

# SUSY2023

17 - 21 July

University of Southampton  
Southampton, UK

The XXX International Conference on Supersymmetry  
and Unification of Fundamental Interactions (SUSY2023)

## Modulus stabilisers and modular invariant models

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21th July, 2023

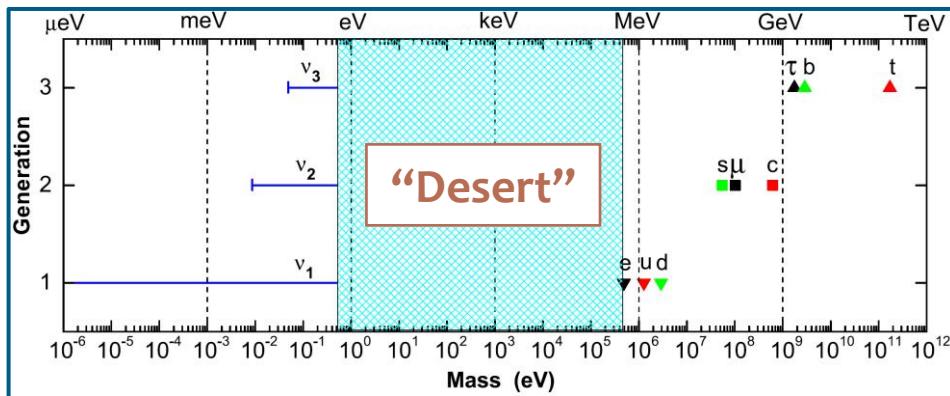


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# Motivation – Neutrino masses, flavour mixing, CP violation

Z. Z. Xing, Phys. Rept., 2020



→ Seesaw mechanism

Traditional discrete flavour symmetry:

Too many flavons; Vacuum alignments

Modular symmetry:

Feruglio, 1706.08749

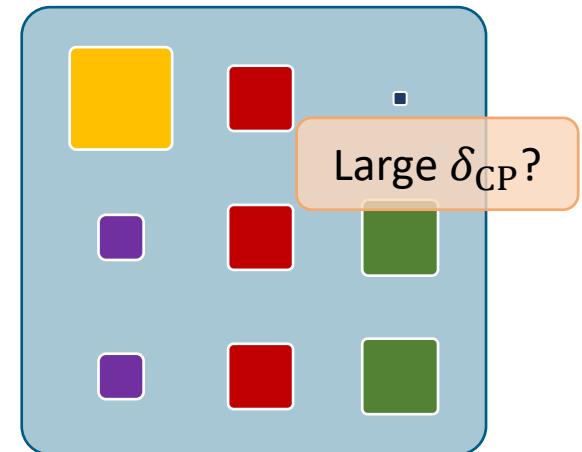
$$\Phi = \begin{pmatrix} \langle \Phi_1 \rangle \\ \langle \Phi_2 \rangle \\ \langle \Phi_3 \rangle \end{pmatrix}$$



$$\begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \\ Y_3(\tau) \end{pmatrix} = Y_r(\tau)$$

Flavon field

Modular form



PMNS matrix

Flavour symmetry

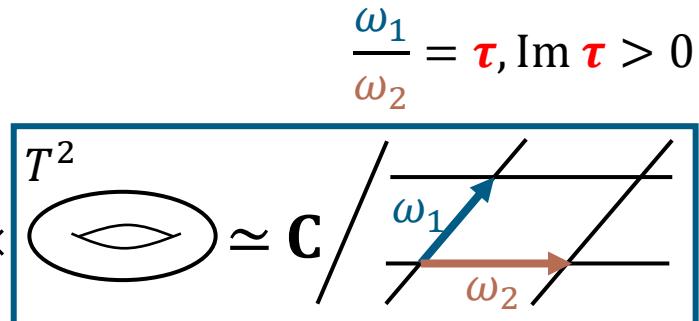
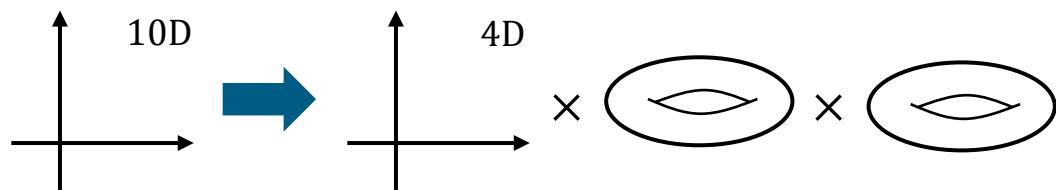
See also talks given by

Gui-jun Ding,  
Hans Peter Nilles,  
Ivo de Medeiros Varzielas,  
Miguel Levy

# Modular transformation

Dixon et al., NPB, 1987  
Hamidi & Vafa, NPB, 1987

## Orbifold compactification



## $SL(2, \mathbb{Z})$ transformation

$$\begin{pmatrix} \omega_1 \\ \omega_2 \end{pmatrix} \rightarrow \begin{pmatrix} \omega'_1 \\ \omega'_2 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \end{pmatrix}$$

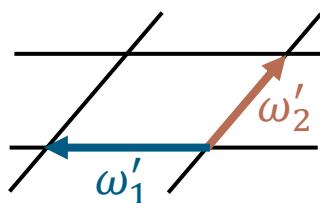
$a, b, c, d \in \mathbb{Z}, ad - bc = 1$



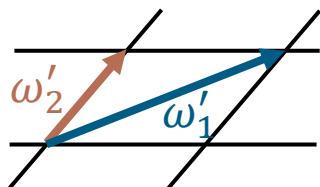
$$\tau \rightarrow \gamma\tau = \frac{\omega'_1}{\omega'_2} = \frac{a\tau + b}{c\tau + d}$$

$$\chi^{(I)} \rightarrow (c\tau + d)^{-k_I} \rho^{(I)}(\gamma) \chi^{(I)}$$

$$S: \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$



$$T: \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$



**Modular group:**  $\Gamma \simeq SL(2, \mathbb{Z})$



Quotient subgroup



Finite modular group  $\Gamma_N$

$$\Gamma_2^{(\prime)} \simeq \mathbf{S}_3^{(\prime)}, \Gamma_3^{(\prime)} \simeq \mathbf{A}_4^{(\prime)}, \Gamma_4^{(\prime)} \simeq \mathbf{S}_4^{(\prime)}, \Gamma_5^{(\prime)} \simeq \mathbf{A}_5^{(\prime)}$$

# Modular forms

Feruglio, 1706.08749

A holomorphic function transforming under modular group as

$$f(\gamma\tau) = (c\tau + d)^k f(\tau), \quad \gamma \in \Gamma_N$$

Weight ←      → Level

The number of linear independent modular forms in a modular space is finite

$$Y_{\mathbf{r}}^{(k)}(\gamma\tau) = (c\tau + d)^k \rho_{\mathbf{r}}(\gamma) Y_{\mathbf{r}}^{(k)}(\tau), \quad \gamma \in \Gamma_N$$

$$Y_{\mathbf{r}}^{(k)} = \begin{pmatrix} f_1^{(k)}(\tau) \\ f_2^{(k)}(\tau) \\ \vdots \\ f_r^{(k)}(\tau) \end{pmatrix}$$

$\mathcal{N} = 1$  MSSM

$$\mathcal{S} = \int d^4x d^2\theta d^2\bar{\theta} \mathcal{K}(\tau, \bar{\tau}, \chi, \bar{\chi}) + \int d^4x d^2\theta \boxed{\mathcal{W}(\tau, \chi)} + \int d^4x d^2\bar{\theta} \overline{\mathcal{W}}(\bar{\tau}, \bar{\chi})$$

Invariant superpotential  $\mathcal{W}(\tau, \chi) \rightarrow \mathcal{W}(\tau, \chi)$

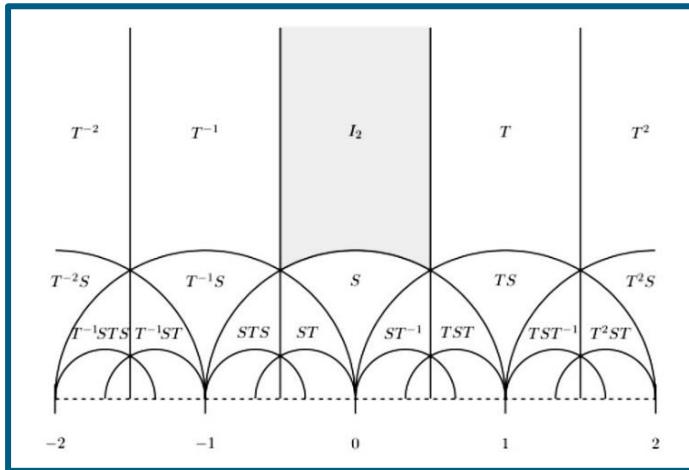
$$\mathcal{W}(\tau, \chi) = \sum_n \sum_{\{I_1, \dots, I_n\}} Y_{I_1 \dots I_n}(\tau) \chi^{(I_1)} \dots \chi^{(I_n)}$$

$$Y_{I_1 \dots I_n}(\tau) \rightarrow (c\tau + d)^{k_Y} \rho_Y(\gamma) Y_{I_1 \dots I_n}(\tau)$$

**Yukawa → Modular forms**

It is the modulus parameter  $\tau$  that determines flavour structure

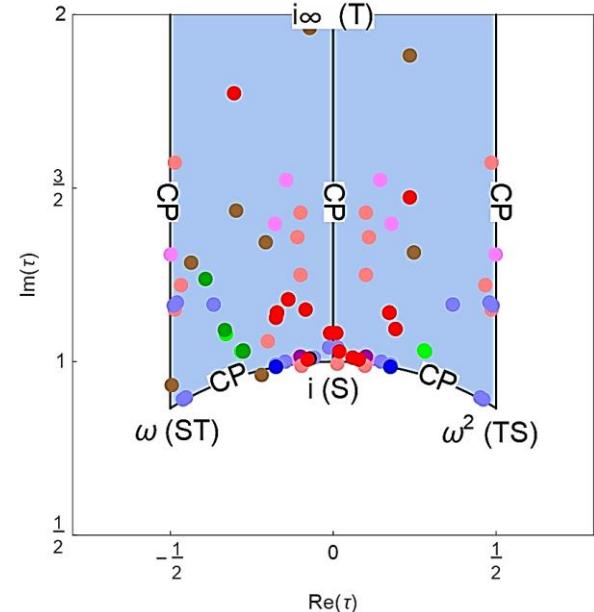
# What is the “correct value” of $\tau$ ?



## Fundamental domain

Enough to investigate the modulus parameter inside the fundamental domain

### - Single modulus case



Feruglio, PRL, 2023

### Bottom-up approach

Free parameter obtained by fitting the experimental data

### Top-down approach

Minimise the scalar potential

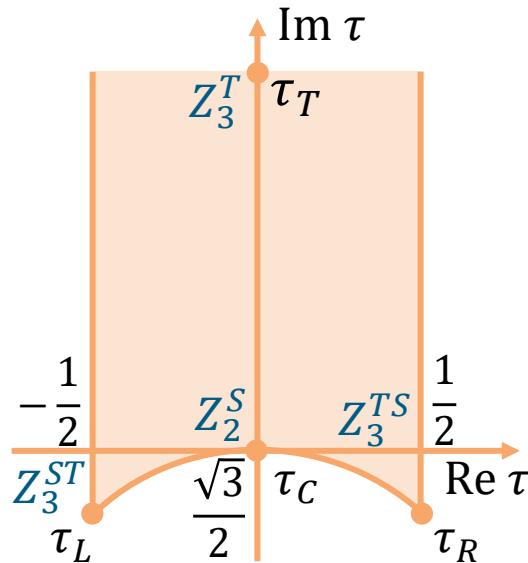
See e.g.,

Kobayashi et al., PRD, 2019

Ishiguro, Kobayashi, Otsuka, JHEP, 2021

Novichkov, Penedo, Petcov, JHEP, 2021

# Modulus stabiliser



**Stabilisers**  $\leftrightarrow$  **Residual symmetries**

$$\tau_L = -1/2 + \sqrt{3}/2i$$

$$\tau_R = +1/2 + \sqrt{3}/2i$$

$$\tau_C = i$$

$$\tau_T = i\infty$$

$$Z_3^{ST} = \{I, ST, (ST)^2\}$$

$$Z_3^{TS} = \{I, TS, (TS)^2\}$$

$$Z_2^S = \{I, S\}$$

$$Z_3^T = \{I, T, T^2\}$$

**Exactly at the stabilisers**

- Single modulus (w/o flavon) **X**
- Multiple moduli

See e.g.,

Ding, King, Liu, Lu, JHEP, 2019

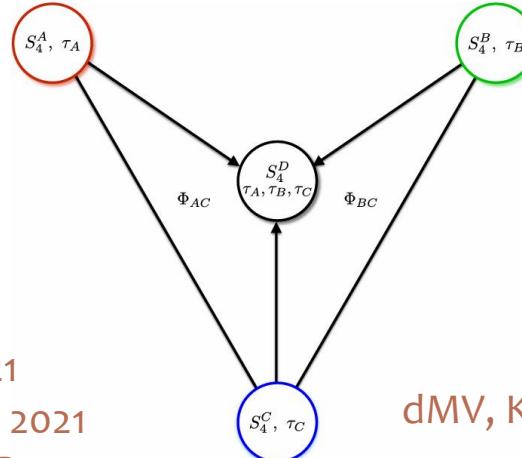
Novichkov et al., PLB, 2019

King, Zhou, PRD, 2020

dMV, Levy, Zhou, JHEP, 2020

Trivial flavour structure

King, Zhou, JHEP, 2021  
dMV, Lourenço, NPB, 2021  
dMV, King, Levy, JHEP, 2023



dMV, King, Zhou  
PRD, 2019

# Modulus stabiliser

If  $\tau$  slightly deviates from stabilisers...

- Viable lepton flavour models with only one modulus
- Series expansion with respect to small deviation parameter

$q$ -expansion:  $f(\tau) = \sum_{n=0}^{\infty} a_n q^{n/N}$     $q = e^{2\pi i \tau}$   
e.g.,

$$Y_3^{(1)}(\tau) = 1 + 12q + 36q^2 + 12q^3 + \dots$$

$$Y_3^{(2)}(\tau) = -6q^{1/3} (1 + 7q + 8q^2 + \dots)$$

$$Y_3^{(3)}(\tau) = -18q^{2/3} (1 + 2q + 5q^2 + \dots)$$

$$\tau = i : \quad s = \frac{\tau - i}{\tau + i}$$

$$\tau = \omega : \quad u = \frac{\tau - \omega}{\tau - \omega^2}$$

$$\tau = i\infty : \quad \epsilon = e^{-2\pi \text{Im}\tau/N}$$

- Generate mass hierarchy of charged leptons or quarks

Example:  $\Gamma'_5 @ \tau = i\infty$  Novichkov, Penedo, Petcov, JHEP, 2021

$$M_e = \begin{pmatrix} 1 & \epsilon^4 & \epsilon \\ \epsilon^3 & \epsilon^2 & \epsilon^4 \\ \epsilon^2 & \epsilon & \epsilon^3 \end{pmatrix}$$

$$(m_\tau, m_\mu, m_e) \sim (1, \epsilon, \epsilon^4)$$

See also

Okada, Tanimoto, PRD, 2021

Feruglio et al., JHEP, 2021

Petcov, Tanimoto, EPJC, 2022

Feruglio, JHEP, 2023

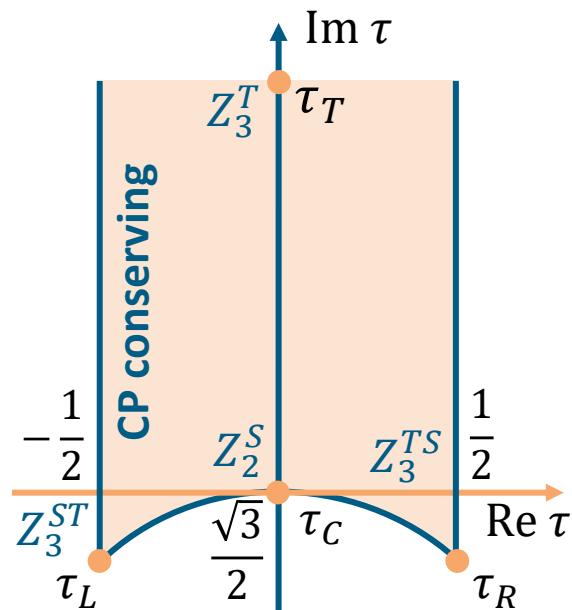
# CP violation near stabilisers

## Generalised CP (gCP) symmetry

$$\begin{aligned}\chi^{(I)}(x) &\xrightarrow{\text{CP}} X_r \bar{\chi}^{(I)}(x_P) \\ \tau &\xrightarrow{\text{CP}} -\tau^* \\ Y_r^{(k)}(\tau) &\xrightarrow{\text{CP}} Y_r^{(k)}(-\tau^*) = [Y_r^{(k)}(\tau)]^*\end{aligned}$$

- All coupling constants should be real
- Modulus  $\tau$  is the only source of CP violation!

Novichkov et al., JHEP, 2019  
Baul et al., PLB, 2019



How does the CP violating phase change if  $\tau$  slightly deviates from stabilisers?

Slowly going to nonzero?  
- Not always the case!

# CP violation near stabilisers

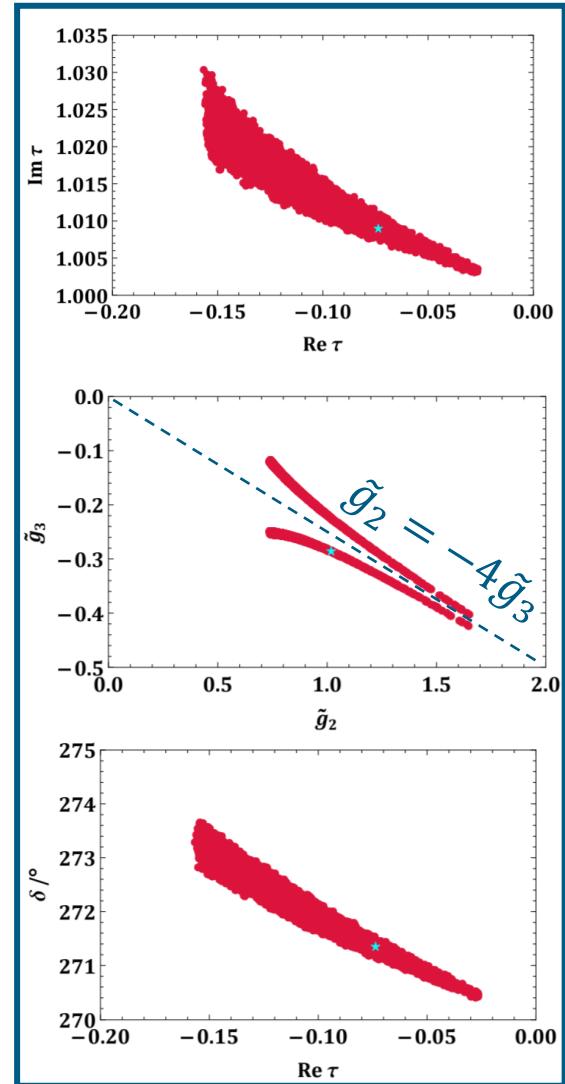
XW & S. Zhou  
JHEP 07 (2021) 093

## A modular $A'_5$ Model – Model building

$$M_l = \frac{v_d}{\sqrt{2}} \begin{pmatrix} \xi_1 & 0 & \xi_2 \left[ \frac{\sqrt{6}}{6} \left( Y_{\mathbf{3}}^{(4)} \right)_1 - \frac{\sqrt{2}}{2} \tilde{\xi} Y_{\mathbf{1}}^{(4)} \right] \\ 0 & \frac{\sqrt{3}}{3} \xi_2 \left( Y_{\mathbf{3}}^{(4)} \right)_2 & -\frac{\sqrt{3}}{3} \xi_2 \left( Y_{\mathbf{3}}^{(4)} \right)_3 \\ 0 & \xi_2 \left[ \frac{\sqrt{6}}{6} \left( Y_{\mathbf{3}}^{(4)} \right)_1 + \frac{\sqrt{2}}{2} \tilde{\xi} Y_{\mathbf{1}}^{(4)} \right] & 0 \end{pmatrix}^*$$

$$M_D = \frac{\sqrt{6} g_1 v_u}{12} \begin{pmatrix} 2 \left( Y_{\mathbf{3}'}^{(2)} \right)_1 & 2 \left( Y_{\mathbf{3}'}^{(2)} \right)_3 & 2 \left( Y_{\mathbf{3}'}^{(2)} \right)_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \tilde{g}_2 \begin{pmatrix} 0 & 0 & 0 \\ -\sqrt{2} \left( Y_{\widehat{\mathbf{6}},1}^{(3)} \right)_4 & -\sqrt{2} \left( Y_{\widehat{\mathbf{6}},1}^{(3)} \right)_2 & \left( Y_{\widehat{\mathbf{6}},1}^{(3)} \right)_1 - \left( Y_{\widehat{\mathbf{6}},1}^{(3)} \right)_6 \\ -\sqrt{2} \left( Y_{\widehat{\mathbf{6}},1}^{(3)} \right)_3 & \left( Y_{\widehat{\mathbf{6}},1}^{(3)} \right)_1 + \left( Y_{\widehat{\mathbf{6}},1}^{(3)} \right)_6 & -\sqrt{2} \left( Y_{\widehat{\mathbf{6}},1}^{(3)} \right)_5 \end{pmatrix}^* + \tilde{g}_3 \begin{pmatrix} 0 & 0 & 0 \\ -\sqrt{2} \left( Y_{\widehat{\mathbf{6}},2}^{(3)} \right)_4 & -\sqrt{2} \left( Y_{\widehat{\mathbf{6}},2}^{(3)} \right)_2 & \left( Y_{\widehat{\mathbf{6}},2}^{(3)} \right)_1 - \left( Y_{\widehat{\mathbf{6}},2}^{(3)} \right)_6 \\ -\sqrt{2} \left( Y_{\widehat{\mathbf{6}},2}^{(3)} \right)_3 & \left( Y_{\widehat{\mathbf{6}},2}^{(3)} \right)_1 + \left( Y_{\widehat{\mathbf{6}},2}^{(3)} \right)_6 & -\sqrt{2} \left( Y_{\widehat{\mathbf{6}},2}^{(3)} \right)_5 \end{pmatrix}^*$$

$$M_R = \frac{\Lambda}{4} \begin{pmatrix} 2 \left( Y_{\mathbf{5}}^{(2)} \right)_1 & -\sqrt{3} \left( Y_{\mathbf{5}}^{(2)} \right)_4 & -\sqrt{3} \left( Y_{\mathbf{5}}^{(2)} \right)_3 \\ -\sqrt{3} \left( Y_{\mathbf{5}}^{(2)} \right)_4 & \sqrt{6} \left( Y_{\mathbf{5}}^{(2)} \right)_2 & -\left( Y_{\mathbf{5}}^{(2)} \right)_1 \\ -\sqrt{3} \left( Y_{\mathbf{5}}^{(2)} \right)_3 & -\left( Y_{\mathbf{5}}^{(2)} \right)_1 & \sqrt{6} \left( Y_{\mathbf{5}}^{(2)} \right)_5 \end{pmatrix}^*$$



# CP violation near stabilisers

XW & S. Zhou  
JHEP 07 (2021) 093

## A modular $A'_5$ Model – Analytical results

- Assume  $\hat{g}_2 = -4\hat{g}_3$  holds, introduce perturbation  $\tau = \mathbf{i} + \epsilon$
- Break the above identity by requiring  $\hat{g}_3 = -\hat{g}_2/4 + \kappa$

### Neutrino masses

$$m_1^2 \approx m_{1,0}^2 - 0.108 \mu_0^2 \hat{g}_2^2 \epsilon_I |\kappa|$$

$$m_2^2 \approx m_{2,0}^2 + 0.108 \mu_0^2 \hat{g}_2^2 \epsilon_I |\kappa|$$

$$m_3^2 \approx (1.583 - 3.971 \epsilon_R - 14.42 \epsilon_I + 35.91 \epsilon_R^2) \hat{g}_2^4 \epsilon_R^2 \mu_0^2$$

### Flavour mixing angles

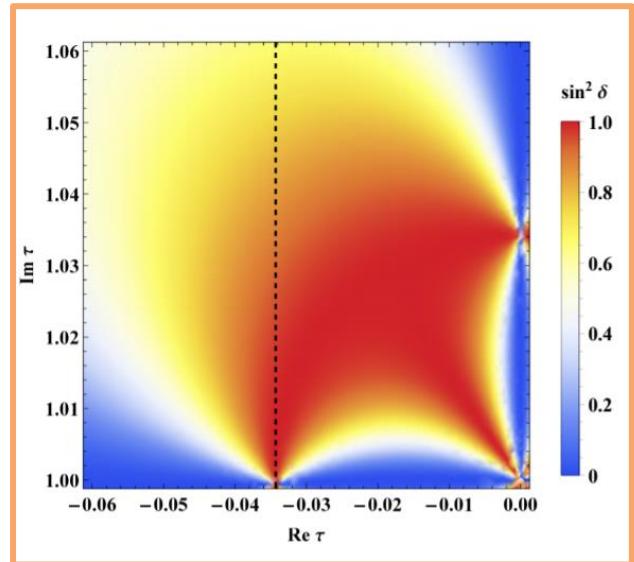
$$\sin \theta_{23} \approx \sqrt{2}/2 + 0.212 \epsilon_R$$

$$\sin \theta_{13} \approx \frac{\mu_0^2 |\kappa| (0.00761 - 0.108 \hat{g}_2^2 \epsilon_R)}{m_{3,0}^2 - m_{1,0}^2}$$

$$\sin^2 \theta_{12} \approx \frac{0.108 \mu_0^2 \hat{g}_2^2 \epsilon_I |\kappa|}{m_{2,0}^2 - m_{1,0}^2 + 0.216 \mu_0^2 \hat{g}_2^2 \epsilon_I |\kappa|}$$

### Leptonic CP violation

$$\delta \approx 2\pi - \arctan \left( \frac{0.0705 - \hat{g}_2^2 \epsilon_R}{\hat{g}_2^2 \epsilon_I} \right)$$



# CP violation near stabilisers

XW & S. Zhou  
JHEP 07 (2021) 093

## A modular $A'_5$ Model – RG running



High-energy scale

RG running

Physical  
Observable

Low-energy scale

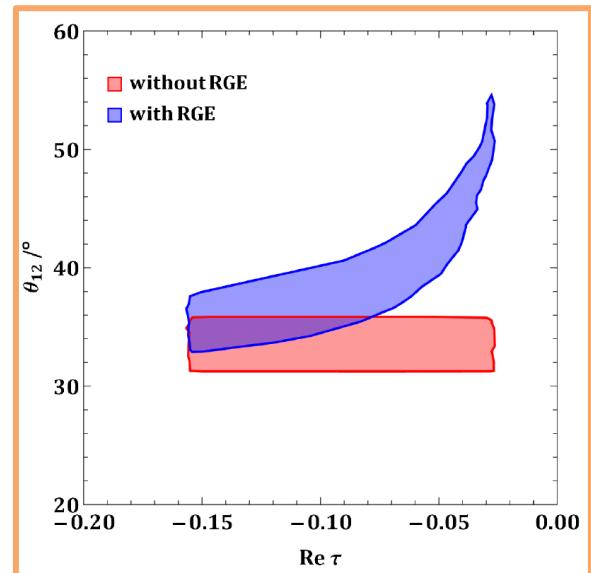
Consider one-loop RG running equation

$$\begin{aligned} 16\pi^2 \frac{d\tilde{Y}_l}{dt} &= \left[ \alpha_l + 3 \left( \tilde{Y}_l \tilde{Y}_l^\dagger \right) \right] \tilde{Y}_l \\ 16\pi^2 \frac{d\mathcal{M}}{dt} &= \alpha_\nu \mathcal{M} + \left[ \left( \tilde{Y}_l \tilde{Y}_l^\dagger \right) \mathcal{M} + \mathcal{M} \left( \tilde{Y}_l \tilde{Y}_l^\dagger \right)^T \right] \end{aligned}$$

Neutrino masses and  $\theta_{12}$  at  $m_Z$  scale

$$\begin{aligned} m_1^2(m_Z) &\approx m_{1,0}^2 - 0.108 I_\tau \mu_0^2 \hat{g}_2^2 \epsilon_I |\kappa| \\ m_2^2(m_Z) &\approx I_\tau^2 m_{2,0}^2 + 0.108 I_\tau \mu_0^2 \hat{g}_2^2 \epsilon_I |\kappa| \\ m_3^2(m_Z) &\approx I_\tau^2 m_{3,0}^2 \\ \sin^2 \theta_{12}(m_Z) &\approx \frac{0.108 I_\tau \mu_0^2 \hat{g}_2^2 \epsilon_I |\kappa|}{I_\tau^2 m_{2,0}^2 - m_{1,0}^2 + 0.216 I_\tau \mu_0^2 \hat{g}_2^2 \epsilon_I |\kappa|} \end{aligned}$$

Machacek and Vaughn, NPB, 1984  
Arason et al., PRD, 1992  
Antusch et al., PLB, 2001, ...



# Modulus stabilisation

## Dynamic origin of the modulus parameter

$\tau$   The vev of modulus field

## Several effects for modulus stabilisation

- Non-Perturbative effects: gaugino condensation, wrapped Euclidean branes,...
- Flux compactification
- Localised objects
- Extra dimensional curvature
- Supersymmetry breaking
- ...

Cicoli, Conlon, Maharana, Parameswaran, Quevedo, Zavala, 2303.04819

# Modulus stabilisation

## $\mathcal{N} = 1$ supergravity model

Kähler function is invariant under modular transformation

$$G(\tau, \bar{\tau}) = \kappa^2 K(\tau, \bar{\tau}) + \log |\kappa^3 W(\tau)|^2$$
$$\mathfrak{n} = \kappa^2 \Lambda_K^2$$

Minimal Kähler potential

$$K(\tau, \bar{\tau}) = -\Lambda_K^2 \log(2 \operatorname{Im} \tau)$$

Superpotential with  $k \neq 0$

$$W(\gamma\tau) = (c\tau + d)^{-\mathfrak{n}} W(\tau)$$

Minimise the scalar potential

$$V = e^{\kappa^2 K} \left( K^{i\bar{j}} D_i W D_{\bar{j}} W^* - 3\kappa^2 |W|^2 \right)$$

Font, Ibáñez, et al., PLB, 1990

Ferrara, Magnoli, et al., PLB, 1990

Binetruy and Galliard, PLB, 1991

Leedom, Righi, Westphal, JHEP, 2023

## Gaugino condensation

$$W \sim \langle \lambda_a \lambda_a \rangle = e^{-f_a/b_a}$$

$$f_a = k_a S + \left( b'_a - \frac{1}{3} k_a \delta_{GS} \right) \ln \eta^6(T) + \dots$$

Tree level:  $f_a = k_a S$



One-loop level: threshold contribution  
and anomaly cancellation

$$W(S, T) = \frac{\Omega(S) H(T)}{\eta^6(T)}$$

$$H(\tau) = (j(\tau) - 1728)^{m/2} j(\tau)^{n/3} P(j(\tau))$$

# Modulus stabilisation

## Single modulus case

Cvetič, Font et al., PLB, 1991

Gonzalo, Ibáñez, Uranga, JHEP, 2018

Novichkov, Penedo, Petcov, JHEP, 2022

The scalar potential (Assuming the dilaton sector decoupled)

$$V(\tau, \bar{\tau}) = \frac{\Lambda_V^4}{8(\text{Im } \tau)^3 |\eta|^{12}} \left[ \frac{4}{3} \left| iH' + \frac{3}{2\pi} H \hat{G}_2 \right|^2 (\text{Im } \tau)^2 - 3|H|^2 \right]$$

At first glance

$V$  – weight zero  
modular function



$\partial V / \partial \tau$  – modular form  
with weight two

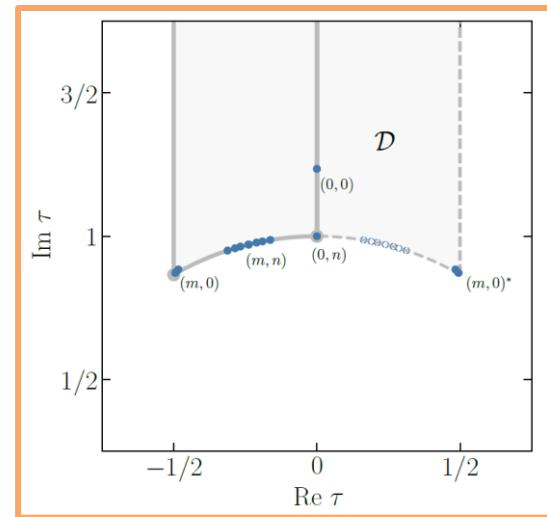


$\frac{\partial V}{\partial \tau} \Big|_{\tau=i} = 0, \frac{\partial V}{\partial \tau} \Big|_{\tau=\omega} = 0$

$\tau = i, \omega$  should be extrema

Numerical calculation

$$H(\tau) = (j(\tau) - 1728)^{m/2} j(\tau)^{n/3} P(j(\tau))$$



Novichkov, Penedo, Petcov, JHEP, 2022

# Modulus stabilisation

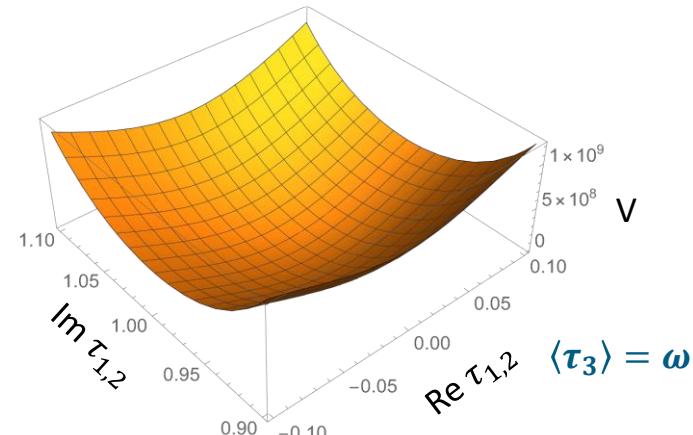
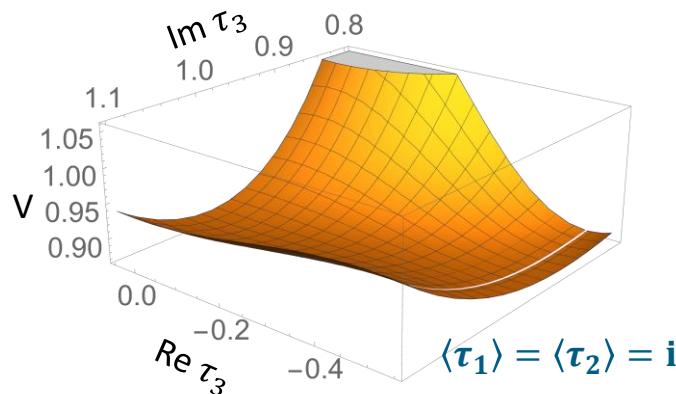
## Triple moduli case

XW & S. King, work in progress

- Extend  $H(\tau)$  to a general form  $\rightarrow \sum H_1^{r_1}(\tau_1)H_2^{r_2}(\tau_2)H_3^{r_3}(\tau_3)$
- Lift the vacuum to dS vacuum by modifying dilaton sector

### A benchmark example

$$H_1(\tau_1) + H_2(\tau_2) + H_3(\tau_3) \quad \text{with } m_{1,2} = 2, m_3 = n_{1,2,3} = 0 \quad \xrightarrow{\text{Vacuum}} \langle\tau_1\rangle = \langle\tau_2\rangle = i \quad \langle\tau_3\rangle = \omega$$



Lead to viable scenarios

TM<sub>1</sub> mixing ✓

dMV, King, Zhou, PRD, 2019

Littlest modular seesaw ✓

dMV, King, Levy  
JHEP, 2023

# Summary

- Modulus stabilisers are very interesting in phenomenological study.
- Modulus stabilisers might not be “stable”.
- Realistic superstring models have a mild preference for finite stabilisers.

*Thanks for your attention!*