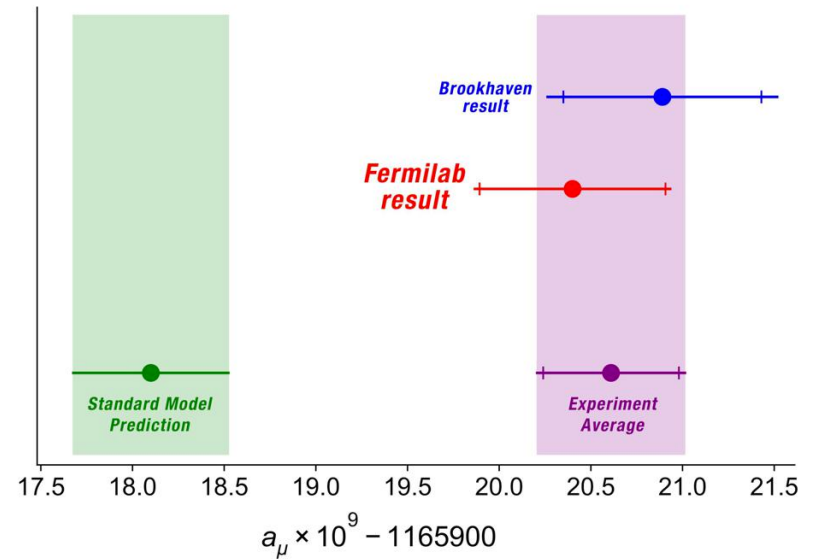
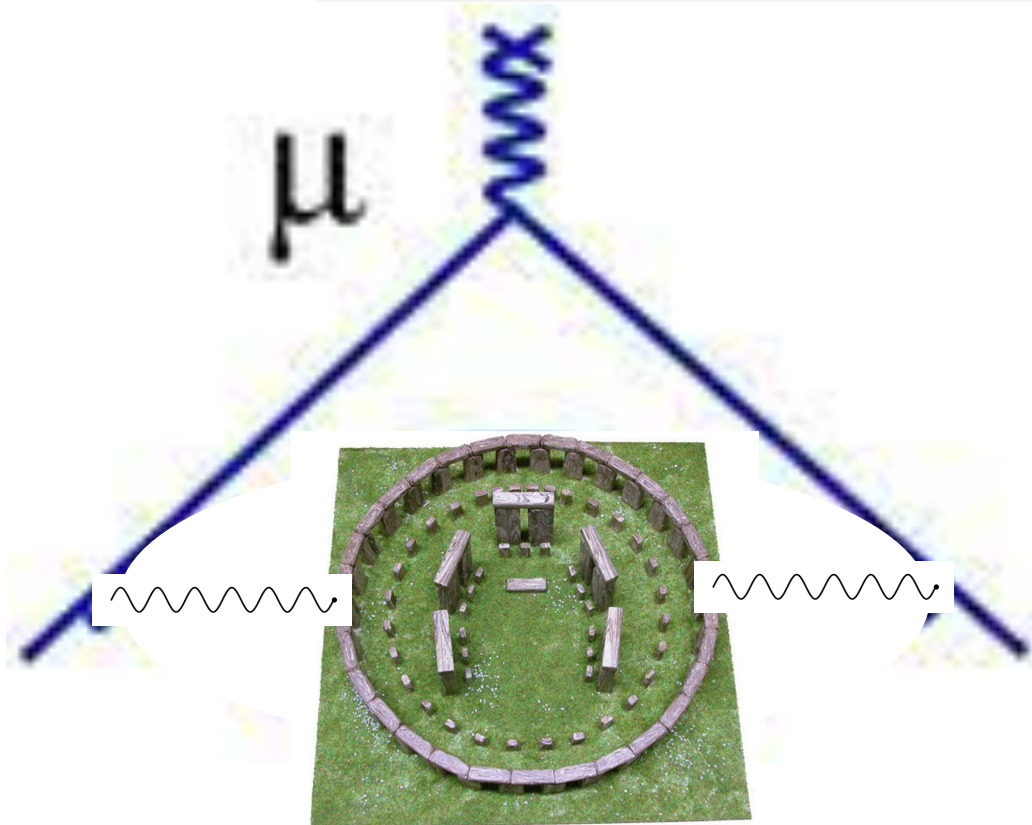


On the Muon g-2 Puzzle

Carlos E. M. Wagner

Physics Department, EFI and KICP, University of Chicago
HEP Division, Argonne National Laboratory



SUSY23 Conference
Univ. of Southampton, Monday, July 17, 2023

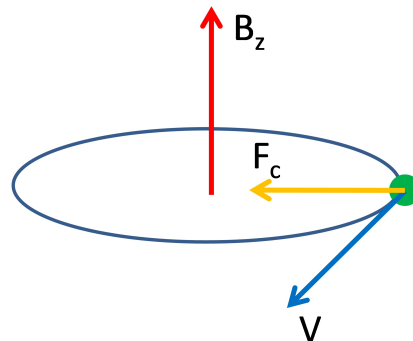
Precision Tests of QED : g-2

- The precession frequency of the lepton spin in a magnetic field is controlled by the so-called g-factor ($g \simeq 2$)

$$\vec{\omega}_S = -\frac{q\vec{B}}{m\gamma} - \frac{q\vec{B}}{2m} (g - 2)$$

- That can be compared with the cyclotron frequency

$$\vec{\omega}_C = -\frac{q\vec{B}}{m\gamma}$$



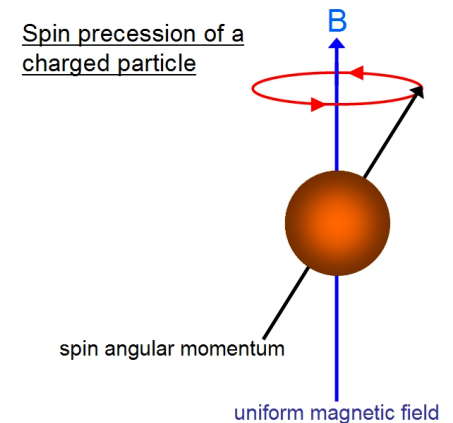
$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- Hence,

$$\vec{\omega}_a = \vec{\omega}_S - \vec{\omega}_C = -\frac{q\vec{B}}{m} \left(\frac{g - 2}{2} \right)$$

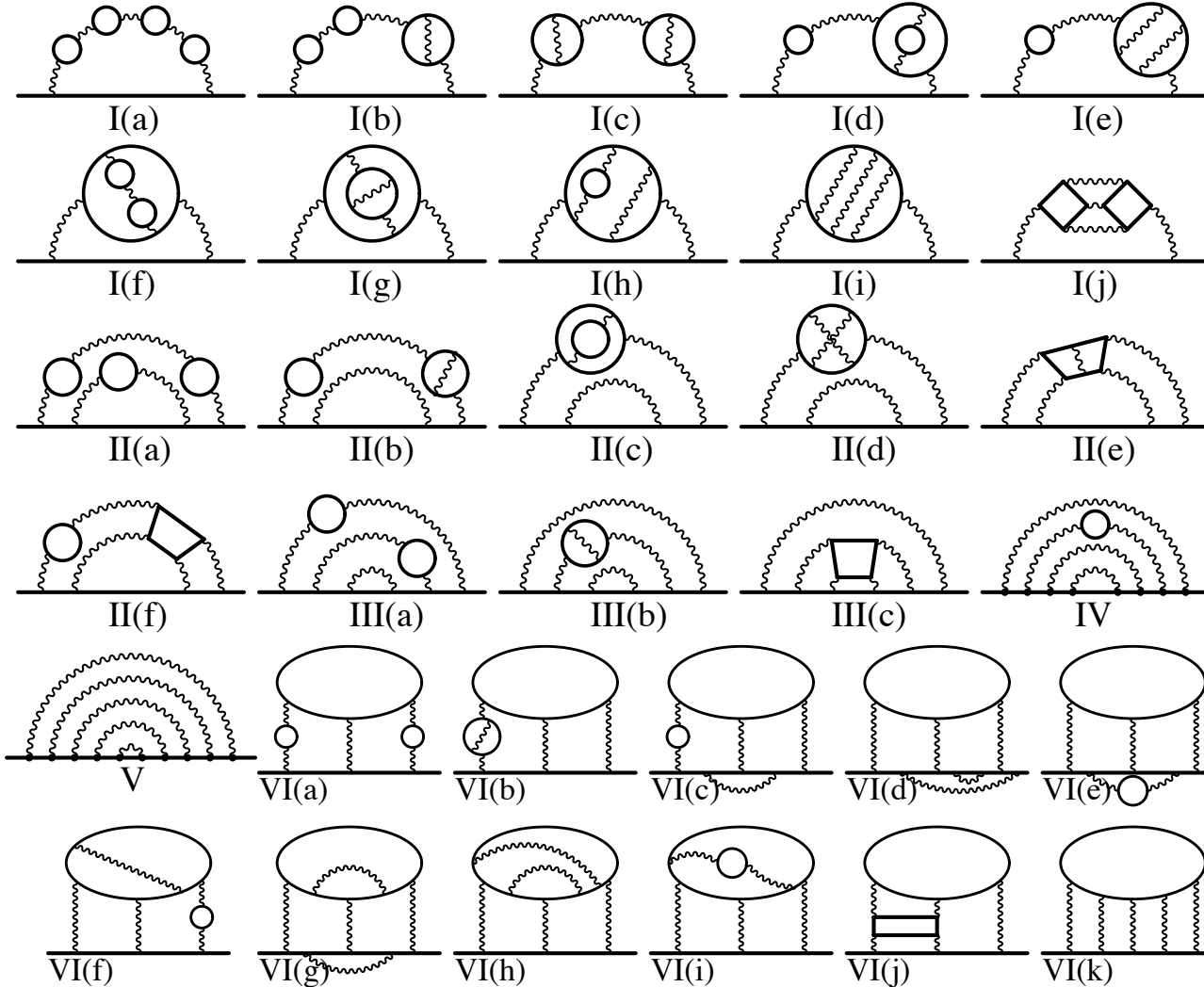
$$a_l = \left(\frac{g - 2}{2} \right) = \frac{\alpha}{2\pi} + \dots$$

- Precise measurement of g-2 is based on a clever way of measuring this frequency difference in a uniform magnetic field.



See, for example, Aoyama, Kinoshita, Nio'17,

5 QED Loop Contributions



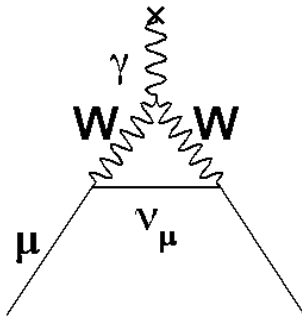
$$a_{\mu}^{\text{QED}}(\alpha(\text{Cs})) = 116\,584\,718.931(7)(17)(6)(100)(23)[104] \times 10^{-11},$$

$$a_{\mu}^{\text{QED}}(\alpha(a_e)) = 116\,584\,718.842(7)(17)(6)(100)(28)[106] \times 10^{-11},$$

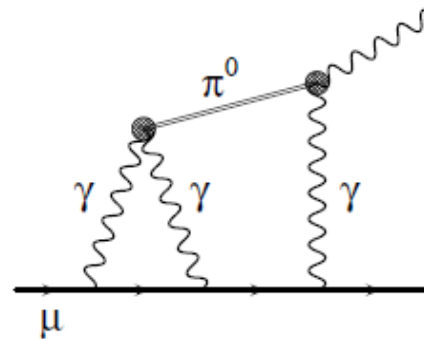
Muon g-2 factor

- The muon is a heavier cousin of the electron with a mass that is about 200 times larger.
- The muon g-2 factor is affected by the same corrections as the electron one, but also by the contribution of weak gauge bosons and heavy mesons in QCD become relevant

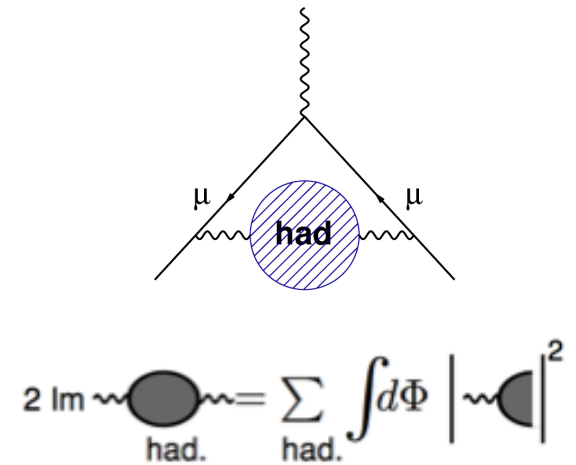
$$\Delta a_l \propto \left(\frac{m_l}{m_{\text{heavy}}} \right)^2$$



$$a_{\mu}^{\text{EW}} = 153.6(1.0) \times 10^{-11},$$



$$a_{\mu}^{\text{LBL}} = 92(19) \times 10^{-11}$$

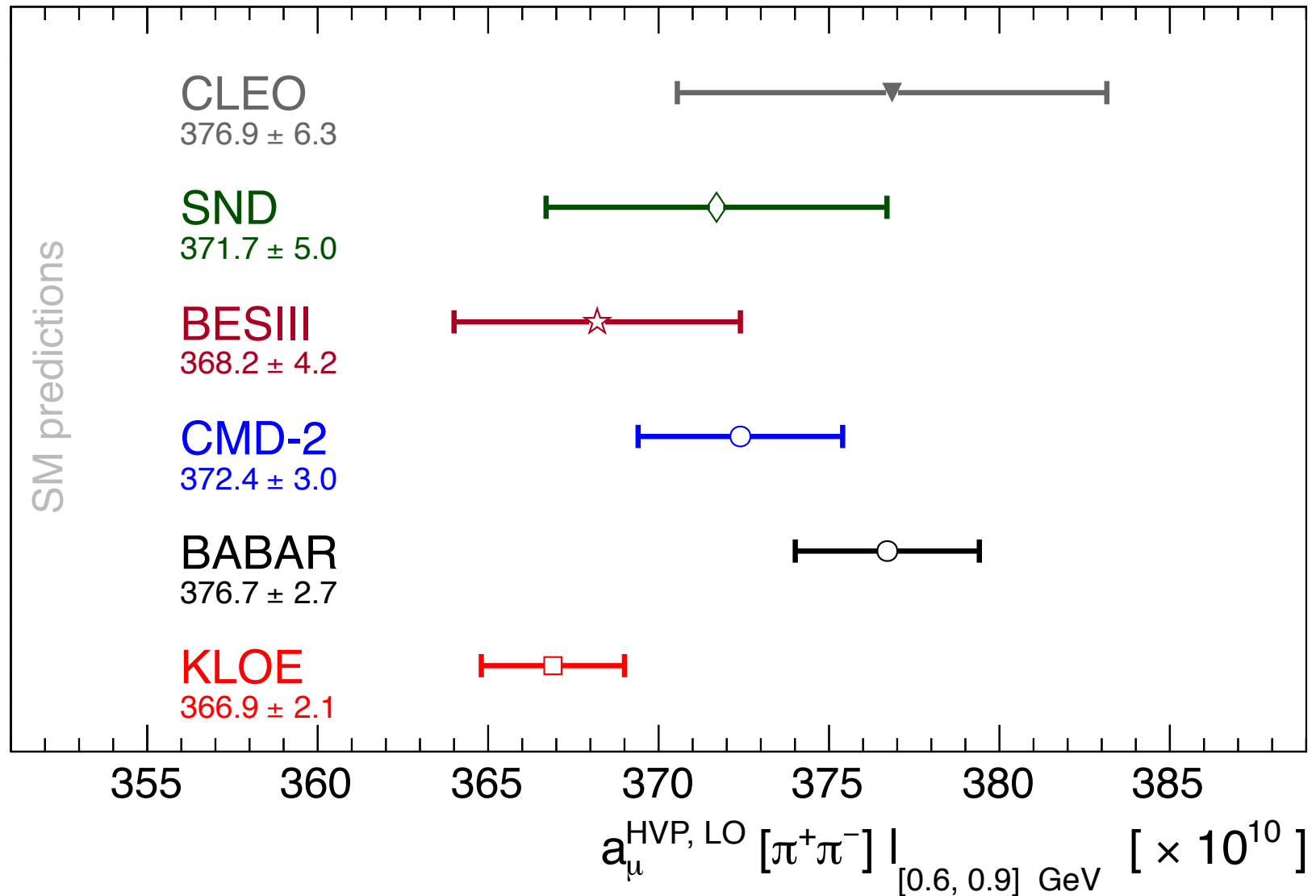


Vacuum polarization contributions computed using hadron cross section data and dispersion relations (optical theorem)

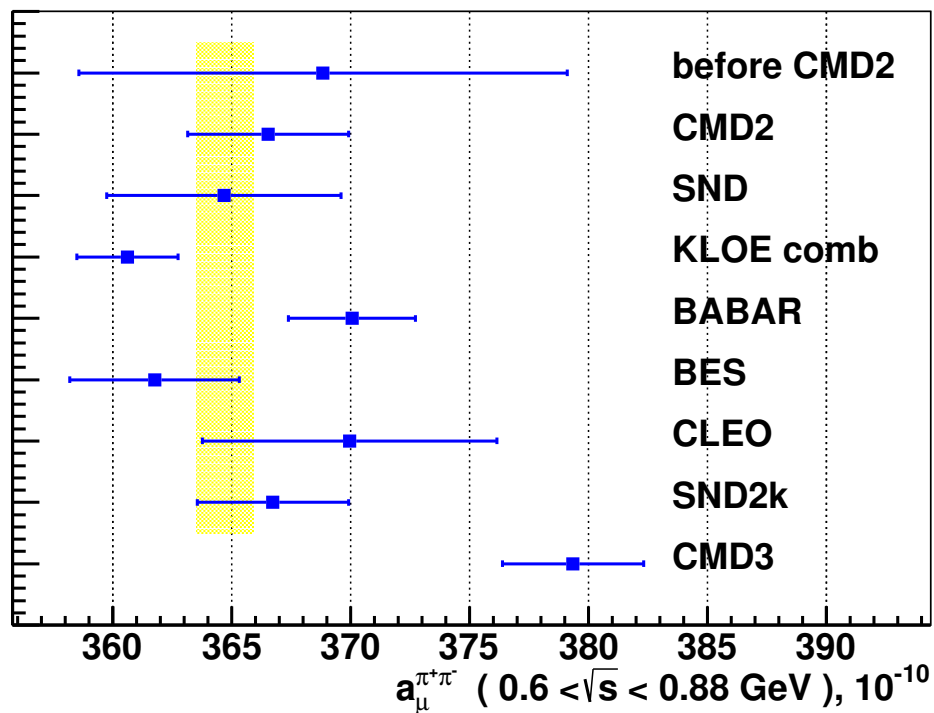
arXiv:2006.04822

Hadronic Vacuum Polarization Contributions based on Data Driven Methods

e^+e^- hadronic cross section + dispersion relations



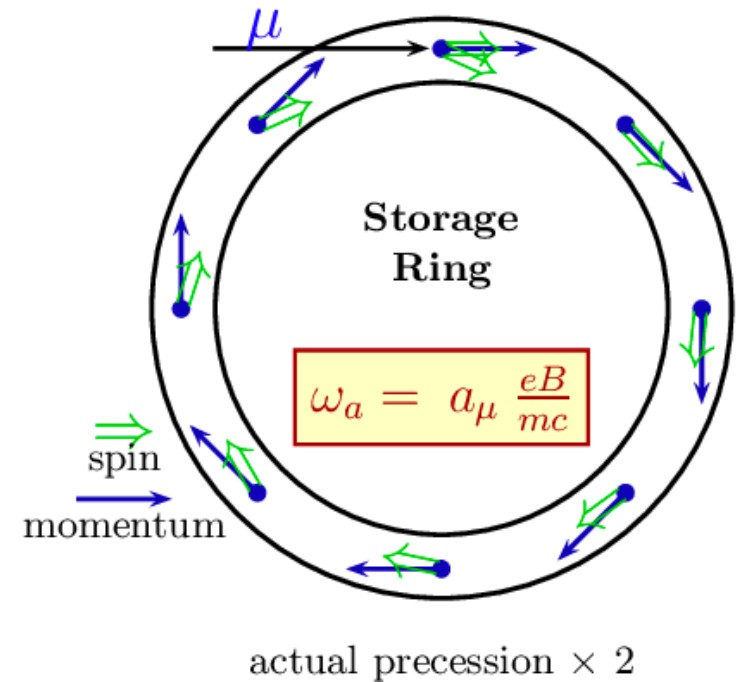
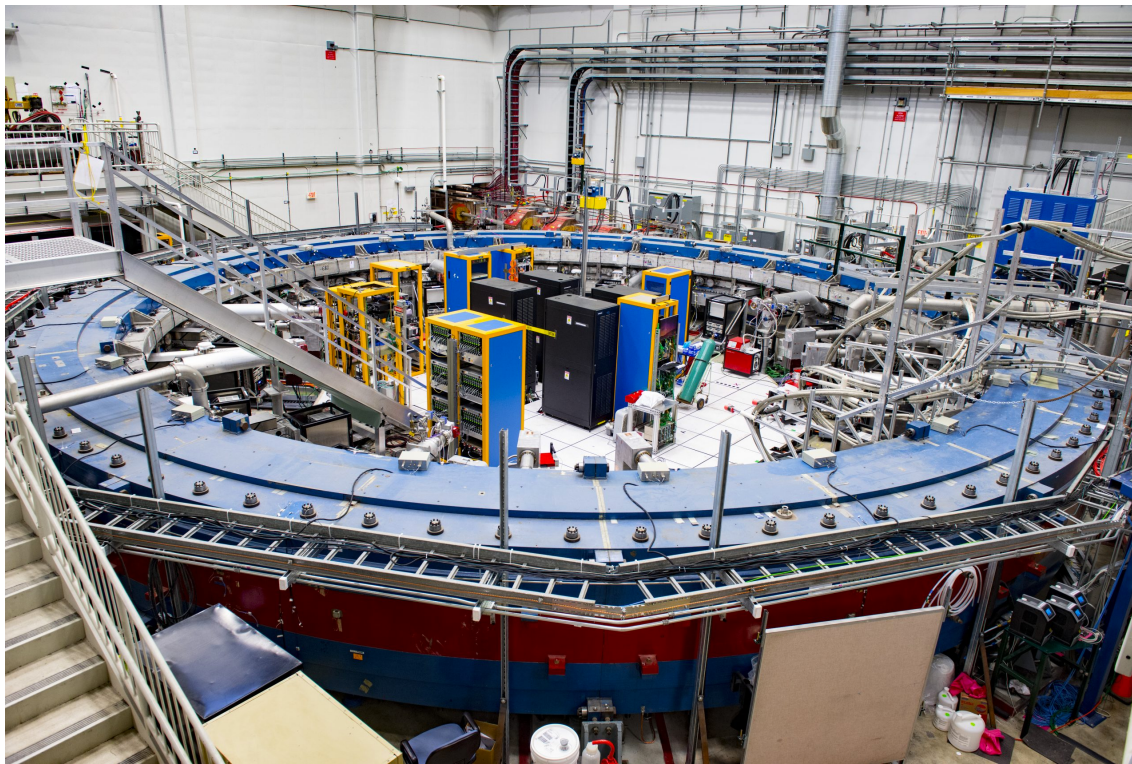
Recent CMD3 Result



Experiment	$a_\mu^{\pi^+\pi^-, LO}, 10^{-10}$
before CMD2	368.8 ± 10.3
CMD2	366.5 ± 3.4
SND	364.7 ± 4.9
KLOE	360.6 ± 2.1
BABAR	370.1 ± 2.7
BES	361.8 ± 3.6
CLEO	370.0 ± 6.2
SND2k	366.7 ± 3.2
CMD3	379.3 ± 3.0

Fermilab g-2 Experiment

Accurate determination of muon Spin precession
in a delicately uniform magnetic field

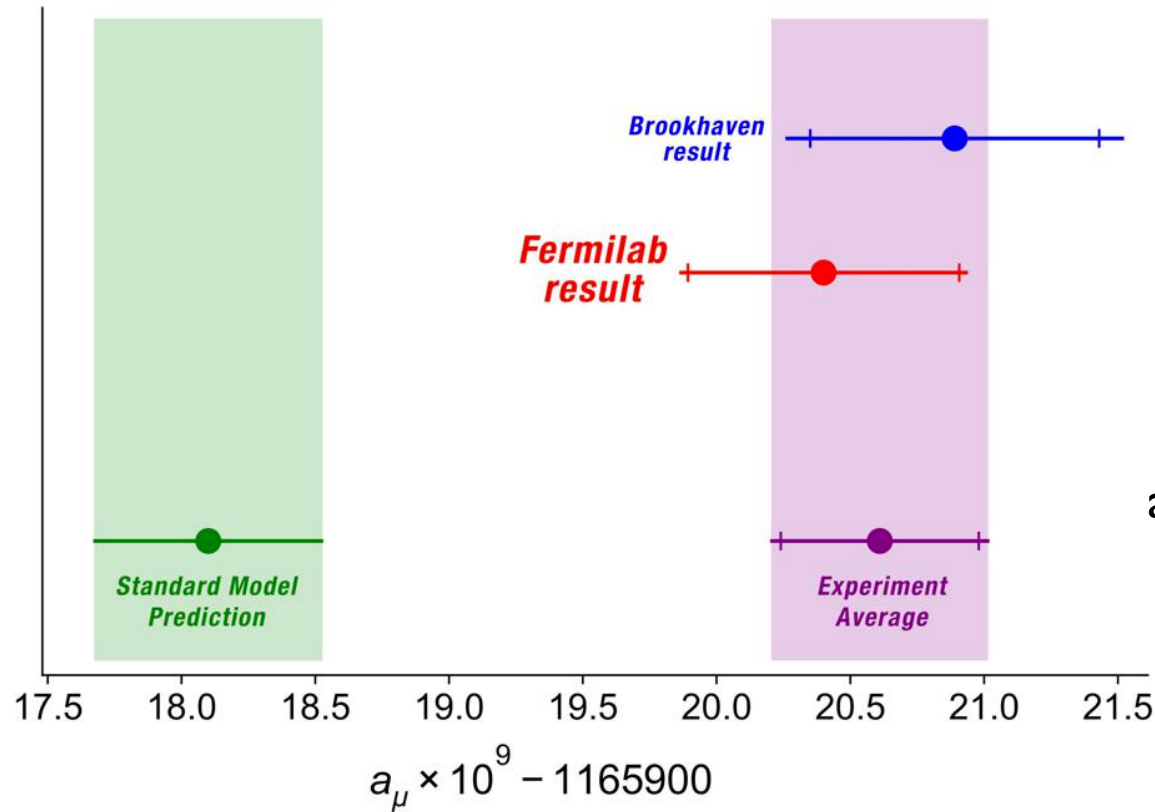


The muon g-2 collaboration confirms the Brookhaven result.

Deviation of 4.2 standard deviations from SM Expectations.

A very important result, that will be further tested in the coming years.

Observe that the g-2 errors are mainly statistical ones.



$$a_\mu^{\text{SM}} = 116\,591\,810(43) \times 10^{-11}$$

arXiv:2006.04822

$$a_\mu^{\text{exp}} = 116\,592\,061(41) \times 10^{-11}$$

$$\Delta a_\mu \equiv (a_\mu^{\text{exp}} - a_\mu^{\text{SM}}) = (251 \pm 59) \times 10^{-11}$$

Comments on the current $g-2$ Anomaly

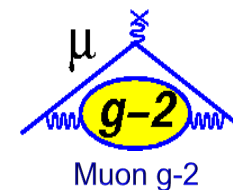
- In a sense, the current discrepancy is between the experimental determination of $g-2$, supported by the Brookhaven and the Fermilab $g-2$ experiments, and the e^+e^- hadronic cross section data.
- In that sense, this anomaly should be taken seriously. The current tension in the hadronic cross section data (KLOE vs BABAR), that cannot lead to an explanation of the measured anomaly, and has already been taken into account in the systematic errors.

arXiv:2006.04822

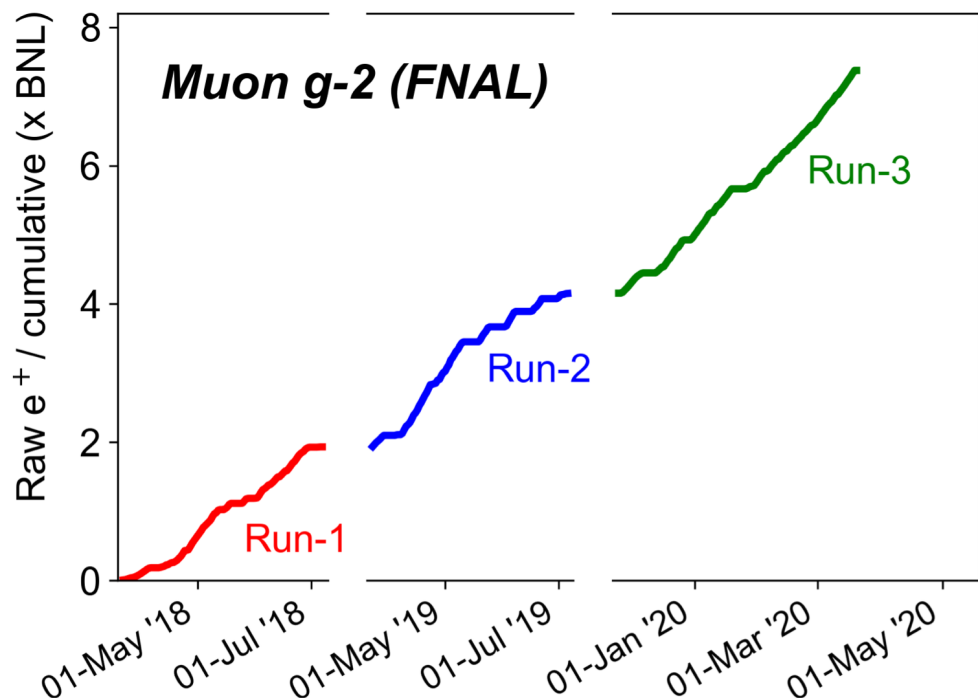
Recent CMD-3 result has not yet been taken into account.

- The good thing is that the $g-2$ collaboration will reduce the error by a factor 2 by next spring and there will be further work on the theoretical estimates.

Data accumulated so far



- Accumulated 7.4xBNL through run 3
- Full run 1 has ~1.2xBNL after Data Quality Cuts
- Improvements between run1 and run 2/3 for:
 - Better beam dynamics
 - Reduced muon loss
 - More stable temperature



RUN1: March-July 2018, ~2x BNL (\rightarrow 1.2 xBNL after data quality)

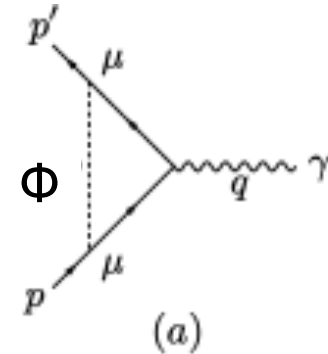
RUN2: March 2019 – July 2019 ~2x BNL

RUN3: Nov 2019 – March 2020 ~3.2 x BNL

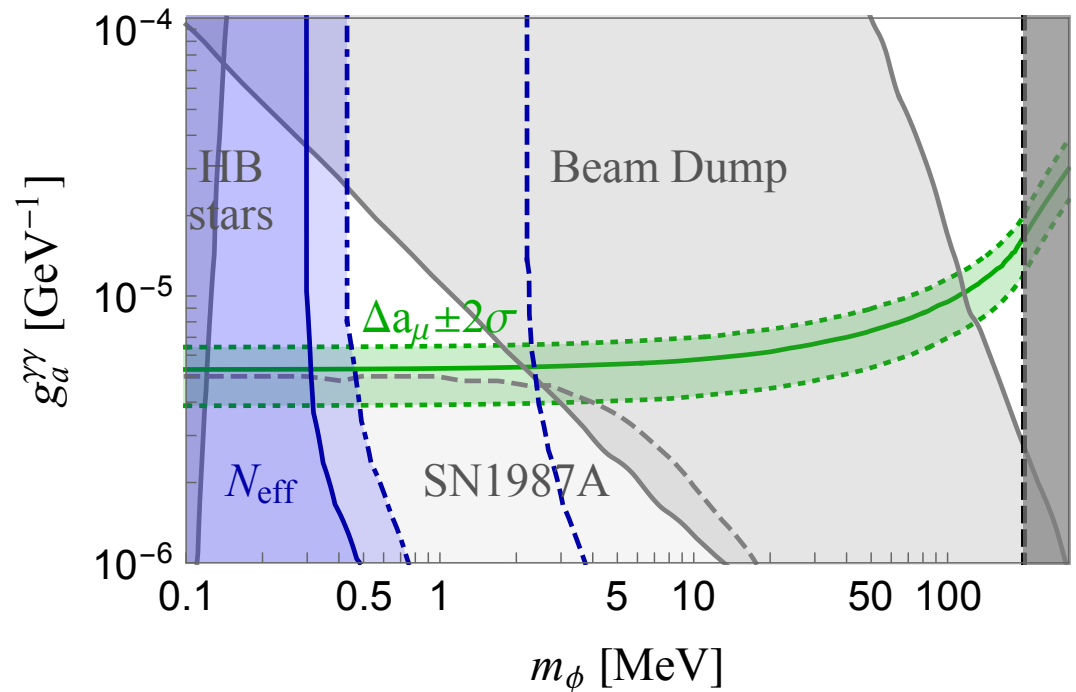
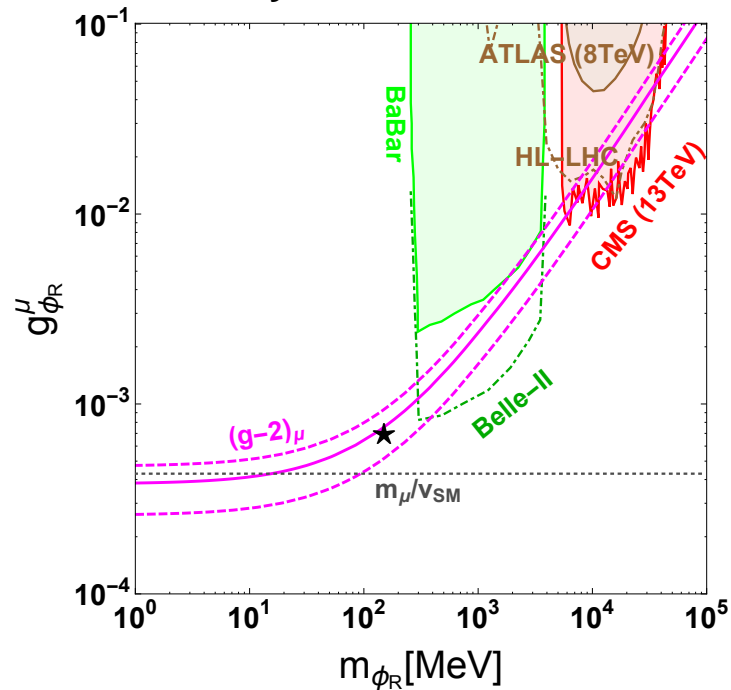
New physics ? Too many possibilities.

Scalar that couples to muons which induces a photon coupling.
Cosmological bound in the 1 MeV region may be avoided if ϕ is the source of neutrino masses

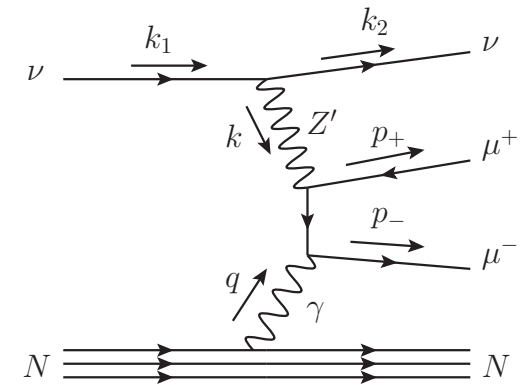
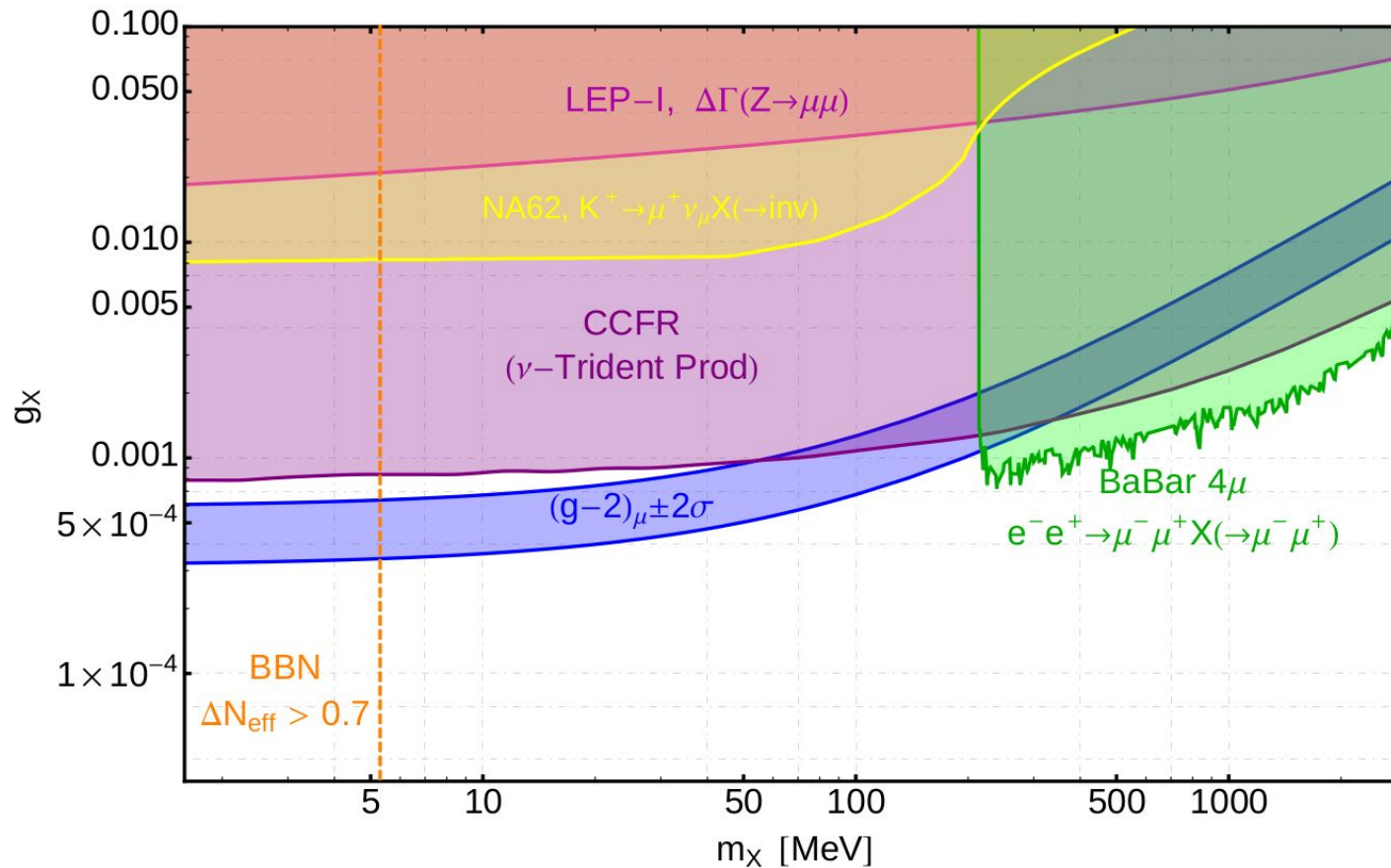
$$\Delta a_\ell = \frac{1}{8\pi^2} \int_0^1 dx \frac{(1-x)^2 ((1+x)g_R^2 - (1-x)g_I^2)}{(1-x)^2 + x(m_S/m_\ell)^2}$$



J. Liu, N. McGinnis, X.Wang, C.W. arXiv:1810.11028, 2110.14665



Bounds on gauge bosons coupled to muons but not electrons or quarks



Altmannshofer, Gori,
Pospelov, Yavin'14

Many other Solutions

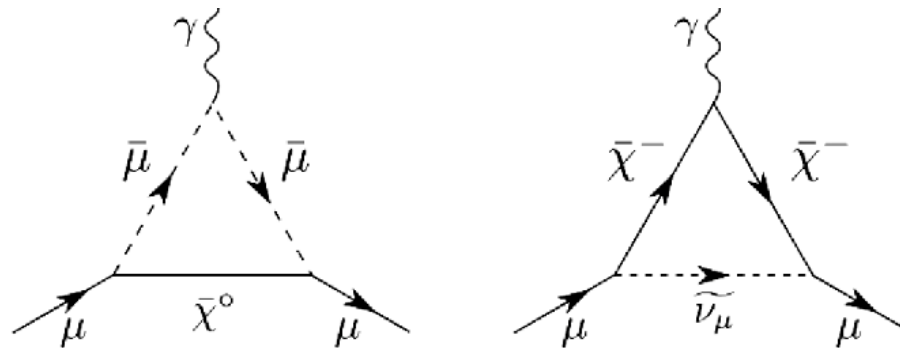
- Axion light particles (beyond the naive one loop solution)
 - Leptoquarks, for suitable arrangement of couplings
 - Two Higgs doublet models, for certain arrangement of the Higgs mass splittings...
-
- Are any of these theories connected to a further understanding of physics at high energies ?

Supersymmetry

Barbieri, Maiani'82, Ellis et al'82, Grifols and Mendez'82
Moroi'95, Carena, Giudice, CW'95, Martin and Wells'00...

$$a_{\mu}^{\tilde{\chi}^{\pm}-\tilde{\nu}_{\mu}} \simeq \frac{\alpha m_{\mu}^2 \mu M_2 \tan \beta}{4\pi \sin^2 \theta_W m_{\tilde{\nu}_{\mu}}^2} \left[\frac{f_{\chi^{\pm}} \left(M_2^2 / m_{\tilde{\nu}_{\mu}}^2 \right) - f_{\chi^{\pm}} \left(\mu^2 / m_{\tilde{\nu}_{\mu}}^2 \right)}{M_2^2 - \mu^2} \right],$$

$$a_{\mu}^{\tilde{\chi}^0-\tilde{\mu}} \simeq \frac{\alpha m_{\mu}^2 M_1 (\mu \tan \beta - A_{\mu})}{4\pi \cos^2 \theta_W (m_{\tilde{\mu}_R}^2 - m_{\tilde{\mu}_L}^2)} \left[\frac{f_{\chi^0} (M_1^2 / m_{\tilde{\mu}_R}^2)}{m_{\tilde{\mu}_R}^2} - \frac{f_{\chi^0} (M_1^2 / m_{\tilde{\mu}_L}^2)}{m_{\tilde{\mu}_L}^2} \right]$$



$$f_{\chi^\pm}(x) = \frac{x^2 - 4x + 3 + 2\ln(x)}{(1-x)^3},$$

$$f_{\chi^0}(x) = \frac{x^2 - 1 - 2x\ln(x)}{(1-x)^3};$$

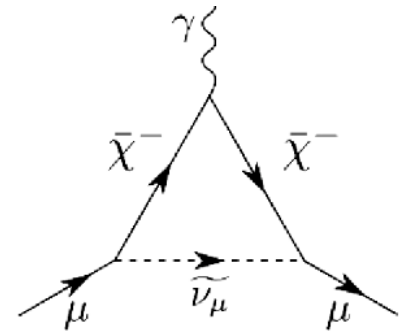
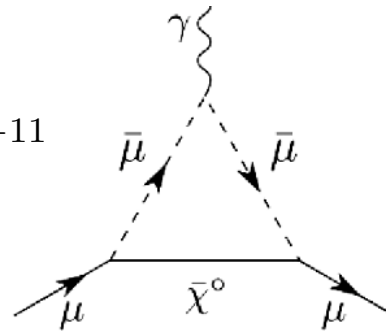
Rough Approximation

- If all **weakly interacting** supersymmetric particle masses were the same, and the gaugino masses had the same sign, then

$$(\Delta a_\mu)^{\text{SUSY}} \simeq 150 \times 10^{-11} \left(\frac{100 \text{ GeV}}{m_{\text{SUSY}}} \right)^2 \tan \beta$$

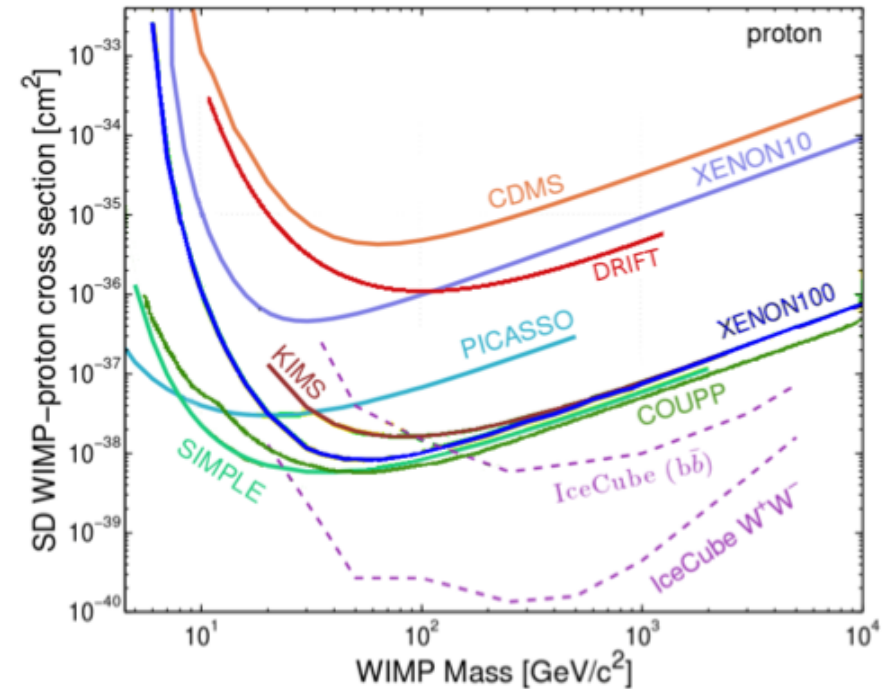
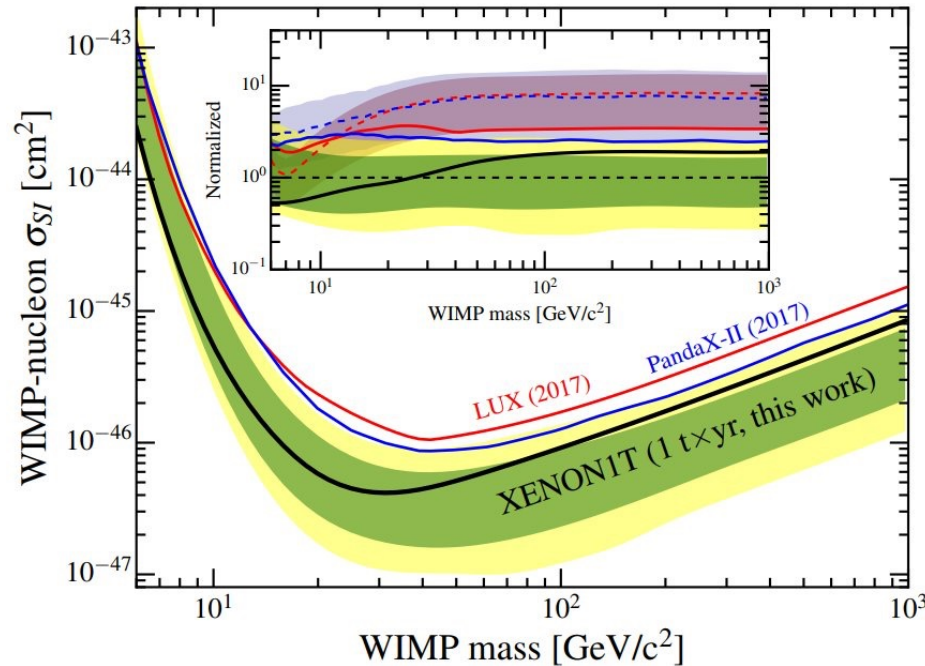
- This implies that, for **$\tan \beta = 10$** , particle masses of order **250 GeV** could explain the anomaly, while for values of **$\tan \beta = 60$** (consistent with the unification of the top and bottom Yukawa) these particle masses could be of order **700 GeV**.

$$\Delta a_\mu \equiv (a_\mu^{\text{exp}} - a_\mu^{\text{SM}}) = (251 \pm 59) \times 10^{-11}$$



Dark Matter

DM : Direct Detection Bounds



$$\sigma_p^{\text{SI}} \propto \frac{m_Z^4}{\mu^4} \left[2(m_{\tilde{\chi}_1^0} + 2\mu/\tan\beta) \frac{1}{m_h^2} + \mu \tan\beta \frac{1}{m_H^2} + (m_{\tilde{\chi}_1^0} + \mu \tan\beta/2) \frac{1}{m_{\tilde{Q}}^2} \right]^2$$

Blind Spot :

$$2 \left(m_{\tilde{\chi}_1^0} + 2 \frac{\mu}{\tan\beta} \right) \frac{1}{m_h^2} \simeq -\mu \tan\beta \left(\frac{1}{m_H^2} + \frac{1}{2m_{\tilde{Q}}^2} \right) \quad \begin{array}{l} \mu \times m_{\tilde{\chi}_1^0} < 0 \\ m_{\tilde{\chi}_1^0} \simeq M_1 \end{array}$$

Cheung, Hall, Pinner, Ruderman'12, Huang, C.W.'14, Cheung, Papucci, Shah, Stanford, Zurek'14, Han, Liu, Mukhopadhyay, Wang'18

$$\sigma^{\text{SD}} \propto \frac{m_Z^4}{\mu^4} \cos^2(2\beta)$$

Dependence of the cross section on the heavy Higgs mass

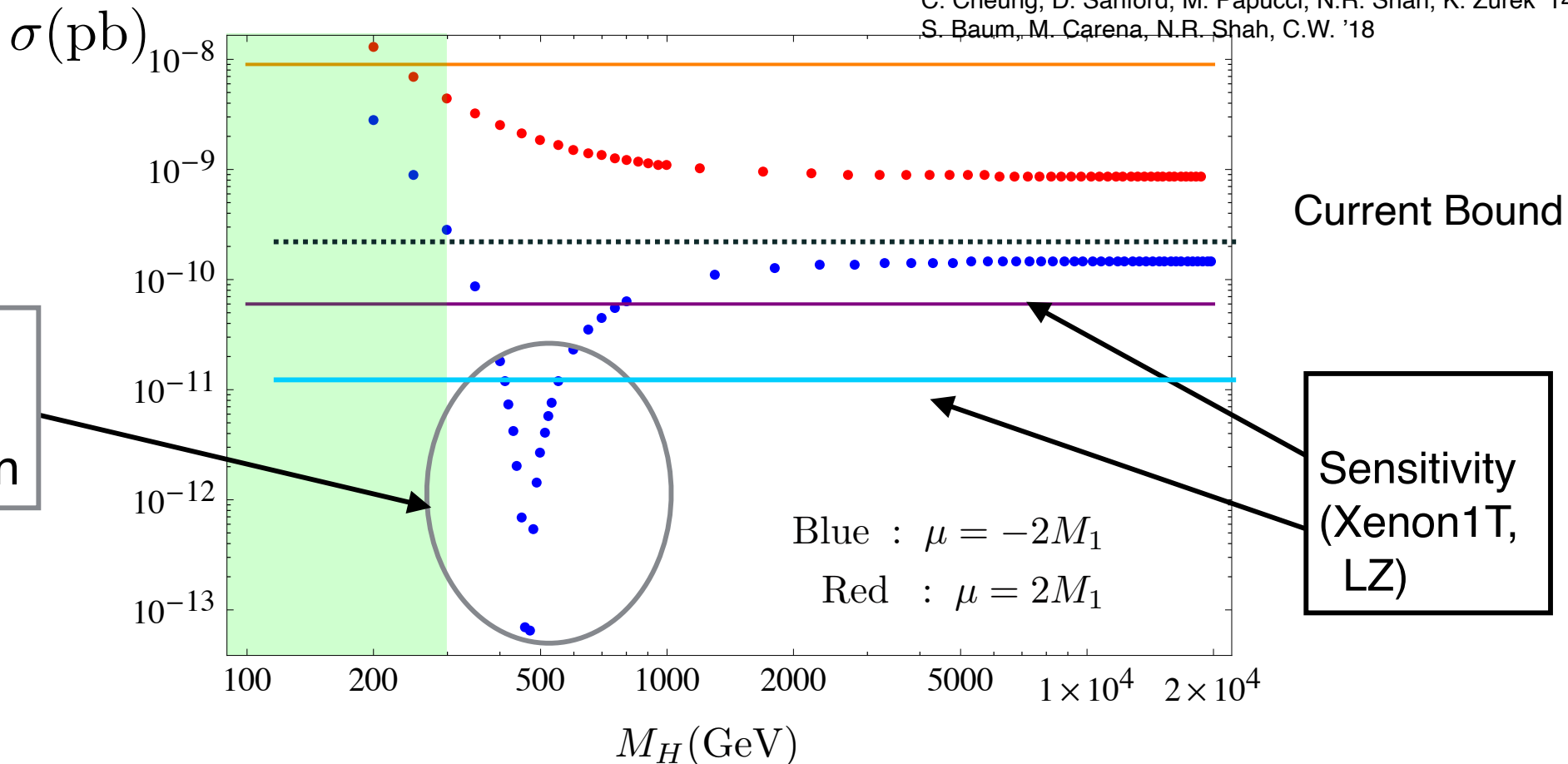
Negative values of μ : Much weaker direct spin-independent detection bounds

Blind Spots : $2(m_{\chi^0} + \mu \sin 2\beta) \frac{1}{m_h^2} = -\mu \tan \beta \frac{1}{M_H^2}$

J. Ellis, K. Olive, Y. Santos, V.C. Spanos'05
 H. Baer, A. Mustafayev, E.K. Park, X. Tata'07
 C. Cheung, L. Hall, D. Pinner, J. Ruderman'12
 P. Huang, C.W.'14
 P. Huang, R. Roglans, D. Spiegel, Y. Sun, C.W.'17
 C. Cheung, D. Sanford, M. Papucci, N.R. Shah, K. Zurek '14
 S. Baum, M. Carena, N.R. Shah, C.W. '18

P. Huang, C.W. 1404.0392

$\tan \beta = 10$



$$m_{\chi^\pm} = |\mu|, \quad m_{\chi^0} = M_1, \quad M_1 = 200 \text{ GeV}$$

g-2 and Direct Detection

Reduction of the cross section is obtained for negative values of $\mu \times M_1$

The direct detection cross sections can also be suppressed for large values of μ

g-2 has two contributions, the Bino one proportional to $\mu \times M_1$ and the other (chargino) proportional to $\mu \times M_2$

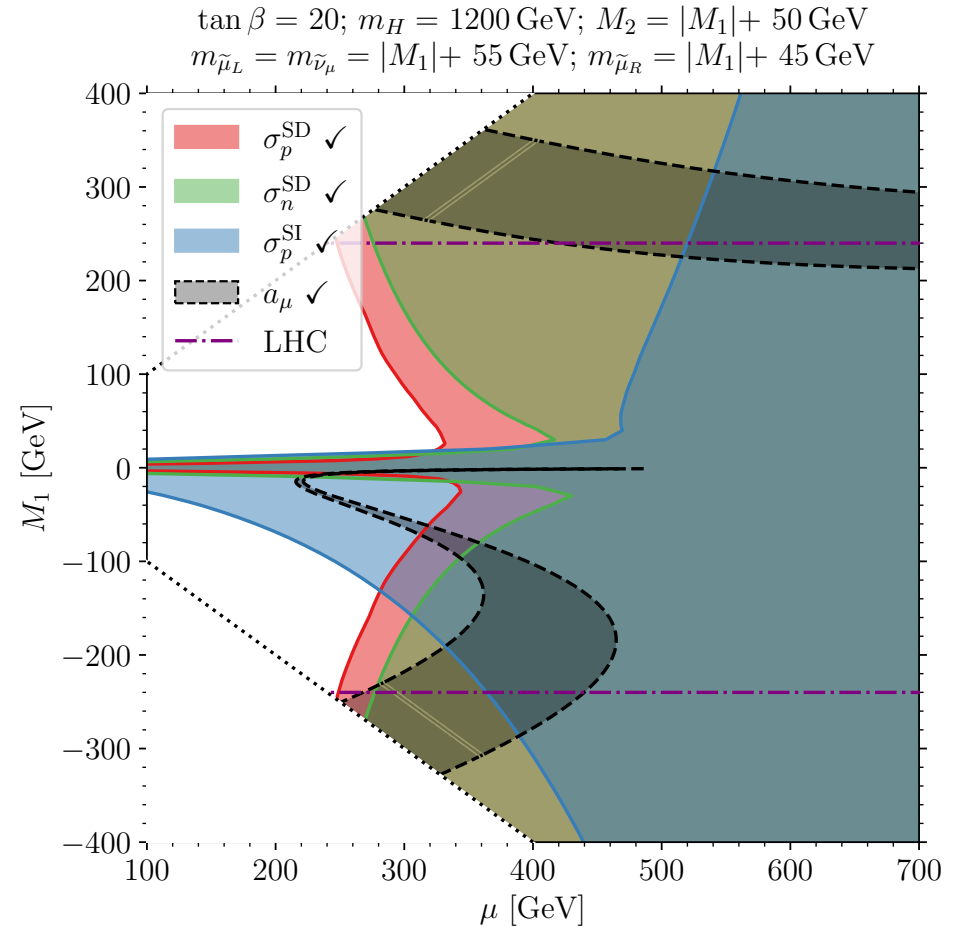
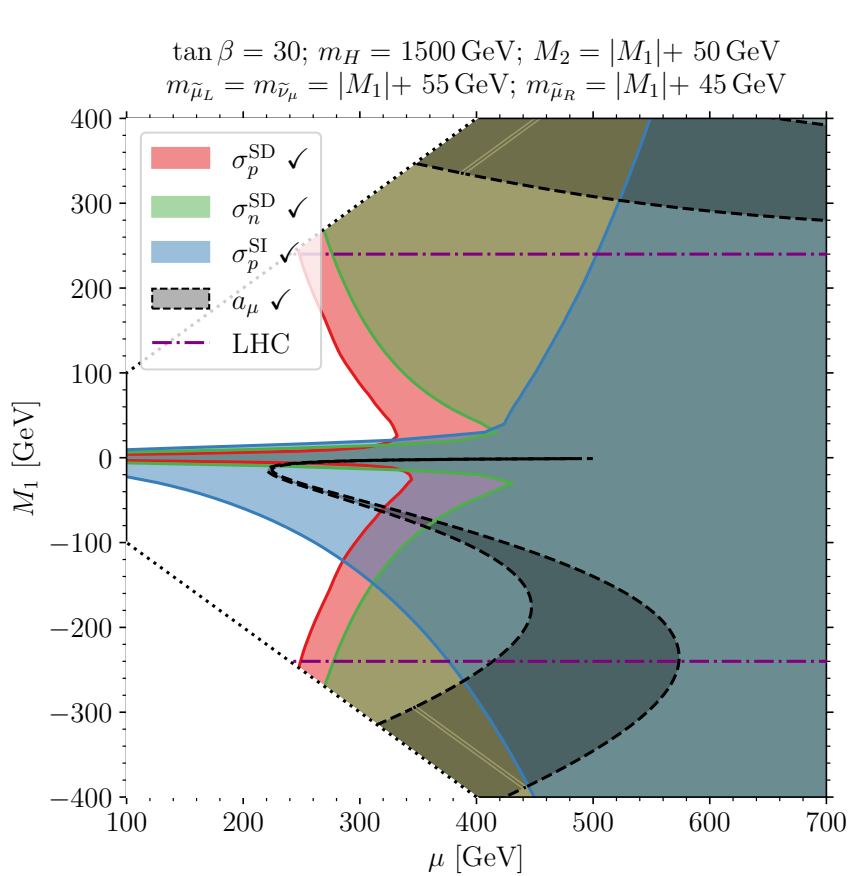
The Bino contribution to g-2 is negative at the proximity of the blind spot but becomes subdominant at smaller values of μ

The chargino contribution is the dominant one for masses of the same order and is suppressed at large μ

Since g-2 needs to be positive, compatibility between g-2 results and Direct detection may be either achieved for large values of μ or for smaller values of μ , when the relative sign of the gaugino masses is opposite.

Compatibility of Dark Matter and g-2 Constraints for a representative example of a compressed spectrum. Stau co-annihilation is assumed

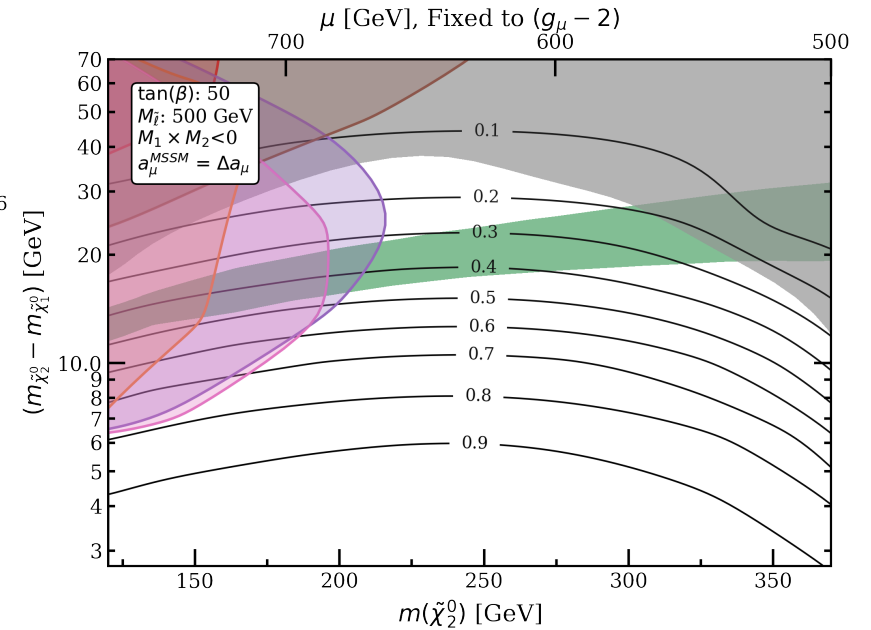
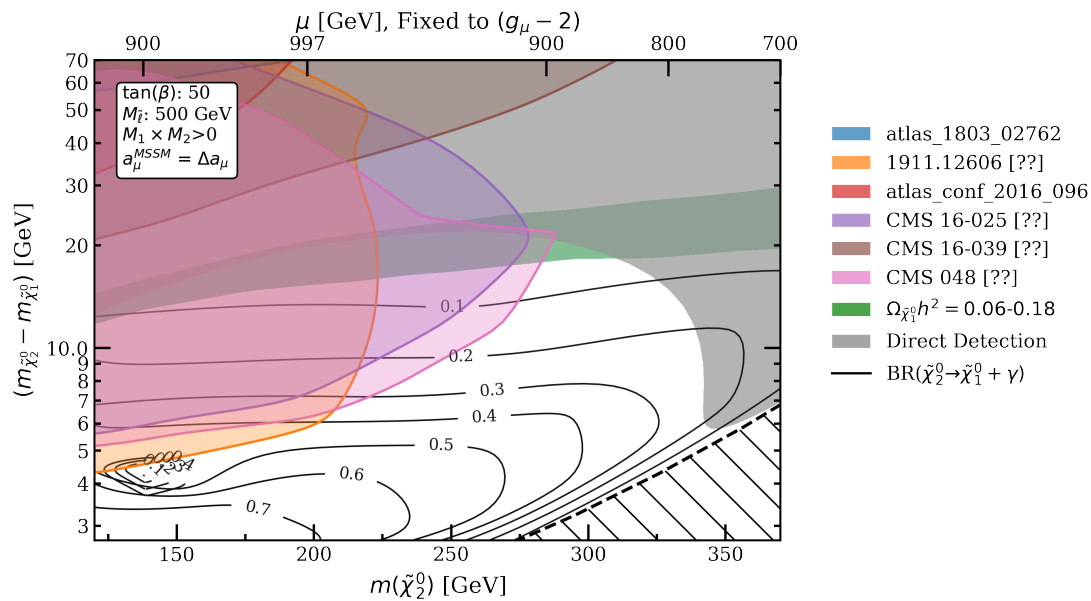
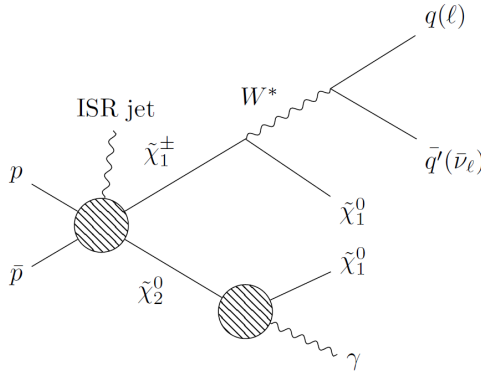
Large hierarchy of values of μ between positive and negative values of the Bino mass parameter is observed.



New LHC Channel

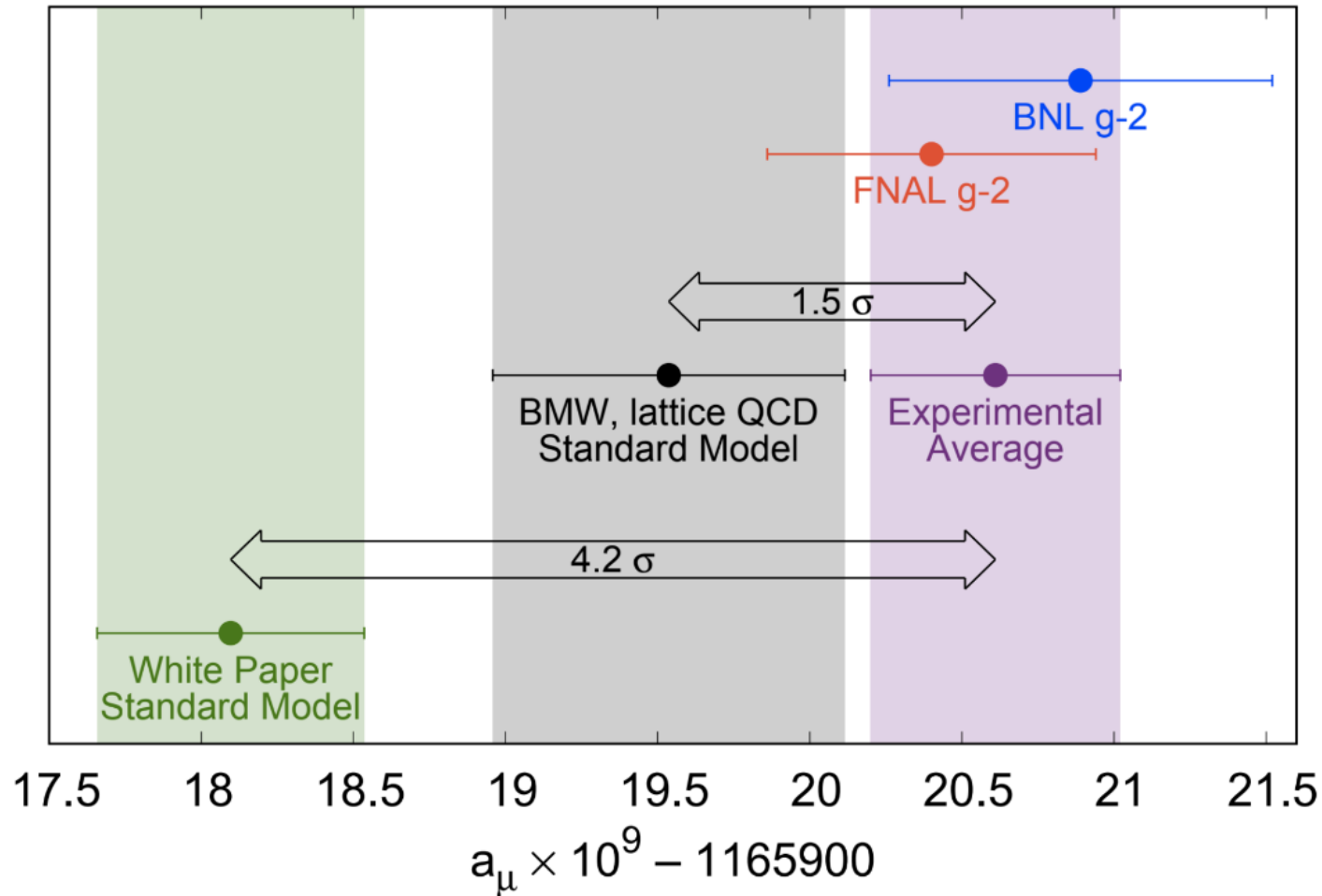
S. Baum, M. Carena, N. Shah, C. Wagner
D. Rocha, T. Ou, arXiv: 2203.01523

(see M. Carena's talk on Thursday)



Comparison of BMW lattice computation with data driven methods

Z. Fodor '21



Comments on the Lattice Evaluation

- The Lattice results should be taken seriously and are a triumph of physics.
- HPV effects would have an impact on the variation of the fine structure constant, affecting precision measurements at M_Z , and any correction from the current values should be limited to energies below 0.9 GeV.

Crivellin et al, 2003.04886; Kezhavarzi, Marciano, Pasera, Sirlin, arXiv: 2006.12666

See also arXiv:2010.07943

- Tension with data could be resolved by a large systematic error in the cross sections evaluation or by new physics contributing to them (Lattice = SM).
- It could also be resolved by some unaccounted systematic error in the lattice evaluations. BMW provides a detailed account of their error estimates and it could be therefore double checked by other lattice groups. So far, all checks have confirmed the BMW results, but in an energy window that represents only a third of the total contribution to the hadronic vacuum polarization.

arXiv: 2202.12347, 2204.12256, 2206.0582,
2206.15084, 2204.01280, 2301.08696, 2301.08274...

- For a recent analysis see arXiv:2306.16808

Dispersion relation

The relation between the hadronic cross section and the anomalous magnetic moment is given by

$$(a_\mu^{\text{had,VP}})_{ee} = \frac{1}{4\pi^3} \int_{m_{\pi^0}^2}^{\infty} ds K(s) \sigma_{\text{had}}(s)$$

$K(s) \simeq m_\mu^2/(3s) \text{ for } s \gg m_\mu^2$

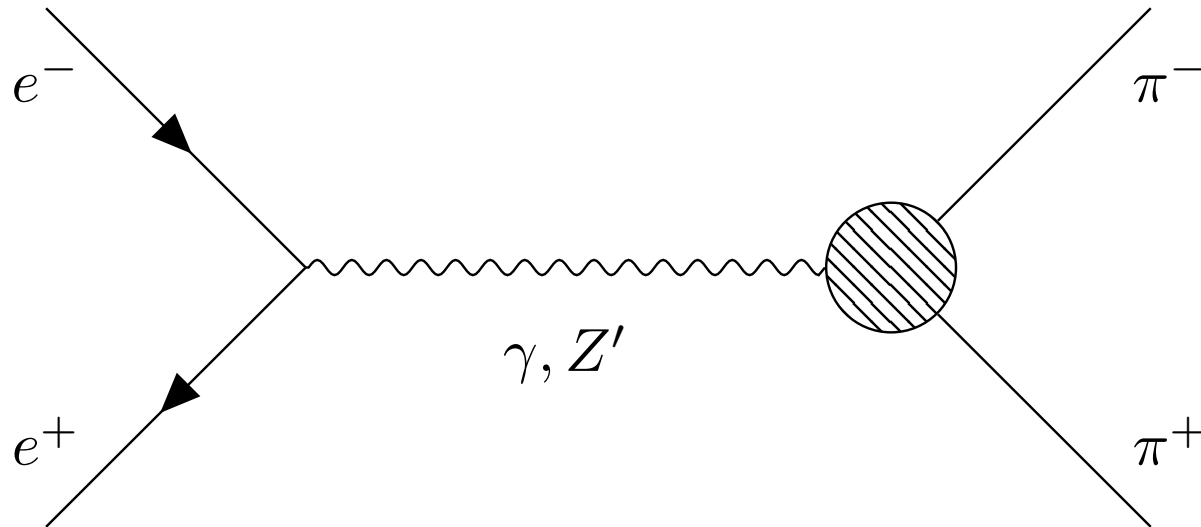
The lattice calculation leads to

$$\Delta a_\mu^{\text{HVP}} = (a_\mu^{\text{had,VP}})_{BMW} - (a_\mu^{\text{had,VP}})_{ee} = 1.44 \times 10^{-9}$$

This implies that, if new physics causes the difference, it should reduce the cross section.

$$\Delta a_\mu^{\text{HVP}} = \frac{1}{4\pi^3} \int_{m_{\pi^0}^2}^{\infty} ds K(s) (-\Delta \sigma_{\text{had}}(s))$$

New Contributions to the Hadronic Cross Section



Vector gauge boson

The simplest model that can induce such an effect is a new vector gauge boson that couples to first generation

Di Luzio et al, 2112.08312

$$\mathcal{L} \supset (g_V^e \bar{e} \gamma^\mu e + g_V^u \bar{u} \gamma^\mu u + g_V^d \bar{d} \gamma^\mu d) Z'_\mu$$

The total hadronic cross section in the 1 GeV region is given by

$$\frac{\sigma_{\pi\pi}^{SM+Z'}}{\sigma_{\pi\pi}^{SM}}(s) = \left| 1 - \frac{g_V^e (g_V^u - g_V^d)}{e^2} \frac{s}{s - m_{Z'}^2 + i m_{Z'} \Gamma_{Z'}} \right|^2$$

$$\Gamma(Z' \rightarrow l^+ l^-) = \frac{1}{3} \frac{(g_V^l)^2}{4\pi} \sqrt{1 - 4 \frac{m_l^2}{m_{Z'}^2}} \left(1 + 2 \frac{m_l^2}{m_{Z'}^2} \right)$$

$$\Gamma(Z' \rightarrow \pi^+ \pi^-) = \frac{1}{3} \frac{(g_V^{ud})^2}{4\pi} \sqrt{1 - 4 \frac{m_\pi^2}{m_{Z'}^2}} \left(1 + 2 \frac{m_\pi^2}{m_{Z'}^2} \right) R(m_{Z'}^2)$$

$$R(s) = \frac{\sigma_{e^+ e^- \rightarrow had}}{\sigma_{e^+ e^- \rightarrow \mu^+ \mu^-}}(s)$$

Fit to Data

$$\sigma_{\text{had}}^{ee}(s) = \sigma_{\pi\pi}^{SM}(s) \left(1 + \frac{\tilde{g}^2 s^2 + 2\tilde{g}s(s - m_{Z'}^2)}{(s - m_{Z'}^2)^2 + m_{Z'}^2 \Gamma_{Z'}^2} \right) \equiv \sigma_{\pi\pi}^{SM}(s) (1 + \delta(\tilde{g}, s))$$

$$\tilde{g} \equiv -g_V^e(g_V^u - g_V^d)/e^2 \qquad g_V^{ud} = g_V^u - g_V^d$$

From here, one can fit for the couplings and the mass of the new gauge boson. Observe that only this coupling combination is determined.

Light gauge bosons are highly restricted due to their contribution to the electron g-2 and isospin violating constraints. For a mass of about 800 MeV

$$g_V^e g_V^{ud} \simeq -4 \times 10^{-4}$$

Isospin Breaking Effects

$$\Delta m_\pi^2 \Big|_{Z'} = \frac{(g_V^{ud})^2}{32\pi^2} \int_0^\infty ds \frac{s}{s + m_{Z'}^2} (F_\pi^V(-s))^2 \left(4W + \frac{s}{m_\pi^2} (W - 1) \right)$$

Crivellin and Hoferichter, 2211.12516

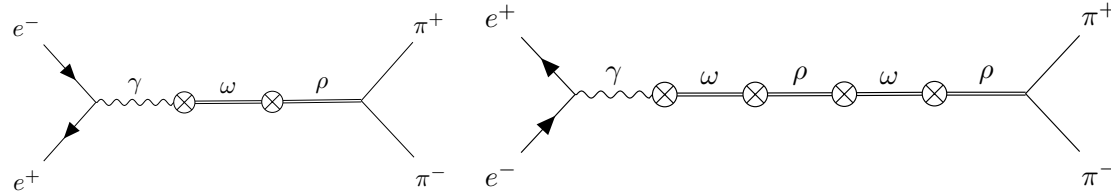
$$W = \sqrt{1 + 4m_\pi^2/s}$$

$$F_\pi^V(-s) = (A_{\rho\gamma}/e) g_{\rho\pi} / (s + m_\rho^2)$$

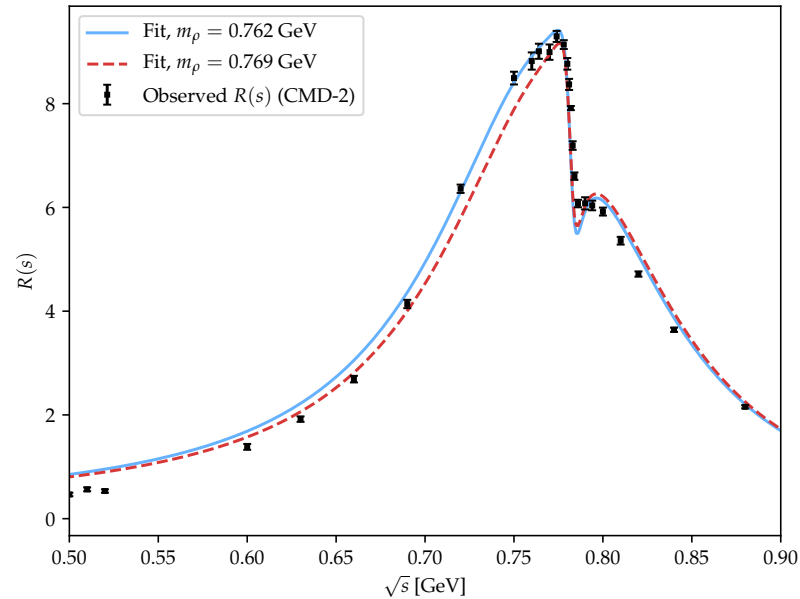
$$g_V^{ud} \lesssim 0.1$$

We therefore restrict the isospin breaking coupling to this range

Pion Form Factor

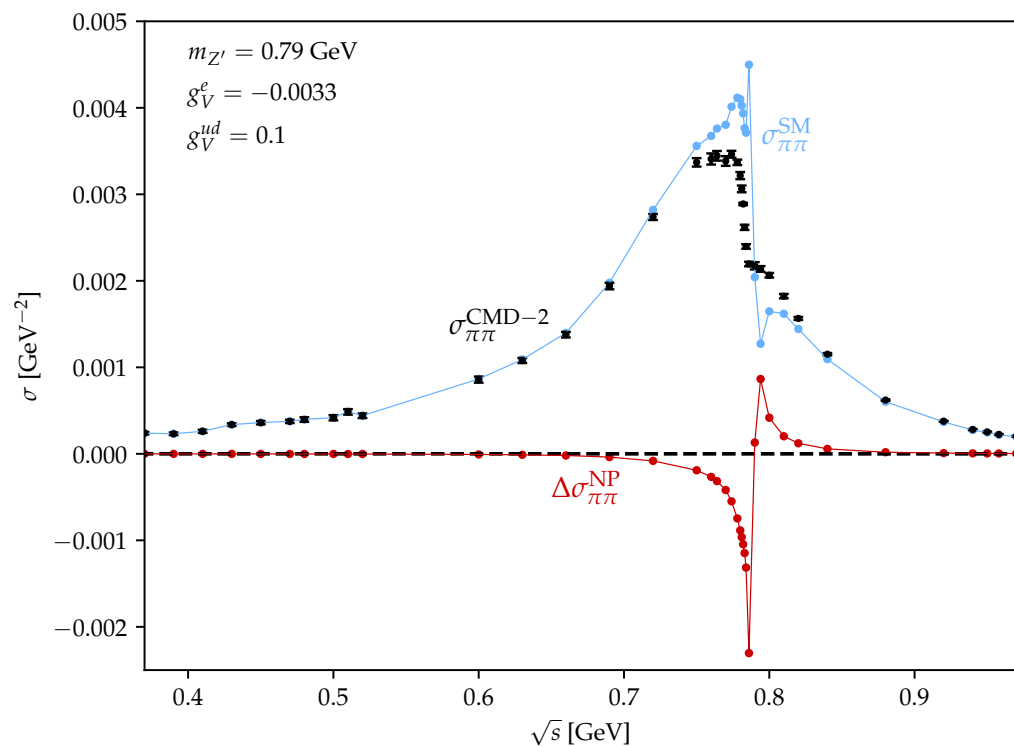


$$R(s) = \frac{1}{4} \left| D(\gamma) \left(\frac{A_{\rho\gamma} + A_{\gamma\omega} D(\omega) A_{\rho\omega}}{1 - A_{\rho\gamma}^2 D(\gamma) D(\rho) - A_{\rho\omega}^2 D(\omega) D(\rho)} \right) D(\rho) \right|^2 \left(\frac{g_{\rho\pi}}{e} \right)^2$$

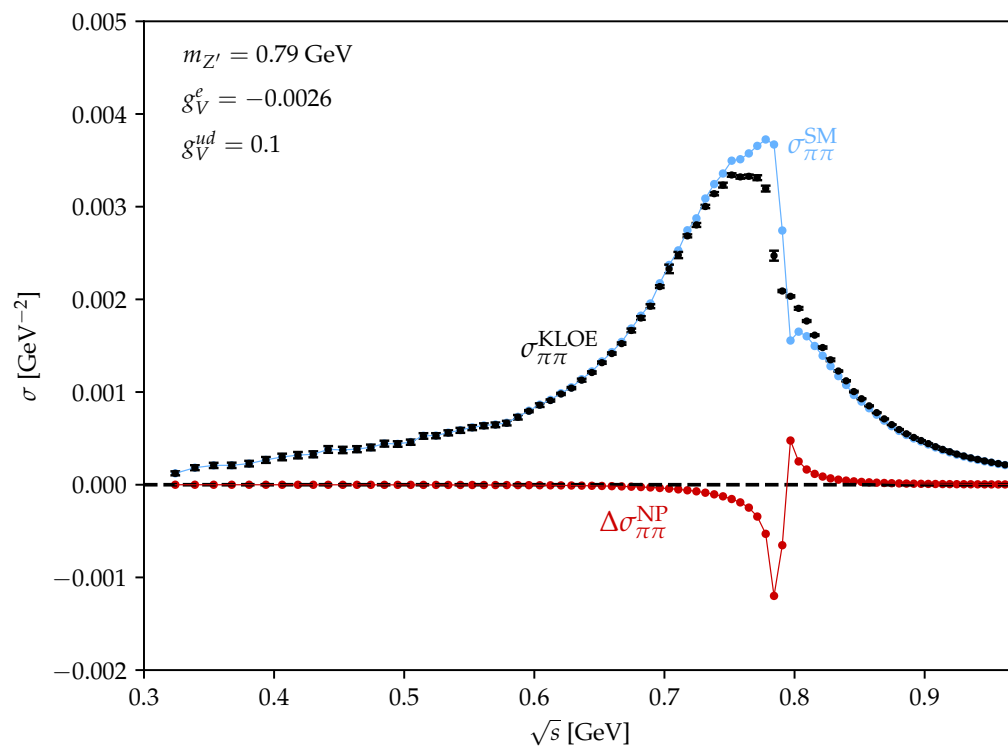


Narrow width Feature

N. Coyle, C.W. arXiv:2305.02354



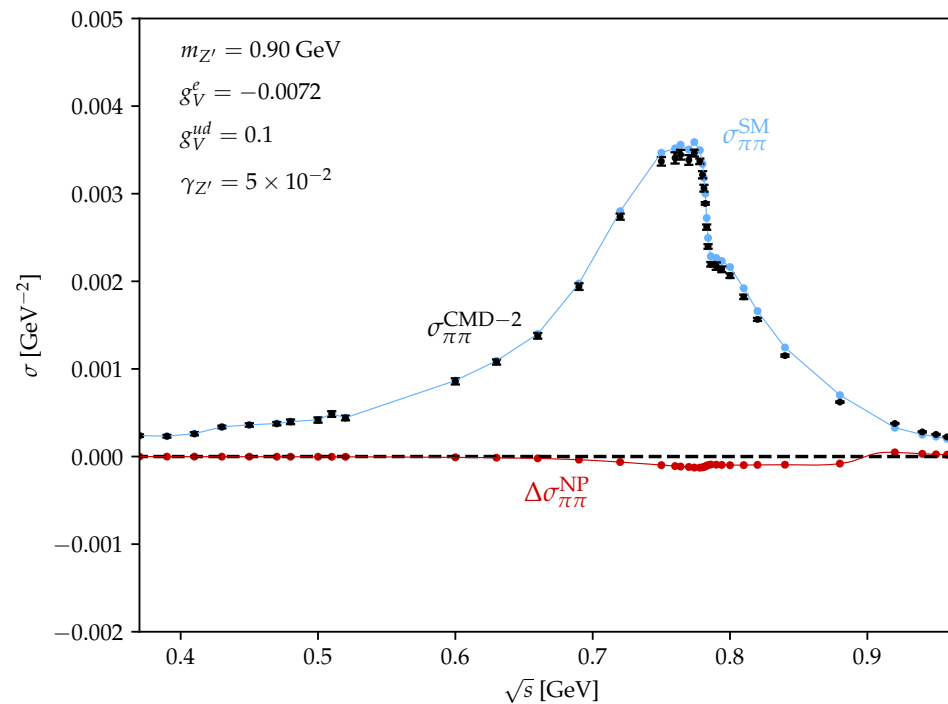
$$\begin{aligned}
 g_V^{ud} &= 0.1 \\
 g_V^e &= -3.3 \times 10^{-3} \\
 m_{Z'} &= 0.79 \text{ GeV}
 \end{aligned}$$



$$\begin{aligned}
 g_V^{ud} &= 0.1 \\
 g_V^e &= -2.6 \times 10^{-3} \\
 m_{Z'} &= 0.79 \text{ GeV}
 \end{aligned}$$

Extra Width Contributions

N. Coyle, C.W. arXiv:2305.02354



$$\gamma_{Z'} = \Gamma_{Z'}/m_{Z'}$$

With an additional contribution to the width the feature disappears. Dark sector challenge.

Benchmarks

N. Coyle, C.W. arXiv:2305.02354

	CMD-2			KLOE		
Benchmark	$m_{Z'} \text{ (GeV)}$	g_V^e	g_V^{ud}	$m_{Z'} \text{ (GeV)}$	g_V^e	g_V^{ud}
1	0.80	-6.6×10^{-3}	0.10	0.80	-6.7×10^{-3}	0.10
2	0.80	-8.3×10^{-3}	0.08	0.80	-8.4×10^{-3}	0.08
3	0.90	-7.2×10^{-3}	0.10	0.90	-7.5×10^{-3}	0.10
4	0.90	-9.0×10^{-3}	0.08	0.90	-9.4×10^{-3}	0.08

Table 1: Example benchmark points with a fixed width of $\gamma_{Z'} = 5 \times 10^{-2}$.

Constraints

Electron g-2

$$\Delta a_e^{Z',\text{loop}} \sim (1.4 \times 10^{-14}) \left(\frac{800 \text{ MeV}}{m_{Z'}} \right)^2 \left(\frac{g_V^e}{2 \times 10^{-3}} \right)^2$$
$$|\Delta a_e^{Z',\text{loop}}| \lesssim 10^{-12}.$$

For Z' masses of order 800 MeV, this is not a serious model building constraint.

BABAR Bounds

Come mainly from the process

N. Coyle, C.W. arXiv:2305.02354

$$e^+e^- \rightarrow Z'\gamma, Z' \rightarrow e^+e^-$$

To avoid the constraint, one should reduce the coupling to electrons, compared to hadrons.

This reduces the production cross section as well as the branching ratio of the decay into electrons

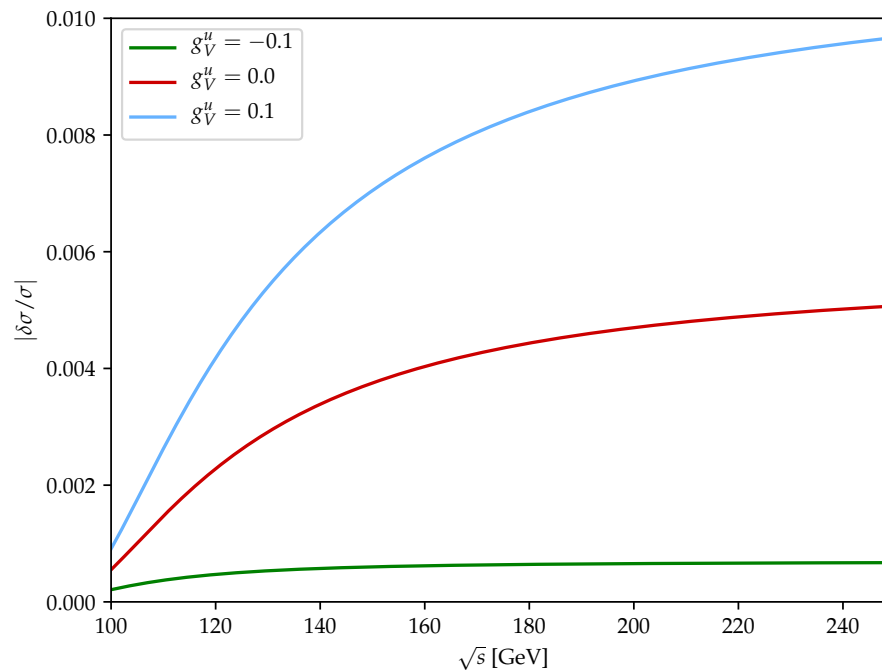
$$g_V^{ud} = 0.1 \rightarrow |g_V^e| \lesssim 5.6 \times 10^{-3}$$

$$\text{Fixed width } \gamma_{Z'} = 5 \times 10^{-2} \rightarrow |g_V^e| \lesssim 1.3 \times 10^{-2}$$

LEP2 Bounds

$$e^+e^- \rightarrow q\bar{q}$$

$$\frac{\delta\sigma}{\sigma} = \frac{\sigma_{Z'\gamma} + \sigma_{Z'Z} + \sigma_{Z'}}{\sigma_Z + \sigma_{Z\gamma} + \sigma_\gamma} \lesssim 0.01$$



$$g_V^{ud} = 0.1$$

g_V^e Fixed to explain lattice – data difference

LEP2 bounds are also satisfied

Origin of coupling hierarchy ?

The coupling difference may be induced by the absence of tree-level lepton couplings. They may be induced by mixing

In such a case,

$$g_V^e = g_V^\mu.$$

$$\Delta a_\mu^{Z',\text{loop}} = (0.55 \times 10^{-9}) \left(\frac{g_V^e}{2 \times 10^{-3}} \right)^2$$

This muon coupling may further reduce the small remaining tension between theory and experiment. The identification of couplings fails in certain cases, leading to a too large one loop contribution

Conclusions

The measurement of $g-2$ has led to a puzzle.

New physics BSM is necessary to explain the 4.2 sigma discrepancy. In this talk I explore some possible explanations and their phenomenological implications. I put emphasis on the SUSY solution.

Alternatively, we need to understand why the hadronic vacuum polarization obtained by data differs from the one obtained by lattice methods.

Three possibilities remain

1. Large systematics in hadronic cross section
2. Large systematics in lattice determination
3. New physics that can explain the difference between two determinations. In this talk, I analyzed an example of this.

The good news is that further experimental and theoretical work is in progress and we will hopefully know the answer soon.