Beyond LO for Hybrid k_T-factorization

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in collaboration with
Leszek Motyka and Grzegorz Ziarko, JHEP 11 (2022) 103, arxiv:2205.09585

and with
Etienne Blanco, Alessandro Giachino, Piotr Kotko, arxiv:2212.03572, https://doi.org/10.1016/j.nuclphysb.2023.116322

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Collinear factorization in QCD at NLO

$$d\sigma^{\mathrm{LO}} = \int \frac{dx_{\mathrm{in}}}{x_{\mathrm{in}}} \frac{d\bar{x}_{\overline{\mathrm{in}}}}{\bar{x}_{\overline{\mathrm{in}}}} \, f_{\mathrm{in}}(x_{\mathrm{in}}) \, f_{\overline{\mathrm{in}}}(\bar{x}_{\overline{\mathrm{in}}}) \, dB(x_{\mathrm{in}}, \bar{x}_{\overline{\mathrm{in}}})$$

general: $K^\mu = x_K P^\mu + \bar{x}_K \bar{P}^\mu + K_\perp^\mu$

one in-state: $k_{\mathrm{in}}^\mu = x_{\mathrm{in}} P^\mu$

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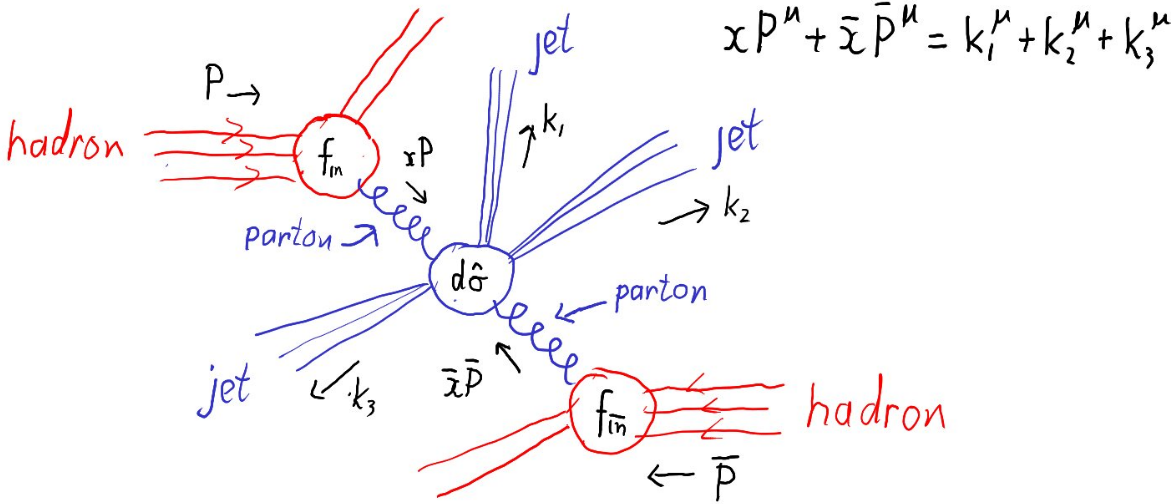

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$$+ \left[f_{in}(x_{in}) \frac{-\alpha_s}{2\pi\epsilon} \int_{\bar{x}_{\overline{in}}}^1 d\bar{z}\, \mathcal{P}_{\overline{in}}(\bar{z}) f_{\overline{in}}\left(\frac{\bar{x}_{\overline{in}}}{\bar{z}}\right) \right.$$
$$\left. + f_{\overline{in}}(\bar{x}_{\overline{in}}) \frac{-\alpha_s}{2\pi\epsilon} \int_{x_{in}}^1 dz\, \mathcal{P}_{in}(z) f_{in}\left(\frac{x_{in}}{z}\right) \right] dB(x_{in}, \bar{x}_{\overline{in}}) \Bigg\}$$

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$$f_{in}^{NLO}(x_{in}) - \frac{1}{\epsilon} \int_{x_{in}}^1 dz\, \mathcal{P}_{in}(z) f_{in}\left(\frac{x_{in}}{z}\right) = \text{finite}$$

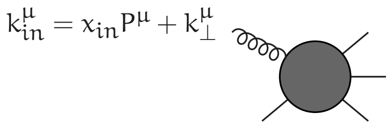
$$f_{\overline{in}}^{NLO}(\bar{x}_{\overline{in}}) - \frac{1}{\epsilon} \int_{\bar{x}_{\overline{in}}}^1 d\bar{z}\, \mathcal{P}_{in}(\bar{z}) f_{\overline{in}}\left(\frac{\bar{x}_{\overline{in}}}{\bar{z}}\right) = \text{finite}$$

Objective

Establish the same within hybrid k_T-factorization,
for which the LO cross section formula is

$$d\sigma^{LO} = \int \frac{dx_{in}}{x_{in}} \frac{d^2k_\perp}{\pi} \frac{d\bar{x}_{\overline{in}}}{\bar{x}_{\overline{in}}} F_{in}(x_{in}, k_\perp)\, f_{\overline{in}}(\bar{x}_{\overline{in}})\, dB^\star(x_{in}, k_\perp, \bar{x}_{\overline{in}})$$

- The amplitudes inside $B^\star(x_{in}, k_\perp, \bar{x}_{\overline{in}})$ depend explicitly on $k_\perp$
- They involve a space-like initial-state gluon with momentum $k_{in}^\mu = x_{in}P^\mu + k_\perp^\mu$



- Such amplitudes need care to be well-defined, to be gauge-invariant.
- We apply the auxiliary-parton method, and our objective is within this constraint.

Auxiliary parton method (tree-level)

$k_{\text{in}} = x_{\text{in}} P + k_\perp$

We desire to obtain the matrix element with one space-like gluon for the process

$g^\star(k_{\text{in}})\ \omega_{\overline{\text{in}}}(k_{\overline{\text{in}}}) \ \rightarrow\ \omega_1(p_1)\ \omega_2(p_2)\ \cdots\ \omega_n(p_n)$ e.g. $g^\star(k_{\text{in}})\ g(k_{\overline{\text{in}}})\ \rightarrow\ g(p_1)\ g(p_2)\ g(p_3)$

and do so by replacing the space-like gluon with an *on-shell auxiliary* quark pair

$q\big(k_1(\Lambda)\big)\ \omega_{\overline{\text{in}}}(k_{\overline{\text{in}}})\ \rightarrow\ q\big(k_2(\Lambda)\big)\ \omega_1(p_1)\ \omega_2(p_2)\ \cdots\ \omega_n(p_n)$

with special momenta

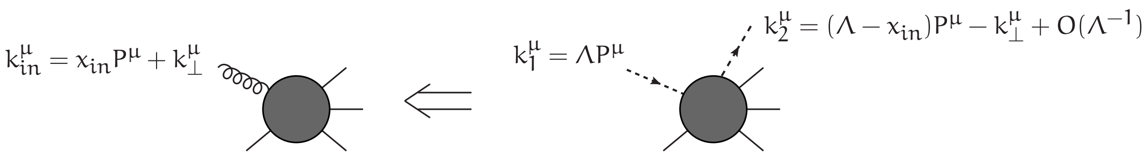
$$k_1^\mu = \Lambda P^\mu \quad , \quad k_2^\mu = p_\Lambda{}^\mu = (\Lambda - x_{\text{in}})P^\mu - k_\perp^\mu + \frac{|k_\perp|^2}{(\Lambda - x_{\text{in}})\nu^2}\,\bar{P}^\mu$$

such that, while individually on-shell, their difference is

$$k_1^\mu - k_2^\mu = x_{\text{in}} P^\mu + k_\perp^\mu + \mathcal{O}\big(\Lambda^{-1}\big) = k_{\text{in}}^\mu + \mathcal{O}\big(\Lambda^{-1}\big)$$

The matrix element with the space-like gluon is obtained by taking $\Lambda \rightarrow \infty$

$$\frac{1}{g_s^2 C_{\text{aux}}}\,\frac{x_{\text{in}}^2 |k_\perp|^2}{\Lambda^2}\,\big|\overline{\mathcal{M}}^{\text{aux}}\big|^2\big(\Lambda P, k_{\overline{\text{in}}}; p_\Lambda, \{p_i\}_{i=1}^n\big) \ \overset{\Lambda\rightarrow\infty}{\longrightarrow}\ \big|\overline{\mathcal{M}}^\star\big|^2\big(k_{\text{in}}, k_{\overline{\text{in}}}; \{p_i\}_{i=1}^n\big)$$



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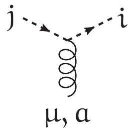
The factor $x_{\text{in}}^2|k_{\perp}|^2$ ensures the correct on-shell limit, $1/\Lambda^2$ selects the leading power, $1/g_s^2$ corrects the power of the coupling.

One can use auxiliary quarks, as well as gluons, by including the color-correction factor

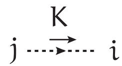
$$C_{\text{aux-q}} = \frac{N_c^2 - 1}{N_c} \quad , \quad C_{\text{aux-g}} = 2N_c$$

Auxiliary parton method

- the auxiliary parton method can be applied to Feynman graphs, from which one can derive eikonal Feynman rules for the auxiliary partons



$$= -\mathrm{i}\, T^{a}_{i,j}\, P^{\mu}$$



$$= \delta_{i,j}\, \frac{\mathrm{i}}{P\cdot K}$$

- this works unambiguously at tree-level (arbitrary number of jets etc.), but needs a treatment of the linear denominators for loop graphs
- the auxiliary parton method can also be applied on closed expressions for complete on-shell amplitudes
- then Λ effectively works as a regulator for linear denominators

$$\frac{1}{P\cdot K} \xleftarrow{\Lambda\to\infty} \frac{2\Lambda}{(\Lambda P + K)^2} \quad \Longrightarrow \quad \ln\Lambda \text{ in loop integrals}$$

- we performed this exercise in (Blanco, Giachino, AvH, Kotko 2023) using on-shell expressions by (Bern, Dixon, Kosower 1994, 1998, and Schmidt 1997) to obtain one-loop amplitudes for $\emptyset \to g^{\star}gg$, $\emptyset \to g^{\star}q\bar{q}$, $\emptyset \to g^{\star}gH$, $\emptyset \to g^{\star}q\bar{q}e^{+}e^{-}$

Auxiliary partons at one loop

We recognize the following pattern:

$$dV^{\star} = dV^{\star\text{fam}} + dV^{\star\text{unf}}$$

$dV^{\star\text{fam}}$ is independent of the type of auxiliary partons
has the correct regular on-shell limit
all $1/\epsilon^2, 1/\epsilon$ poles look as if the space-like gluon were on-shell

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For example, apply Λ limit on $A^{\mathrm{loop}}(1_{\bar{Q}}, 6_Q, 2_{\bar{q}}, 3_q, 4_{e^+}, 5_{e^-})$ (Bern, Dixon, Kosower 1998) to get $A^{\mathrm{loop}}(1^{\star}, 2_{\bar{q}}, 3_q, 4_{e^+}, 5_{e^-})$. The pole-part is proportional to the tree-level amplitude with factor

$$\left\{ -\frac{1}{\epsilon^2}\left[\left(\frac{\mu^2}{-s_{p3}}\right)^{\epsilon} + \left(\frac{\mu^2}{-s_{p2}}\right)^{\epsilon}\right] - \frac{3}{2\epsilon}\right\} A^{\mathrm{tree}}(1^{\star}, 2_{\bar{q}}, 3_q, 4_{e^+}, 5_{e^-}) \ ,$$

with s_{p2} and s_{p3} involving only the longitudinal part of $k_1 = p + k_{\perp}$.

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$dV^{\star\mathrm{unf}} = a_\epsilon N_c \,\mathrm{Re}\big(\mathcal{V}_{\mathrm{aux}}\big)\, dB^{\star}$ is proportional to Born result $\quad a_\epsilon = \frac{\alpha_s}{2\pi}\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}$

$$\mathcal{V}_{\mathrm{aux}} = \left(\frac{\mu^2}{|k_\perp|^2}\right)^{\epsilon}\left[\frac{2}{\epsilon}\ln\frac{\Lambda}{x_{\mathrm{in}}} - i\pi + \bar{\mathcal{V}}_{\mathrm{aux}}\right] + \mathcal{O}(\epsilon) + \mathcal{O}\big(\Lambda^{-1}\big)$$

$$\bar{\mathcal{V}}_{\text{aux-q}} = \frac{1}{\epsilon}\frac{13}{6} + \frac{\pi^2}{3} + \frac{80}{18} + \frac{1}{N_c^2}\left[\frac{1}{\epsilon^2} + \frac{3}{2}\frac{1}{\epsilon} + 4\right] - \frac{n_f}{N_c}\left[\frac{2}{3}\frac{1}{\epsilon} + \frac{10}{9}\right]$$

$$\bar{\mathcal{V}}_{\text{aux-g}} = -\frac{1}{\epsilon^2} + \frac{\pi^2}{3}$$

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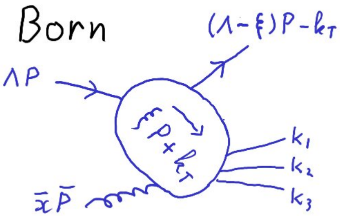
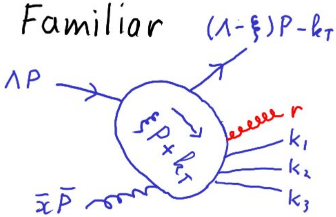
More-or-less proven using known universal collinear limits of one-loop amplitudes (Bern, Chalmers 1995, Bern, Del Duca, Kilgore, Schmidt 1999).

Before the large-Λ, the small-$|k_\perp|$ corresponds to a collinear limit of auxiliary partons.
While the large-Λ and small-$|k_\perp|$ limit commute at tree-level, they do not at one loop.

Real radiation with auxiliary partons





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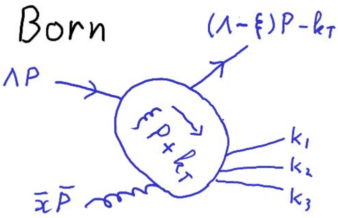

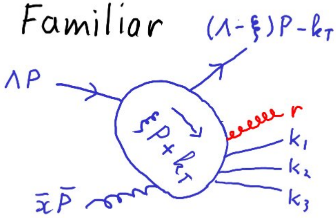

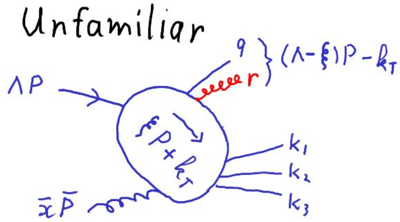

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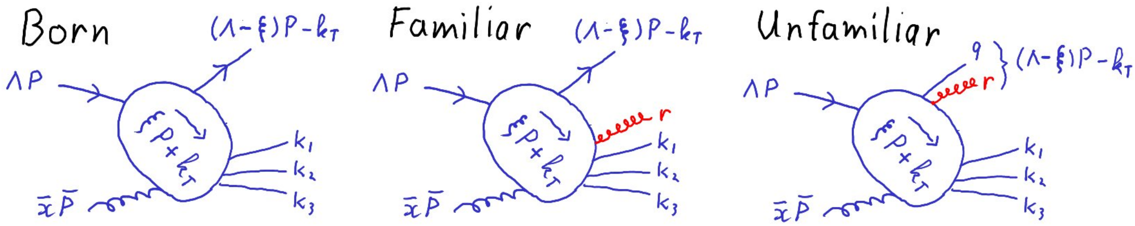

The differential phase space and the matrix element factorize for the *unfamiliar* case, where the radiative gluon participates in the consumption of Λ.

$$\frac{1}{\mathcal{C}_{\mathrm{aux}}}\left|\overline{\mathcal{M}}^{\mathrm{aux}}\right|^2\big((\Lambda+x_{\mathrm{in}})P,k_{\overline{\mathrm{in}}}\,;x_r\Lambda P+r_\perp+\bar{x}_r\bar{P}\,,\,x_q\Lambda P+q_\perp+\bar{x}_q\bar{P}\,,\,\{p_i\}_{i=1}^n\big)$$
$$\overset{\Lambda\to\infty}{\longrightarrow}\;\mathcal{Q}_{\mathrm{aux}}(x_q,q_\perp,x_r,r_\perp)\,\frac{\Lambda^2\left|\overline{\mathcal{M}}^\star\right|^2(x_{\mathrm{in}}P-q_\perp-r_\perp,k_{\overline{\mathrm{in}}}\,;\{p_i\}_{i=1}^n)}{x_{\mathrm{in}}^2|q_\perp+r_\perp|^2}$$

$$\mathcal{Q}_{\mathrm{aux}}(x_q,q_\perp,x_r,r_\perp)=x_qx_r\,\mathcal{P}_{\mathrm{aux}}(x_q,x_r)\,|q_\perp+r_\perp|^2$$
$$\times\left[\frac{c_{\bar{q}}}{|q_\perp|^2|r_\perp|^2}+\frac{1}{x_r|q_\perp|^2+x_q|r_\perp|^2-x_qx_r|q_\perp+r_\perp|^2}\left(\frac{c_r\,x_r^2}{|r_\perp|^2}+\frac{c_q\,x_q^2}{|q_\perp|^2}\right)\right]$$

- Phase space also factorizes and the contribution can be calculated analytically.
- The result contains $\ln\Lambda$ and depends on the type of auxiliary partons.

Complete unfamiliar contribution

Combining the unfamiliar contributions and organizing them suggestively, we can write

$$\mathrm{d}R^{\star\,\mathrm{unf}} + \mathrm{d}V^{\star\,\mathrm{unf}} = \Delta_{\mathrm{unf}}\,\mathrm{d}B^{\star}\ ,$$

where

$$\Delta_{\mathrm{unf}} = \frac{a_\epsilon N_c}{\epsilon}\left(\frac{\mu^2}{|k_\perp|^2}\right)^{\epsilon}\left[\mathcal{J}_{\mathrm{aux}} + \mathcal{J}_{\mathrm{univ}} + \mathcal{J}_{\mathrm{univ}} - 2\ln\frac{2P\cdot\bar{P}x_{\mathrm{in}}}{|k_\perp|^2} + \frac{1}{\epsilon}\right]\ ,$$

with

$$\mathcal{J}_{\mathrm{univ}} = \frac{11}{6} - \frac{n_f}{3N_c} - \frac{\mathcal{K}}{N_c}(-\epsilon) \quad \text{writing} \quad \mathcal{K} = N_c\left(\frac{67}{18} - \frac{\pi^2}{6}\right) - \frac{5n_f}{9}\ ,$$

and

$$\mathcal{J}_{\mathrm{aux\text{-}q}} = \frac{3}{2} - \frac{1}{2}(-\epsilon) \quad , \quad \mathcal{J}_{\mathrm{aux\text{-}g}} = \frac{11}{6} + \frac{n_f}{3N_c^3} + \frac{n_f}{6N_c^3}(-\epsilon)\ .$$

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$$dR^{\star\,\mathrm{unf}} + dV^{\star\,\mathrm{unf}} = \Delta_{\mathrm{unf}}\, dB^{\star}\ ,$$

where

$$\Delta_{\mathrm{unf}} = \frac{a_\epsilon N_c}{\epsilon}\left(\frac{\mu^2}{|k_\perp|^2}\right)^{\epsilon}\left[\mathcal{J}_{\mathrm{aux}} + \mathcal{J}_{\mathrm{univ}} + \mathcal{J}_{\mathrm{univ}} - 2\ln\frac{2P\cdot\bar{P}x_{\mathrm{in}}}{|k_\perp|^2} + \frac{1}{\epsilon}\right]\ ,$$

with

$$\mathcal{J}_{\mathrm{univ}} = \frac{11}{6} - \frac{n_f}{3N_c} - \frac{\mathcal{K}}{N_c}(-\epsilon) \quad \text{writing} \quad \mathcal{K} = N_c\left(\frac{67}{18} - \frac{\pi^2}{6}\right) - \frac{5n_f}{9}\ ,$$

and

$$\mathcal{J}_{\mathrm{aux\text{-}q}} = \frac{3}{2} - \frac{1}{2}(-\epsilon) \quad , \quad \mathcal{J}_{\mathrm{aux\text{-}g}} = \frac{11}{6} + \frac{n_f}{3N_c^3} + \frac{n_f}{6N_c^3}(-\epsilon)\ .$$

- No $\ln\Lambda$ present. $\mathcal{O}(\alpha_s)$ contribution to the space-like gluon Regge trajectory.

Complete unfamiliar contribution

Combining the unfamiliar contributions and organizing them suggestively, we can write

$$dR^{\star\,\mathrm{unf}} + dV^{\star\,\mathrm{unf}} = \Delta_{\mathrm{unf}}\, dB^{\star}\ ,$$

where

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- Target impact factor corrections as in Ciafaloni, Colferai 1999.

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- Target impact factor corrections as in Ciafaloni, Colferai 1999.

- No $\ln\Lambda$ present. $\mathcal{O}(\alpha_s)$ contribution to the space-like gluon Regge trajectory.
- Related to renormalization of the coupling constant (virtual was not UV-subtracted).

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- Target impact factor corrections as in Ciafaloni, Colferai 1999.
- Must be removed because the region where radiation is collinear to P is double-counted.
- No $\ln\Lambda$ present. $\mathcal{O}(\alpha_s)$ contribution to the space-like gluon Regge trajectory.
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Combining the unfamiliar contributions and organizing them suggestively, we can write

$$dR^{\star\,\mathrm{unf}} + dV^{\star\,\mathrm{unf}} = \Delta_{\mathrm{unf}}\, dB^{\star} \ ,$$

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with

$$\mathcal{J}_{\mathrm{univ}} = \frac{11}{6} - \frac{n_f}{3N_c} - \frac{\mathcal{K}}{N_c}(-\epsilon) \quad \text{writing} \quad \mathcal{K} = N_c\left(\frac{67}{18} - \frac{\pi^2}{6}\right) - \frac{5n_f}{9} \ ,$$

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- Target impact factor corrections as in Ciafaloni, Colferai 1999.
- Must be removed because the region where radiation is collinear to P is double-counted.

We follow the prescription of Ciafaloni, Colferai 1999 and remove the collinear region with and angular ordering condition on the relevant $1/x_r$-term

$$\theta\left(\frac{|r_{\perp}|}{\nu\sqrt{\Lambda}} < x_r < \frac{|r_{\perp}|}{|r_{\perp} + k_{\perp}|}\right)$$

Familiar divergencies

Familiar (UV-subtracted) virtual divergencies involving the space-like gluon look as if it were on-shell, with only the longitudinal momentum component $x_{\text{in}}P$ in the soft log:

$$-\frac{C_A}{\epsilon^2}\left|\overline{\mathcal{M}}^\star\right|^2+\frac{2}{\epsilon}\sum_{i\neq\star}\ln\!\left(\frac{\mu^2}{2x_{\text{in}}P\cdot p_i}\right)\left(\overline{\mathcal{M}}^\star\right)^2_{i\star}-\frac{11N_c-2n_f}{6\epsilon}\left|\overline{\mathcal{M}}^\star\right|^2$$

Familiar real soft behavior with the space-like gluon acting as "spectator" looks as if it were on-shell, with only the longitudinal momentum component $x_{\text{in}}P$ in the eikonal terms:

$$\frac{(x_{\text{in}}P\cdot p_i)}{(x_{\text{in}}P\cdot r)(r\cdot p_i)}\left(\overline{\mathcal{M}}^\star\right)^2_{i\star}$$

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Familiar (UV-subtracted) virtual divergencies involving the space-like gluon look as if it were on-shell, with only the longitudinal momentum component $x_{in}P$ in the soft log:

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$$\frac{(x_{in}P\cdot p_i)}{(x_{in}P\cdot r)(r\cdot p_i)}\left(\overline{\mathcal{M}}^\star\right)^2_{i\star}$$

Soft and collinear singularities in the the real radiation integral of the tree-level matrix element are typically dealt with using subtraction (Catani-Seymour, FKS, etc.):

$$\int d\Phi_{n+1}\,|\overline{\mathcal{M}}^\star|^2_{n+1}\,J_R = \int d\Phi_{n+1}\left[|\overline{\mathcal{M}}^\star|^2_{n+1}\,J_R - \sum \mathcal{R}(p_r)\otimes\mathcal{A}\,J_B\right] + \sum\int d\Phi_n\int[dp_r]\mathcal{R}(p_r)\otimes\mathcal{A}\,J_B$$

where the p_r-integrals are supposed to be performable analytically within dimreg.

Familiar real collinear singularity

Tree-level matrix elements with a space-like gluon still have a singularity when a radiative gluon becomes collinear to P.

$$\left|\overline{\mathcal{M}}^{\star}\right|^2\left(x_{\text{in}}P + k_\perp, k_{\overline{\text{in}}}; r, \{p_i\}_{i=1}^n\right)$$
$$\xrightarrow{r\to x_r P} \frac{2N_c}{P\cdot r}\,\frac{x_{\text{in}}^2}{x_r(x_{\text{in}}-x_r)^2}\left|\overline{\mathcal{M}}^{\star}\right|^2\left((x_{\text{in}}-x_r)P + k_\perp, k_{\overline{\text{in}}}; \{p_i\}_{i=1}^n\right)$$

Collinear splitting function with only the $1/z/(1-z)$ part.

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Collinear splitting function with only the $1/z/(1-z)$ part.

Integrate over relevant phase space with restriction

$$\frac{\bar{x}_r}{\bar{x}_{\overline{\mathrm{in}}}}<a\,\frac{x_r}{x_{\mathrm{in}}}\quad\text{with}\quad a=\frac{|k_\perp-r_\perp|^2}{Sx_{\mathrm{in}}\bar{x}_{\overline{\mathrm{in}}}}\quad\text{and}\quad|r_\perp|^2=Sx_r\bar{x}_r\quad\Rightarrow\quad|r_\perp|<|k_\perp-r_\perp|\,\frac{x_r}{x_{\mathrm{in}}}$$

which is the complement of the restriction on the unfamiliar phase space.

$$\int_0^1\frac{dx_{\mathrm{in}}}{x_{\mathrm{in}}}\int d^2k_\perp\,F(x_{\mathrm{in}},k_\perp)\,dR_{\mathrm{coll}}^{\star\,\mathrm{fam}}\left(x_{\mathrm{in}}P_A+k_\perp,k_{\overline{\mathrm{in}}};\{p_i\}_{i=1}^n\right)$$

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Tree-level matrix elements with a space-like gluon still have a singularity when a radiative gluon becomes collinear to P.

$$\left|\overline{\mathcal{M}}^\star\right|^2\left(x_{in}P+k_\perp,k_{\overline{in}}\,;r,\{p_i\}_{i=1}^n\right)$$
$$\xrightarrow{r\to x_rP}\frac{2N_c}{P\cdot r}\,\frac{x_{in}^2}{x_r(x_{in}-x_r)^2}\left|\overline{\mathcal{M}}^\star\right|^2\left((x_{in}-x_r)P+k_\perp,k_{\overline{in}}\,;\{p_i\}_{i=1}^n\right)$$

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which is the complement of the restriction on the unfamiliar phase space.

$$\int_0^1\frac{dx_{in}}{x_{in}}\int d^2k_\perp\,F(x_{in},k_\perp)\,dR_{coll}^{\star\,fam}\left(x_{in}P_A+k_\perp,k_{\overline{in}}\,;\{p_i\}_{i=1}^n\right)$$
$$=\int_0^1\frac{dx_{in}}{x_{in}}\int d^2k_\perp\,\tilde{F}(x_{in},k_\perp)\,dB^\star\left(x_{in},k_\perp,\bar{x}_{\overline{in}}\,;\{p_i\}_{i=1}^n\right)$$

$$\tilde{F}(x_{in},k_\perp)=\frac{2a_\epsilon N_c}{\pi_\epsilon\mu^{-2\epsilon}}\int_{x_{in}}^1\frac{dz}{z(1-z)}\int\frac{d^{2-2\epsilon}r_\perp}{|r_\perp|^2}\,\frac{|k_\perp|^2}{|k_\perp+r_\perp|^2}\,F\left(\frac{x_{in}}{z},k_\perp+r_\perp\right)\theta_{|r_\perp|<|k_\perp|(1-z)}$$

Essentially identical to formula from Nefedov 2020 for multi-Regge evolution.

Familiar real collinear singularity

Tree-level matrix elements with a space-like gluon still have a singularity when a radiative gluon becomes collinear to P.

$$\left|\overline{\mathcal{M}}^\star\right|^2\left(x_{in}P + k_\perp, k_{\overline{in}}; r, \{p_i\}_{i=1}^n\right) \xrightarrow{r\to x_r P} \frac{2N_c}{P\cdot r}\,\frac{x_{in}^2}{x_r(x_{in}-x_r)^2}\left|\overline{\mathcal{M}}^\star\right|^2\left((x_{in}-x_r)P + k_\perp, k_{\overline{in}}; \{p_i\}_{i=1}^n\right)$$

Collinear splitting function with only the $1/z/(1-z)$ part.

Integrate over relevant phase space with restriction

$$\frac{\bar{x}_r}{\bar{x}_{\overline{in}}} < a\,\frac{x_r}{x_{in}} \quad \text{with} \quad a = \frac{|k_\perp - r_\perp|^2}{S x_{in}\bar{x}_{\overline{in}}} \quad \text{and} \quad |r_\perp|^2 = S x_r \bar{x}_r \quad \Rightarrow \quad |r_\perp| < |k_\perp - r_\perp|\,\frac{x_r}{x_{in}}$$

which is the complement of the restriction on the unfamiliar phase space.

$$\int_0^1 \frac{dx_{in}}{x_{in}} \int d^2k_\perp\, F(x_{in}, k_\perp)\, dR_{coll}^{\star\,fam}\left(x_{in}P_A + k_\perp, k_{\overline{in}}; \{p_i\}_{i=1}^n\right) = \int_0^1 \frac{dx_{in}}{x_{in}} \int d^2k_\perp\, \tilde{F}(x_{in}, k_\perp)\, dB^\star\left(x_{in}, k_\perp, \bar{x}_{\overline{in}}; \{p_i\}_{i=1}^n\right)$$

$$\tilde{F}(x_{in}, k_\perp) = a_\epsilon N_c \left(\frac{\mu^2}{|k_\perp|^2}\right)^\epsilon \left\{\frac{F(x_{in}, k_\perp)}{\epsilon^2} - \frac{2}{\epsilon}\int_{x_{in}}^1 dz \left[\frac{1}{[1-z]_+} + \frac{1}{z}\right] F\left(\frac{x_{in}}{z}, k_\perp\right)\right\} + \mathcal{O}(\epsilon^0)$$

Real contribution to the non-cancelling collinear remnant.

Summary

$$d\sigma^{\mathrm{NLO}} = \int \frac{dx_{\mathrm{in}}}{x_{\mathrm{in}}}\, d^2k_\perp \frac{d\bar{x}_{\overline{\mathrm{in}}}}{\bar{x}_{\overline{\mathrm{in}}}} \Big\{ F(x_{\mathrm{in}}, k_\perp)\, f(\bar{x}_{\overline{\mathrm{in}}}) \Big[dV^\star(x_{\mathrm{in}}, k_\perp, \bar{x}_{\overline{\mathrm{in}}}) + dR^\star(x_{\mathrm{in}}, k_\perp, \bar{x}_{\overline{\mathrm{in}}}) \Big]_{\mathrm{cancelling}}$$

$$+ \Big[F^{\mathrm{NLO}}(x_{\mathrm{in}}, k_\perp) + F(x_{\mathrm{in}}, k_\perp)\Delta_{\mathrm{unf}}(x_{\mathrm{in}}, k_\perp) + \Delta^\star_{\mathrm{coll}}(x_{\mathrm{in}}, k_\perp) \Big] f(\bar{x}_{\overline{\mathrm{in}}})\, dB^\star(x_{\mathrm{in}}, k_\perp, \bar{x}_{\overline{\mathrm{in}}})$$

$$+ \Big[f^{\mathrm{NLO}}(\bar{x}_{\overline{\mathrm{in}}}) + \Delta_{\overline{\mathrm{coll}}}(\bar{x}_{\overline{\mathrm{in}}}) \Big] F(x_{\mathrm{in}}, k_\perp) dB^\star(x_{\mathrm{in}}, k_\perp, \bar{x}_{\overline{\mathrm{in}}}) \Big\}$$

$$\Delta_{\overline{\mathrm{coll}}}(\bar{x}_{\overline{\mathrm{in}}}) = -\frac{a_\epsilon}{\epsilon} \int_{\bar{x}_{\overline{\mathrm{in}}}}^{1} dz \left[\mathcal{P}^{\mathrm{reg}}_{\overline{\mathrm{in}}}(z) + \gamma_{\overline{\mathrm{in}}} \delta(1-z) \right] f\left(\frac{\bar{x}_{\overline{\mathrm{in}}}}{z}\right)$$

$$\Delta^\star_{\mathrm{coll}}(x_{\mathrm{in}}, k_\perp) = -\frac{a_\epsilon}{\epsilon} \int_{x_{\mathrm{in}}}^{1} dz \left[\frac{2N_c}{[1-z]_+} + \frac{2N_c}{z} + \gamma_g \delta(1-z) \right] F\left(\frac{x_{\mathrm{in}}}{z}, k_\perp\right)$$

$$\Delta_{\mathrm{unf}}(x_{\mathrm{in}}, k_\perp) = \frac{a_\epsilon N_c}{\epsilon} \left(\frac{\mu^2}{|k_\perp|^2}\right)^\epsilon \left[\mathrm{impactFactCorr} + \mathrm{couplingRenorm} - 2\ln\frac{2P\cdot\bar{P}x_{\mathrm{in}}}{|k_\perp|^2} \right]$$

$$f^{\mathrm{NLO}}(\bar{x}_{\overline{\mathrm{in}}}) + \Delta_{\overline{\mathrm{coll}}}(\bar{x}_{\overline{\mathrm{in}}}) = \mathrm{finite}$$

$$F^{\mathrm{NLO}}(x_{\mathrm{in}}, k_\perp) + F(x_{\mathrm{in}}, k_\perp)\Delta_{\mathrm{unf}}(x_{\mathrm{in}}, k_\perp) + \Delta^\star_{\mathrm{coll}}(x_{\mathrm{in}}, k_\perp) \stackrel{?}{=} \mathrm{finite}$$