\right)^2 = \frac{\left(Y_{126}^{422}(M_I)\right)^2 \left(3Y_b(M_I) + Y_\tau(M_I)\right)^2}{16\left[\left(Y_{126}^{422}(M_I)\right)^2 - \left(Y_b(M_I) - Y_\tau(M_I)\right)^2\right]} \,.$$

Constraints from Yukawa unification



Solutions of matching conditions

Constraints from Yukawa unification



(Numerical) Solutions of RGEs + matching conditions

Constraints from Yukawa unification

- The constraint from unification of Yukawa couplings imposes non-trivial relations in the parameters of the scalar sector, which is described by the (numerical) solution of the RGEs of Yukawa couplings with particular boundary conditions (matching conditions).
- The original dimesionless parameters (Yukawa couplings) will be related to the ratio of vevs ($\tan\beta$), which is the parameters in the scalar sector.
- As an example, we construct a specific non-SUSY SO(10) model, with the observed fermion masses, the unification of Yukawa coupling implies that the ratio of vevs $\tan\beta \lesssim 30$, which can be tested easily.

4. Conclusions

Conclusions

- We discuss how we do model building based on first principles. Certain constraints must be imposed to obtain a phenomenologically-consistent model. Our motivation is to study the model-independent consequences of imposing these constraints.

- In particular, we discussed the particular constraints of unification of gauge and Yukawa couplings in non-supersymmetric SO(10) models.

- The constraint from unification of couplings imposes non-trivial relations in the bare parameters, which is described by the solution of the RGEs with particular boundary conditions (matching conditions). We are still lack of understanding of the analytical structures of RGEs in these cases, especially for the Yukawa couplings for non-trivial BSM models.

- For example, our result shows unification of Yukawa couplings for 422 breaking chains in non-SUSY SO(10) models can be achieved when $\tan\beta \lesssim 30$.

- We expect similar effects when imposing the constraint of unification in other GUT models.

Proton decay

- The proton decay is a function of unification scale as well as the unified coupling, for example:

$$\tau(p \to e^+\pi^0) \simeq (7.47 \times 10^{35}\mathrm{yr})\left(\frac{M_U}{10^{16}\ \mathrm{GeV}}\right)^4 \left(\frac{0.03}{\alpha_U}\right)^2$$

Meloni-Ohlsson-Pernow ’20

Proton decay

- Numerical result: proton decay only preferred the Pati-Salam (422) and Minimal Left-Right (3221) breaking chains of SO(10).

Breaking chain	$\log\left(\frac{M_{Ic}}{\mathrm{GeV}}\right)^{2-\mathrm{loop}}$	$\log\left(\frac{M_{Uc}}{\mathrm{GeV}}\right)^{2-\mathrm{loop}}$	$\alpha_U^{2-\mathrm{loop}}$	$\tau(p \to e^+\pi^0)/\mathrm{yr}$
422	10.03	16.19	0.032	3.82×10^{36}
3221	10.66	15.45	0.023	7.84×10^{33}
422D	13.65	14.66	0.026	4.22×10^{30}
3221D	11.82	14.63	0.024	3.89×10^{30}

Table 3: A summary table of the numerical results of the intermediate scale, the unification scale, and the universal gauge coupling at the two-loop level, neglecting all the threshold corrections as well as the estimated proton lifetimes obtained for each considered breaking chain with two Higgs doublets at the electroweak scale. The ratio of vevs is fixed to $\tan\beta = 65$ as the results do not change significantly for lower values of $\tan\beta$.

Scalar multiplets in different breaking chains

Intermediate symmetry	Scalar Multiplets
422	$\Phi_{10} \oplus \Sigma_{126} \oplus \Delta_R \oplus \Delta_{45R}$
422D	$\Phi_{10} \oplus \Sigma_{126} \oplus \Delta_L \oplus \Delta_R \oplus \Delta_{45L} \oplus \Delta_{45R}$
3221	$\Phi_{10} \oplus \Sigma_{126} \oplus \Delta_R$
3221D	$\Phi_{10} \oplus \Sigma_{126} \oplus \Delta_L \oplus \Delta_R \oplus \Delta_{45L} \oplus \Delta_{45R}$

Table 1: List of scalar multiplets containing light fields, for each intermediate symmetry. They are the only ones which are not integrated out below the SO(10) symmetry breaking scale mass M_U.