Gravitational Waves and Gravitino Mass in No-Scale SUGRA Wess-Zumino model with Polonyi Term

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LABORATÓRIO DE INSTRUMENTAÇÃO
E FÍSICA EXPERIMENTAL DE PARTÍCULAS

University of Southampton

Outline

1. Introduction and Motivation
2. Parameter Space Scan
   a. Random Sampling
   b. Artificial Intelligence Guided
3. Gravitational Wave Spectrum
4. Conclusions

1) Introduction and Motivation

Introduction and Motivation
No-Scale SUGRA Inflation

- 2013 - Ellis, Nanopoulos, Olive

$$K_{ENO} = -3\log\left(T+\bar{T}-\frac{\Phi\bar{\Phi}}{3}\right)$$

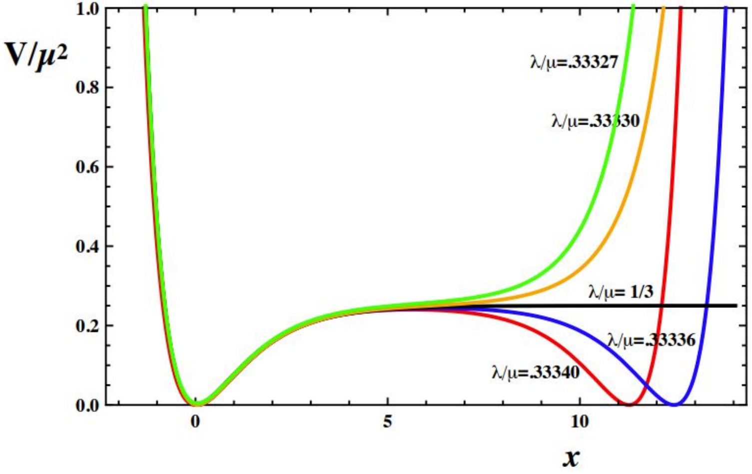
$$W_{ENO} = \frac{\hat{\mu}}{2}\Phi^2 - \frac{\lambda}{3}\Phi^3$$

- Starobinsky Inflation:

$$\chi = \frac{1}{\sqrt{2}}(x+i\,y),\; T+\bar{T}=c$$

$$\Phi = \sqrt{3\,c}\tanh\left(\frac{\chi}{\sqrt{3}}\right)$$

$$\hat{\mu} = \mu\sqrt{c/3}$$



No-Scale Supergravity Realization of the Starobinsky Model of Inflation - Ellis, Nanopoulos, Olive - Phys. Rev. Lett. 111, 111301 [1305.1247]

C.f. Olive talk yesterday morning

Introduction and Motivation

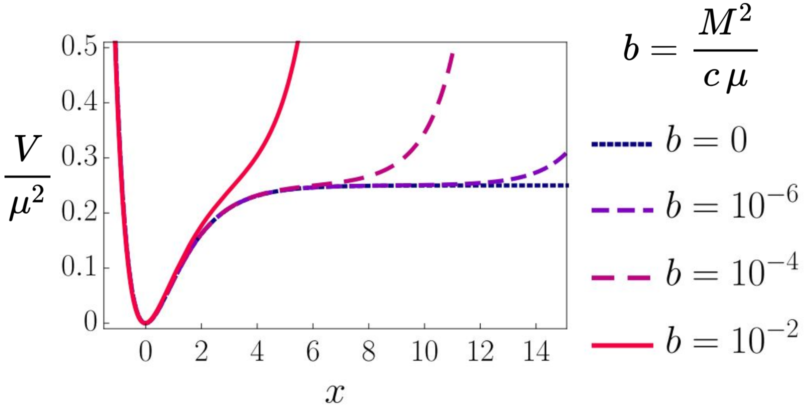
No-Scale SUGRA Inflation

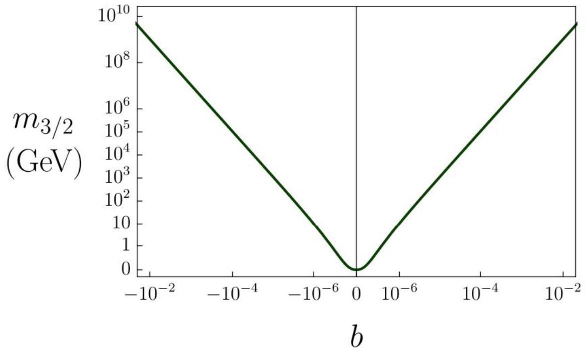
Starobinsky-like inflation in no-scale supergravity Wess-Zumino model with Polonyi term - MCR, Stephen F. King - JHEP 07 (2017) 033 [1703.08333]

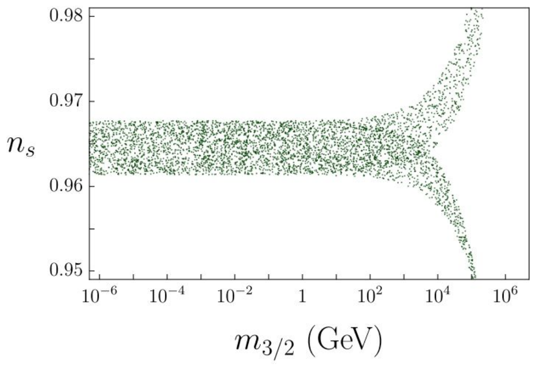
- 2017 - MCR, Stephen F. King

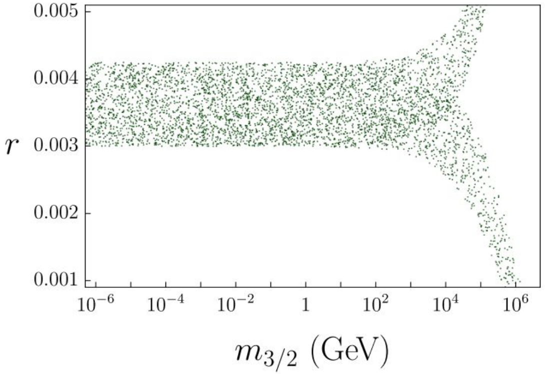
$$W_{CRK} = W_{ENO} + M^2\Phi = M^2\Phi + \frac{\hat{\mu}}{2}\Phi^2 - \frac{\lambda}{3}\Phi^3$$

- Starobinsky-like Inflation **with SUSY breaking**









Gravitational waves from no-scale supergravity - Spanos, Stamou - Eur.Phys.J.C 83 (2023) 1, 4 [2205.05595] (see also 2008.01457, 2104.08654)

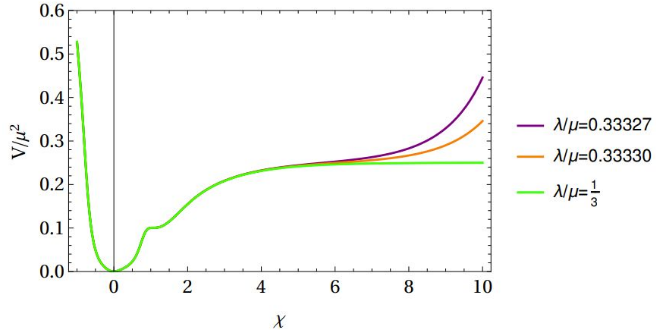
Introduction and Motivation
No-Scale SUGRA Inflation

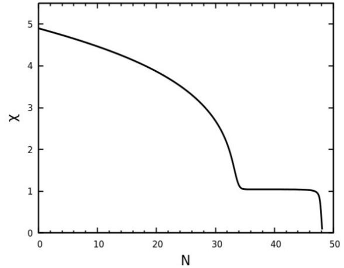
- 2022 - Spanos, Stamou

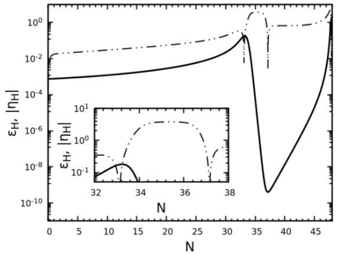
$$W = W_{ENO}$$

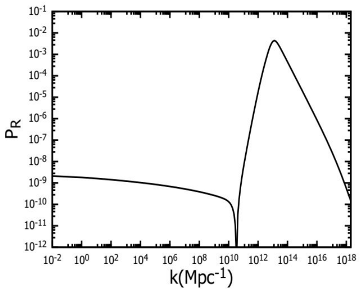
$$K_{NSS} = -3\log\left(T+\bar{T}-\frac{\Phi\bar{\Phi}}{3}+ae^{-b\,(\Phi+\bar{\Phi})^2}(\Phi+\bar{\Phi})^4\right)$$

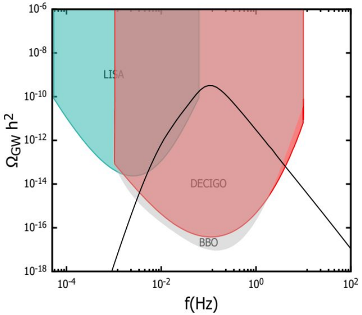
- Kahler potential modification produces a **kink in the inflaton potential**
- Kink slows down the inflaton, leading to a **second inflationary stage** with an **enhanced power spectrum** which leads to the production of **Gravitational Waves (GW)**











Introduction and Motivation
No-Scale SUGRA Inflation

- 2023 - MCR, Stephen F. King

$$W_{CRK} = W_{ENO} + M^2\Phi = M^2\Phi + \frac{\hat{\mu}}{2}\Phi^2 - \frac{\lambda}{3}\Phi^3$$

$$K_{NSS} = -3\log\left(T + \bar{T} - \frac{\Phi\bar{\Phi}}{3} + ae^{-b\,(\Phi+\bar{\Phi})^2}(\Phi+\bar{\Phi})^4\right)$$

- Study the impact of the Polonyi term in GW production
- Study the impact of the kink in SUSY breaking
- Assess whether the phenomenology of GW and the phenomenology of SUSY breaking interact

2) Parameter Space Scan

Parameter Space Scan
The Dynamic Inflaton Solution

- Need to find the dynamical background solution

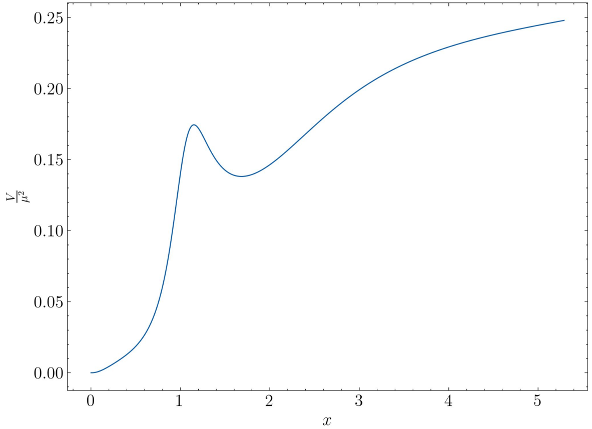
$$\frac{d^2 x}{d N^2} + 3\,\frac{d\,x}{d\,N} - \frac{1}{2}\left(\frac{d\,x}{d\,N}\right)^3 + \left(3 - \frac{1}{2}\left(\frac{d\,x}{d\,N}\right)^2\right)\partial_x \ln V = 0$$

- Keep track of field redefinitions

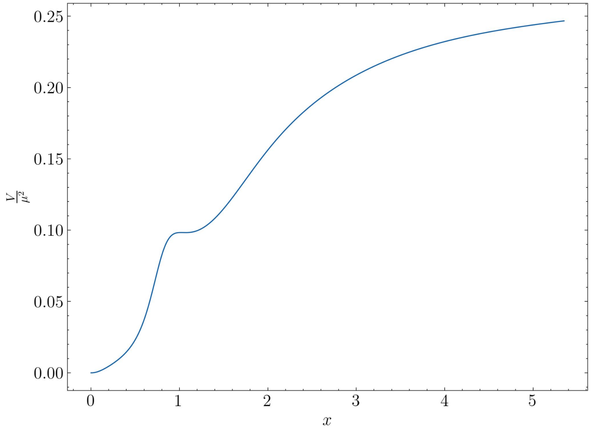
$$\mathcal{L}_{kin} \supset K_{\Phi\bar{\Phi}}\Big|_{\Phi=\phi} |\partial_\mu \phi|^2,\ \frac{1}{2}(\partial_\mu x)^2 = \left|\frac{d\,x}{d\,\Phi}\right|^2\Bigg|_{\Phi=\phi} |\partial_\mu \phi|^2$$

$$dx = \sqrt{2\,K_{\Phi\bar{\Phi}}\Big|_{\Phi=\phi}}\,d\phi,\ \partial_x \to \frac{1}{\sqrt{2\,K_{\Phi\bar{\Phi}}\Big|_{\Phi=\phi}}}\partial_\phi$$

Bad shape



Good shape



Parameter Space Scan

Planck and $m_{3/2}$

- Planck-constrained observables at pivot scale, k*=0.05 Mpc^{-1}, from slow-roll at tip of the potential

$$r \simeq 16\,\epsilon_V,\; n_s \simeq 1 - 6\,\epsilon_V + 2\,\eta$$

$$\epsilon_V = \frac{1}{2}\left(\frac{\partial_x V}{V}\right)^2\Big|_{x=x^*},\; \eta_V = \frac{\partial_{x,x} V}{V}\Big|_{x=x^*}$$

- Gravitino mass computed at the global minimum

$$V \propto |\partial_\chi W|^2 = 0$$

$$\langle\Phi\rangle = \frac{1}{2\,\lambda}\left(\hat{\mu} \pm \sqrt{\hat{\mu}^2 + 4\,M^2\lambda}\right)$$

$$m_{3/2}^2 = e^K |W|^2\Big|_{\Phi=\langle\Phi\rangle}$$

Parameter Space Scan

Planck and $m_{3/2}$

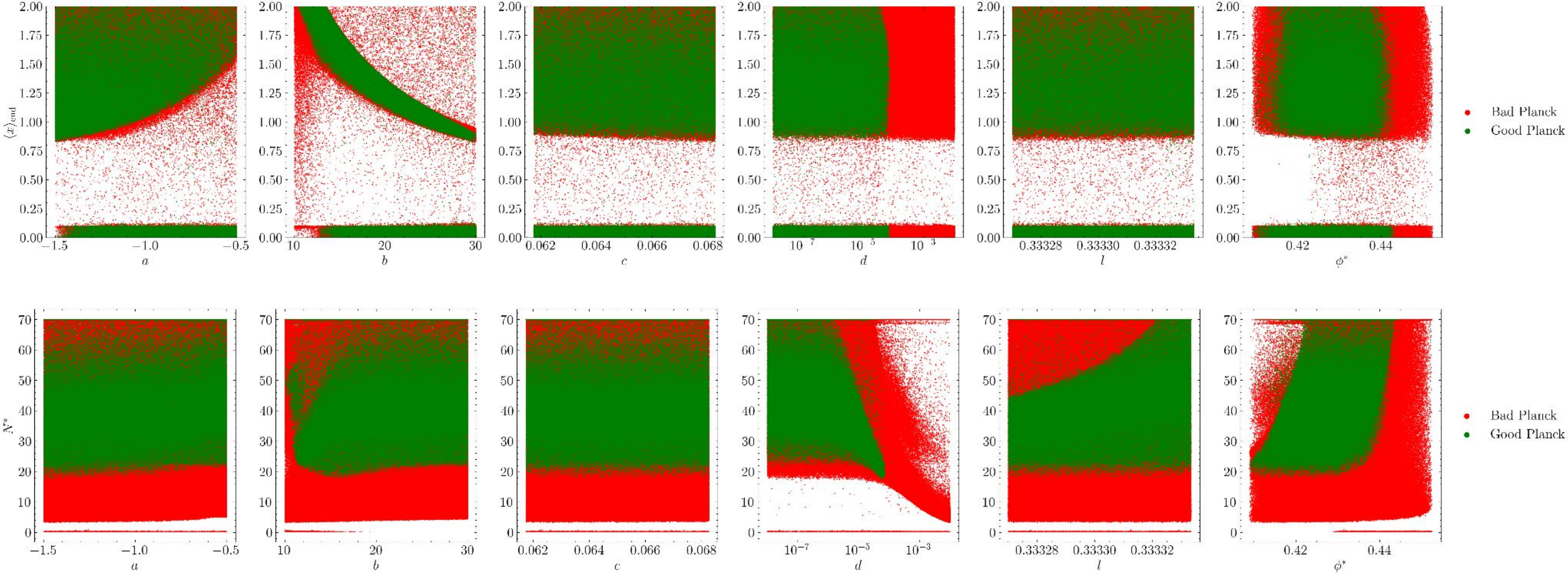
- Allow for more parametric freedom around the points found in [1703.08333, 2205.05595]

$$a \in [-1.5, -0.5],\ b \in [10, 30],\ c \in 0.065 \times [0.95, 1.05]$$
$$M = \sqrt{\mu\, d} \ \text{ with } \ d \in 10^{[-8,-2]}$$
$$\lambda = \mu\, l \ \text{ with } l \in [0.33327, 1/3]$$
$$\phi^* \in \sqrt{3\, c} \times [0.95, 1.00]$$

- Scanned just over 1M points
- For all points in the this scan, the potential was found to be always positive semi-definite
    - The global minimum is found at V=0
    - Real and imaginary components never mix

Parameter Space Scan

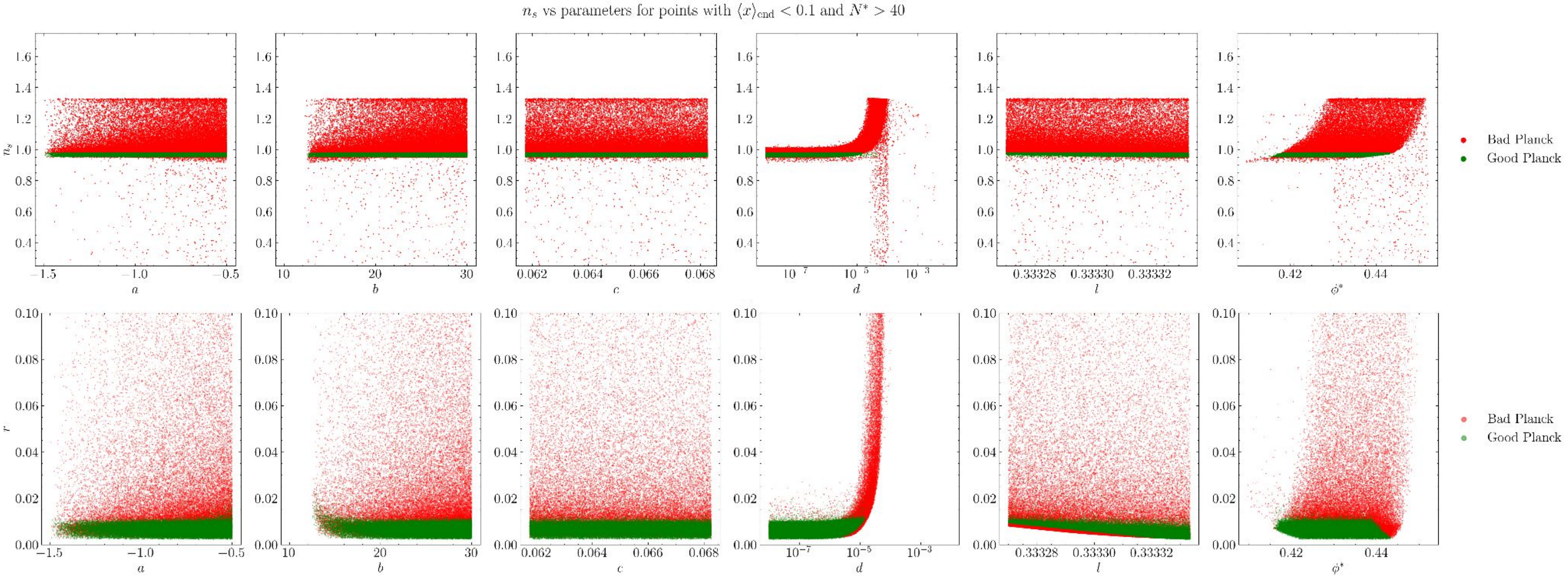
Planck and $m_{3/2}$



Parameter Space Scan

Planck and $m_{3/2}$

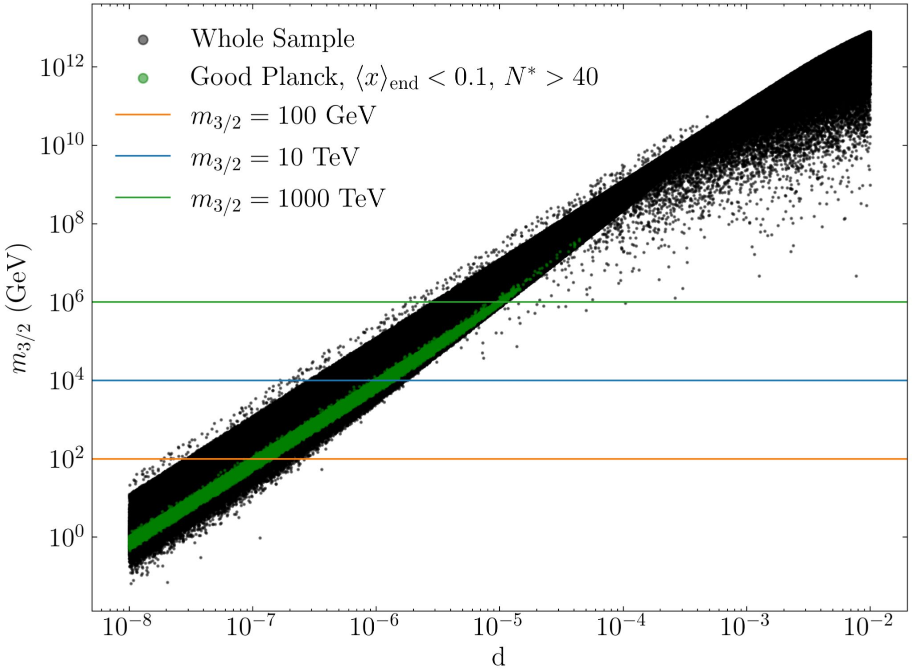
Planck observables are mostly constrained by **d**, the parameter that controls the Gravitino mass, and **b**, the (main) parameter that controls the kink



Parameter Space Scan

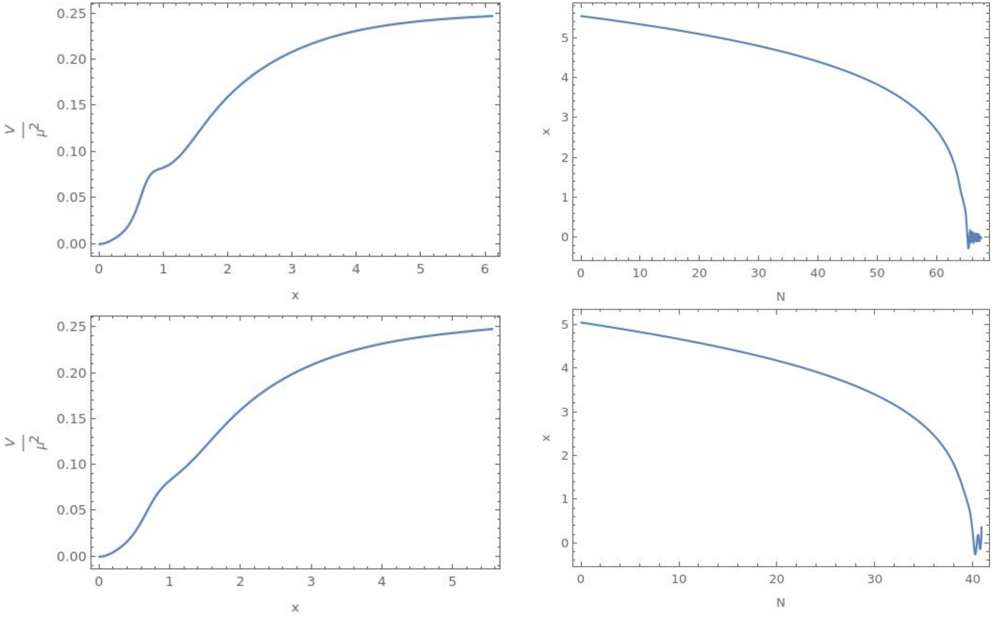
Planck and $m_{3/2}$

Gravitino mass is bounded at O(1000) TeV



Parameter Space Scan
Potential Shapes

Unfortunately, most of the points do not exhibit the desired potential features to generate GW



Parameter Space Scan
Challenges to Produce Gravitational Waves

- None of the points exhibit a power spectrum enhancement large enough to produce GW
- Inflaton needs to be slowed down enough in the kink (~ two-stage inflation)
- It has been extensively discussed by Stamou and Spanos that one needs to **tweak** and **fine tune** the **b** parameter
    - Manual tweaking the value of **b** of points from the scan would then spoil Planck observables, N^*, $\langle x \rangle_{final}$

Need a better approach to find points with **good inflation** and with a **pronounced enough kink** to produce **GW**

Parameter Space Scan
Artificial Intelligence Guided Scan

Exploring Parameter Spaces with Artificial Intelligence and Machine Learning Black-Box Optimisation Algorithms - Fernando Abreu de Souza, MCR, Nuno Filipe Castro, Mehraveh Nikjoo, Werner Porod, Phys.Rev.D 107 (2023) 3, 035004 [2206.09223]

- It was recently proposed that one can scan parameter spaces with Artificial Intelligence and Machine Learning black-box optimisation algorithms
- The methodology:
    - Choose a set of **criteria**: Planck observables and good inflation, location and length of the kink, (slow-roll approximation) power spectrum enhancement at kink
    - **Quantify** how badly a point is performing under the criteria
    - Employ an **intelligent search algorithm** that looks for points that fit all the criteria (we used CMA-ES, an evolutionary strategy)

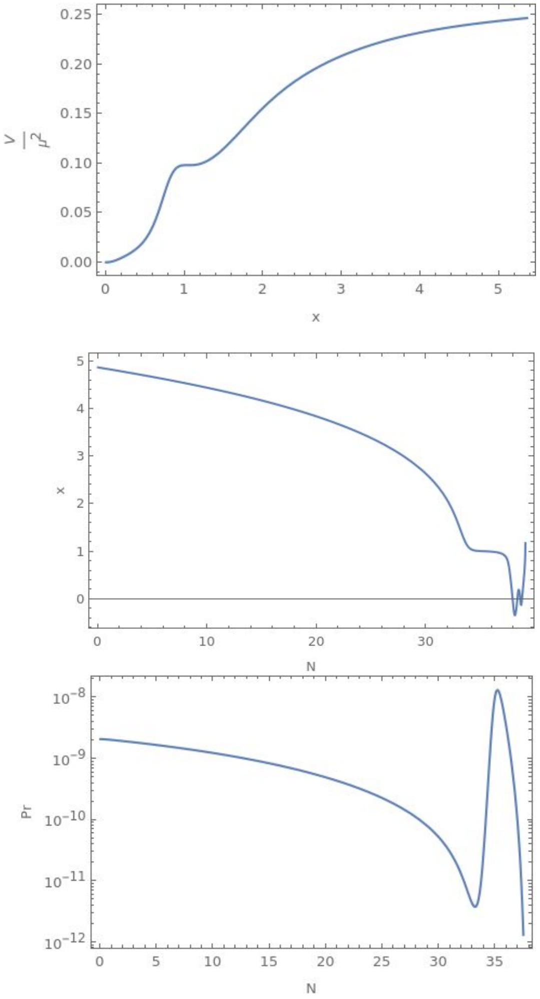
$$P_{R,sr} \simeq \frac{1}{8\,\pi^2}\frac{H^2}{\varepsilon_H}$$

$$\varepsilon_H = \frac{1}{2}\left(\frac{dx}{dN}\right)^2$$

$$H^2 = \frac{V}{3-\varepsilon_H}$$

Parameter Space Scan
Artificial Intelligence Guided Scan

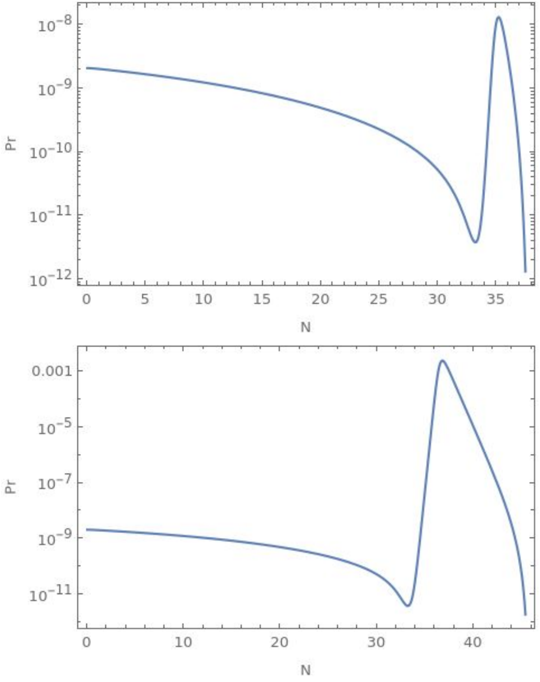
- The algorithm could not find points that completely satisfied all the criteria
- **However**, the **best** points found were **easy to manually tune** to satisfy all the criteria!
- Point found (significant digits are important)
  - a= -1.0056852832254333
  - b=22.25263984321967
  - c=0.0680509749369046 9
  - d=5.3438352750262 7 x 10^{-07}
  - l=0.33328322912577735
  - ϕ^*=0.43520666936934826
- Kink does not slow down the inflation enough



Parameter Space Scan
Artificial Intelligence Guided Scan

- But this point can be easily tuned!
  - b=22.25263984321967 **x 0.998343**
- With minimal impact on observables
  - r: 0.116302 -> 0.116312
  - n_s: 0.957572 -> 0.957567
  - $m_{3/2}$: 2791.03 GeV -> 2791.15 GeV

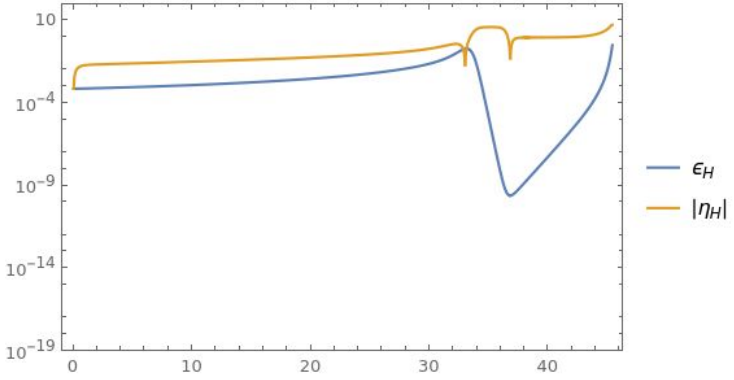
This is a promising point to produce GW! However, we need to go beyond the slow-roll approach to compute the power spectrum



3) Gravitational Waves Spectrum

Gravitational Waves Spectrum
Beyond Slow-Roll

- When "rolling through" the kink, the inflaton dynamics **violate the slow-roll assumptions**
- In fact, it was observed [Adams, Creswell, Easther, 0102236] that the slow-roll approximation actually **underestimates** the peak of power spectrum
- We need to go **beyond the slow-roll** approximation



Gravitational Waves Spectrum
Beyond Slow-Roll

- Following [Ringeval 07, Stamou 21, SS 21,22], for each GW mode with wave-number **k** the contribution to the power spectrum is

$$P_R = \frac{k^3}{2\,\pi^2}\left|R_k\right|^2,\; R_k = \Psi + \frac{\delta x}{dx/dN}$$

where Ψ is the Bardeen potential, and δx the inflaton perturbations, obtained by solving (with appropriate initial conditions)

$$\frac{d^2\delta x}{dN^2} = -\left(3 - \frac{1}{2}\left(\frac{dx}{dN}\right)^2\right)\frac{d\delta x}{dN} - \frac{1}{H^2}\partial^2_{x,x}V\,\delta x - \frac{k^2}{a^2H^2}\delta x + 4\frac{d\Psi}{dN}\frac{dx}{dN} - \frac{2\Psi}{H^2}\partial_x V$$

$$\frac{d^2\Psi}{dN^2} = -\left(7 - \frac{1}{2}\left(\frac{dx}{dN}\right)^2\right)\frac{d\Psi}{dN} - \left(2\frac{V}{H^2} + \frac{k^2}{a\,H}\right)\Psi - \frac{1}{H}\partial^2_{x,x}V\,\delta x$$

Gravitational Waves Spectrum
Beyond Slow-Roll

- Following [Ringeval 07, NSS 20, Stamou 21, SS 21, 22], the steps to obtain present day power spectrum
  - Compute the background solution, **x(N)**
  - For each mode **k** interest
    - Obtain N_k, the "time" at which **k** crosses the Hubble radius
    - Evolve Ψ, δx from N_{ic} until the end of inflation
      - We notice that initial conditions (not shown here) only depend on $x(N_{ic})$, k, $a(N_{ic})$
      - N_{ic} can be set "close" to N_k, say $k_{ic} \sim 10^{-3}$ k
    - The power spectrum for the mode **k** is

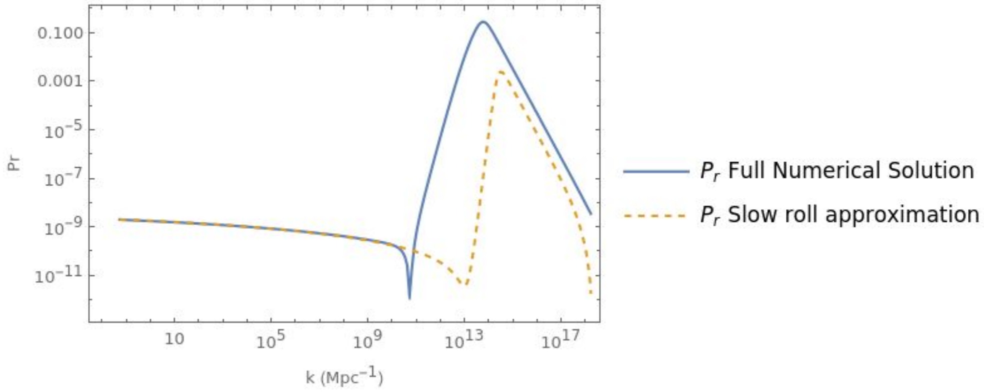
$$P_R = \frac{k^3}{2\,\pi^2}|R_k|^2,\; R_k = \Psi + \frac{\delta x}{dx/dN}$$

Gravitational Waves Spectrum
Beyond Slow-Roll

- Peak gets further enhanced when compared to slow-roll approximation
- Peak is also wider
- Using

$$1\,\mathrm{Mpc}^{-1} = 0.97154 \times 10^{-14}\mathrm{s}$$
$$k = 2\pi f$$

we expect the peak of GW spectrum to be around 0.1 Hz -> Space-based interferometers



Gravitational Waves Spectrum
Final Computation

- Having P_R, we can finally compute the (present day) GW spectrum. Following [Espinosa, et al 1804.07732, SS 21, 22]

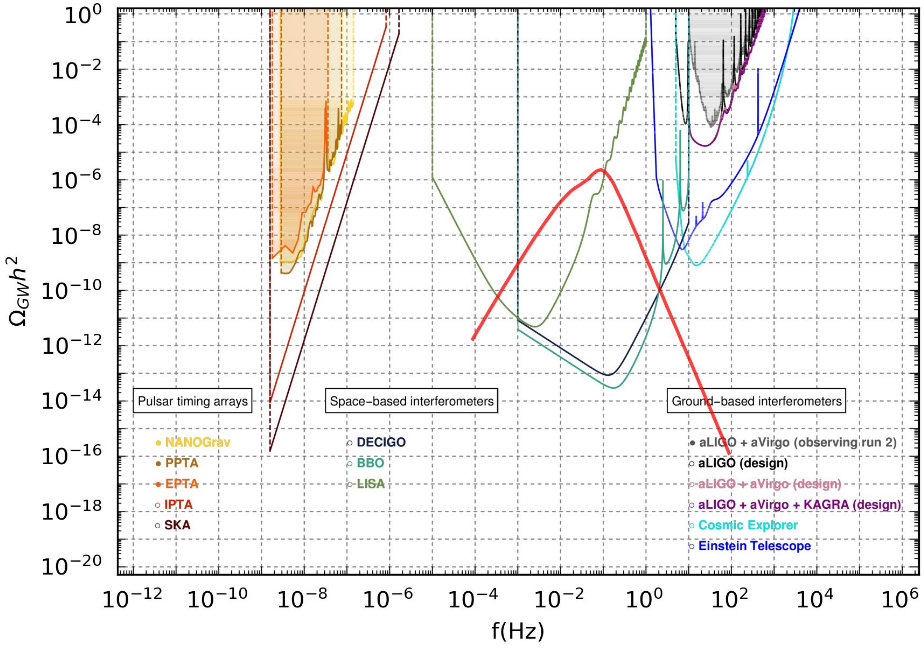
$$h^2\Omega_{GW}(k) = \frac{\Omega_r}{36}\int_0^{1/\sqrt{3}} \mathrm{d}d \int_{1/\sqrt{3}}^{\infty} \mathrm{d}s \left[\frac{(s^2-1/3)(d^2-1/3)}{s^2-d^2}\right]^2 P_R(k\,X)P_R(k\,Y)\left(I_c^2+I_s^2\right)$$

$$X = \frac{\sqrt{3}}{2}(s+d),\; Y = \frac{\sqrt{3}}{2}(s-d)$$

$$I_c = -36\,\pi\frac{\left(s^2+d^2-2\right)^2}{\left(s^2-d^2\right)^3}\Theta(s-1)$$

$$I_s = -36\frac{\left(s^2+d^2-2\right)^2}{\left(s^2-d^2\right)^2}\left[\frac{\left(s^2+d^2-2\right)}{\left(s^2-d^2\right)}\log\left|\frac{d^2-1}{s^2-1}\right|+2\right]$$

Gravitational Waves Spectrum
Present Day Spectrum



With
$m_{3/2} = 2791.15\,\mathrm{GeV}$

Sensitivity curves from [Schmitz 2002.04615]

4) Conclusions

Conclusions
And Future Work

- We have presented a No-Scale SUGRA inflationary potential which has a Starobinsky limit
  - We extend the Wess-Zumino superpotential to include a Polonyi term => SUSY breaking and gravitino mass after inflation
  - We considered a non-minimal Kahler potential which imprints a kink on the inflaton potential => Production of GW
- We found the parameter space region with GW production to be very restricted, but found points with the aid of an Artificial Intelligence search algorithm
  - GW spectrum detectable by space-based interferometers
  - SUSY breaking has a natural upper bound for successful inflation $m_{3/2} < 1000$ TeV
  - Gravitino mass and GW spectrum are mostly uncorrelated
- Future work
  - Study Primordial Black Hole production in the model
  - Understand if we can improve the AI search to fine tune the relevant parameters

Thanks!

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