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Higher-Form Symmetries in (Non-)Compact M-/F-Theory Constructions

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Motivation

Studying M-/string theory on special holonomy spaces X:

- Non-compact spaces X → Geometric engineering of supersymmetric quantum field theories (SQFTs):

- Build dictionary:{operators, symmetries} ⟺ {geometry,topology}

- Focus: higher-form global symmetries ⟺ topology (associated with ``flavor'' branes)

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c.f., plenary talks by Ignatios Antoniadis, Eran Palti…

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Goals

- Identify geometric origin of higher-form symmetries (0-form, 1-form & 2-group) for M-/string theory on non-compact & compact special holonomy spaces

  Punchline:

  - Higher form symmetries via cutting & gluing of singular boundary of non-compact X^{loc}
  - The fate of higher-form symmetries (gauged or broken) via gluing of X^{loc} to compact X

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- Highlight: M-theory on elliptically fibered Calabi-Yau (CY) manifolds, dual to F-theory

  - Confront results with those, obtained via resolutions & arithmetic properties of elliptic curves

Based on

non-compact geometries:

- M. C., J. J. Heckman, M. Hübner and E. Torres: "0-Form, 1-Form and 2-Group Symmetries via Cutting and Gluing of Orbifolds," 2203.10102

& compact geometries:

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Outline

- Introduction: Defect Group in geom. engineering

- Focus: Defect group and higher-form symmetries $\rightarrow$ Topology of flavor symmetry group

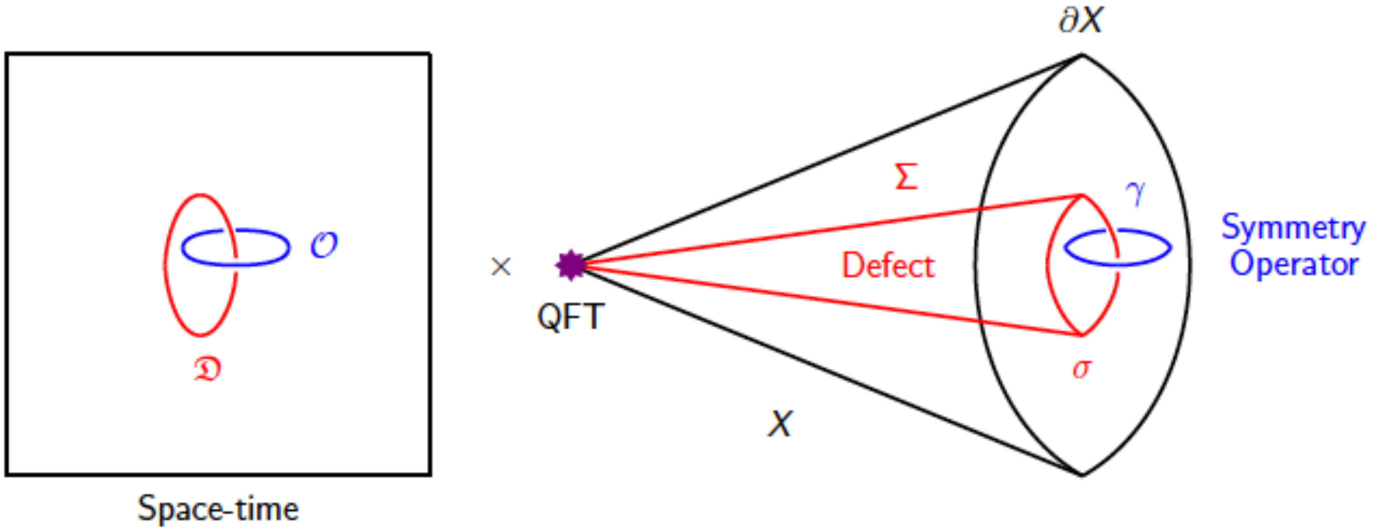
- Compact (elliptic) examples $\rightarrow$ fate of higher-form symmetries

- Concluding remarks

I. Defect group for M-theory on non-compact X

- Defect Group for extended p-dim operators associated with M2 and M5 branes: $\mathcal{D}_p = \mathcal{D}_p^{\mathrm{M2}} \oplus \mathcal{D}_p^{\mathrm{M5}}$

- Schematically:



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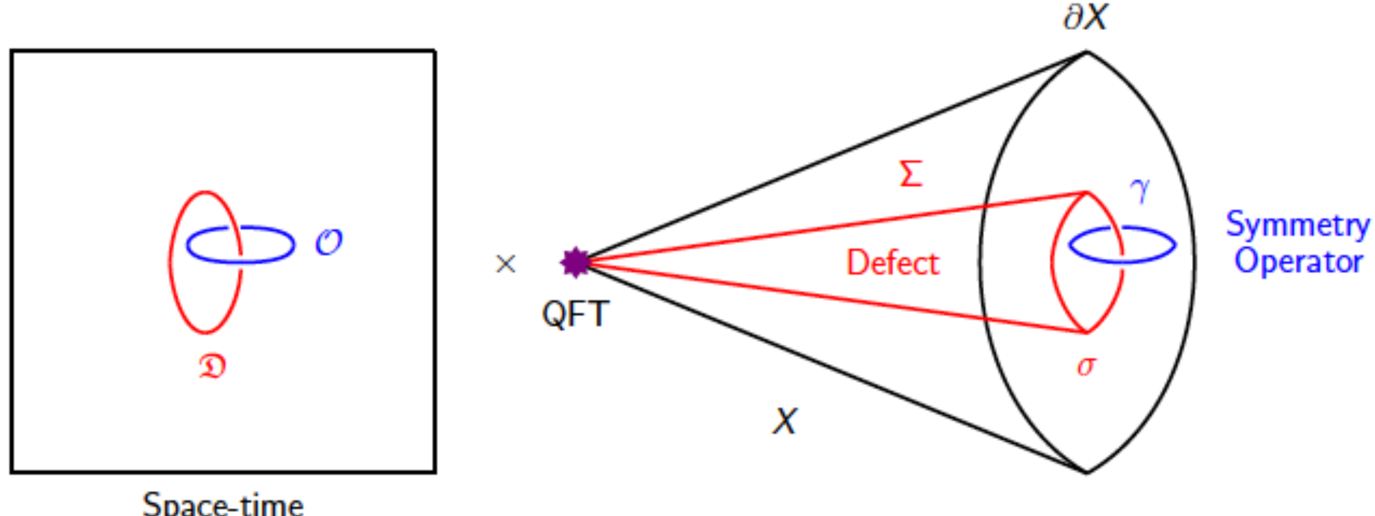

- M2, M5 in X wrapping relative cycles:

[Morrison, Schäfer-Nameki, Willett, 2020], [Albertini, Del Zotto, Garcia Etxebarria, Hosseini, 2020], [Del Zotto, Heckman, Park, Rudelius, 2015]

$$\mathcal{D}_p^{\mathrm{M2}} = \frac{H_{3-p}(X,\partial X)}{H_{3-p}(X)} \cong H_{3-p-1}(\partial X)|_{\mathrm{triv}}$$

[p-dim el. operators in SCFT]

$$\mathcal{D}_p^{\mathrm{M5}} = \frac{H_{6-p}(X,\partial X)}{H_{6-p}(X)} \cong H_{6-p-1}(\partial X)|_{\mathrm{triv}}$$

[p-dim mag. operators in SCFT]

[focus on torsional]

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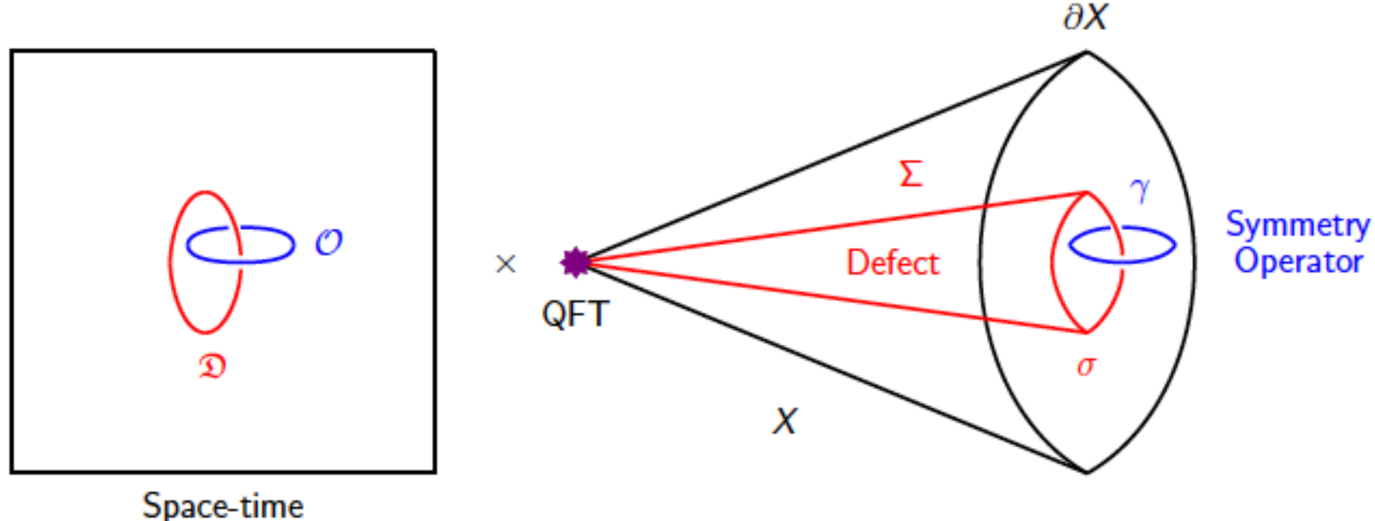

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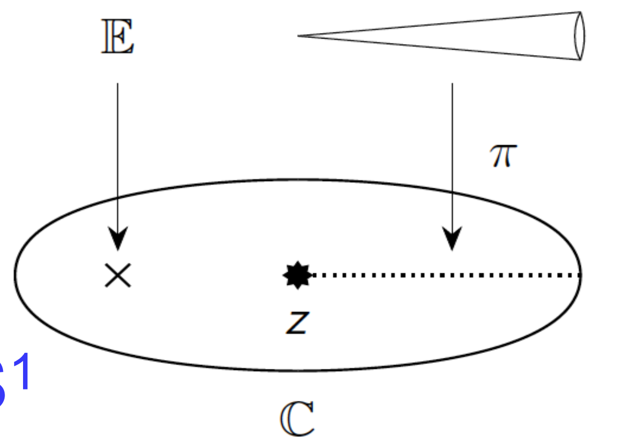
- Focus on defect ops.; symmetry ops., c.f., [Heckman, Hübner, Torres, Zhang 2022,..; M.C., Heckman, Hübner, Torres, Zhang 2023]

Example: non-compact *K3*

[M.C., Dierigl, Lin, Zhang, 2021, 2022]

- Local elliptically fibered *K3* $\mathbb{E} \hookrightarrow X \rightarrow \mathbb{C}$
- Singular fiber of Kodaira type ϕ at $z \in \mathbb{C}$ w/ monodromy M



[torsional two-cycle]

- Boundary $\mathbb{E} \hookrightarrow \partial X \rightarrow S^1$
- Exact sequence for spaces fibered over circles:

$$0 \rightarrow \mathrm{coker}\,(M_n - 1) \rightarrow H_n(X) \rightarrow \ker\,(M_{n-1} - 1) \rightarrow 0$$

M_n - monodromy in homology in degree n

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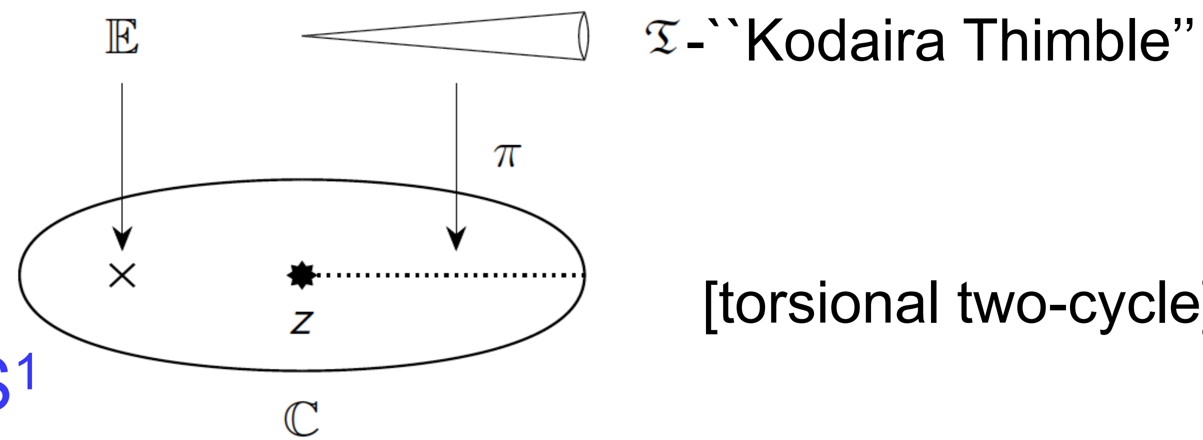

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$$\mathcal{D}_1^{M2} = \mathcal{D}_4^{M5} = H_2(X, \partial X)/H_2(X) \cong \mathrm{Tor\,Coker}(M-1) = \langle \mathfrak{T} \rangle$$

- X engineers 7D SYM with gauge algebra g_ϕ w/ Defect group $\mathcal{D} = \langle \mathfrak{T} \rangle_1^{M2} \oplus \langle \mathfrak{T} \rangle_4^{M5}$

Example (continued):

- Non-trivial self-linking/intersection: $\ell(\partial\mathfrak{T}, \partial\mathfrak{T}) = \mathfrak{T} \cdot \mathfrak{T} \neq 0$
- Elements of $\mathcal{D}_1^{M2}$, $\mathcal{D}_4^{M5}$ typically mutually non-local
- Choose electric polarization $\mathcal{D}_1^{M2}$ [main focus of the talk]

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- Gauge group is simply connected G_ϕ w/algebra g_ϕ (ADE)

- Resulting 7D SYM theory w/ gauge group G_ϕ

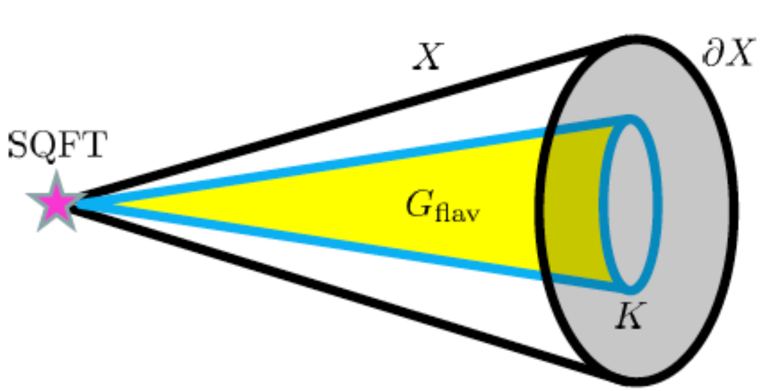
- (Wilson) Line operators $\mathcal{D}_1^{M2}$ acted on by 1-form symmetry $Z_{G\phi}$ fixes group topology [Pontyagin dual to line operators]

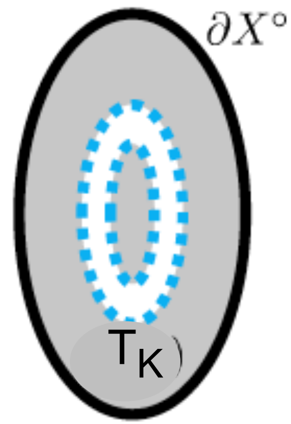
Now, turn to higher-form structures for non-compact spaces X in higher dimensions (D $\geq$ 6)

→ leads to new phenomena

II. Boundary geometry of flavor branes

- Singular non-compact space X w/ $K = \cup_i K_i$ - ADE flavor in boundary ∂X w/ naïve flavor symmetry $\widetilde{G}_F$ w/ simply connected ADE algebra $\mathfrak{g}_i$
- Smooth boundary $\partial X^\circ = \partial X \setminus K$ & a tubular region T_K (excise K)





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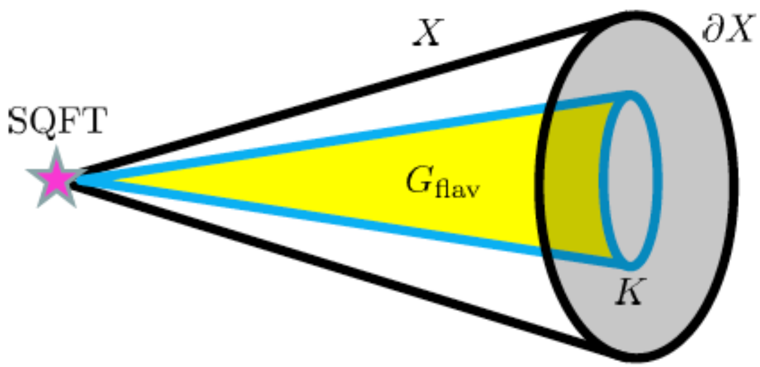

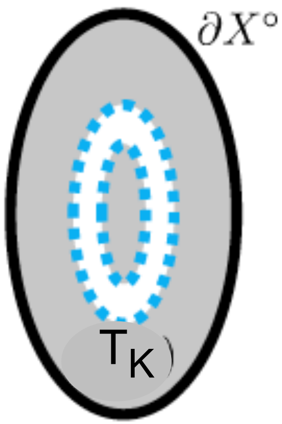

$\partial X = \partial X^\circ \cup T_K$

- True flavor&1-form: Mayer-Vietoris sequence for singular boundary

true flavor center naïve flavor center

$Z_{\text{0-form}}$: $$0 \to \ker(\iota_1) \to H_1(\partial X^\circ \cap T_K) \xrightarrow{\iota_1} \frac{H_1(\partial X^\circ \cap T_K)}{\ker(\iota_1)} \to 0\,,$$

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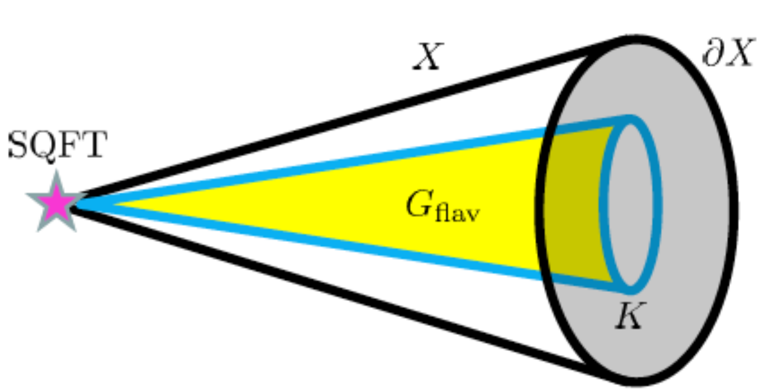

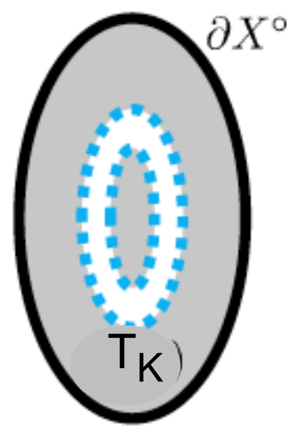

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Which can be collapse into:

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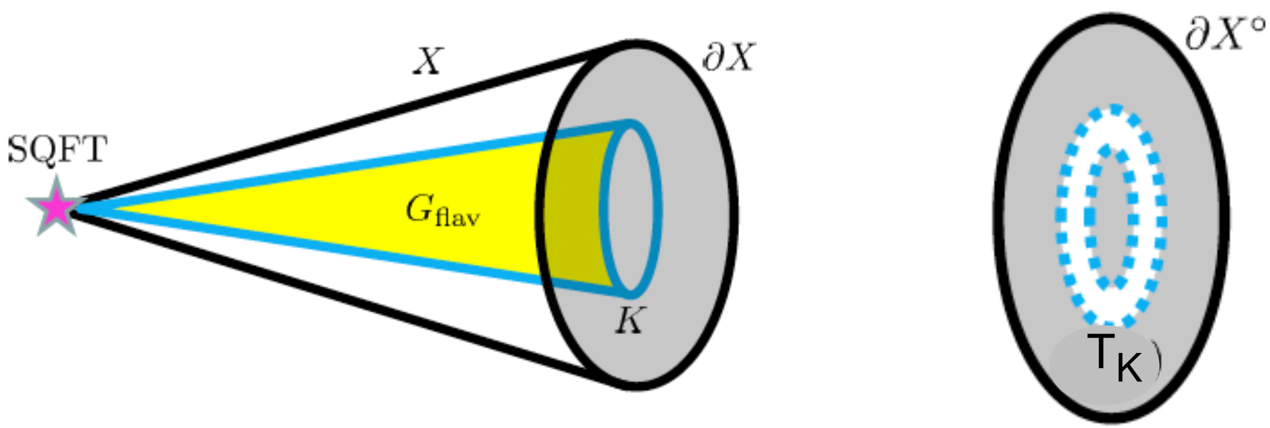

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$$0 \to \underbrace{\ker(\iota_1)}_{\text{true flavor center}} \to \underbrace{H_1(\partial X^\circ \cap T_K)}_{\text{naïve flavor center}} \longrightarrow$$

Which can be collapse into single exact sequence, relating 0-form and 1-form symmetries

$$\to \underbrace{H_1(\partial X^\circ) \oplus H_1(T_K)}_{\text{naïve 1-form}} \to \underbrace{H_1(\partial X)}_{\text{true 1-form}} \to 0$$

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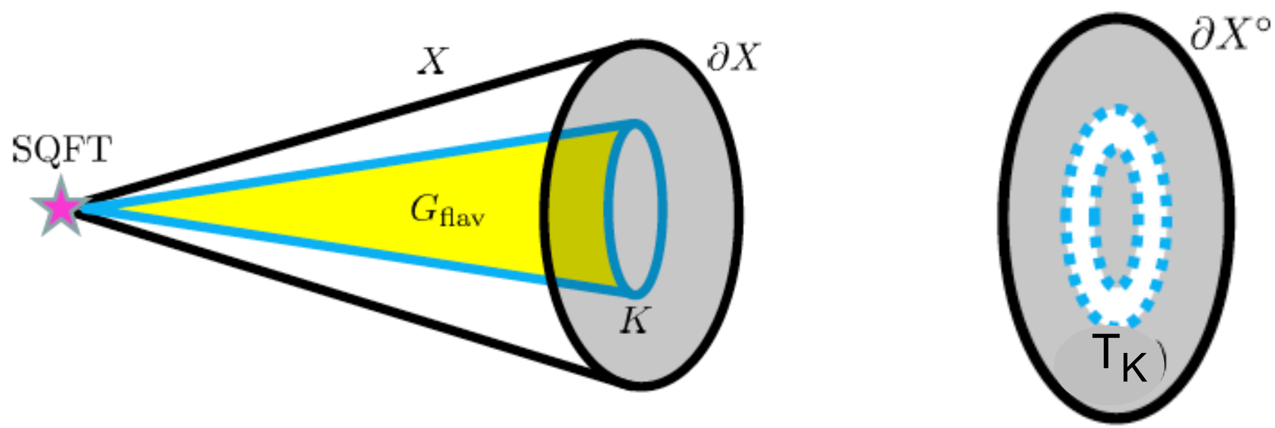

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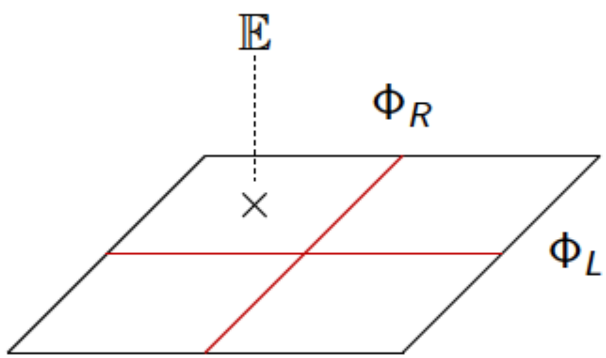
- When the bottom sequence does not split → 0-form and 1-form intertwined leading to 2-group!

[Benini, Cordova, Hsin, 2019]
[Lee, Ohmori, Tachikawa, 2021]

[Del Zotto, Heckman, Tomasiello, Vafa, 2014]

Example: Elliptically fibered threefold $\mathbb{E} \hookrightarrow X_3 \to B = \mathbb{C}^2$

w/non-compact base



- Two non-compact discriminant loci: ϕ_L on $\mathbb{C}$ x {0} & ϕ_R on {0} x $\mathbb{C}$

  two monodromies $M_{L,}$ & M_R ; two linking circles $S^1{}_L$ & $S^1{}_R$ on the boundary

- Again, exact spectral sequences for spaces fibered over circles $S^1{}_{L\ R}$ in terms of $M_{L,R}$

[Del Zotto, Heckman, Tomasiello, Vafa, 2014]

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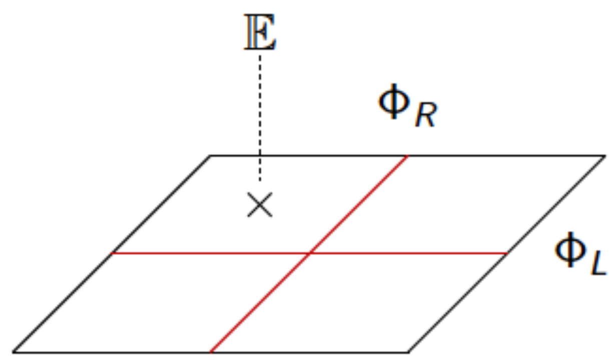

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- Again, exact spectral sequences for spaces fibered over circles $S^1_{L\,R}$ in terms of $M_{L,R}$ w/ results:

For SU(n) x SU(m) group algebra the group: $G_F = \dfrac{SU(n) \times SU(m)}{\mathbb{Z}_{\mathrm{gcd}(n,m)}}$

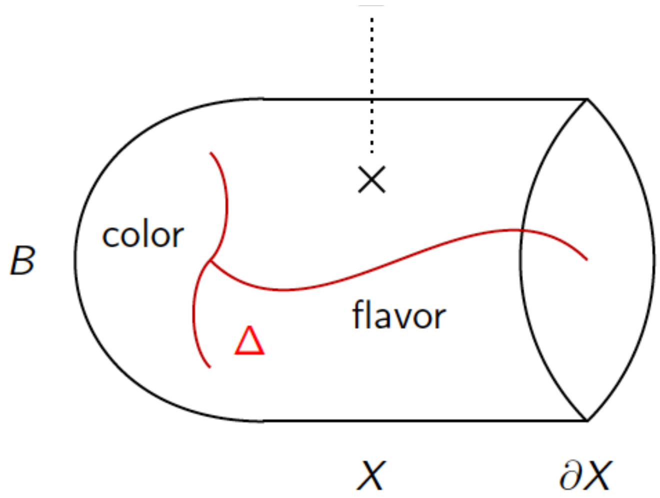
General: $G_F = \dfrac{G_L \times G_R}{Z_{\mathrm{diag}}}$

Further examples

- General elliptically fibered Calabi-Yau's

  Examples with 2-group

$$\mathbb{E} \hookrightarrow X_n \rightarrow B_{n-1}$$



- Systematic study of orbifolds, G_2,…

No time

IV. Compact Models

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- What is M-theory gauge group?
- For elliptically fibered geometries via M-/F-theory duality!

  - Non-Abelian group algebras – ADE Kodaira classification
  group topology → Mordell-Weil torsion

[Aspinwall, Morrison, 1998],
[Mayrhofer, Morrison,Till, Weigand, 2014], [M.C., Lin, 2017]

IV. Compact Models

- Compact singular space X → theory that includes quantum gravity & global symmetries gauged or broken
- What is M-theory gauge group?
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IV. Compact Models

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- Total group topology → Shoida map of Mordell-Weil

[M.C., Lin, 2017]

$$\frac{U(1)^r \times G_{\text{non-ab}}}{\prod_{i=1}^{r} \mathbb{Z}_{m_i} \times \prod_{j=1}^{t} \mathbb{Z}_{k_j}}$$

Digression: F-theory on elliptically fibered Calabi-Yau fourfolds
w/specific elliptic fibration (F_{11} polytope)
led to D=4 N=1effective theory

[M.C., Klevers, Peña, Oehlmann, Reuter, '15]

Standard Model gauge group $\dfrac{SU(3) \times SU(2) \times U(1)}{\mathbb{Z}_6}$

w/ gauge group topology
(geometric - encoded in Shioda Map of MW)

[M.C., Lin, '17]

w/ toric bases B_3 (3D polytopes)

[M.C., Halverson, Lin, Liu, Tian, '19, PRL]

Quadrillion Standard Models (QSMs)
with 3-chiral families & gauge coupling unification

[gauge divisors – in class of *anti-canonical divisor* $\overline{K}_C$]

Current efforts: determination the exact matter spectra
(including vector pair & # of Higgs pairs)
→ Studies of (limit) root bundles on matter curves

[Bies, M.C., Donagi,(Liu), Ong, '21,'22,'23]

No time, but the latest status in [Bies, M.C., Donagi, Ong, 2307.02535]

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  quantified conditions under which no anomalies due to gauged 1-form symmetry
  [magnetic version – preferred polarization!]

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- True for all 8D N=1 string compactifications (beyond F-theory) via (refined) string junction construction

[M.C., Diriegl, Ling, Zhang, '22]

[M. C., Heckman, Hübner, Torres,
"Generalized Symmetries, Gravity, and the Swampland," 2307...]

Fate of higher-form structures in Compact Geometries

- Decompose $X \rightarrow \cup_i X_i^{loc}$ into local models X_i^{loc}

  Converse: Glue $\{X_i^{loc}\}$ to X $\Leftrightarrow$ Couple $\{SQFT_i\}$ to a resulting QFT & include gravity

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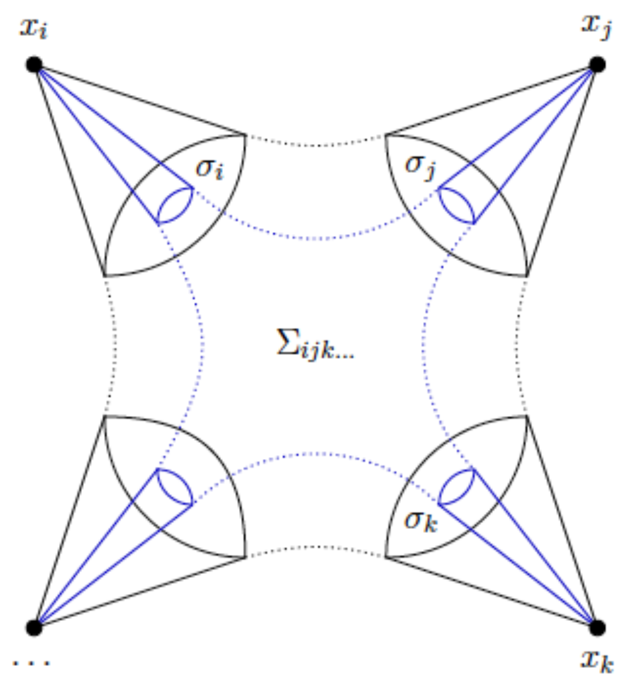

Defect Operators:
M2-/M5-brane wrapped on cones

Σ_{ijk} – two-chain glues defects

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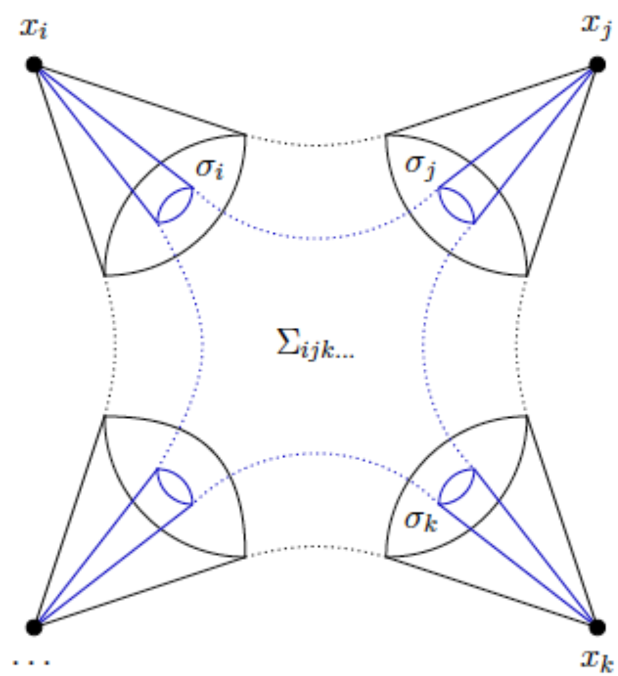

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- Some relative Cycles in X_i^{loc} survive & compactify → (Some) defects in $SQFT_i$ become dynamical - ``gauged'' → determine the global gauge group in compact models

Fate of higher-form structures in Compact Geometries (continued):

- Quantify: Mayer-Vietoris long exact sequence for covering $\{X_i^{\text{loc}}\}$ relates homologies of X_i^{loc} to X

$$\dots \xrightarrow{\jmath_n} H_n(X) \xrightarrow{\partial_n} H_{n-1}(\partial X^{\text{loc}}) \xrightarrow{\imath_{n-1}} H_{n-1}(X^\circ) \oplus H_{n-1}(X^{\text{loc}}) \to \dots$$

w/ respect to the covering: $X = X^{\text{loc}} \cup X^\circ\,, \qquad X^\circ = X \setminus X^{\text{loc}}$

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- Example of K3 Surfaces → Mayer-Vietoris short exact sequence:

$$0 \to H_2(X^\circ) \to H_2(X) \to H_1(\partial X^{\mathrm{loc}}) \to H_1(X^\circ) \to 0$$

$$\cong \oplus_i H_1(\partial X_i^{\mathrm{loc}})$$

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Fate of higher-form structures in Compact Geometries (continued):

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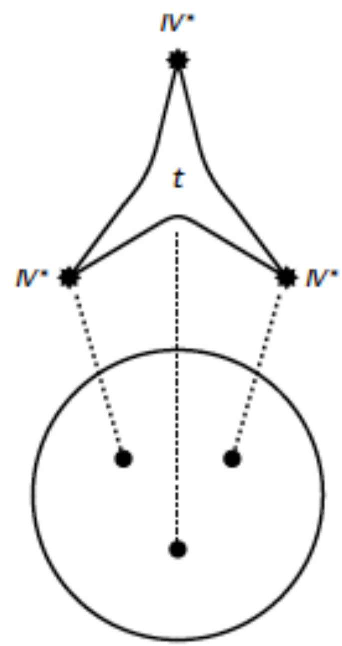
Decomposition of compact two-cycles into a sum of relative cycles associated with each local model ∂_2: $H_2(X) \to \bigoplus_i H_1(\partial X_i^{loc})$

Fate of higher-form structures in Compact Geometries (continued):

- Elliptic K3 example: $\pi : X \rightarrow B$ w/ three IV^* Kodaira singularities



Three Kodaira thimbles

$$t = \mathfrak{T}_{E_{6,1}} + \mathfrak{T}_{E_{6,2}} + \mathfrak{T}_{E_{6,3}}$$

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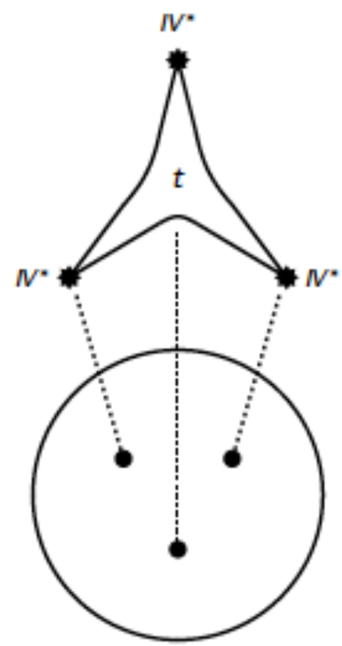

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Torsional cycles associated w/ Mordell-Weil torsion decompose into relative cycles of $\{X_i^{loc}\}$ w/ group $G_{ADE} = (E_6 \times E_6 \times E_6)/\mathbb{Z}_3$

→ complementary geometric results!

V. Concluding Remarks

- Systematic determination of gauge group, including Abelian factors via cutting and gluing techniques

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& higher dimensional CY's: some T^6/Γ_i orbifolds (subtleties)…

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  Fate of 2-group → ``dissolved'' in global models
  Non-invertible symmetries→ fusion algebra of TFT,
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Work in progress w/ J. Heckman, M. Hübner, E. Torres, H. Zhang

Thank you!