Bounding/Characterizing String Geometries

Lara B. Anderson – Virginia Tech

Based on work with James Gray and Callum Brodie
arXiv:2211.05804,23xx.xxxxx
Gray, Patil, Scanlon (In progress)

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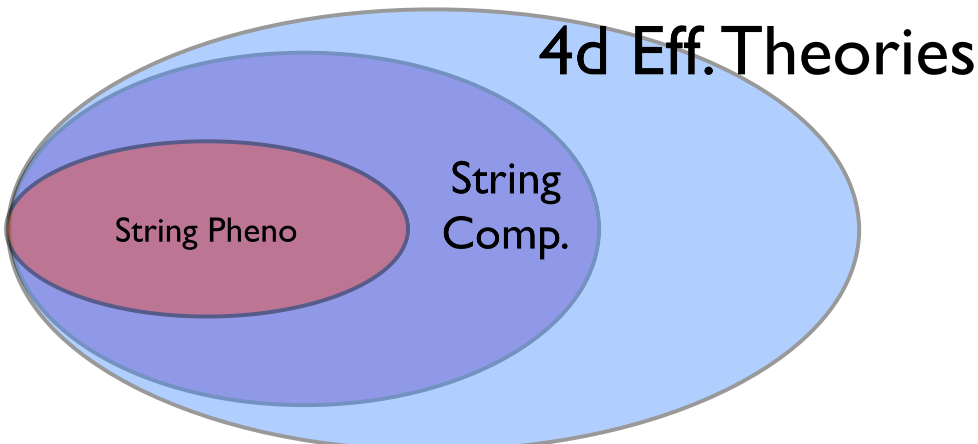
Invent the Future

Motivation

- String theory is a powerful extension of quantum gravity, but extracting the effective 4D physics is challenging:

Higher dimensional geometry —> **String Comp.** —> 4D Physics?

- Need a robust toolkit in any corner of string theory to extract the full low energy physics
- In string compactifications: which field theories <-> which geometries?
- What can learned from the top down setting?

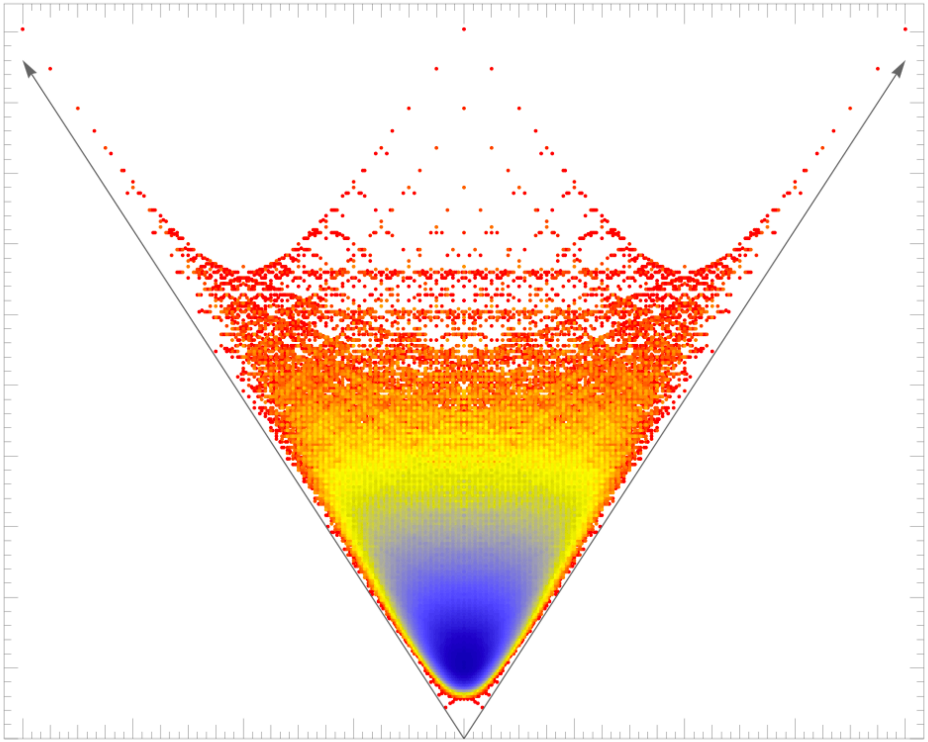


Which theories/geometries?

- In string compactifications: which field theories <-> which geometries?
- Broad goal: bound and characterize background geometries.
- What "geometry" matters? Geometries vs. topologies?
- Bounds? Field theoretic nature?
- Different answers for different theories (i.e. N=1,2,.., bundles, branes/singularities, fluxes, etc).

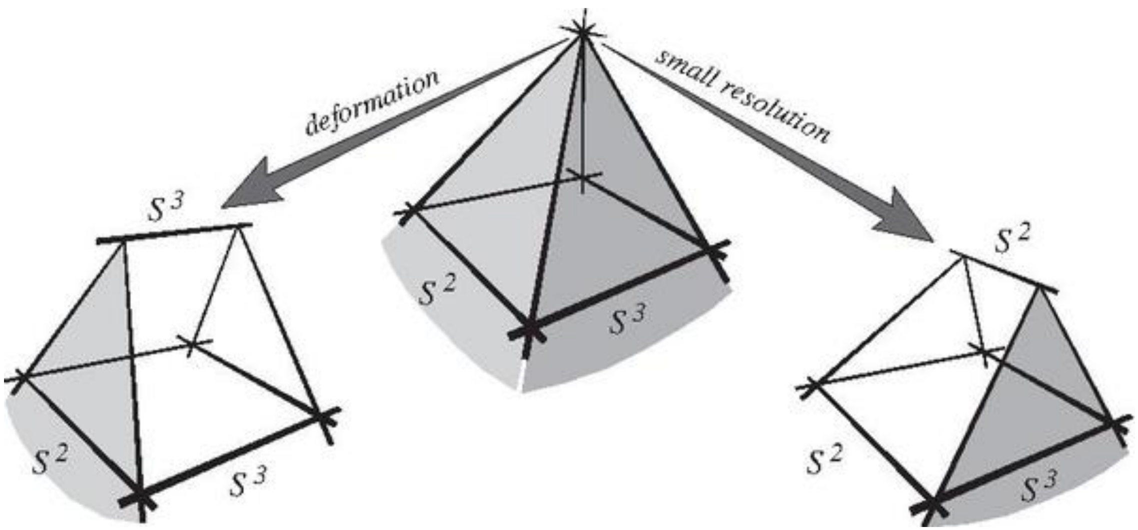
CY Manifolds

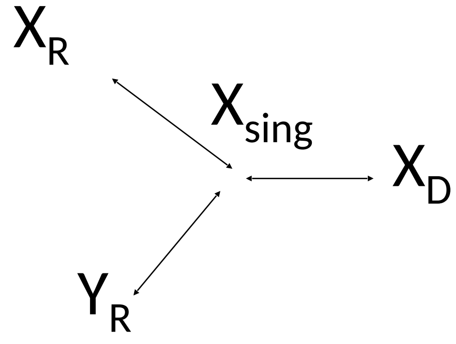
- Finite?
- How to tell if two manifolds are the same? Wall's data: $(h^{1,1}, h^{2,1}, c_2(X), d_{rst})$ different then X, X' not diffeomorphic.
- **Geometric Transitions:**
- Connect (most) Known CY manifolds
- All CY manifolds? All SU(3) Structure manifolds?? (Reid)



Geometric Transitions

- Topology changing transitions: Conifolds and Flops





- Geometrically these transitions are often well understood.
- They are also well understood field-theoretically in some contexts (i.e. Type II). But not in 4D, N=1 theories…

- What about in the N=1 context?
- In $E_8 x E_8$ heterotic string theory things are more difficult. The theory has a Bianchi Identity:

$$dH = -\alpha' \mathrm{tr} F \wedge F + \alpha' \mathrm{tr} R \wedge R$$

Three-form

Negative sources from gauge and positive sources from gravitational sectors

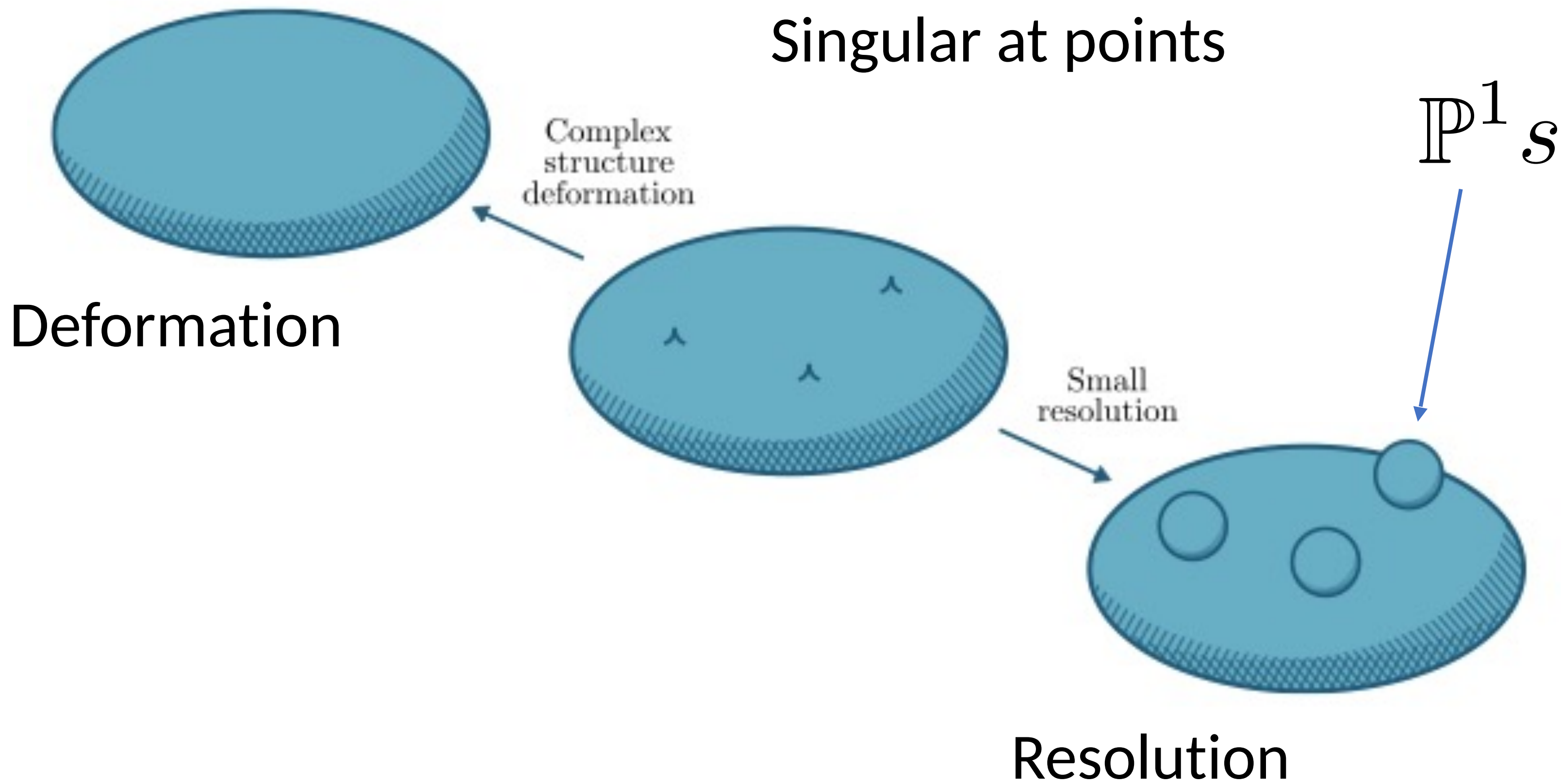
- Chern class condition:

$$[\mathrm{tr} R \wedge R] = [\mathrm{tr} F \wedge F] \Rightarrow c_2(\Omega_X) = c_2(V)$$

- With 5-branes: $c_2(\Omega_X) = c_2(V) + [C]$
- This means that we cannot set the gauge fields to zero.

Conifold transitions

- At the level of geometry we have schematically:



- In an example:

Smooth quintic

$$X_D = \left[\ \mathbb{P}^4 \mid 5\ \right]^{(1,101)}$$

Nodal quintic

Deform complex structure

$$X_0 = \left[\ \mathbb{P}^4 \mid 5\ \right]$$

$$X_R = \left[\begin{array}{c|cc} \mathbb{P}^1 & 1 & 1 \\ \mathbb{P}^4 & 1 & 4 \end{array}\right]^{(2,86)}$$

Blow up singular points of nodal quintic with (16) $\mathbb{P}^1 s$

- You can also think of this transition as a brane separating from the cotangent bundle (c.f. small instanton transitions where a brane separates from the gauge bundle):

$$0 \to f^*\Omega_{X_0} \to \Omega_{X_R} \to \mathcal{O}_{\mathbb{P}^1{}_s}(-2) \to 0$$

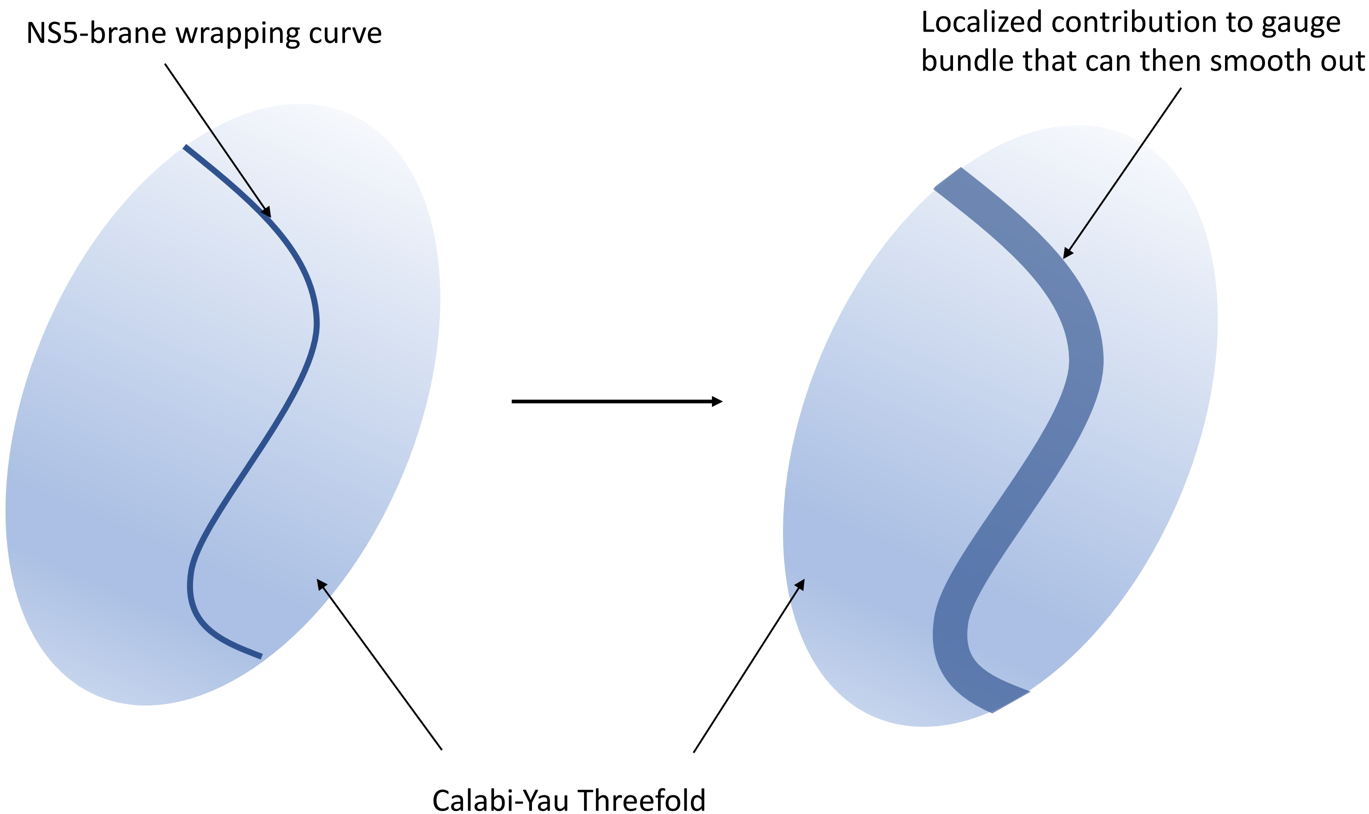
Deformation manifold cotangent bundle

Resolution manifold cotangent bundle

Sheaf localized on exceptional curves representing branes

Small Instanton Transitions (SITs)

- SITs are the process whereby a five-brane is “dissolved” into the gauge bundle:



Including a gauge bundle in the transition

- Recall the anomaly cancelation condition in heterotic string-theory:

$$c_2(\Omega_{X_R}) = c_2(V_R)$$

- How does this change during the transition?

$$c_2(\Omega_{X_R}) + [\mathbb{P}^1 s] = c_2(V_R) + [\mathbb{P}^1 s]$$

This is how the gravitational sector changes given the transition we have seen in the cotangent bundle.

The gauge sector must change in the same way.

- A natural guess is to add the same brane into the gauge bundle!
- The process actually turns out to be a complicated series of brane recombinations and small instanton transitions.
- Gives us a geometric description of the heterotic conifold transition.
- Example of some dual bundles on the example geometries:

Deformation side bundle:

$$0 \to \mathcal{O}(-5) \to \mathcal{O}(-1)^{\oplus 5} \to V_D \to 0$$

Resolution side bundle:

$$0 \to \begin{array}{c} \mathcal{O}(-1,-5) \\ \oplus \\ \mathcal{O}(0,-4) \end{array} \to \begin{array}{c} \mathcal{O}(-1,0) \oplus \mathcal{O}(0,-5) \\ \oplus \\ \mathcal{O}(0,-1)^{\oplus 4} \end{array} \to V_R \to 0$$

The full structure of the transition

- On the next slide is the map of the transition, presented at the level of classes for clarity.

- All of the sequences of sheaves (and Hecke Transforms) required for this process to occur exist and can be written down explicitly.

- In what follows V_s is a "spectator bundle" which goes through the transition in a trivial manner.

$$\mathrm{c}_2(\Omega_{X_R}) = \mathrm{c}_2(V_R)$$

Pair create curve supported sheaves | Pair create curve supported sheaves

$$\mathrm{c}_2(\Omega_{X_R}) + [\mathbb{P}^1 s] = \mathrm{c}_2(V_R) + [\mathbb{P}^1 s]$$

SIT in cotangent bundle | SIT in gauge bundle

$$c_2(f^*\Omega_{X_0}) = c_2(V_s) + [\mathcal{C}_R] + [\mathbb{P}^1 s]$$

“Brane” recombination

$$c_2(f^*\Omega_{X_0}) = c_2(V_s) + [\mathcal{C}_D]$$

SIT in gauge bundle

$$\mathrm{c}_2(f^*\Omega_{X_0}) = \mathrm{c}_2(V_D)$$

What happens to the degrees of freedom of the theory?

Deformation side:

Kahler moduli: $h^{1,1} = 1$

Complex Structure: $h^{2,1} = 101$

Bundle Moduli: $h^1(\mathrm{End}_0(V)) = 324$

Total = 426

Resolution side:

Kahler moduli: $h^{1,1} = 2$

Complex Structure: $h^{2,1} = 86$

Bundle Moduli: $h^1(\mathrm{End}_0(V)) = 338$

Total = 426

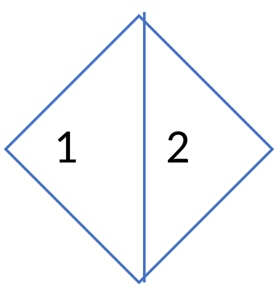
These “dual” pairs are related to a phenomenon that has appeared before

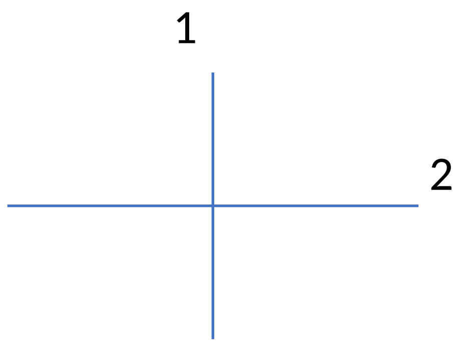
- (0,2) Target Space Duality (Distler + Kachru, Blumenhagen…)
- (The two bundles I displayed earlier are TSD)
- However, our approach is more general than TSD: applies to arbitrary bundle constructions and to 5-brane theories.

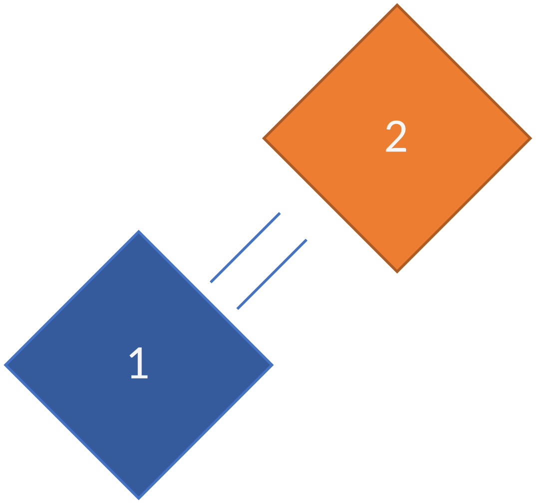
Distler and Kachru hep-th/9707198, Blumenhagen hep-th/9707198 and hep-th/9710021, Blumenhagen and Rahn 1106.4998, Anderson and Feng 1607.04628

How are theories related? New string duality?

- Key question: Is this branch change, connected field space, or duality?







E.g.s:

Type II
CY Flops

IIA CY Conifold
(Higgs & Coulomb branches)

Type II
CY Mirror Symmetry

Exploring the effective theories

- Zero-mode spectrum matches
- What about further structure of the theory? Potentials? (LA/Feng, LA/Brodie/Gray to appear)
- Compare perturbative Yukawa coupling, non-perturbative effects.
- There are clearly interesting/problematic new contributions in resolution geometry

$$W_{C_i} \sim \left(\mathrm{Pfaff}_{C_i}\right) e^{-\int_{C_i} J_{new}}$$

Perturbative Yukawa couplings?

- To understand these we need information about the mapping on field space (for both moduli and charged matter).
- Some cases are easy to understand (if boring).
- Previous Example: SO(10) theory with no 10's – all couplings vanish.
- For more interesting cases we can simply compute on both sides of the duality and see if we can spot the structure we need.
- For E_6 theories:

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$$\lambda_{abc}\phi^a{}_{\mathbf{27}}\phi^b{}_{\mathbf{27}}\phi^c{}_{\mathbf{27}} \qquad H^1(V)\times H^1(V)\times H^1(V)\rightarrow H^3(\wedge^3 V)$$

An E6 example:

Deformation side:

$$\left[\mathbb{P}^4 \middle| 5\right]$$

$$0 \to V_D \to \mathcal{O}(1)^3 \oplus \mathcal{O}(2) \to \mathcal{O}(5) \to 0$$

Resolution side:

$$\left[\begin{array}{ccccccc|cc} y_0 & y_1 & y_2 & y_3 & y_4 & x_0 & x_1 & p_1 & p_2 \\ \hline 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 3 & 0 & 4 & 4 \end{array}\right]$$

$$0 \to V_R \to \mathcal{O}(0,2)^2 \oplus \mathcal{O}(0,1) \oplus \mathcal{O}(1,0) \to \mathcal{O}(1,5) \to 0$$

- In this example we find that all 147,440 independent, non-vanishing, Yukawa couplings correctly match as holomorphic functions of the moduli on either side of the duality (there are 95 families in this case).
- This matching works for a class of monad bundles where the rank of the final line bundle sum in the sequence is 1.

- More generally we have been able to show that a more complex mapping of family structures is necessary.

- In some of these more complex examples, however, we know that the Yukawa couplings must match for other reasons… so work continues!

Summary

- We have proposed a consistent/minimal geometric process by which compactifications of the heterotic string can undergo conifold transitions.

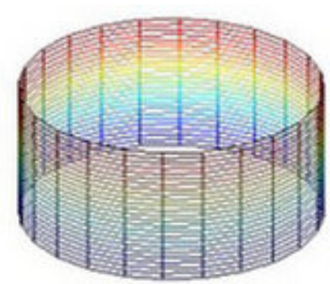
- Compactifications which are related by such transitions —> identical four dimensional physics (spectra and superpotential couplings).
- For certain types of compactifications (w/o five-branes) this equivalence of theories corresponds to TSD – so we have given that duality a geometric interpretation.

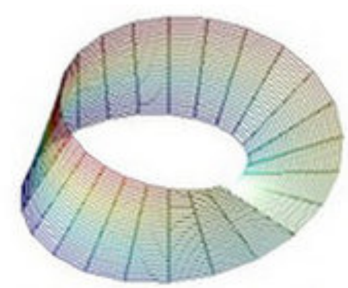
- More generally this duality is novel and has several potential consequences.

Heterotic Wall's Data?

- Not so simple as manifold topology + bundle topology.
- What topological data characterizes the theory?
- Initial observations: ch3(V), Singlets
  $h^{1,1} + h^{2,1} + h^1(End_0(V))$
- Approach: Topology of the bundle as a manifold

$$Z = \mathbb{P}(\pi : V \to X)$$





Further Topics

- The pair creation effect we discussed should be studied in isolation in a simpler setting (and other theories).
- Moduli Mapping (in progress with Gray, Patil and Scanlon)
- Other geometric transitions? (E.g. flops)
- F-theory duals?
- Broader moduli space of heterotic compactifications? (Important for model building/searching for examples of phenomena).