

Woes of the “minimal” non-supersymmetric SO(10) GUT

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Unification of Fundamental Interactions

Introduction

The latest (and final?!) chapter in the long story of the “minimal” non-SUSY $\text{SO}(10)$ GUT: $45 + 126 + 10_{\mathbb{C}}$.

- Woe #1: tree-level tachyoncities
[curable at 1-loop]
- Woe #2: Higgs fine-tuning is unsuitable for fermion fit
[incurable in perturbative regions]

The model: $45 + 126 + 10_{\mathbb{C}}$

- non-SUSY renormalizable $\text{SO}(10)$ GUT
- Field content:
 - fermions: $3 \times 16_F$
 - scalars: $45 + 126 + 10_{\mathbb{C}}$

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 \rightarrow scalars: $45 + 126 + 10_{\mathbb{C}}$
- Motivation: “**minimality**” and “**calculability**” of proton decay:
no $(45_G \cdot 45_G \cdot S)/M_{Pl}$ operator, GUT scale more robust

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- **Breaking sector:** 2-stage breaking (GUT & seesaw scale)

$$\text{SO}(10) \xrightarrow{\langle 45 \rangle} G \xrightarrow{\langle 126 \rangle} \text{SU}(3) \times \text{SU}(2) \times \text{U}(1)$$

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- **Yukawa sector:** realistic, $Y_{10}, \tilde{Y}_{10}, Y_{126}$ symmetric

$$\mathcal{L}_{Yuk} = 16_F 16_F (Y_{10} \textcolor{blue}{10} + \tilde{Y}_{10} \textcolor{blue}{10}^* + Y_{126} \textcolor{blue}{126}^*)$$

Symmetry breaking

- The $45 + 126$ Higgs model

$$\text{SO}(10) \xrightarrow{\langle 45 \rangle} G \xrightarrow{\langle 126 \rangle} \text{SU}(3) \times \text{SU}(2) \times \text{U}(1)$$

- 3 SM-singlet VEVs: $\omega_{BL}, \omega_R \in 45$ and $\sigma \in 126$
- What are the possibilities for G ?

$\langle 45 \rangle$ direction

$$\omega_{BL} \approx \omega_R$$

$$\omega_{BL} \approx -\omega_R$$

$$\omega_{BL} \sim \omega_R$$

$$\omega_R \ll \omega_{BL}$$

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|---------------------------------|-----------------------|
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| $\omega_{BL} \approx -\omega_R$ | $5' 1'$ |
| $\omega_{BL} \sim \omega_R$ | $3_c 2_L 1_R 1_{B-L}$ |
| $\omega_R \ll \omega_{BL}$ | $3_c 2_L 2_R 1_{B-L}$ |
| $\omega_{BL} \ll \omega_R$ | $4_C 2_L 1_R$ |

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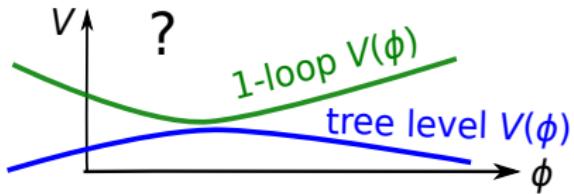
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| $\omega_{BL} \sim \omega_R$ | $3_c 2_L 1_R 1_{B-L}$ | V ? |
| $\omega_R \ll \omega_{BL}$ | $3_c 2_L 2_R 1_{B-L}$ | 1-loop $V(\phi)$ |
| $\omega_{BL} \ll \omega_R$ | $4_C 2_L 1_R$ | tree level $V(\phi)$ |



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| $\omega_{BL} \sim \omega_R$ | $3_c 2_L 1_R 1_{B-L}$ | ✗ (non-perturbative $\frac{\omega_{BL}\omega_R}{ \sigma ^2}$) |
| $\omega_R \ll \omega_{BL}$ | $3_c 2_L 2_R 1_{B-L}$ | ✓ ⊕ ($M_{GUT} \approx 10^{18}$ GeV) |
| $\omega_{BL} \ll \omega_R$ | $4_C 2_L 1_R$ | ✓ ⊕ ($M_{GUT} \approx 10^{15}$ GeV) |

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leads to

$$M_U = Y_{10} v_{10}^u + \tilde{Y}_{10} v_{10}^d{}^* + Y_{126} v_{126}^u,$$

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- (a) SM mass fit known to work already with just Y_{10} , Y_{126}
- (b) SM Higgs must be **admixture** of $\textcolor{blue}{10}$ - and $\textcolor{blue}{126}$ -doublets

Doublet fine-tuning

- Doublet mass matrix:

$$M_{(1,2,\pm 1/2)}^2 = \begin{pmatrix} M_{126}^2 & M_{\text{mix}}^2 \\ M_{\text{mix}}^{2\dagger} & M_{10}^2 \end{pmatrix}$$

SM Higgs obtained by fine-tuning: $\det M_{(1,2,\pm 1/2)}^2 = 0$.

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- Way out: tune down **only to next-order corrections**

→ What happens at 1-loop?

→ If 2-loop order not computed: necessary but not sufficient condition

Numerical analysis — setup

For a point in parameter space:

- (1) Compute **1-loop** and **tree-level** spectrum

[interesting problem on its own, but technical; will not discuss further]

- (2) **Viability** considerations:

- (2a) **non-tachyonicity** for all states

- (2b) **unification** of gauge couplings g_i @ **2-loop**

- (2c) **perturbativity** (degree of arbitrariness in definition)

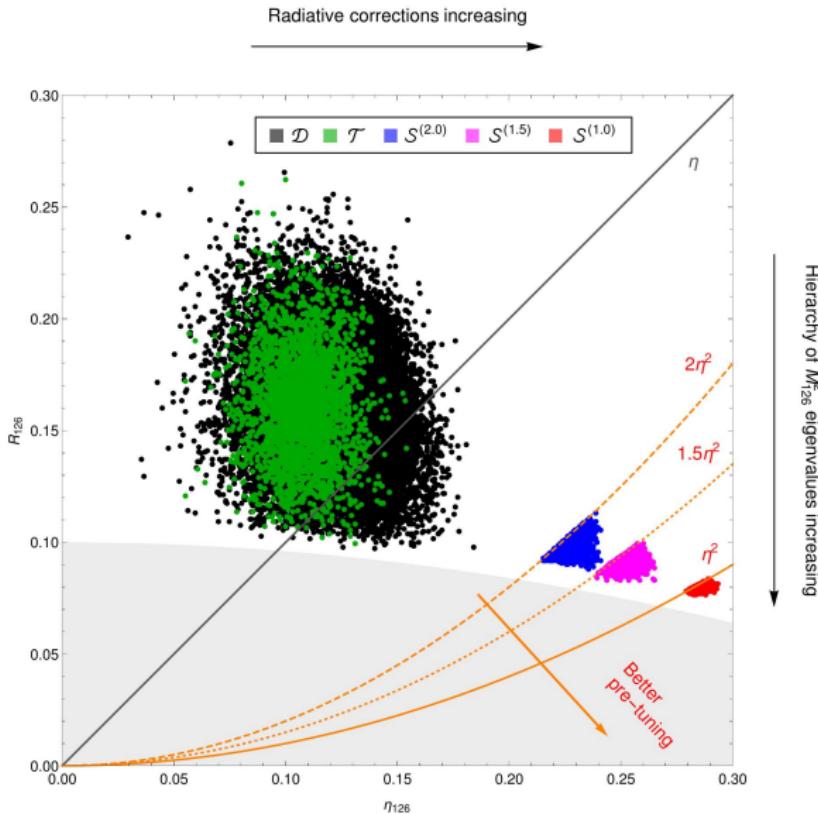
→ corrections to masses under control: “ $|\delta m^2/m^2| < 1$ ”

→ RGE for λ under control: up to $M_{GUT} \cdot 10^{+0.5}$

- (3) Doublet **fine-tuning**: considered through quantities

$$R := \frac{m_{(1)}^2}{m_0^2}, \quad \eta := \frac{\delta m_{(1)}^2}{m_0^2}, \quad \text{where } m_0^2 \text{ "typical" tree-level mass.}$$

Fine-tuning vs perturbativity I



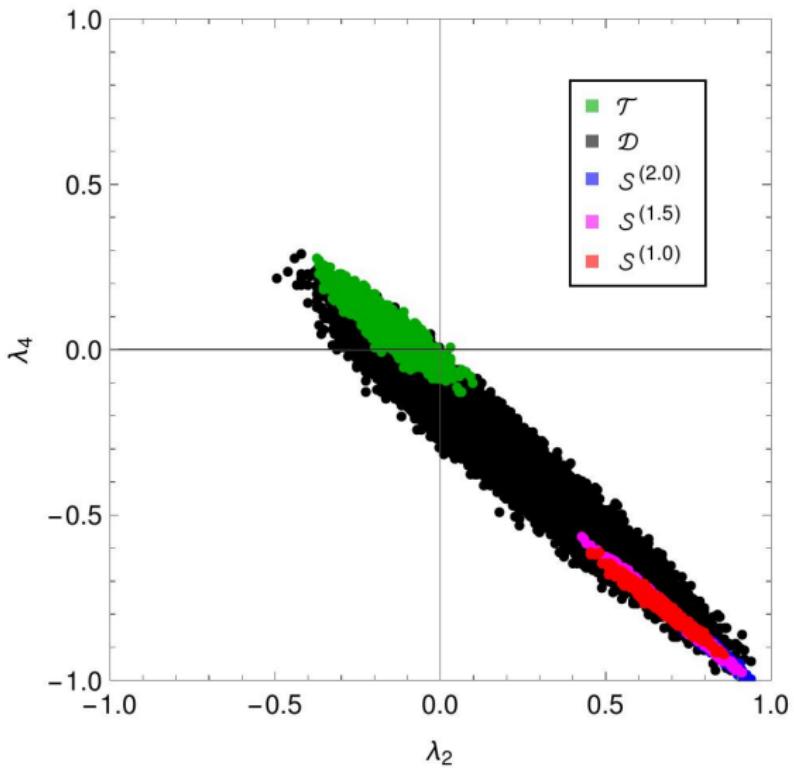
Datasets of viable points:

- \mathcal{D} : generic point
- \mathcal{T} : enhanced RGE perturbativity $10^{\pm 1} M_{GUT}$
- \mathcal{S} : M_{126}^2 tuned to 2-loop expectation

Plot: η vs R

Tuning happens at the expense of perturbativity!

Fine-tuning vs perturbativity II



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Regions with
enhanced perturbativity
and doublet tuning
incompatible!

(best demonstrated in the
 λ_2 - λ_4 parameter plane)

Summary

SO(10) GUT model with scalar sector $45 + 126 + 10_C$

→ Tricky: 1-loop is first consistent perturbative order

- Suitable vacuum only at 1-loop
 - Small patch of parameter space viable
 - Doublet fine-tuning (1-loop): clashes with perturbativity

→ Model perturbatively not viable

→ A more general message:

large # of fields \Rightarrow perturbativity vs other demands
(possible clash)

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Thank you for your attention!

Backup: technical challenges

- A lot of particles: scalar mass matrix $M_S^2(\phi)$ in V_1 is 297×297 in Higgs model, 317×317 in full theory
- A lot of parameters: the scalar potential written schematically is

$$\begin{aligned} V(45, 126) = & \mu^2 45^2 + a 45^4 + \\ & + \nu^2 |126|^2 + \lambda |126|^4 + \eta 126^4 + \tau 45 \cdot |126|^2 + \\ & + (\alpha, \beta) 45^2 \cdot |126|^2 + \gamma 45^2 \cdot 126^2 + h.c., \end{aligned}$$

$$\begin{aligned} V(45, 126, 10) = & V(45, 126) + \xi^2 10^2 + h 10^4 \\ & + \kappa 10^2 45^2 + \zeta 45^2 \cdot 126 \cdot 10 + \rho 10^2 |126|^2 + \\ & + \rho' 10^2 126^2 + \varphi |126|^2 \cdot 126 \cdot 10 + h.c. \end{aligned}$$

(possibly >1 independent contraction, for brevity 10^* was written as 10)

- Parameters in full theory:
 $(15 \mathbb{R} + 14 \mathbb{C})$ dimensionless, $(5 \mathbb{R} + 1 \mathbb{C})$ massive