Multiple Modular Models

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The Standard Model is very successful but...

- Neutrinos have masses (νSM)
- Dark matter (no viable explanation)
- Matter / antimatter asymmetry (no viable explanation)
- Hierarchy problem (fine-tuning between parameters)
- Strong CP problem (fine-tuning between parameters)
- Gauge couplings (additional free parameters) - GUT?
- Flavour problem (many additional free parameters) - FS?

BSM solutions involve additional fields and symmetries

The Standard Model flavour problem

3 fermion generations? Masses span orders of magnitude?
3 generations of quarks, small mixing
3 generations of leptons, large and peculiar mixing
(mixing between weak and mass eigenstates)

Beyond the Standard Model with family symmetries

Without $y_f H F f_R$, $\mathcal{L}_{\nu SM}$ has accidental symmetry $SU(3)^6$

FS: upgrade subgroup of $SU(3)^6$ to actual symmetry of $\mathcal{L}$

1. Generations charged differently under FS
2. Yukawa couplings no longer invariant
3. FS must be broken somehow...

Non-Abelian?

3 reasons

- 3 generations explained naturally
- νSM: FS $\subset SU(3)^6$; $SO(10)$ GUT: FS $\subset SU(3)$
- Lepton mixing strongly suggests non-Abelian FS

Modular Hierarchies

See e.g.

King, King
https://arxiv.org/abs/2002.00969
Hierarchies with weightons

Novichkov, Penedo, Petcov
https://arxiv.org/abs/2102.07488
Hierarchies near stabilisers

Stabilisers / Fixed points

See:
Ding, King, Liu, Lu `https://arxiv.org/abs/1910.03460`
IdMV, Levy, Zhou `https://arxiv.org/abs/2008.05329`

Example: $S_\tau \tau = -\frac{1}{\tau} \quad S_\tau[\tau = i] = -\frac{1}{i} = i = \tau$

Stabilisers S_3

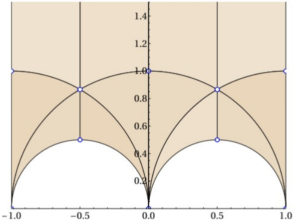


Figure 2: The fundamental domain $\mathcal{D}(2)$ of $\bar{\Gamma}(2)$ (i.e., the full target space of $\Gamma_2 \simeq S_3$) with the stabilisers of modular transformations of Γ_2 denoted as dots.

	γ	τ_γ
$\mathcal{C}_2$	$T_\tau C_\tau$	$0, 1+i$
	T_τ	$i\infty, \frac{1}{2}+\frac{i}{2}$
	S_τ	$i, 1$
$\mathcal{C}_3$	$T_\tau S_\tau$	$-\frac{1}{2}+\frac{i\sqrt{3}}{2}, \frac{1}{2}+\frac{i\sqrt{3}}{2}$
	C_τ	$-\frac{1}{2}+\frac{i\sqrt{3}}{2}, \frac{1}{2}+\frac{i\sqrt{3}}{2}$

Table 1: The non-identity elements of Γ_2 and respective stabilisers.

Stabilizers method

Method

We know what τ stabilizes →

1. Take $\tau = \tau_i$, where $\tau_i = \gamma_i \tau_i, i = 1, ..., 4$ is a stabiliser of $\mathcal{D}$;
2. Act γ on τ: $\tau' = \gamma\tau$. Compute γ^{-1}; → Go from τ to τ' with γ
3. The element that stabilises τ' is given by $\gamma^{-1}\gamma_i\gamma$.

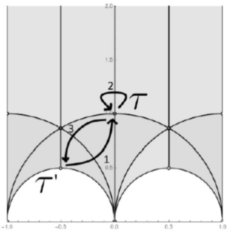


Figure 1: An example of the applied methodology to find the stabilisers of Γ_N. The example shown is for Γ_2, where the arrows denote the actions of different elements, γ^{-1}, γ_i, γ, for 1,2,3 respectively, following the convention of the text.

Modular forms at stabilisers

$$\rho_I(\gamma)\, Y_I(\tau_\gamma) = (c\tau_\gamma + d)^{-2k}\, Y_I(\tau_\gamma)\,.$$

$Y_I(\tau_\gamma)$ at stabiliser is eigenvector of $\rho_I(\gamma)$, eigenvalue $(c\tau_\gamma + d)^{-2k}$ (phase)

Multiple Modular?

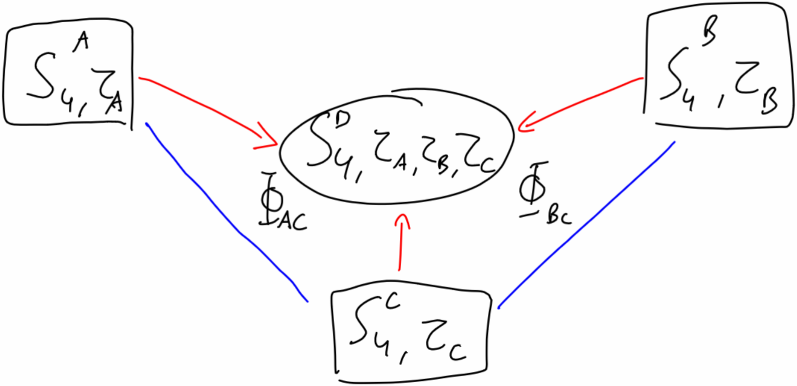
Multiple Modulus enables

- Multiple stabilizers / fixed points
- Stabilizers outside fundamental domain

Modular Models with residual symmetries!

The framework

IdMV, King, Zhou `https://arxiv.org/abs/1906.02208`
Basically, break multiples to diagonal subgroup.



The framework (paper version)

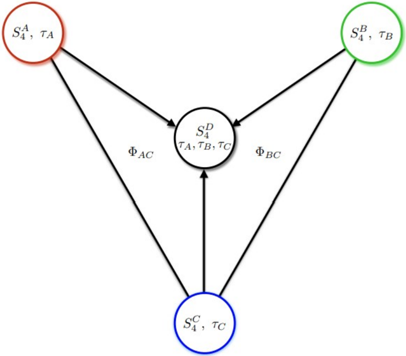


Figure 1: Illustration of the breaking of $S_4^A \times S_4^B \times S_4^C \to S_4^D$, identified as the diagonal subgroup, via the VEVs of Φ_{AC} and Φ_{BC}.

Example model with 3 S_4

Field	S_4^A	S_4^B	S_4^C	$2k_A$	$2k_B$	$2k_C$
L	**1**	**1**	**3**	0	0	0
e^c	**1**	**1**	**1**	0	0	-6
μ^c	**1**	**1**	**1**	0	0	-4
τ^c	**1**	**1**	**1**	0	0	-2
N_A^c	**1**	**1**	**1**	-6	0	0
N_B^c	**1**	**1**	**1**	0	-4	0
Φ_{AC}	**3**	**1**	**3**	0	0	0
Φ_{BC}	**1**	**3**	**3**	0	0	0

Example superpotential

$$
\begin{aligned}
w_\ell \;=\; & \frac{1}{\Lambda}\left[L\Phi_{AC} Y_A(\tau_A) N^c_A + L\Phi_{BC} Y_B(\tau_B) N^c_B\right] H_u \\
& + \left[L Y_e(\tau_C) e^c + L Y_\mu(\tau_C)\mu^c + L Y_\tau(\tau_C)\tau^c\right] H_d \\
& + \frac{1}{2} M_A(\tau_A) N^c_A N^c_A + \frac{1}{2} M_B(\tau_B) N^c_B N^c_B + M_{AB}(\tau_A,\tau_B) N^c_A N^c_B
\end{aligned}
$$

Example effective superpotential

$$w_\ell^{\text{eff}} = \left[\frac{v_{AC}}{\Lambda} L Y_A(\tau_A) N_A^c + \frac{v_{BC}}{\Lambda} L Y_B(\tau_B) N_B^c\right] H_u + (\ldots)$$

Multiple Multiple Modular Models

King, Zhou
`https://arxiv.org/abs/1908.02770` (2 S_4)
King, Zhou
`https://arxiv.org/abs/2103.02633` ($SU(5)$)

More Multiple Multiple Modular Models

IdMV, Lourenço
https://arxiv.org/abs/2107.04042 (2 A_4)
IdMV, Lourenço
https://arxiv.org/abs/2206.14869 (2 A_5)

IdMV, King, Levy
https://arxiv.org/abs/2211.00654 (Littlest Modular)

See talk by Levy!

Conclusions

- Multiple Modular Models have specific advantages
- Modular symmetries are promising as the origin of flavour
- Models using Stabilisers enable residual symmetries