Tri-hypercharge: a path to the origin of flavour

Mario Fernández Navarro[†]

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Based on arXiv [2305.07690] [hep-ph] in collaboration with Steve King

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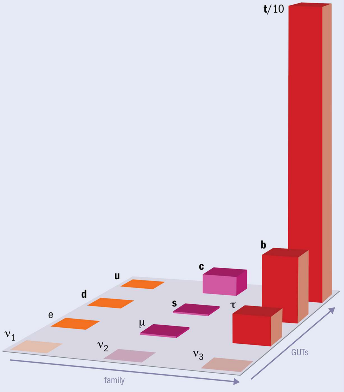
The flavour puzzle



$$m_t \sim \frac{v_{\rm SM}}{\sqrt{2}}\,, \qquad m_c \sim \lambda^{3.3}\frac{v_{\rm SM}}{\sqrt{2}}\,, \qquad m_u \sim \lambda^{7.5}\frac{v_{\rm SM}}{\sqrt{2}}\,,$$
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$$m_\tau \sim \lambda^{3.0}\frac{v_{\rm SM}}{\sqrt{2}}\,, \qquad m_\mu \sim \lambda^{4.9}\frac{v_{\rm SM}}{\sqrt{2}}\,, \qquad m_e \sim \lambda^{8.4}\frac{v_{\rm SM}}{\sqrt{2}}\,,$$

$$V_{us} \sim \lambda\,, \qquad V_{cb} \sim \lambda^2\,, \qquad V_{ub} \sim \lambda^3\,,$$

$$\tan\theta^\nu_{23} \sim 1\,, \qquad \tan\theta^\nu_{12} \sim \frac{1}{\sqrt{2}}\,, \qquad \sin\theta^\nu_{13} \sim \frac{\lambda}{\sqrt{2}}\,,$$

where $v_{\rm SM} \simeq 246\,\text{GeV}$ and $\lambda = \sin\theta_C \simeq 0.224$

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- Why the three families interact so differently with the Higgs?
- What is the origin of very small neutrino masses, and why the PMNS mixing is so different from the CKM mixing?

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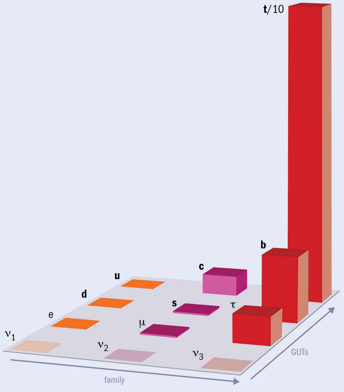

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  ⇒ A theory of flavour is needed!

Tri-hypercharge in a nutshell

$$\boxed{SU(3)_c \times SU(2)_L \times U(1)_{Y_1} \times U(1)_{Y_2} \times U(1)_{Y_3}}$$
$$\to SU(3)_c \times SU(2)_L \times U(1)_{Y_1+Y_2+Y_3}$$

Field	$SU(3)_c$	$SU(2)_L$	$U(1)_{Y_1}$	$U(1)_{Y_2}$	$U(1)_{Y_3}$
Q_1	$\mathbf{3}$	$\mathbf{2}$	$1/6$	0	0
u_1^c	$\bar{\mathbf{3}}$	$\mathbf{1}$	$-2/3$	0	0
d_1^c	$\bar{\mathbf{3}}$	$\mathbf{1}$	$1/3$	0	0
L_1	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	0	0
e_1^c	$\mathbf{1}$	$\mathbf{1}$	1	0	0
Q_2	$\mathbf{3}$	$\mathbf{2}$	0	$1/6$	0
u_2^c	$\bar{\mathbf{3}}$	$\mathbf{1}$	0	$-2/3$	0
d_2^c	$\bar{\mathbf{3}}$	$\mathbf{1}$	0	$1/3$	0
L_2	$\mathbf{1}$	$\mathbf{2}$	0	$-1/2$	0
e_2^c	$\mathbf{1}$	$\mathbf{1}$	0	1	0
Q_3	$\mathbf{3}$	$\mathbf{2}$	0	0	$1/6$
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- Light charged fermion masses and CKM mixing are naturally small because they arise from non-renormalisable operators (involving the high scale scalar SM singlet fields breaking $U(1)^3_Y$ down to SM hypercharge, which act as a link between the different $U(1)_{Y_i}$).

Tri-hypercharge gauge theory

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- The Yukawa couplings above preserve an accidental $U(2)^5$ flavour symmetry acting on the light families,

$$U(2)^5 \equiv U(2)_Q \times U(2)_u \times U(2)_d \times U(2)_L \times U(2)_e \, .$$

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- However, the hierarchies $y_{b,\tau}/y_t \approx 0.01$ not explained $\Rightarrow$ Consider type II 2HDM (could be enforced by either Z_2 or supersymmetry)

$$H_u(\mathbf{1},\mathbf{2})_{(0,0,\frac{1}{2})}, \qquad H_d(\mathbf{1},\mathbf{2})_{(0,0,-\frac{1}{2})}\,,$$

with $\tan\beta = v_u/v_d \approx 20$.

Tri-hypercharge gauge theory

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- Third family with $\mathcal{O}(1)$ renormalisable Yukawa couplings

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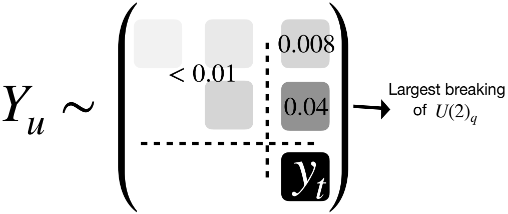
- High scale scalar SM singlet fields (hyperons ϕ) breaking $U(1)_Y^3$ down to SM hypercharge can provide light charged fermion masses at the non-renormalisable level (small breaking of $U(2)^5$).
- Similar mechanism as other non-universal gauge theories of flavour (see talks by Joe Davighi and Javier Lizana).



Tri-hypercharge gauge theory

2-step symmetry breaking

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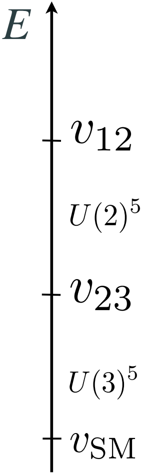
$$\xrightarrow{v_{23}} SU(3)_c \times SU(2)_L \times U(1)_{Y_1+Y_2+Y_3} + {\color{blue}Z'_{23}} + {\color{red}Z'_{12}}$$



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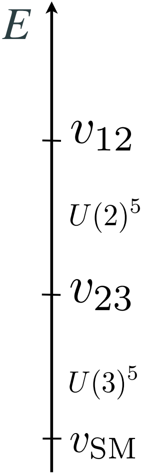
- Dynamics at v_{12} explain m_1/m_2. Gauge symmetry above v_{12} explicitly breaks $U(2)^5$

  $\Rightarrow Z'_{12}$ potentially mediates 1-2 FCNCs (e.g. $\mu \to e\gamma$), but it is heavier than Z'_{23}.



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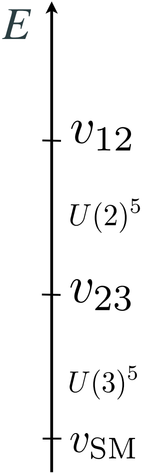

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- Dynamics at v_{23} explain m_2/m_3. Gauge symmetry preserves $U(2)^5$.

  $\Rightarrow$ Z'_{23} FCNCs suppressed by the approximate $U(2)^5$ symmetry, can be as light as a few TeV.

Charged fermions: spurion formalism

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$$\Phi(0, \frac{1}{2}, -\frac{1}{2}) Q_2 H_u u_2^c = (0, 0, 0)\,.$$

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- If $\langle \phi^{(0, \frac{1}{2}, -\frac{1}{2})} \rangle / \Lambda = \lambda^3$ where $\lambda = \sin\theta_C = 0.223$, then

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- Repeat this process systematically for all charged fermion masses.

Charged fermions: spurion formalism

$$\mathcal{L} = \begin{pmatrix} Q_1 & Q_2 & Q_3 \end{pmatrix} \begin{pmatrix} \Phi(\frac{1}{2},0,-\frac{1}{2}) & \Phi(-\frac{1}{6},\frac{2}{3},-\frac{1}{2}) & \Phi(-\frac{1}{6},0,\frac{1}{6}) \\ \Phi(\frac{2}{3},-\frac{1}{6},-\frac{1}{2}) & \Phi(0,\frac{1}{2},-\frac{1}{2}) & \Phi(0,-\frac{1}{6},\frac{1}{6}) \\ \Phi(\frac{2}{3},0,-\frac{2}{3}) & \Phi(0,\frac{2}{3},-\frac{2}{3}) & 1 \end{pmatrix} \begin{pmatrix} u_1^c \\ u_2^c \\ u_3^c \end{pmatrix} H_u$$

$$+ \begin{pmatrix} Q_1 & Q_2 & Q_3 \end{pmatrix} \begin{pmatrix} \Phi(-\frac{1}{2},0,\frac{1}{2}) & \Phi(-\frac{1}{6},-\frac{1}{3},\frac{1}{2}) & \Phi(-\frac{1}{6},0,\frac{1}{6}) \\ \Phi(-\frac{1}{3},-\frac{1}{6},\frac{1}{2}) & \Phi(0,-\frac{1}{2},\frac{1}{2}) & \Phi(0,-\frac{1}{6},\frac{1}{6}) \\ \Phi(-\frac{1}{3},0,\frac{1}{3}) & \Phi(0,-\frac{1}{3},\frac{1}{3}) & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} H_d$$

$$+ \begin{pmatrix} L_1 & L_2 & L_3 \end{pmatrix} \begin{pmatrix} \Phi(-\frac{1}{2},0,\frac{1}{2}) & \Phi(\frac{1}{2},-1,\frac{1}{2}) & \Phi(\frac{1}{2},0,-\frac{1}{2}) \\ \Phi(-1,\frac{1}{2},\frac{1}{2}) & \Phi(0,-\frac{1}{2},\frac{1}{2}) & \Phi(0,\frac{1}{2},-\frac{1}{2}) \\ \Phi(-1,0,1) & \Phi(0,-1,1) & 1 \end{pmatrix} \begin{pmatrix} e_1^c \\ e_2^c \\ e_3^c \end{pmatrix} H_d + \text{h.c.}\,,$$

in the EFT framework (neglecting $\mathcal{O}(1)$ dimensionless coefficients)

$$\Phi = \frac{\phi_1 ... \phi_N}{\Lambda_1 .. \Lambda_N}\,.$$

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$$+ \begin{pmatrix} Q_1 & Q_2 & Q_3 \end{pmatrix} \begin{pmatrix} \Phi(-\frac{1}{2},0,\frac{1}{2}) & \Phi(-\frac{1}{6},-\frac{1}{3},\frac{1}{2}) & \Phi(-\frac{1}{6},0,\frac{1}{6}) \\ \Phi(-\frac{1}{3},-\frac{1}{6},\frac{1}{2}) & \Phi(0,-\frac{1}{2},\frac{1}{2}) & \Phi(0,-\frac{1}{6},\frac{1}{6}) \\ \Phi(-\frac{1}{3},0,\frac{1}{3}) & \Phi(0,-\frac{1}{3},\frac{1}{3}) & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} H_d$$

$$+ \begin{pmatrix} L_1 & L_2 & L_3 \end{pmatrix} \begin{pmatrix} \Phi(-\frac{1}{2},0,\frac{1}{2}) & \Phi(\frac{1}{2},-1,\frac{1}{2}) & \Phi(\frac{1}{2},0,-\frac{1}{2}) \\ \Phi(-1,\frac{1}{2},\frac{1}{2}) & \Phi(0,-\frac{1}{2},\frac{1}{2}) & \Phi(0,\frac{1}{2},-\frac{1}{2}) \\ \Phi(-1,0,1) & \Phi(0,-1,1) & 1 \end{pmatrix} \begin{pmatrix} e_1^c \\ e_2^c \\ e_3^c \end{pmatrix} H_d + \mathrm{h.c.}\,,$$

in the EFT framework (neglecting $\mathcal{O}(1)$ dimensionless coefficients)

$$\Phi = \frac{\phi_1 ... \phi_N}{\Lambda_1 .. \Lambda_N} \,.$$

Assuming all Λ_i(s) to be universal

- Similar m_2/m_3 hierarchy in all charged sectors (same for m_1/m_3) $\Rightarrow$ 2HDM also motivated for $m_{s,\mu}/m_c$ hierarchy

Charged fermions: spurion formalism

$$\mathcal{L} = \begin{pmatrix} Q_1 & Q_2 & Q_3 \end{pmatrix} \begin{pmatrix} \Phi(\frac{1}{2},0,-\frac{1}{2}) & \Phi(-\frac{1}{6},\frac{2}{3},-\frac{1}{2}) & \Phi(-\frac{1}{6},0,\frac{1}{6}) \\ \Phi(\frac{2}{3},-\frac{1}{6},-\frac{1}{2}) & \Phi(0,\frac{1}{2},-\frac{1}{2}) & \Phi(0,-\frac{1}{6},\frac{1}{6}) \\ \Phi(\frac{2}{3},0,-\frac{2}{3}) & \Phi(0,\frac{2}{3},-\frac{2}{3}) & 1 \end{pmatrix} \begin{pmatrix} u_1^c \\ u_2^c \\ u_3^c \end{pmatrix} H_u$$

$$+ \begin{pmatrix} Q_1 & Q_2 & Q_3 \end{pmatrix} \begin{pmatrix} \Phi(-\frac{1}{2},0,\frac{1}{2}) & \Phi(-\frac{1}{6},-\frac{1}{3},\frac{1}{2}) & \Phi(-\frac{1}{6},0,\frac{1}{6}) \\ \Phi(-\frac{1}{3},-\frac{1}{6},\frac{1}{2}) & \Phi(0,-\frac{1}{2},\frac{1}{2}) & \Phi(0,-\frac{1}{6},\frac{1}{6}) \\ \Phi(-\frac{1}{3},0,\frac{1}{3}) & \Phi(0,-\frac{1}{3},\frac{1}{3}) & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} H_d$$

$$+ \begin{pmatrix} L_1 & L_2 & L_3 \end{pmatrix} \begin{pmatrix} \Phi(-\frac{1}{2},0,\frac{1}{2}) & \Phi(\frac{1}{2},-1,\frac{1}{2}) & \Phi(\frac{1}{2},0,-\frac{1}{2}) \\ \Phi(-1,\frac{1}{2},\frac{1}{2}) & \Phi(0,-\frac{1}{2},\frac{1}{2}) & \Phi(0,\frac{1}{2},-\frac{1}{2}) \\ \Phi(-1,0,1) & \Phi(0,-1,1) & 1 \end{pmatrix} \begin{pmatrix} e_1^c \\ e_2^c \\ e_3^c \end{pmatrix} H_d + \mathrm{h.c.}\,,$$

in the EFT framework (neglecting $\mathcal{O}(1)$ dimensionless coefficients)

$$\Phi = \frac{\phi_1 \ldots \phi_N}{\Lambda_1 .. \Lambda_N}\,.$$

Assuming all Λ_i(s) to be universal

- Similar m_2/m_3 hierarchy in all charged sectors (same for m_1/m_3) $\Rightarrow$ 2HDM also motivated for $m_{s,\mu}/m_c$ hierarchy
- $\mu_L - \tau_L$ mixing connected to the hierarchy $m_2/m_3 \approx \lambda^3$. Similarly $e_L - \tau_L$ mixing connected to $m_1/m_3 \approx \lambda^6$.

Charged fermions: spurion formalism

$$\mathcal{L} = \begin{pmatrix} Q_1 & Q_2 & Q_3 \end{pmatrix} \begin{pmatrix} \Phi(\frac{1}{2},0,-\frac{1}{2}) & \Phi(-\frac{1}{6},\frac{2}{3},-\frac{1}{2}) & \Phi(-\frac{1}{6},0,\frac{1}{6}) \\ \Phi(\frac{2}{3},-\frac{1}{6},-\frac{1}{2}) & \Phi(0,\frac{1}{2},-\frac{1}{2}) & \Phi(0,-\frac{1}{6},\frac{1}{6}) \\ \Phi(\frac{2}{3},0,-\frac{2}{3}) & \Phi(0,\frac{2}{3},-\frac{2}{3}) & 1 \end{pmatrix} \begin{pmatrix} u_1^c \\ u_2^c \\ u_3^c \end{pmatrix} H_u$$

$$+ \begin{pmatrix} Q_1 & Q_2 & Q_3 \end{pmatrix} \begin{pmatrix} \Phi(-\frac{1}{2},0,\frac{1}{2}) & \Phi(-\frac{1}{6},-\frac{1}{3},\frac{1}{2}) & \Phi(-\frac{1}{6},0,\frac{1}{6}) \\ \Phi(-\frac{1}{3},-\frac{1}{6},\frac{1}{2}) & \Phi(0,-\frac{1}{2},\frac{1}{2}) & \Phi(0,-\frac{1}{6},\frac{1}{6}) \\ \Phi(-\frac{1}{3},0,\frac{1}{3}) & \Phi(0,-\frac{1}{3},\frac{1}{3}) & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} H_d$$

$$+ \begin{pmatrix} L_1 & L_2 & L_3 \end{pmatrix} \begin{pmatrix} \Phi(-\frac{1}{2},0,\frac{1}{2}) & \Phi(\frac{1}{2},-1,\frac{1}{2}) & \Phi(\frac{1}{2},0,-\frac{1}{2}) \\ \Phi(-1,\frac{1}{2},\frac{1}{2}) & \Phi(0,-\frac{1}{2},\frac{1}{2}) & \Phi(0,\frac{1}{2},-\frac{1}{2}) \\ \Phi(-1,0,1) & \Phi(0,-1,1) & 1 \end{pmatrix} \begin{pmatrix} e_1^c \\ e_2^c \\ e_3^c \end{pmatrix} H_d + \text{h.c.}\,,$$

in the EFT framework (neglecting $\mathcal{O}(1)$ dimensionless coefficients)

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Assuming all Λ_i(s) to be universal

- Similar m_2/m_3 hierarchy in all charged sectors (same for m_1/m_3) $\Rightarrow$ 2HDM also motivated for $m_{s,\mu}/m_c$ hierarchy
- $\mu_L - \tau_L$ mixing connected to the hierarchy $m_2/m_3 \approx \lambda^3$. Similarly $e_L - \tau_L$ mixing connected to $m_1/m_3 \approx \lambda^6$.
- Alignment of V_{cb} and V_{ub} depends on dimensionless coefficients. Alignment of V_{us} is model-dependent.

Charged fermions: from spurions to hyperons

Simplest example model: promote spurions to hyperons and assume one EFT cut-off Λ (neglecting $\mathcal{O}(1)$ coefficients)

$$\mathcal{L}^{d\leq 5} = \begin{pmatrix} Q_1 & Q_2 & Q_3 \end{pmatrix} \begin{pmatrix} \phi_{\ell 13}^{(\frac{1}{2},0,-\frac{1}{2})}/\Lambda & 0 & \phi_{q13}^{(-\frac{1}{6},0,\frac{1}{6})}/\Lambda \\ 0 & \phi_{\ell 23}^{(0,\frac{1}{2},-\frac{1}{2})}/\Lambda & \phi_{q23}^{(0,-\frac{1}{6},\frac{1}{6})}/\Lambda \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u_1^c \\ u_2^c \\ u_3^c \end{pmatrix} H_u$$

$$+ \begin{pmatrix} Q_1 & Q_2 & Q_3 \end{pmatrix} \begin{pmatrix} \tilde{\phi}_{\ell 13}^{(-\frac{1}{2},0,\frac{1}{2})}/\Lambda & \phi_{d12}^{(-\frac{1}{6},-\frac{1}{3},\frac{1}{2})}/\Lambda & \phi_{q13}^{(-\frac{1}{6},0,\frac{1}{6})}/\Lambda \\ 0 & \tilde{\phi}_{\ell 23}^{(0,-\frac{1}{2},\frac{1}{2})}/\Lambda & \phi_{q23}^{(0,-\frac{1}{6},\frac{1}{6})}/\Lambda \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} H_d$$

$$+ \begin{pmatrix} L_1 & L_2 & L_3 \end{pmatrix} \begin{pmatrix} \tilde{\phi}_{\ell 13}^{(-\frac{1}{2},0,\frac{1}{2})}/\Lambda & 0 & \phi_{\ell 13}^{(\frac{1}{2},0,-\frac{1}{2})}/\Lambda \\ 0 & \tilde{\phi}_{\ell 23}^{(0,-\frac{1}{2},\frac{1}{2})}/\Lambda & \phi_{\ell 23}^{(0,\frac{1}{2},-\frac{1}{2})}/\Lambda \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e_1^c \\ e_2^c \\ e_3^c \end{pmatrix} H_d \,.$$

- Each hyperon ϕ above develops an arbitrary VEV breaking $U(1)_Y^3$.

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Simplest example model: promote spurions to hyperons and assume one EFT cut-off Λ (neglecting $\mathcal{O}(1)$ coefficients)

$$\mathcal{L}^{d\leq 5} = \begin{pmatrix} Q_1 & Q_2 & Q_3 \end{pmatrix} \begin{pmatrix} \phi_{\ell 13}^{(\frac{1}{2},0,-\frac{1}{2})}/\Lambda & 0 & \phi_{q13}^{(-\frac{1}{6},0,\frac{1}{6})}/\Lambda \\ 0 & \phi_{\ell 23}^{(0,\frac{1}{2},-\frac{1}{2})}/\Lambda & \phi_{q23}^{(0,-\frac{1}{6},\frac{1}{6})}/\Lambda \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u_1^c \\ u_2^c \\ u_3^c \end{pmatrix} H_u$$

$$+ \begin{pmatrix} Q_1 & Q_2 & Q_3 \end{pmatrix} \begin{pmatrix} \tilde{\phi}_{\ell 13}^{(-\frac{1}{2},0,\frac{1}{2})}/\Lambda & \phi_{d12}^{(-\frac{1}{6},-\frac{1}{3},\frac{1}{2})}/\Lambda & \phi_{q13}^{(-\frac{1}{6},0,\frac{1}{6})}/\Lambda \\ 0 & \tilde{\phi}_{\ell 23}^{(0,-\frac{1}{2},\frac{1}{2})}/\Lambda & \phi_{q23}^{(0,-\frac{1}{6},\frac{1}{6})}/\Lambda \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} H_d$$

$$+ \begin{pmatrix} L_1 & L_2 & L_3 \end{pmatrix} \begin{pmatrix} \tilde{\phi}_{\ell 13}^{(-\frac{1}{2},0,\frac{1}{2})}/\Lambda & 0 & \phi_{\ell 13}^{(\frac{1}{2},0,-\frac{1}{2})}/\Lambda \\ 0 & \tilde{\phi}_{\ell 23}^{(0,-\frac{1}{2},\frac{1}{2})}/\Lambda & \phi_{\ell 23}^{(0,\frac{1}{2},-\frac{1}{2})}/\Lambda \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e_1^c \\ e_2^c \\ e_3^c \end{pmatrix} H_d \,.$$

- Each hyperon ϕ above develops an arbitrary VEV breaking $U(1)_Y^3$.
- We have the freedom to choose the ratios $\langle\phi\rangle/\Lambda$ in order to explain charged fermion masses and CKM mixing.

Charged fermions: from spurions to hyperons

Ratios $\langle \phi \rangle / \Lambda$ fixed as powers of the Wolfenstein parameter $\lambda = \sin\theta_C \simeq 0.224$

$$
\begin{aligned}
\mathcal{L}^{d\leq 5} = &\begin{pmatrix} u_1 & u_2 & u_3 \end{pmatrix} \begin{pmatrix} \lambda^6 & 0 & \lambda^3 \\ 0 & \lambda^3 & \lambda^2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u_1^c \\ u_2^c \\ u_3^c \end{pmatrix} \frac{v_{\mathrm{SM}}}{\sqrt{2}} \\
&+ \begin{pmatrix} d_1 & d_2 & d_3 \end{pmatrix} \begin{pmatrix} \lambda^6 & \lambda^4 & \lambda^3 \\ 0 & \lambda^3 & \lambda^2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} \lambda^2 \frac{v_{\mathrm{SM}}}{\sqrt{2}} \\
&+ \begin{pmatrix} e_1 & e_2 & e_3 \end{pmatrix} \begin{pmatrix} \lambda^6 & 0 & \lambda^6 \\ 0 & \lambda^3 & \lambda^3 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e_1^c \\ e_2^c \\ e_3^c \end{pmatrix} \lambda^2 \frac{v_{\mathrm{SM}}}{\sqrt{2}} \,.
\end{aligned}
$$

- Whithin the limitations of the EFT framework, we obtain a good description of charged fermion masses and CKM mixing.

Charged fermions: from spurions to hyperons

Ratios $\langle\phi\rangle/\Lambda$ fixed as powers of the Wolfenstein parameter $\lambda = \sin\theta_C \simeq 0.224$

$$\mathcal{L}^{d\leq 5} = \begin{pmatrix} u_1 & u_2 & u_3 \end{pmatrix} \begin{pmatrix} \lambda^6 & 0 & \lambda^3 \\ 0 & \lambda^3 & \lambda^2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u_1^c \\ u_2^c \\ u_3^c \end{pmatrix} \frac{v_{\mathrm{SM}}}{\sqrt{2}}$$
$$+ \begin{pmatrix} d_1 & d_2 & d_3 \end{pmatrix} \begin{pmatrix} \lambda^6 & \lambda^4 & \lambda^3 \\ 0 & \lambda^3 & \lambda^2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} \lambda^2 \frac{v_{\mathrm{SM}}}{\sqrt{2}}$$
$$+ \begin{pmatrix} e_1 & e_2 & e_3 \end{pmatrix} \begin{pmatrix} \lambda^6 & 0 & \lambda^6 \\ 0 & \lambda^3 & \lambda^3 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e_1^c \\ e_2^c \\ e_3^c \end{pmatrix} \lambda^2 \frac{v_{\mathrm{SM}}}{\sqrt{2}} .$$

- Whithin the limitations of the EFT framework, we obtain a good description of charged fermion masses and CKM mixing.
- However, no $\phi_{12}^{(q,-q,0)}$ hyperon breaking $U(1)_{Y_1} \times U(1)_{Y_2} \xrightarrow{v_{12}} U(1)_{Y_1+Y_2}$ at a high scale v_{12} is specified.
- Moreover, unexplained large hierarchy of VEVs $\langle\phi_{\ell 13}^{(\frac{1}{2},0,-\frac{1}{2})}\rangle / \langle\phi_{\ell 23}^{(0,\frac{1}{2},-\frac{1}{2})}\rangle \approx \lambda^3$, where both hyperons participate in 23 breaking $U(1)_{Y_1+Y_2} \times U(1)_{Y_3} \xrightarrow{v_{23}} U(1)_{Y_1+Y_2+Y_3}$.

Charged fermions: Minimal model with 3 hyperons

Introduce the minimal set of hyperons ($\mathcal{O}(1)$ coefficients for each entry are implicit)

$$\phi_{\ell 23}^{(0,\frac{1}{2},-\frac{1}{2})}\,, \qquad \phi_{q23}^{(0,-\frac{1}{6},\frac{1}{6})}\,, \qquad \phi_{q12}^{(-\frac{1}{6},\frac{1}{6},0)}\,.$$

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$$\mathcal{L} = \begin{pmatrix} Q_1 & Q_2 & Q_3 \end{pmatrix} \begin{pmatrix} \tilde{\phi}_{q12}^3 \phi_{\ell 23}/\Lambda^4 & \phi_{q12}\phi_{\ell 23}/\Lambda^2 & \phi_{q12}\phi_{q23}/\Lambda^2 \\ \tilde{\phi}_{q12}^4 \phi_{\ell 23}/\Lambda^5 & \phi_{\ell 23}/\Lambda & \phi_{q23}/\Lambda \\ \tilde{\phi}_{q12}^4 \phi_{\ell 23}\tilde{\phi}_{q23}/\Lambda^6 & \phi_{\ell 23}\tilde{\phi}_{q23}/\Lambda^2 & 1 \end{pmatrix} \begin{pmatrix} u_1^c \\ u_2^c \\ u_3^c \end{pmatrix} H_u$$

$$+ \begin{pmatrix} Q_1 & Q_2 & Q_3 \end{pmatrix} \begin{pmatrix} \phi_{q12}^3 \tilde{\phi}_{\ell 23}/\Lambda^4 & \phi_{q12}\tilde{\phi}_{\ell 23}/\Lambda^2 & \phi_{q12}\phi_{q23}/\Lambda^2 \\ \phi_{q12}^2 \tilde{\phi}_{\ell 23}/\Lambda^3 & \tilde{\phi}_{\ell 23}/\Lambda & \phi_{q23}/\Lambda \\ \phi_{q12}^2 \phi_{q23}^2/\Lambda^4 & \phi_{q23}^2/\Lambda^2 & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} H_d$$

$$+ \begin{pmatrix} L_1 & L_2 & L_3 \end{pmatrix} \begin{pmatrix} \phi_{q12}^3 \tilde{\phi}_{\ell 23}/\Lambda^4 & \tilde{\phi}_{q12}^3 \tilde{\phi}_{\ell 23}/\Lambda^4 & \tilde{\phi}_{q12}^3 \phi_{\ell 23}/\Lambda^4 \\ \phi_{q12}^6 \tilde{\phi}_{\ell 23}/\Lambda^7 & \tilde{\phi}_{\ell 23}/\Lambda & \phi_{\ell 23}/\Lambda \\ \phi_{q12}^6 \tilde{\phi}_{\ell 23}^2/\Lambda^8 & \tilde{\phi}_{\ell 23}^2/\Lambda^2 & 1 \end{pmatrix} \begin{pmatrix} e_1^c \\ e_2^c \\ e_3^c \end{pmatrix} H_d\,,$$

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$$\mathcal{L} = \begin{pmatrix} Q_1 & Q_2 & Q_3 \end{pmatrix} \begin{pmatrix} \tilde{\phi}_{q12}^3 \phi_{\ell 23}/\Lambda^4 & \phi_{q12}\phi_{\ell 23}/\Lambda^2 & \phi_{q12}\phi_{q23}/\Lambda^2 \\ \tilde{\phi}_{q12}^4 \phi_{\ell 23}/\Lambda^5 & \phi_{\ell 23}/\Lambda & \phi_{q23}/\Lambda \\ \tilde{\phi}_{q12}^4 \phi_{\ell 23}\tilde{\phi}_{q23}/\Lambda^6 & \phi_{\ell 23}\tilde{\phi}_{q23}/\Lambda^2 & 1 \end{pmatrix} \begin{pmatrix} u_1^c \\ u_2^c \\ u_3^c \end{pmatrix} H_u$$

$$+ \begin{pmatrix} Q_1 & Q_2 & Q_3 \end{pmatrix} \begin{pmatrix} \phi_{q12}^3 \tilde{\phi}_{\ell 23}/\Lambda^4 & \phi_{q12}\tilde{\phi}_{\ell 23}/\Lambda^2 & \phi_{q12}\phi_{q23}/\Lambda^2 \\ \phi_{q12}^2 \tilde{\phi}_{\ell 23}/\Lambda^3 & \tilde{\phi}_{\ell 23}/\Lambda & \phi_{q23}/\Lambda \\ \phi_{q12}^2 \phi_{q23}^2/\Lambda^4 & \phi_{q23}^2/\Lambda^2 & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} H_d$$

$$+ \begin{pmatrix} L_1 & L_2 & L_3 \end{pmatrix} \begin{pmatrix} \phi_{q12}^3 \tilde{\phi}_{\ell 23}/\Lambda^4 & \tilde{\phi}_{q12}^3\tilde{\phi}_{\ell 23}/\Lambda^4 & \tilde{\phi}_{q12}^3\phi_{\ell 23}/\Lambda^4 \\ \phi_{q12}^6 \tilde{\phi}_{\ell 23}/\Lambda^7 & \tilde{\phi}_{\ell 23}/\Lambda & \phi_{\ell 23}/\Lambda \\ \phi_{q12}^6 \tilde{\phi}_{\ell 23}^2/\Lambda^8 & \tilde{\phi}_{\ell 23}^2/\Lambda^2 & 1 \end{pmatrix} \begin{pmatrix} e_1^c \\ e_2^c \\ e_3^c \end{pmatrix} H_d\,,$$

- Second family masses and V_{cb} arise at dimension 5.

Charged fermions: Minimal model with 3 hyperons

Introduce the minimal set of hyperons ($\mathcal{O}(1)$ coefficients for each entry are implicit)

$$\phi_{\ell 23}^{(0,\frac{1}{2},-\frac{1}{2})}\,, \qquad \phi_{q23}^{(0,-\frac{1}{6},\frac{1}{6})}\,, \qquad \phi_{q12}^{(-\frac{1}{6},\frac{1}{6},0)}\,.$$

$$\mathcal{L} = \begin{pmatrix} Q_1 & Q_2 & Q_3 \end{pmatrix} \begin{pmatrix} \tilde{\phi}_{q12}^{3}\phi_{\ell 23}/\Lambda^4 & \phi_{q12}\phi_{\ell 23}/\Lambda^2 & \phi_{q12}\phi_{q23}/\Lambda^2 \\ \tilde{\phi}_{q12}^{4}\phi_{\ell 23}/\Lambda^5 & \phi_{\ell 23}/\Lambda & \phi_{q23}/\Lambda \\ \tilde{\phi}_{q12}^{4}\phi_{\ell 23}\tilde{\phi}_{q23}/\Lambda^6 & \phi_{\ell 23}\tilde{\phi}_{q23}/\Lambda^2 & 1 \end{pmatrix} \begin{pmatrix} u_1^c \\ u_2^c \\ u_3^c \end{pmatrix} H_u$$

$$+ \begin{pmatrix} Q_1 & Q_2 & Q_3 \end{pmatrix} \begin{pmatrix} \phi_{q12}^{3}\tilde{\phi}_{\ell 23}/\Lambda^4 & \phi_{q12}\tilde{\phi}_{\ell 23}/\Lambda^2 & \phi_{q12}\phi_{q23}/\Lambda^2 \\ \phi_{q12}^{2}\tilde{\phi}_{\ell 23}/\Lambda^3 & \tilde{\phi}_{\ell 23}/\Lambda & \phi_{q23}/\Lambda \\ \phi_{q12}^{2}\phi_{q23}^{2}/\Lambda^4 & \phi_{q23}^{2}/\Lambda^2 & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} H_d$$

$$+ \begin{pmatrix} L_1 & L_2 & L_3 \end{pmatrix} \begin{pmatrix} \phi_{q12}^{3}\tilde{\phi}_{\ell 23}/\Lambda^4 & \tilde{\phi}_{q12}^{3}\tilde{\phi}_{\ell 23}/\Lambda^4 & \tilde{\phi}_{q12}^{3}\phi_{\ell 23}/\Lambda^4 \\ \phi_{q12}^{6}\tilde{\phi}_{\ell 23}/\Lambda^7 & \tilde{\phi}_{\ell 23}/\Lambda & \phi_{\ell 23}/\Lambda \\ \phi_{q12}^{6}\tilde{\phi}_{\ell 23}^{2}/\Lambda^8 & \tilde{\phi}_{\ell 23}^{2}/\Lambda^2 & 1 \end{pmatrix} \begin{pmatrix} e_1^c \\ e_2^c \\ e_3^c \end{pmatrix} H_d\,,$$

- Second family masses and V_{cb} arise at dimension 5.
- V_{ub} at dimension 6 and first family masses at dimension 8.

Charged fermions: Minimal model with 3 hyperons

Introduce the minimal set of hyperons ($\mathcal{O}(1)$ coefficients for each entry are implicit)

$$\phi_{\ell 23}^{(0,\frac{1}{2},-\frac{1}{2})}, \quad \phi_{q23}^{(0,-\frac{1}{6},\frac{1}{6})}, \quad \phi_{q12}^{(-\frac{1}{6},\frac{1}{6},0)}.$$

$$\mathcal{L} = \begin{pmatrix} Q_1 & Q_2 & Q_3 \end{pmatrix} \begin{pmatrix} \tilde{\phi}_{q12}^3 \phi_{\ell 23}/\Lambda^4 & \phi_{q12}\phi_{\ell 23}/\Lambda^2 & \phi_{q12}\phi_{q23}/\Lambda^2 \\ \tilde{\phi}_{q12}^4 \phi_{\ell 23}/\Lambda^5 & \phi_{\ell 23}/\Lambda & \phi_{q23}/\Lambda \\ \tilde{\phi}_{q12}^4 \phi_{\ell 23}\tilde{\phi}_{q23}/\Lambda^6 & \phi_{\ell 23}\tilde{\phi}_{q23}/\Lambda^2 & 1 \end{pmatrix} \begin{pmatrix} u_1^c \\ u_2^c \\ u_3^c \end{pmatrix} H_u$$

$$+ \begin{pmatrix} Q_1 & Q_2 & Q_3 \end{pmatrix} \begin{pmatrix} \phi_{q12}^3 \tilde{\phi}_{\ell 23}/\Lambda^4 & \phi_{q12}\tilde{\phi}_{\ell 23}/\Lambda^2 & \phi_{q12}\phi_{q23}/\Lambda^2 \\ \phi_{q12}^2 \tilde{\phi}_{\ell 23}/\Lambda^3 & \tilde{\phi}_{\ell 23}/\Lambda & \phi_{q23}/\Lambda \\ \phi_{q12}^2 \phi_{q23}^2/\Lambda^4 & \phi_{q23}^2/\Lambda^2 & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} H_d$$

$$+ \begin{pmatrix} L_1 & L_2 & L_3 \end{pmatrix} \begin{pmatrix} \phi_{q12}^3 \tilde{\phi}_{\ell 23}/\Lambda^4 & \tilde{\phi}_{q12}^3 \tilde{\phi}_{\ell 23}/\Lambda^4 & \tilde{\phi}_{q12}^3 \phi_{\ell 23}/\Lambda^4 \\ \phi_{q12}^6 \tilde{\phi}_{\ell 23}/\Lambda^7 & \tilde{\phi}_{\ell 23}/\Lambda & \phi_{\ell 23}/\Lambda \\ \phi_{q12}^6 \tilde{\phi}_{\ell 23}^2/\Lambda^8 & \tilde{\phi}_{\ell 23}^2/\Lambda^2 & 1 \end{pmatrix} \begin{pmatrix} e_1^c \\ e_2^c \\ e_3^c \end{pmatrix} H_d ,$$

- Second family masses and V_{cb} arise at dimension 5.
- V_{ub} at dimension 6 and first family masses at dimension 8.
- RH fermion mixing generally suppressed.

Charged fermions: Minimal model with 3 hyperons

Introduce the minimal set of hyperons ($\mathcal{O}(1)$ coefficients for each entry are implicit)

$$\phi_{\ell 23}^{(0,\frac{1}{2},-\frac{1}{2})}\,, \qquad \phi_{q23}^{(0,-\frac{1}{6},\frac{1}{6})}\,, \qquad \phi_{q12}^{(-\frac{1}{6},\frac{1}{6},0)}\,.$$

$$\begin{aligned}\mathcal{L} = &\begin{pmatrix} Q_1 & Q_2 & Q_3 \end{pmatrix} \begin{pmatrix} \tilde{\phi}^3_{q12}\phi_{\ell 23}/\Lambda^4 & \phi_{q12}\phi_{\ell 23}/\Lambda^2 & \phi_{q12}\phi_{q23}/\Lambda^2 \\ \tilde{\phi}^4_{q12}\phi_{\ell 23}/\Lambda^5 & \phi_{\ell 23}/\Lambda & \phi_{q23}/\Lambda \\ \tilde{\phi}^4_{q12}\phi_{\ell 23}\tilde{\phi}_{q23}/\Lambda^6 & \phi_{\ell 23}\tilde{\phi}_{q23}/\Lambda^2 & 1 \end{pmatrix} \begin{pmatrix} u^c_1 \\ u^c_2 \\ u^c_3 \end{pmatrix} H_u \\
&+ \begin{pmatrix} Q_1 & Q_2 & Q_3 \end{pmatrix} \begin{pmatrix} \phi^3_{q12}\tilde{\phi}_{\ell 23}/\Lambda^4 & \phi_{q12}\tilde{\phi}_{\ell 23}/\Lambda^2 & \phi_{q12}\phi_{q23}/\Lambda^2 \\ \phi^2_{q12}\tilde{\phi}_{\ell 23}/\Lambda^3 & \tilde{\phi}_{\ell 23}/\Lambda & \phi_{q23}/\Lambda \\ \phi^2_{q12}\phi^2_{q23}/\Lambda^4 & \phi^2_{q23}/\Lambda^2 & 1 \end{pmatrix} \begin{pmatrix} d^c_1 \\ d^c_2 \\ d^c_3 \end{pmatrix} H_d \\
&+ \begin{pmatrix} L_1 & L_2 & L_3 \end{pmatrix} \begin{pmatrix} \phi^3_{q12}\tilde{\phi}_{\ell 23}/\Lambda^4 & \tilde{\phi}^3_{q12}\tilde{\phi}_{\ell 23}/\Lambda^4 & \tilde{\phi}^3_{q12}\phi_{\ell 23}/\Lambda^4 \\ \phi^6_{q12}\tilde{\phi}_{\ell 23}/\Lambda^7 & \tilde{\phi}_{\ell 23}/\Lambda & \phi_{\ell 23}/\Lambda \\ \phi^6_{q12}\tilde{\phi}^2_{\ell 23}/\Lambda^8 & \tilde{\phi}^2_{\ell 23}/\Lambda^2 & 1 \end{pmatrix} \begin{pmatrix} e^c_1 \\ e^c_2 \\ e^c_3 \end{pmatrix} H_d\,,\end{aligned}$$

- Second family masses and V_{cb} arise at dimension 5.
- V_{ub} at dimension 6 and first family masses at dimension 8.
- RH fermion mixing generally suppressed.
- Fixing ratios $\langle\phi\rangle/\Lambda$ from data

$$\frac{\langle\phi_{\ell 23}\rangle}{\Lambda} \sim \frac{m_c}{m_t} \simeq \lambda^3\,, \qquad \frac{\langle\phi_{q23}\rangle}{\Lambda} \sim V_{cb} \simeq \lambda^2\,, \qquad \frac{\langle\phi_{q12}\rangle}{\Lambda} \sim V_{us} \simeq \lambda,$$

Charged fermions: Minimal model with 3 hyperons

Introduce the minimal set of hyperons

$$\frac{\langle \phi_{\ell 23}^{(0,\frac{1}{2},-\frac{1}{2})} \rangle}{\Lambda} \sim \frac{m_c}{m_t} \simeq \lambda^3 \,, \qquad \frac{\langle \phi_{q23}^{(0,-\frac{1}{6},\frac{1}{6})} \rangle}{\Lambda} \sim V_{cb} \simeq \lambda^2 \,, \qquad \frac{\langle \phi_{q12}^{(-\frac{1}{6},\frac{1}{6},0)} \rangle}{\Lambda} \sim V_{us} \simeq \lambda,$$

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$$+ \begin{pmatrix} d_1 & d_2 & d_3 \end{pmatrix} \begin{pmatrix} \lambda^6 & \lambda^4 & \lambda^3 \\ \lambda^5 & \lambda^3 & \lambda^2 \\ \lambda^6 & \lambda^4 & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} \lambda^2 \frac{v_{\mathrm{SM}}}{\sqrt{2}}$$
$$+ \begin{pmatrix} e_1 & e_2 & e_3 \end{pmatrix} \begin{pmatrix} \lambda^6 & \lambda^6 & \lambda^6 \\ \lambda^9 & \lambda^3 & \lambda^3 \\ \lambda^{12} & \lambda^6 & 1 \end{pmatrix} \begin{pmatrix} e_1^c \\ e_2^c \\ e_3^c \end{pmatrix} \lambda^2 \frac{v_{\mathrm{SM}}}{\sqrt{2}} \,.$$

- Largest VEV is $\langle \phi_{q12} \rangle \approx v_{12}$ triggering $U(1)_{Y_1} \times U(1)_{Y_2} \xrightarrow{v_{12}} U(1)_{Y_1+Y_2}$.

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Charged fermions: Minimal model with 3 hyperons

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- Both 23-breaking VEVs at the same scale v_{23}, mild hierarchy $v_{23}/v_{12} \approx \lambda$.
- Alignment of V_{us} not specified, plus large RH mixing $s_{12}^{d_R} \simeq \mathcal{O}(\lambda^2) \Rightarrow$ This model is minimally modified to predict V_{us} from down sector and suppressed $s_{12}^{d_R} \simeq \mathcal{O}(\lambda^5)$ (in the paper!).

Neutrinos: General considerations

Spurions for the Weinberg operator (carrying M^{-1} dimension)

$$\mathcal{L}_{\text{Weinberg}} = \begin{pmatrix} L_1 & L_2 & L_3 \end{pmatrix} \begin{pmatrix} \Phi(1,0,-1) & \Phi(\frac{1}{2},\frac{1}{2},-1) & \Phi(\frac{1}{2},0,-\frac{1}{2}) \\ \Phi(\frac{1}{2},\frac{1}{2},-1) & \Phi(0,1,-1) & \Phi(0,\frac{1}{2},-\frac{1}{2}) \\ \Phi(\frac{1}{2},0,-\frac{1}{2}) & \Phi(0,\frac{1}{2},-\frac{1}{2}) & 1 \end{pmatrix} \begin{pmatrix} L_1 \\ L_2 \\ L_3 \end{pmatrix} H_u H_u \,,$$

- $U(2)^5$ is present in the neutrino sector $\Rightarrow$ naive expectation is a heavier active neutrino with tiny mixing with the others.

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- E.g. introduce $U(1)^3_Y$ singlet $N(\mathbf{1},\mathbf{1})_{(0,0,0)} \Rightarrow \mathcal{L}_N \supset L_3 H_u N + m_N N N$ with $L_2 H_u N$ small as it originates from higher dimensional operators $\Rightarrow$ unsuccesfull atmospheric mixing

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- Solution $\Rightarrow$ Add SM singlet neutrinos which carry $U(1)^3_Y$ charges (with vanishing SM hypercharge $Y_1 + Y_2 + Y_3 = 0$).

Seesaw mechanism *à la tri-hypercharge*

- Consider adding $N_{\mathrm{atm}}^{(0,\frac{1}{4},-\frac{1}{4})}$ and $\phi_{\mathrm{atm}}^{(0,\frac{1}{4},-\frac{1}{4})}$, then (remember $\phi_{\ell 23}^{(0,1/2,-1/2)}$)

$$\mathcal{L}_{N_{\mathrm{atm}}} \supset \frac{1}{\Lambda_{\mathrm{atm}}}(\phi_{\mathrm{atm}} L_2 + \tilde{\phi}_{\mathrm{atm}} L_3) H_u N_{\mathrm{atm}} + \tilde{\phi}_{\ell 23} N_{\mathrm{atm}} N_{\mathrm{atm}} \,,$$

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- However, gauge anomalies via $N_{\rm atm}^{(0,\frac{1}{4},-\frac{1}{4})}$ $\Rightarrow$introduce conjugate neutrino $\overline{N}_{\rm atm}^{(0,-1/4,1/4)}$

$$\begin{aligned}\mathcal{L}_{N_{\rm atm}} =& \frac{1}{\Lambda_{\rm atm}}(\phi_{\rm atm} L_2 + \tilde{\phi}_{\rm atm} L_3) H_u N_{\rm atm} + \frac{\phi_{\rm atm}}{\Lambda_{\rm atm}} L_3 H_u \overline{N}_{\rm atm} \\ &+ \phi_{\ell 23} N_{\rm atm} N_{\rm atm} + \tilde{\phi}_{\ell 23} \overline{N}_{\rm atm} \overline{N}_{\rm atm} + M_{N_{\rm atm}} \overline{N}_{\rm atm} N_{\rm atm} \,,\end{aligned}$$

Notice $L_3 H_u = (0,0,0)$.

Seesaw mechanism *à la tri-hypercharge*

- Play the same game to obtain solar mixing

$$N_{\rm sol}^{(\frac{1}{4},\frac{1}{4},-\frac{1}{2})}\,, \qquad \overline{N}_{\rm sol}^{(-\frac{1}{4},-\frac{1}{4},\frac{1}{2})}\,, \qquad \phi_{\rm sol}^{(-\frac{1}{2},-\frac{1}{2},1)}\,, \qquad \phi_{e12}^{(\frac{1}{4},-\frac{1}{4},0)}\,, \qquad \phi_{\nu 13}^{(-\frac{1}{4},-\frac{1}{4},\frac{1}{2})}\,,$$

$$\mathcal{L}_{N_{\rm sol}} = \frac{1}{\Lambda_{\rm sol}}(\phi_{e12}L_1 + \tilde{\phi}_{e12}L_2 + \phi_{\nu 13}L_3)H_u N_{\rm sol} + \frac{\phi_{\nu 13}}{\Lambda_{\rm sol}} L_3 H_u \overline{N}_{\rm sol} + \phi_{\rm sol} N_{\rm sol} N_{\rm sol} + \tilde{\phi}_{\rm sol}\overline{N}_{\rm sol}\overline{N}_{\rm sol} + M_{N_{\rm sol}}\overline{N}_{\rm sol} N_{\rm sol}\,,$$

Seesaw mechanism *à la tri-hypercharge*

- Assume $M_{N_{\mathrm{sol}}} \approx M_{N_{\mathrm{atm}}} \equiv M_{\mathrm{VL}}$ for simplicity, $\langle \phi_{e12} \rangle \approx \mathcal{O}(v_{12})$ and the rest $\langle \phi \rangle \approx \mathcal{O}(v_{23})$, apply seesaw formula

$$m_\nu = m_D M_N^{-1} m_D^{\mathrm{T}} = \left[\begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ \times & \times & \times \end{pmatrix} M_{\mathrm{VL}} + \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix} v_{23} \right] \frac{1}{v_{23}^2 - M_{\mathrm{VL}}^2} .$$

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- $M_{\mathrm{VL}} \lesssim v_{23}$ required to describe PMNS mixing $\Rightarrow$ SM singlet neutrinos with mass at the lower scale v_{23}.

Seesaw mechanism *à la tri-hypercharge*

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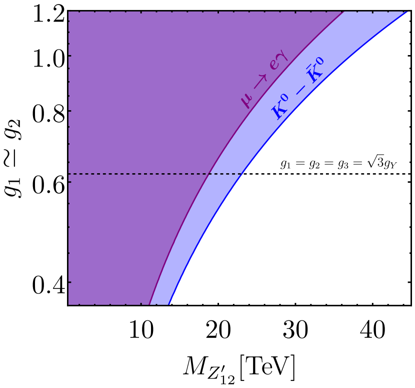
- $M_{\mathrm{VL}} \lesssim v_{23}$ required to describe PMNS mixing $\Rightarrow$ SM singlet neutrinos with mass at the lower scale v_{23}.
- Finally, up to dimensionless coefficients

$$\boxed{m_\nu \simeq \begin{pmatrix} 1 & 1 & \lambda \\ 1 & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix} v_{23} \frac{v_{\mathrm{SM}}^2}{\Lambda_{\mathrm{atm}}^2} \,.}$$

the texture above can reproduce best fit PMNS mixing (NuFit 5.1) with $\mathcal{O}(1)$ coefficients. If $v_{23} \approx \mathcal{O}(\mathrm{TeV})$ then $\Lambda_{\mathrm{atm}} \approx \mathcal{O}(10^6\ \mathrm{TeV})$.

Phenomenology: Z'_{12}



$$\mathcal{L}_{Z'_{12}} = Y_{\psi_{L,R}} \overline{\psi}_{L,R} \gamma^\mu \begin{pmatrix} -g_1 \sin\theta_{12} & 0 & 0 \\ 0 & g_2 \cos\theta_{12} & 0 \\ 0 & 0 & 0 \end{pmatrix} \psi_{L,R} Z'_{12\mu} \,,$$

$$\sin\theta_{12} = \frac{g_1}{\sqrt{g_1^2 + g_2^2}} \,,$$

- Z'_{12} with intrinsical flavour non-universal couplings, breaking explicitly $U(2)^5$. Flavour-violating couplings arise from CKM and charged lepton mixing.

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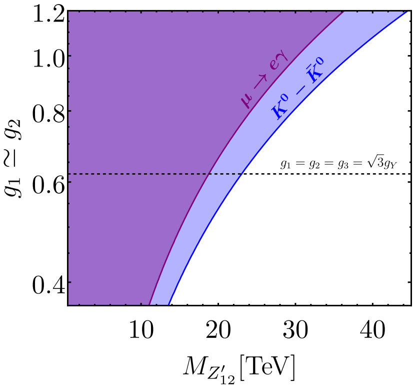

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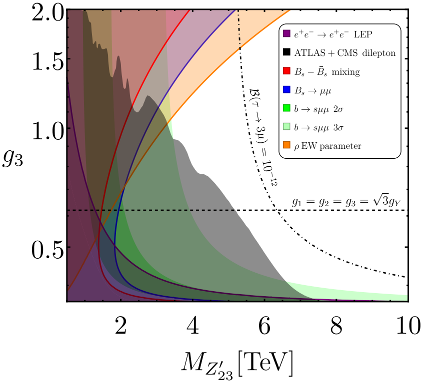
- Z'_{12} with intrinsical flavour non-universal couplings, breaking explicitly $U(2)^5$. Flavour-violating couplings arise from CKM and charged lepton mixing.
- $K - \bar{K}$ mixing most constraining, but depends on alignment of V_{us} and $d_R - s_R$ mixing (model-dependent). We find $M_{Z'_{12}} \gtrsim 20\,\text{TeV}$ for some models in the paper (worst case scenario $M_{Z'_{12}} \gtrsim 300\,\text{TeV}$).



$$\mathcal{L}_{Z'_{23}} = Y_{\psi_{L,R}} \overline{\psi}_{L,R} \gamma^\mu \begin{pmatrix} -g_{12}\sin\theta_{23} & 0 & 0 \\ 0 & -g_{12}\sin\theta_{23} & 0 \\ 0 & 0 & g_3\cos\theta_{23} \end{pmatrix} \psi_{L,R} Z'_{23\mu}$$

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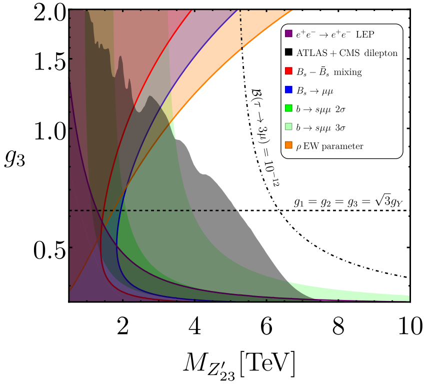
$$g_Y = \frac{g_{12} g_3}{\sqrt{g_{12}^2 + g_3^2}}\,, \qquad g_{12} = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}}\,,$$

$$\frac{v_{23}}{v_{12}} \approx \lambda\,.$$

- LHC dilepton searches $pp \to Z'_{23} \to e^+e^-,\ \mu^+\mu^-$ exclude light Z'_{23} due to large couplings to light quarks and leptons. Flavour observables usually not competitive.

Phenomenology: Z'_{23}



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- LHC dilepton searches $pp \to Z'_{23} \to e^+e^-, \mu^+\mu^-$ exclude light Z'_{23} due to large couplings to light quarks and leptons. Flavour observables usually not competitive.
- $Z - Z'_{23}$ mixing connected to g_3. Small shift on the mass of $Z \Rightarrow \rho$ EW parameter larger than 1

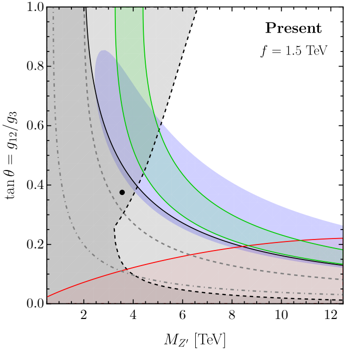
$$\sin\theta_{Z-Z'_{23}} = \frac{g_3 \cos\theta_{23}}{\sqrt{g_Y^2 + g_L^2}} \left(\frac{M_Z^0}{M_{Z'_{23}}^0}\right)^2 \Rightarrow \rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = \frac{1}{1 - g_3^2\cos^2\theta_{23}\left(\frac{v_{\rm SM}}{2M_{Z'_{23}}^0}\right)^2} > 1\,,$$

Phenomenology: Z'_{23}

- "Deconstructed hypercharge" Davighi and Stefanek [2305.16280]

$$SU(3)_c \times SU(2)_L \times U(1)_{Y_1+Y_2} \times U(1)_{Y_3}\,, \qquad g_Y = \frac{g_{12} g_3}{\sqrt{g_{12}^2 + g_3^2}} \Rightarrow g_{12}, g_3 \geq g_Y\,.$$

- UV complete model and study of naturalness: EWPOs global fit and 95% CL bound with m_W^{old} (solid dashed line), preferred region with m_W^{new} (blue region). Green region can explain m_c/m_t in their particular UV completion. Solid black line are LHC limits (see talk by Joe Davighi)



Take home messages

$$\boxed{SU(3)_c \times SU(2)_L \times U(1)_{Y_1} \times U(1)_{Y_2} \times U(1)_{Y_3}}$$

- Tri-hypercharge gauge group might be the first step towards understanding the origin of three flavours, the hierarchical charged fermion masses and CKM mixing.

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- If seesaw mechanism is implemented via adding SM singlet neutrinos, the $U(1)_Y^3$ setup leads to a low scale seesaw where SM singlet neutrinos might be as light as a few TeV.

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- If seesaw mechanism is implemented via adding SM singlet neutrinos, the $U(1)_Y^3$ setup leads to a low scale seesaw where SM singlet neutrinos might be as light as a few TeV.
- Rich phenomenology via Z' bosons if NP scales are low: from flavour-violating observables to LHC physics and EW precision physics.

Acknowledgements

The speaker acknowledges support from the European Union's Horizon 2020 Research and Innovation programme under the Marie Skłodowska-Curie grant agreement No. 860881-HIDDeN.

Neutrinos: Example of seesaw mechanism

Getting ready to apply the seesaw formula (neglecting $\mathcal{O}(1)$ coefficients)

$$m_{D_L} = \begin{pmatrix} & \overline{N}_{\rm sol} & \overline{N}_{\rm atm} \\ \hline L_1| & 0 & 0 \\ L_2| & 0 & 0 \\ L_3| & \frac{\tilde{\phi}_{\nu 13}}{\Lambda_{\rm sol}} & \frac{\phi_{\rm atm}}{\Lambda_{\rm atm}} \end{pmatrix} H_u \,, \qquad m_{D_R} = \begin{pmatrix} & N_{\rm sol} & N_{\rm atm} \\ \hline L_1| & \frac{\phi_{e12}}{\Lambda_{\rm sol}} & 0 \\ L_2| & \frac{\tilde{\phi}_{e12}}{\Lambda_{\rm sol}} & \frac{\phi_{\rm atm}}{\Lambda_{\rm atm}} \\ L_3| & \frac{\phi_{\nu 13}}{\Lambda_{\rm sol}} & \frac{\tilde{\phi}_{\rm atm}}{\Lambda_{\rm atm}} \end{pmatrix} H_u \,,$$

$$M_L = \begin{pmatrix} & \overline{N}_{\rm sol} & \overline{N}_{\rm atm} \\ \hline \overline{N}_{\rm sol}| & \tilde{\phi}_{\rm sol} & 0 \\ \overline{N}_{\rm atm}| & 0 & \tilde{\phi}_{\ell 23} \end{pmatrix} \approx v_{23} \mathbb{I}_{2\times 2} \,, \quad M_R \approx \begin{pmatrix} & N_{\rm sol} & N_{\rm atm} \\ \hline N_{\rm sol}| & \phi_{\rm sol} & 0 \\ N_{\rm atm}| & 0 & \phi_{\ell 23} \end{pmatrix} \approx v_{23} \mathbb{I}_{2\times 2} \,,$$

$$M_{LR} = \begin{pmatrix} & N_{\rm sol} & N_{\rm atm} \\ \hline \overline{N}_{\rm sol}| & M_{N_{\rm sol}} & 0 \\ \overline{N}_{\rm atm}| & 0 & M_{N_{\rm atm}} \end{pmatrix} \approx M_{\rm VL} \mathbb{I}_{2\times 2} \,,$$

Backup: Neutrinos

Full neutrino mass matrix (remember $M_L \approx M_R \approx v_{23}\mathbb{I}_{2\times 2}\,, \quad M_{LR} \approx M_{\rm VL}\mathbb{I}_{2\times 2}$)

$$M_\nu = \begin{pmatrix} & \nu & \overline{N} & N \\ \hline \nu| & 0 & m_{D_L} & m_{D_R} \\ \overline{N}| & m_{D_L}^{\rm T} & M_L & M_{LR} \\ N| & m_{D_R}^{\rm T} & M_{LR}^{\rm T} & M_R \end{pmatrix} \equiv \begin{pmatrix} 0 & m_D \\ m_D^{\rm T} & M_N \end{pmatrix}\,.$$

$$m_{D_L} = \begin{pmatrix} & \overline{N}_{\rm sol} & \overline{N}_{\rm atm} \\ \hline L_1| & 0 & 0 \\ L_2| & 0 & 0 \\ L_3| & \frac{\tilde{\phi}_{\nu 13}}{\Lambda_{\rm sol}} & \frac{\phi_{\rm atm}}{\Lambda_{\rm atm}} \end{pmatrix} H_u\,, \qquad m_{D_R} = \begin{pmatrix} & N_{\rm sol} & N_{\rm atm} \\ \hline L_1| & \frac{\phi_{e12}}{\Lambda_{\rm sol}} & 0 \\ L_2| & \frac{\tilde{\phi}_{e12}}{\Lambda_{\rm sol}} & \frac{\phi_{\rm atm}}{\Lambda_{\rm atm}} \\ L_3| & \frac{\phi_{\nu 13}}{\Lambda_{\rm sol}} & \frac{\tilde{\phi}_{\rm atm}}{\Lambda_{\rm atm}} \end{pmatrix} H_u\,,$$

Backup: Neutrinos

Full neutrino mass matrix (remember $M_L \approx M_R \approx v_{23}\mathbb{I}_{2\times 2}\,, \quad M_{LR} \approx M_{\rm VL}\mathbb{I}_{2\times 2}$)

$$M_\nu = \left(\begin{array}{c|ccc} & \nu & \overline{N} & N \\ \hline \nu & 0 & m_{D_L} & m_{D_R} \\ \overline{N} & m_{D_L}^{\rm T} & M_L & M_{LR} \\ N & m_{D_R}^{\rm T} & M_{LR}^{\rm T} & M_R \end{array}\right) \equiv \begin{pmatrix} 0 & m_D \\ m_D^{\rm T} & M_N \end{pmatrix} .$$

$$m_{D_L} = \left(\begin{array}{c|cc} & \overline{N}_{\rm sol} & \overline{N}_{\rm atm} \\ \hline L_1 & 0 & 0 \\ L_2 & 0 & 0 \\ L_3 & \frac{\tilde{\phi}_{\nu 13}}{\Lambda_{\rm sol}} & \frac{\phi_{\rm atm}}{\Lambda_{\rm atm}} \end{array}\right) H_u\,, \qquad m_{D_R} = \left(\begin{array}{c|cc} & N_{\rm sol} & N_{\rm atm} \\ \hline L_1 & \frac{\phi_{e12}}{\Lambda_{\rm sol}} & 0 \\ L_2 & \frac{\tilde{\phi}_{e12}}{\Lambda_{\rm sol}} & \frac{\phi_{\rm atm}}{\Lambda_{\rm atm}} \\ L_3 & \frac{\phi_{\nu 13}}{\Lambda_{\rm sol}} & \frac{\tilde{\phi}_{\rm atm}}{\Lambda_{\rm atm}} \end{array}\right) H_u\,,$$

Provided that $m_D \ll M_N$, we can apply the seesaw formula

$$m_\nu = m_D M_N^{-1} m_D^{\rm T} = \begin{pmatrix} m_{D_L} & m_{D_R} \end{pmatrix} \begin{pmatrix} v_{23} & -M_{\rm VL} \\ -M_{\rm VL} & v_{23} \end{pmatrix} \begin{pmatrix} m_{D_L}^{\rm T} \\ m_{D_R}^{\rm T} \end{pmatrix} \frac{1}{v_{23}^2 - M_{\rm VL}^2}$$

$$= \left[m_{D_L} m_{D_L}^{\rm T} v_{23} - m_{D_L} m_{D_R}^{\rm T} M_{\rm VL} - m_{D_R} m_{D_L}^{\rm T} M_{\rm VL} + m_{D_R} m_{D_R}^{\rm T} v_{23} \right] \frac{1}{v_{23}^2 - M_{\rm VL}^2} .$$

Neutrinos: Example of seesaw mechanism

Provided that $m_D \ll M_N$, we can apply the seesaw formula

$$m_\nu = m_D M_N^{-1} m_D^{\mathrm{T}} = \begin{pmatrix} m_{D_L} & m_{D_R} \end{pmatrix} \begin{pmatrix} v_{23} & -M_{\mathrm{VL}} \\ -M_{\mathrm{VL}} & v_{23} \end{pmatrix} \begin{pmatrix} m_{D_L}^{\mathrm{T}} \\ m_{D_R}^{\mathrm{T}} \end{pmatrix} \frac{1}{v_{23}^2 - M_{\mathrm{VL}}^2}$$

$$= \left[m_{D_L} m_{D_L}^{\mathrm{T}} v_{23} - m_{D_L} m_{D_R}^{\mathrm{T}} M_{\mathrm{VL}} - m_{D_R} m_{D_L}^{\mathrm{T}} M_{\mathrm{VL}} + m_{D_R} m_{D_R}^{\mathrm{T}} v_{23} \right] \frac{1}{v_{23}^2 - M_{\mathrm{VL}}^2} \, .$$

- Contributions proportional to M_{VL} do not provide a successful m_ν

$$m_{D_L} m_{D_R}^{\mathrm{T}} M_{\mathrm{VL}} + m_{D_R} m_{D_L}^{\mathrm{T}} M_{\mathrm{VL}} = \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ \times & \times & \times \end{pmatrix} M_{\mathrm{VL}} \, ,$$

$$m_{D_R} m_{D_R}^{\mathrm{T}} v_{23} + m_{D_L} m_{D_L}^{\mathrm{T}} v_{23} = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix} v_{23} \, .$$

Neutrinos: Example of seesaw mechanism

Provided that $m_D \ll M_N$, we can apply the seesaw formula

$$m_\nu = m_D M_N^{-1} m_D^{\mathrm{T}} = \begin{pmatrix} m_{D_L} & m_{D_R} \end{pmatrix} \begin{pmatrix} v_{23} & -M_{\mathrm{VL}} \\ -M_{\mathrm{VL}} & v_{23} \end{pmatrix} \begin{pmatrix} m_{D_L}^{\mathrm{T}} \\ m_{D_R}^{\mathrm{T}} \end{pmatrix} \frac{1}{v_{23}^2 - M_{\mathrm{VL}}^2}$$

$$= \left[m_{D_L} m_{D_L}^{\mathrm{T}} v_{23} - m_{D_L} m_{D_R}^{\mathrm{T}} M_{\mathrm{VL}} - m_{D_R} m_{D_L}^{\mathrm{T}} M_{\mathrm{VL}} + m_{D_R} m_{D_R}^{\mathrm{T}} v_{23} \right] \frac{1}{v_{23}^2 - M_{\mathrm{VL}}^2} \,.$$

- Contributions proportional to M_{VL} do not provide a successful m_ν

$$m_{D_L} m_{D_R}^{\mathrm{T}} M_{\mathrm{VL}} + m_{D_R} m_{D_L}^{\mathrm{T}} M_{\mathrm{VL}} = \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ \times & \times & \times \end{pmatrix} M_{\mathrm{VL}} \,,$$

$$m_{D_R} m_{D_R}^{\mathrm{T}} v_{23} + m_{D_L} m_{D_L}^{\mathrm{T}} v_{23} = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix} v_{23} \,.$$

$M_{\mathrm{VL}} \lesssim v_{23}$ required to describe PMNS mixing $\Rightarrow$ SM singlet neutrinos with mass at the lower scale v_{23}

Neutrinos: Example of seesaw mechanism

Provided that $m_D \ll M_N$, we can apply the seesaw formula

$$m_\nu = m_D M_N^{-1} m_D^{\mathrm{T}} = \begin{pmatrix} m_{D_L} & m_{D_R} \end{pmatrix} \begin{pmatrix} v_{23} & -M_{\mathrm{VL}} \\ -M_{\mathrm{VL}} & v_{23} \end{pmatrix} \begin{pmatrix} m_{D_L}^{\mathrm{T}} \\ m_{D_R}^{\mathrm{T}} \end{pmatrix} \frac{1}{v_{23}^2 - M_{\mathrm{VL}}^2}$$

$$= \left[m_{D_L} m_{D_L}^{\mathrm{T}} v_{23} - m_{D_L} m_{D_R}^{\mathrm{T}} M_{\mathrm{VL}} - m_{D_R} m_{D_L}^{\mathrm{T}} M_{\mathrm{VL}} + m_{D_R} m_{D_R}^{\mathrm{T}} v_{23} \right] \frac{1}{v_{23}^2 - M_{\mathrm{VL}}^2} .$$

- $M_{\mathrm{VL}} \lesssim v_{23}$ required to describe oscillation data. For simplicity we consider $M_{\mathrm{VL}} \ll v_{23}$ (and $v_{23}/v_{12} \approx \lambda$ obtained in the charged fermion sector) and obtain (neglecting $\mathcal{O}(1)$ coefficients)

$$m_\nu \approx m_{D_R} m_{D_R}^{\mathrm{T}} v_{23}^{-1} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} v_{23} \frac{H_u H_u}{\Lambda_{\mathrm{atm}}^2} + \begin{pmatrix} 1 & 1 & \lambda \\ 1 & 1 & \lambda \\ \lambda & \lambda & \lambda^2 \end{pmatrix} v_{23} \frac{H_u H_u}{\lambda^2 \Lambda_{\mathrm{sol}}^2} .$$

Neutrinos: Example of seesaw mechanism

Provided that $m_D \ll M_N$, we can apply the seesaw formula

$$m_\nu = m_D M_N^{-1} m_D^{\mathrm{T}} = \begin{pmatrix} m_{D_L} & m_{D_R} \end{pmatrix} \begin{pmatrix} v_{23} & -M_{\mathrm{VL}} \\ -M_{\mathrm{VL}} & v_{23} \end{pmatrix} \begin{pmatrix} m_{D_L}^{\mathrm{T}} \\ m_{D_R}^{\mathrm{T}} \end{pmatrix} \frac{1}{v_{23}^2 - M_{\mathrm{VL}}^2}$$

$$= \left[m_{D_L} m_{D_L}^{\mathrm{T}} v_{23} - m_{D_L} m_{D_R}^{\mathrm{T}} M_{\mathrm{VL}} - m_{D_R} m_{D_L}^{\mathrm{T}} M_{\mathrm{VL}} + m_{D_R} m_{D_R}^{\mathrm{T}} v_{23} \right] \frac{1}{v_{23}^2 - M_{\mathrm{VL}}^2} .$$

- $M_{\mathrm{VL}} \lesssim v_{23}$ required to describe oscillation data. For simplicity we consider $M_{\mathrm{VL}} \ll v_{23}$ (and $v_{23}/v_{12} \approx \lambda$ obtained in the charged fermion sector) and obtain (neglecting $\mathcal{O}(1)$ coefficients)

$$m_\nu \approx m_{D_R} m_{D_R}^{\mathrm{T}} v_{23}^{-1} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} v_{23} \frac{H_u H_u}{\Lambda_{\mathrm{atm}}^2} + \begin{pmatrix} 1 & 1 & \lambda \\ 1 & 1 & \lambda \\ \lambda & \lambda & \lambda^2 \end{pmatrix} v_{23} \frac{H_u H_u}{\lambda^2 \Lambda_{\mathrm{sol}}^2} .$$

- If $\Lambda_{\mathrm{atm}}/\Lambda_{\mathrm{sol}} \simeq \lambda$ then (remember $\langle H_u \rangle = v_{\mathrm{SM}}/\sqrt{2}$)

$$\boxed{m_\nu \simeq \begin{pmatrix} 1 & 1 & \lambda \\ 1 & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix} v_{23} \frac{v_{\mathrm{SM}}^2}{\Lambda_{\mathrm{atm}}^2} ,}$$

scales $\Lambda_{\mathrm{atm}} \approx 10^6\,\mathrm{TeV}$ and $v_{23} \approx \mathcal{O}(1\,\mathrm{TeV})$ provide enough suppression for $m_\nu \approx \mathcal{O}(0.05\,\mathrm{eV})$. SM singlet neutrinos with mass at scale v_{23}.

Model 2: RH mixing suppressed

$$\phi_{\ell 23}^{(0,\frac{1}{2},-\frac{1}{2})}, \quad \phi_{q23}^{(0,-\frac{1}{6},\frac{1}{6})}, \quad \phi_{q13}^{(-\frac{1}{6},0,\frac{1}{6})}, \quad \phi_{d12}^{(-\frac{1}{6},-\frac{1}{3},\frac{1}{2})}, \quad \phi_{e12}^{(\frac{1}{4},-\frac{1}{4},0)}.$$

$$\mathcal{L} = \begin{pmatrix} Q_1 & Q_2 & Q_3 \end{pmatrix} \begin{pmatrix} \phi_{e12}^2\tilde{\phi}_{\ell 23} & \phi_{q13}\tilde{\phi}_{q23}\phi_{\ell 23} & \phi_{q13} \\ \phi_{e12}^2\tilde{\phi}_{q13}\phi_{q23}\phi_{\ell 23} & \phi_{\ell 23} & \phi_{q23} \\ \phi_{e12}^2\tilde{\phi}_{q13}\phi_{\ell 23} & \phi_{\ell 23}\tilde{\phi}_{q23} & 1 \end{pmatrix} \begin{pmatrix} u_1^c \\ u_2^c \\ u_3^c \end{pmatrix} H_u$$

$$+ \begin{pmatrix} Q_1 & Q_2 & Q_3 \end{pmatrix} \begin{pmatrix} \phi_{e12}^2\tilde{\phi}_{\ell 23} & \phi_{d12} & \phi_{q13} \\ \phi_{q13}^2\phi_{q23} & \tilde{\phi}_{\ell 23} & \phi_{q23} \\ \phi_{q13}^2 & \phi_{q23}^2 & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} H_d$$

$$+ \begin{pmatrix} L_1 & L_2 & L_3 \end{pmatrix} \begin{pmatrix} \tilde{\phi}_{e12}^2\tilde{\phi}_{\ell 23} & \phi_{e12}^2\tilde{\phi}_{\ell 23} & \phi_{e12}^2\phi_{\ell 23} \\ \tilde{\phi}_{e12}^4\tilde{\phi}_{\ell 23} & \tilde{\phi}_{\ell 23} & \phi_{\ell 23} \\ \tilde{\phi}_{e12}^4\tilde{\phi}_{\ell 23}^2 & \tilde{\phi}_{\ell 23}^2 & 1 \end{pmatrix} \begin{pmatrix} e_1^c \\ e_2^c \\ e_3^c \end{pmatrix} H_d\,.$$

$$\frac{\langle\phi_{\ell 23}\rangle}{\Lambda} = \frac{\langle\phi_{q13}\rangle}{\Lambda} \simeq \lambda^3\,, \quad \frac{\langle\phi_{q23}\rangle}{\Lambda} \simeq \lambda^2\,, \quad \frac{\langle\phi_{d12}\rangle}{\Lambda} \simeq \lambda^4\,, \quad \frac{\langle\phi_{e12}\rangle}{\Lambda} \simeq \lambda\,.$$

$$\mathcal{L} = \begin{pmatrix} u_1 & u_2 & u_3 \end{pmatrix} \begin{pmatrix} \lambda^5 & \lambda^8 & \lambda^3 \\ \lambda^{10} & \lambda^3 & \lambda^2 \\ \lambda^7 & \lambda^5 & 1 \end{pmatrix} \begin{pmatrix} u_1^c \\ u_2^c \\ u_3^c \end{pmatrix} \frac{v_{\rm SM}}{\sqrt{2}}$$

$$+ \begin{pmatrix} d_1 & d_2 & d_3 \end{pmatrix} \begin{pmatrix} \lambda^5 & \lambda^4 & \lambda^3 \\ \lambda^8 & \lambda^3 & \lambda^2 \\ \lambda^6 & \lambda^4 & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} \lambda^2 \frac{v_{\rm SM}}{\sqrt{2}}$$

$$+ \begin{pmatrix} e_1 & e_2 & e_3 \end{pmatrix} \begin{pmatrix} \lambda^5 & \lambda^5 & \lambda^5 \\ \lambda^7 & \lambda^3 & \lambda^3 \\ \lambda^{10} & \lambda^6 & 1 \end{pmatrix} \begin{pmatrix} e_1^c \\ e_2^c \\ e_3^c \end{pmatrix} \lambda^2 \frac{v_{\rm SM}}{\sqrt{2}}\,.$$