



The 30th International Conference on Supersymmetry  
and Unification of Fundamental Interactions

2023-07-17  
~ 07-21

# Complementary Test of GUTs in Neutrino and GW Observatories

Ye-Ling Zhou (HIAS) 2023-07-19

Based on works with B. Fu, S.F. King, L. Marsili, S. Pascoli and J. Turner



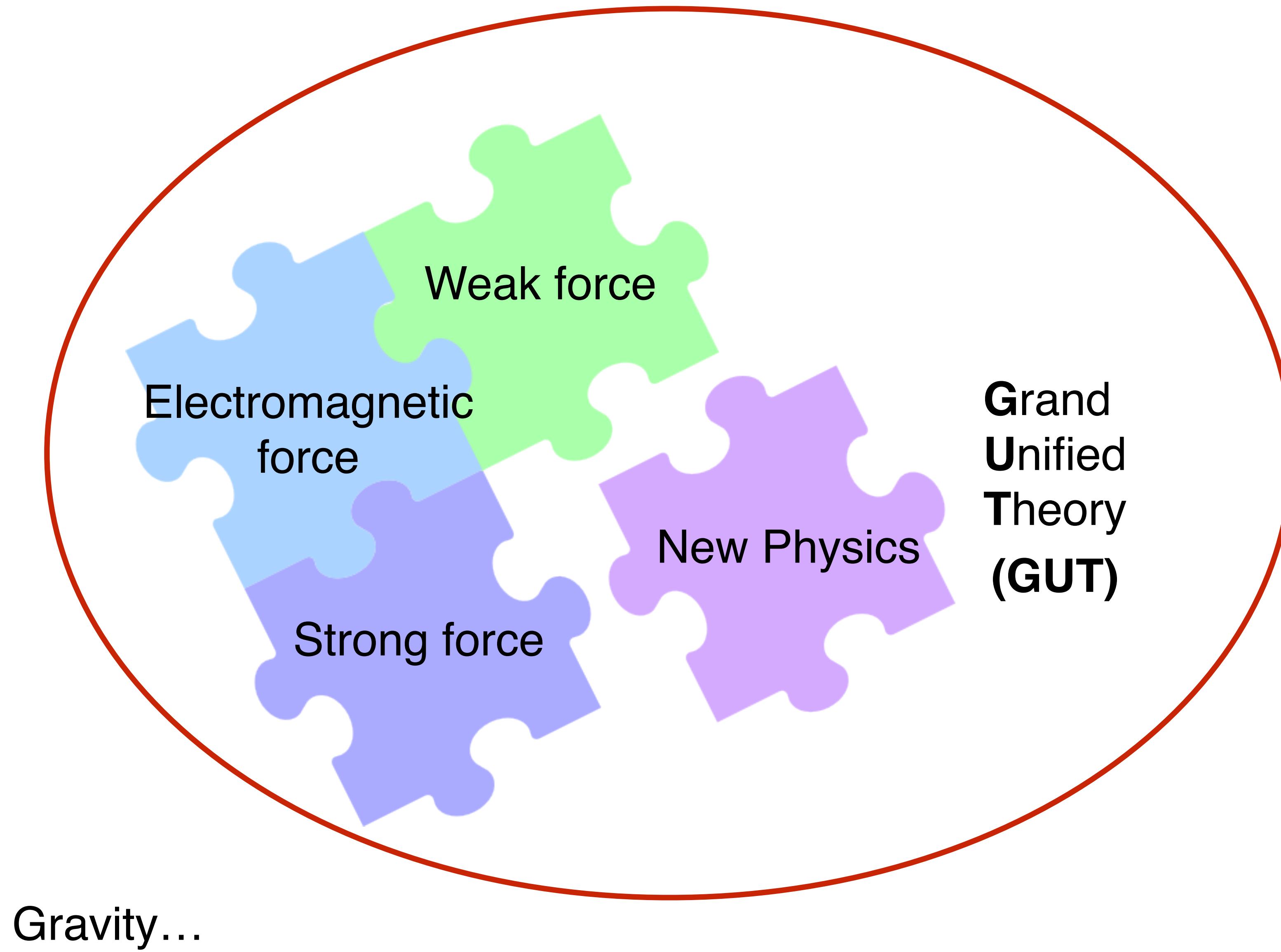
國科大杭州高學研究院  
Hangzhou Institute for Advanced Study, UCAS



ICTP-AP  
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for Theoretical Physics Asia-Pacific  
国际理论物理中心-亚太地区

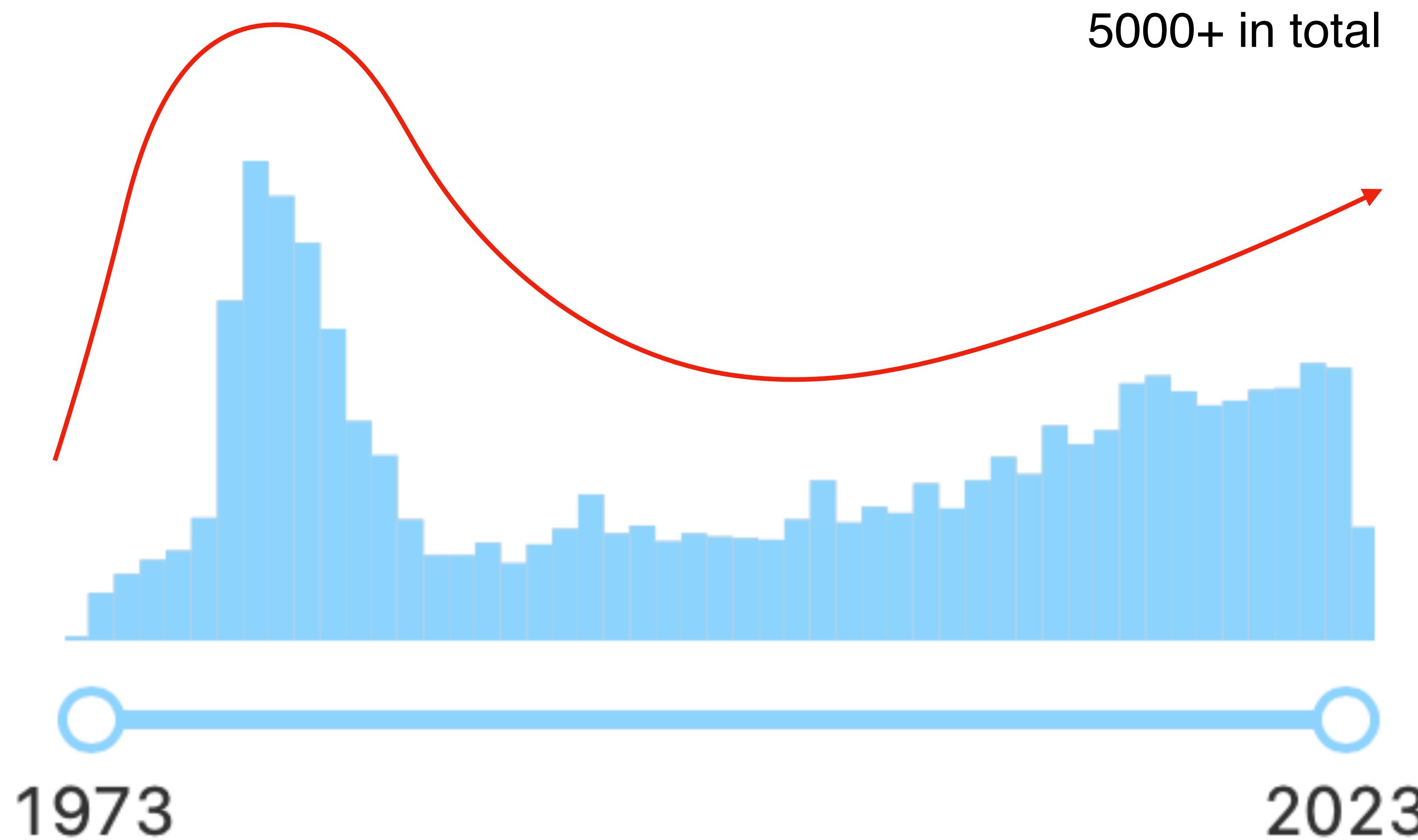
# Framework of GUTs

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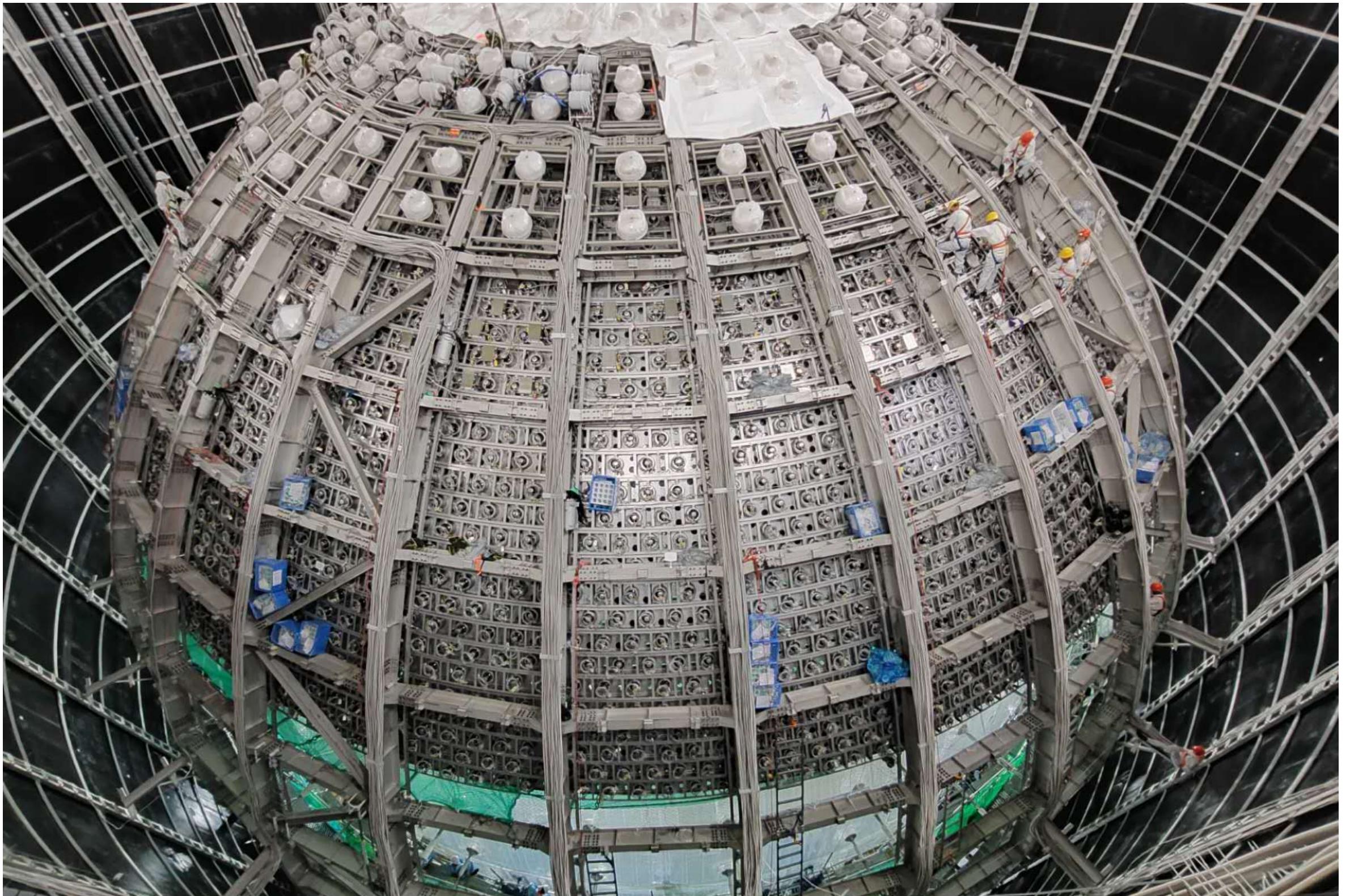
# Why do we study GUTs today?

# # of citations of Georgi-Glashow model from its born to yesterday



# Reason I: upcoming neutrino experiments

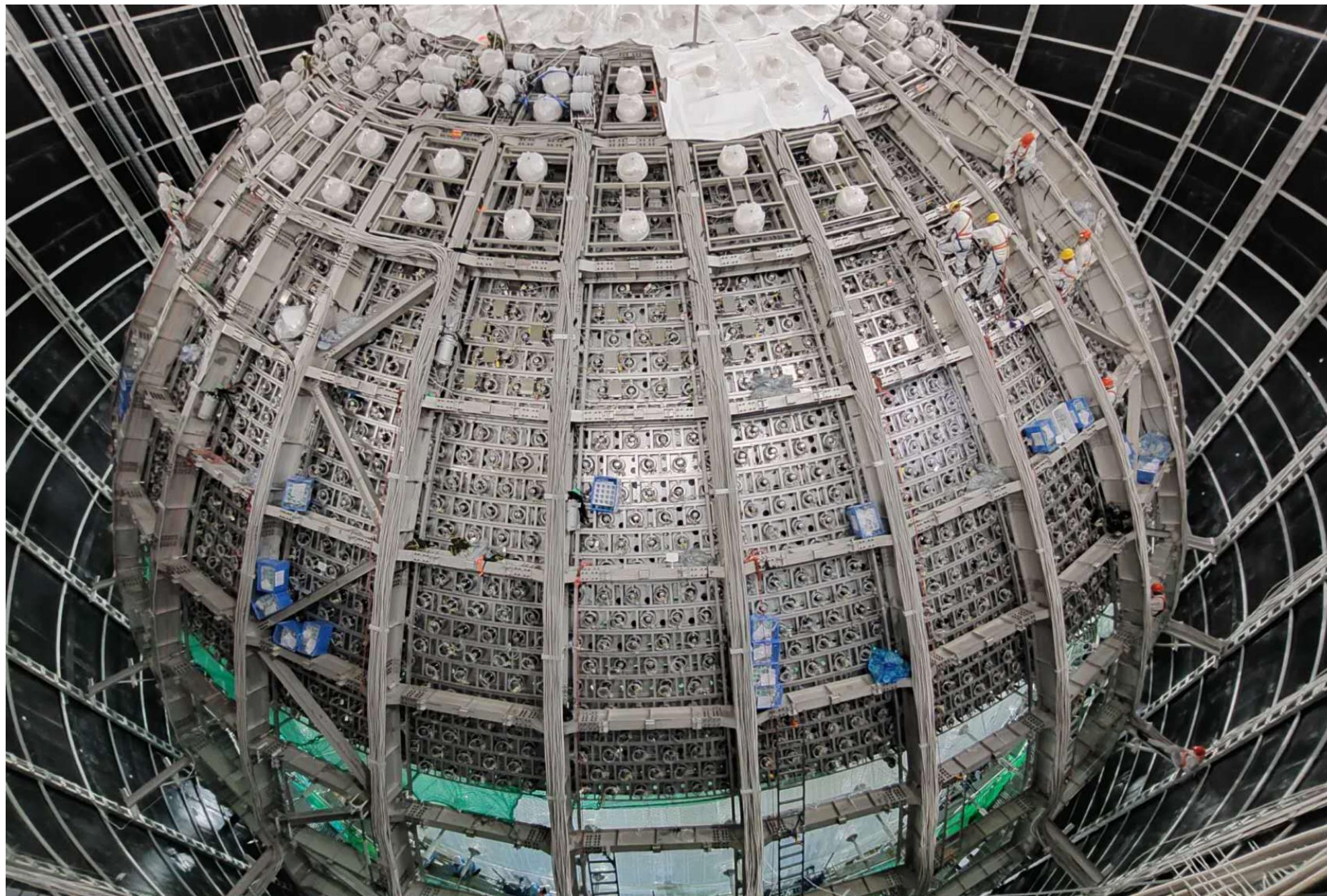
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JUNO, run next year

20kt FV  $\sim 7 \times 10^{33}$  proton

# Reason I: upcoming neutrino experiments

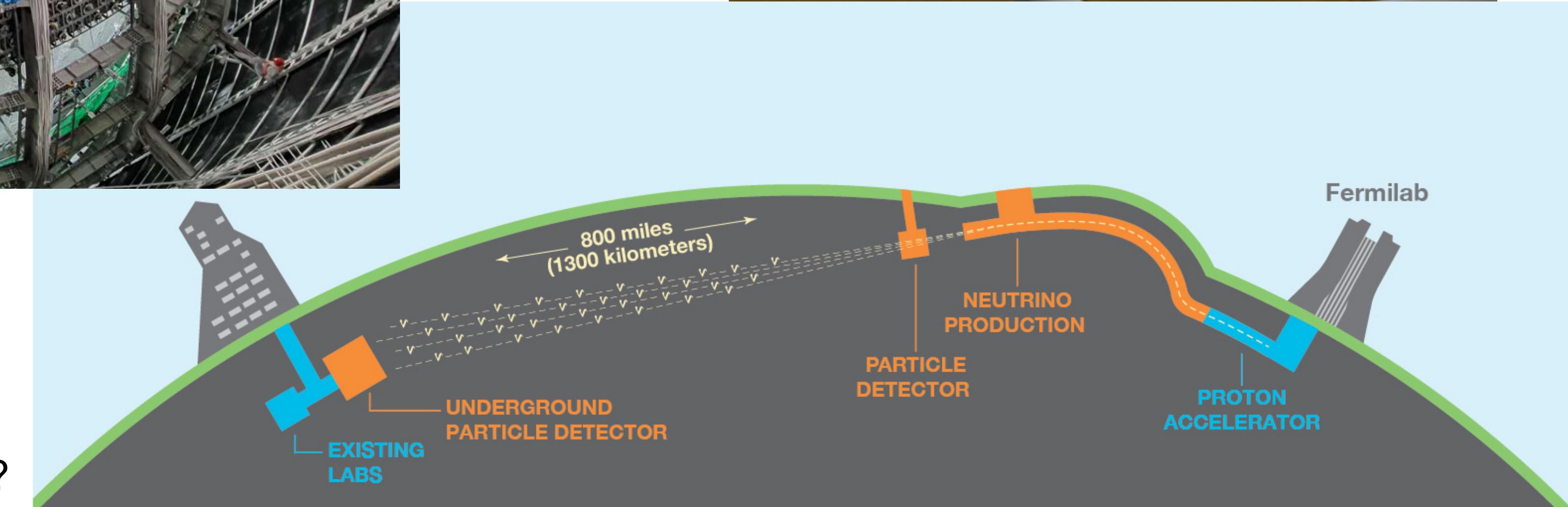
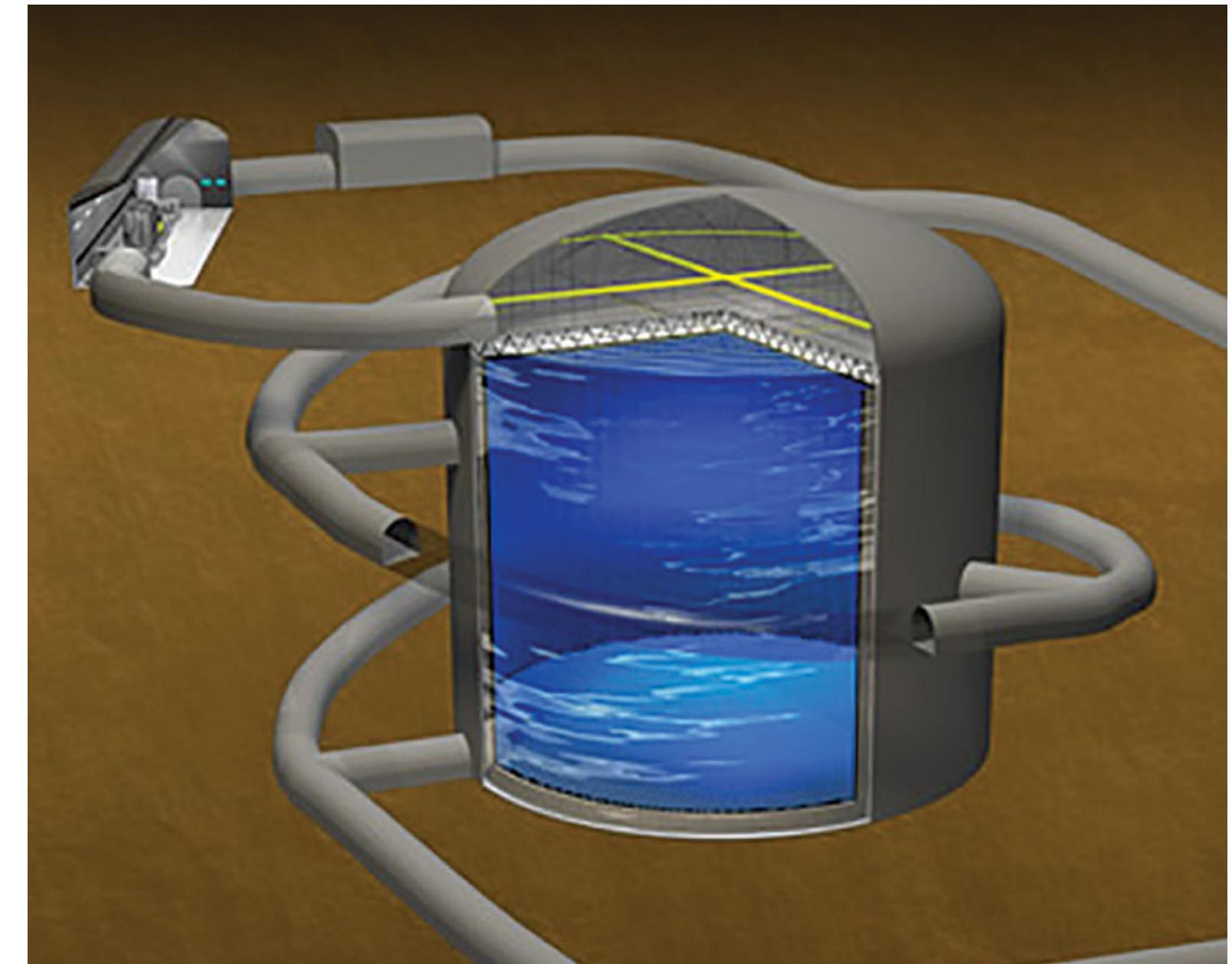


JUNO, run next year

20kt FV  $\sim 7 \times 10^{33}$  proton

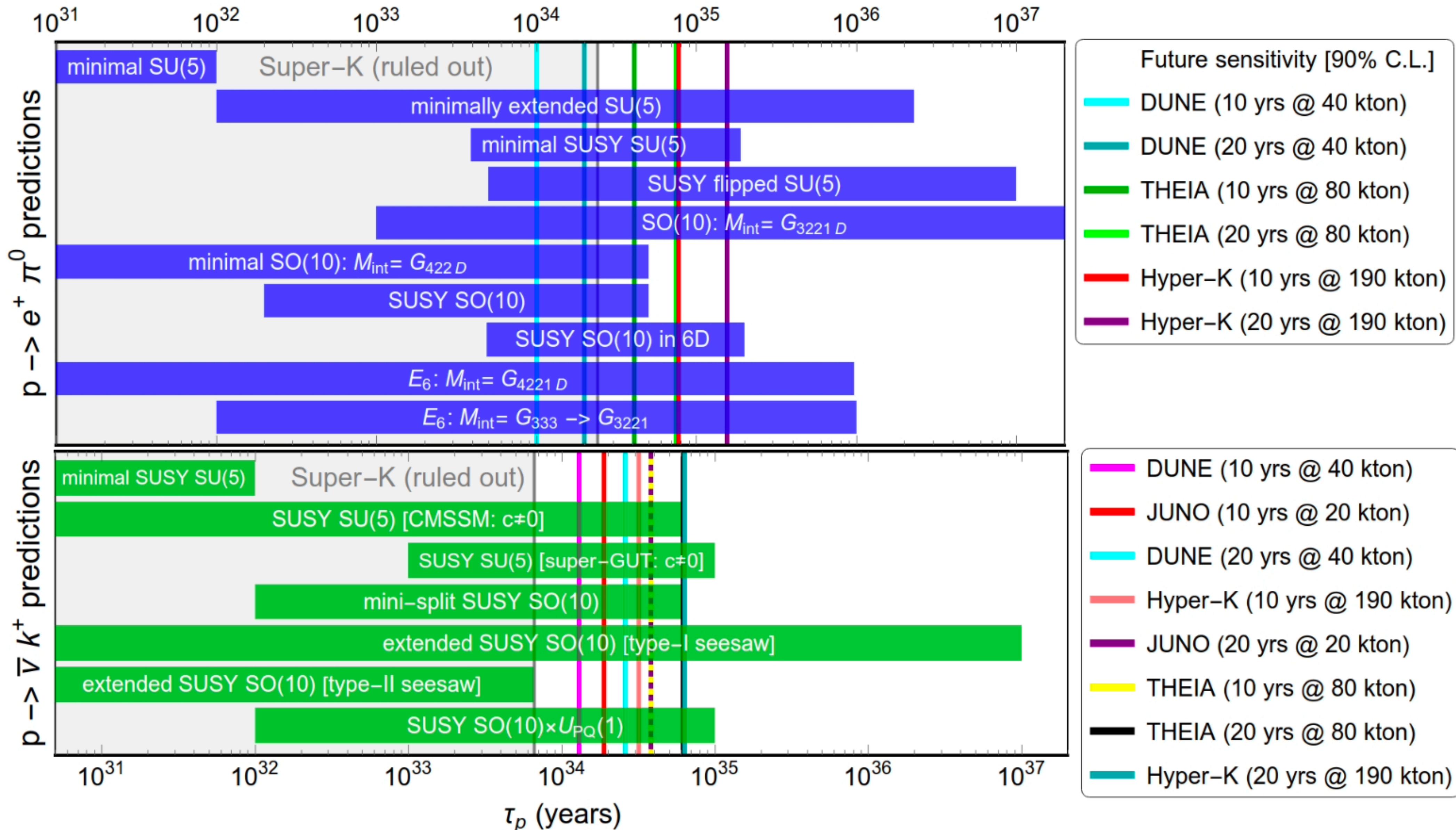
DUNE, run in 2030?

Hyper-K  
188 kt FV  
expected to  
run in 2027



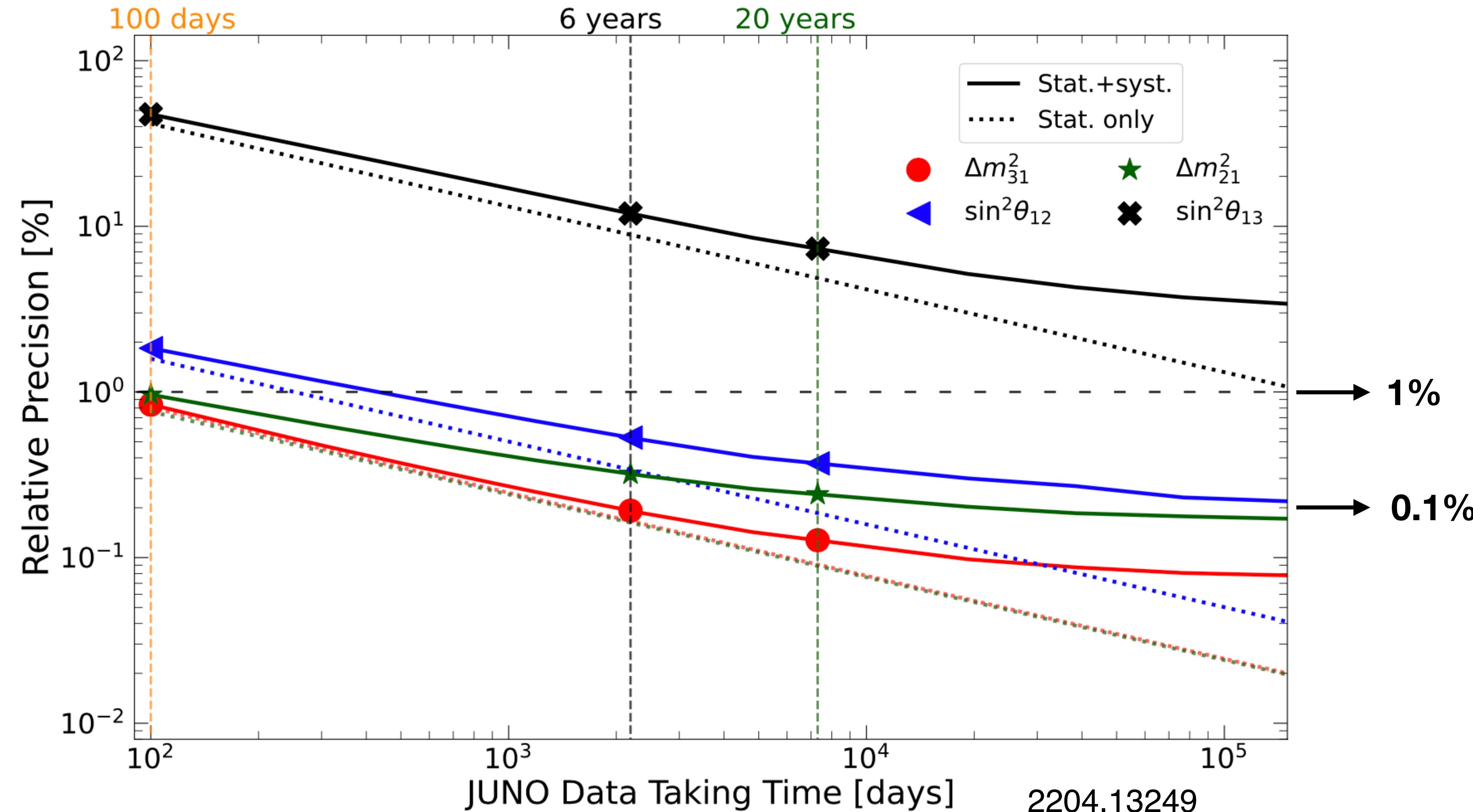
# Reason I: upcoming neutrino experiments

Snowmass, 2203.08771

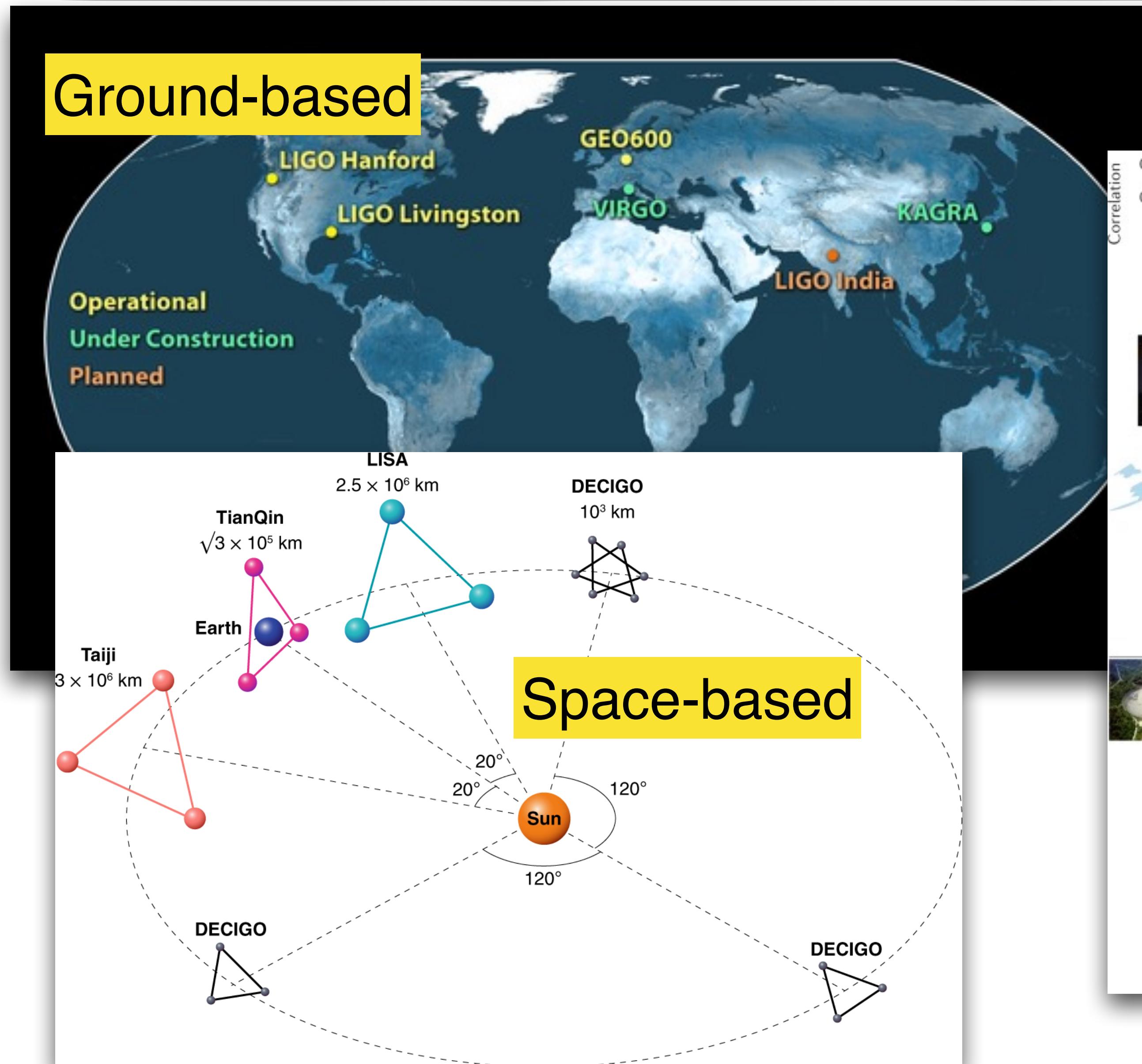


# Reason I: upcoming neutrino experiments

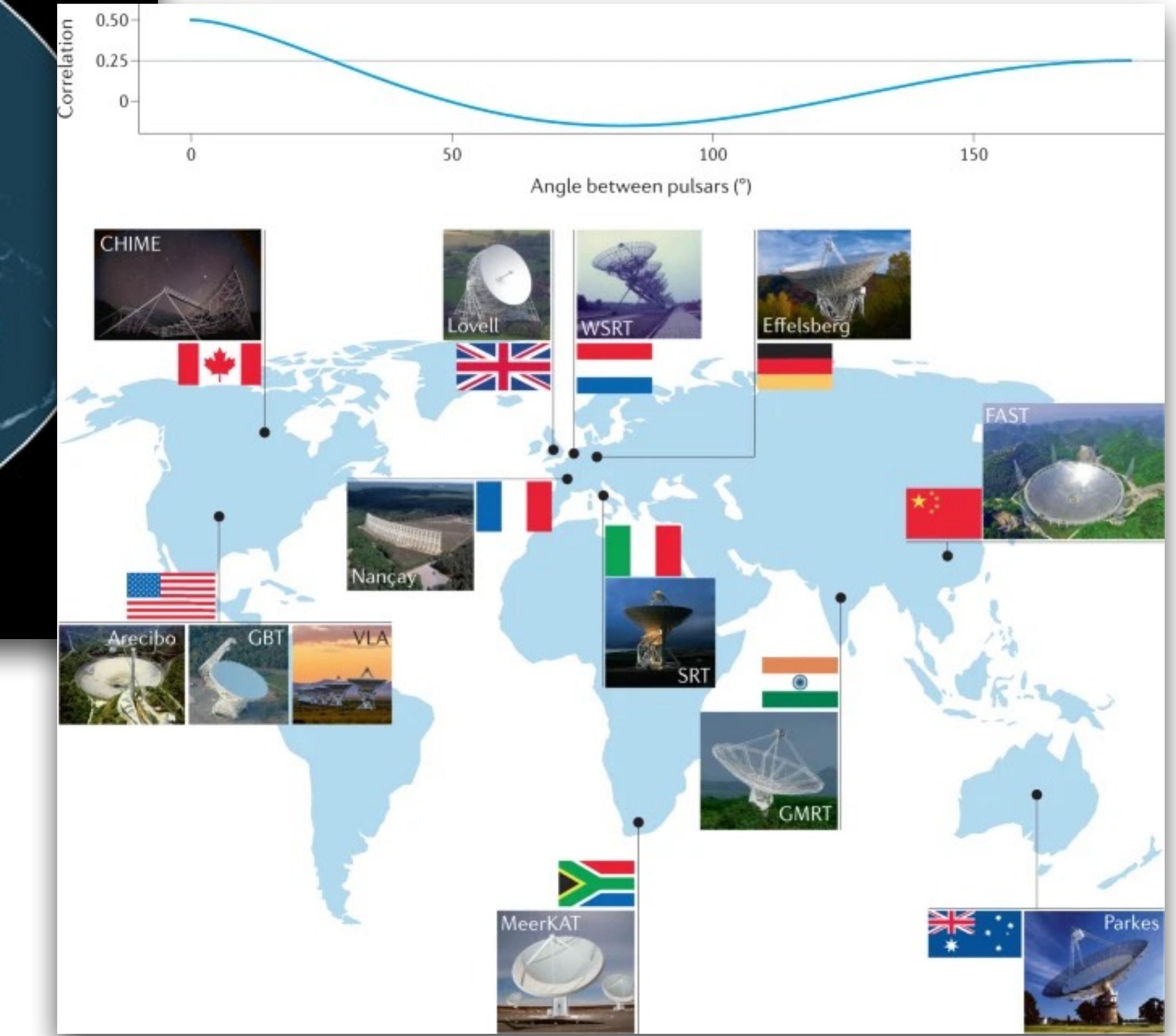
An era of precision measurement of neutrino oscillations!



# Reason II: Undergoing and upcoming GW measurements

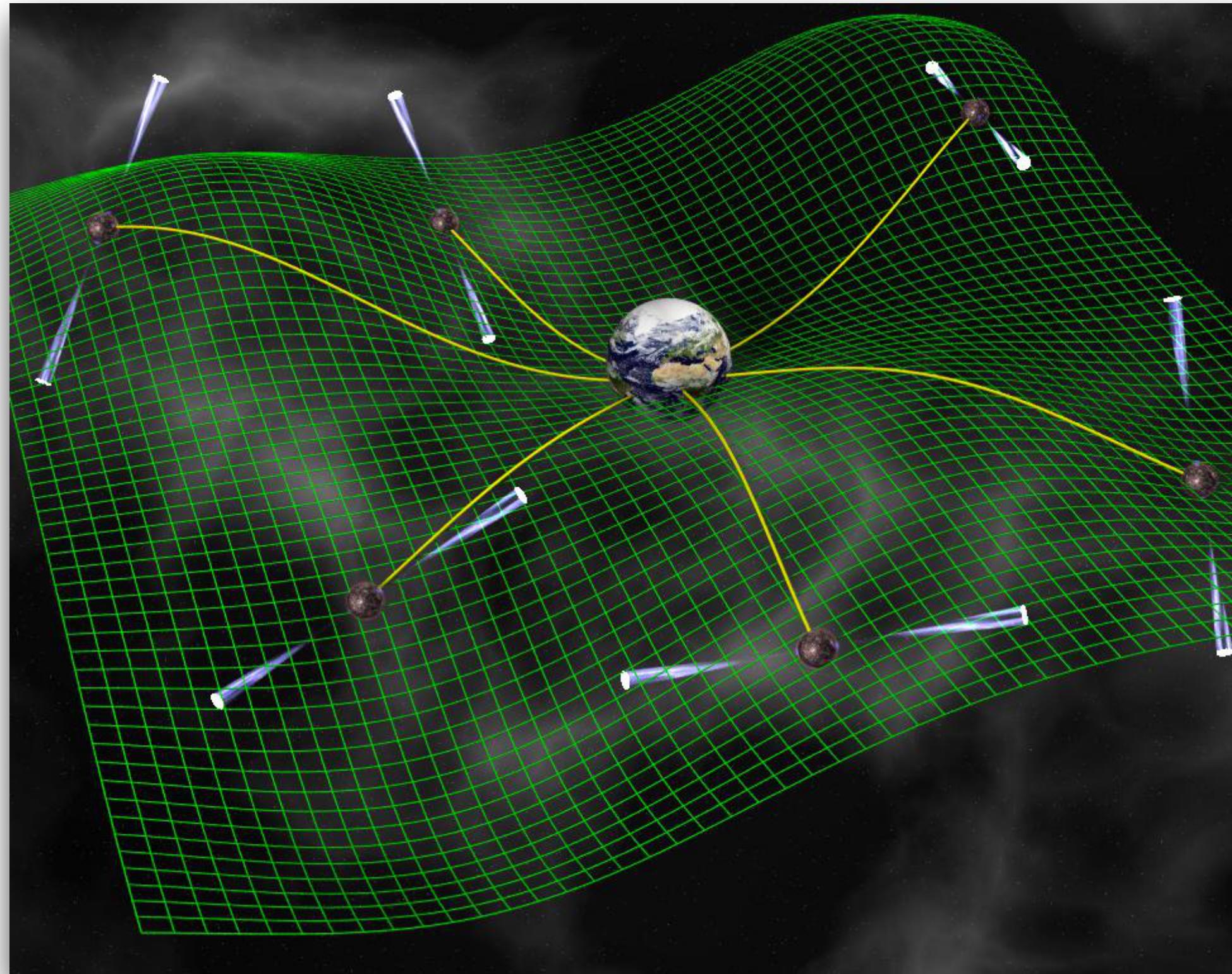


## Pulsar-Timing Arrays (PTAs)



# Reason II: Undergoing and upcoming GW measurements

On 28 Jun 2023



The **NANOGrav** 15 yr Data Set: Evidence for a  
Gravitational-wave Background  
2306.16213

The second data release from the **European Pulsar  
Timing Array III**. Search for gravitational wave signals  
2306.16214

Search for an isotropic gravitational-wave background with  
the **Parkes Pulsar Timing Array**  
2306.16215

Searching for the nano-Hertz stochastic gravitational wave  
background with the **Chinese Pulsar Timing Array** Data  
Release I  
2306.16216

If cosmic GW background is observed, then what is the origin?

# Roads to GUTs

- Unification of symmetries

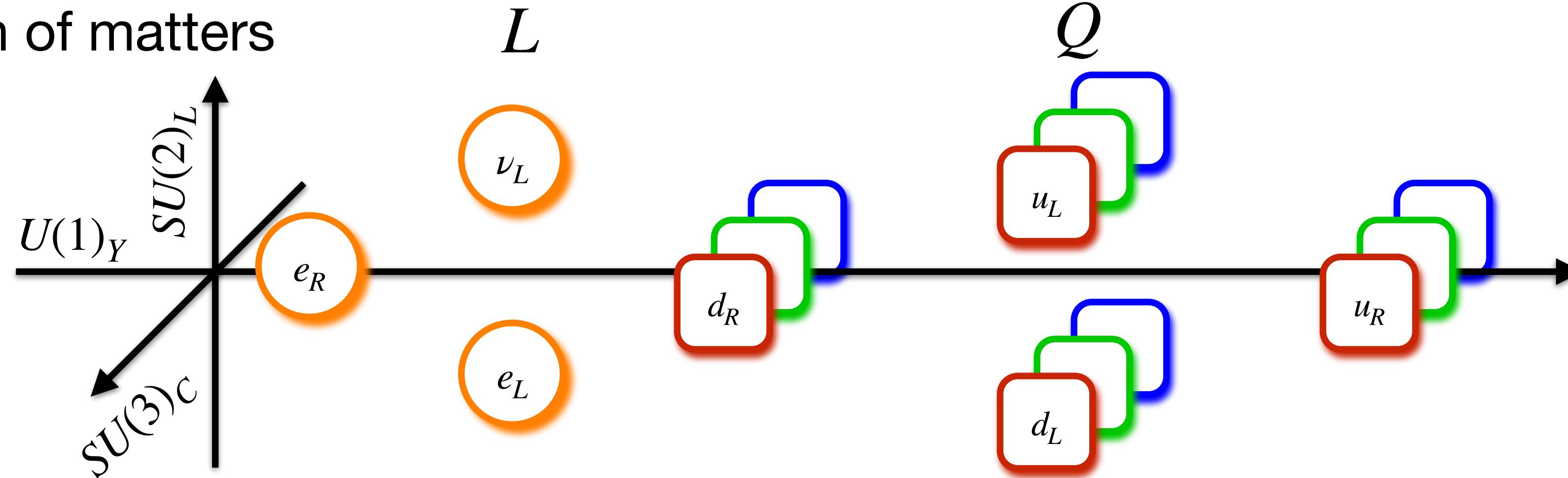
$$G_{\text{GUT}} \supset G_{\text{SM}} = SU(3)_C \times SU(2)_L \times U(1)_Y$$

- Unification of couplings

$$\begin{array}{c} \downarrow \\ g_3 = g_2 = g_1 \end{array} \xrightarrow{\text{EW}} \text{up to a loop correction factor}$$

The scale where three gauge couplings are unified, denoted as  $M_{\text{GUT}}$  in this talk

- Unification of matters



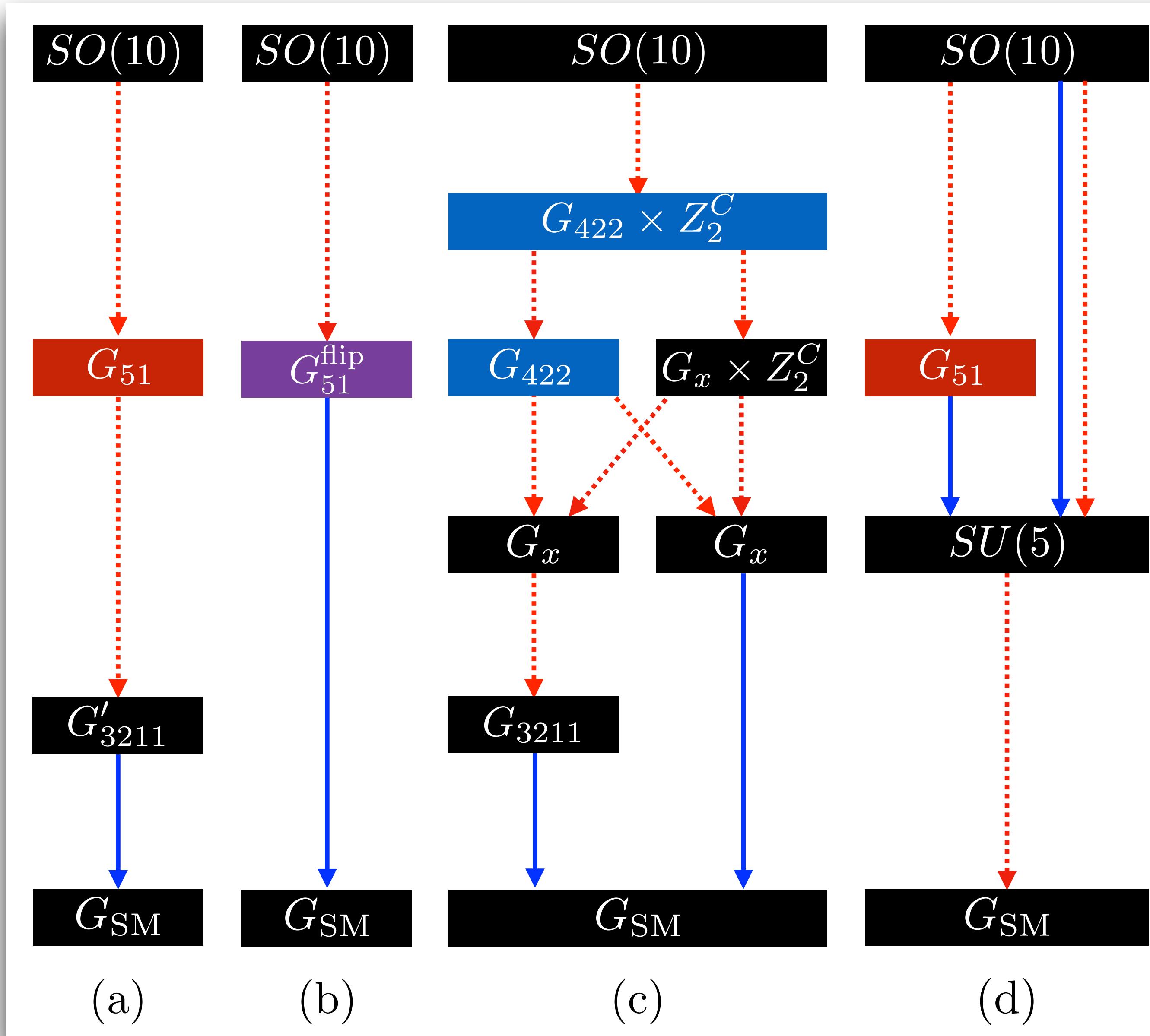
Weak hypercharge:  $Y = -1$     $Y = -\frac{1}{2}$     $Y = -\frac{1}{3}$

$Y = \frac{1}{6}$     $Y = \frac{2}{3}$

# Roads to GUTs

- Georgi-Glashow (1972)  $SU(5)$  Quarks & leptons  $\sim \bar{\textbf{5}} + \textbf{10}$
  - More realistic  $SU(5)$  Talks by Ilja Doršner, Kevin Hinze, Jonathan Steiner, Shihwen Hor
  - $SU(5) \times U(1)_{B-L}$   $\bar{\textbf{5}} + \textbf{10} + \textbf{1}, \nu_R \sim \textbf{1}$
  - Flipped  $SU(5) \times U(1)_X$   $u \leftrightarrow d, \nu \leftrightarrow e$  Talk by George Leontaris  
Rujula, Georgi, Glashow (1980); Barr,(1982); Derendinger, Kim, Nanopoulos (1984); Antoniadis, Ellis, Hagelin, Nanopoulos (1989)
  - Pati-Salam (1973),  $SU(4)_c \times SU(2)_L \times SU(2)_R = G_{422}$   
 $(\mathbf{4}, \mathbf{2}, \mathbf{1}) : \psi_L = \begin{pmatrix} u^1 & u^2 & u^3 & \nu \\ d^1 & d^2 & d^3 & e \end{pmatrix}_L, \quad (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}) : \psi_R = \begin{pmatrix} u^1 & u^2 & u^3 & \nu \\ d^1 & d^2 & d^3 & e \end{pmatrix}_R^c$
  - $SO(10)$  GUTs Fritzsch, Minkowski (1975) Talk by Vasja Susič, Shaikh Saad  
 $\mathbf{16} = \bar{\mathbf{5}} + \mathbf{10} + \mathbf{1} = (\mathbf{4}, \mathbf{2}, \mathbf{1}) + (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$  More on Friday by Luca Marsili, Ruiwen Ouyang...  
 $SO(10) \quad SU(5) \quad SU(4)_c \times SU(2)_L \times SU(2)_R$

# SO(10) breaking chains



$$G_{422} = SU(4)_C \times SU(2)_L \times SU(2)_R$$

$$G_{51} = SU(5) \times U(1)_X$$

$$G_{51}^{\text{flip}} = SU(5)_{\text{flip}} \times U(1)_{\text{flip}}$$

$$Z_2^C: \quad \psi_L \leftrightarrow \psi_R^c$$

$$G_x = G_{421} \text{ or } G_{3221}$$

$$G_{3221} = SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

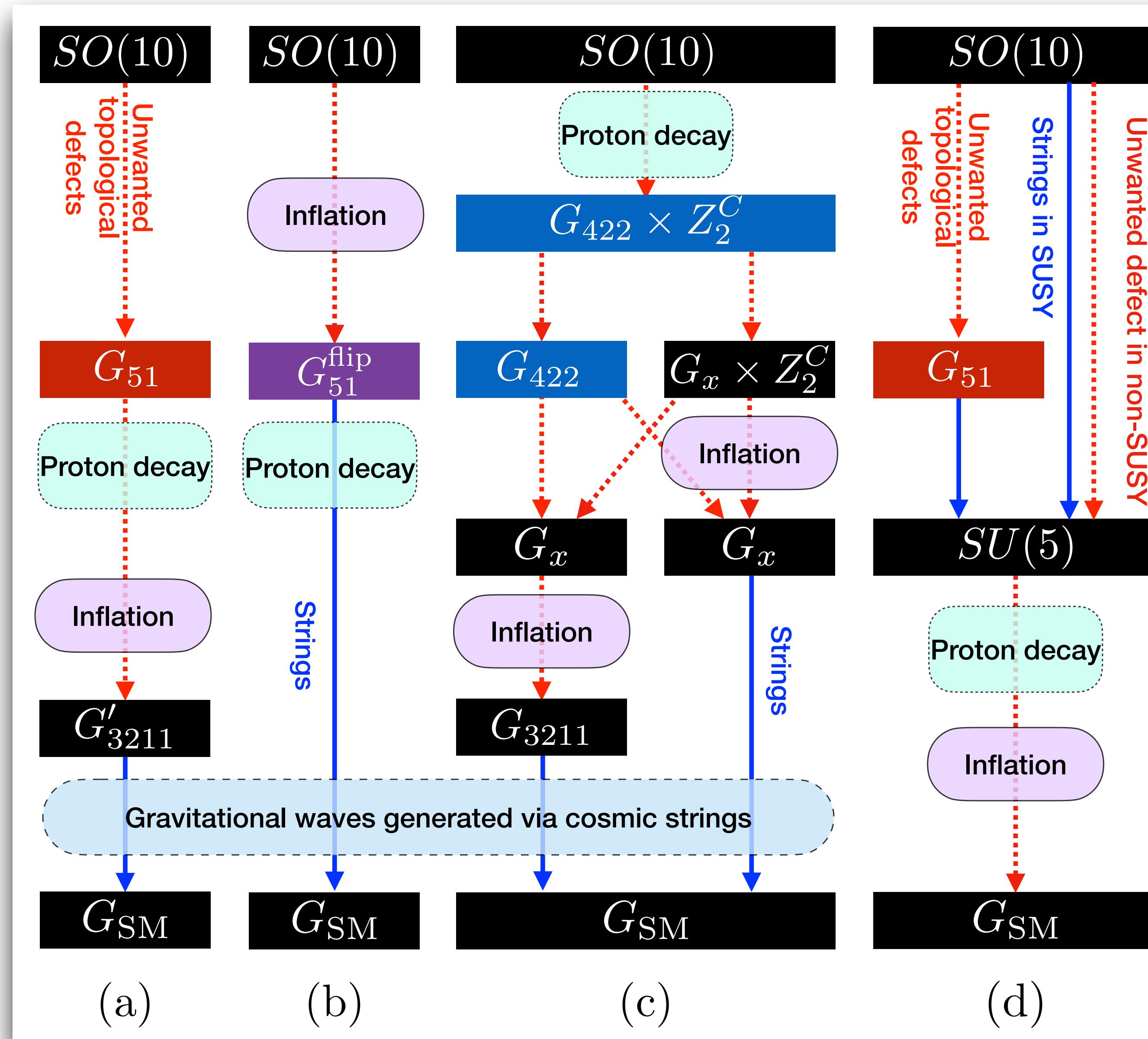
$$G_{421} = SU(4)_C \times SU(2)_L \times U(1)_Y$$

$$G_{3211} = SU(3)_C \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$$

$$G'_{3211} = SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$$

$$G_{\text{SM}} = SU(3)_C \times SU(2)_L \times U(1)_Y$$

# SO(10) phenos



**Unwanted topological defects:  
monopoles and domain walls**

In any breaking chains, inflation has to be introduced to inflate unwanted defects

$$G_{422} = SU(4)_C \times SU(2)_L \times SU(2)_R$$

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$$G_x = G_{421} \text{ or } G_{3221}$$

$$G_{3221} = SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

$$G_{421} = SU(4)_C \times SU(2)_L \times U(1)_Y$$

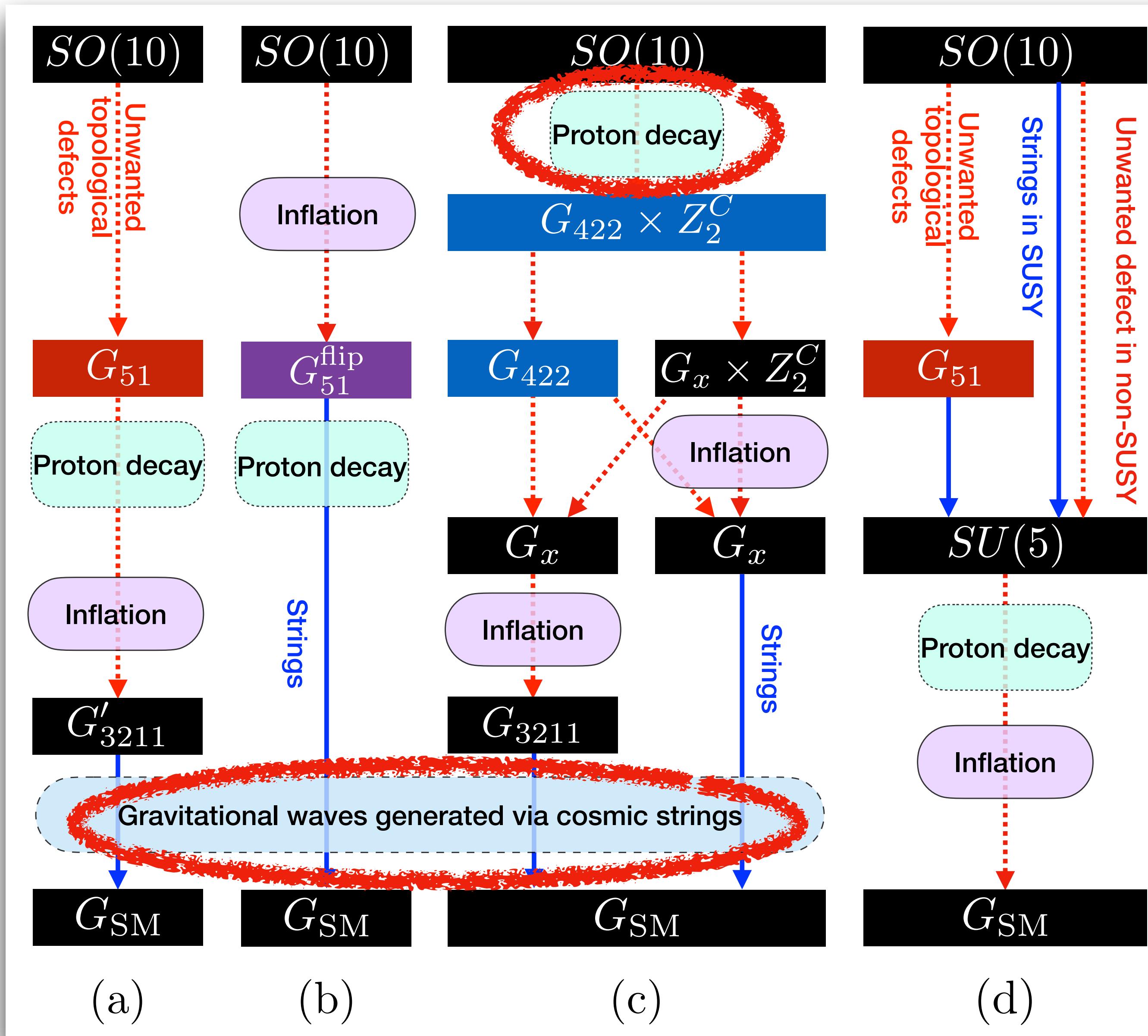
$$G_{3211} = SU(3)_C \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$$

$$G'_{3211} = SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$$

$$G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$$

# SO(10) phenos

## Fermion masses and mixing



## Unwanted topological defects: monopoles and domain walls

In any breaking chains, inflation has to be introduced to inflate unwanted defects

$$G_{422} = SU(4)_C \times SU(2)_L \times SU(2)_R$$

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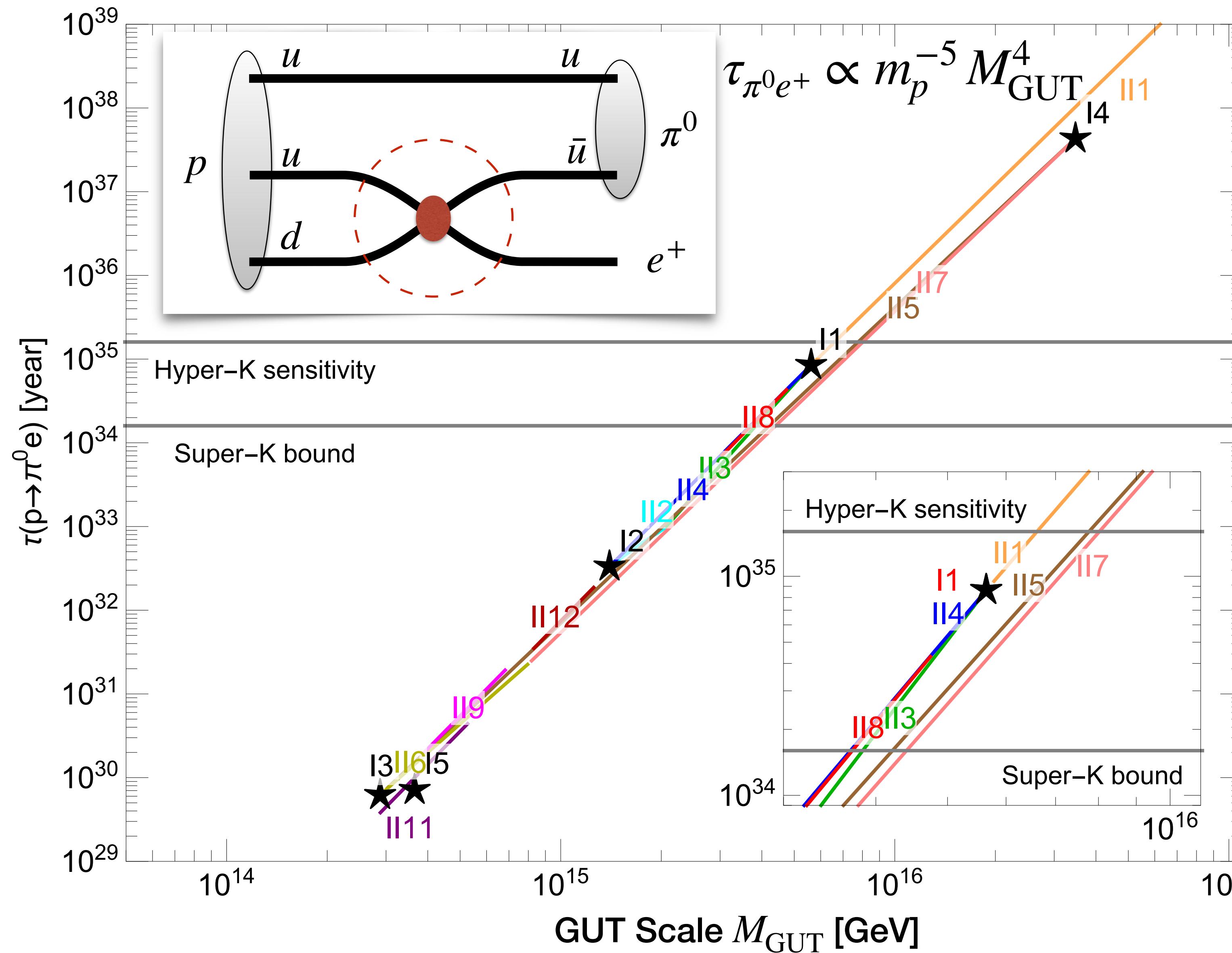
$$G_{3211} = SU(3)_C \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$$

$$G'_{3211} = SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$$

$$G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$$

King, Pascoli, Turner, **YLZ**, 2005.13549

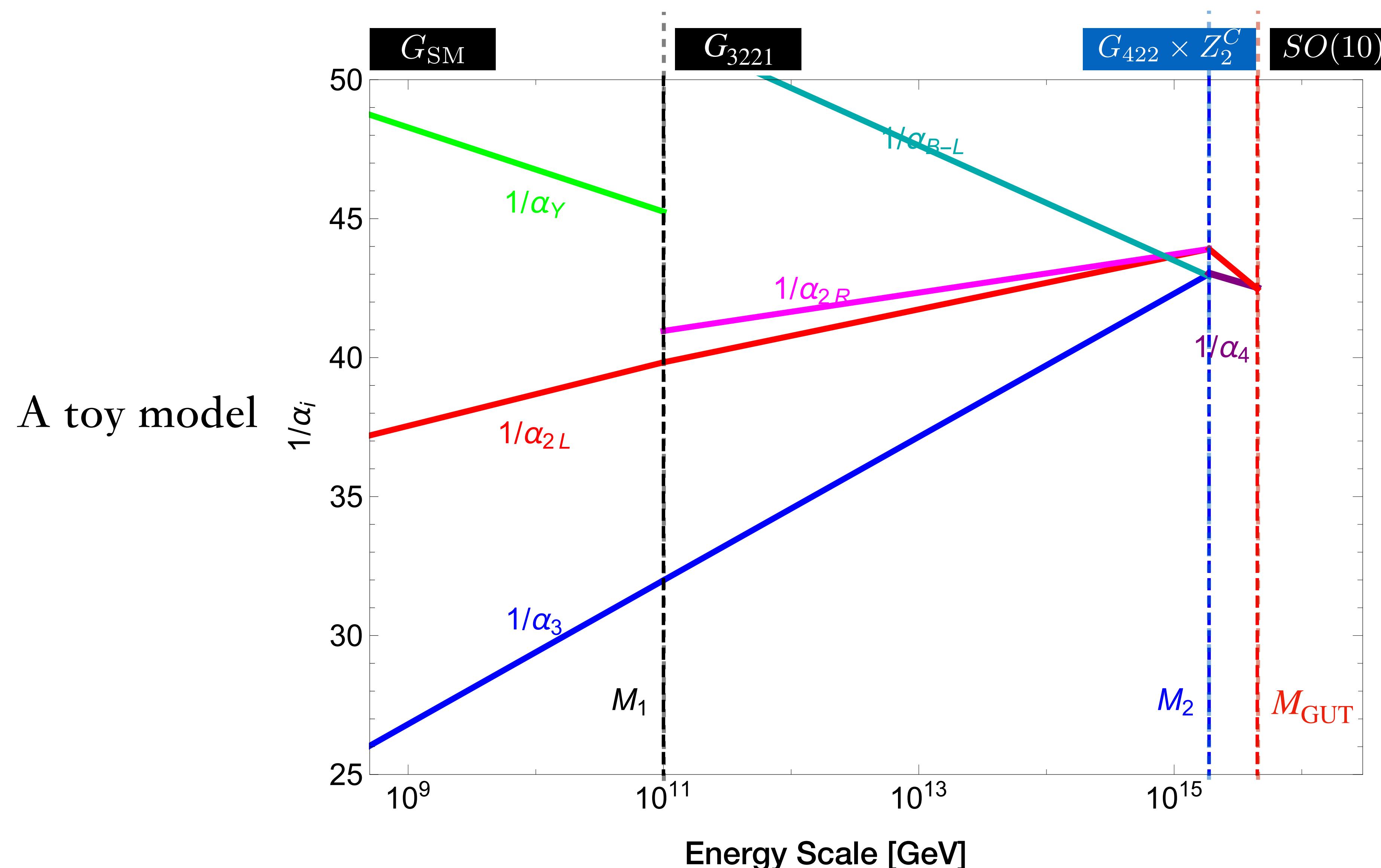
# Proton decay in SO(10) GUTs



$SO(10)$	$\xrightarrow{\text{defect}} Higgs$	$G_2$	$\xrightarrow{\text{defect}} Higgs$	$G_1$	$\xrightarrow{\text{defect}} Higgs$	$G_{\text{SM}}$
II1:	$\xrightarrow{m} \mathbf{210}$	$G_{422}$	$\xrightarrow{m} \mathbf{45}$	$G_{3221}$	$\xrightarrow{s} \mathbf{126}$	
II2:	$\xrightarrow{m,s} \mathbf{54}$	$G_{422}^C$	$\xrightarrow{m} \mathbf{210}$	$G_{3221}^C$	$\xrightarrow{s,w} \mathbf{126}$	
II3:	$\xrightarrow{m,s} \mathbf{54}$	$G_{422}^C$	$\xrightarrow{m,w} \mathbf{45}$	$G_{3221}$	$\xrightarrow{s} \mathbf{126}$	
II4:	$\xrightarrow{m,s} \mathbf{210}$	$G_{3221}^C$	$\xrightarrow{w} \mathbf{45}$	$G_{3221}$	$\xrightarrow{s} \mathbf{126}$	
II5:	$\xrightarrow{m} \mathbf{210}$	$G_{422}$	$\xrightarrow{m} \mathbf{45}$	$G_{421}$	$\xrightarrow{s} \mathbf{126}$	
II6:	$\xrightarrow{m,s} \mathbf{54}$	$G_{422}^C$	$\xrightarrow{m} \mathbf{45}$	$G_{421}$	$\xrightarrow{s} \mathbf{126}$	
II7:	$\xrightarrow{m,s} \mathbf{54}$	$G_{422}^C$	$\xrightarrow{w} \mathbf{45}$	$G_{422}$	$\xrightarrow{m} \mathbf{210}$	$\mathbf{126}, \mathbf{45}$
II8:	$\xrightarrow{m} \mathbf{45}$	$G_{3221}$	$\xrightarrow{m} \mathbf{45}$	$G_{3211}$	$\xrightarrow{s} \mathbf{126}$	
II9:	$\xrightarrow{m,s} \mathbf{210}$	$G_{3221}^C$	$\xrightarrow{m,w} \mathbf{45}$	$G_{3211}$	$\xrightarrow{s} \mathbf{126}$	
II10:	$\xrightarrow{m} \mathbf{210}$	$G_{422}$	$\xrightarrow{m} \mathbf{210}$	$G_{3211}$	$\xrightarrow{s} \mathbf{126}$	
II11:	$\xrightarrow{m,s} \mathbf{54}$	$G_{422}^C$	$\xrightarrow{m,w} \mathbf{210}$	$G_{3211}$	$\xrightarrow{s} \mathbf{126}$	
II12:	$\xrightarrow{m} \mathbf{45}$	$G_{421}$	$\xrightarrow{m} \mathbf{45}$	$G_{3211}$	$\xrightarrow{s} \mathbf{126}$	

King, Pascoli, Turner, **YLZ**,  
2106.15634

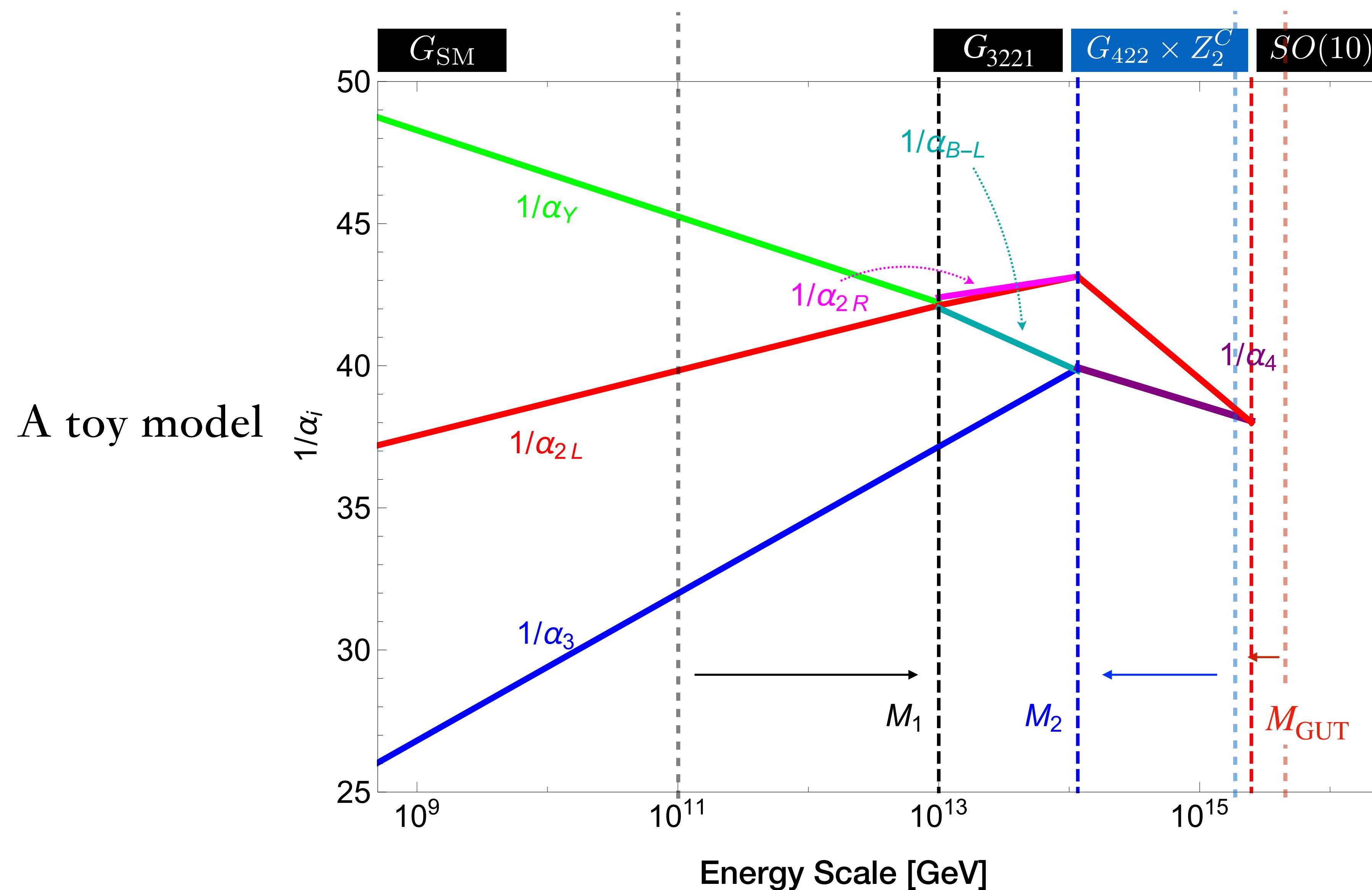
# Correlation between the GUT scale and intermediate scales



$M_i$  understood  
as the mass of  
heavy gauge  
boson there

No new particle  
introduced if not  
necessary

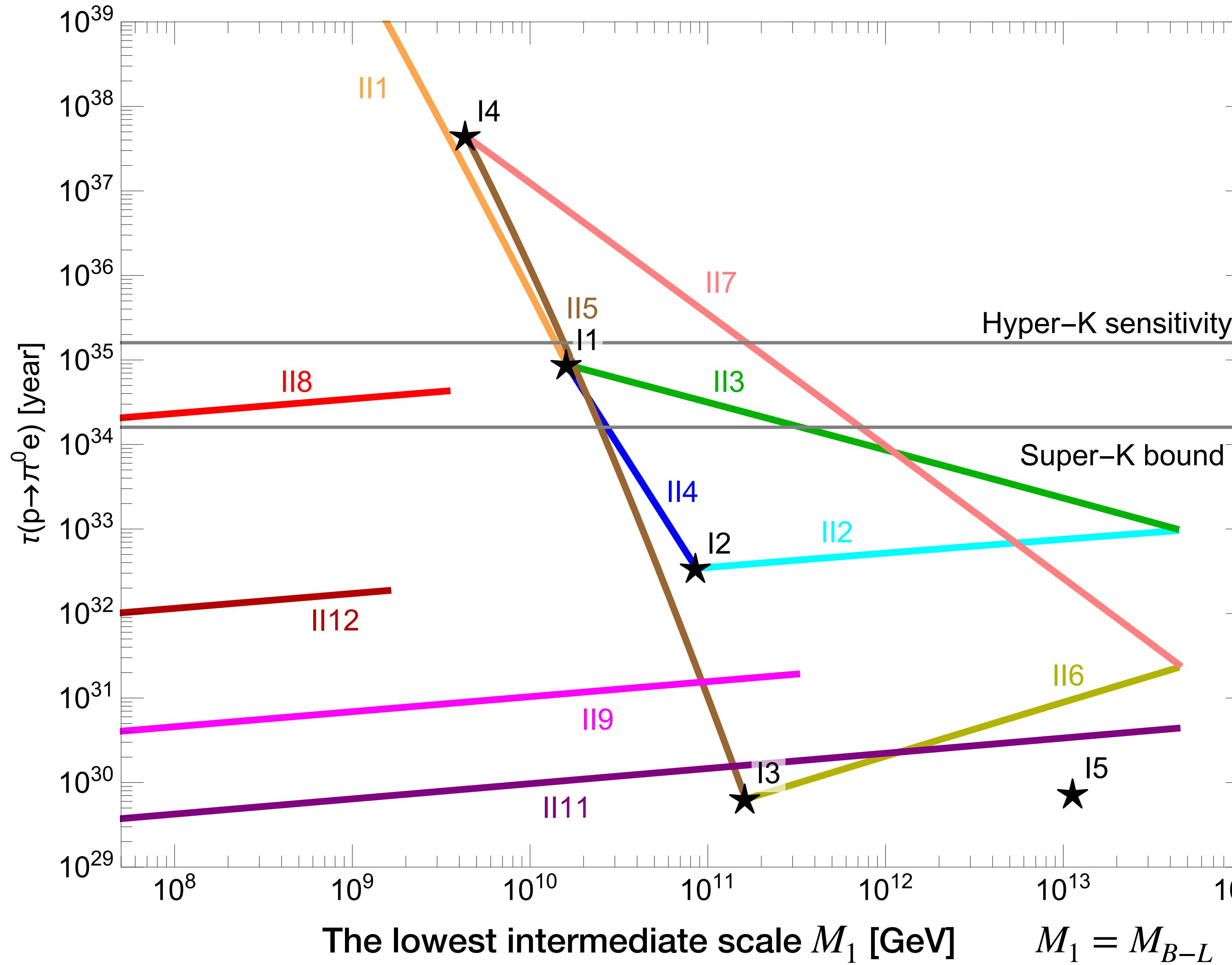
# Correlation between the GUT scale and intermediate scales



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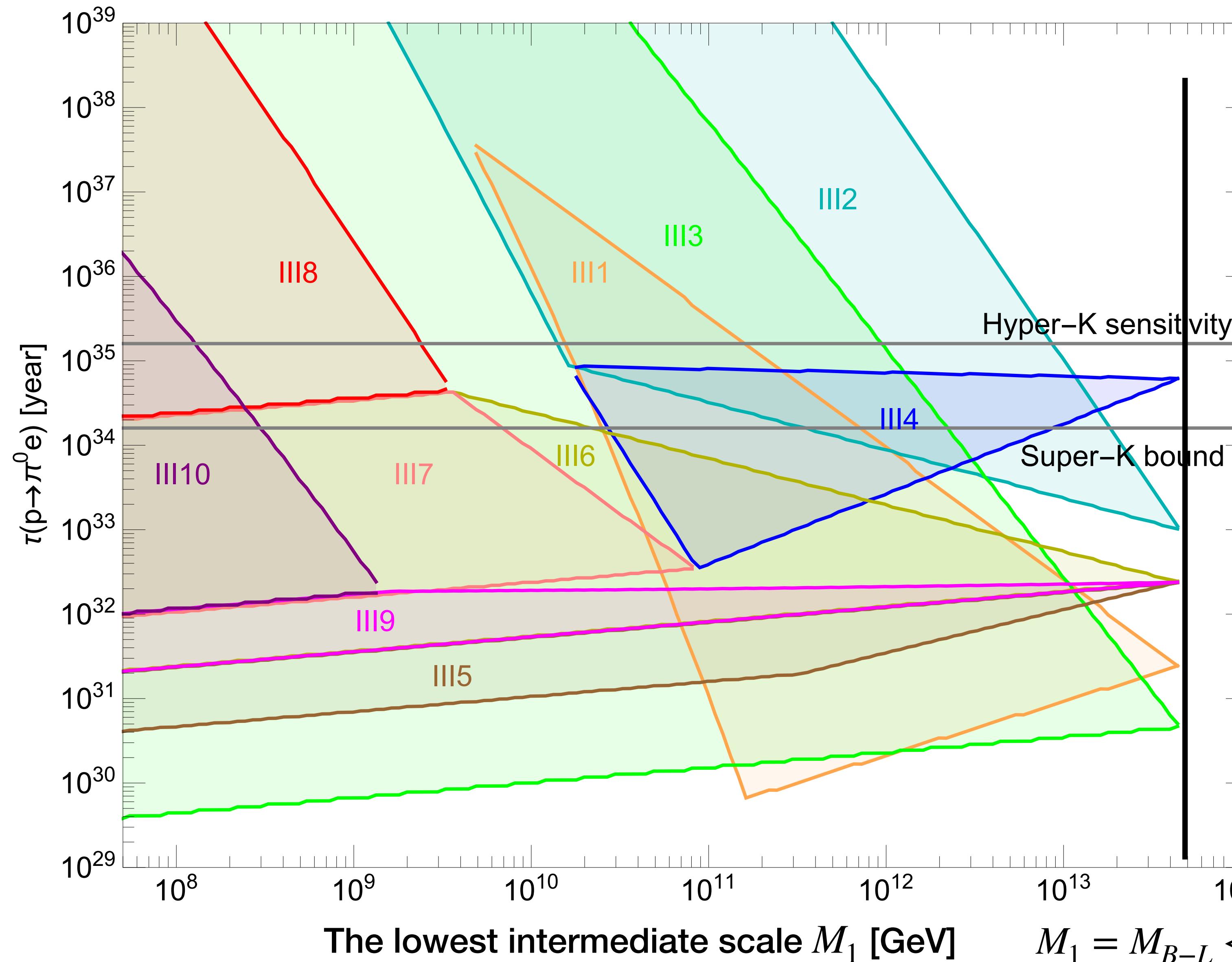
# Intermediate scales of SO(10) GUTs restricted by proton decay



$SO(10)$	$\xrightarrow{\text{defect}} H_{\text{iggs}}$	$G_2$	$\xrightarrow{\text{defect}} H_{\text{iggs}}$	$G_1$	$\xrightarrow{\text{defect}} H_{\text{iggs}}$	$G_{\text{SM}}$
II1:	$\frac{m}{210} \rightarrow$	$G_{422}$	$\frac{m}{45} \rightarrow$	$G_{3221}$	$\frac{s}{126} \rightarrow$	
II2:	$\frac{m,s}{54} \rightarrow$	$G_{422}^C$	$\frac{m}{210} \rightarrow$	$G_{3221}^C$	$\frac{s,w}{126} \rightarrow$	
II3:	$\frac{m,s}{54} \rightarrow$	$G_{422}^C$	$\frac{m,w}{45} \rightarrow$	$G_{3221}$	$s \rightarrow$	$\frac{126}{126}$
II4:	$\frac{m,s}{210} \rightarrow$	$G_{3221}^C$	$\frac{w}{45} \rightarrow$	$G_{3221}$	$s \rightarrow$	$\frac{126}{126}$
II5:	$\frac{m}{210} \rightarrow$	$G_{422}$	$\frac{m}{45} \rightarrow$	$G_{421}$	$s \rightarrow$	$\frac{126}{126}$
II6:	$\frac{m,s}{54} \rightarrow$	$G_{422}^C$	$\frac{m}{45} \rightarrow$	$G_{421}$	$s \rightarrow$	$\frac{126}{126}$
II7:	$\frac{m,s}{54} \rightarrow$	$G_{422}^C$	$\frac{w}{210} \rightarrow$	$G_{422}$	$m \rightarrow$	$\frac{126,45}{126,45}$
II8:	$\frac{m}{45} \rightarrow$	$G_{3221}$	$\frac{m}{45} \rightarrow$	$G_{3211}$	$s \rightarrow$	$\frac{126}{126}$
II9:	$\frac{m,s}{210} \rightarrow$	$G_{3221}^C$	$\frac{m,w}{45} \rightarrow$	$G_{3211}$	$s \rightarrow$	$\frac{126}{126}$
II10:	$\frac{m}{210} \rightarrow$	$G_{422}$	$\frac{m}{210} \rightarrow$	$G_{3211}$	$s \rightarrow$	$\frac{126}{126}$
II11:	$\frac{m,s}{54} \rightarrow$	$G_{422}^C$	$\frac{m,w}{210} \rightarrow$	$G_{3211}$	$s \rightarrow$	$\frac{126}{126}$
II12:	$\frac{m}{45} \rightarrow$	$G_{421}$	$\frac{m}{45} \rightarrow$	$G_{3211}$	$s \rightarrow$	$\frac{126}{126}$

King, Pascoli, Turner, **YLZ**,  
2106.15634

# Intermediate scales of SO(10) GUTs restricted by proton decay



$SO(10)$	defect Higgs	$G_3$	defect Higgs	$G_2$	defect Higgs	$G_1$	defect Higgs	$G_{SM}$	Observable strings?
III1:	$\xrightarrow{m,s} 54$	$G_{422}^C$	$\xrightarrow{w} 210$	$G_{422}$	$\xrightarrow{m} 45$	$G_{421}$	$\xrightarrow{s} 126$		✓
III2:	$\xrightarrow{m,s} 54$	$G_{422}^C$	$\xrightarrow{w} 210$	$G_{422}$	$\xrightarrow{m} 45$	$G_{3221}$	$\xrightarrow{s} 126$		✓
III3:	$\xrightarrow{m,s} 54$	$G_{422}^C$	$\xrightarrow{w} 210$	$G_{422}$	$\xrightarrow{m} 210$	$G_{3211}$	$\xrightarrow{s} 126$		✓
III4:	$\xrightarrow{m,s} 54$	$G_{422}^C$	$\xrightarrow{m} 210$	$G_{3221}^C$	$\xrightarrow{w} 45$	$G_{3221}$	$\xrightarrow{s} 126$		✓
III5:	$\xrightarrow{m,s} 54$	$G_{422}^C$	$\xrightarrow{m} 210$	$G_{3221}^C$	$\xrightarrow{m,w} 45$	$G_{3211}$	$\xrightarrow{s} 126$		✓
III6:	$\xrightarrow{m,s} 54$	$G_{422}^C$	$\xrightarrow{m,w} 45$	$G_{3221}$	$\xrightarrow{m} 45$	$G_{3211}$	$\xrightarrow{s} 126$		✓
III7:	$\xrightarrow{m,s} 210$	$G_{3221}^C$	$\xrightarrow{w} 45$	$G_{3221}$	$\xrightarrow{m} 45$	$G_{3211}$	$\xrightarrow{s} 126$		✓
III8:	$\xrightarrow{m} 210$	$G_{422}$	$\xrightarrow{m} 45$	$G_{3221}$	$\xrightarrow{m} 45$	$G_{3211}$	$\xrightarrow{s} 126$		✓
III9:	$\xrightarrow{m,s} 54$	$G_{422}^C$	$\xrightarrow{m} 45$	$G_{421}$	$\xrightarrow{m} 45$	$G_{3211}$	$\xrightarrow{s} 126$		✓
III10:	$\xrightarrow{m} 210$	$G_{422}$	$\xrightarrow{m} 45$	$G_{421}$	$\xrightarrow{m} 45$	$G_{3211}$	$\xrightarrow{s} 126$		✓

King, Pascoli, Turner, **YLZ**,  
2106.15634

$M_1 = M_{B-L} < 5 \times 10^{13}$  GeV in non-SUSY GUTs

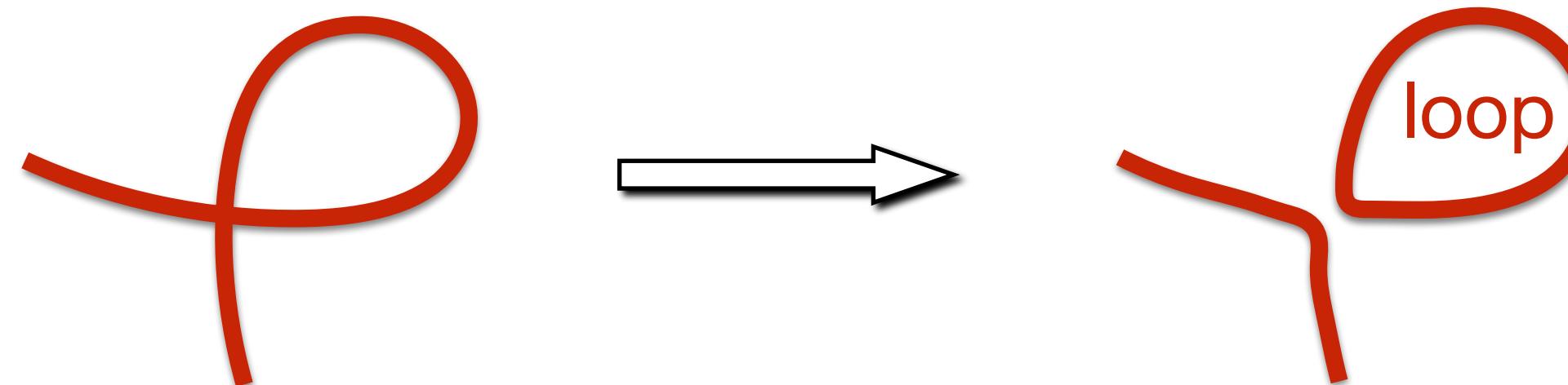
# Gravitational waves predicted in SO(10) GUTs

- $SO(10) \supset U(1)_{B-L}$
- Cosmic string is a topological defect predicted by  $U(1)$  spontaneous breaking.

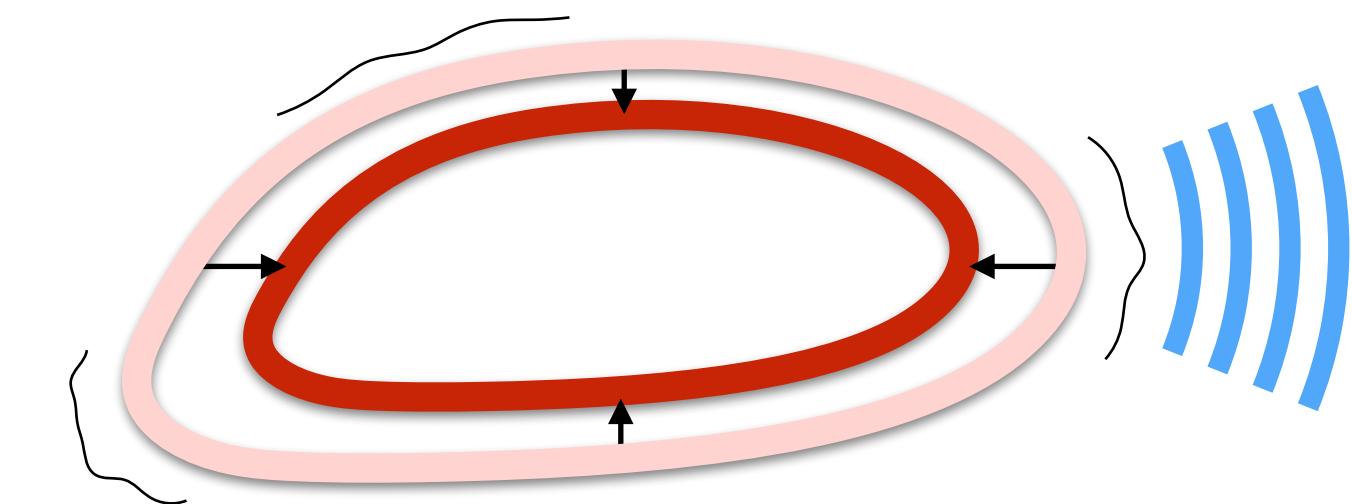
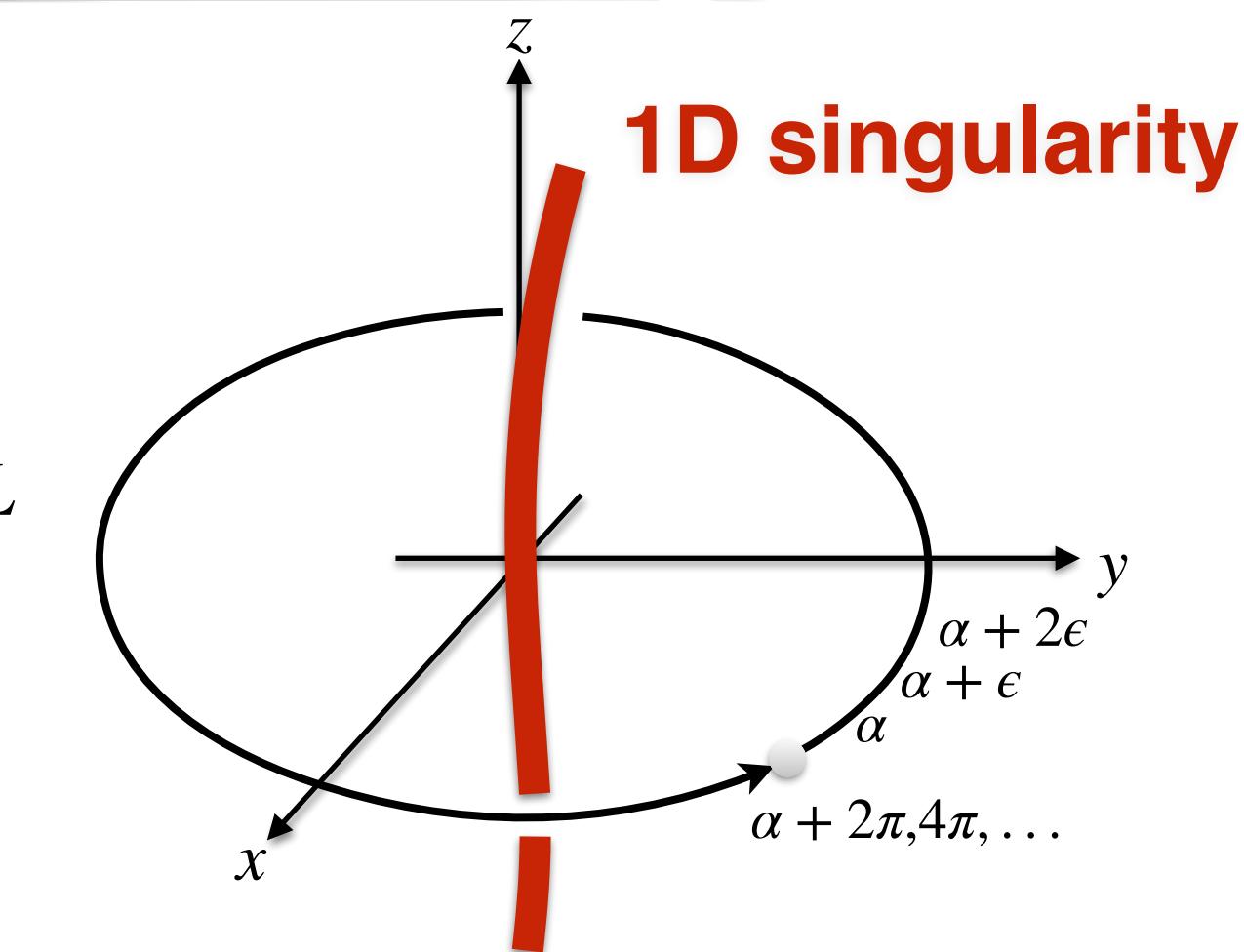
Tension of the string  
(energy per length)

$$\mu = 2\pi v_{B-L}^2 \sim M_{B-L}^2$$

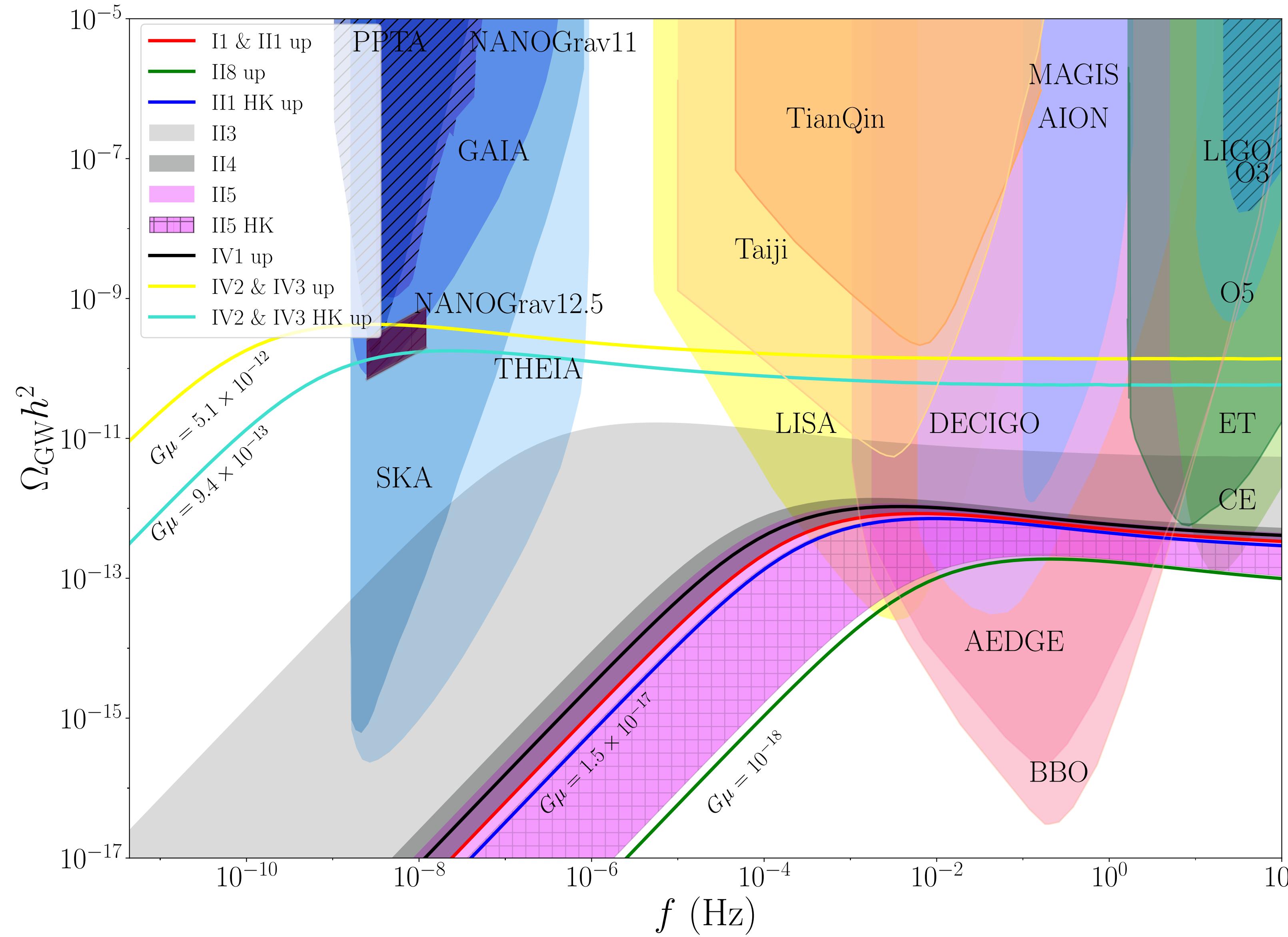
- Network of cosmic strings generates string loops via intercommunications and intersections.



- Loops oscillate, getting shrunk via gravitational radiation. The radiation appears as a cosmic background today.
- Simulation of energy spectrum of GW background,  
**BOS model (Blanco-Pillado, Olum & Shlaer), 1101.5173, 1309.6637, 1709.02693**  
**Simplified code, Cui, Lewicki, Morrissey, Well, 1808.08968**  
Other simulations, Lorenz, Ringeval, Sakellariadou, 1006.0931, 1709.03845, ...



# Predictions for each breaking chain of SO(10) GUTs



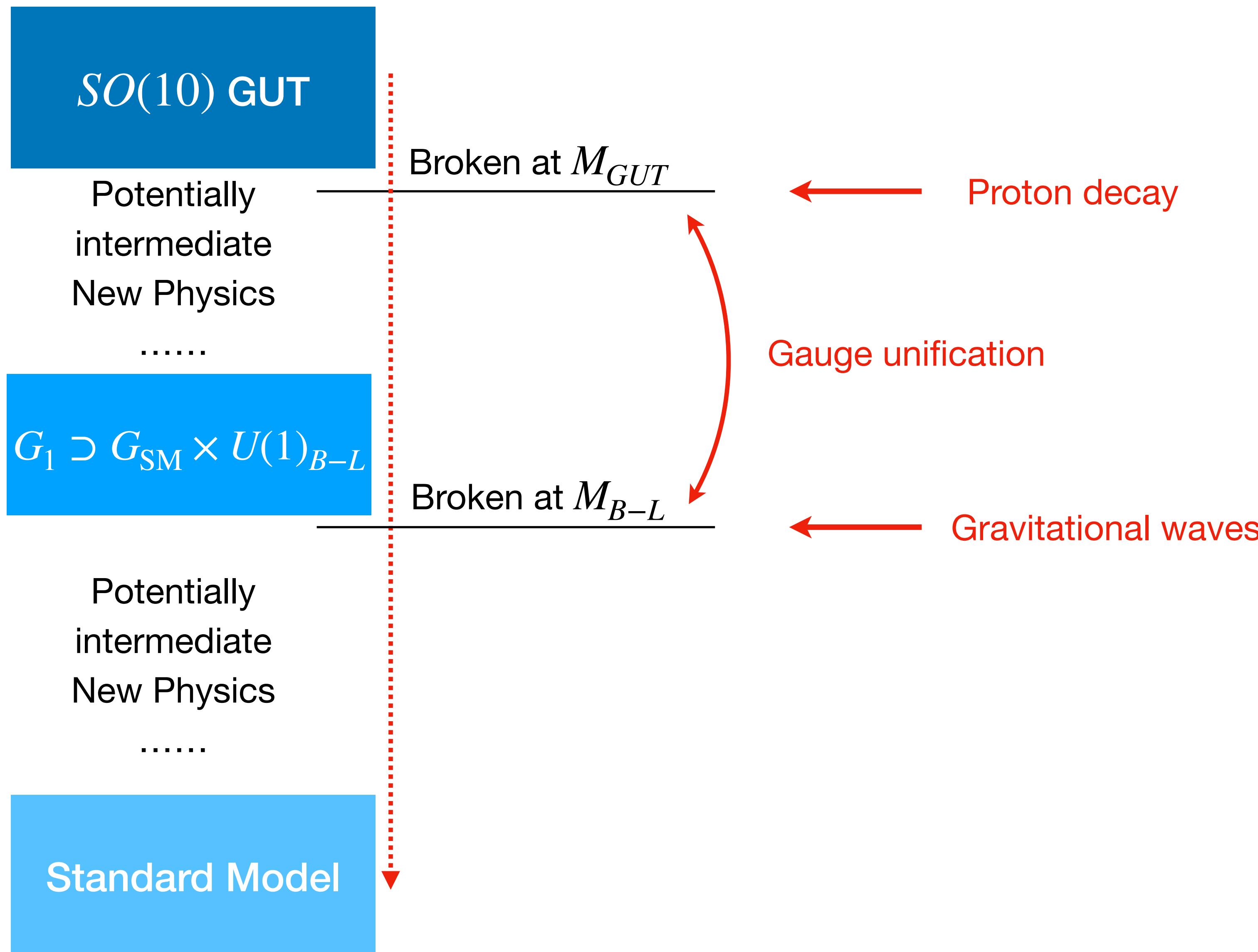
Based on BOS model

$$\Omega h^2 \propto \sqrt{\mu} \propto M_{B-L}$$

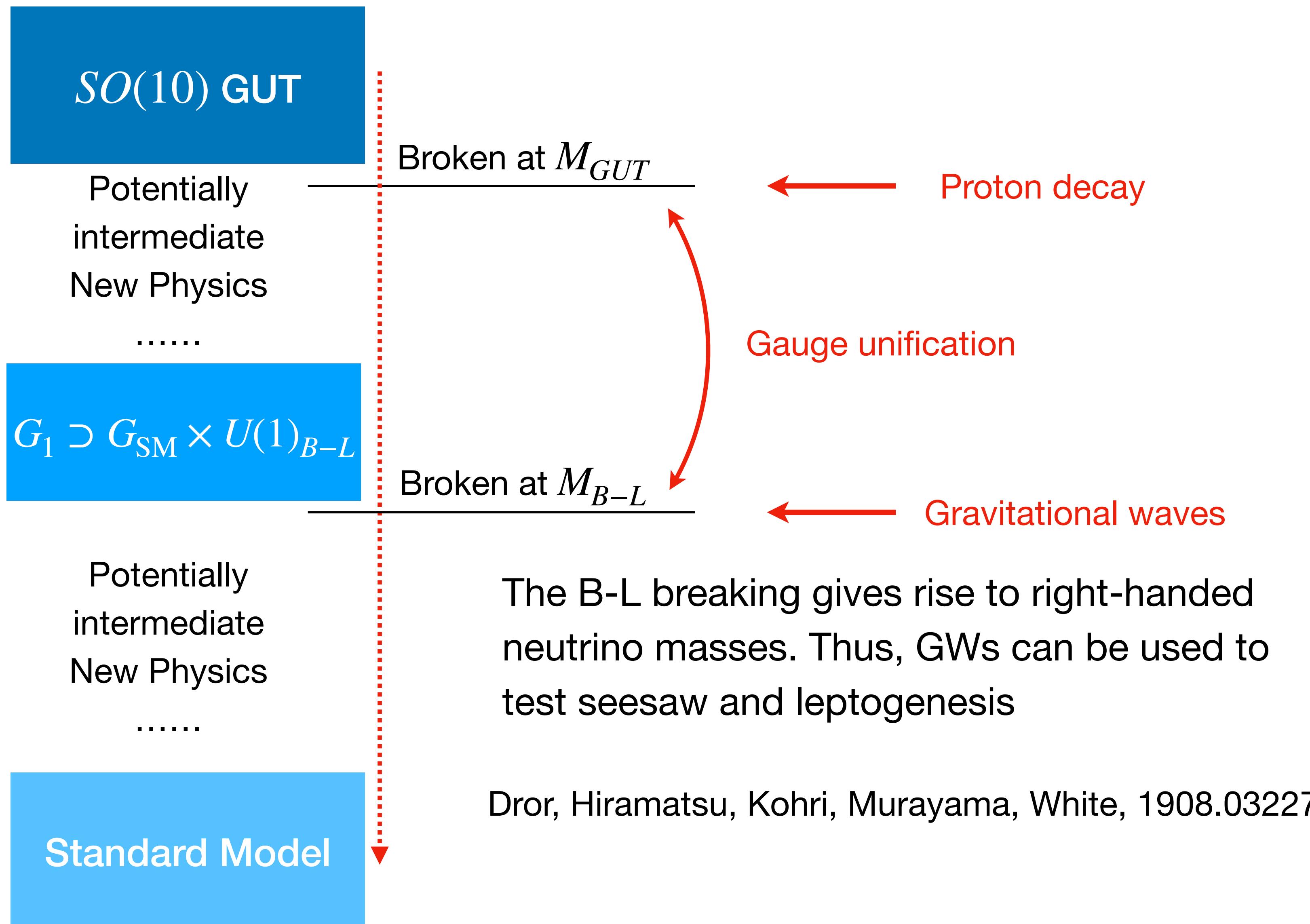
$$\mu \simeq \frac{2M_{B-L}^2}{\alpha_1}$$

King, Pascoli, Turner, **YLZ**,  
2106.15634

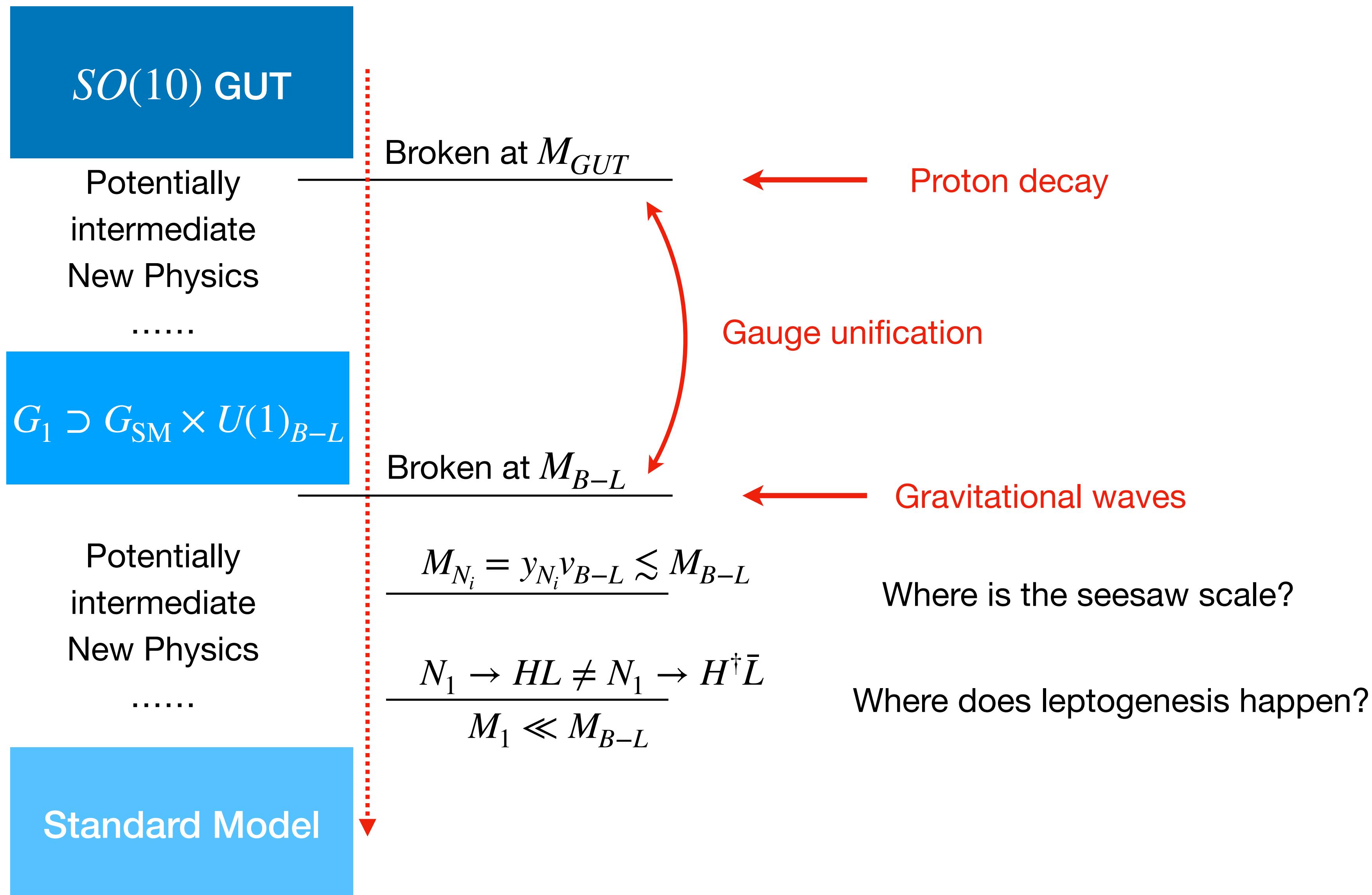
# Complementary tests of GUTs



# Complementary tests of GUTs



# Complementary tests of GUTs



# Fermion masses in SO(10) GUTs

- All matter fields  $\{Q = (u_L, u_R), u_R, d_R, L = (\nu_L, e_L), e_R, \nu_R\}$  are arranged in **16**-plet of SO(10)

$$\mathbf{16} \times \mathbf{16} = \mathbf{10} + \mathbf{126} + \mathbf{120}$$

- To get a gauge-invariant Yukawa couplings, 3 Higgs can be introduced  $\mathbf{10}_H, \overline{\mathbf{126}}_H, \mathbf{120}_H$

$$\mathcal{L}_Y = Y_{\mathbf{10}}^* \mathbf{16} \cdot \mathbf{16} \cdot \mathbf{10}_H + Y_{\overline{\mathbf{126}}}^* \mathbf{16} \cdot \mathbf{16} \cdot \overline{\mathbf{126}}_H + Y_{\mathbf{120}}^* \mathbf{16} \cdot \mathbf{16} \cdot \mathbf{120}_H + \text{h.c.}$$

- The SM Higgs is the lightest component of the mixture of these Higgs

Dutta, Mimura, Mohapatra, 0412105

$$Y_d = r_1(h + f + i h')$$

$$Y_u = h + r_2 f + i r_3 h'$$

$$M_{\nu_R} = m_{\nu_R} f$$

$$Y_e = r_1(h - 3f + i c_e h')$$

$$Y_\nu = h - 3r_2 f + i c_\nu h'$$

Yukawa/mass matrices in SO(10)

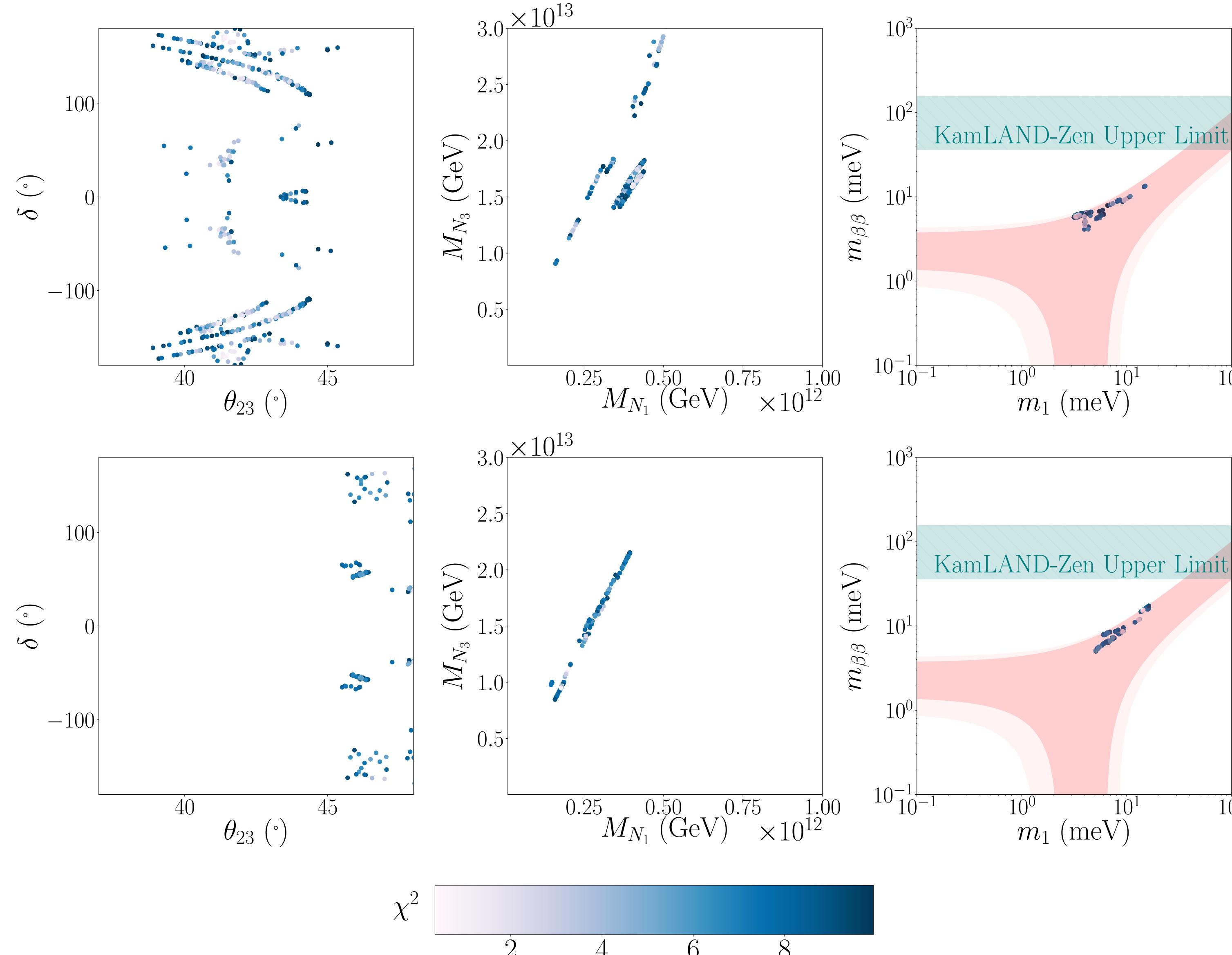
$3 \times 3$  matrices

$h \propto Y_{\mathbf{10}}$

$f \propto Y_{\overline{\mathbf{126}}}$

$h' \propto Y_{\mathbf{120}}$

# Prediction of lepton mixing in SO(10)

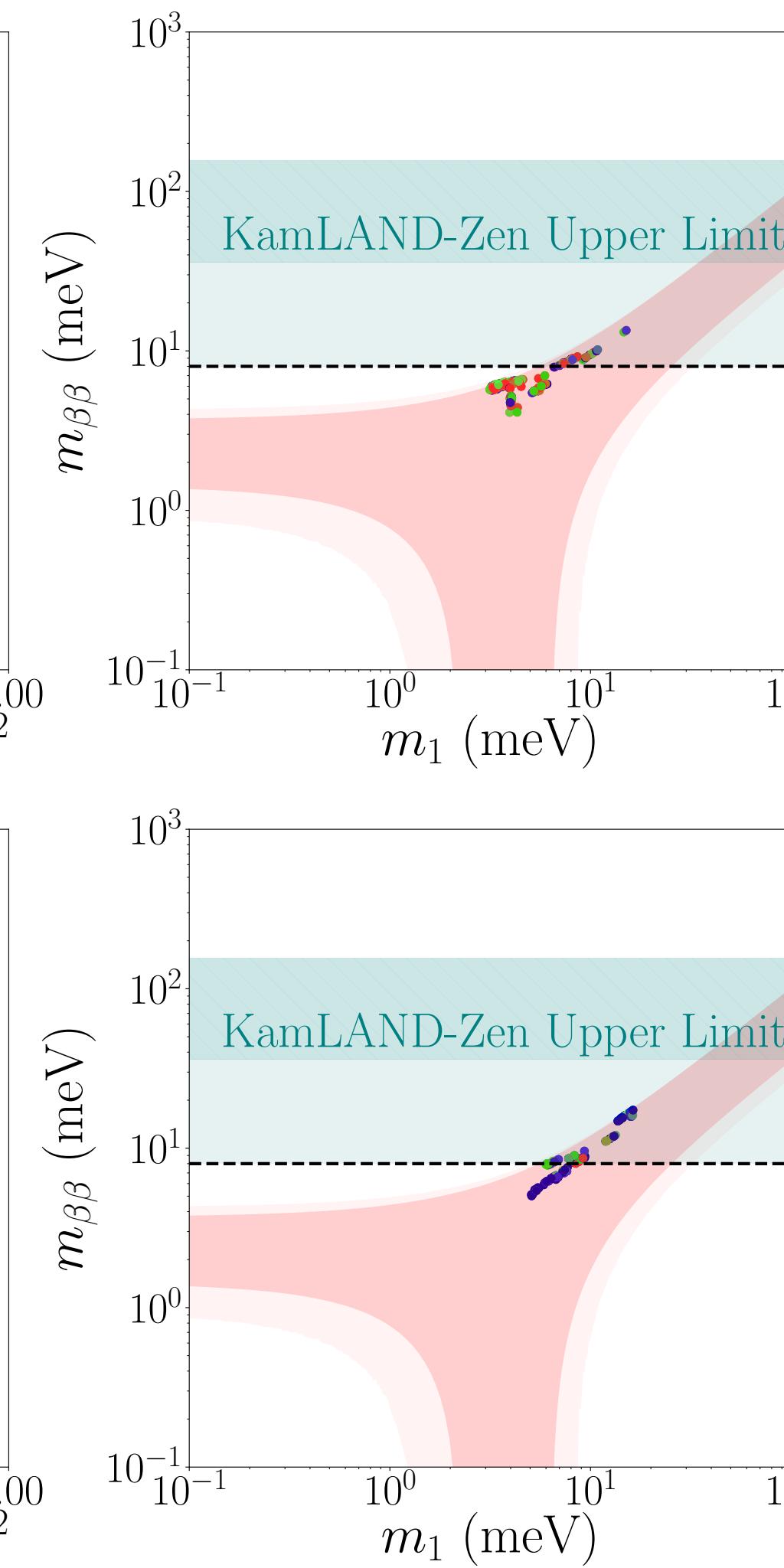
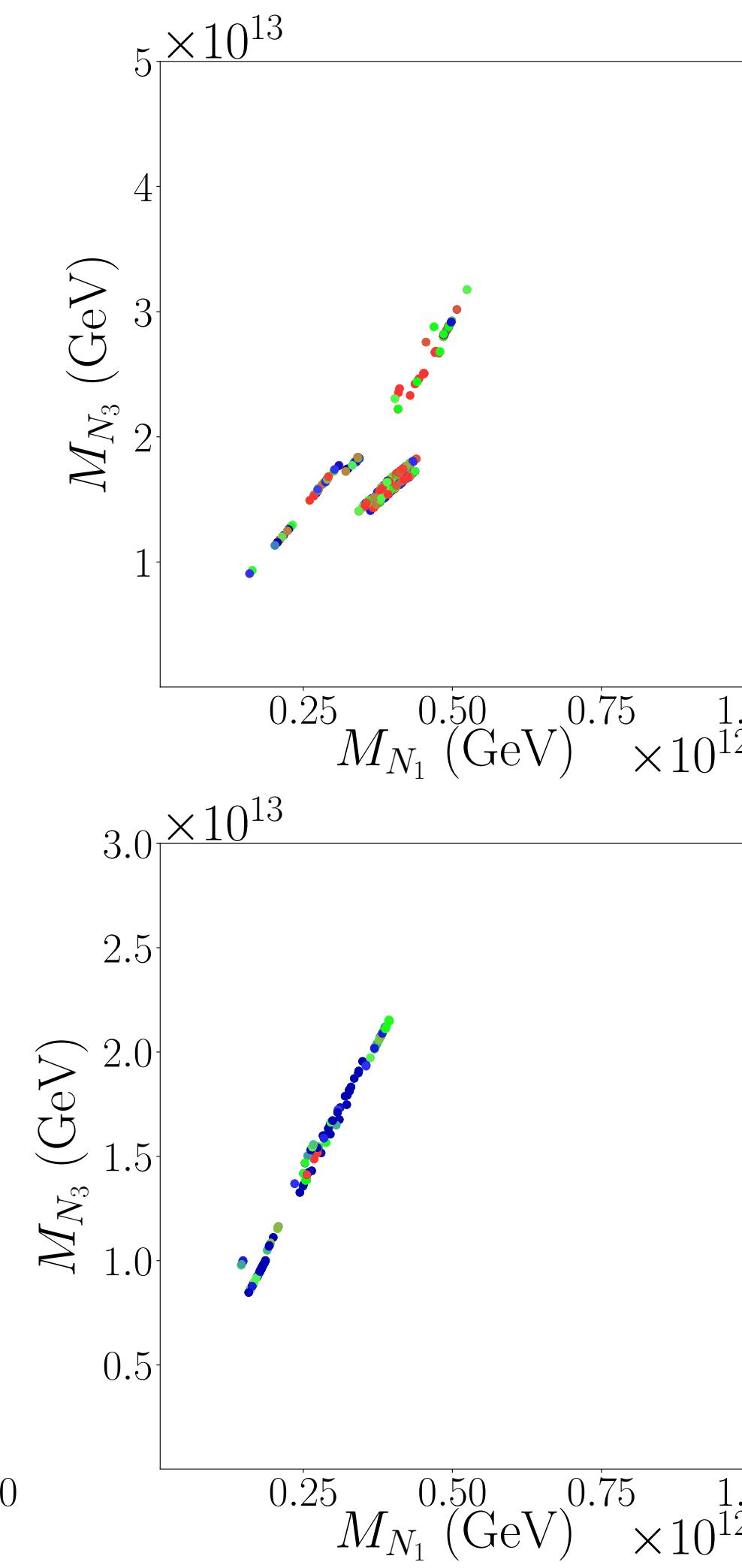
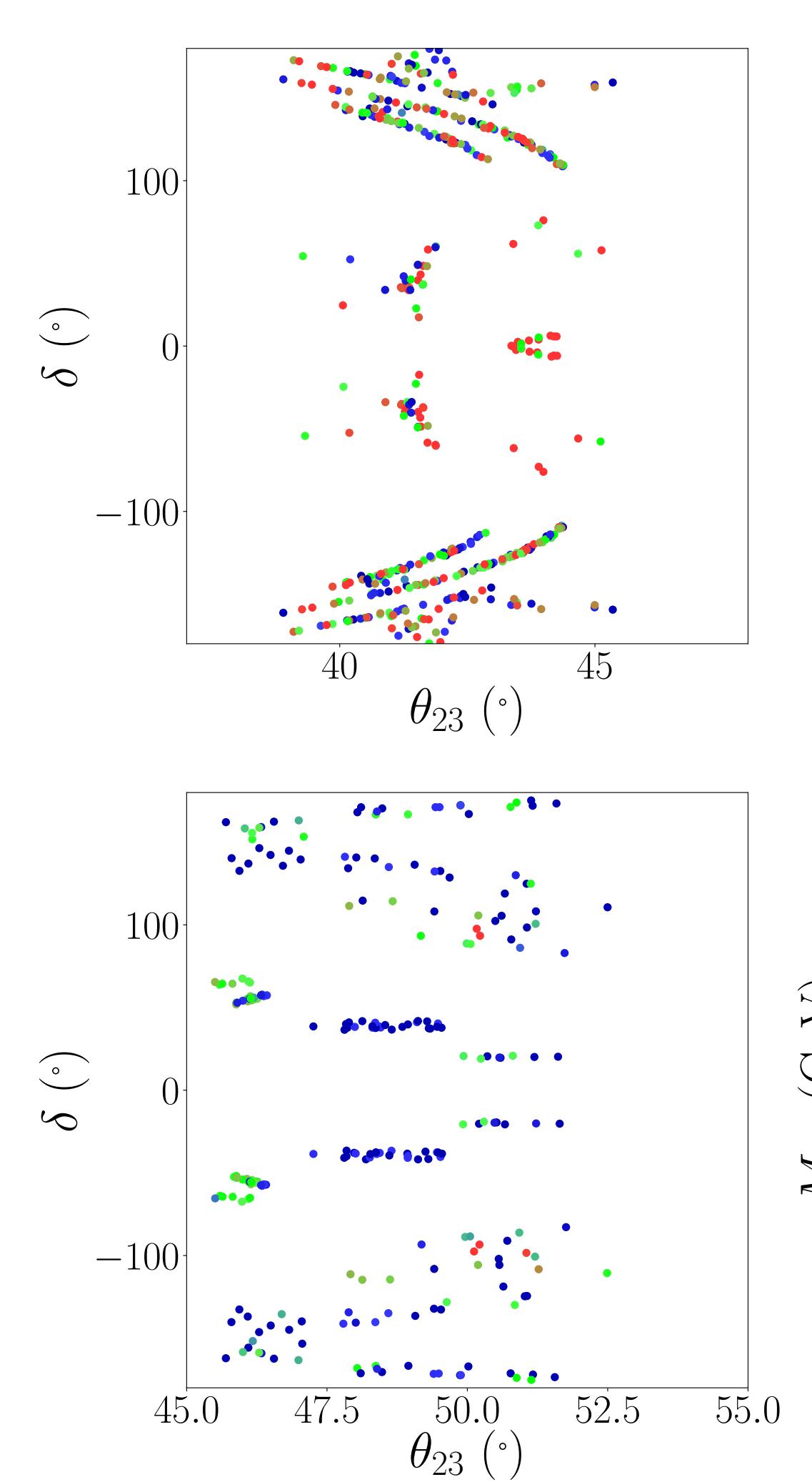


**Take-away message:**

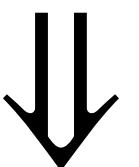
$M_1 \sim (2, 5) \times 10^{11} \text{ GeV}$   
 $M_3 \sim (1, 3) \times 10^{13} \text{ GeV}$   
 $m_{\beta\beta} \sim 10^{-2} \text{ eV}$   
 Normal ordering of  $\nu$  masses are preferred

Fu, King, Marsili, Pascoli, Turner, **YLZ**, arXiv:2209.00021

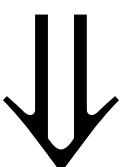
# Thermal leptogenesis in SO(10)



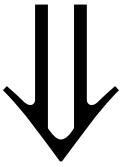
Data of quark masses,  
CKM mixing, lepton  
masses, PMNS mixing



Heavy neutrino masses  
and Dirac v Yukawa  
couplings

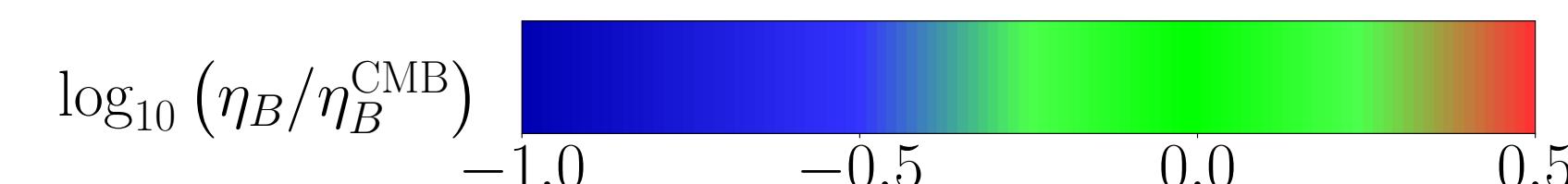


CP violation in heavy  
neutrino decay

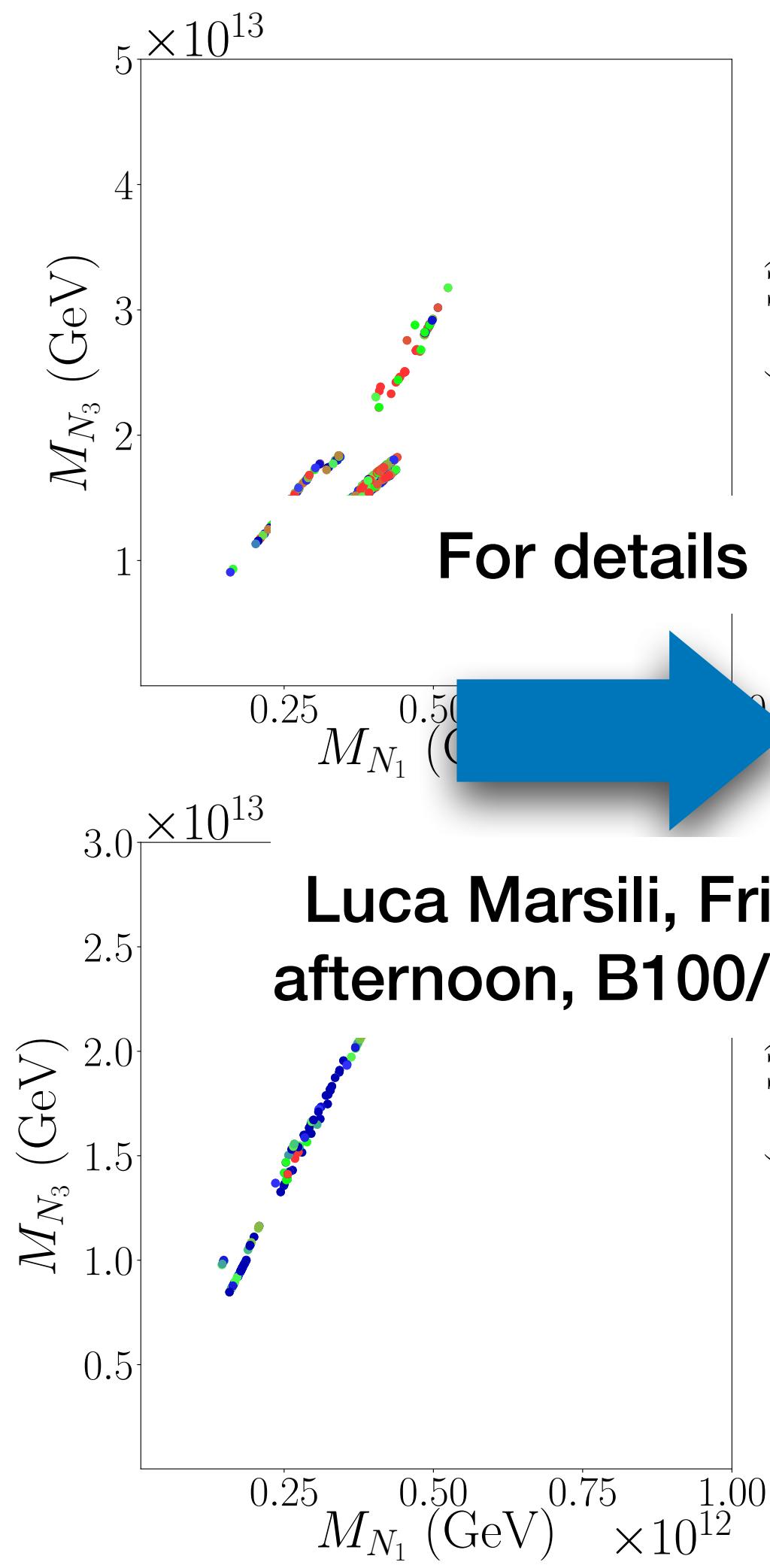
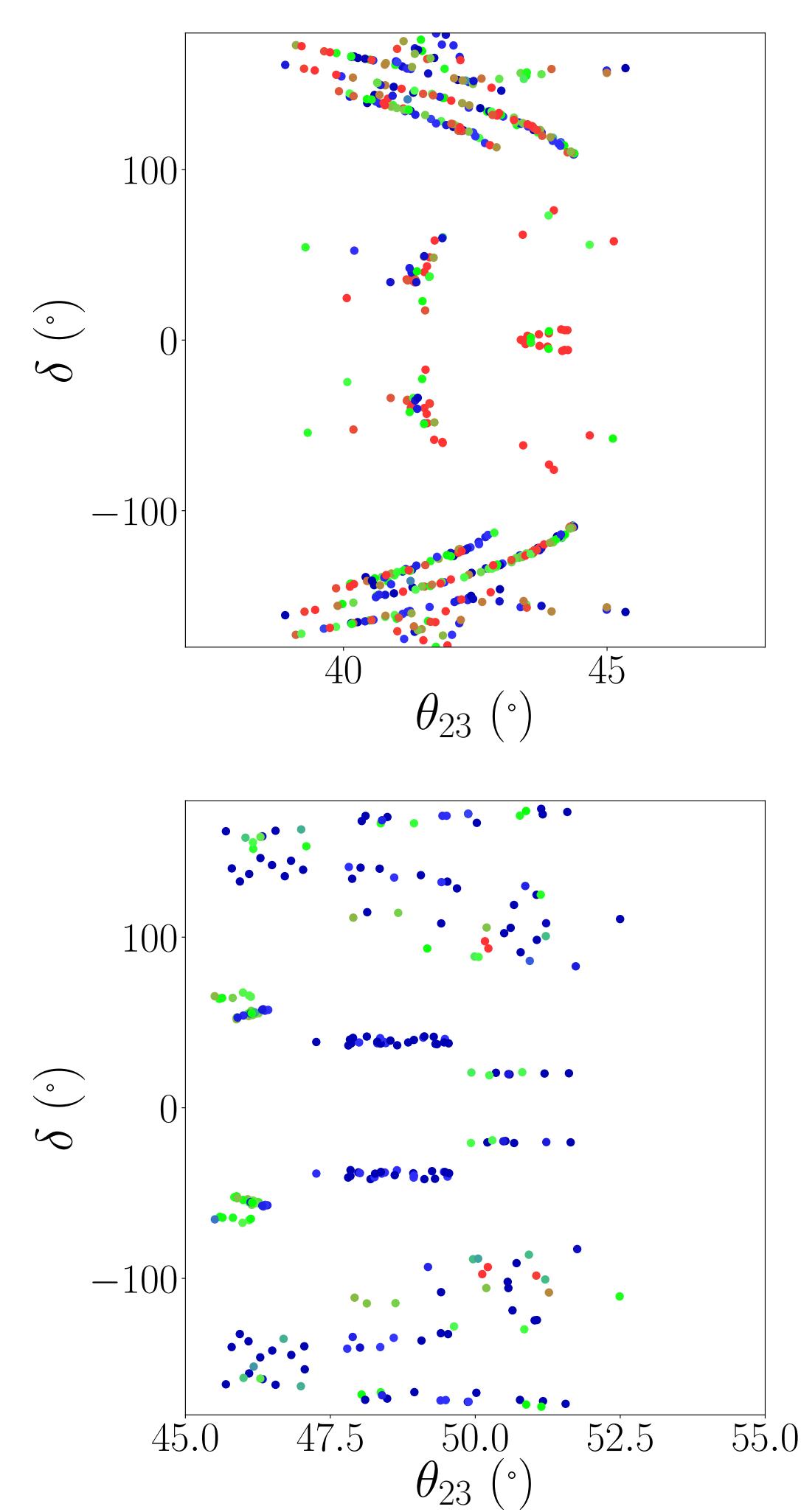


**Thermal leptogenesis**

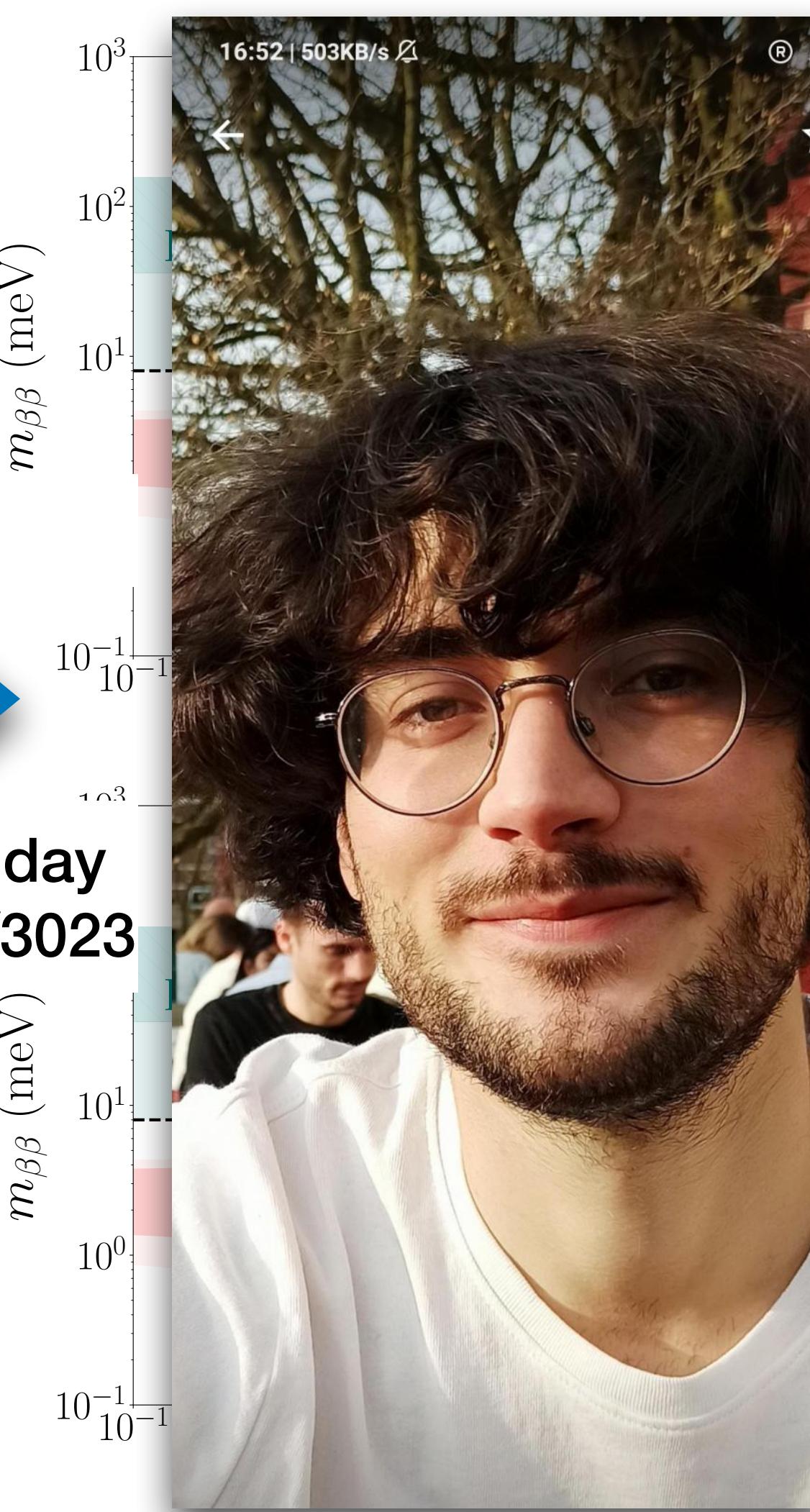
Fu, King, Marsili, Pascoli,  
Turner, **YLZ**, arXiv:2209.00021



# Thermal leptogenesis in SO(10)



Luca Marsili, Friday  
afternoon, B100/3023



Data of quark masses,  
CKM mixing, lepton  
masses, PMNS mixing

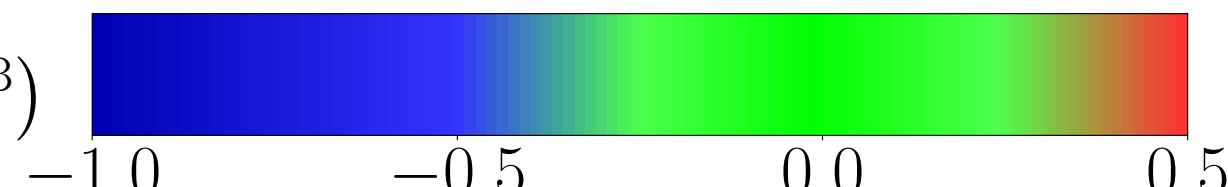
Heavy neutrino masses  
and Dirac v Yukawa  
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CP violation in heavy  
neutrino decay

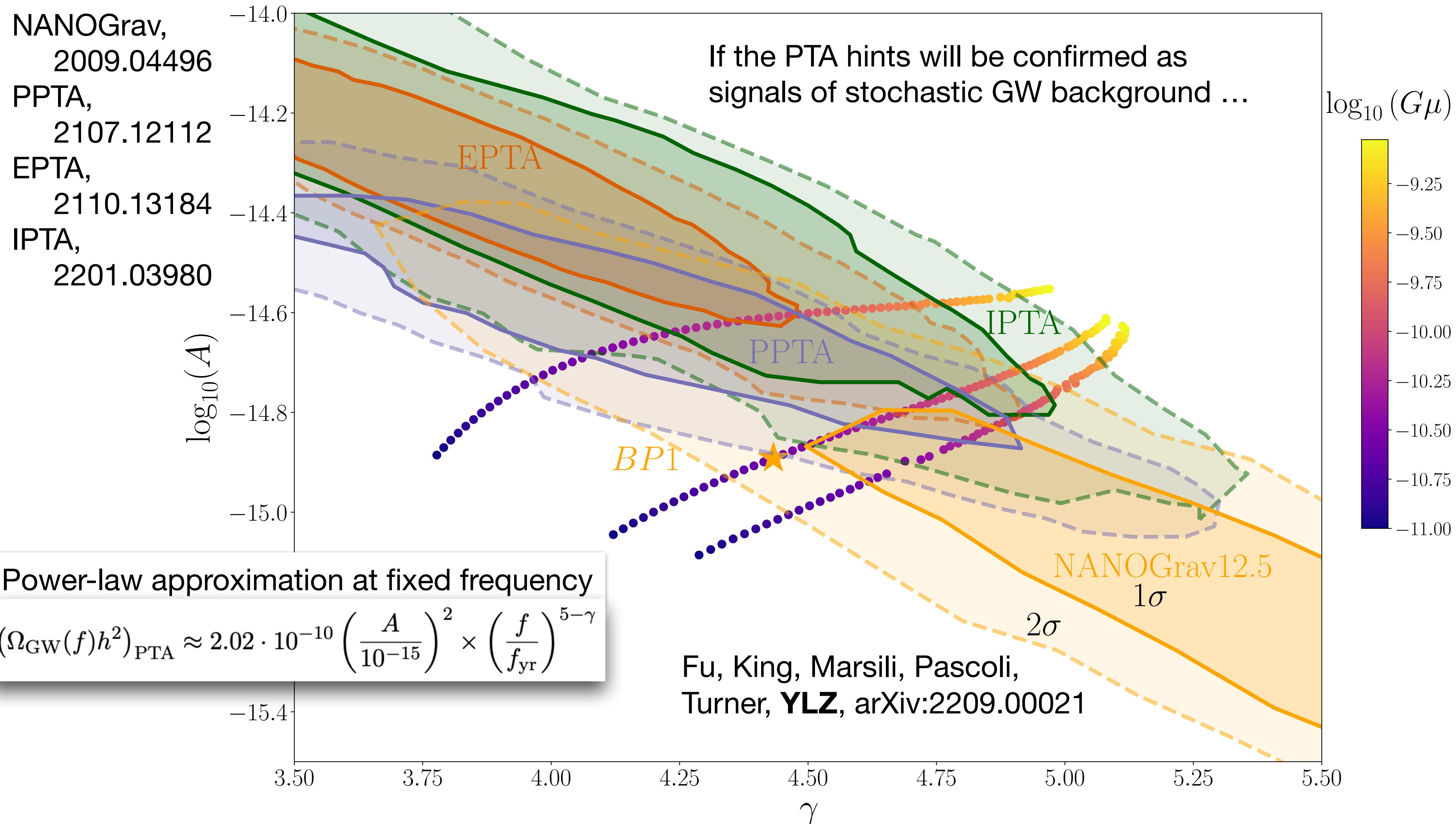
**Thermal leptogenesis**

Fu, King, Marsili, Pascoli,  
Turner, **YLZ**, arXiv:2209.00021

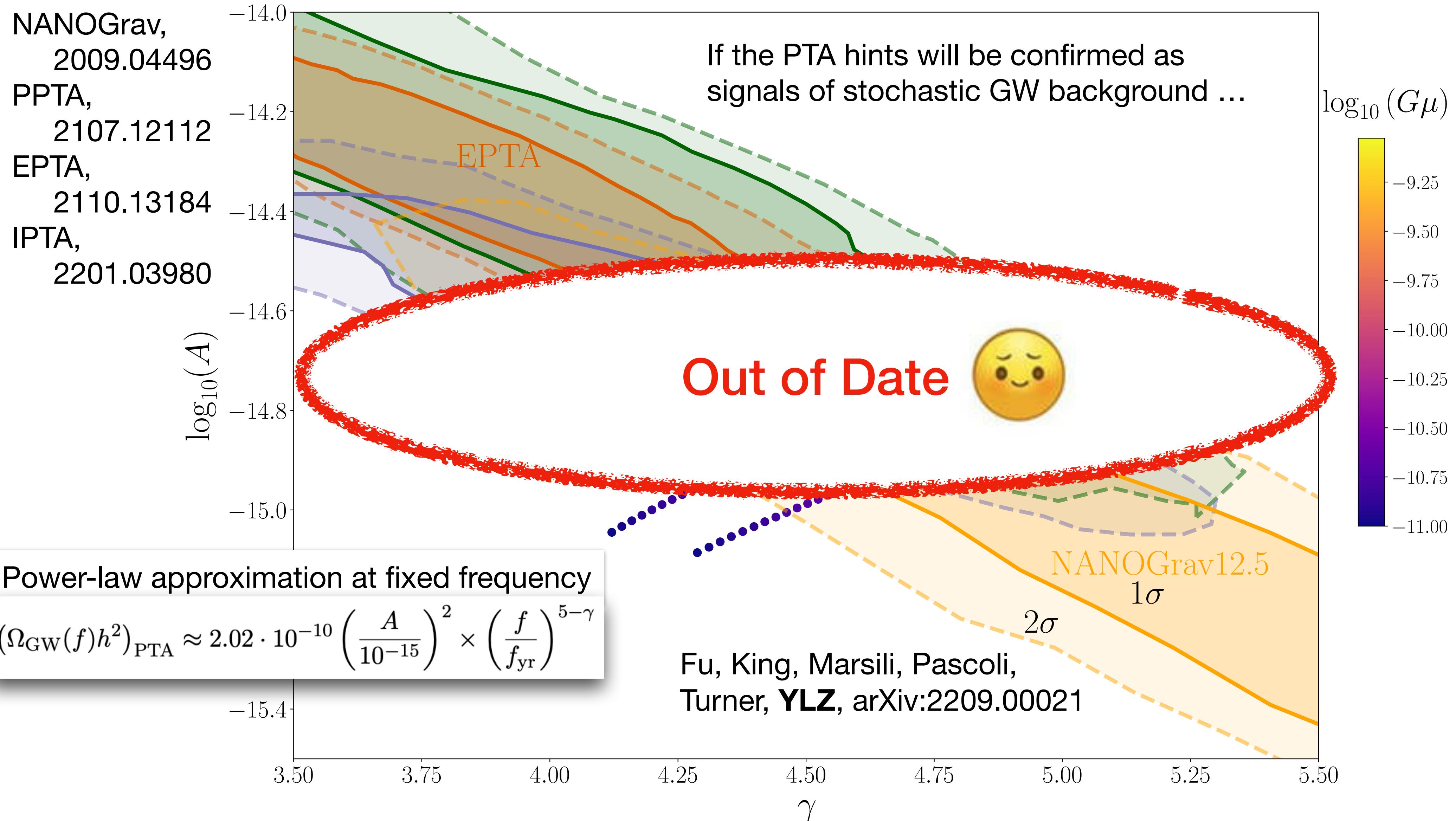
$$\log_{10} \left( \eta_B / \eta_B^{\text{CMB}} \right)$$



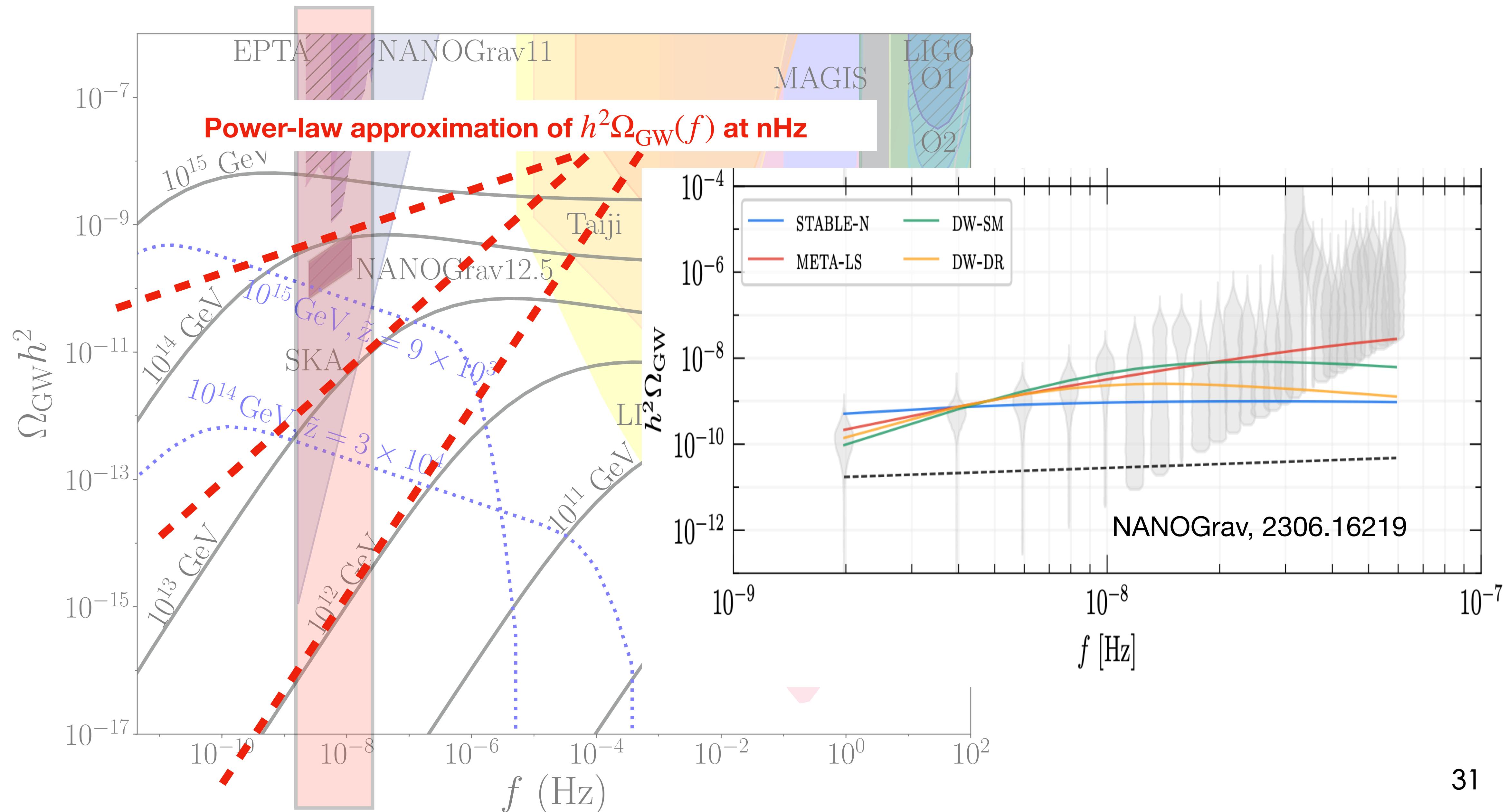
# Pulsar Timing Arrays (PTAs) hints on the model



# Pulsar Timing Arrays (PTAs) hints on the model

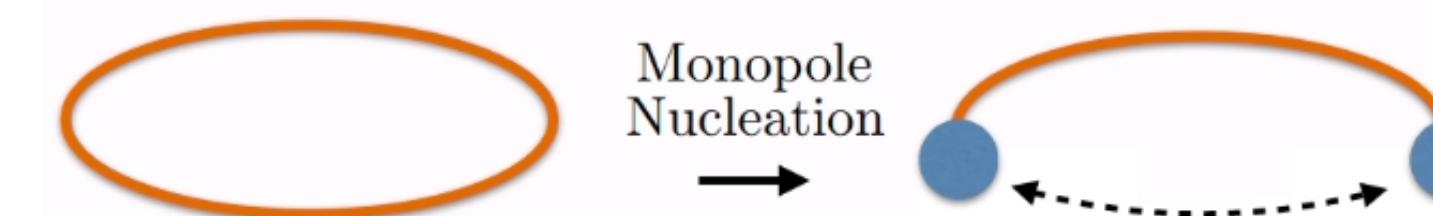


# Tensions between NANOGrav and GWs via Nambu-Goto strings

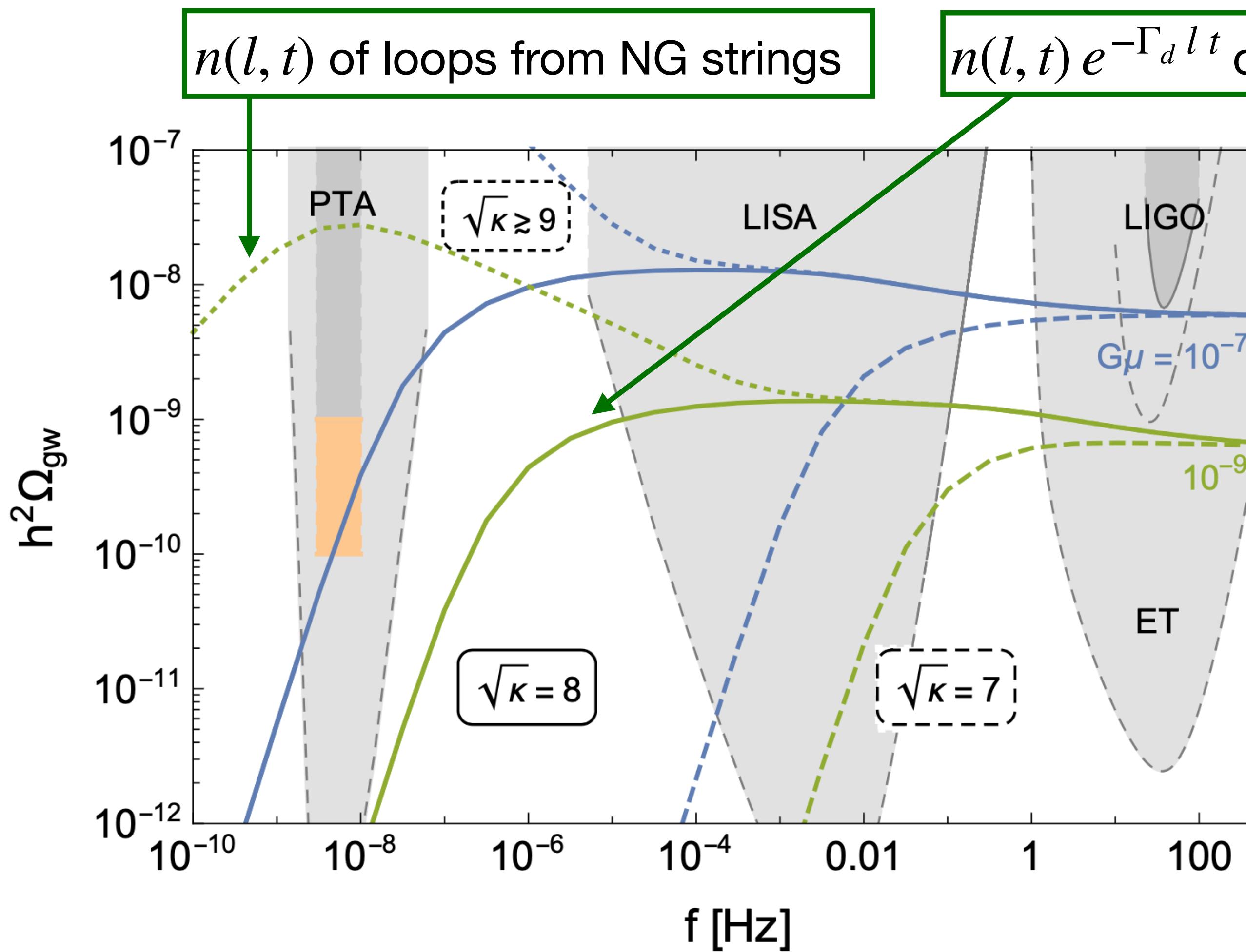


# Tensions between NANOGrav and GWs via Nambu-Goto strings

Way out: metastable strings



Talk by Shaikh Saad



$$\Gamma_d = \frac{\mu}{2\pi} e^{-\pi\kappa}$$

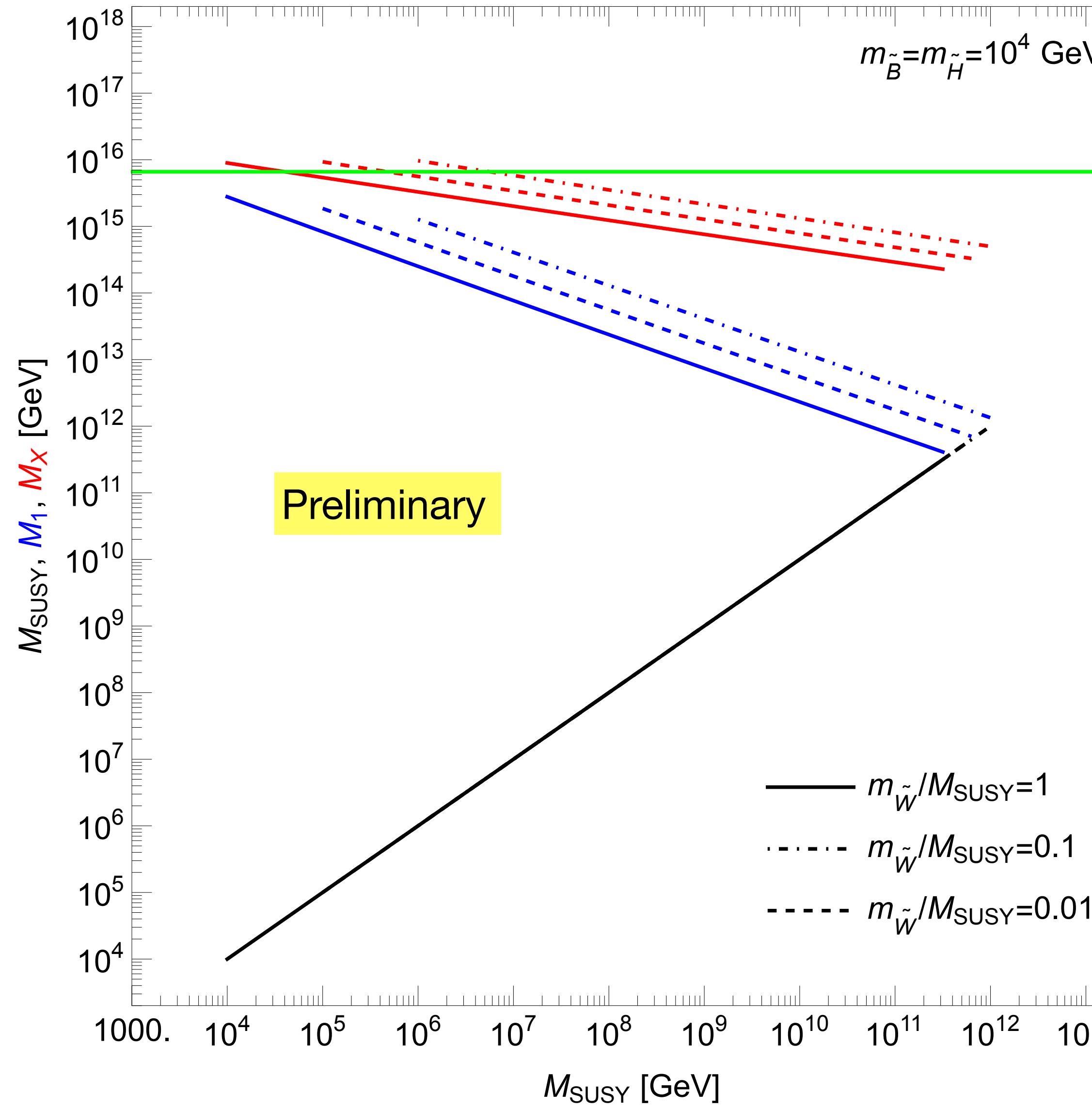
$$\sqrt{\kappa} = \frac{m_{\text{monopole}}}{\sqrt{\mu_{\text{string}}}} \sim \alpha_{\text{GUT}}^{-1/2} \frac{M_{\text{GUT}}}{M_{B-L}}$$

$$\sqrt{\kappa} \simeq (8, 9) \Rightarrow M_{\text{GUT}} \sim M_{B-L}$$

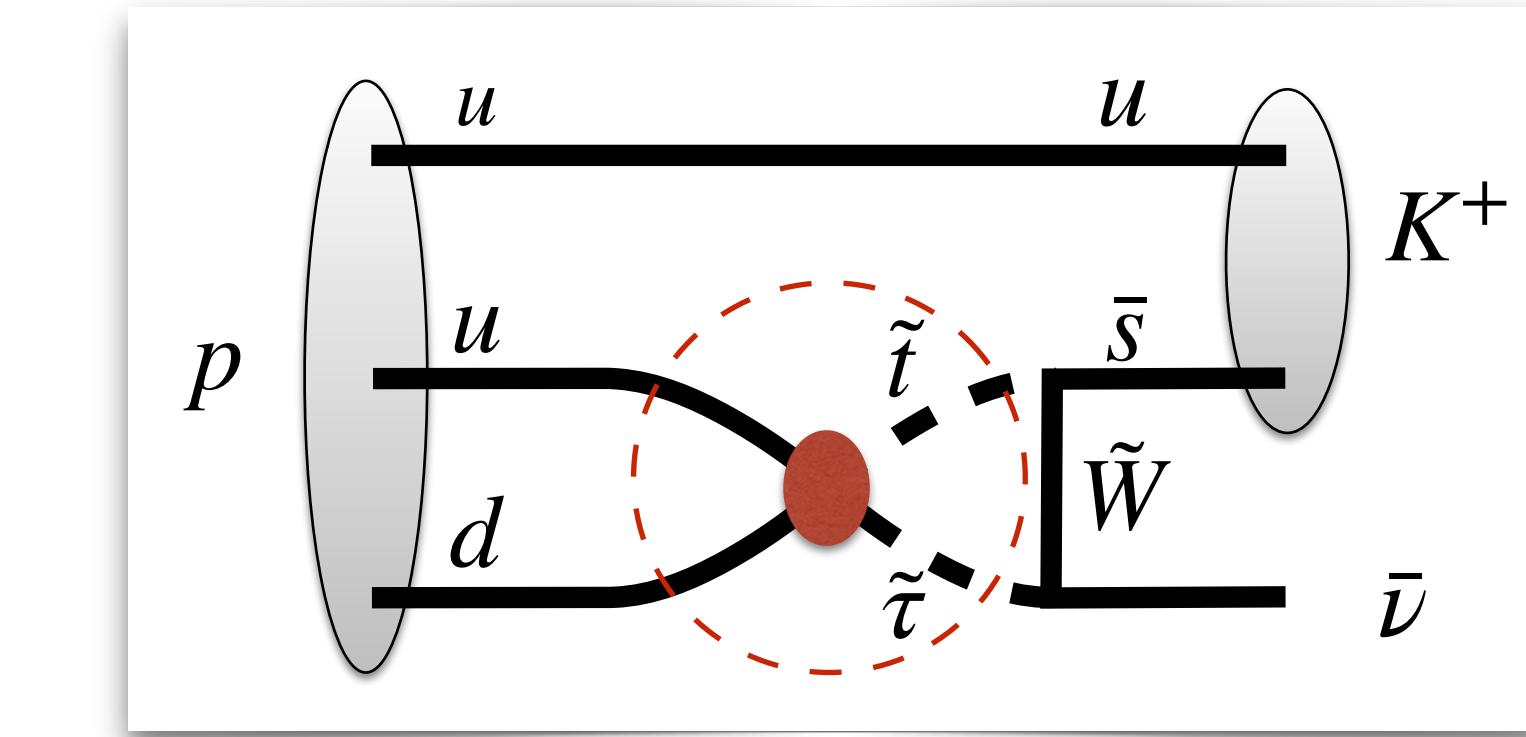
A GUT inflation separates the GUT breaking and B-L breaking in the time scale is required.

Antusch, Hinze, Saad, Steiner, 2307.04595

# How SUSY helps?



SUSY enhances the decay channel  
 $p \rightarrow K^+ \bar{\nu}$



Restriction from Super-K and  
sensitivities of JUNO and Hyper-K  
should be considered

Fu, King, Marsili, Pascoli,  
Turner, **YLZ**, in progress

# Summary

- ✓ Complementary tests of GUTs via measurements of
  - ◆ proton decay
  - ◆ fermion masses and mixing, in particular, neutrino masses and mixing
  - ◆ GWs via cosmic strings from spontaneous breaking of  $U(1)_{B-L}$
- ✓ Realistic SO(10) GUT models consistent with all established results in particle experiments are viable. They can also explain the baryon-anti baryon asymmetry in the Observed Universe via thermal leptogenesis.
- ✓ The most recent NANOGrav data does not support GW via stable Nambu-Goto strings, but that from metastable strings is allowed. A metastable string requires the B-L breaking scale very close to the GUT scale. Such case is supported in SUSY GUTs.

*Thank you for listening*

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Back up

# A benchmark point (BP1) study

Inputs	$a_1$ $63.57^\circ$	$a_2$ $84.17^\circ$	$c_\nu$ -1.945	$m_0$ 82.82 meV	$(\eta_u, \eta_c, \eta_t; \eta_d, \eta_s, \eta_b)$ $(+, +, -; +, -, +)$
Outputs	$\theta_{13}$ $8.53^\circ$	$\theta_{12}$ $32.7^\circ$	$\theta_{23}$ $41.9^\circ$	$\delta$ $-125^\circ$	$m_1$ $3.36$ meV
$(\chi^2 = 0.33)$	$m_{\beta\beta}$ $5.83$ meV		$M_{N_1}$ $4.23 \cdot 10^{11}$ GeV	$M_{N_2}$ $5.32 \cdot 10^{11}$ GeV	$M_{N_3}$ $1.66 \cdot 10^{13}$ GeV

$$Y_u = \begin{pmatrix} 2.54 \cdot 10^{-6} & 0 & 0 \\ 0 & -0.00137 & 0 \\ 0 & 0 & -0.428 \end{pmatrix}, \quad Y_e = 10^{-2} \cdot \begin{pmatrix} -0.0018 & 0.0012 - 0.0111i & -0.0008 + 0.004i \\ 0.0012 + 0.0111i & -0.0003 & -0.0009 - 0.2304i \\ -0.0008 - 0.0004i & -0.0009 + 0.2304i & 0.9155 \end{pmatrix},$$

$$Y_d = 10^{-2} \cdot \begin{pmatrix} 0.0056 & -0.0039 + 0.0014i & 0.0024 - 0.0i \\ -0.0039 - 0.0014i & -0.0100 & 0.0029 + 0.0281i \\ 0.0024 + 0.0i & 0.0029 - 0.0281i & 0.5686 \end{pmatrix}, \quad Y_\nu = 10^{-2} \cdot \begin{pmatrix} -0.7743 & 0.5374 + 0.1348i & -0.3379 - 0.0049i \\ 0.5374 - 0.1348i & 1.1586 & -0.3979 + 2.8068i \\ -0.3379 + 0.0049i & -0.3979 - 2.8068i & -6.4066 \end{pmatrix},$$

$$M_{\nu_R} = 10^{13} \cdot \begin{pmatrix} -0.0354 & 0.0246 & -0.0154 \\ 0.0246 & 0.0467 & -0.0182 \\ -0.0154 & -0.0182 & 1.6650 \end{pmatrix} \text{ GeV}. \quad M_\nu = 10^{-2} \cdot \begin{pmatrix} -0.5269 + 0.0i & 0.3628 + 0.0090i & -0.0434 - 0.0446i \\ 0.3628 + 0.0090i & 0.7407 - 0.0058i & -0.3755 - 2.417i \\ -0.0434 - 0.0446i & -0.3755 - 2.4168i & -2.9181 + 2.0125i \end{pmatrix} \text{ eV}$$