SUSY 2023 University of Southampton **July, 17-21, 2023**

A Road map for Model Building in IIB/F-theory

George K. Leontaris

University of Ioannina

$I\omega\alpha\nu\nu\iota\nu\alpha$

GREECE

Outline of the Talk

- Introductory remarks
- $\mathcal{F}$-Theory basics
- Building $\mathcal{F}$-Theory GUTs
- Minimal $\mathcal{F}$*lipped* $\mathcal{F}$-$SU(5)$
- Concluding Remarks

$\mathcal{A}$

A few remarks

F-theory is an exciting reformulation of String Theory in a twelve dimensional space

It involves a number of beautiful mathematical subjects including : Topology, Algebraic Geometry and Elliptic Fibrations.

Here, our principal task is to describe the methodology in building effective unified theories (GUTs) with predictive power

F-GUTs vs *Ordinary* GUTs

Field theory GUTs	F-theory GUTs
Predictions	New ingredients and features
▲ Gauge Coupling Unification	▲ Magnetised Fluxes
▲ Charge Quantisation	induce chirality
▲ Fermions "assembled" in	break gauge groups
simple GUT representations	▲ Topological properties:
Issues	# fermion generations
▲ Origin of fermions,	New $U(1)$ symmetries:
mass hierarchy and mixing ?	▲ Put additional constraints on $\mathcal{L}_Y$
▲ Rapid proton decay due to	Protect baryon number etc $\rightarrow$
insufficiently constrained $\mathcal{L}_Y$	Robust model building framework

$\mathcal{B}$

★ **F-theory** and Elliptic Fibration★

C. Vafa, hep-th/9602022

& *reviews*:

F. Denef hep-th/0803.1194 ; T. Weigand 1806.01854

$\Downarrow$

F-theory

(**Defining Features**)

-

i) **Non-perturbative formulation of Type II-B string compactifications**

-

ii) **Presence of 7-branes which backreact on the geometry**

-

in particular

iii) **D7 branes are magnetic sources for the RR axion C_0.**

-

iv) **Inherits $SL(2, Z)$ invariance from Type II-B**

$SL(2,Z)$-invariance

1. The dilaton ϕ determines the *string coupling:*

$$g_{IIB} = e^{\phi}$$

2. *The RR axion C_0, and the dilaton ϕ are combined to one modulus, the axio-dilaton field:*

$$\boxed{\tau = C_0 + i\, e^{-\phi} \to C_0 + \frac{i}{g_{IIB}}}$$

3. The importance of τ is that it can be used to write the type **IIB** action in an $SL(2,Z)$ invariant way

$$S_{IIB} \quad \propto \quad \int d^{10}x\sqrt{-g}\left(R - \frac{1}{2}\frac{\partial_\mu \tau \partial^\mu \bar{\tau}}{(\mathrm{Im}\tau)^2} - \frac{1}{2}\frac{|G_3|^2}{\mathrm{Im}\tau} - \frac{1}{4}|F_5|^2\right)$$

$$-\frac{i}{4}\int \frac{1}{\mathrm{Im}\tau} C_4 + G_3 \wedge \tilde{G}_3$$

4. Indeed, it can be checked that this action in invariant under the trasformations:

$$\tau \to \frac{a\tau + b}{c\tau + d}$$

5. Due to $SL(2, Z)$ invariace, the modulus τ can vary accordingly, while leaving the action invariant.

6. Notice also that the imaginary part is

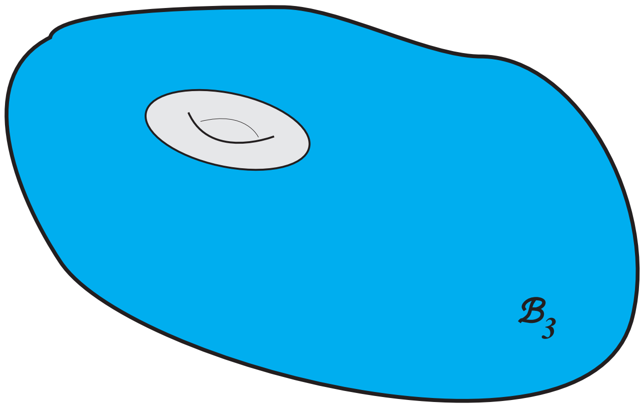
$$\boxed{\mathrm{Im}\tau = \frac{1}{g_{IIB}}}$$

which implies that there exist values of τ leading to strongly coupled regions.

A few words about

Elliptic Curves & Elliptic Fibration

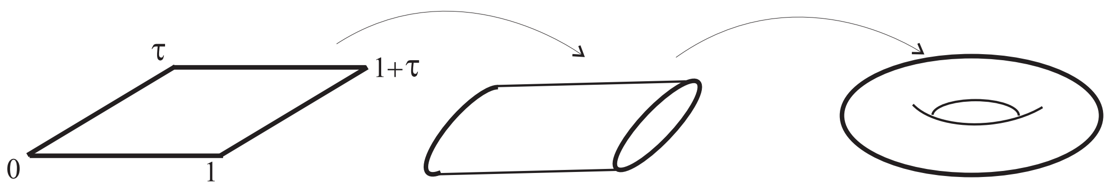
An extremely important implication of the variation of the axio-dilaton τ is that it gives rise to an elliptic fibration over the physical space-time. In order to see this, let's start with II-B theory which is defined in 10-d space described by: $\mathcal{R}^{3,1} \times \mathcal{B}_3$



- $\mathcal{R}^{3,1}$ is the usual 4-d space-time
- $\mathcal{B}_3$ Calabi-Yau (CY) manifold of 3 complex dimensions (3-fold)

▲ ▲ **F-theory** is compactified on an **elliptically fibered manifold** where $\mathcal{B}_3$ is the base of the fibration.

Fibration *is implemented by the axio-dilaton modulus* $\tau = C_0 + \imath\, e^{-\phi}$ *which can be thought as describing a torus*



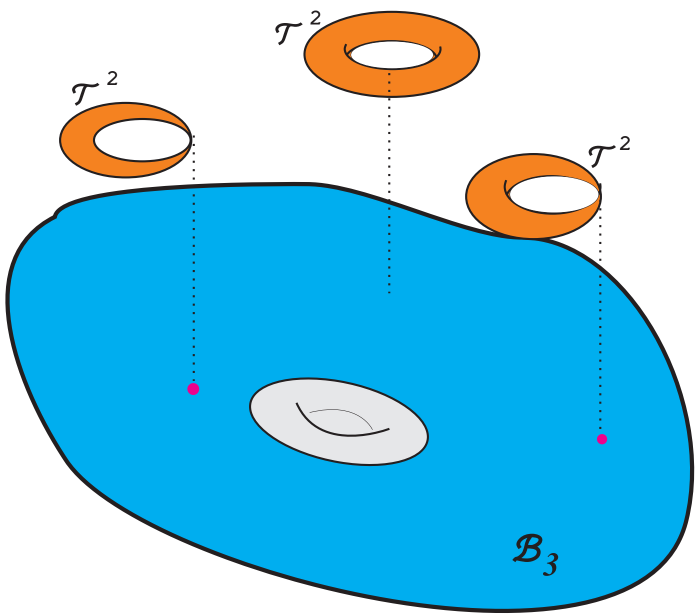
More precisely, we make a continuous mapping of τ to the points of the base B_3. We say that:

▲ F-theory is defined on $\mathcal{R}^{3,1} \times \mathcal{X}$ ▲

where $\mathcal{X}$, elliptically fibered **CY** 4-fold over the base B_3

This is depicted below where τ-tori are associated with points of B_3.

Red points correspond to possible geometric singularities of the fiber



Mathematically, the **Elliptic Fibration** is described by the vanishing locus of the $\mathcal{W}$eierstraß $\mathcal{E}$quation

$$y^2 = x^3 + f(z)\, x w^4 + g(z) w^6$$

1. $f(z)$, $g(z) \rightarrow 8^{th}$ and 12^{th} degree polynomials.
2. Equivalence relations of homogeneous (projective) coordinates $(x, y, w, z) \simeq (\lambda^2 x, \lambda^3 y, \lambda w, z)$ and $(x, y, z, w) \simeq (\lambda^4 x, \lambda^6 y, \lambda z, w)$
3. The zero section σ_0 is described by the intersection $w = 0$ which marks the point $[x : y : w] \rightarrow [1 : 1 : 0]$.
4. The elliptic fibration is CY, as long as $f(z)$ and $g(z)$ are holomorphic sections of line bundles[a] $\mathcal{O}(K_B^{-4})$ and $\mathcal{O}(K_B^{-6})$ respectively.

[a] K_B is the canonical class of the base B_3.

Two important quantities characterise the fibration:

These are

▲ The discriminant: (24^{th}*-degree in* z)

$$\Delta(z) = 4\, f(z)^3 + 27 g(z)^2$$

and

▲▲ the j-invariant:

$$j(\tau) = \frac{4(24 f(z))^3}{\Delta(z)}$$

▲ The zeros of the discriminant determine the fiber singularities:

$$\Delta = \prod_{i=1}^{24} (z - z_i) = 0 \;\Rightarrow\; 24 \text{ roots } z_i$$

▲▲ In addition, the j-invariant provides a relation between the modulus τ and the *coordinate* z:[a]

$$j(\tau(z)) = 4\,\frac{(24f(z))^3}{\Delta(z)} \propto e^{-2\pi i\tau} + \cdots \tag{1}$$

Its solution determines the axio-dilaton τ around the zeros z_i of Δ:

$$\tau \approx \frac{1}{2\pi i}\log(z - z_i)$$

Encircling a root z_i, due to the multivalued log function, τ shifts:

$$\tau \to \tau + 1 \;\Rightarrow C_0 \to C_0 + 1 \to$$

In other words, τ, C_0 undergo Monodromy.

[a] $j(\tau) \sim e^{-2\pi i\tau} + 744 + \mathcal{O}(e^{2\pi i\tau}) \sim e^{2\pi/g_s}e^{-2\pi i C_0} + 744 + \mathcal{O}(e^{-2\pi/g_s})$.

The **Interpretation** of this picture is that at the root $z = z_i$ there is a source of RR-flux which is associated with a $D7$-brane **perpendicular** to the "tangent plane" $\Rightarrow$ $D7$ branes are magnetic sources for the RR axion C_0

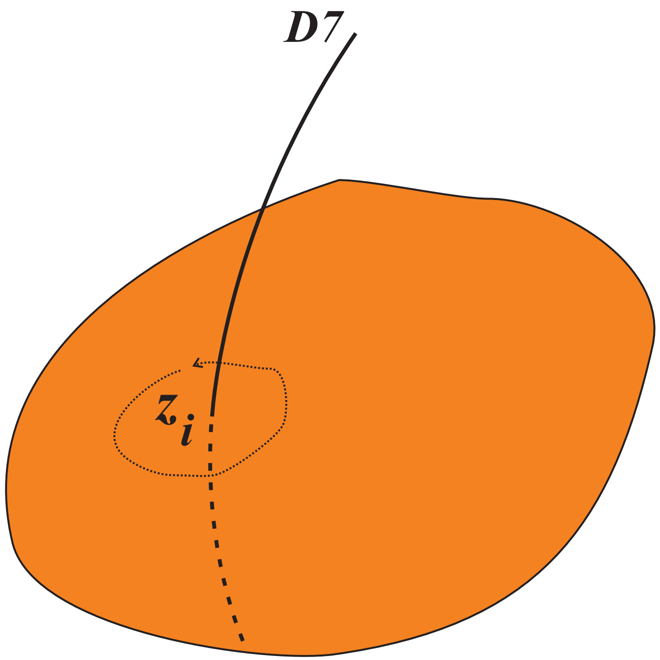


Figure 1: Moving around z_i, $\log(z) \rightarrow \log|z| + i(2\pi + \theta)$ and $\tau \rightarrow \tau + 1$

Observation

This monodromic behaviour can be further understood by noting that it can fit into the more general $SL(2,Z)$ invariance of IIB string theory.

Indeed this shift can be obtained by the following $SL(2,Z)$ element:

$$M_{[0,1]} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \Rightarrow M_{[0,1]} \begin{pmatrix} \tau \\ 1 \end{pmatrix} = \begin{pmatrix} \tau + 1 \\ 1 \end{pmatrix}$$

We denote a $D7$-brane with $[1,0]$ on which a $(1,0)$ string ends.

We note in passing that

this motivates the generalisation to 7-branes of the type $[p,q]$ [a]

[a] To be distinguised from a (p,q) string being a BPS bound state of p fundamental strings and q D1-strings.

Geometric Singularities

Summarising the analysis so far, the elliptic fibration is represented by the Weierstraß equation (*fixing* $w = 1$):

$$y^2 = x^3 + f(z)\,x + g(z)$$

- At the points where the discriminant $\Delta = 27g^2 + 4f^3$ vanishes, the elliptic fiber degenerates.
- The type of Manifold **singularity** is specified by the vanishing order of Δ and the polynomials $f(z)$, $g(z)$ of Weierstraß eqn
- It was shown (in '60s) by Kodaira that these **geometric singularities** are classified in terms of $\mathcal{A}\,\mathcal{D}\,\mathcal{E}$ Lie groups.

In F-theory these singularities are interpreted as:

$$\Downarrow$$

$$\boxed{CY_4\text{-}\textbf{Singularities} \rightleftarrows \textbf{gauge symmetries}}$$

▲ The above description concerns the non-abelian part of the effective theory which according to $\mathcal{A}\mathcal{D}\mathcal{E}$ classification will result to an effective model with one of the following gauge groups (in standard notation)

$$\begin{array}{c} Non\,Abelian \\ Gauge\,Groups \end{array} \quad \Rightarrow \quad \left\{ \begin{array}{c} SU(n) \\ SO(m) \\ \mathcal{E}_n \end{array} \right.$$

▲▲ Note: There are also Abelian symmetries associated with the elliptic fibers of CY_4 and will be discussed shortly

$\mathcal{C}$

The Non Abelian Sector

Rôle of Geometric Singularities on EFTs

Kodaira classified the type of singularities in terms of the vanishing order of $f(z)$, $g(z)$ and $\Delta(z) = 4f(z)^3 + 27g(z)^2$.
(see details in Morrison, Vafa hep-th/9603161)

$\mathrm{ord}(f(z))$	$\mathrm{ord}(g(z))$	$\mathrm{ord}(\Delta(z))$	fiber type	Singularity
0	0	n	I_n	A_{n-1}
≥ 1	1	2	II	none
1	≥ 2	3	III	A_1
≥ 2	2	4	IV	A_2
2	≥ 3	$n+6$	I_n^*	D_{n+4}
≥ 2	3	$n+6$	I_n^*	D_{n+4}
≥ 3	4	8	IV^*	$\mathcal{E}_6$
3	≥ 5	9	III^*	$\mathcal{E}_7$
≥ 4	5	10	II^*	$\mathcal{E}_8$

Perhaps, a more pheno-friendly approach is **Tate's Algorithm**

$$\boxed{y^2 + a_1 x\, y + a_3 y = x^3 + a_2\, x^2 + a_4 x + a_6}$$

$$a_n = \sum_{\ell = k \geq 0} a_{n,\ell}\, z^\ell$$

Table: *Geometric Singularities w.r.t. vanishing order of a_i and Δ*

Group	a_1	a_2	a_3	a_4	a_6	Δ
$SU(2n)$	0	1	n	n	$2n$	$2n$
$SU(2n+1)$	0	1	n	$n+1$	$2n+1$	$2n+1$
$SU(5)$	0	1	2	3	5	5
$SO(10)$	1	1	2	3	5	7
$\mathcal{E}_6$	1	2	3	3	5	8
$\mathcal{E}_7$	1	2	3	3	5	9
$\mathcal{E}_8$	1	2	3	4	5	10

$\mathcal{EXAMPLE}$

Defining $b_k = b_{k,0} + b_{k,1} z + \cdots$, $(b_{k,0} \neq 0)$ we choose a_i to be:

$$a_1 = -b_5,\ a_2 = b_4 z,\ a_3 = -b_3 z^2,\ a_4 = b_2 z^3,\ a_6 = b_0 z^5$$

Then, the vanishing orders of each a_n is:

Vanishing order	a_1	a_2	a_3	a_4	a_6	Δ	
	–	z^1	z^2	z^3	z^5	z^5	$\rightarrow \mathbf{SU(5)}$

$\Rightarrow$ Weierstraß' equation for the $SU(5)$ singularity

$$y^2 = x^3 + b_0 z^5 + b_2 x z^3 + b_3 y z^2 + b_4 x^2 z + b_5 x y \tag{2}$$

⋆ A useful notion for local model building is the spectral cover obtained by defining homogeneous coordinates $z \to U$, $x \to V^2$, $y \to V^3$ and affine parameter $s = \frac{U}{V}$, so that (2) implies:

$$\mathcal{C}_5 : \boxed{0 \ = \ b_0 s^5 + b_2 s^3 + b_3 s^2 + b_4 s + b_5}$$

$\mathcal{D}$

The Abelian Sector

Our interest in continuous Abelian Groups (and other discrete symmetries) arises from phenomenological considerations, in particular the necessity to

constrain the Yukawa Lagrangian

In F-theory such symmetries appear naturally

Recall that F-theory is defined on elliptically fibred fourfold.

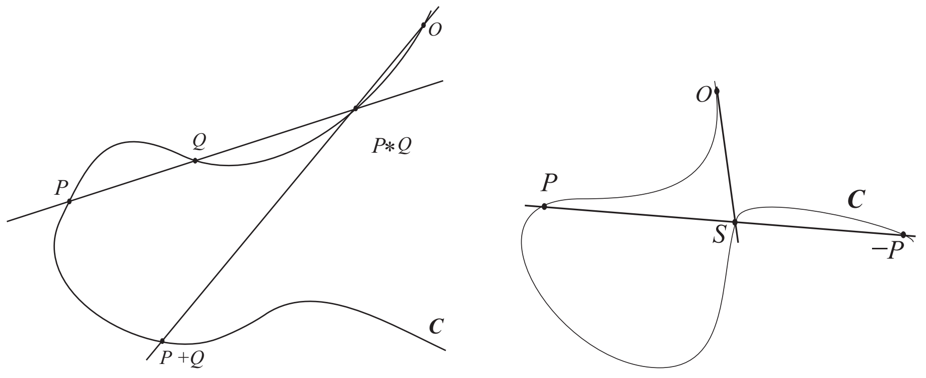
Hence,

Elliptic Curves are fundamental in F-theory constructions.

In particular the Rational Points on the Elliptic Curves are associated with the Abelian Sector of the theory

First, let's start with coefficients $\in \mathbb{R}$.

The rational points [a] on Elliptic Curves form a Group



The addition law: Given two rational points P, Q, we define $P + Q$ as in the left plot which is rational. ($\mathcal{O}$ the neutral element).
The opposite of P is defined by $P + (-P) = \mathcal{O}$ (right plot)

[a] A point is said to be rational if its coordinates are rational. A rational curve is defined by an equation with rational coefficients.

Mordell Theorem

$\Downarrow$

The Rational Points on Elliptic Curves constitute an Abelian Group which is generated by a finite number of elements.

It is called:

$\Downarrow$

Mordell - Weil Group

Returning now to the case of the elliptic fibration:

The Rational Points are *'promoted'* to *Rational* Sections

★ *As a consequence, a new class of Abelian symmetries -associated with Rational Sections - appear in the effective theory*

$$\underbrace{\mathbb{Z}\oplus\mathbb{Z}\oplus\cdots\oplus\mathbb{Z}}_{r}\oplus\mathcal{G}$$

Here, r is the rank of the abelian group and $\mathcal{G}$ is the *Torsion* part:[a]

$$\mathcal{G}=\begin{cases}\mathbb{Z}_n & n=1,2,\ldots,10,12\\ \mathbb{Z}_k\times\mathbb{Z}_2 & k=2,4,6,8\end{cases}$$

[a] *D.S. Kubert: Universal bounds on the torsion of elliptic curves, Proc. London Math. Soc., third series, 33 (1976) 193-237. For relations to Tate-Shafarevich group see Braun et.al 1401.7844, Cvetic et.al 1502.06953*

To wrap things up:

In $\mathcal{F}$-Theory, Abelian gauge symmetries (other than those embedded in E_8) are encoded in rational sections of the Elliptic Fibration and constitute the so called Mordell-Weil group.

Simplest (*and perhaps most viable*) Case:
Rank-1 Mordell-Weil

References:

Morrison-Park: 1208.2695, Cvetic et al: 1210.6094; Mayhofer et al, 1211.6742; Borchmann et al 1307.2902; Antoniadis, GKL: 1404.6720; Mayhofer,Palti, Weigand: 1410.7814; Krippendorf et al: 1401.7844, GKL: 1501.06499; Cvetic and Lin: 1809.00012

$\mathcal{E}$

F-theory Model Building

(*Original papers: Beasley, Heckman, Vafa : 0802.3391, 0806.0102*

Donagi et al 0808.2223, 0904.1218)

Early reviews: 1001.0577, 1203.6277, 1212.0555

Recent: 1806.01854; 2212.07443

A Class of 'semi-local' constructions

The final effective (GUT) model depends on the choice of:

$\Downarrow$

1) Manifold 2) Fluxes 3) Monodromies

Let's examine the role of each one of them

$\Downarrow$

▲▼ *The role of* the manifold: ▲▼

▲ The candidate GUT is embedded in $\mathcal{E}_8$ which is the maximal exceptional group in elliptic fibration.
So, we suppose that there is a divisor, accommodating our choice while the rest is the symmetry commutant to it.

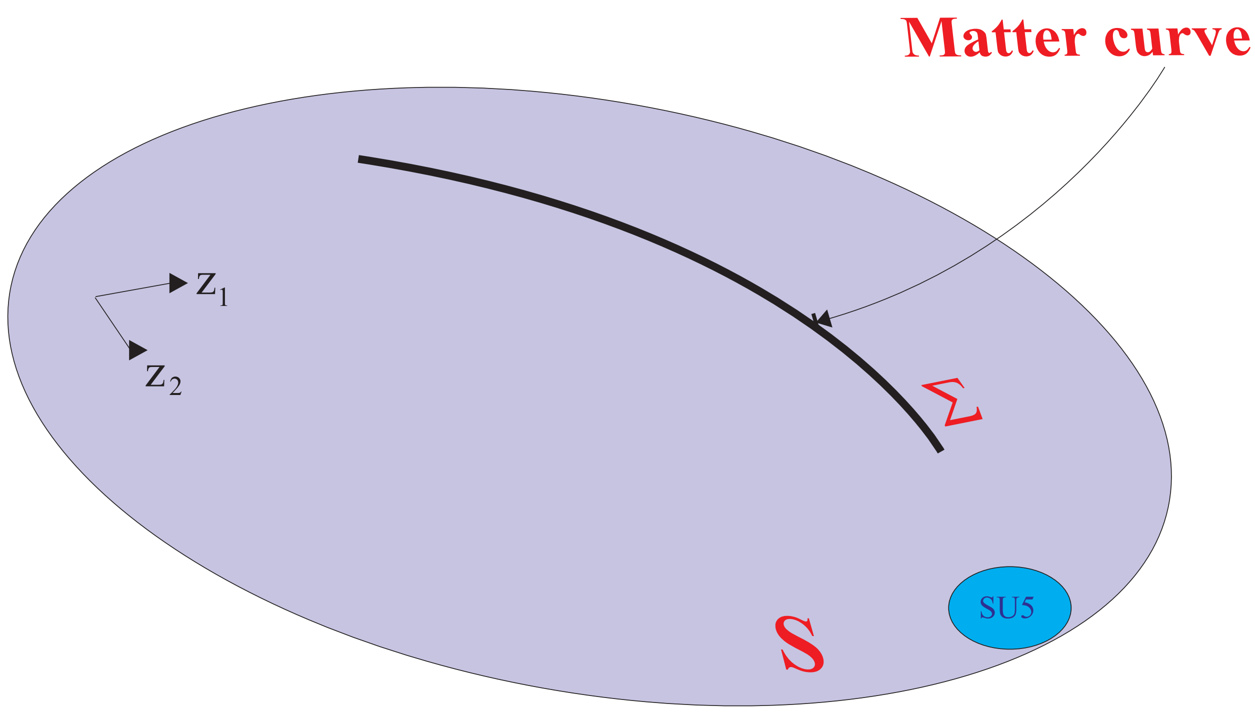
$$\mathcal{E}_8 \to \mathbf{G_{GUT}} \times \mathcal{C}$$

Example: *Assuming a Manifold with $SU(5)$ divisor*:

$$\begin{aligned} \mathcal{E}_8 \quad &\to \quad SU(5) \times SU(5)_\perp \\ &\to \quad SU(5) \times U(1)_\perp^4 \end{aligned}$$

Matter descends from the $\mathcal{E}_8$-Adjoint which decomposes as:

$$248 \to (24,1) + (1,24) + (10,5) + (\overline{5},10) + (\overline{10},\overline{5}) + (5,\overline{10})$$

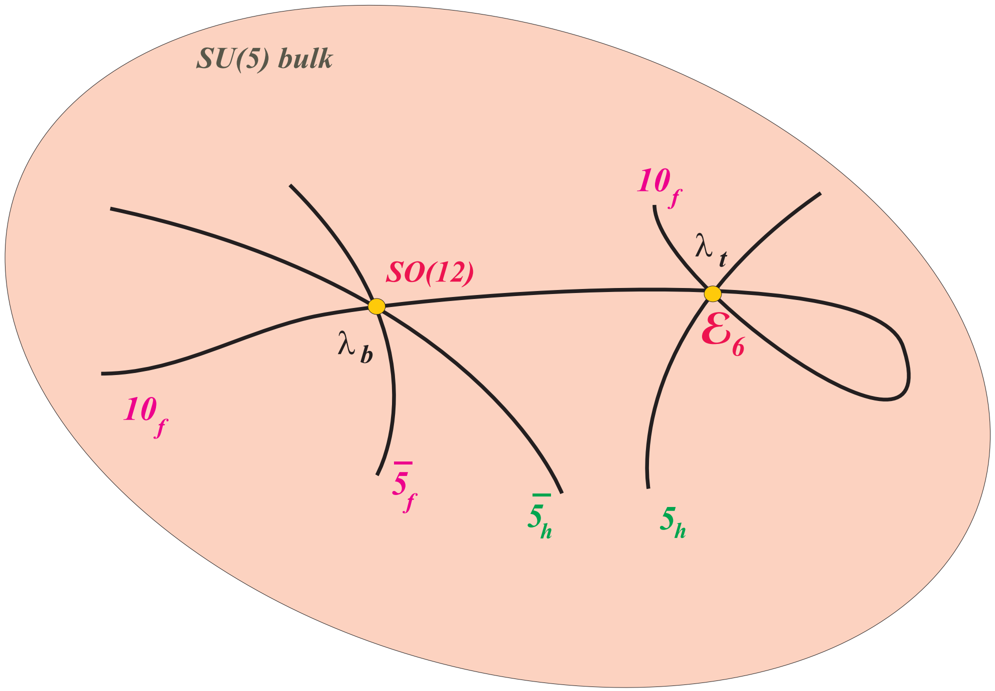


In F-theory, matter is localised along intersections with other 7-branes.

When 7-branes intersect S*, the discriminant vanishes,* $\Delta = 0$*, and therefore along a matter curve* Σ *the gauge symmetry is enhanced*

When branes intersect, the singularity increases and the **gauge symmetry** is further enhanced. Yukawa couplings are formed at tripple intersections . For example, in the $\mathbf{SU(5)}$ case:[a]

$$\lambda_b\, 10\cdot\bar{5}\cdot\bar{5}\ \in \mathbf{SO(12)},\ \ \lambda_t\, 10\cdot 10\cdot 5 \in \mathbf{E_6}$$



[a] Here we assume that there is a Z_2 monodromy so that λ_t exists.

The role of fluxes:

Three important implications

- determine $SU(5)$ chirality
- trigger $SU(5)$ Symmetry Breaking

(fluxes act as the surrogate of the Higgs vev)

- Split the $SU(5)$-representations

There are two distinct sets of fluxes with discrete roles:

i) The integers M_{10}, M_5: are associated with $U(1)_\perp$ fluxes and determine the chirality $\#(10-\overline{10})$ and $\#(5-\bar{5})$ of $SU(5)$

ii) The hypercharge flux N_Y, *turned on along* $U(1)_Y \in SU(5)$, breaks $SU(5)$ and **splits** *the* $SU(5)$*-representations*

$SU(5)$ chirality from perpendicular $U(1)_\perp$ Flux

$U(1)_\perp$–Flux on $\in$ **10**'s:

$$\#10 - \#\overline{10} \quad = \quad M_{10}$$

$U(1)_\perp$– Flux on $\in$ **5**'s:

$$\#5 - \#\overline{5} \quad = \quad M_5$$

SM chirality form Hypercharge Flux

$U(1)_Y -$**Flux**-splitting of **10**'s:

$$n_{(3,2)_{\frac{1}{6}}} - n_{(\bar{3},2)_{-\frac{1}{6}}} = M_{10}$$
$$n_{(\bar{3},1)_{-\frac{2}{3}}} - n_{(3,1)_{\frac{2}{3}}} = M_{10} - N_{Y_{10}}$$
$$n_{(1,1)_1} - n_{(1,1)_{-1}} = M_{10} + N_{Y_{10}}$$

$U(1)_Y -$ **Flux**-splitting of **5**'s:

$$n_{(3,1)_{-\frac{1}{3}}} - n_{(\bar{3},1)_{\frac{1}{3}}} = M_5$$
$$n_{(1,2)_{\frac{1}{2}}} - n_{(1,2)_{-\frac{1}{2}}} = M_5 + N_{Y_5}$$

Splitting $5/\bar{5}$ Higgses with the Hypercharge Flux

For the **Higgs 'curve'** *in particular choose:* $M_5 = 0$, $N_{Y_5} = \pm 1$.

$U(1)_Y-$ **Flux**-splitting of $\mathbf{5_{H_u}}$:

$$n_{(3,1)_{-\frac{1}{3}}} - n_{(\bar{3},1)_{\frac{1}{3}}} = M_5 = 0$$

$$n_{(1,2)_{\frac{1}{2}}} - n_{(1,2)_{-\frac{1}{2}}} = M_5 + N_{Y_5} = 0 + 1 = 1 \ (H_u)$$

$U(1)_Y-$ **Flux**-splitting of $\mathbf{\bar{5}_{H_d}} \rightarrow$:

$$n_{(3,1)_{-\frac{1}{3}}} - n_{(\bar{3},1)_{\frac{1}{3}}} = M_5 = 0$$

$$n_{(1,2)_{\frac{1}{2}}} - n_{(1,2)_{-\frac{1}{2}}} = M_5 + N_{Y_5} = 0 - 1 = -1 \ (H_d)$$

This is the analogue of the Doublet-Triplet splitting

Fermion mass hierarchy (*rank-1 mass matrices*)

▼ If families are distributed on different matter curves:
Implementation of Froggatt-Nielsen mechanism in F-models:
Dudas and Palti, 0912.0853, 1007.1297, Camara et al, 1110.2206
SF King, GKL and G.G. Ross, 1005.1025, 1009.6000

▼ If all three families are on the same matter curve, masses to lighter families can be generated by:
i) non-commutative fluxes *Cecotti et al, 0910.0477*
ii) non-perturbative effects, *Aparicio et al, 1104.2609*

▼ Higher rank mass matrices, (global models) *Cvetic,M et al hep-th/1906.10119*

Flipped SU(5) from F-theory

(*with V. Basiouris, Eur.Phys.J.C 82 (2022) 11, 1041*)

It follows according to the following breaking pattern:

$$E_8 \supset SO(10) \times SU(4)_\perp \supset [SU(5) \times U(1)_\chi] \times SU(4)_\perp \ , \qquad (3)$$

MONODROMIES

focusing on $SU(4)_\perp$ $\rightarrow$ *locally described by Cartan roots:*

$$t_i \ = \ SU(4)_\perp - \text{roots} \rightarrow \ \sum_{i=1}^{4} t_i = 0$$

$SU(5)_{GUT}$ *representations in Effective Theory transform according to:*

$$(10,4) \to 10_{t_i} \quad (\overline{5},6) \to \overline{5}_{t_i+t_j}$$

roots t_i *obey a* 4^{th}*-degree polynomial* ($SU(4)$ spectral cover)

$$\sum_{k=0}^{4} b_k t^{4-k} = 0$$

with b_k *'conveying' topological properties to the effective model*

Solving for $t_i = t_i(b_k)$ $\Rightarrow$ possible branchcuts: $\to$ Monodromies

Minimum case :

$$Z_2 : \ t_1 \leftrightarrow t_2 \Rrightarrow U(1)_{\perp}^{3} \to U(1)_{\perp}^{2}$$

A few remarks

▲ **Flipped** $SU(5)$ **needs only** $10+\overline{10}$ **for symmetry breaking**.

▲ *No need to turn on* $U(1)_{Y_0} \in SU(5)$ *flux which requires special conditions to keep* $U(1)_{Y_0}$*-boson massless. Under these assumptions:*

▲ **"Flipped"** $SU(5)$ **one of the few possible viable choices!**

$$10_{t_1} \to F_i,\ \bar{5}_{t_1} \to \bar{f}_i,\ 1_{t_1} \to e_j^c, \tag{4}$$

$$\mathbf{5_{-t_1-t_4}} \to \mathbf{h},\ \mathbf{\bar{5}_{t_3+t_4}} \to \mathbf{\bar{h}}\ . \tag{5}$$

$$10_{t_3} \to H,\ \overline{10}_{-t_4} \to \overline{H}, \tag{6}$$

$$1_{t_3} \to E_m^c,\ 1_{-t_4} \to \bar{E}_n^c, \tag{7}$$

The model predictes the existence of singlets

$$\mathbf{1}_{t_i - t_j} \to \theta_{ij},\ i, j = 1, 2, 3, 4$$

(modulo the Z_2 monodromy $t_1 \leftrightarrow t_2$), dubbed here:

$$\theta_{12} \equiv \theta_{21} = S,\ \theta_{13} = \chi,\ \theta_{31} = \bar{\chi},$$

$$\theta_{14} \to \psi,\ \theta_{41} = \bar{\psi},\ \theta_{34} \to \zeta,\ \theta_{43} \to \bar{\zeta}$$

The Z_2 monodromy allows a tree-level top-Yukawa coupling.
Superpotential terms:

$$\begin{aligned}\mathcal{W} &= \lambda_{ij}^{u} F_i \bar{f}_j \bar{h} + \lambda_{ij}^{d} F_i F_j h\, \bar{\psi} + \lambda_{ij}^{e} e_i^{c} \bar{f}_j h\, \bar{\psi} + \kappa_i \overline{H} F_i S\, \bar{\psi} \\ &\quad + \lambda_{\bar{H}} \overline{H}\overline{H} \bar{h} \bar{\zeta} + \lambda_H H H h \bar{\zeta}(\chi + \bar{\zeta}\psi) + \lambda_\mu (\chi + \lambda' \bar{\zeta}\psi) \bar{h}\, h\end{aligned}$$

Mass terms

$$\lambda^u_{ij} F_i \bar{f}_j \bar{h} \to Q\, u^c\, h_u + \ell \nu^c\, h_u \to m_D^T = m_u \propto \lambda^u \langle h_u \rangle$$

$$\lambda^d_{ij} F_i\, F_j h \bar{\psi} \to m_d = \lambda^d \langle h_d \rangle$$

$$H\, H\, h + \bar{H}\, \bar{H}\, \bar{h} \to \langle H \rangle d^c_H D + \langle \bar{H} \rangle \bar{d}^c_H \bar{D}$$

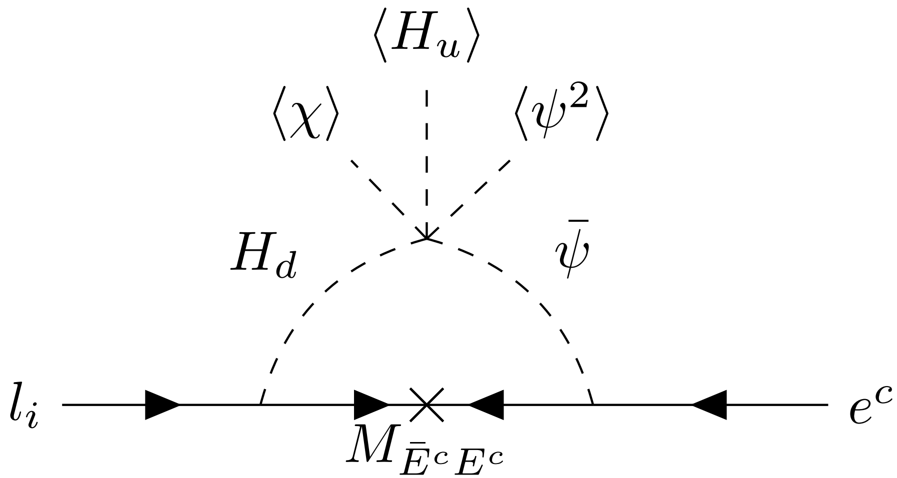
see-saw with extra (sterile) 'neutrino' S:

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m_{\nu_D} & 0 \\ m^T_{\nu_D} & 0 & M_{\nu^c S} \\ 0 & M^T_{\nu^c S} & M_S \end{pmatrix} \tag{8}$$

A prediction

The extra vector-like pair $(E_n^c, \bar{E}_n^c)$ acts as a source for

$g_\mu - 2$ enhancement [a]



Feynman diagram for the $(E_n^c, \bar{E}_n^c)$ contribution to $g_\mu - 2$

[a] See yesterday's talk by Carlos E.M. Wagner

Future Perspectives

★ Exploit $SL(2,Z)$ **invariance → fermion mass hierarchy** ★

▲▼ Examine the implications on superpotential $\mathcal{W}$ assuming that the various matter fields, and the Yukawa couplings are transformed under the appropriate congruence group of the modular group. (V. Basiouris, M. Crispim-Romão, S.F. King, GKL *to appear*) [a]

★ **Generalised Fluxes and the Superpotential** ★

▲ Viable phenomenological models must be free of massless moduli

▲ The standard geometric fluxes imply $\mathcal{W} \sim \int (F_3 - \tau H_3) \wedge \Omega_3$ which is usually not adequate to fix all kinds of moduli.

▲ However, we may use T and S-dualities, to extend $\mathcal{W}$ by including non-geometric fluxes.

(*P. Shukla 1603.01290 and PS & GKL to appear...*)

[a] For modular fermions in Heterotic String see H.P. Nilles talk.

To summarise:

•

<u>In F-theory $\exists$ *interesting connections between*</u>

- GUT Symmetry and *elliptically fibred* CY Manifold
- Abelian Symmetries and Rational sections
- Topological properties of internal manifold $\Rightarrow$ robust predictions of *Effective F-Theory Models*

★ **Thank you for your attention** ★