

The Relaxion: An update.

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CLUSTER OF EXCELLENCE
QUANTUM UNIVERSE



Universität Hamburg

This talk.

based on work together with Aleksandr Chatrchyan



[arXiv: 2210.01148](#) & [2211.15694](#)

Motivation: Origin of the Electroweak Scale .

The Hierarchy problem

The hierarchy problem.

If Standard Model is an effective field theory below M_{Planck}

$$V = m_H^2 h^2 + \lambda h^4 \quad \text{Why } |m_H^2| \ll M_{\text{Planck}}^2 \quad ?$$

Why does the Higgs vacuum reside so close to the critical line separating the phase with unbroken ($\langle h \rangle = 0$) from the phase with broken ($\langle h \rangle \neq 0$) electroweak symmetry?

Solutions to the Hierarchy Problem .

Adding a symmetry

- > **Supersymmetry**

- > **Global symmetry ...**

Experimental signals: partners

Lowering the cutoff

- > **Randall-Sundrum / Composite Higgs,**

- > **Large Extra Dimensions ...**

Experimental signals: resonances

Selecting a vacuum : **Relaxation (dynamics),**

Experimental signals: typically through cosmology

Motivation.

What if the weak scale is selected by cosmological dynamics, not symmetries?

Special point in parameter space:

$m^2_{\text{H}} = 0$ *not* related to a symmetry

Instead, related to early-universe dynamics!

Relaxion idea: Higgs mass parameter is field-dependent

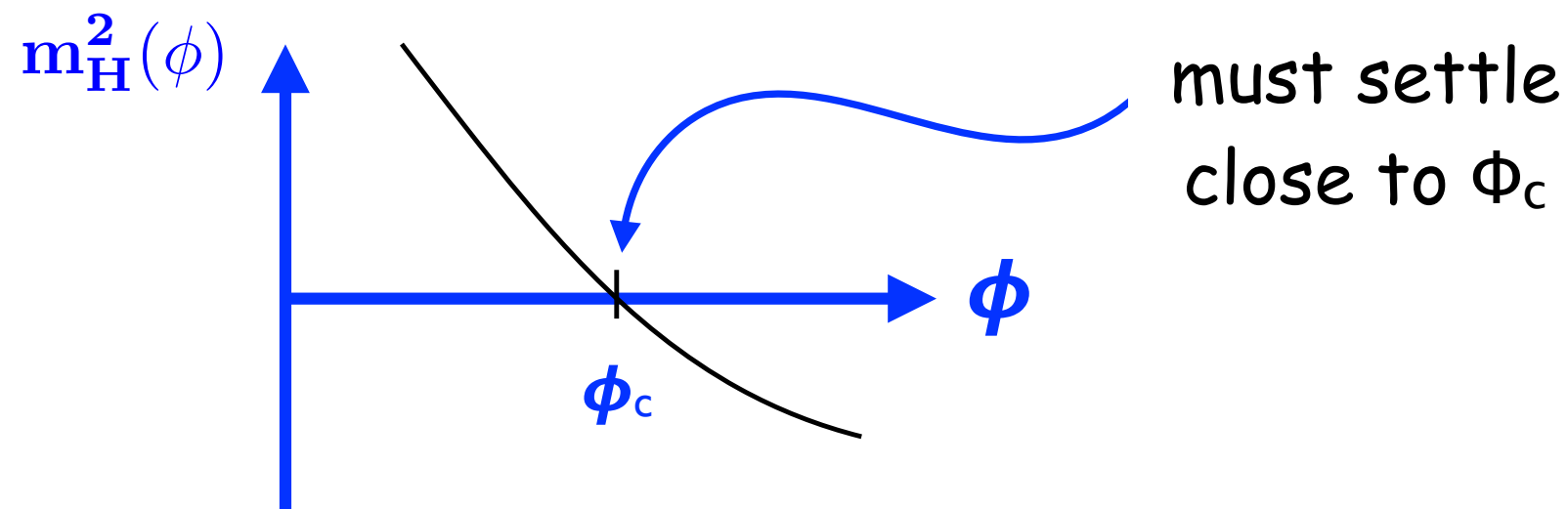
$$m_H^2 |H|^2 \rightarrow m_H^2(\phi) |H|^2$$

a new scalar field

ϕ can get a value such that $m_H^2(\phi) \ll \Lambda^2$

from a dynamical interplay between H and ϕ

UV cutoff



m_H naturally stabilized due to back-reaction of the Higgs field after EW symmetry breaking !

Relaxion mechanism.

[GKR: Graham, Kaplan, Rajendran '15]

inspired by Abbott's attempt to solve the Cosmological Constant problem, '85

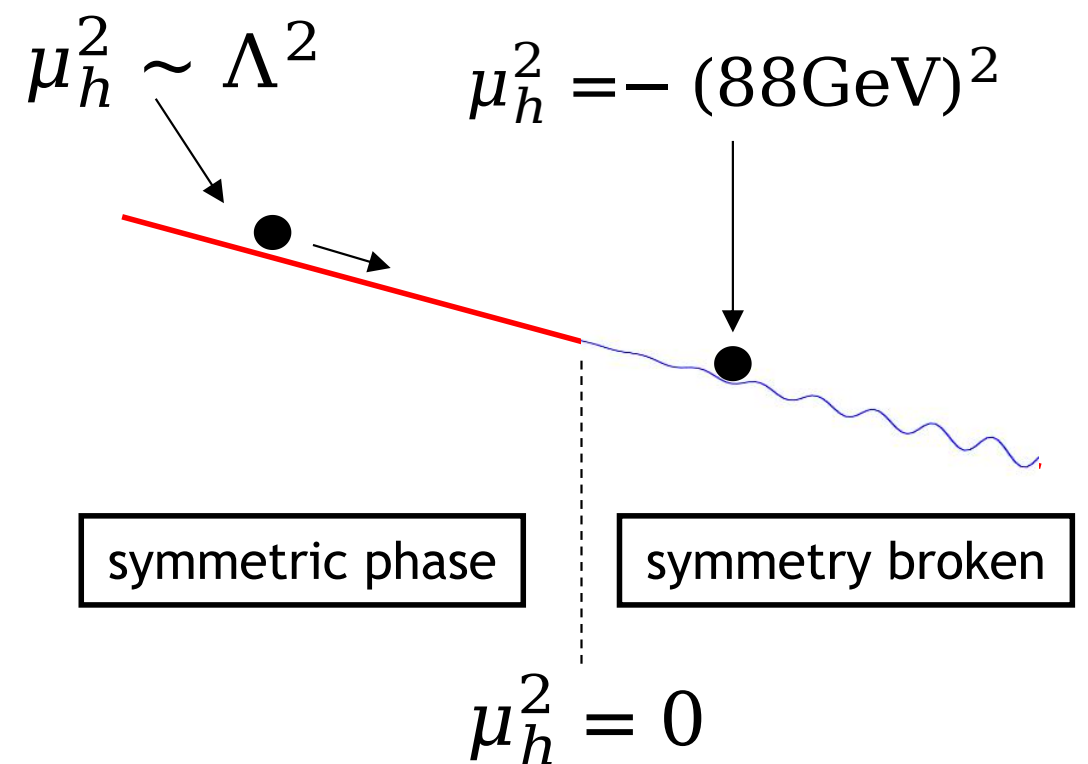
[for a recent update see

ϕ : relaxion, classically evolving pNGB.

Dynamical Higgs mass, controlled by vev of ϕ :

$$\mu_h^2 \rightarrow \mu_h^2(\phi) = \Lambda^2 - g\Lambda\phi$$

Λ : cutoff of the Higgs effective theory



Relaxion mechanism.

potential: $U(\phi) = -g\Lambda^3\phi + \Lambda_b^4(v_h)[1 - \cos(\phi/f)]$

**Rolling
potential**

Higgs-vev-dependent barriers

stopping mechanism:

Slow-roll dynamics during **inflation** $\dot{\phi}_{SR} = \frac{U'}{3H_I}$

Relaxion stops near the first minimum

$$0 = V'(\phi_0) = -g\Lambda^3 + \frac{\Lambda_b^4(\phi_0)}{f} \sin\left(\frac{\phi_0}{f}\right). \longrightarrow \Lambda_b^4 \sim g\Lambda^3 f$$

Relaxion mechanism.

[GKR: Graham, Kaplan, Rajendran '15]

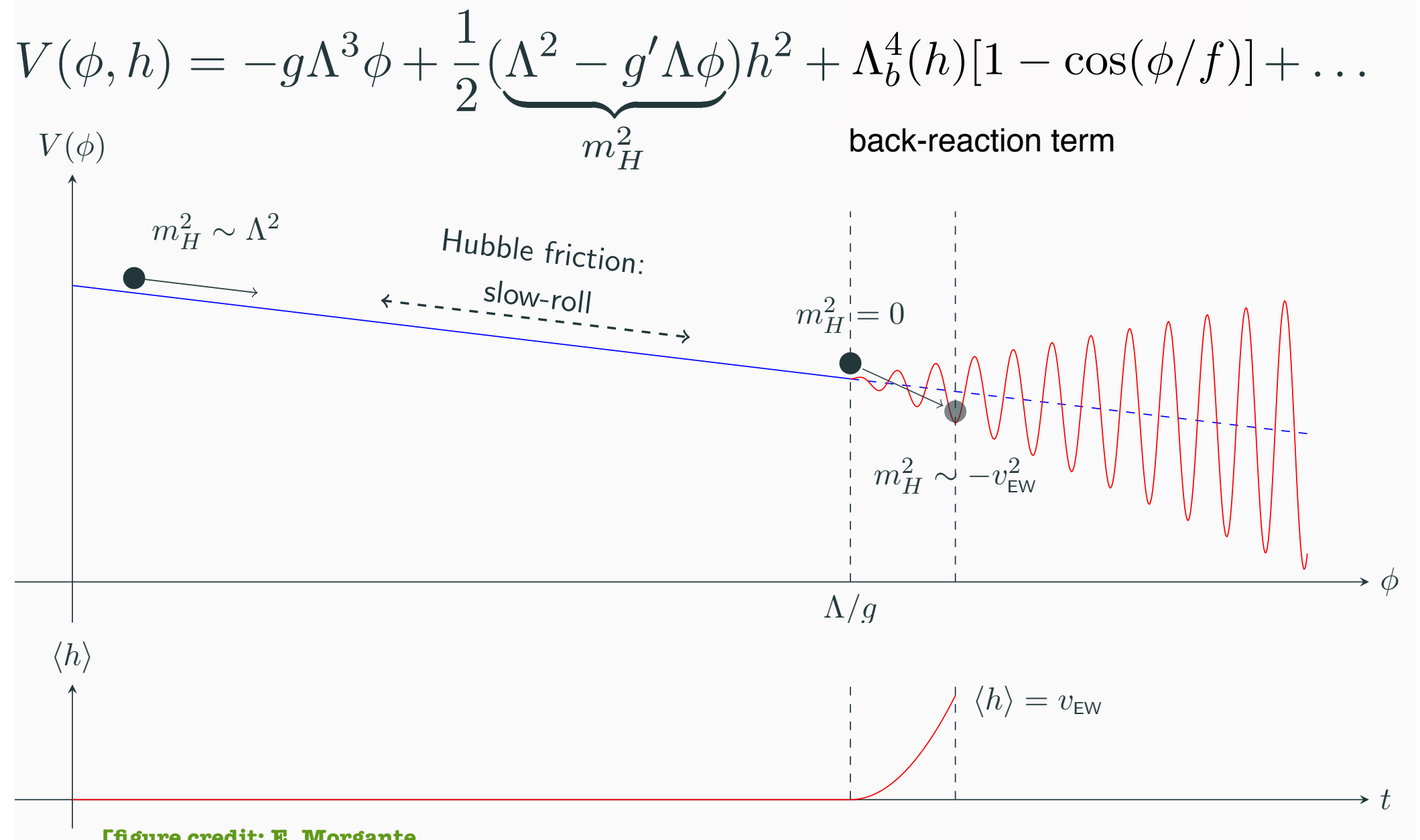
inspired by Abbott's attempt to solve the Cosmological Constant problem, '85

[for a recent update see

ϕ : relaxion, classically evolving pNGB.

Higgs-relaxion potential

Λ : cutoff of the Higgs effective theory



[figure credit: E. Morgante]

The QCD and non-QCD models.

The **QCD** relaxion model

- Higgs-dependent barriers from the **QCD anomaly**,

$$\Lambda_b^4(v_h) \approx \Lambda_{QCD}^3 m_u$$

- Problem: the relaxion no longer solves the **strong CP problem!**

$$\theta_{QCD} \sim \mathcal{O}(1)$$

The **nonQCD** relaxion model

- Higgs-dependent barriers from a **hidden gauge group**

$$\Lambda_b(v_h) < \sqrt{4\pi} v_h \quad (\text{stability of the potential})$$

The classical non-QCD relaxion window.

1) Vacuum energy

The **change of relaxion energy** much less compared to the **energy scale of inflation**

$$\Delta U \sim \Lambda^4 < H_I^2 M_{Pl}^2$$

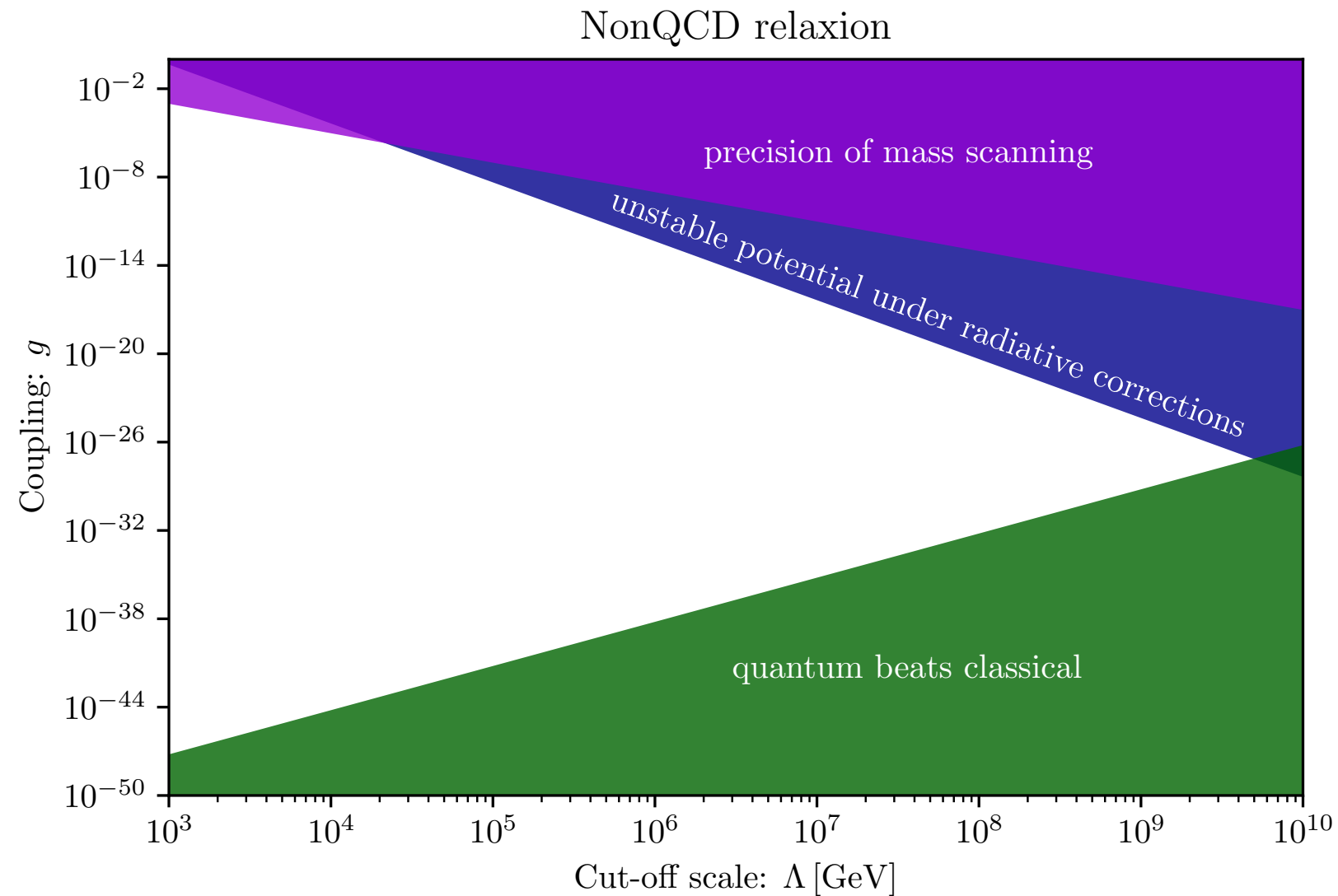
2) Classical beats quantum

The **slow-roll** ($\dot{\phi} = g\Lambda^3/3H_I$) per unit Hubble time dominates over the random walk ($\Delta\phi \sim H_I$)

$$H_I < (g\Lambda^3)^{1/3}$$

1) + 2) \longrightarrow

$$\frac{\Lambda^2}{M_{Pl}} < H_I < g^{1/3} \Lambda$$

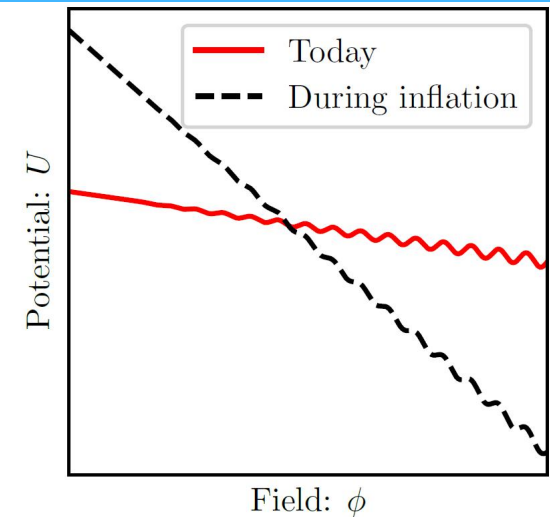


$$\Lambda < 4 \times 10^9 \text{ GeV} \left(\frac{\Lambda_b}{\sqrt{4\pi} v_h} \right)^{4/7}$$

The classical QCD relaxion window

Local minima are not CP conserving

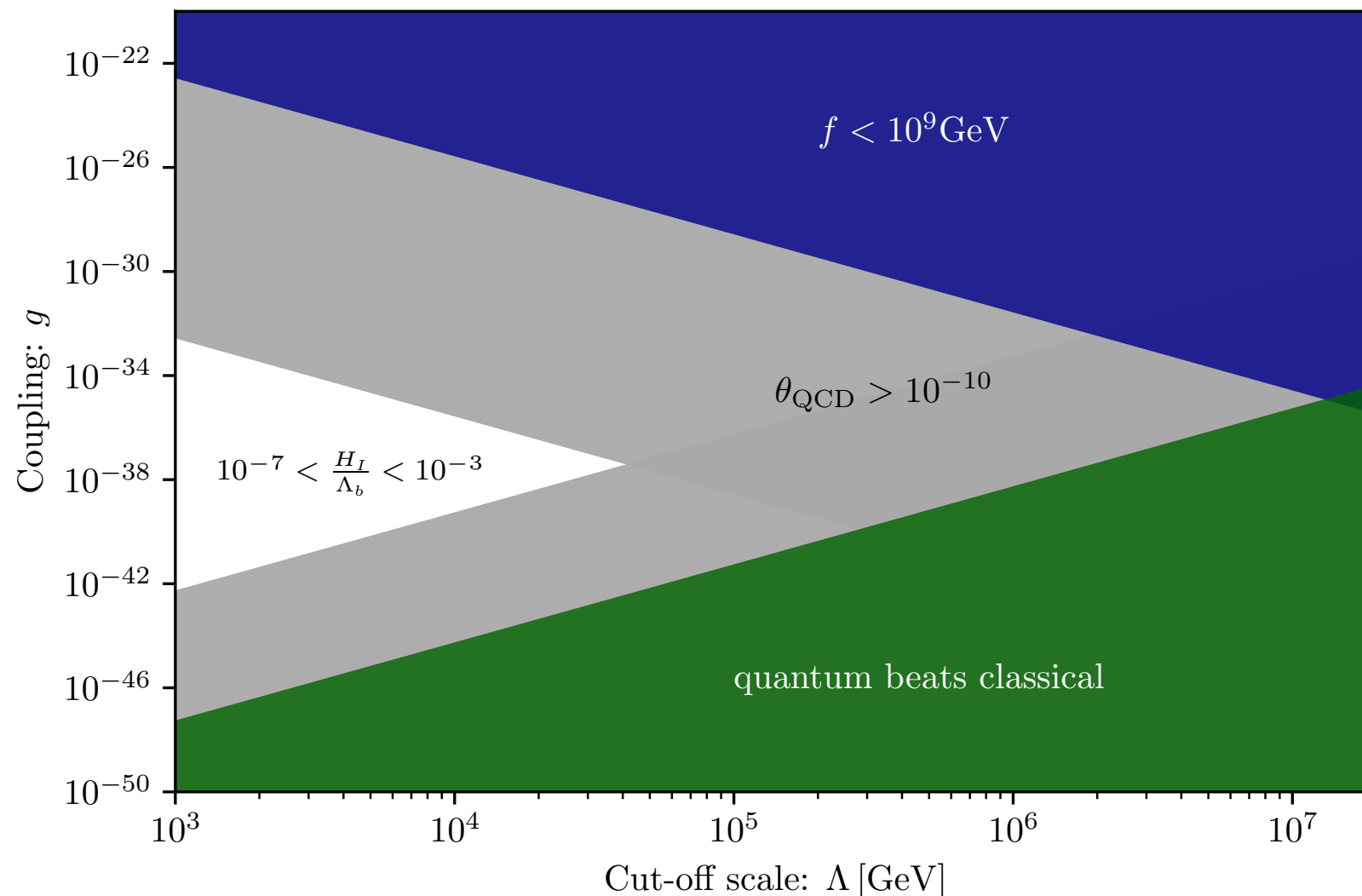
$$0 = U'(\bar{\theta}) = -g\Lambda^3 + \frac{\Lambda_b^4}{f} \sin \bar{\theta} \quad \longrightarrow \quad \bar{\theta} = \arcsin\left(\frac{g\Lambda^3 f}{\Lambda_b^4}\right)$$



Solution: the slope of the potential drops after inflation to reduce CP violation

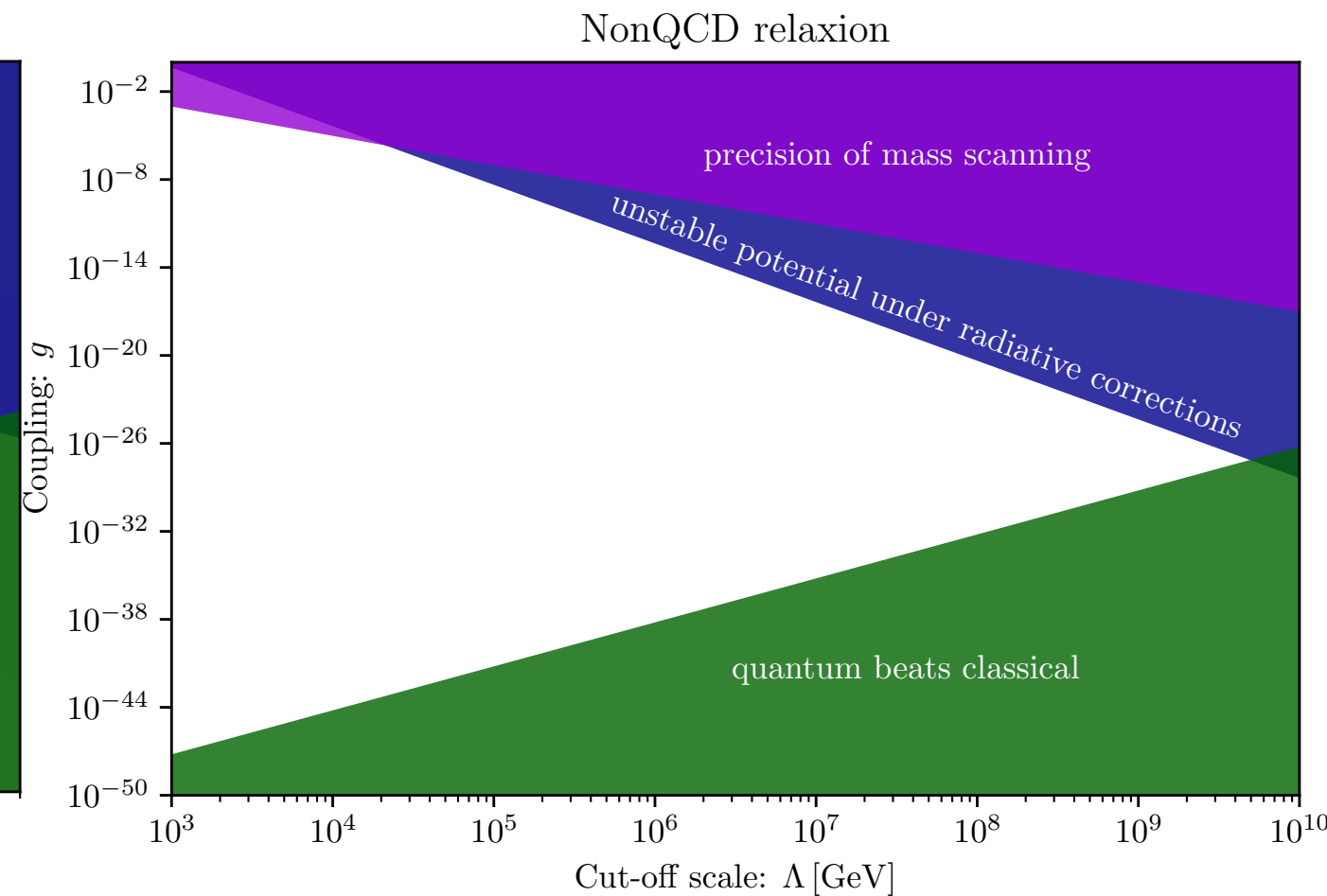
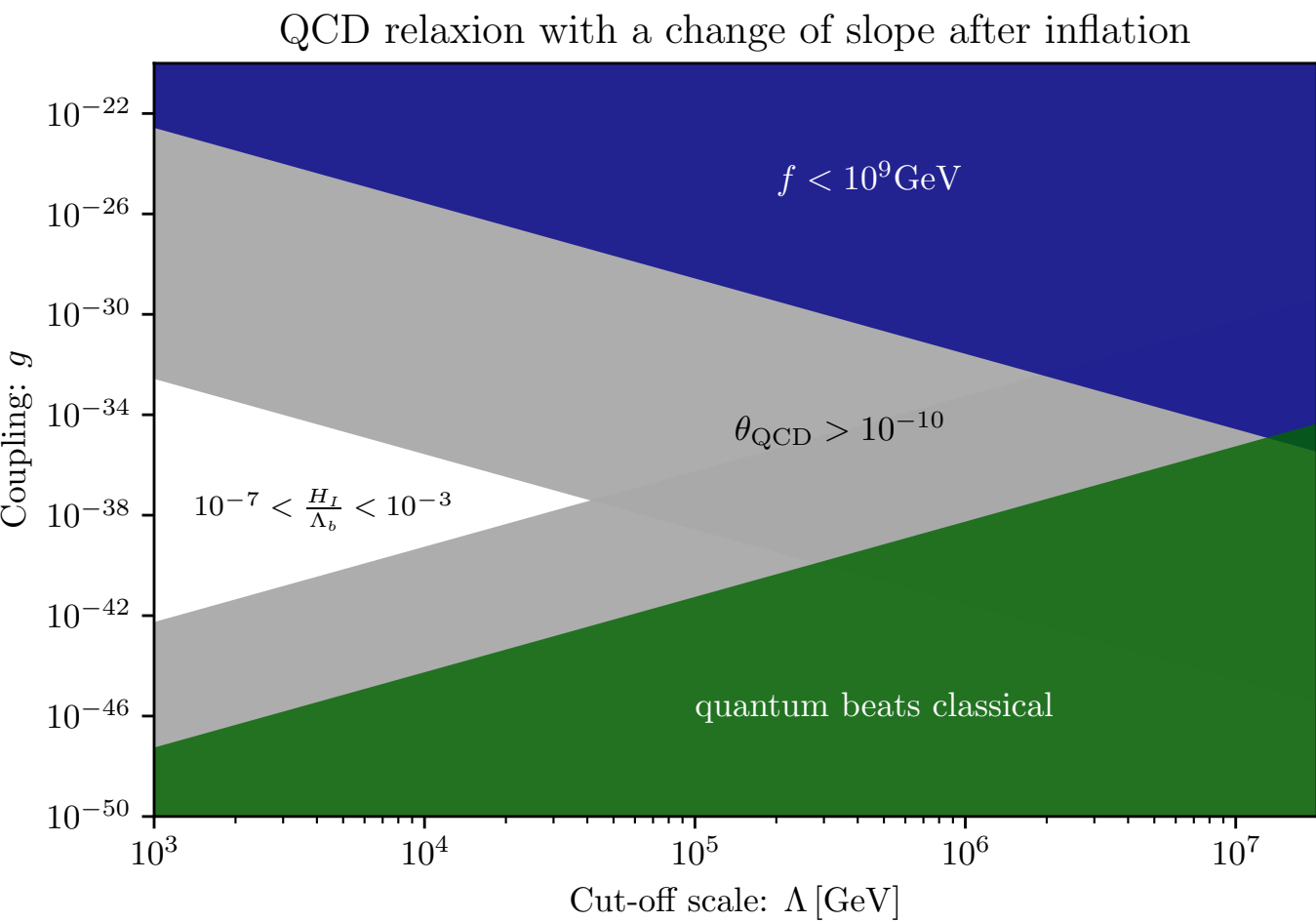
$$g = \xi g_I, \quad \xi < 10^{-10}, \quad \bar{\theta} = \xi \bar{\theta}_I < 10^{-10}.$$

QCD relaxion with a change of slope after inflation



$$\Lambda < 3 \times 10^4 \text{ GeV} \left(\frac{10^9 \text{ GeV}}{f} \right)^{1/6} \left(\frac{\xi}{10^{-10}} \right)^{1/4}.$$

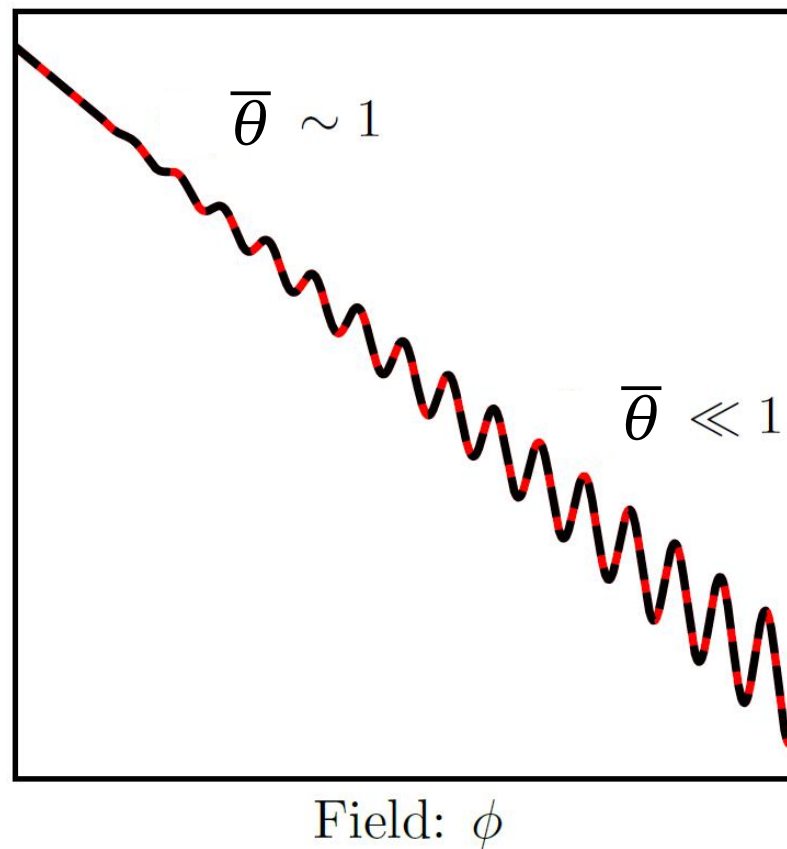
The classical relaxion windows



Role of quantum fluctuations during inflation: The Stochastic Relaxion .

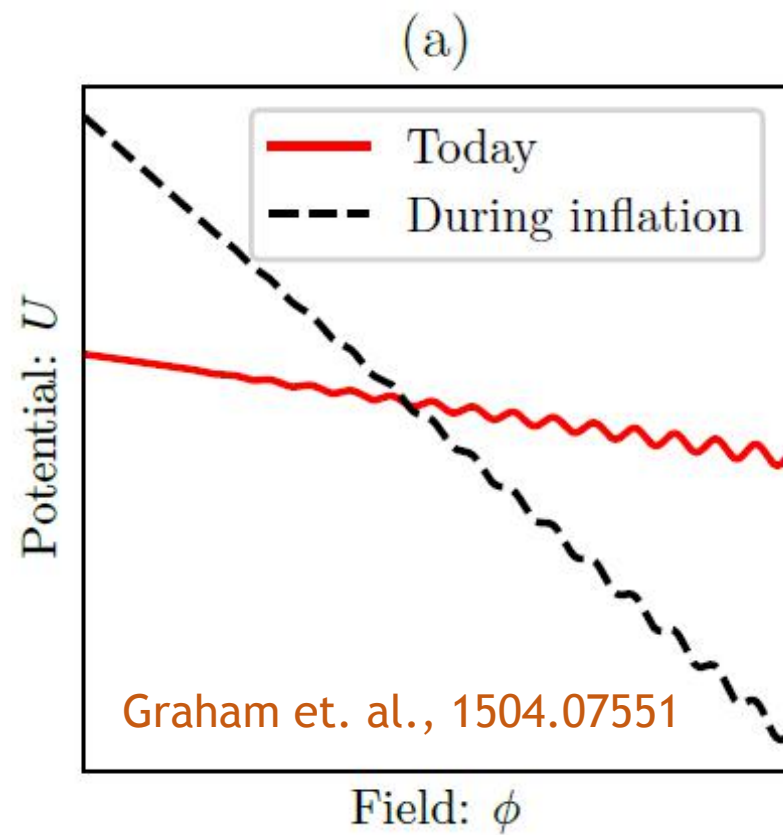
The relaxion stops near the first local minimum, unless the Hubble parameter during inflation is large enough so that the random walk prevents it from getting trapped.

CP is less violated if the relaxion stops at a much deeper minimum .

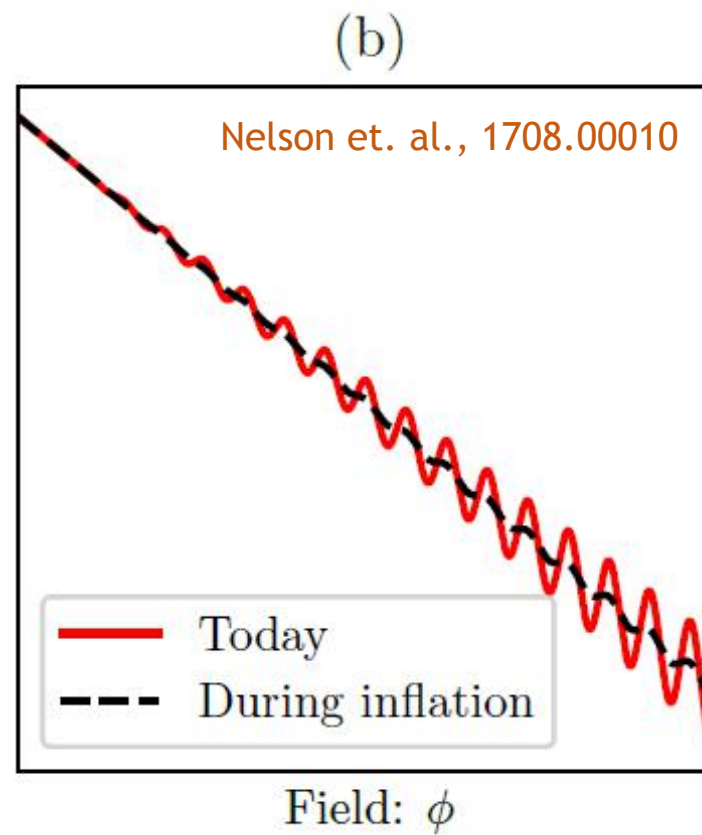


Can fluctuations during inflation modify the stopping condition?

Different approaches to the QCD relaxion



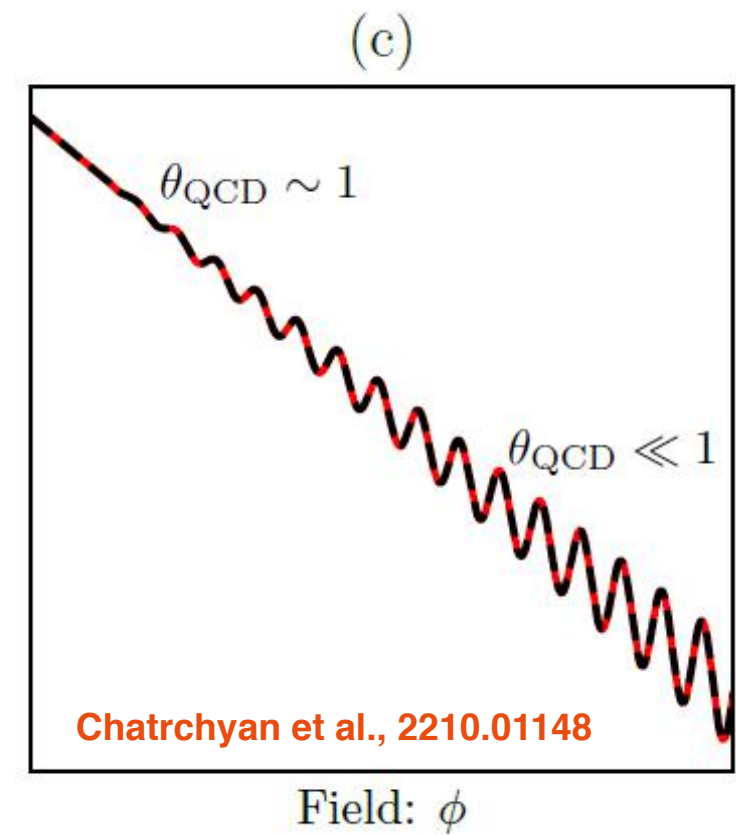
$$10^{-7} \Lambda_b < H_I < 10^{-3} \Lambda_b$$



$$3\text{GeV} < H_I < 100\text{GeV}$$

$$\Lambda_b^4(T, h) \approx \frac{\Lambda_b^4(0, h)}{1 + (T/\Lambda_{\text{QCD}})^m}$$

Wrong stopping condition
used, $\Lambda_b^4(\phi; T_I) \sim g\Lambda^3 f$



$$H_I \sim \Lambda_b \approx 75\text{MeV}$$

Preview of this talk.

- **We revisit the original relaxion mechanism including the stochastic behavior of the relaxion**
- **Important consequences even in the “classical-beats-quantum” regime**
- **We explore the regime “quantum-beats-classical”**
- **Large new region of parameter space**
- **Relaxion can naturally be dark matter**

What if the “Classical-beats-Quantum” (CbQ) condition is dropped ?

The relaxion stops near the first local minimum, unless the Hubble parameter during inflation is large enough so that the random walk prevents it from getting trapped.

The Fokker-Planck formalism .

Dynamics of quantum fluctuations of a light scalar field, $m \ll H_I$, in de Sitter spacetime can be described in terms a Fokker-Planck equation:

$$\frac{d\rho}{dt} = \boxed{\frac{1}{3H_I} \frac{\partial(\rho \partial_\phi V)}{\partial \phi}} + \boxed{\frac{H_I^3}{8\pi^2} \frac{\partial^2 \rho}{\partial \phi^2}}$$

Drift term

Diffusion term

$\rho(\phi)$ - probability distribution of the relaxion having its average field value inside the Hubble patch equal to ϕ at time t .

Random kicks from low- k superhorizon fluctuations

$$\phi = \langle \varphi(x) \rangle_{|x| < (aH_I)^{-1}} \sim \int_{k < aH_I} \varphi(k)$$

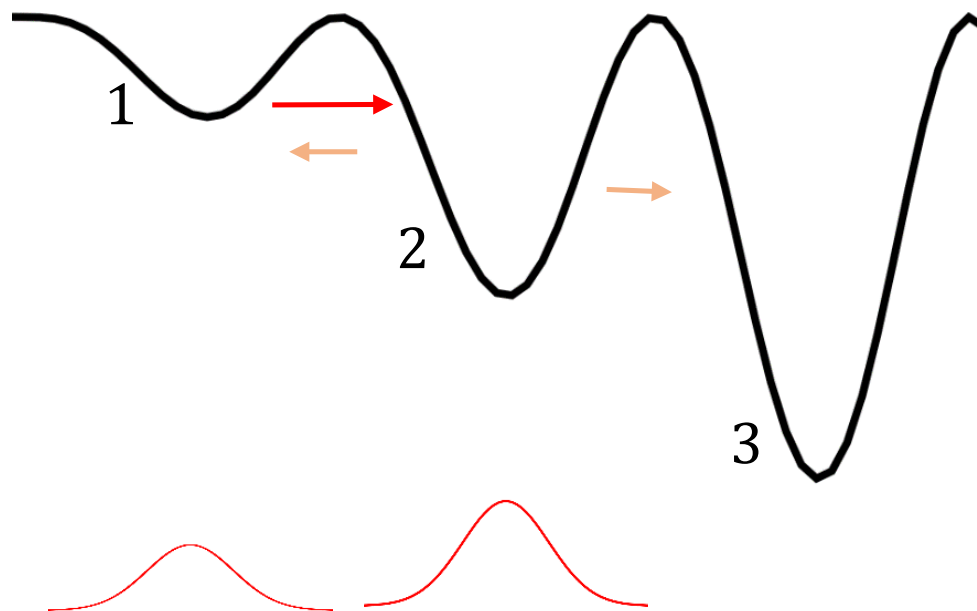
e.g. 9407016 [Starobinsky-Yokoyama]

Stochastic dynamics of the relaxion .

In the relaxion potential, each local minimum is followed by a deeper one.
Diffusion effects + slope of the potential \rightarrow nonzero flux for the distribution function.

Backwards flux of probability from the lower minimum is generated as well but is smaller due to the larger barriers in the backwards direction.

Diffusion generates a **flux of probability** to a lower minimum



$$\frac{dN_1}{dt} = -k_{12}^{\rightarrow} N_1 + k_{21}^{\leftarrow} N_2$$

$$k \approx \frac{\sqrt{V_0''|V_b''|}}{6\pi H_I} e^{-\frac{8\pi^2 \Delta V_b}{3H_I^4}}$$

Hawking-Moss
instanton

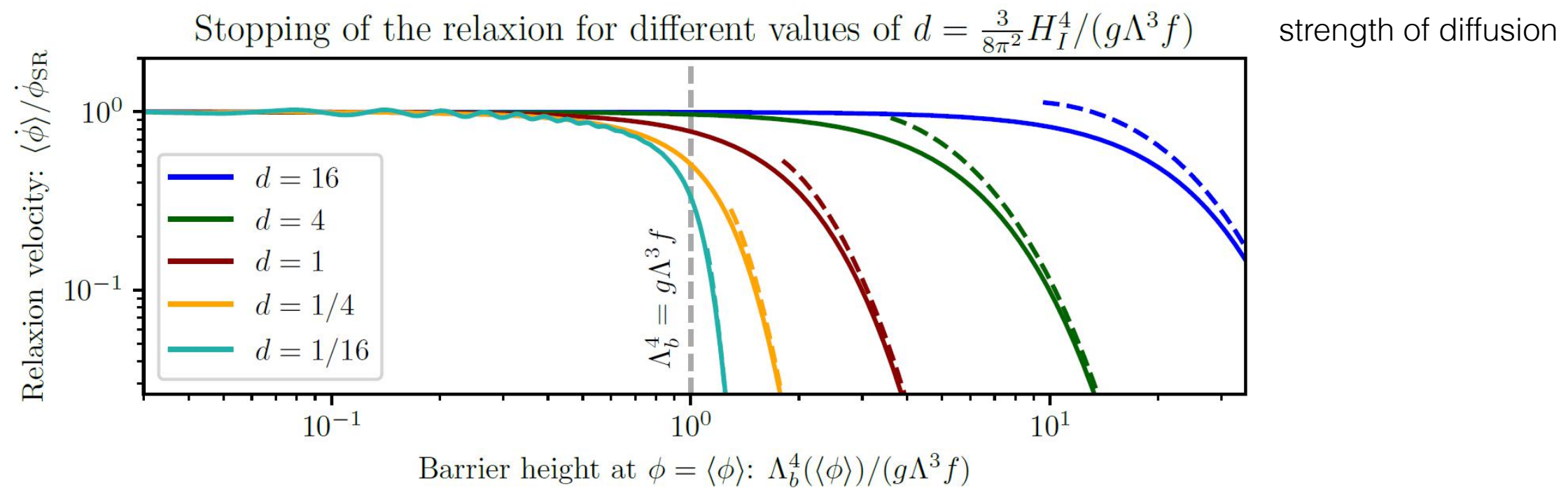
PLB 110 (1982) 35.

broadening of the distribution

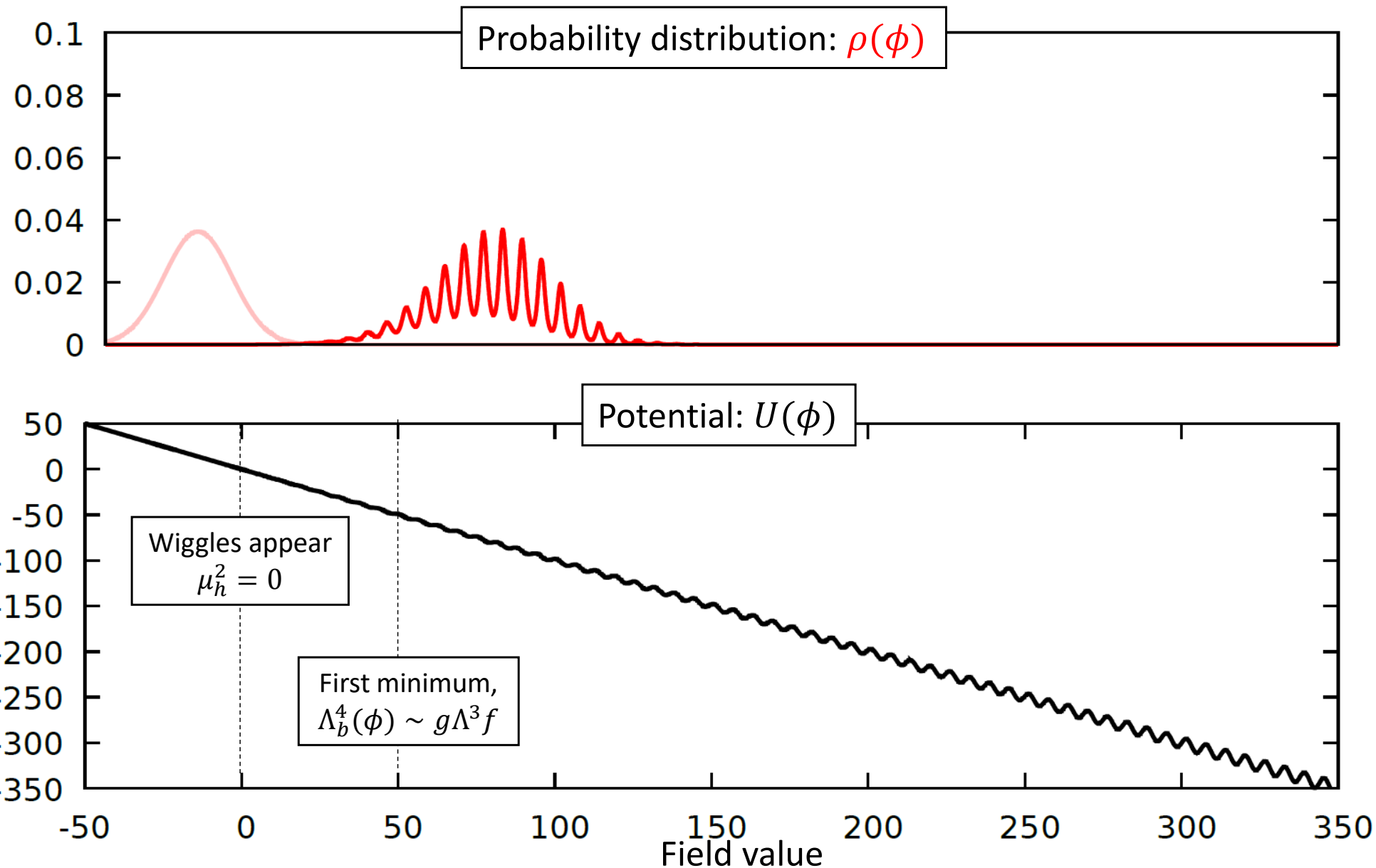
$$\sigma(t) = \sqrt{\frac{H_I^3}{4\pi^2} t}$$

Stochastic dynamics of the relaxion

**Modified stopping condition:
The relaxion is trapped at the minimum whose lifetime
is longer than the duration of inflation.**



Real-time numerical simulation of the FP equation .



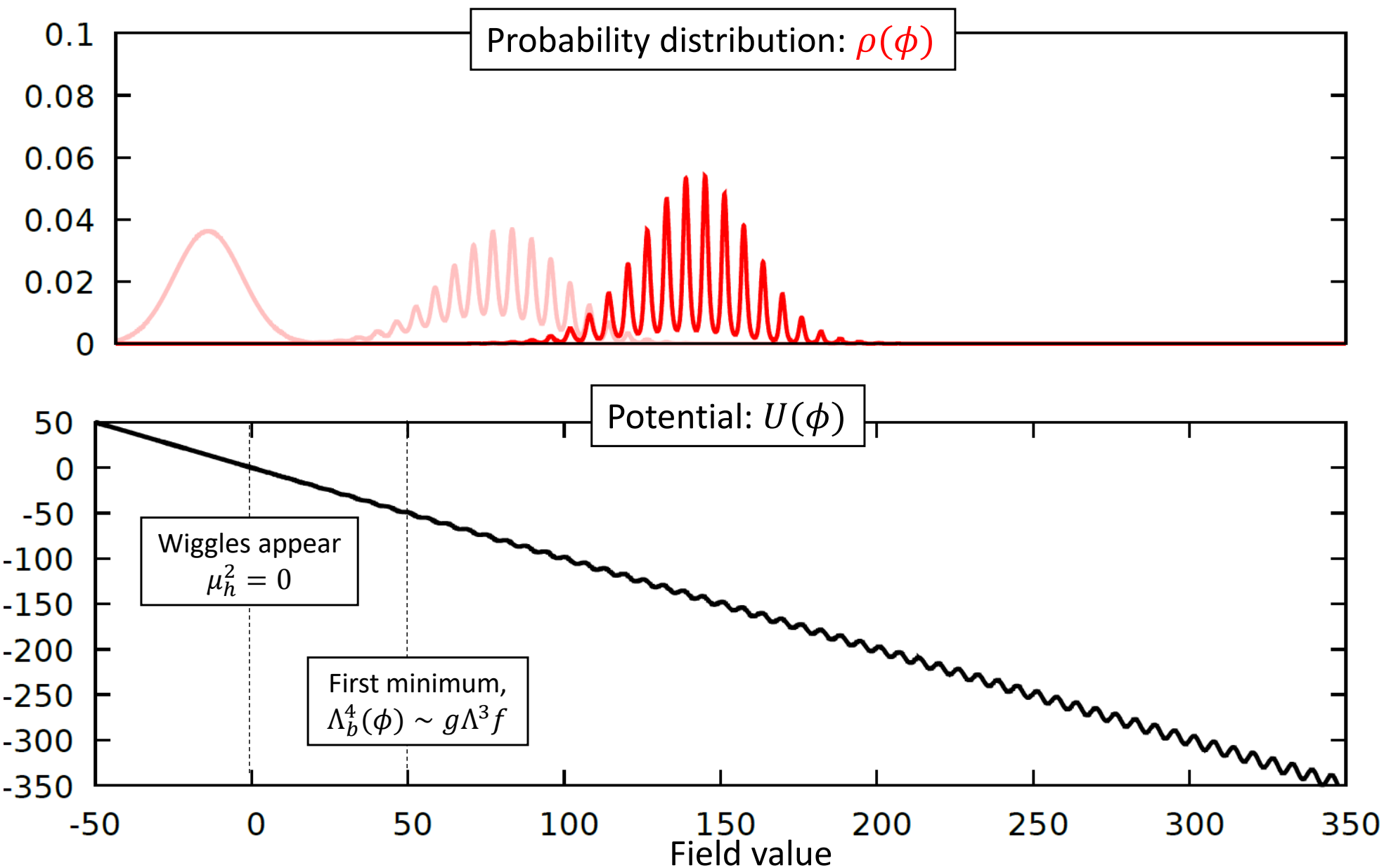
The relaxation slows down after

$$\frac{8\pi^2 \Delta V_b}{3H_I^4} < 1$$

The new stopping condition,

$$\Lambda_b^4 \sim \max\left(g\Lambda^3 f, \frac{3H_I^4}{8\pi^2}\right).$$

Real-time numerical simulation of the FP equation .



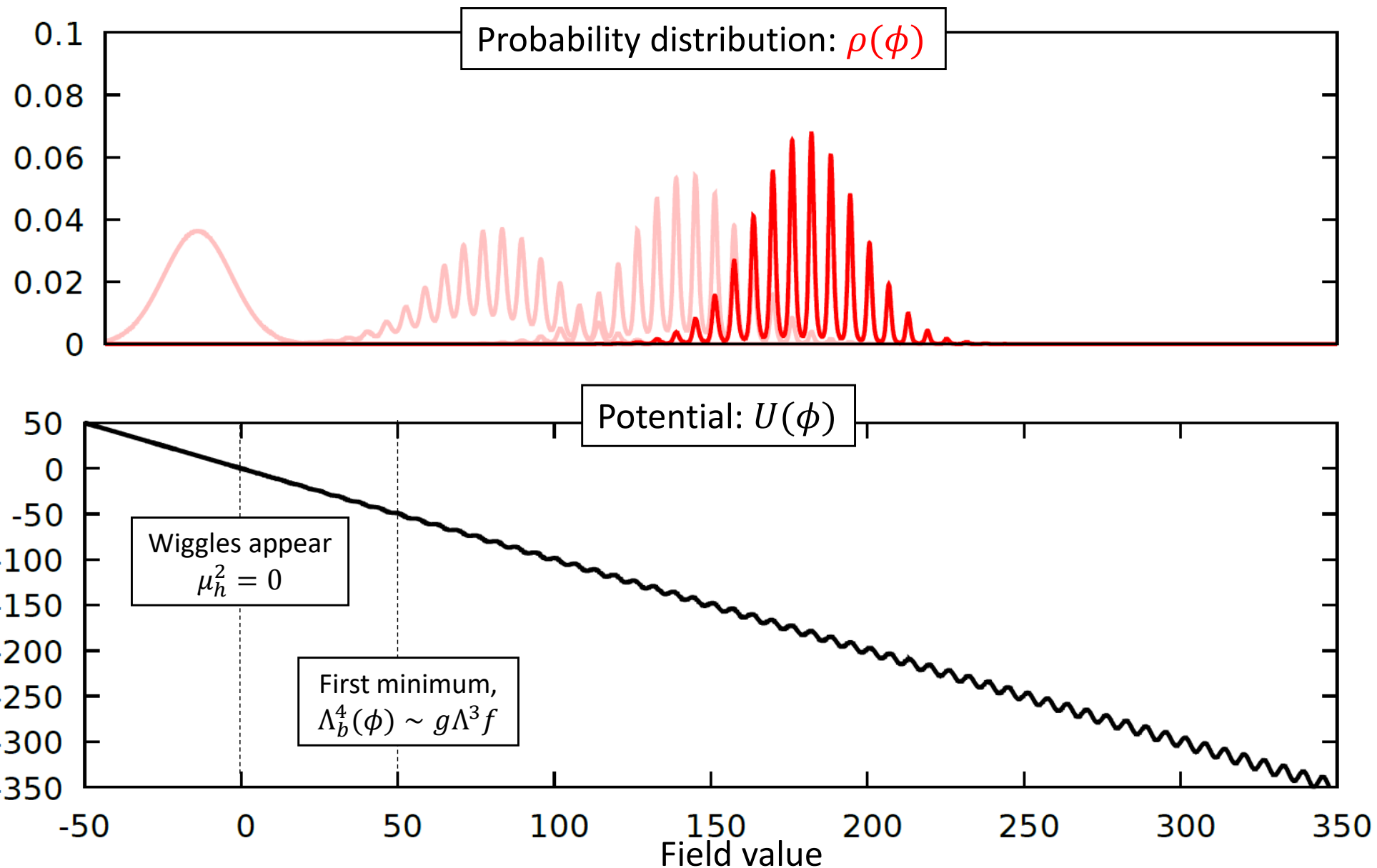
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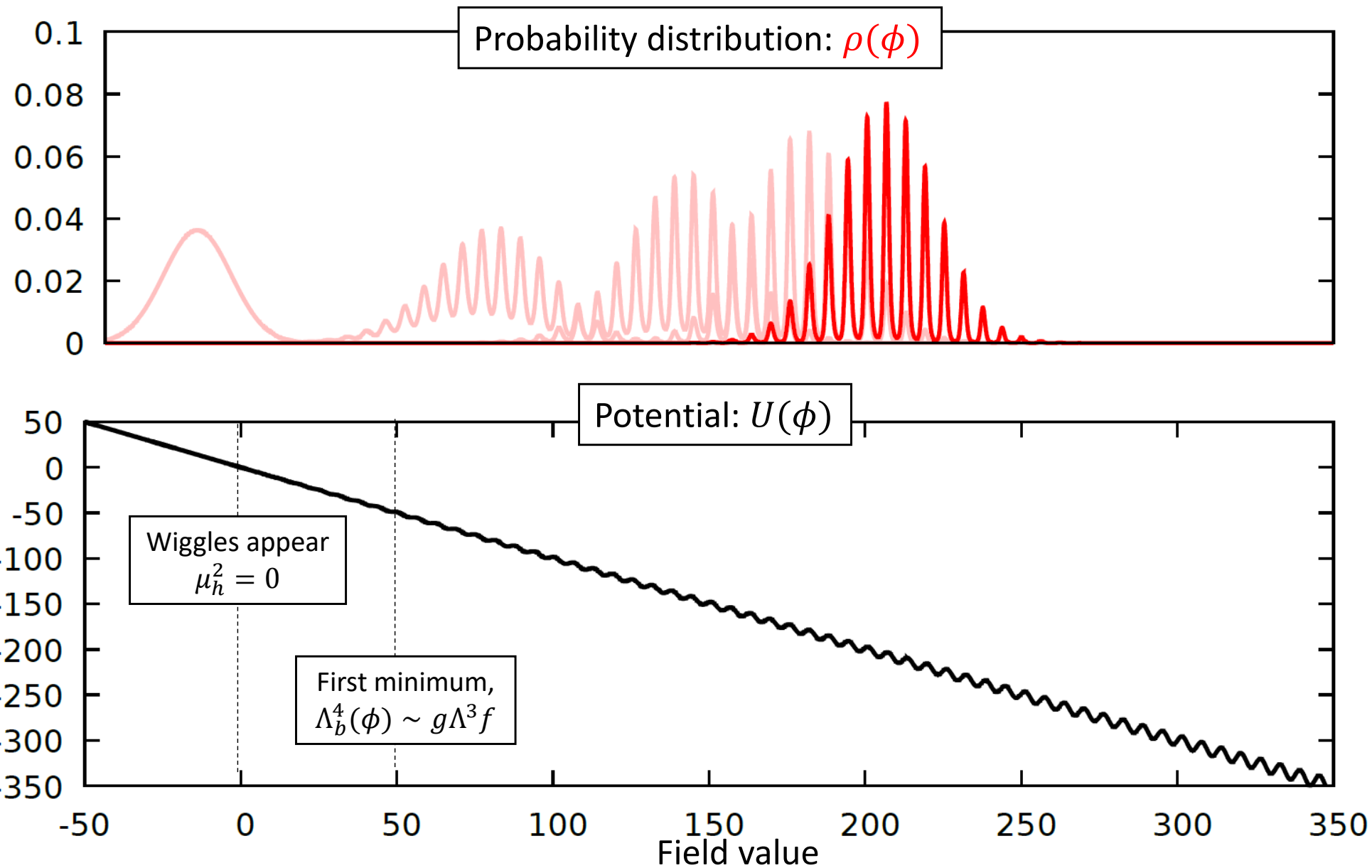
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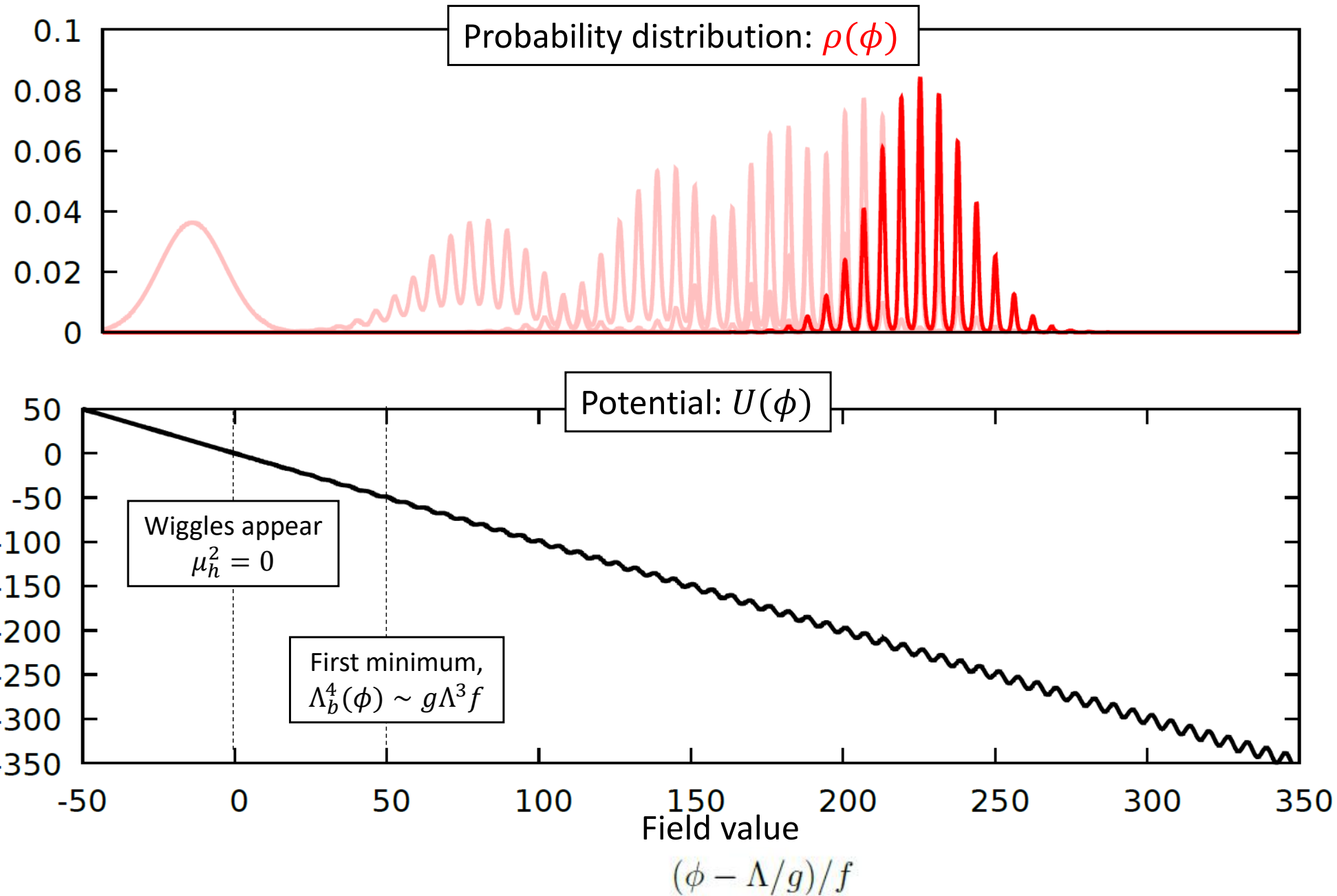
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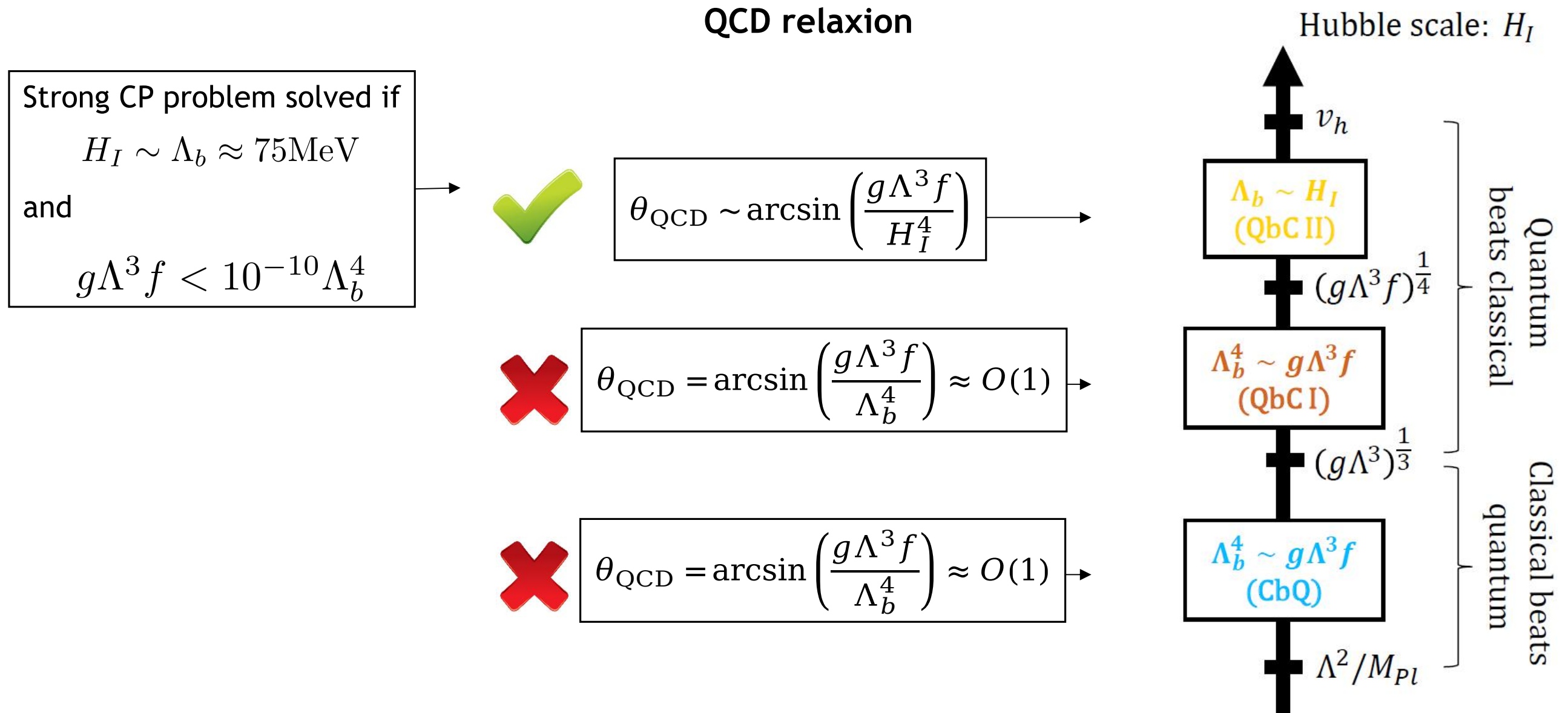
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$$\Lambda_b^4 \sim \max\left(g\Lambda^3 f, \frac{3H_I^4}{8\pi^2}\right).$$

Can the Relaxion be a QCD axion/solve the strong CP problem ?

former discussion: **A. Nelson and C. Prescod-Weinstein, 1708.00010.**

QCD Relaxion parameter space



Eternal Inflation

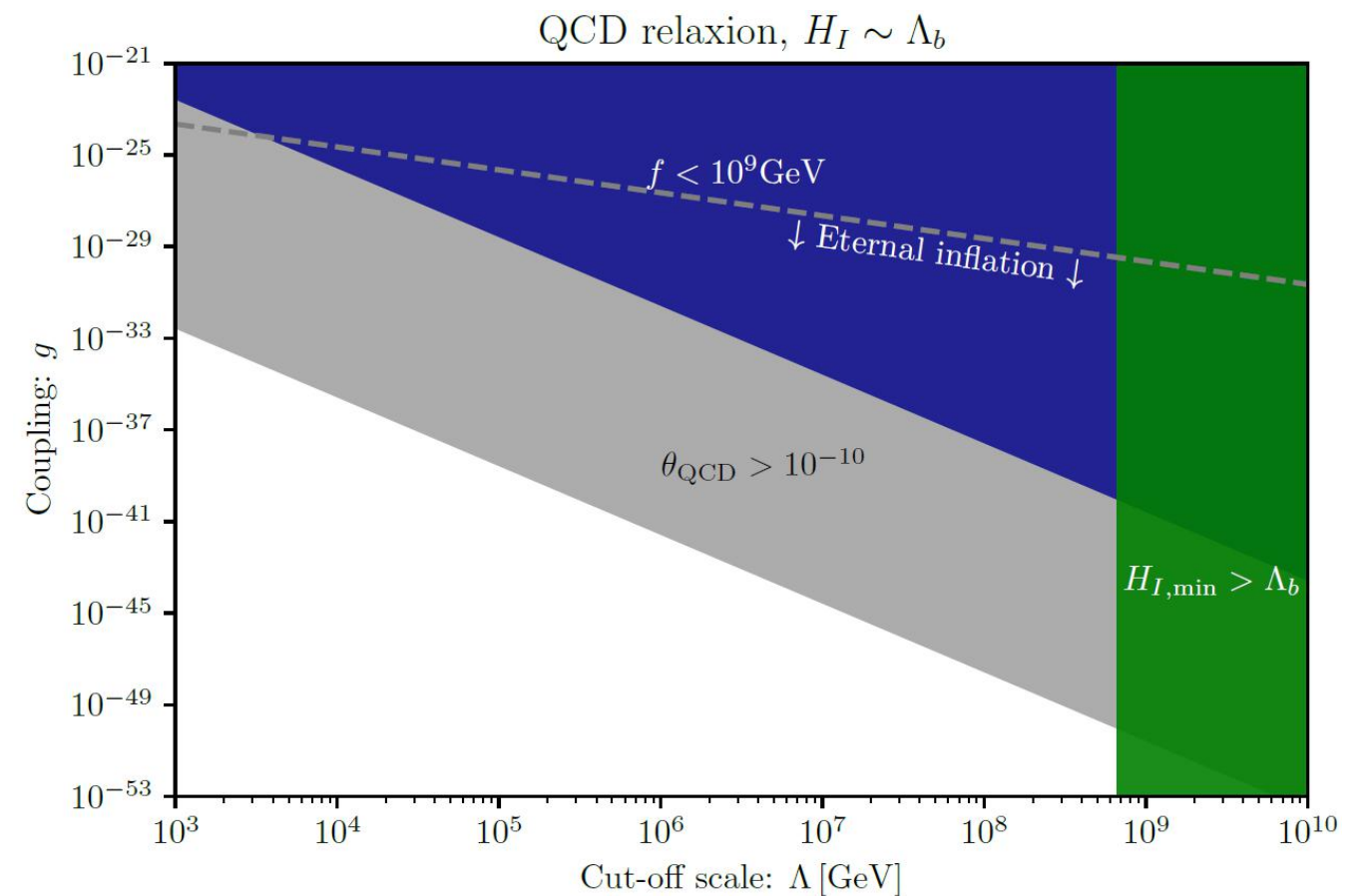
- The minimum number of e-folds of inflation required to relax the Higgs mass from $\mu_h \sim \Lambda$ to $\mu_h = 0$ is given by

$$N_I = H_I t_I > N_{\text{req}} = \frac{3H_I^2}{g^2 \Lambda^2}$$

- If $N_I > N_c \sim \frac{2\pi^2}{3} \frac{M_{\text{Pl}}^2}{H_I^2}$, inflation is **eternal**.

0802.1067

- Eternal inflation has associated measure problems.

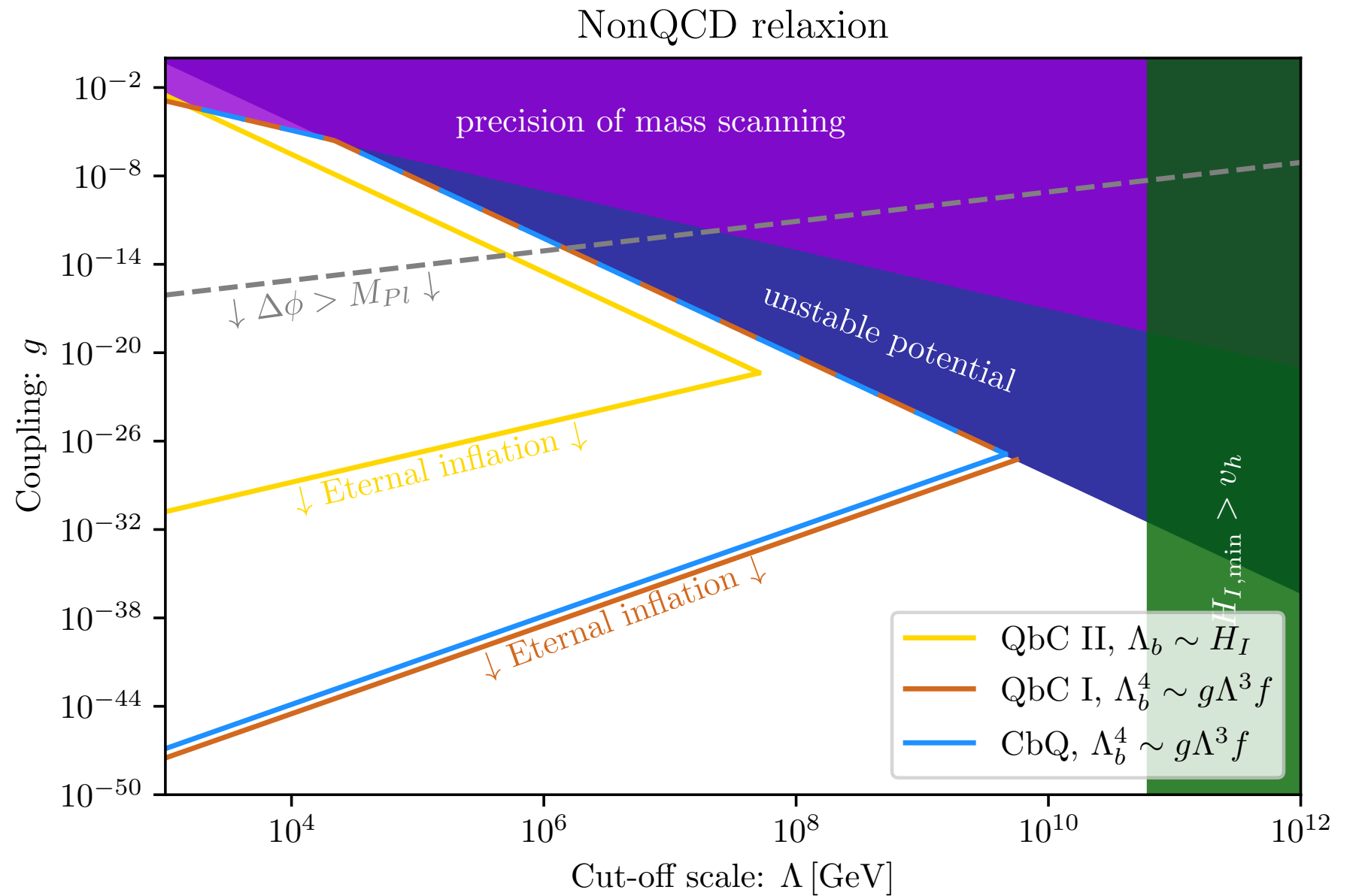
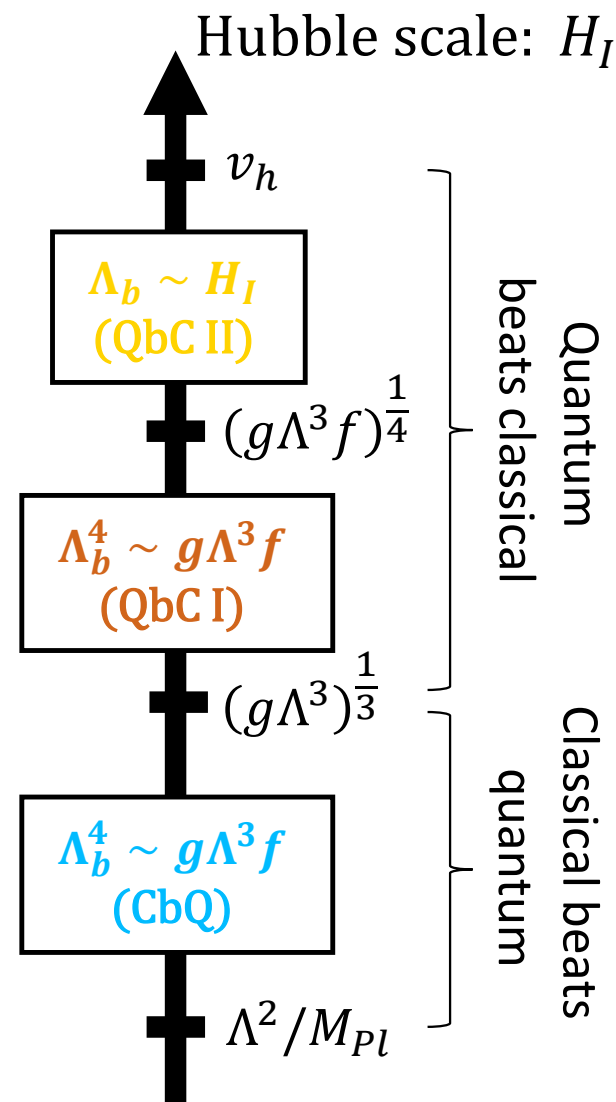


- Possible solution: using scale factor cut-off measure.

Nelson et. al., 1708.00010

Non-QCD Relaxion.

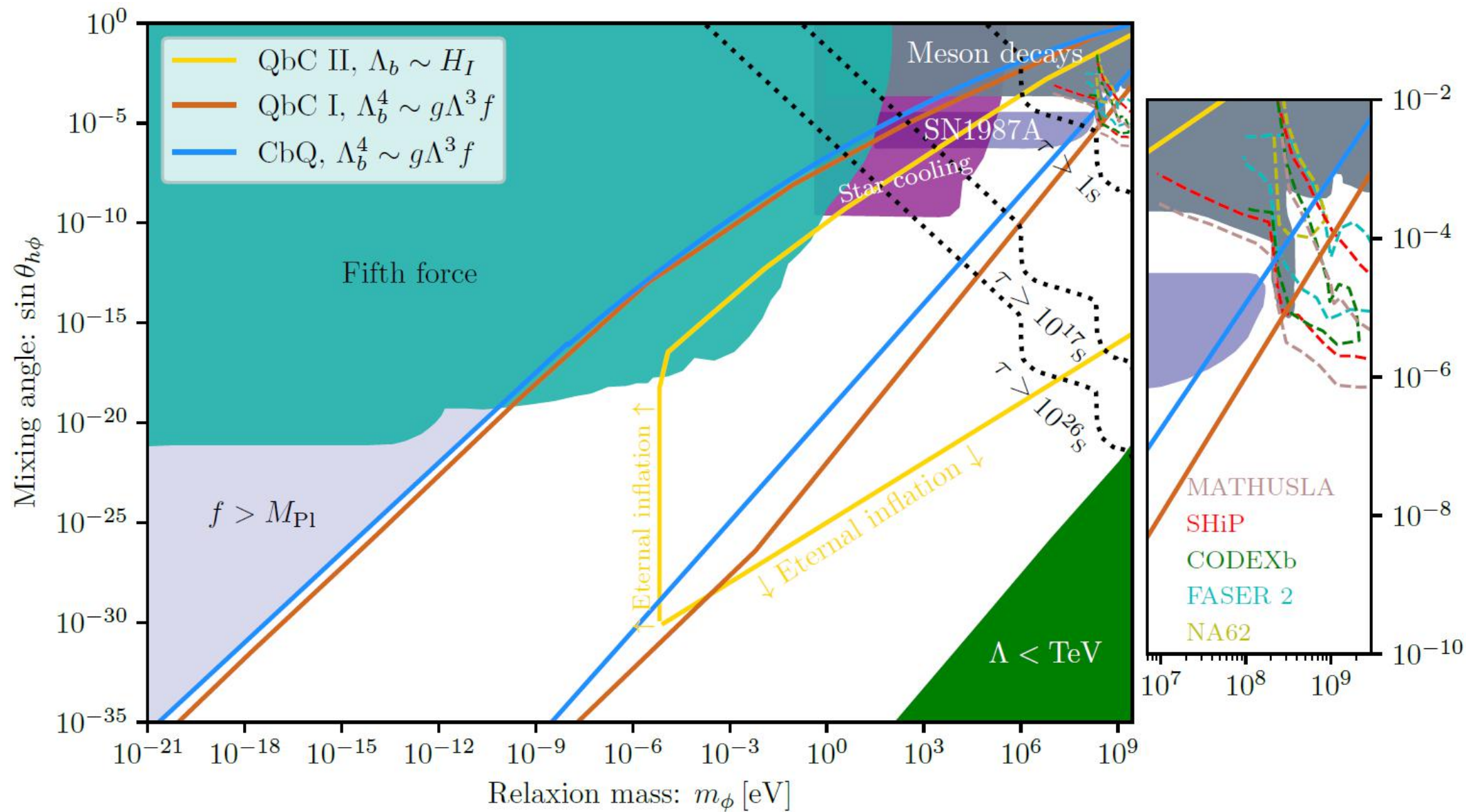
Dropping the Classical-beats-Quantum condition for the non-QCD relaxion



Interactions of the relaxion

The relaxion interacts via its mixing with the Higgs

$$\sin(2\theta_{h\phi}) = -\frac{2m_{h\phi}^2}{\sqrt{(2m_{h\phi}^2)^2 + (m_h^2 - m_\phi^2)^2}}.$$



Flacke et. al.,
1610.02025

Hardy et al.,
1611.05852

Balaji et al.,
2205.01699

Light and stable in most of the parameter space:
Can the relaxion be Dark Matter?

Relaxion Dark Matter from Stochastic Misalignment .

Relaxion Dark Matter .

Typical displacement of ϕ from the minimum: $\sigma_\phi^2 = \frac{3H_I^4}{8\pi^2 m_\phi^2}$

Can the relaxion explain **dark matter**?

“Classical beats quantum” regime (GKR):

Small relic density

$$\Omega_{\phi,0} < 2.6 \times 10^{-12} \left(\frac{\text{eV}}{m} \right)^{\frac{1}{6}}$$

unless a high reheating temperature

See Banerjee et. al., 1810.01889

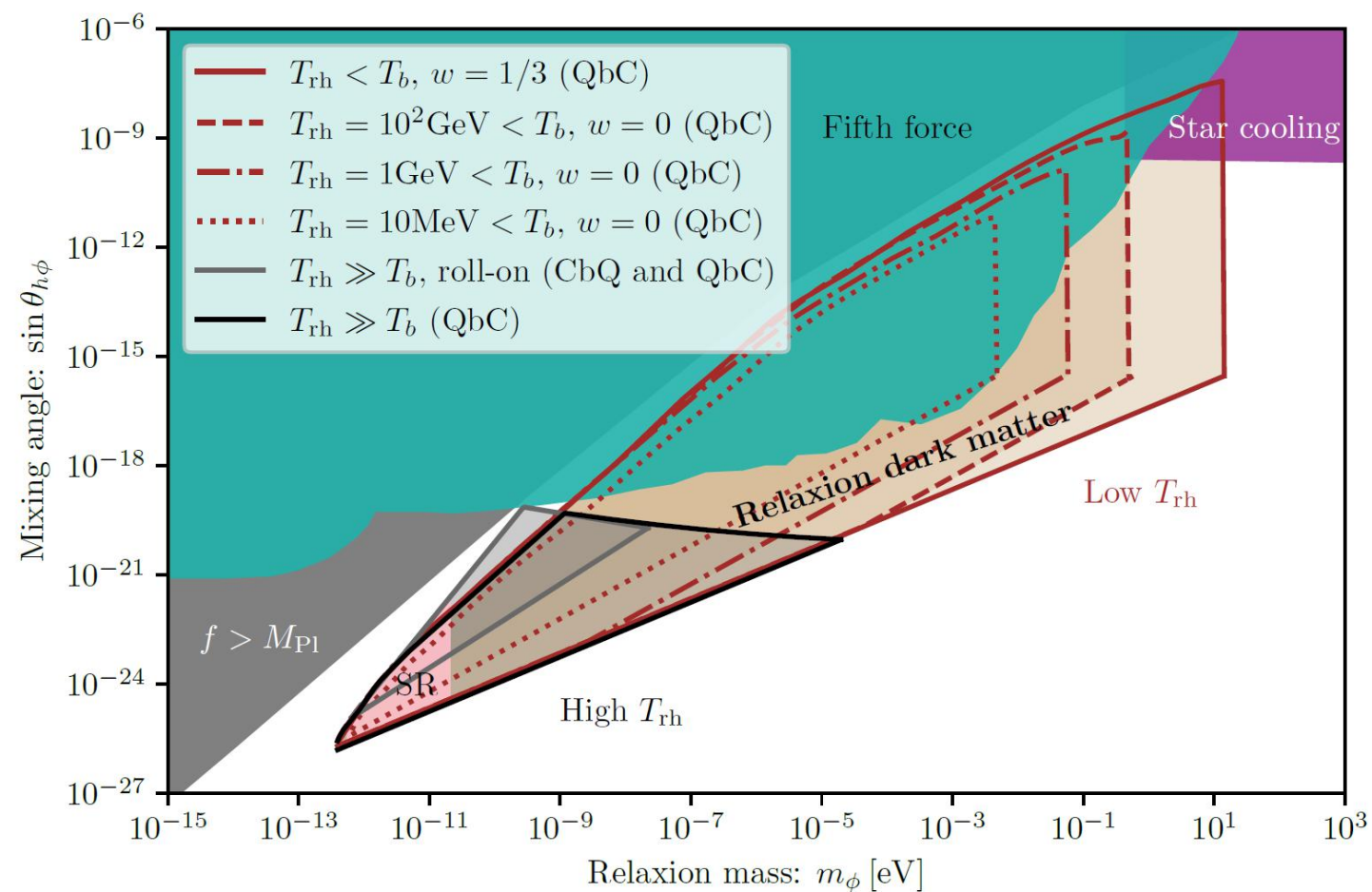


“Quantum beats classical” regime:



$$\frac{\Omega_{\phi,0}}{\Omega_{DM}} \approx 0.2 \times 10^{-6} \left(\frac{m}{\text{eV}} \right)^{-3/2} \left(\frac{H_I}{\text{GeV}} \right)^4 \min \left\{ 1, \left(\frac{H_{rh}}{H_{osc}} \right)^{\frac{1-3w}{2(1+w)}} \right\}.$$

Relaxion dark matter window



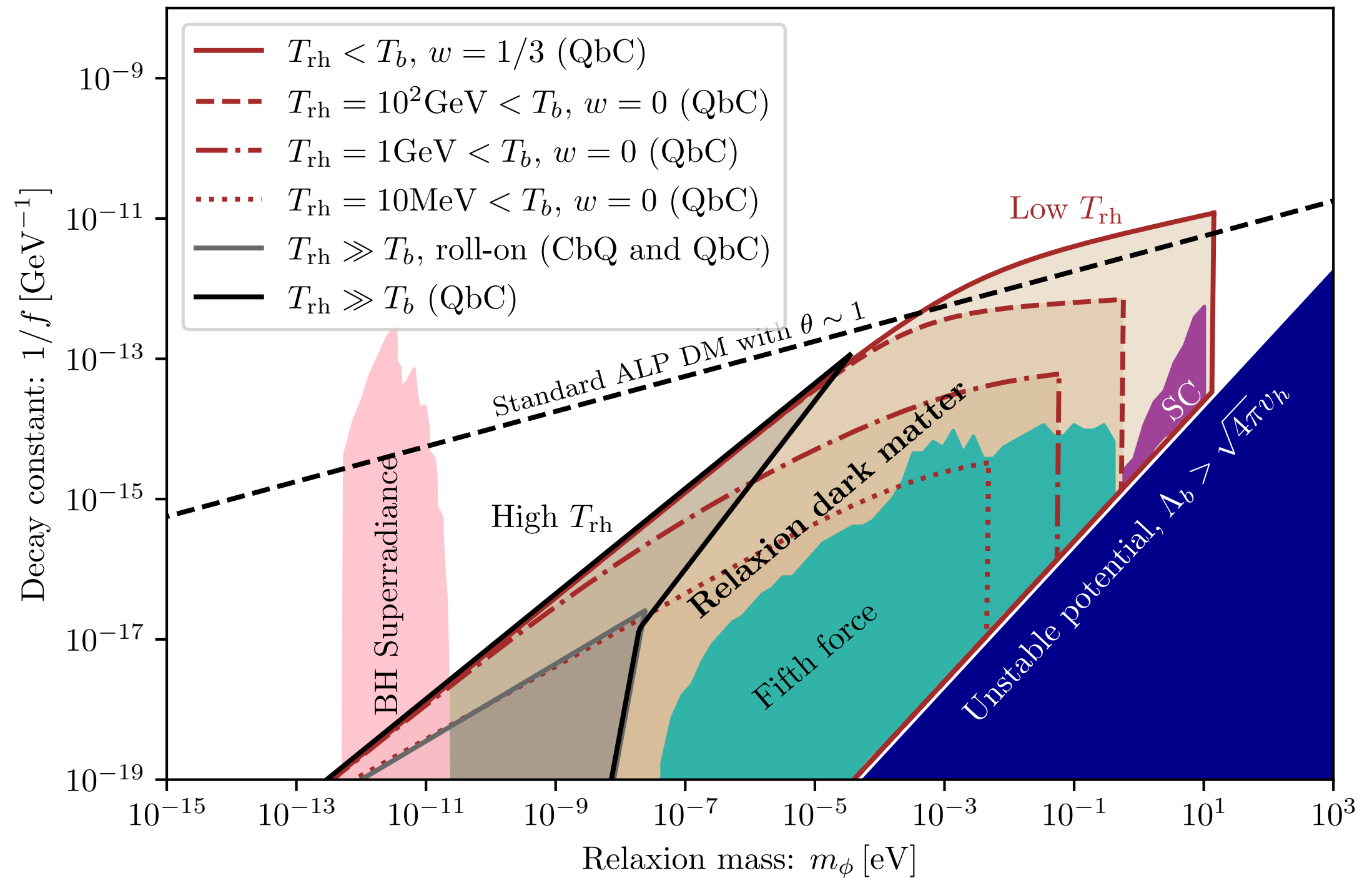
Brown: low reheating temperature, stochastic misalignment

Grey: high reheating temperature, misalignment from roll-on after reheating

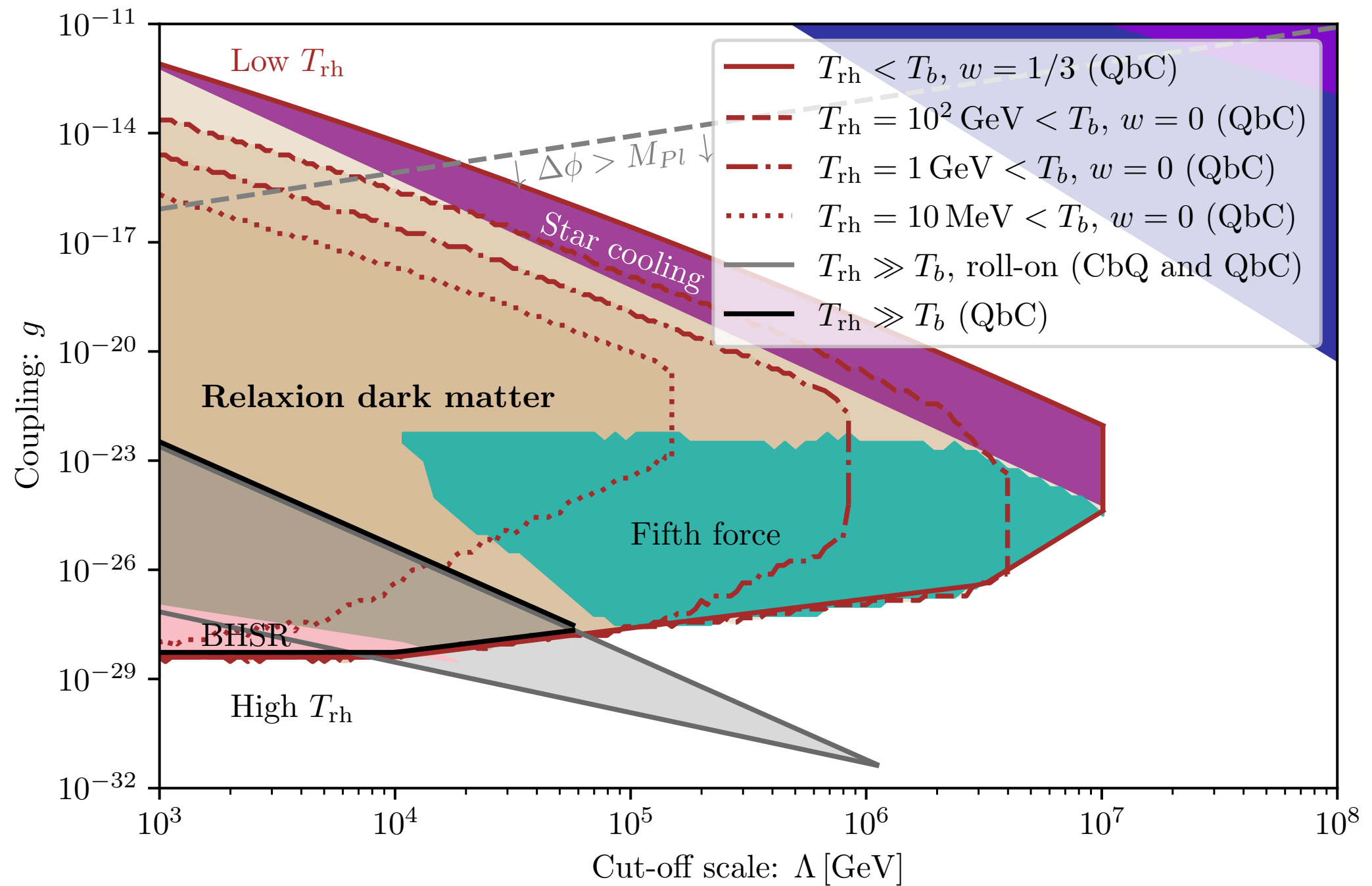
Banerjee et. al., 1810.01889

Black: high reheating temperature, stochastic misalignment

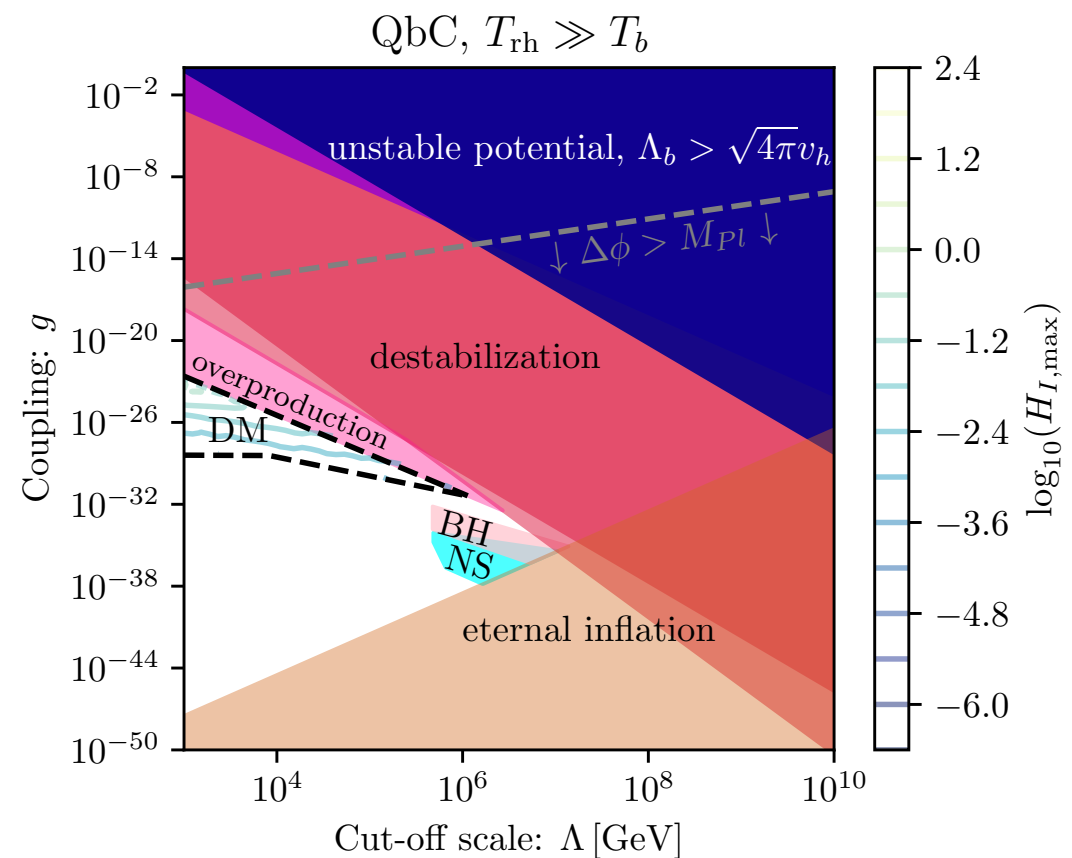
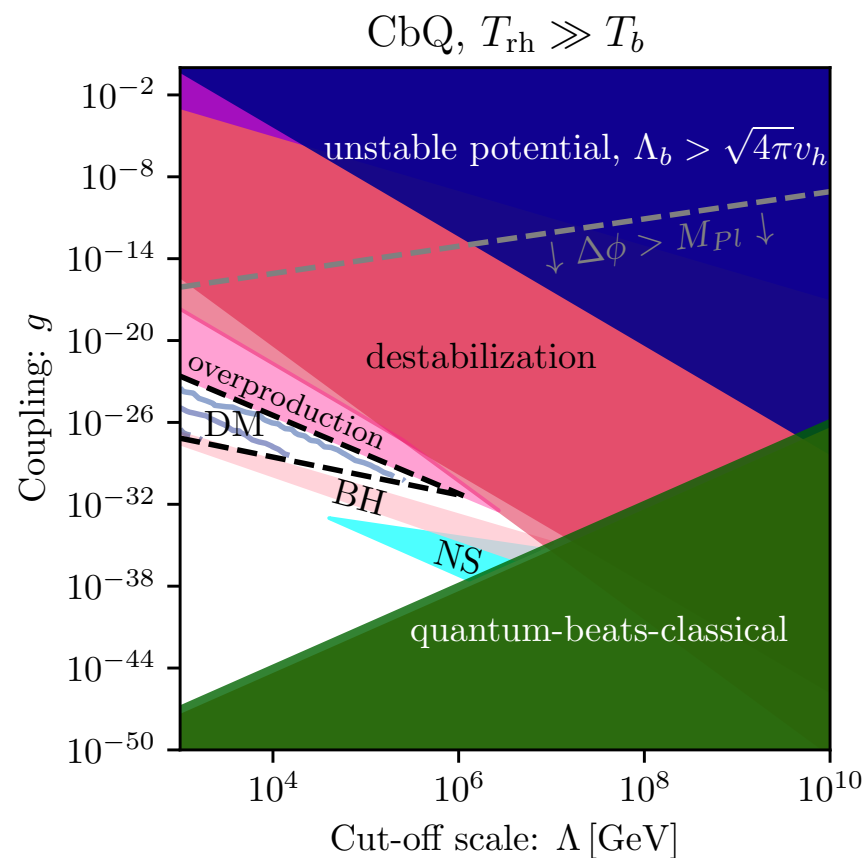
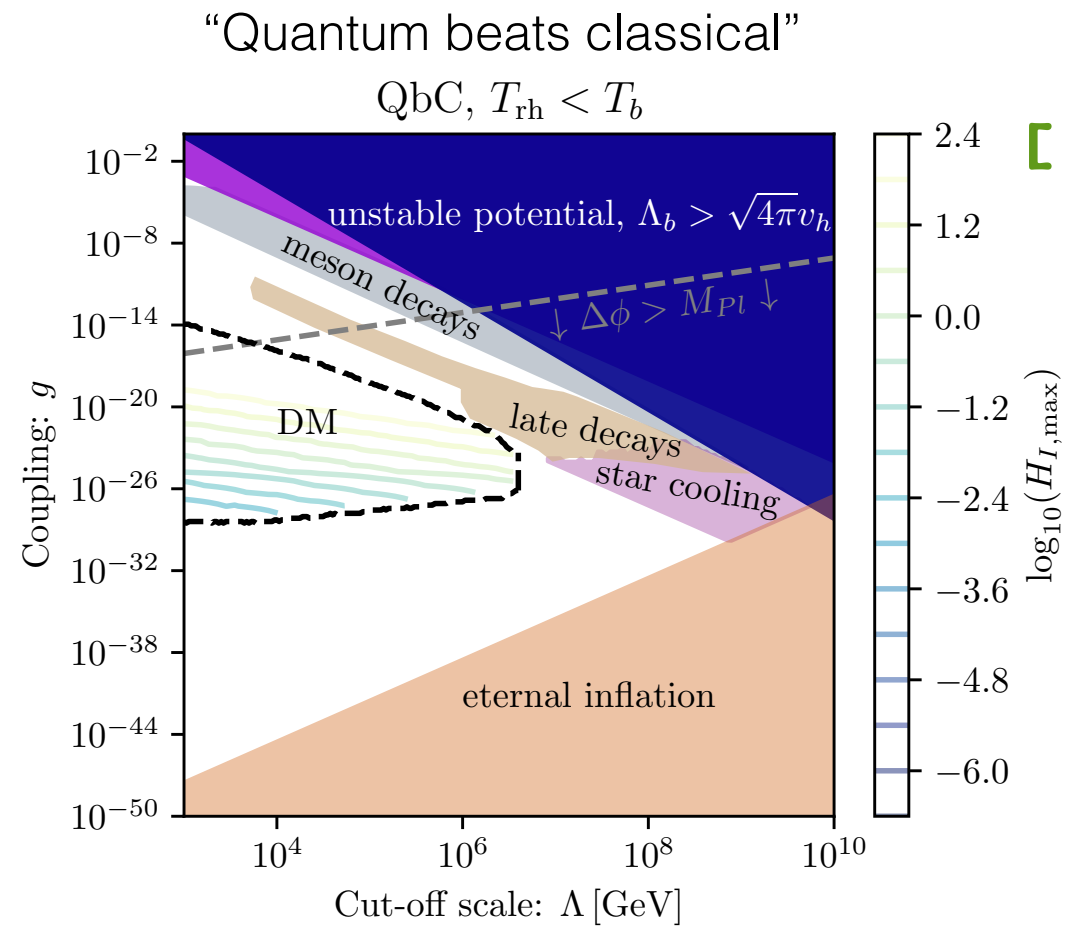
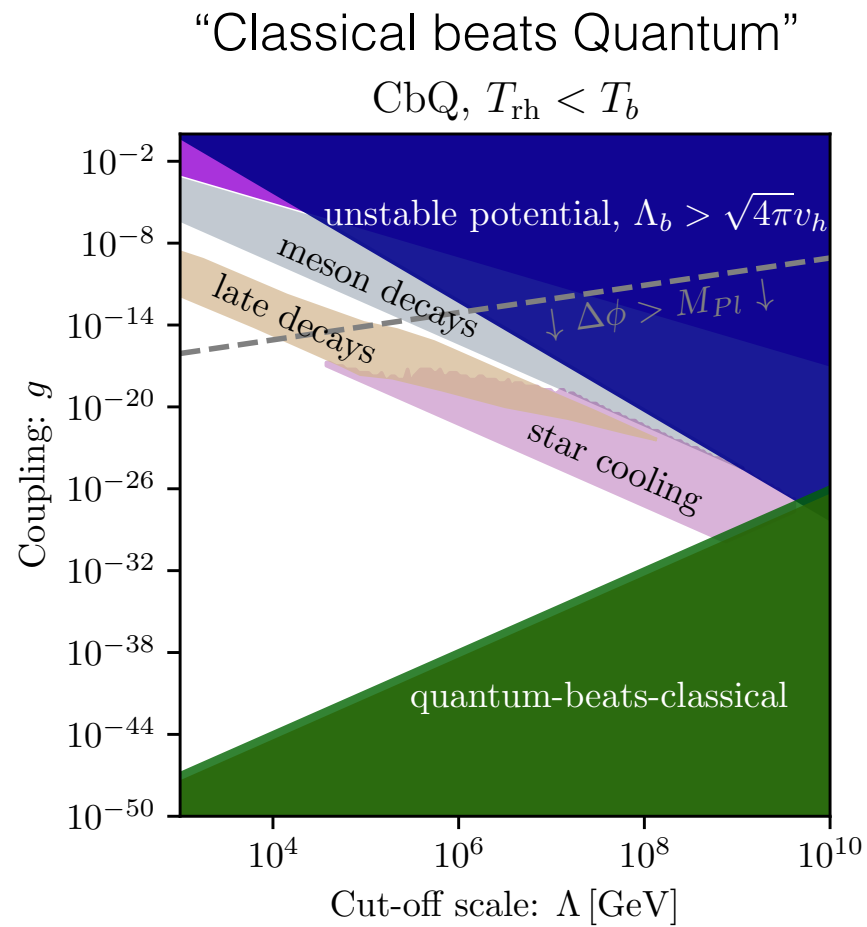
Relaxion dark matter window



Relaxion dark matter window

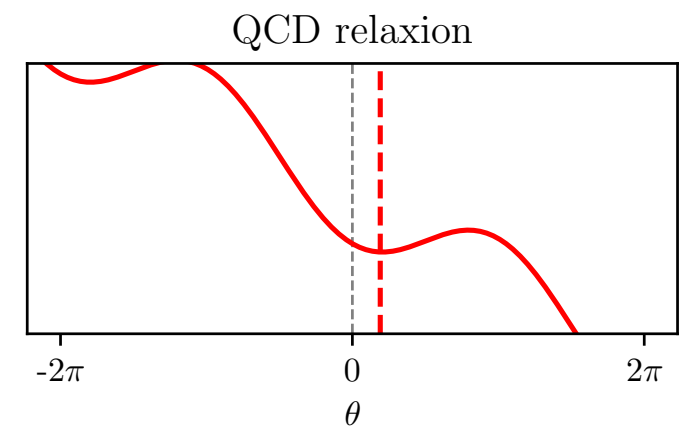
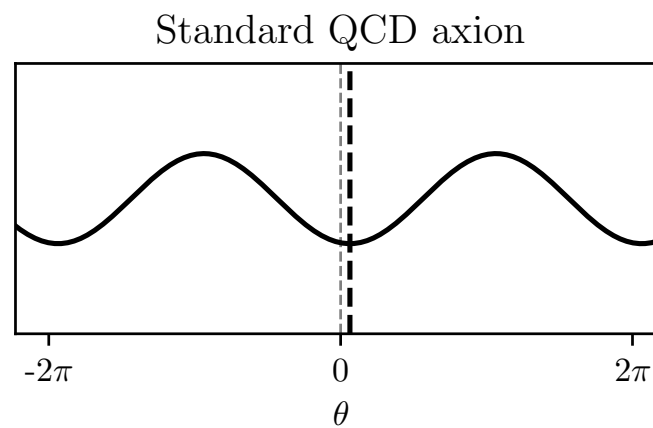
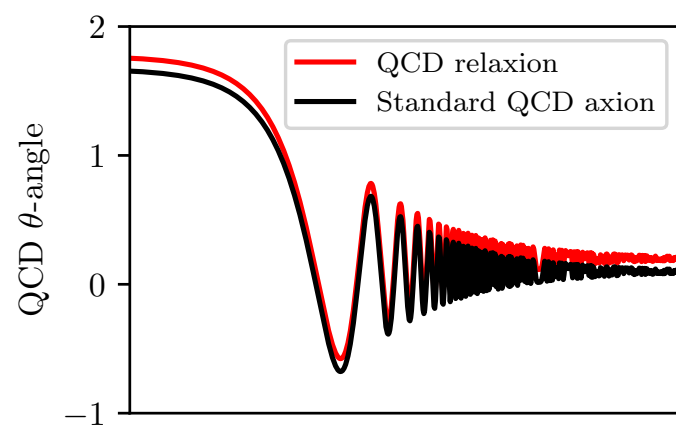
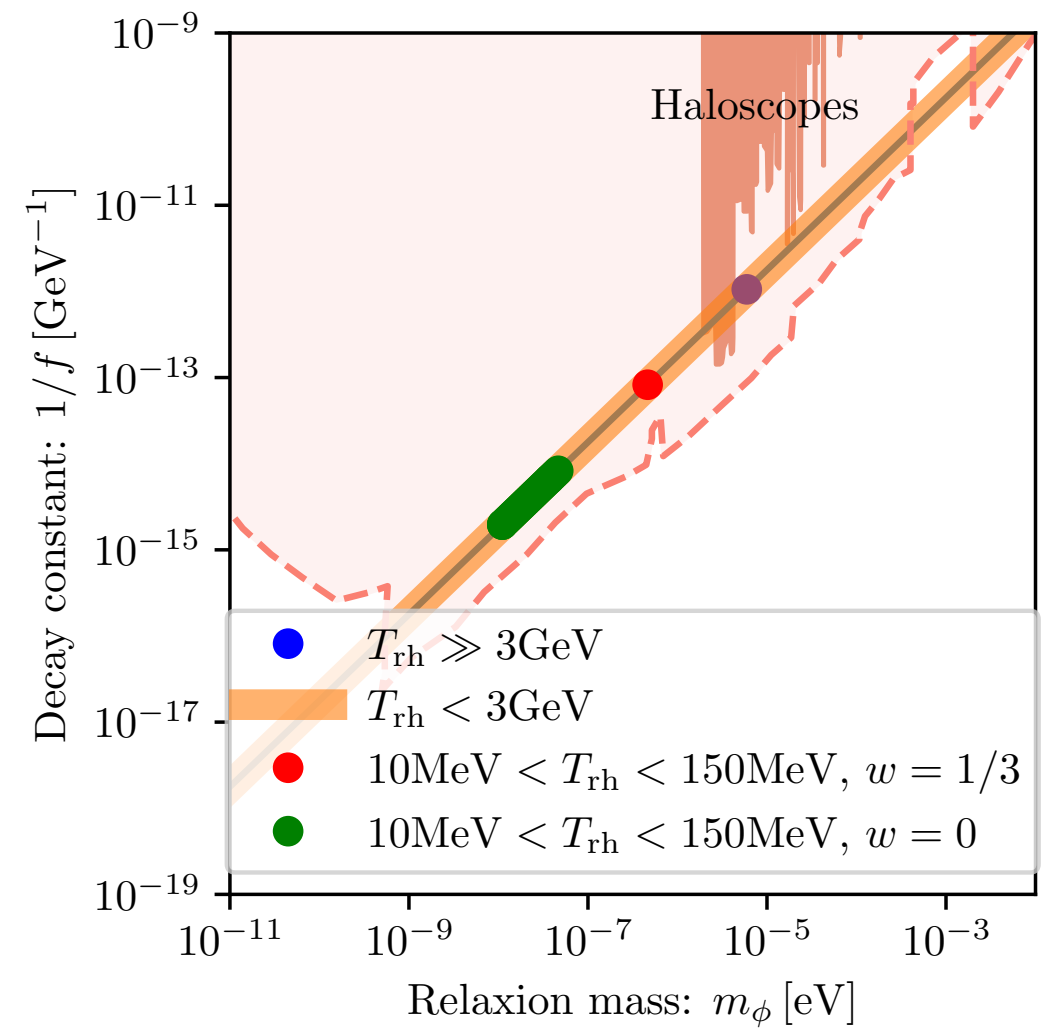
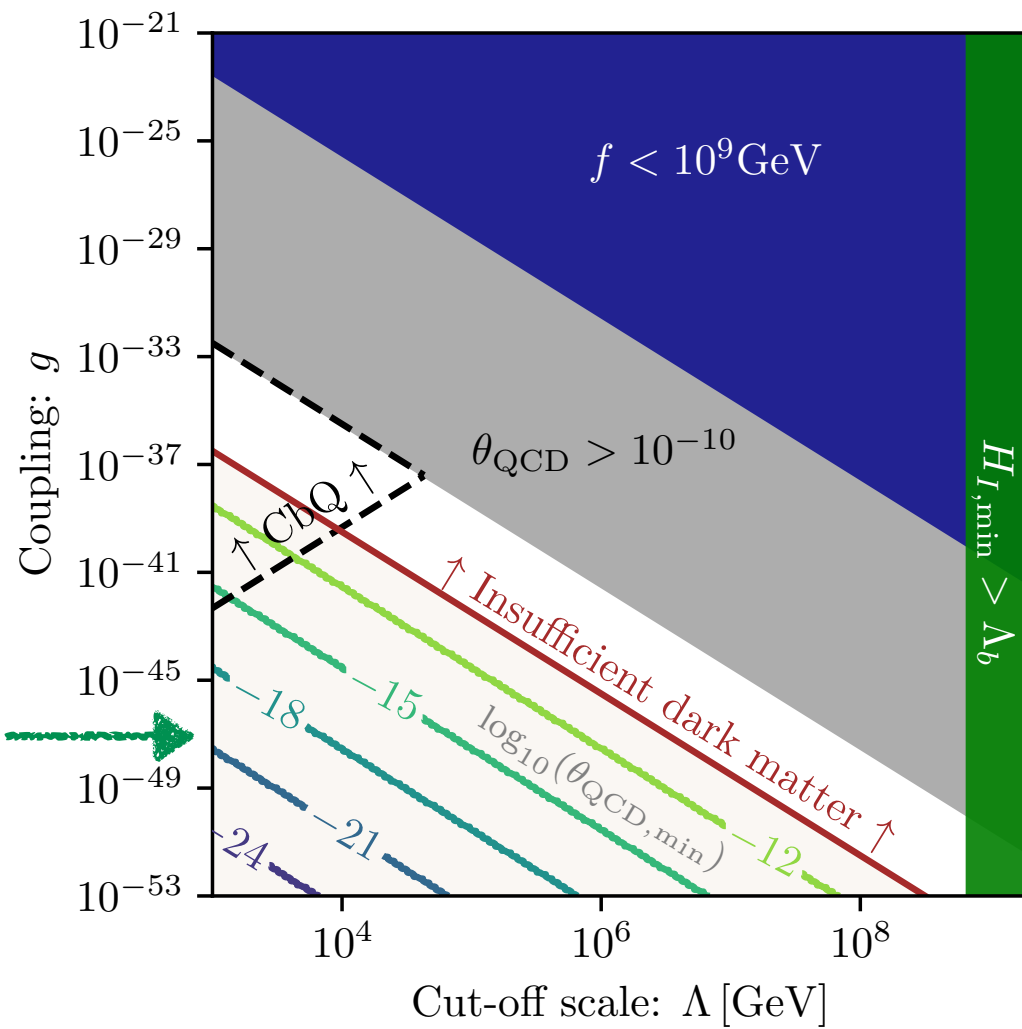


Summary Non-QCD relaxion: A rich spectrum of possibilities



QCD Relaxion Dark Matter window

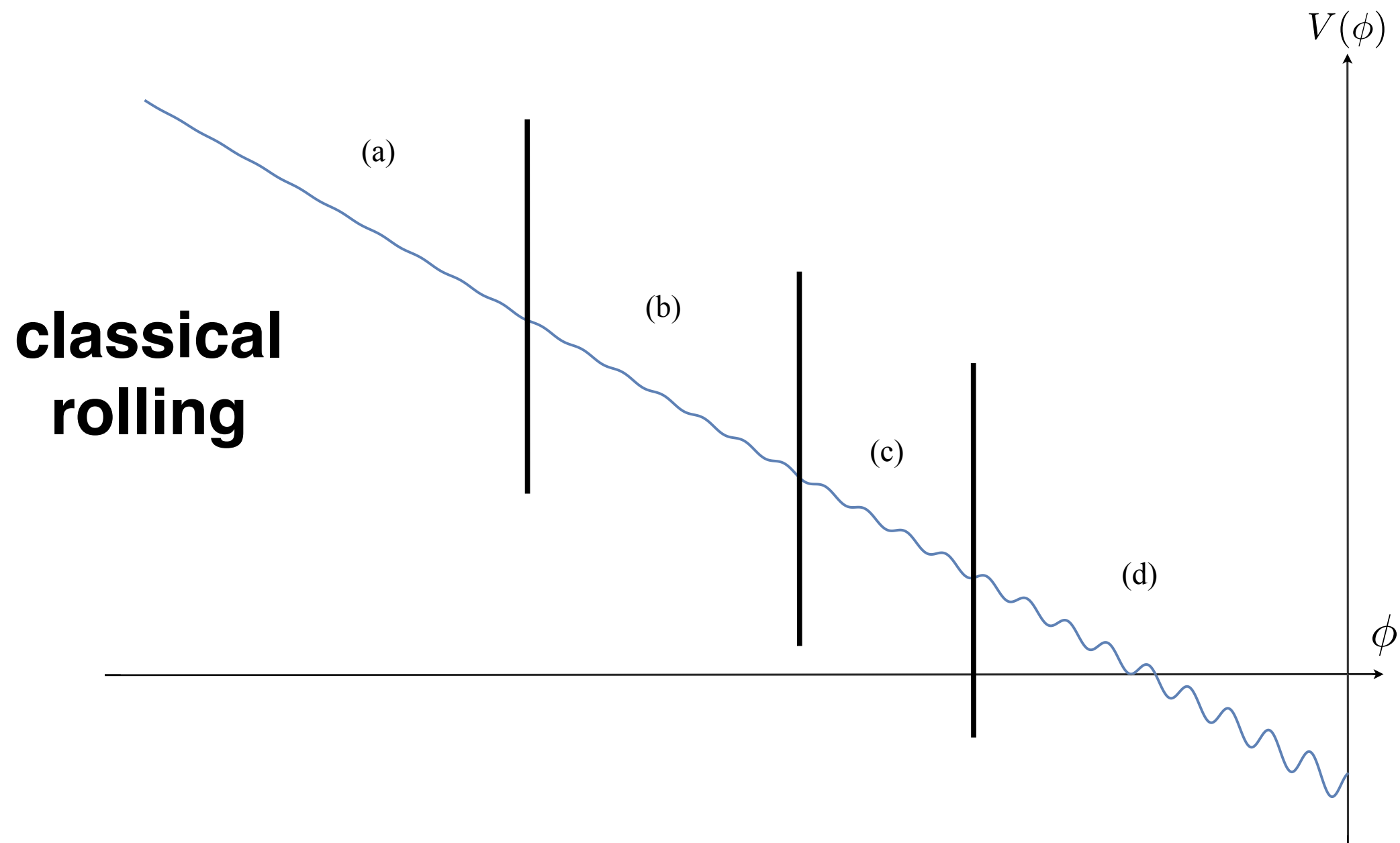
contours of $\log_{10} \theta_{\text{QCD}}$



**Fluctuations are important
even in the ‘Classical-
beats-Quantum’ regime !**

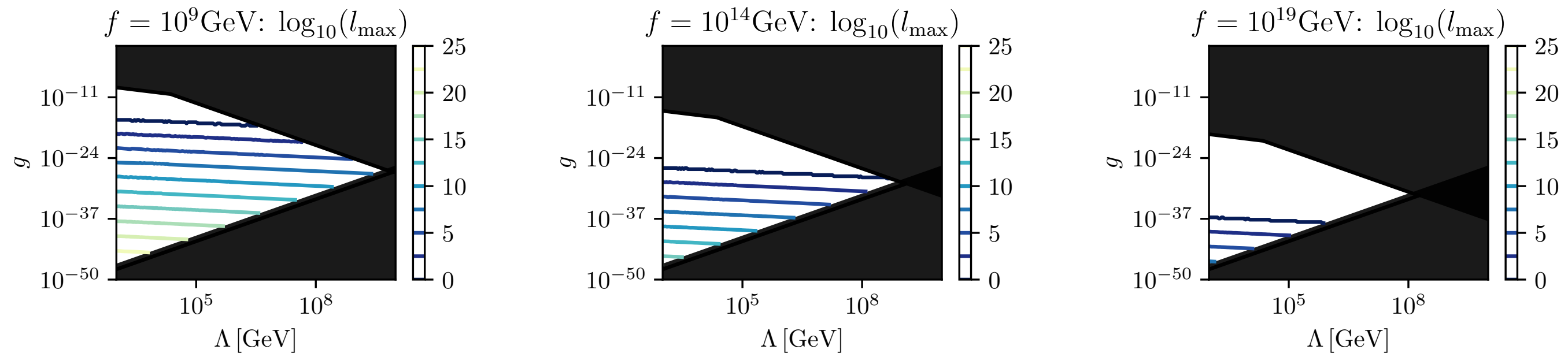
Classical-beats-quantum regime .

The relaxion does not stop at the first minimum!



Classical-beats-Quantum regime .

The relaxion does not stop at the first minimum but at the 10^ℓ th !



Implications for the “Runaway relaxion from finite density”.

[Balkin, Serra, Springmann, Stelzl, Weiler, 2106.11320]

Knowing in which minimum the relaxion ends up is crucial to study the stability of that local minimum after inflation.

In particular: the behavior of the relaxion in dense environments, such as stars.

Height of barriers depends on Higgs vev which depends on the density of fermion fields coupled to the Higgs, including baryons.

—>

Finite density effects can modify the effective relaxion potential and suppress the height of its barriers.

Implications for the “Runaway relaxion from finite density”.

[Balkin, Serra, Springmann, Stelzl, Weiler, 2106.11320]

Require that no relaxion bubbles of lower local minima can form in neutron stars, white dwarfs and sun-like stars.

**condition for barrier to
disappear inside the
core of the star:**

**average
baryonic
density**

$$n > 3 \times 10^{-3} \text{MeV}^3 \left(\frac{\text{TeV}}{\Lambda / \sqrt{l}} \right)^2 \left(\frac{\Lambda_b}{\text{MeV}} \right)^4$$

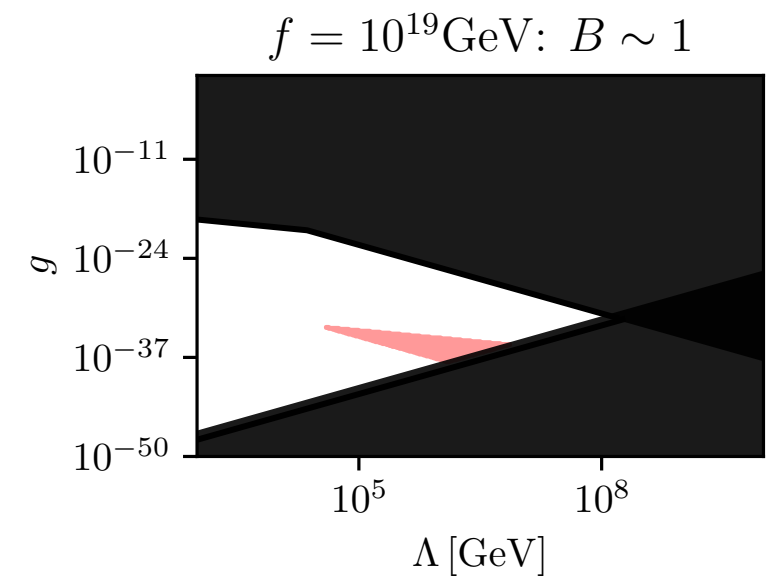
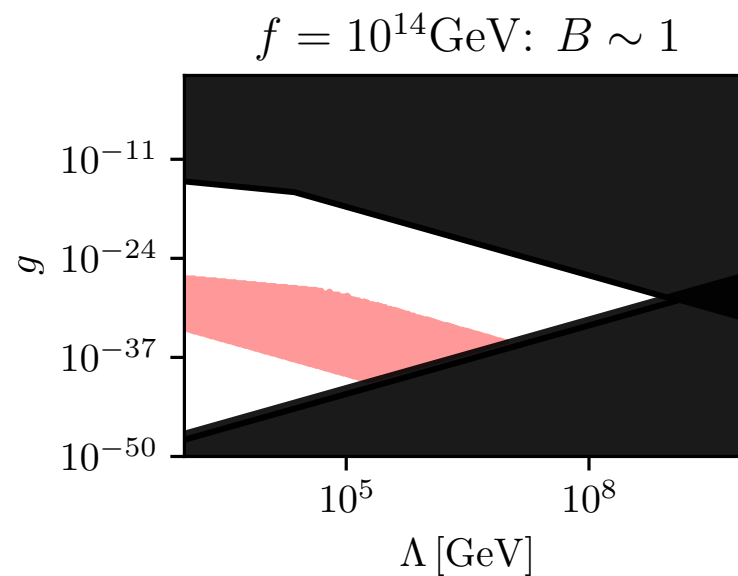
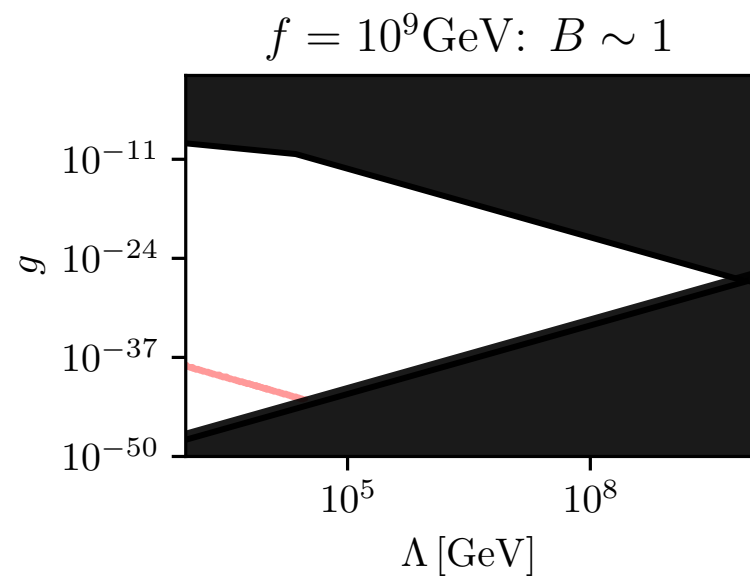
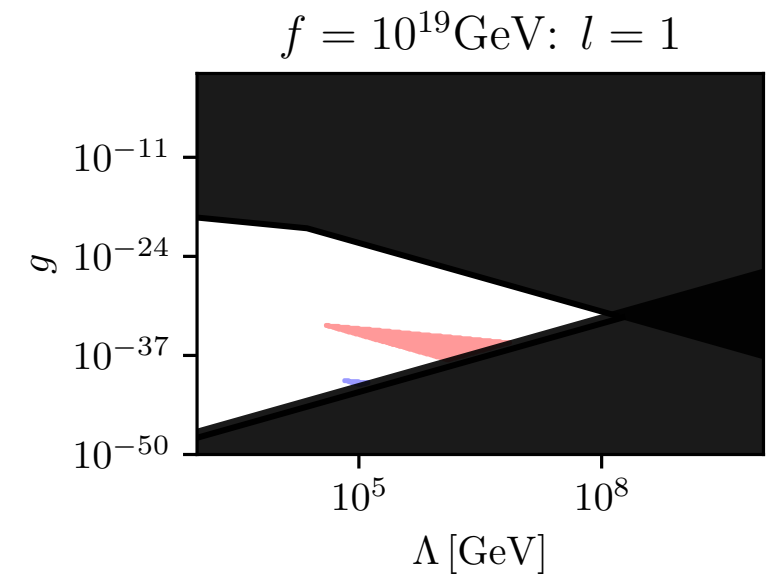
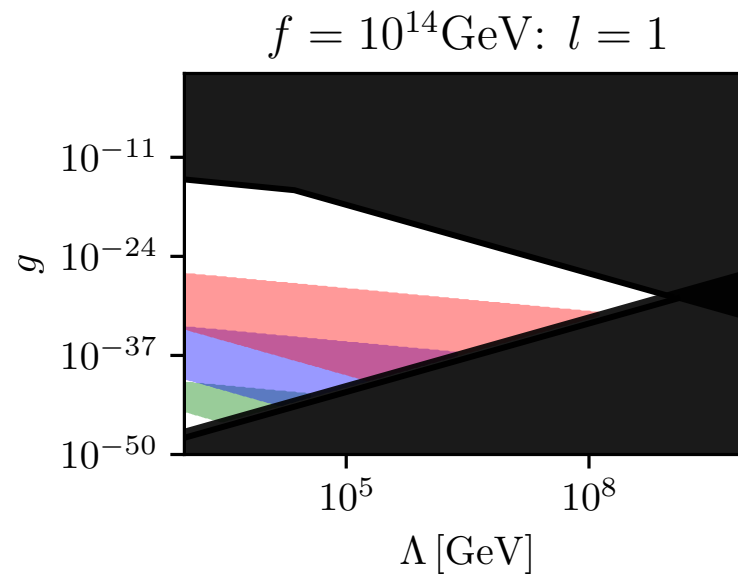
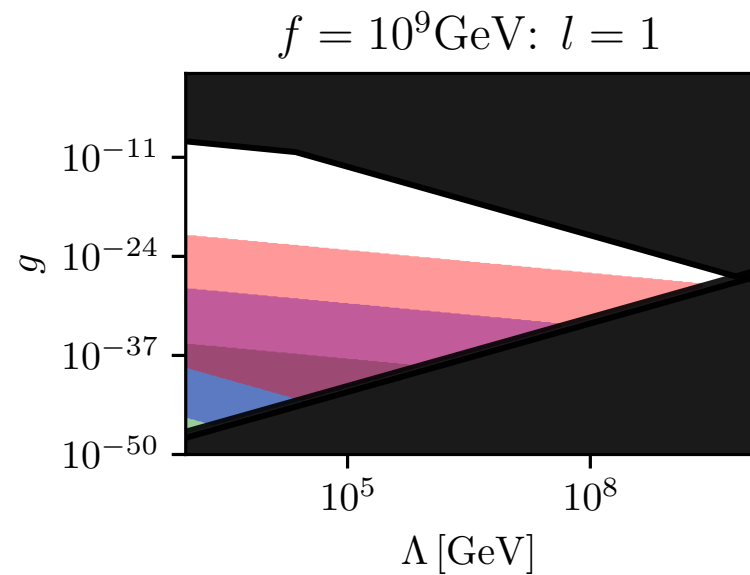
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**condition for bubble
overcoming the pressure and
expanding outwards:**

radius if object

$$r > \frac{\Lambda_b^2}{g\Lambda^3}$$

Relaxion safe from finite density effects! ■



Almost no dangerous runaway-relaxion region.

Conclusion.

- We explored the stochastic window for the relaxion.
- We derived a new stopping condition.
- We determined precisely the stopping minimum (very far from the first one even in the Classical-beats-Quantum regime \rightarrow no runaway from high-density effects)
- We explore the regime “Quantum-beats-Classical”
- Full determination of the viable regions of parameter space (H_I , f , g , Λ)
- Relaxion can naturally be dark matter

General Summary on Relaxion.

- **A new approach to the hierarchy problem based on intertwined cosmological history of Higgs and axion-like states. Connects Higgs physics with inflation & (DM) axions.**
- **An existence proof that technical naturalness does not require new physics at the weak scale**

$$\Lambda < \left(v^4 M_P^3\right)^{1/7} = 3 \times 10^9 \text{ GeV}$$

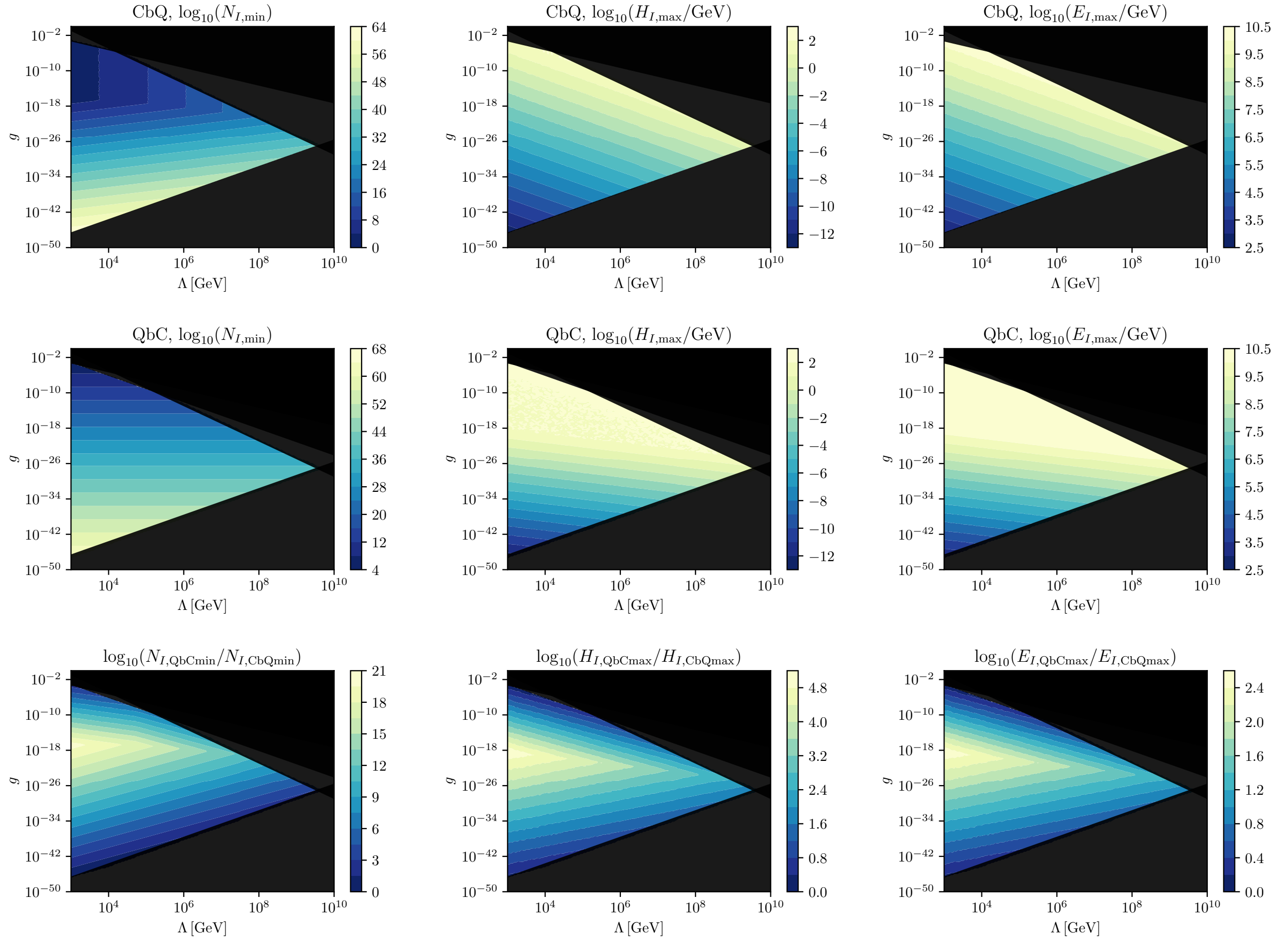
- **Change of paradigm:**

no signature at the LHC , new physics are weakly coupled light states which couple to the Standard Model through their tiny mixing with the Higgs.

- **Experimental tests from cosmological overabundances, late decays, Big Bang Nucleosynthesis, Gamma-rays, Cosmic Microwave Background...**

Annexes.

Required number of e-folds & scale of inflation.



Volume-weighting.

- Volume-weighted Fokker-Planck equation

$$\frac{dP}{dt} = \frac{1}{3H_I} \frac{\partial(P \partial_\phi V)}{\partial \phi} + \frac{H_I^3}{8\pi^2} \frac{\partial^2 P}{\partial \phi^2} + \frac{4\pi}{M_{\text{Pl}}^2} \frac{V}{H_I} P \quad P(\phi, t) = e^{3(H(\phi) - H_I)t} \rho(\phi, t)$$

- Does the relaxion climb up during inflation?

No, if $N_I < N_c$

$$\phi_{\text{peak}}(t) = \dot{\phi}_{\text{SR}} t - \frac{g\Lambda^3 H_I^2 t^2}{M_{\text{Pl}}^2 \pi}$$

- The fate of “wrong” Hubble patches ($\mu_h \sim \Lambda$) after inflation

The field slow-rolls down to the region with a small Higgs vev.

Fate of the relaxion after inflation .

1. **Reheating:** the relaxion can be destabilized if $T_{rh} > T_b$

Require $T_{rh} < T_b$

$T_b < v_h$ is the barrier
reappearance temperature

2. **Onset of oscillations:** $H_{osc} \approx \frac{m_\phi}{3}$

3. **Relaxion decay:**

$$\Gamma_\phi = \sin^2 \theta \times \Gamma_h(m_\phi)$$

If $\Gamma_\phi < 10^{17} \text{s}^{-1}$, relaxion oscillations behave as **dark matter**.

4. **Typical displacement from the minimum,**

$$\sigma_\phi^2 = \frac{3H_I^4}{8\pi^2 m^2}$$

Barriers for Non-QCD relaxion .

$$V = -(\Lambda^2 - g'\Lambda\phi)H^2 + \lambda H^4 + g\Lambda^3\phi + \Lambda_b^4(H) \cos\frac{\phi}{f}$$

New strong dynamics gives wiggles

$$L_{eff} = m_N \bar{N}N + m_L \bar{L}L + yH\bar{N}L + \tilde{y}H^*\bar{L}N + \frac{\phi}{f} G' \widetilde{G'}$$



$$V \simeq \frac{y\tilde{y}\Lambda_s^3}{m_L} |H|^2 \cos\frac{\phi}{f}$$

$$(m_L > \Lambda_s > m_N)$$